Studying light flavour resonances in $\pi^+\pi^-$ photoproduction

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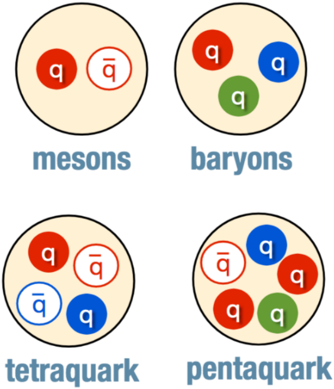
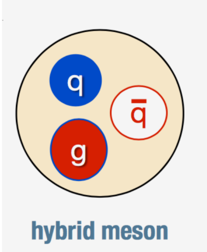
Introduction-Ordinary and Exotic Hadrons

According to quark models, quarks can be organized as triplets to form **Baryons** and $q\bar{q}$ pairs to form **Mesons**.

➜ However the QCD predict that other kind of resonances could be formed i.e. multi-quarks configurations such as tetraquarks and pentaquarks.



Introduction- Hybrid Mesons



The gluonic field excitations

$$\downarrow$$

Certain sets of quantum numbers cannot be formed from a quark and antiquark pair, such as:

$$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-} ...$$

Motivation

Searching for exotic mesons allows us to:

- confirm the theory of quantum chromodynamics,
- gain a better understanding of the fundamental quark-antiquark interactions,
- understand the role of gluons and the origin of confinement,nucleon mass.etc..

Hybrids can be photproduced through $\gamma + p \rightarrow \eta\pi + p$ at JLab.

➡ Our main motivation is to understand the non exotic channel $\pi\pi$ because without this we can not proceed to the exotic channels. So we are focusing on computing distributions and moments of the $\pi\pi$ photoproduction.

Deck Mechanism :

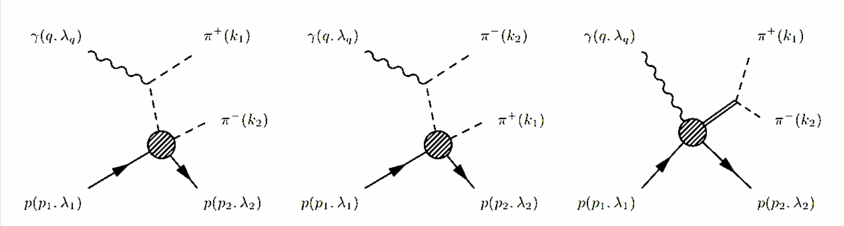


Figure: The Deck Mechanism for two pion photoproduction. Note that either charged pions may couple with the incoming photon.

Model Description

For $\gamma(q, \lambda_\gamma) + p(p_1, \lambda_1) \to \pi^+(k_1) + \pi^-(k_2) + p(p_2, \lambda_2)$.

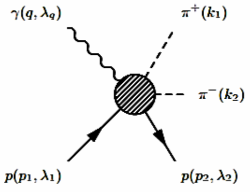


Figure: Kinematics

We have worked with the following kinematic invariants:

$$s = (p_1 + q)^2, \quad (1)$$

$$s_i = (k_i + p_2)^2, \quad (2)$$

$$t = (p_1 - p_2)^2, \quad (3)$$

$$s_{12} = (k_1 + k_2)^2. \quad (4)$$

The Deck Model amplitude can be written as:

$$\mathcal{M}^{\text{Deck}}_{\lambda_1\lambda_2\lambda_q}(s,t,s_{12},\Omega) = e\Big[-\frac{\epsilon(q,\lambda_q)\cdot k_2}{q\cdot k_2}\beta(t_2)\mathcal{T}^{+}_{\lambda_1\lambda_2}(s_1,t;t_2) + \frac{\epsilon(q,\lambda_q)\cdot k_1}{q\cdot k_1}\beta(t_1)\mathcal{T}^{-}_{\lambda_1\lambda_2}(s_2,t;t_1)\Big]. \tag{5}$$

Where $\beta(t_i) = \exp((t_\pi - t_i^{\text{min}})/\Lambda_\pi^2)$, $\Lambda_\pi = 0.9GeV$, $t_\pi = (q-k_i)^2$ and

$$t_1^{\text{min}} = m_\pi^2 - \frac{1}{2s}\Big[(s-m_p^2)(s-s_2+m_\pi^2) - \lambda^{1/2}(s,0,m_p^2))\lambda^{1/2}(s,s_2,m_\pi^2)\Big], \tag{6}$$

$$t_2^{\text{min}} = m_\pi^2 - \frac{1}{2s}\Big[(s-m_p^2)(s-s_1+m_\pi^2) - \lambda^{1/2}(s,0,m_p^2)\lambda^{1/2}(s,s_1,m_\pi^2)\Big] \tag{7}$$

where $\lambda(a,b,c) = a^2+b^2+c^2-2(ab+bc+ca)$.

Momentum-vectors in the Helicity and Gottfried-Jackson (GJ) Frames:

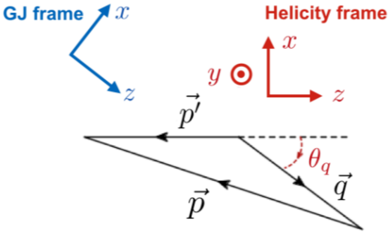
$$\boldsymbol{p_1^H} = |\vec{p_1}|(\sin\theta_1, 0, \cos\theta_1) \quad ; \quad \boldsymbol{p_1^{GJ}} = |\vec{p_1}|(-\sin\theta_1, 0, \cos\theta_1) \tag{8}$$

$$\boldsymbol{p_2^H} = |\vec{p_2}|(0, 0, -1) \quad ; \quad \boldsymbol{p_2^{GJ}} = |\vec{p_2}|(-\sin\theta_2, 0, \cos\theta_2) \tag{9}$$

$$\boldsymbol{q^H} = |\vec{q}|(-\sin\theta_q, 0, \cos\theta_q) \quad ; \quad \boldsymbol{q^{GJ}} = |\vec{q}|(0, 0, 1) \tag{10}$$

$$\boldsymbol{k_1^H} = |\vec{k_1}|(\sin\theta^H\cos\phi^H, \sin\theta^H\sin\phi^H, \cos\theta^H) = -\boldsymbol{k_2^H} \tag{11}$$

$$\boldsymbol{k_1^{GJ}} = |\vec{k_1}|(\sin\theta^{GJ}\cos\phi^{GJ}, \sin\theta^{GJ}\sin\phi^{GJ}, \cos\theta^{GJ}) = -\boldsymbol{k_2^{GJ}} \tag{12}$$



Pion-proton Scattering:

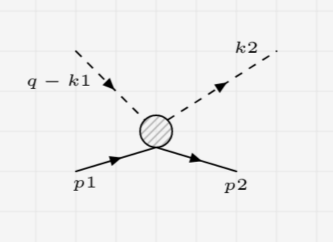


Figure: Feynman diagram for $\pi^- p \to \pi^- p$

Assuming that the intermediate pion is offshell, then the pion-proton scattering amplitude will read:

$$T^-_\lambda = \bar{u}_\lambda(p_2)\left[A^-(s,t,t_\pi) + \frac{1}{2}\gamma_\mu(q - k_1 + k_2)^\mu B^-(s,t,t_\pi)\right]u_\lambda(p_1), \quad (13)$$

where $t_\pi = (q - k_1)$

Pion-proton Scattering:

Similarly for the positive exchanged pion:

$$T^+_\lambda = \bar{u}_\lambda(p_2)\left[A^+(s,t,t_\pi) + \frac{1}{2}\gamma_\mu(q-k_2+k_1)^\mu B^+(s,t,t_\pi)\right]u_\lambda(p_1), \quad (14)$$

where $t_\pi = (q-k_2)$.
In the πN center of mass frame the t-channel A and B defined as follows:

$$\frac{1}{4\pi}A^\pm = \frac{\sqrt{s}+m_p}{Z_1^+Z_2^+}f_1^\pm - \frac{\sqrt{s}-m_p}{Z_1^-Z_2^-}f_2^\pm, \quad (15)$$

$$\frac{1}{4\pi}B^\pm = \frac{1}{Z_1^+Z_2^+}f_1^\pm - \frac{1}{Z_1^-Z_2^-}f_2^\pm. \quad (16)$$

Where f_1 and f_2 are called the reduced helicity amplitudes, $Z_i^\pm = \sqrt{E_i \pm m_p}$.

Pion-proton scattering

The partial wave decomposition:

$$f_1 = \frac{1}{\sqrt{|\mathbf{p_1}||\mathbf{p_2}|}} \sum_{l=0}^{\infty} f_{l+}(s) P'_{l+1}(\cos\theta) - \frac{1}{\sqrt{|\mathbf{p_1}||\mathbf{p_2}|}} \sum_{l=2}^{\infty} f_{l-}(s) P'_{l-1}(\cos\theta), \tag{17}$$

$$f_2 = \frac{1}{\sqrt{|\mathbf{p_1}||\mathbf{p_2}|}} \sum_{l=1}^{\infty} [f_{l-}(s) - f_{l+}(s)] P'_l(\cos\theta). \tag{18}$$

The scalar functions A and B in our case depends on the pion virtuality, in which it enters in Z_1 and hence P_1 since

$$E_1 = \frac{s_i - t_\pi + m_p^2}{2\sqrt{s_i}} \tag{19}$$

Also our scattering angle is proportional to the virtuality as follows:

$$\cos\theta = \frac{2s_i(t - 2m_p^2) + (s_i - t_\pi + m_p^2)(s_i - m_\pi^2 + m_p^2)}{\sqrt{\lambda(s_i, t_\pi, m_p^2)}\,\sqrt{\lambda(s_i, m_\pi^2, m_p^2)}}. \tag{20}$$

Resonance Production:

Adding the P wave to our amplitude i.e. the ρ resonance which comes as a result of the exchange of both pomeron and f_2. *
The amplitude can be defined as:

$$\mathcal{M}^{\mathsf{P}}_{\lambda,\lambda_1,\lambda_2} = \frac{-1}{s} g_{\rho\pi} \beta_N^{\gamma\rho} e^{\beta_N t} BW(m_{\pi\pi}) R(s,t) \bar{u}(p_2,\lambda_2)\gamma^{\alpha} u(p_1,\lambda_1) v_{\alpha}^{\lambda} \quad (21)$$

Where $v_{\alpha}^{\lambda} = k_{\alpha}\epsilon^{\lambda} \cdot (k_1 - k_2) - q \cdot (k_1 - k_2)\epsilon_{\alpha}^{\lambda}$, and $k = k_1 + k_2$.
The Regge propagator is given by:

$$R(s,t) = \frac{\alpha_N(t)}{\alpha_N(0)} \frac{1 + e^{i\pi\alpha_N(t)}}{\sin(\pi\alpha(t))} \left(\frac{s}{s_0}\right)^{\alpha_N(t)} \quad (22)$$

with the following linear trajectories for both pomeron and f_2 exhanges:

$$\alpha_P(t) = 1.08 + 0.2t \quad (23)$$

$$\alpha_{f_2}(t) = 0.5 + 0.9t \quad (24)$$

*https://doi.org/10.1103/PhysRevD.97.094003

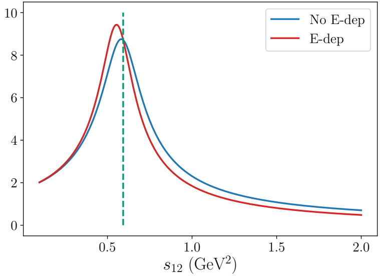
Resonance Production:

We used the energy dependent width BW distribution:

$$BW(s) = \frac{1}{m_\rho^2 - m_{\pi\pi}^2 - i m_\rho \Gamma(s)} \quad \text{with} \quad \Gamma(s) = \left(\frac{s - 4m_\pi^2}{m^2 - 4m_\pi^2} \right)^{3/2} \tag{25}$$

We also used the following constants:†

$$g_{\rho\pi} = 5.96 \quad \beta_P^{\gamma\rho} = 2.506 \quad \beta_P = 3.6 \quad \beta_{f_2}^{\gamma\rho} = 2.47 \quad \beta_{f_2} = 0.55 \quad s_0 = 1 \;\; GeV^2 \tag{26}$$



† http://cgl.soic.indiana.edu/jpac/sdme.php

Differential Cross Section

The P-wave differential cross section can be written as:

$$\frac{d\sigma}{dtdm_{\pi\pi}d\Omega} = \frac{1}{2}(2\pi)\kappa \sum_{\lambda_q\lambda_1\lambda_2} \left| \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \mathcal{M}^{lm}_{\lambda_1\lambda_2\lambda_q}(s,t,s_{12}) Y_{lm}(\Omega) \right|^2, \quad (27)$$

where

$$\kappa = \frac{1}{(2\pi)^3}\frac{1}{4\pi}\frac{1}{2\pi}\frac{\lambda^{1/2}(s_{12},m_\pi^2,m_\pi^2)}{16\sqrt{s_{12}}(s-m^2)^2}\frac{1}{2}. \quad (28)$$

Thus we obtain:

$$\frac{d\sigma_l}{dt} = \sum_{m=-l}^{l} \int_{m_{\pi\pi}^{\text{min}}}^{m_{12}^{\text{max}}} dm_{\pi\pi}\, \frac{1}{2}(2\pi)\kappa \sum_{\lambda_q\lambda_1\lambda_2} |\mathcal{M}^{lm}_{\lambda_1\lambda_2\lambda_q}(s,t,s_{12})|^2 \quad (29)$$

Differential Cross Section

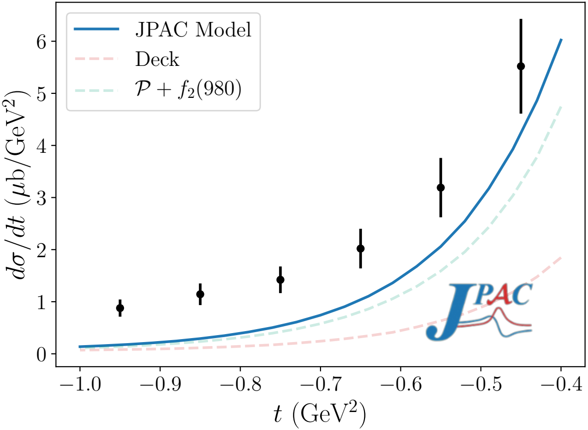


Figure: Differential cross section for P-wave ($l = 1$), Deck and the full Model in the $M_{\pi\pi}$ range $0.4 - 1.2$ GeV and beam energy $E_\gamma = 3.4$ GeV

Modifying t-dependence of the Model:

$$\mathcal{M}^{\mathsf{P}}_{\lambda,\lambda_1,\lambda_2} = \frac{-1}{s} g_{\rho\pi}\beta_N^{\gamma\rho} BW(m_{\pi\pi}) R(s,t)\bar{u}(p_2,\lambda_2)\gamma^{\alpha}u(p_1,\lambda_1)\nu_{\alpha}^{\lambda} * e^{\beta_N t} \quad (30)$$

Hence total P-wave amplitude can be written as:

$$M = M_P e^{\beta_p t} + M_{f_2}((1-\epsilon)e^{\beta^1_{f_2} t} + \epsilon e^{\beta^2_{f_2} t}) \quad (31)$$

Where

$$\beta_p = 3.6, \qquad \beta^1_{f_2} = 0.55, \qquad \beta^2_{f_2} = -0.20923732, \qquad \epsilon = 1.3710881 \quad (32)$$

UPDATED Differential Cross Section

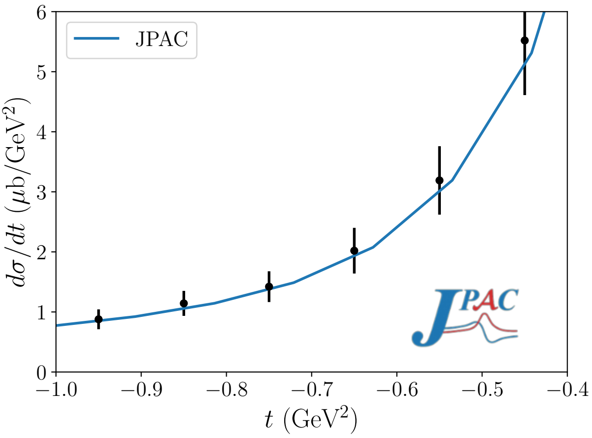


Figure: Differential cross section for our improved model in the $M_{\pi\pi}$ range $0.4 - 1.2$ GeV and beam energy $E_\gamma = 3.4$ GeV

Moments of Angular Distribution

We used the following moments convention[‡]

$$Y_{LM}(s,t,m_{\pi\pi}) = \sqrt{4\pi}\int d\Omega \frac{d\sigma}{dtdm_{\pi\pi}d\Omega}\text{Re}\,Y_{LM}(\Omega), \qquad (33)$$

this normalization ensures that

$$Y_{00}(s,t,m_{\pi\pi}) = \frac{d\sigma}{dtdm_{\pi\pi}}. \qquad (34)$$

One can use the other definition of the moments given in [Phys.Rev.D 100, 054017], they are related to the above definition via

$$Y_{LM} = 2\pi\sqrt{2L+1}H^{0}_{LM}$$

.

‡arXiv:0907.1021 [hep-ex]

Moments of Angular Distribution

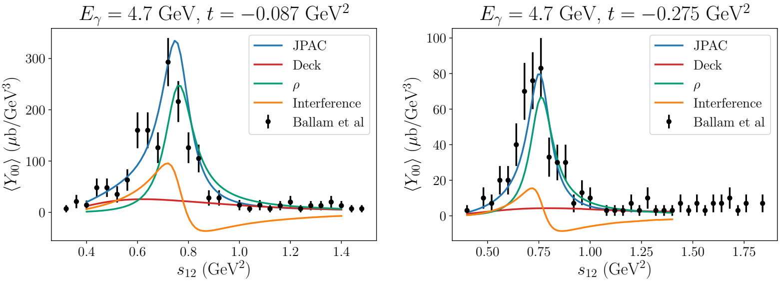


Figure: Comparing the prediction of JPAC model with experimental data from Ballam and CLAS.

Summary

We have presented:

- the definition of our model which composed of two mechanisms i.e Deck model arises from diffractive scattering of photon on the target proton via one pion exchange and the ρ resonance production piece.
- the computation of both Deck and the P wave contribution amplitudes.
- the calculations of P- wave differential cross section of Deck model, ρ resonance (pomeron $+f_2$) as well as the full model.
- We also showed some results of the moments of angular distribution at different momentum transfer t.

  Currently we are calculating all the remaining moments and comparing it with the data. We also are in the last steps of finalizing our paper to be published soon. So stay tuned!


Thank you