Lifetime and confinement of a quasi-gluon

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- Found by many different approaches (e.g. the screened loop expansion, RGZ, numerical reconstruction, SDE)
- Poles (and Residues) seem to be gauge invariant
- Related to String Tension
- Related to Condensates
- Define an (observable?) physical mass

- Complex Poles and Spectrum (Minkowski)

- Generalized Källen-Lehmann
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Confinement and Complex Poles

QCD is a confining theory (phenomenological evidence)
but no formal proof yet!

Yang-Mills (no quark):
Center Symmetry $\longrightarrow$ Order Parameter (Polyakov Loop)

Gluons are confining for heavy quarks in a theory without quarks!

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But, what about the confining mechanism of gluons?

A dynamical mechanism would arise from their complex mass:
finite damping rate $\longrightarrow$ $c\,\tau \approx 10^{-15}$ m

Are there complex poles in the gluon propagator?
Are the poles “phsyical” or artefacts?

Screened Expansion in a generic covariant gauge

Same standard, BRST invariant, SU(N) YM Lagrangian:

$$S = S_0 + S_I = \left[S_0 + \frac{1}{2} \int A_\mu \, \delta\Gamma^{\mu\nu} \, A_\nu \right] + \left[S_I - \frac{1}{2} \int A_\mu \, \delta\Gamma^{\mu\nu} \, A_\nu \right]$$

$\nwarrow$ not BRST inv. $\nearrow$

P.T. does not satisfy exact relations imposed by BRST at any finite order

$$\begin{cases} \Delta_m^{\mu\nu}(p) = \dfrac{1}{p^2 + m^2} \, t^{\mu\nu}(p) + \dfrac{\xi}{p^2} \, \ell^{\mu\nu}(p) & \text{(free propagator)} \\ \delta\Gamma^{\mu\nu} = \left[\Delta_m^{-1\,\mu\nu} - \Delta_0^{-1\,\mu\nu} \right] = m^2 \, t^{\mu\nu}(p) & \text{(2-point vertex)} \end{cases}$$

$\nwarrow$ Exact since $\Pi^L = 0$

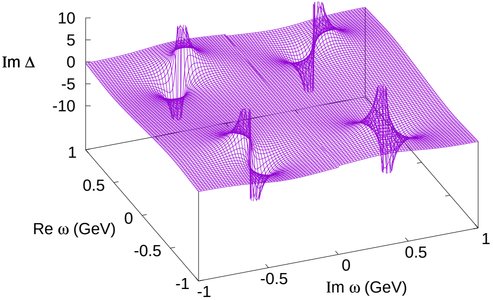
Exact identities (BRST) measure the accuracy: variational method!

1 free parameter $m/\mu \rightarrow$ { gauge inv. of Poles; gauge inv. of Residues $\leftarrow$ NEW }

Gauge invariance of Poles and Residues

In the long wave-length limit $p^2 = \omega^2 - \mathbf{k}^2 \to \omega^2$ the poles are at

$\omega = \pm(M \pm i\gamma)$ where $\mathbf{M} = \mathbf{0.581}$ **GeV** and $\gamma = \mathbf{0.375}$ **GeV.**

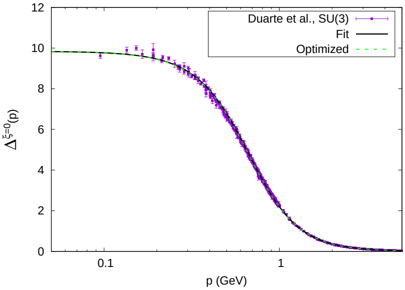


$$\Delta_E(p^2) \approx \frac{R}{p^2 + M^2} + \frac{R^\star}{p^2 + (M^2)^\star}$$

$$\frac{\mathrm{Im}\, R}{\mathrm{Re}\, R} = \mathbf{3.132}$$ [F.S+G.C, PRD 98 (2018)]

Stingl (1986); Dudal et al. (2008); Dudal et al.(2020);

Hayashi+Kondo (2019); Binosi+Tripolt (2020);



Gauge invariance $\Longrightarrow$
Predictions from first principles
(no free parameters)

String tension (short distance limit)

Static quark potential at tree-level ($r \to 0$)

$$V(r) \approx -C_F(4\pi\alpha_s)\int \frac{d^3\mathbf{k}}{(2\pi)^3}\Delta(\mathbf{k}^2)e^{i\mathbf{k}\cdot\mathbf{r}} = -C_F\frac{1}{\pi\, i\, r}\,\alpha_s \int_{-\infty}^{+\infty} k\mathrm{d}k\Delta_E(k^2)e^{ikr}$$

$$V(r) = -C_F\frac{\alpha_s}{r}\left[R\exp(-Mr) + R^\star\exp(-M^\star r)\right]$$

Expanding in powers of r, up to an irrelevant additive constant

$$V(r) \approx C_F\,(2\,\mathrm{Re}\{R\})\,\alpha_s\left[-\frac{1}{r} - \frac{\mathrm{Re}\{RM^2\}}{2\,\mathrm{Re}\{R\}}\,r + \dots\right] \approx \text{const.}\times\left(-\frac{1}{r} + kr\right)$$

where ($\tan\theta\ \tan\phi > 1$)

$$k = \frac{-\mathrm{Re}\{RM^2\}}{2\,\mathrm{Re}\{R\}} = 0.584\mathrm{GeV}^2 > 0, \left(\sigma = \frac{4}{3}\alpha_s\, k \approx 0.2\ \mathrm{GeV}^2 \quad \textit{if} \quad \alpha_s \approx 0.3\right)$$

while expanding in powers of $1/p^2$

$$\Delta_E(p^2) = (2\,\mathrm{Re}\,R)\left[\frac{1}{p^2} + \frac{2k}{p^4} + \mathcal{O}(1/p^6)\right]$$

Condensates and OPE

Boucaud et al. [Phys. Lett. B **493**:315-324,(2000)]
In the Landau gauge, by OPE

$$\Delta_E(p^2) = \Delta_0(p^2) + \frac{N_c g^2}{4(N_c^2-1)} \frac{\langle A^2 \rangle}{p^4} + \mathcal{O}(1/p^6)$$

good fit of data in the 2-10 GeV window.

Taking $\Delta_0(p^2) \approx Z/p^2$ for the *perturbative* propagator

$$\Delta_E(p^2) \approx Z\left[\frac{1}{p^2} + \frac{N_c g^2}{4Z(N_c^2-1)} \frac{\langle A^2 \rangle}{p^4}\right] \Leftrightarrow \Delta_E(p^2) = (2\,\mathrm{Re}\,R)\left[\frac{1}{p^2} + \frac{2k}{p^4}\right]$$

Then, up to an irrelevant renormalization factor

$$k = \frac{N_c}{8Z(N_c^2-1)}\left[g^2\langle A^2 \rangle\right] = \frac{-\,\mathrm{Re}\{RM^2\}}{2\,\mathrm{Re}\{R\}} = 0.584\,\mathrm{GeV}^2 > 0$$

The phases of R and M are essential for predicting the correct condensates and string tension

Observable gluon mass

Is the gluon mass observable? And how is it defined?

Complex Poles ⇒ **define** a mass scale $|M|$

Many definitions of a “gluon mass” [J.H. Field, PRD 66 (2002)]:

Author	Reference	Estimation Method	Gluon Mass
Parisi, Petronzio	[12]	$J/\psi \to \gamma X$	800 MeV
Cornwall	[8]	Various	500 ± 200 MeV
Donnachie, Landshoff	[59]	Pomeron parameters	687-985 MeV
Hancock, Ross	[61]	Pomeron slope	800 MeV
Nikolaev *et al.*	[62]	Pomeron parameters	750 MeV
Spiridonov, Chetyrkin	[63]	$\Pi^{em}_{\mu\nu}$, $\langle TrG^2_{\mu\nu}\rangle$	750 MeV
Lavelle	[64]	$qq \to qq$, $\langle TrG^2_{\mu\nu}\rangle$	$640\ \mathrm{MeV}^2/Q(\mathrm{MeV})$
Kogan, Kovner	[67]	QCD vacuum energy, $\langle TrG^2_{\mu\nu}\rangle$	1.46 GeV
Field	[68]	pQCD at low scales (various)	$1.5^{+1.2}_{-0.6}$ GeV
Liu, Wetzel	[39]	$\Pi^{em}_{\mu\nu}$, $\langle TrG^2_{\mu\nu}\rangle$	570 MeV
		Glue ball current, $\langle TrG^2_{\mu\nu}\rangle$	470 MeV
Ynduráin	[66]	QCD potential	10^{-10}-20 MeV
Leinweber *et al.*	[69]	Lattice Gauge	1.02 ± 0.10 GeV
Field	This paper	$J/\psi \to \gamma X$	$0.721^{+0.016}_{-0.068}$ GeV
		$\Upsilon \to \gamma X$	$1.18^{+0.09}_{-0.29}$ GeV

Table 15: *Estimates of the value of the gluon mass from the literature. For Donnachie*

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For $t > 0$,

$$i\,\Delta^{\mu\nu}(\mathbf{x},t) = \langle 0|A^\mu(0)e^{i\mathbf{P}\cdot\mathbf{x}}\,e^{-i\hat{H}t}\,A^\nu(0)|0\rangle = \sum_n \rho_n^{\mu\nu}\;e^{i\mathbf{p}_n\cdot\mathbf{x}}\;e^{-iE_n t}$$

Complex poles $\Leftrightarrow$ Complex $E_n = \pm(\omega_n \pm i\gamma_n)$, with $E_n^2 = M^2, M^{\star 2}$

But the F.T. reads

$$i\Delta(\mathbf{p},p_0) = \sum_n (2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}_n)\;\rho_n(p)\int_0^\infty \left[e^{ip_0 t} + e^{-ip_0 t}\right]\;e^{-iE_n t}\mathrm{d}t$$

"convergence principle" requires $E_n = -\omega - i\gamma$, $E_n' = \omega - i\gamma = -E_n^\star$

$$\Delta(p) = \sum_n \frac{(2\pi)^3[2\rho_n(p)\,E_n]\delta^3(\mathbf{p}-\mathbf{p_n})}{p^2-M_n^2} - \sum_n \frac{(2\pi)^3[2\rho_n(p)\,E_n^*]\delta^3(\mathbf{p}-\mathbf{p_n})}{p^2-M_n^{*\,2}}$$

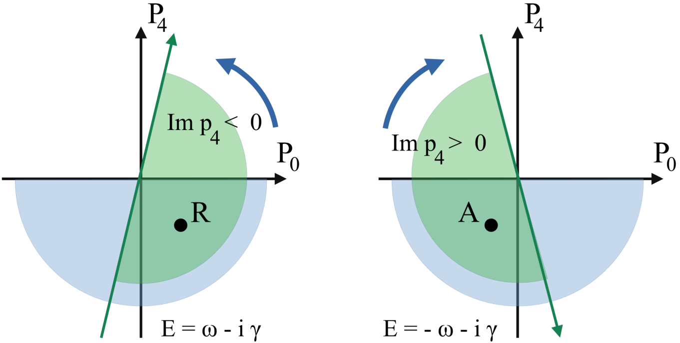
$$\Delta(p) = \frac{R}{p^2-M^2} \overset{\downarrow}{-} \frac{R^*}{p^2-M^{2\,*}} \quad\rightarrow\quad -\frac{1}{p^2-m^2+i\epsilon}$$ for $R=0,\quad R^\star = 1$

$$\begin{cases}\text{Anomalous Pole:} & E = -\sqrt{\mathbf{p}^2+M^2} = -\omega - i\gamma\\ \text{Regular Pole:} & E' = -E^\star = \sqrt{\mathbf{p}^2+M^{\star 2}} = \omega - i\gamma\end{cases}$$

Clockwise Wigner rotation

$$\begin{cases} p_0 = ip_4 \\ t = -i\tau \end{cases} \rightarrow \boxed{e^{ip_0 t} = e^{ip_4 \tau}} \rightarrow \begin{cases} t > 0 \leftrightarrow \mathrm{Im}\, p_0 < 0 \\ \tau > 0 \leftrightarrow \mathrm{Im}\, p_4 < 0 \end{cases}$$

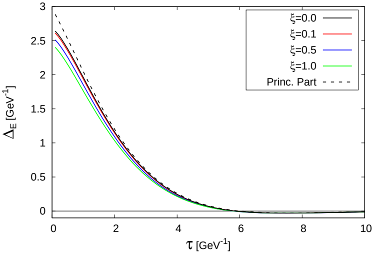
"Convergence principle":



$\theta(t) \rightarrow \theta(\tau)$ and $\int_{-\infty}^{+\infty} dp_4$ $\theta(t) \rightarrow \theta(-\tau)$ and $\int_{+\infty}^{-\infty} dp_4$

The gluon as a quasi-particle

Continuation in direct space by $\theta(t) \to \theta(\pm\tau)$:



$$[\Delta_E(\tau)]_{\mathbf{p}=0} \sim \exp(-\tau\,\mathrm{Re}\,M)\ \cos(\phi - \tau\,\mathrm{Im}\,M), \qquad \tau > 0$$

$$\begin{cases} \phi = \arctan\left(\mathrm{Im}\,R/\,\mathrm{Re}\,R\right) - \arctan\left(\mathrm{Im}\,M/\,\mathrm{Re}\,M\right) \approx 0.69 \\ \tau_0 = (\pi/2 + \phi)/\,\mathrm{Im}\,M \approx 2.26/(0.375\,\mathrm{GeV}) = 6.0\,\mathrm{GeV}^{-1} \approx 1.2\ \mathrm{fm} \end{cases}$$

$$[\Delta(t)]_{\mathbf{p}=0} \sim \exp(-t\,\mathrm{Im}\,M)\ \cos(\phi - t\,\mathrm{Re}\,M), \qquad t > 0$$

The quasi-gluon is confined: $t_0 = 1/\,\mathrm{Im}\,M \approx 0.5\ \mathrm{fm}$

Are the intermediate states physical?

Single particle zero-norm eigenstates: $\hat{H}|\pm\rangle = E_\pm|\pm\rangle$

$$E_\pm = \omega \pm i\eta, \qquad \langle +|+\rangle = \langle -|-\rangle = 0, \qquad \langle +|-\rangle = 1$$

Interacting vacuum:
$$\begin{cases} |\Omega\rangle = |0\rangle + C\,|+\rangle & \sim \text{ Gupta Bleuler} \\ \langle\Omega|\,\hat{A}\,|\Omega\rangle = \langle 0|\,\hat{A}\,|0\rangle & \end{cases}$$

$$\langle n|e^{-i\hat{H}t}|\Omega\rangle \sim e^{-i\delta E_n\, t}$$

$|+,-\rangle$ $\quad E = 2\omega, \quad \langle +,-|+,-\rangle = 1$

$\nearrow$ $\delta E = E_- = \omega - i\eta$

$|\Omega\rangle = |0\rangle + C\,|+\rangle$ $\qquad$ *PHYSICAL* $|n\rangle$

$\searrow$ $\delta E = -(E_+) = -\omega - i\eta$

$|0\rangle$ $\quad E = 0, \quad \langle 0|0\rangle = 1$

But also $|0\rangle \longrightarrow |+,-\rangle \qquad \delta E = 2\omega = 2\,\mathrm{Re}\,M \approx 1.2$ GeV (Glueball?)

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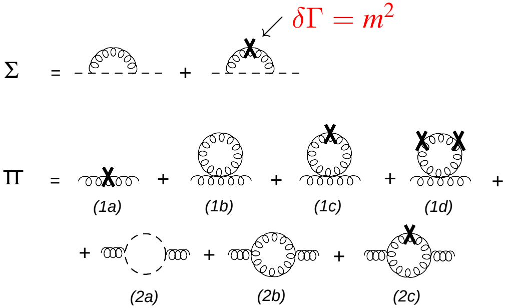
THANK YOU

BACKUP SLIDES

At variance with Curci-Ferrari model (Tissier and Wschebor, 2011):

$$\Delta_T(p) = \frac{1}{(p^2+m^2) - \Pi^T} = \frac{1}{(p^2+m^2) - ({\color{red}m^2} + \Pi^T_{Loops})} = \frac{1}{p^2 - \Pi^T_{Loops}}$$



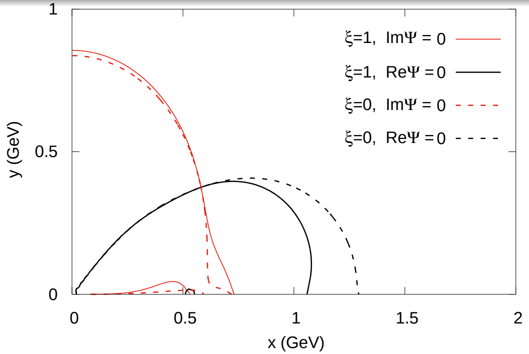
- The pole shift cancels at tree level
- All spurious diverging mass terms cancel without counterterms and/or parameters
- Standard UV behavior

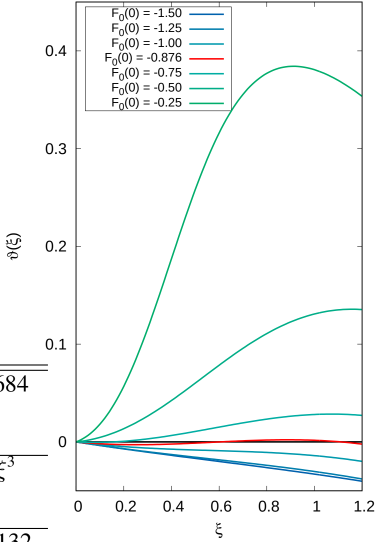
In the $\overline{MS}$ scheme: $\Pi^{diverg.} = \frac{Ng^2}{(4\pi)^2}\left(\frac{2}{\epsilon} + \log\frac{\mu^2}{m^2}\right) p^2 \left(\frac{13}{6} - \frac{\xi}{2}\right)$

Standard UV behavior $\Longrightarrow \Pi^{finite} \sim -\frac{Ng^2}{(4\pi)^2} p^2 \left(\frac{13}{6} - \frac{\xi}{2}\right) \log\frac{p^2}{\mu^2}$

Optimized Screened Expansion

Optimization by ξ-independence of principal part





$$F_0(0) = -0.876, \quad m_0 = m(0) = 0.656 \text{ GeV}, \quad Z(0) = 2.684$$

$$|\theta(\xi)| < 2.76 \cdot 10^{-3}, \qquad 0 < \xi < 1.2$$

$$F_0(\xi) \approx -0.8759 - 0.01260\xi + 0.009536\xi^2 + 0.009012\xi^3$$

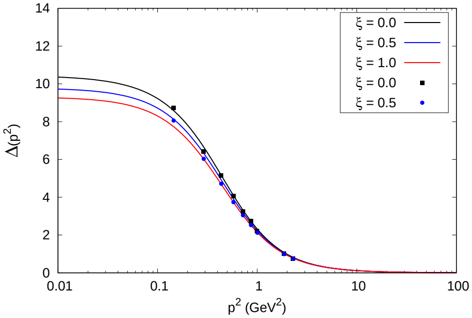
$$m^2(\xi)/m_0^2 \approx 1 - 0.39997\xi + 0.064141\xi^2$$

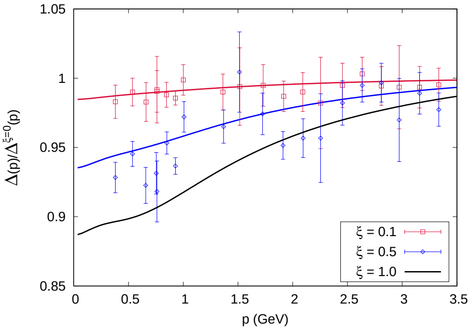
$$z_0/m_0 = 0.8857 + 0.5718\, i, \quad t_R = \text{Im}\, R(0)/\,\text{Re}\, R(0) = 3.132$$

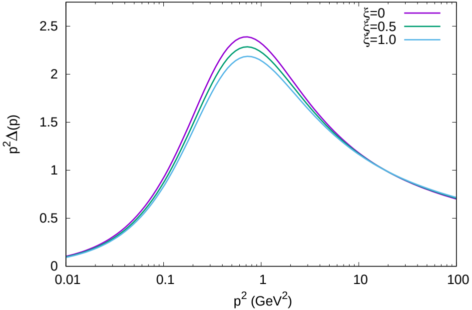
$$M = 0.581 \text{ GeV}, \gamma = 0.375 \text{ GeV} \quad \text{(invariant pole)}$$

Back to Euclidean Space: generic covariant gauge $\xi \neq 0$

Optim. S.E. vs. Lattice data of Bicudo,Binosi,Cardoso,Oliveira,Silva PRD 92 (2015)





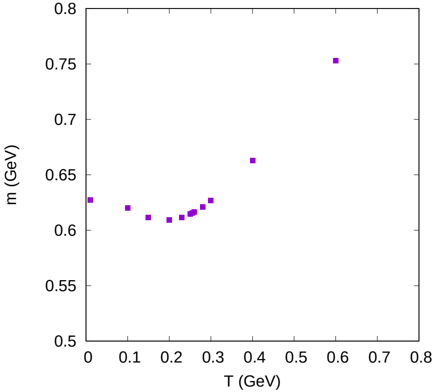


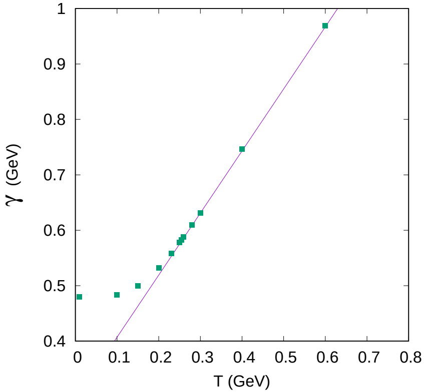
- Optim. in Complex pl. $\Rightarrow$ Euclidean
- Quantitative agreement with lattice
- Qual. agreem. with DS if N.I. are used: Aguilar, Binosi, Papavassiliou (2015)
- Not a fit! No free parameters.
- Quantitative prediction up to and beyond the Feynman gauge ($\xi = 1$) (not accessible by other methods)

Finite T

Trajectory of poles in the complex plane

In the limit $\mathbf{k} \to 0$ the pole $\omega = \pm(m \pm i\gamma)$ is the same for Δ_L, Δ_T.
Using $m_0 = 0.73$ GeV and $F_0 = -1.05$ (fixed at $T = 0$):

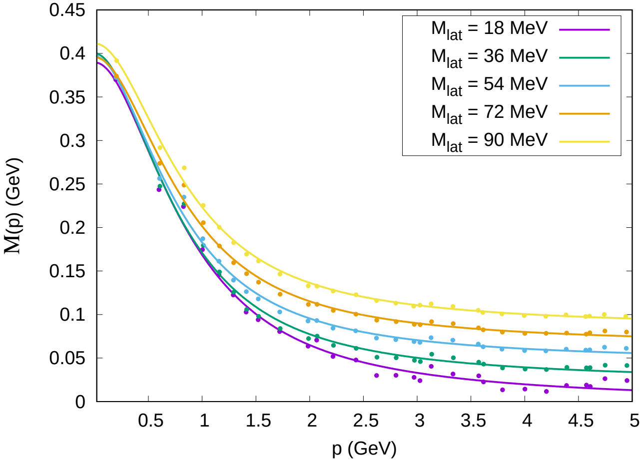




The line is the fit $\gamma = \gamma_0 + bT$ with $\gamma_0 = 0.295$ GeV and $b = 1.12$.
(Hard thermal loops: $\gamma/T = 3.3\alpha_s$)

FULL QCD

Quark sector - light quarks - c.c. scheme (G.C., D. Rizzo, M. Battello, F.S, 2021)



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