QCD and the Pion Form Factor

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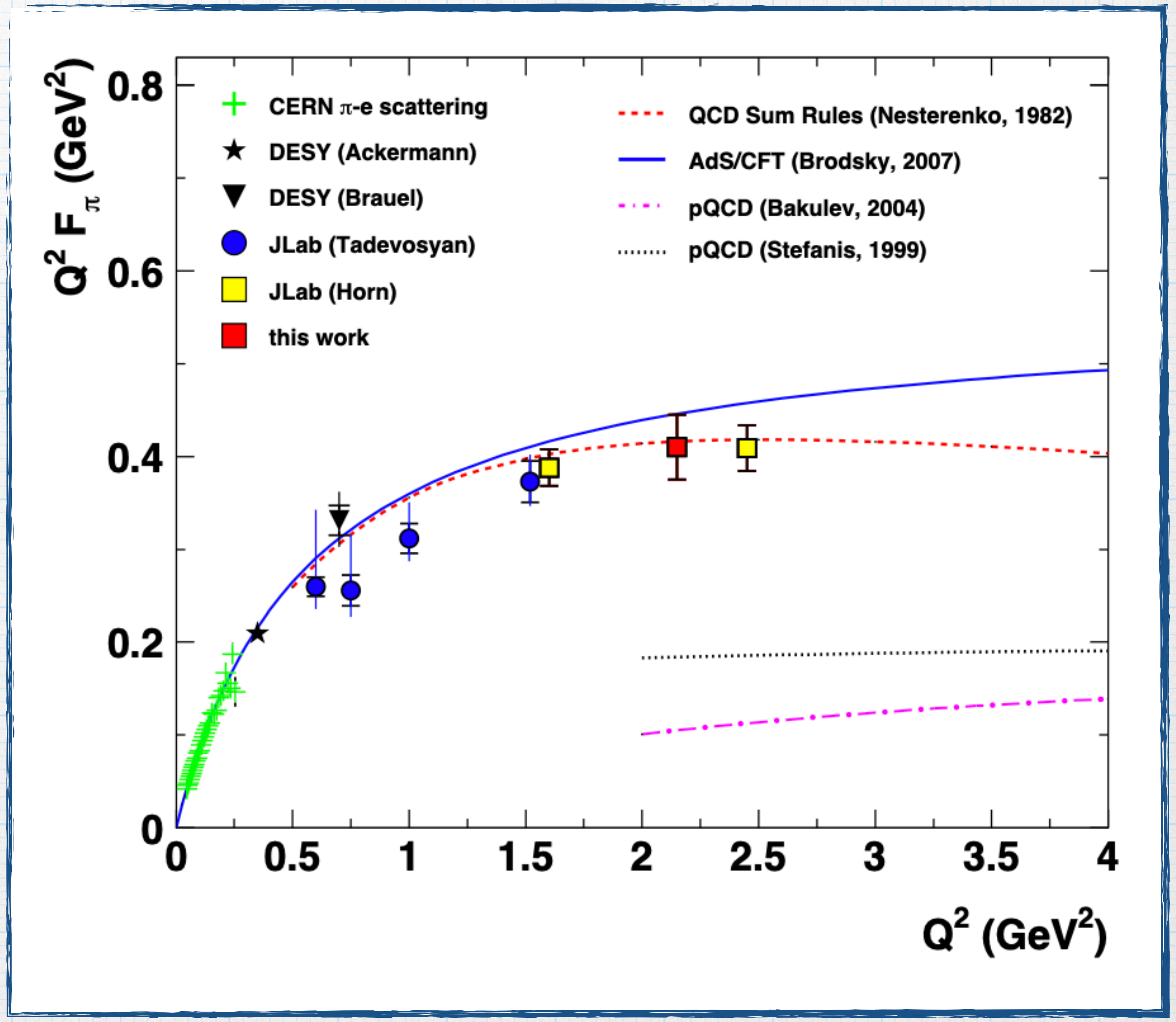
PQCD vs. Non-PQCD

- Already at $Q^2 = 2\, GeV^2$ α_s should be small enough to produce a reasonable estimate for the pion form factor using PQCD

- By comparing PQCD results with experiment it is clear that PQCD is not the entire story

- We can learn about QCD by examining how the addition various *non-perturbative* effects change the asymptotic behavior of the pion form factor



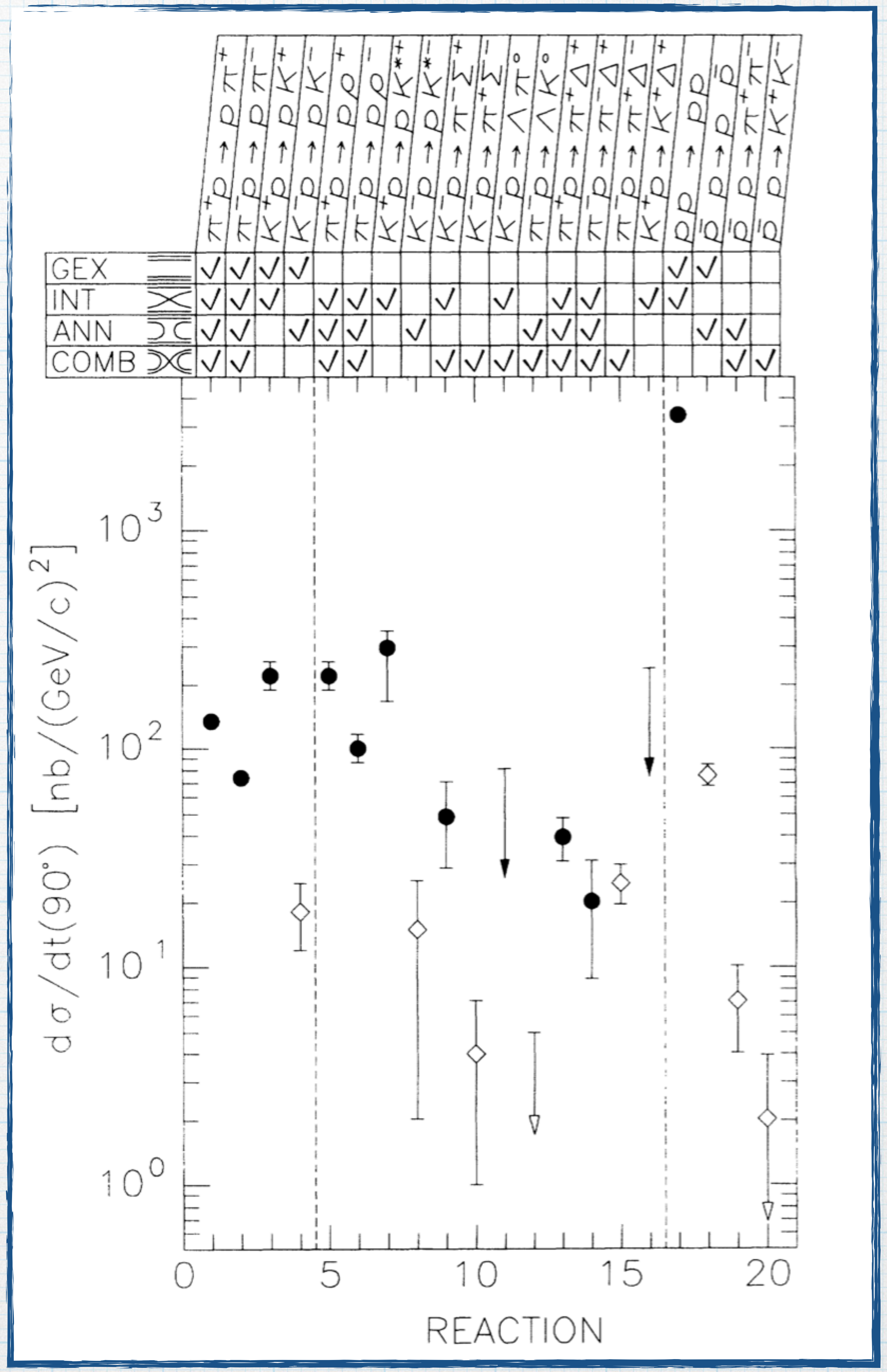
Horn et al (2007) 0707.1795

Quark Interchange

- It is expected that form factors decrease with momentum transfer

- Dependence of the form factor on energy implies dominant contribution asymptotically is the lightest intermediate state $\gamma \rightarrow q\bar{q} \rightarrow \pi\pi$

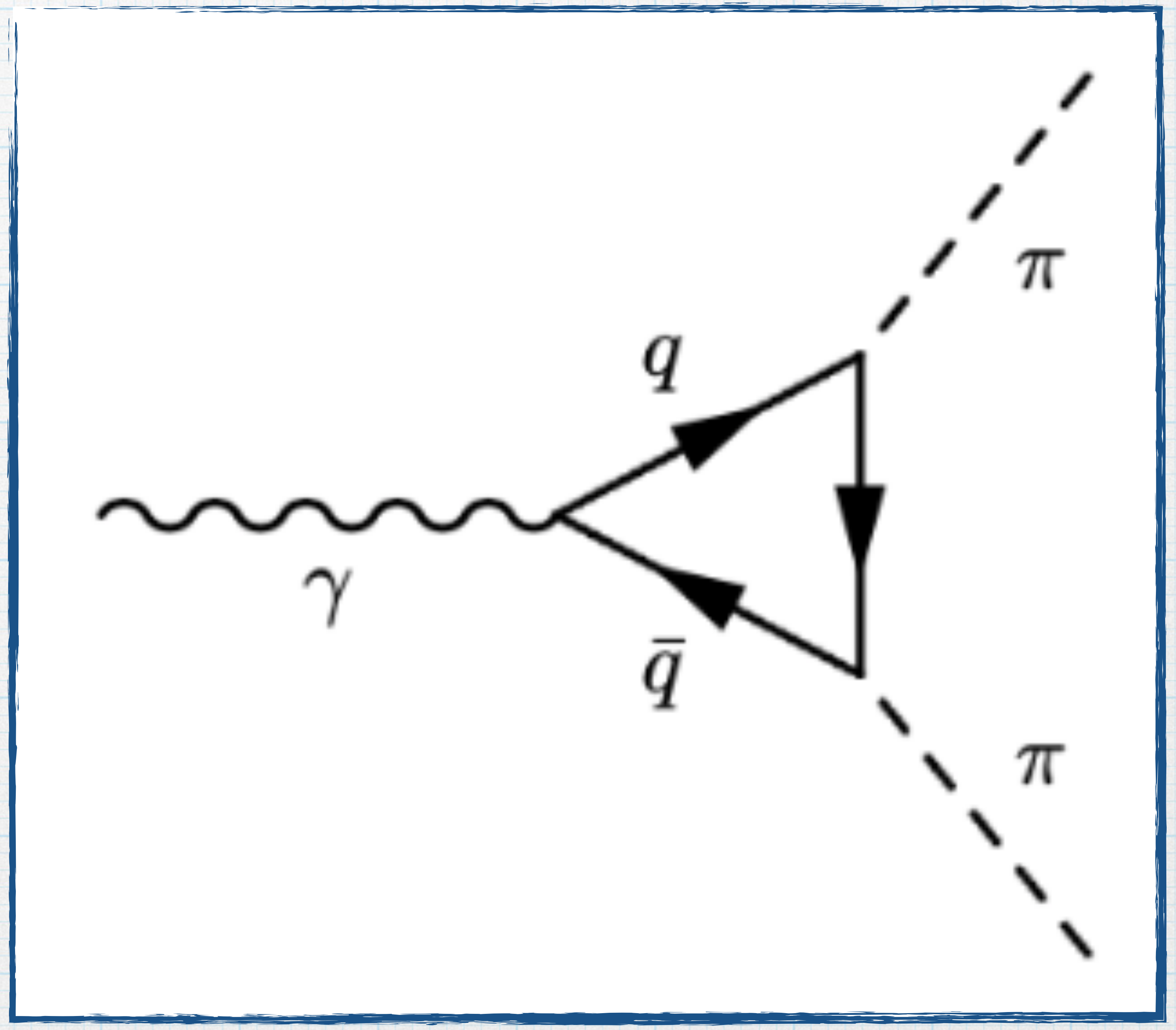
- Dominance of quark interchange has been observed experimentally



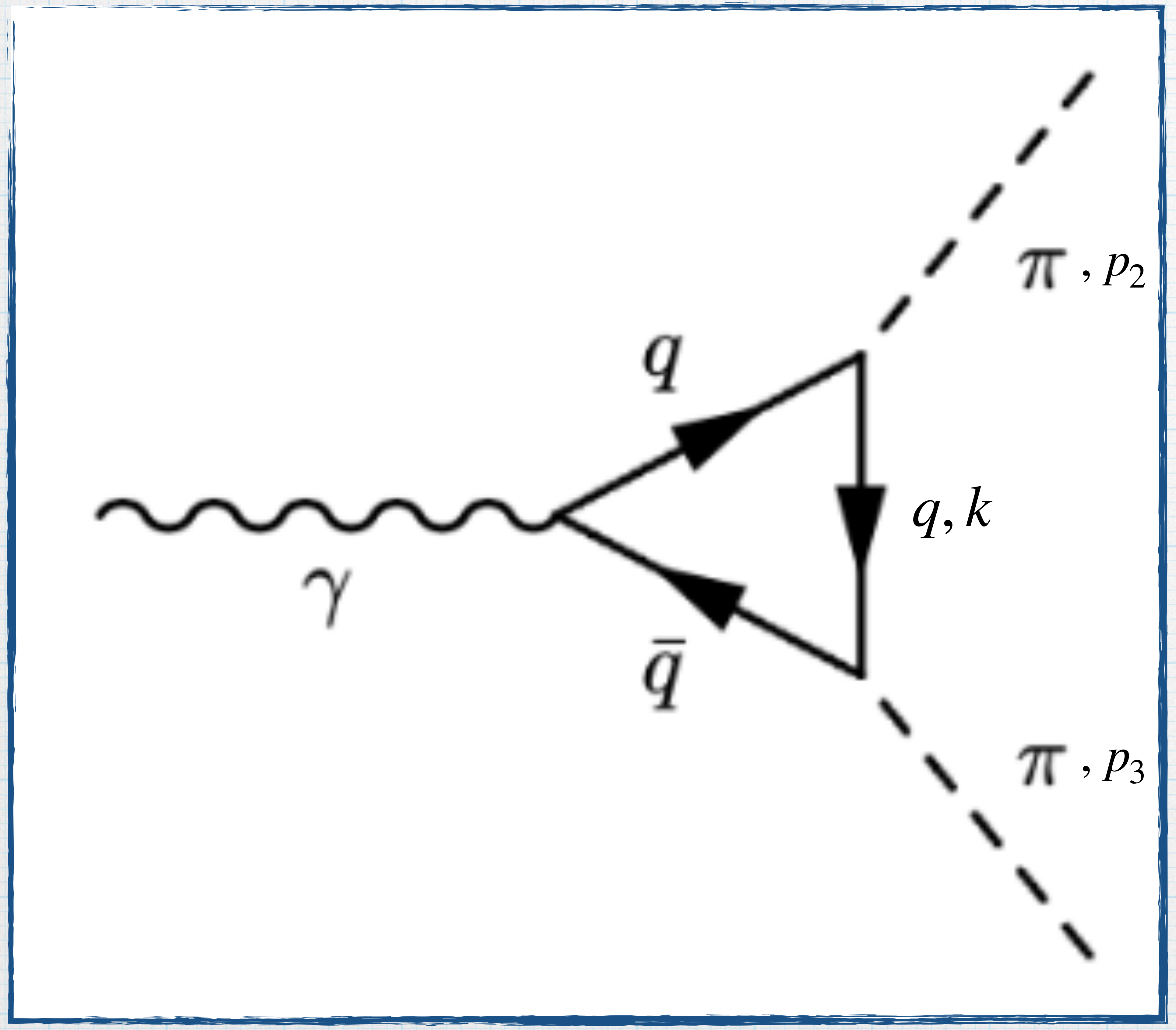
White et al *Phys.Rev.D* 49 (1994) 58–78

Quark Interchange

- Want to study the effects of adding non-perturbative effects to basic quark interchange model
  - What effect does gluon exchange have?
  - What effect does modifying the $q\bar{q}$ propagators have?
  - What effect does modifying the transverse propagator have?



All Scalar Model



$$s = (p_2 + p_3)^2$$

- Consider the case where all particles are scalars at large s

$$F(s) \simeq \int \frac{d^4k}{(k^2-\mu^2)((k+p_2)^2-m_q)^2((k-p_3)^2-m_q^2)} \sim \frac{\ln^2 s}{s}$$

- This is the hardest quark interchange process

- Any non-perturbative modifications will make the process softer by modifying powers of s or powers of $\ln s$

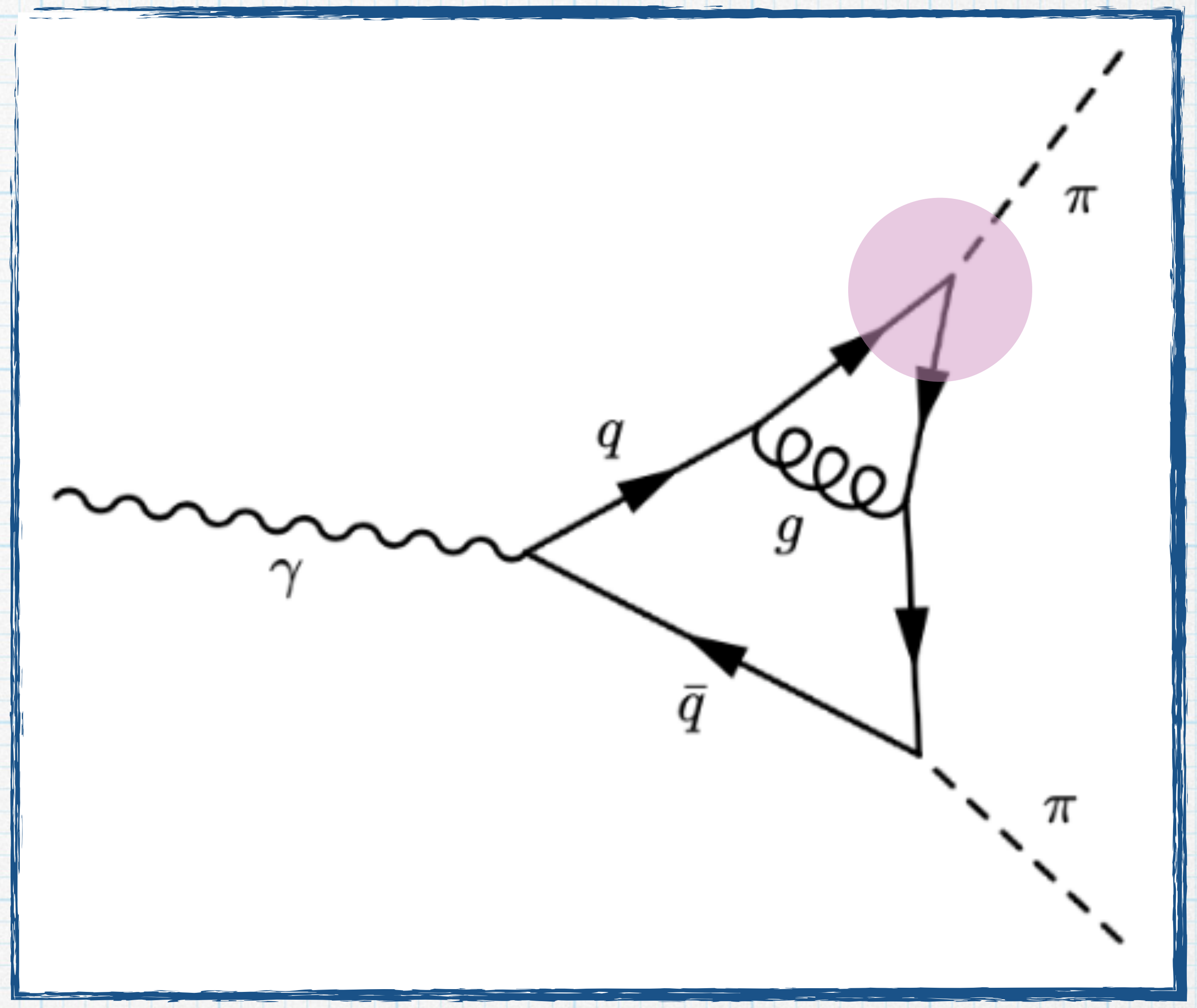
Gluon Exchange

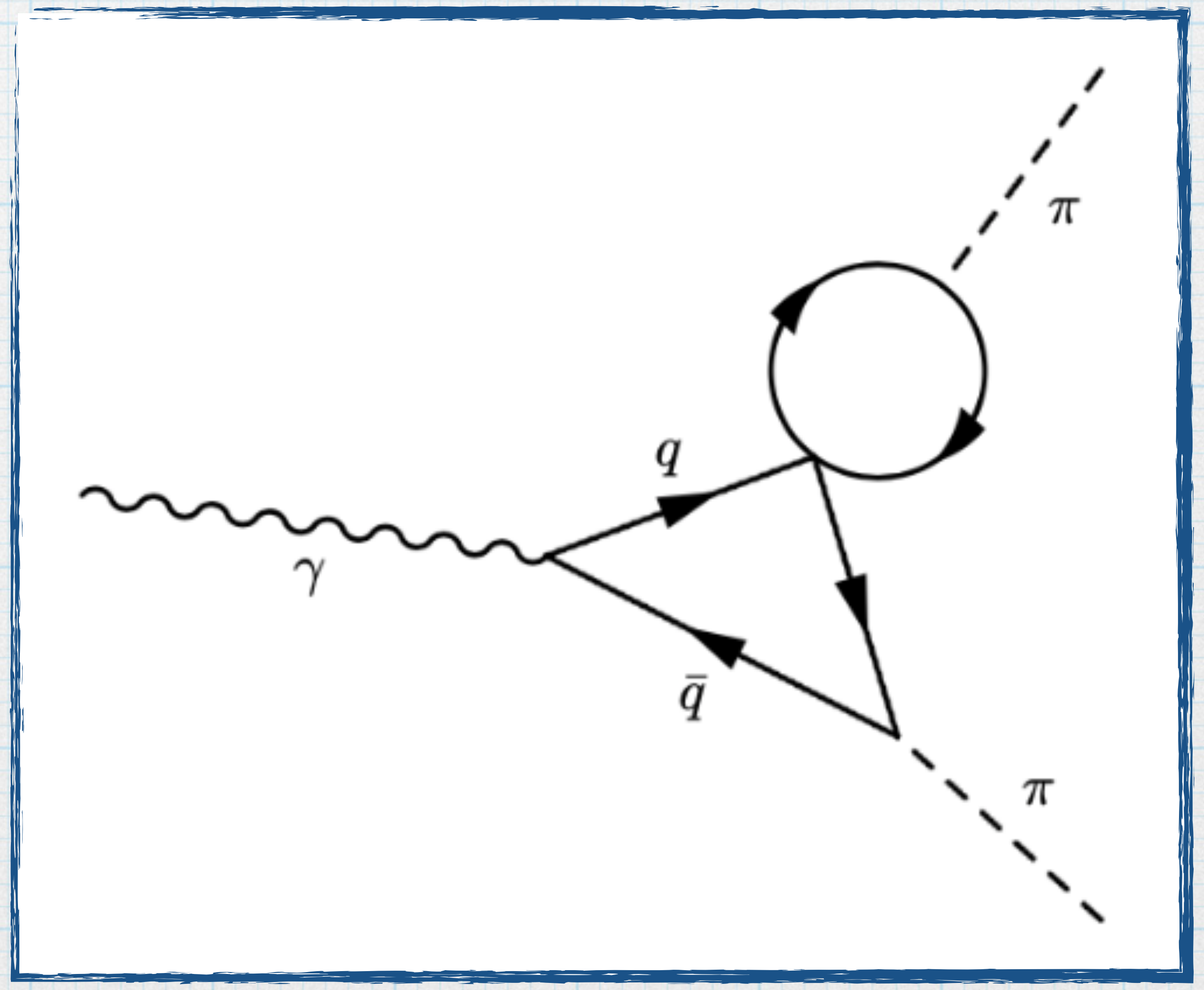
- Gluon exchange adds factor of $\alpha_s(s) \sim \frac{1}{\ln s}$

- At large s gluon propagator shrinks to a point. This decouples the two loop momenta so that what happens at the $q\bar{q}\pi$ vertex does not effect the leading large s behavior of the FF

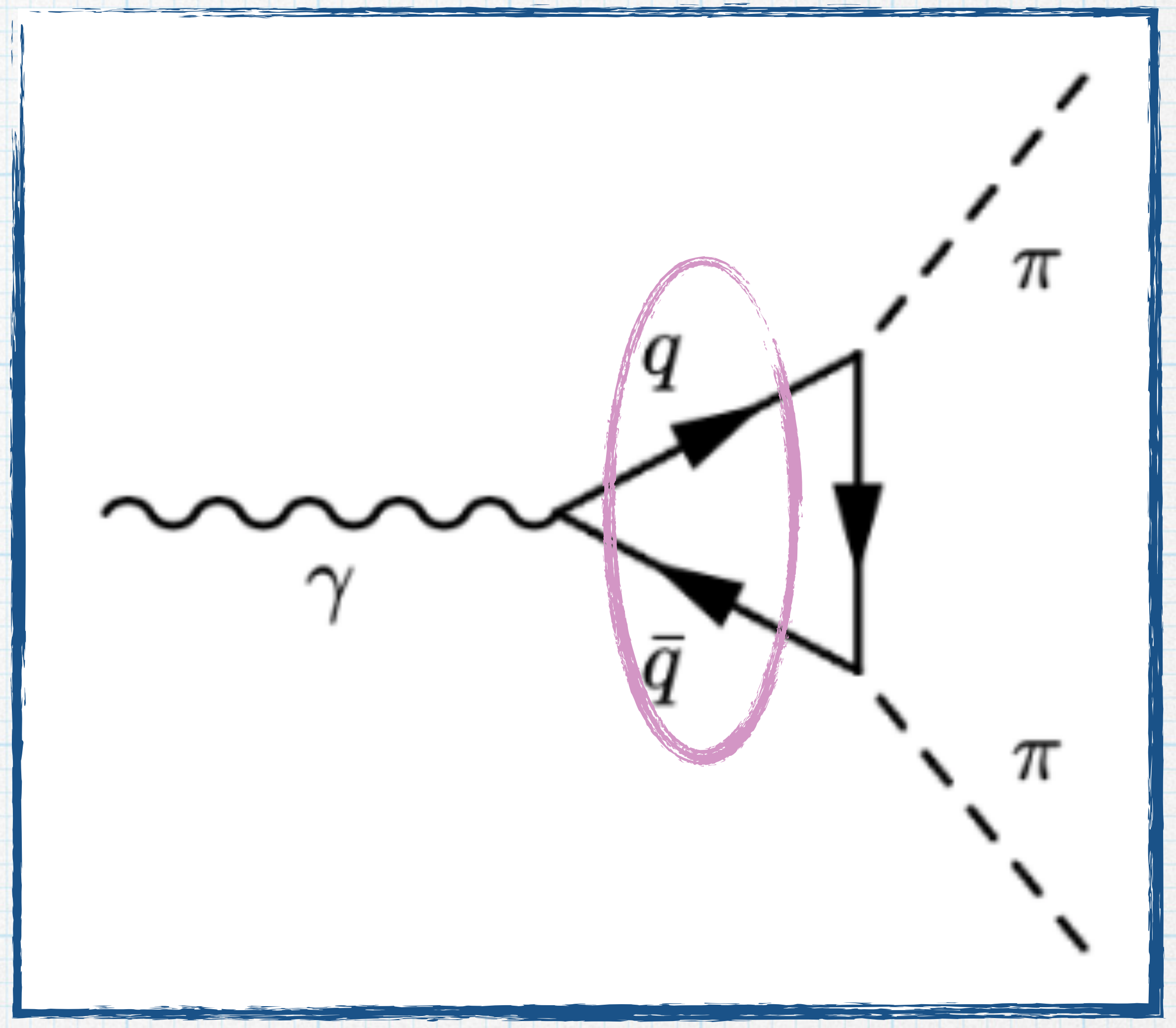
- This decoupling blocks further softening of the FF from non-perturbative effects at the $q\bar{q}\pi$ vertex

- Gluon loops soften asymptotic behavior by *at most* a factor of $\alpha(s) \sim \frac{1}{\ln s}$





Non–Perturbative Pion Vertices



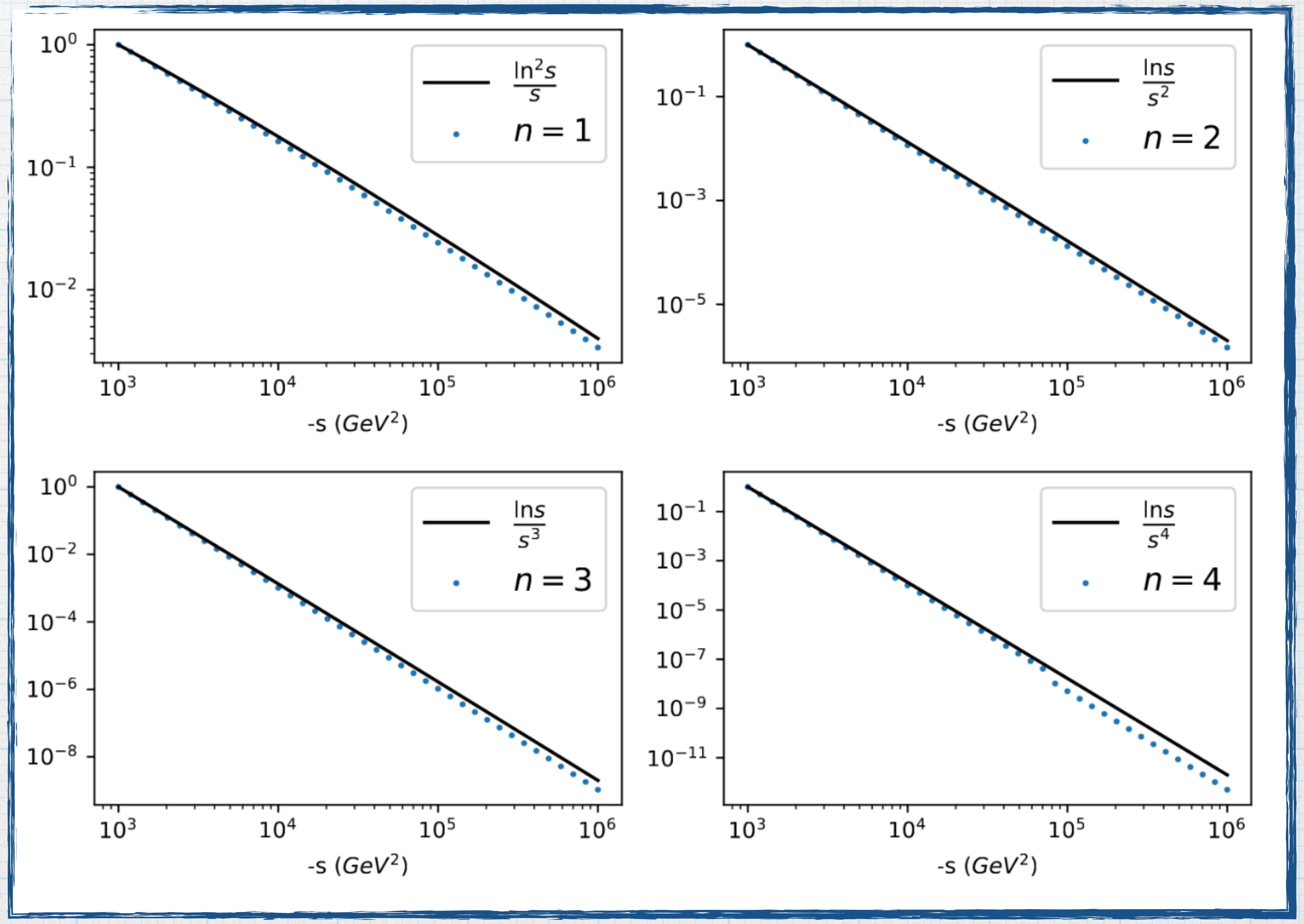
- Make FF softer by modifying the horizontal propagators. These propagators contain info about the total energy

- Try $\dfrac{1}{p^2 - m^2} \rightarrow \dfrac{1}{(p^2 - m^2)^n}$

- This effectively modifies the $q\bar{q}\pi$ interaction to make the FF softer

- Modifies the overall factor of s

$$F(s) \sim \frac{\ln s}{s^n}$$



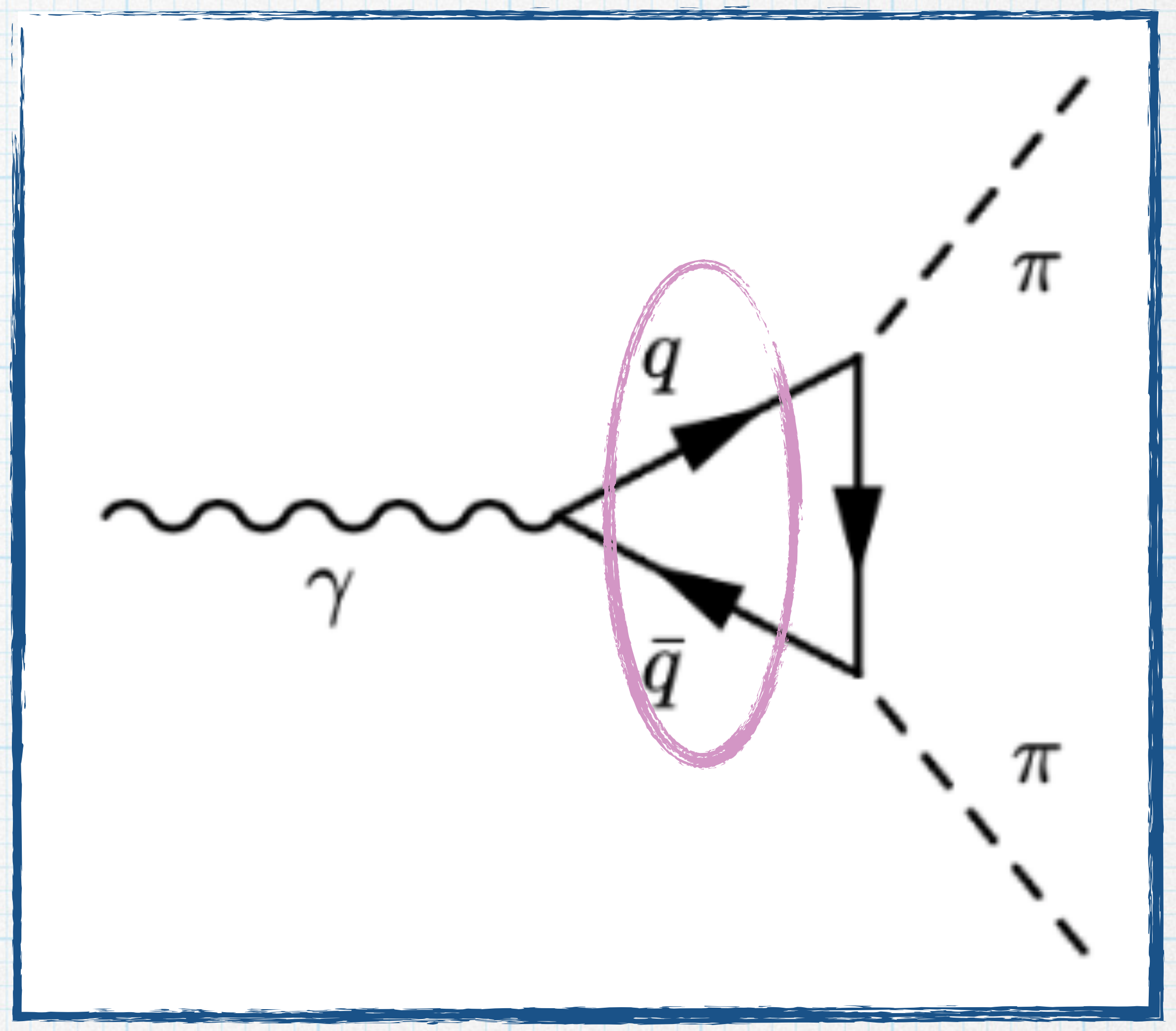
Non-Perturbative Pion Vertices

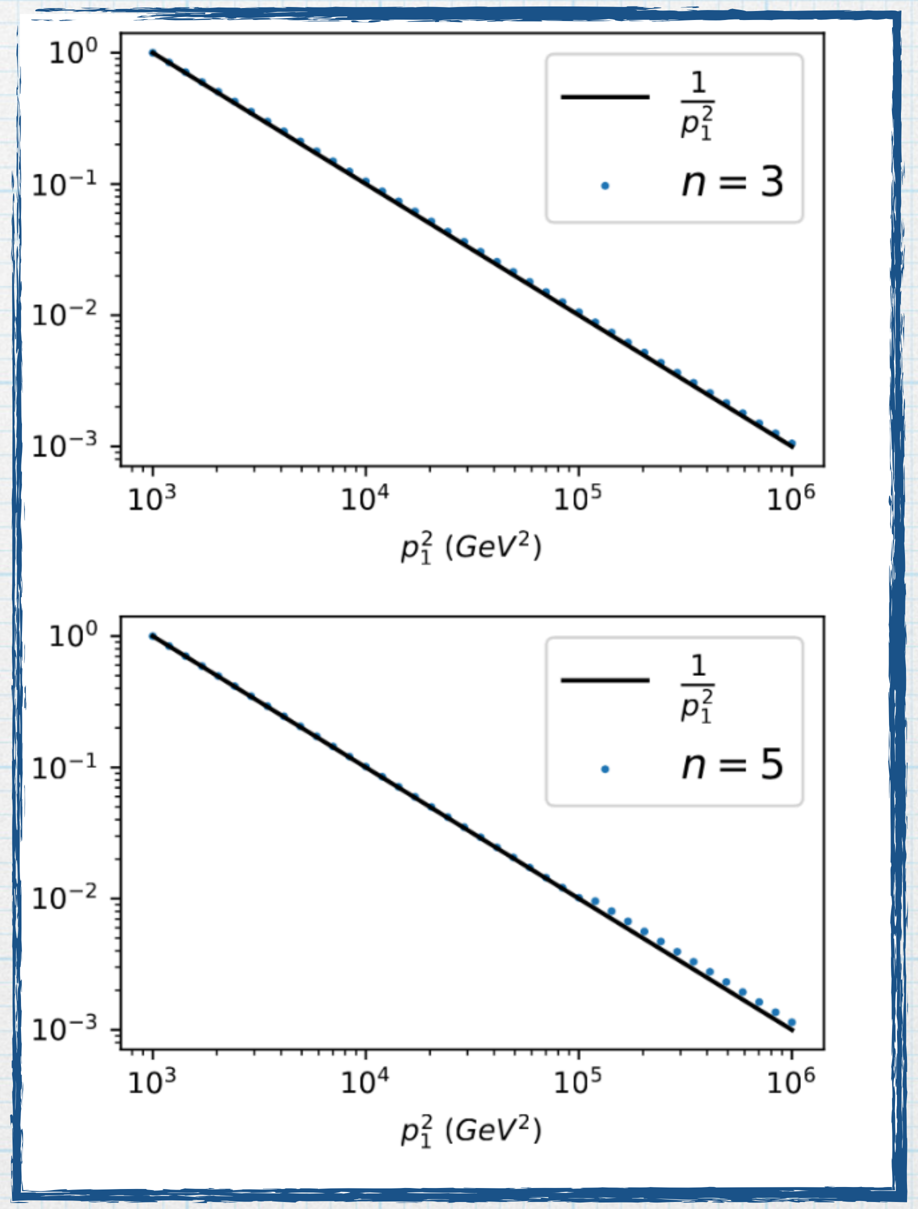
- Make FF softer by modifying the horizontal propagators. These propagators contain info about the total energy

- Try $\frac{1}{p^2 - m^2} \to \frac{1}{(p^2 - m^2)^n}$

- However, if a gluon is exchanged at the vertex then this effect is not seen

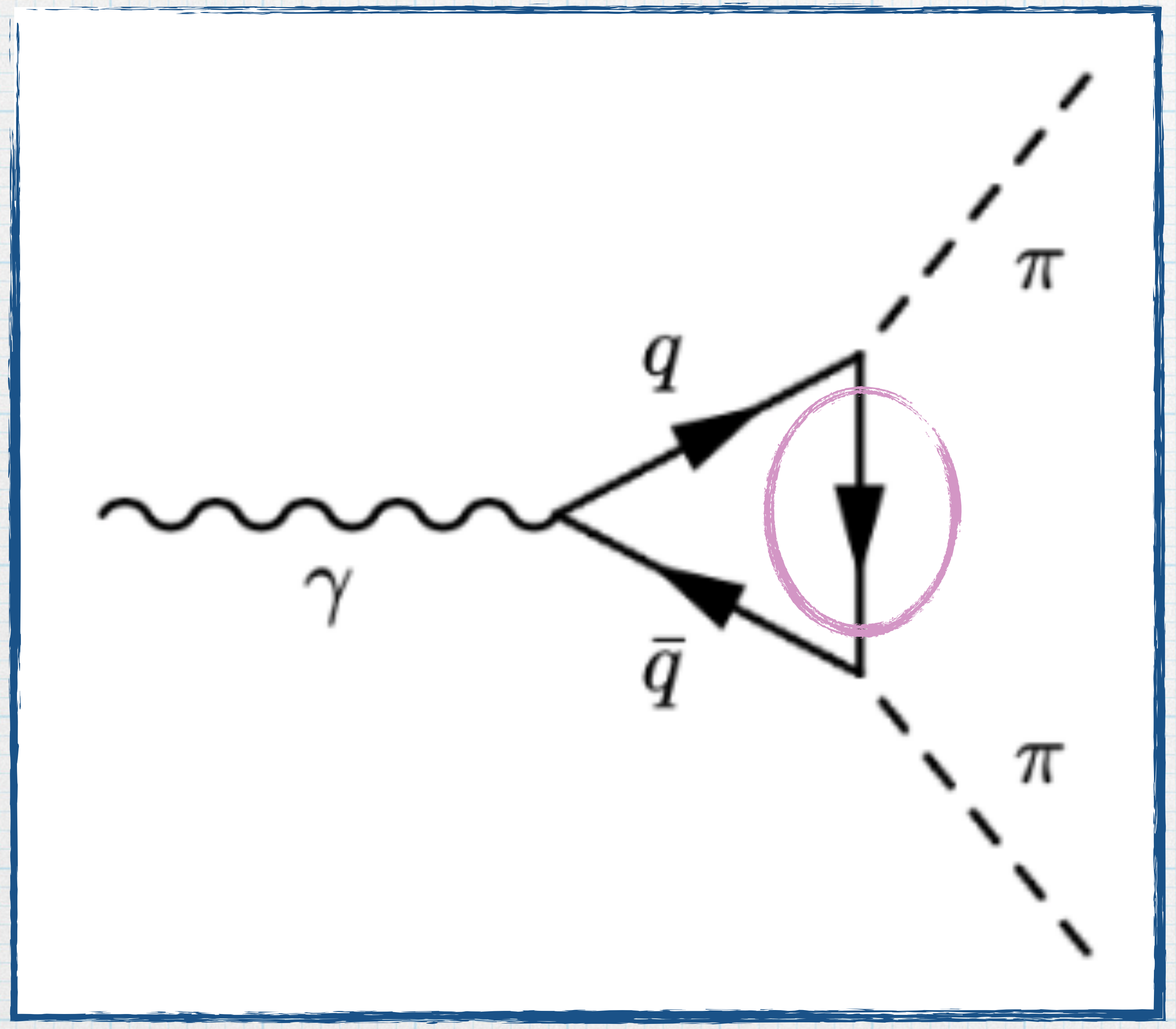
$$F(s) \sim \frac{\ln s}{s}$$





p_1 = momentum of horizontal quark

Modification of Transverse Propagator



- Investigate the transverse propagator at large values of s

$$F(s) = \frac{1}{\pi}\int ds' \frac{\Delta F(s')}{s'-s} \quad \rightarrow \quad \Delta F(s') \sim \int dz \frac{1}{m^2 - \frac{s'}{2}(1-z)}$$

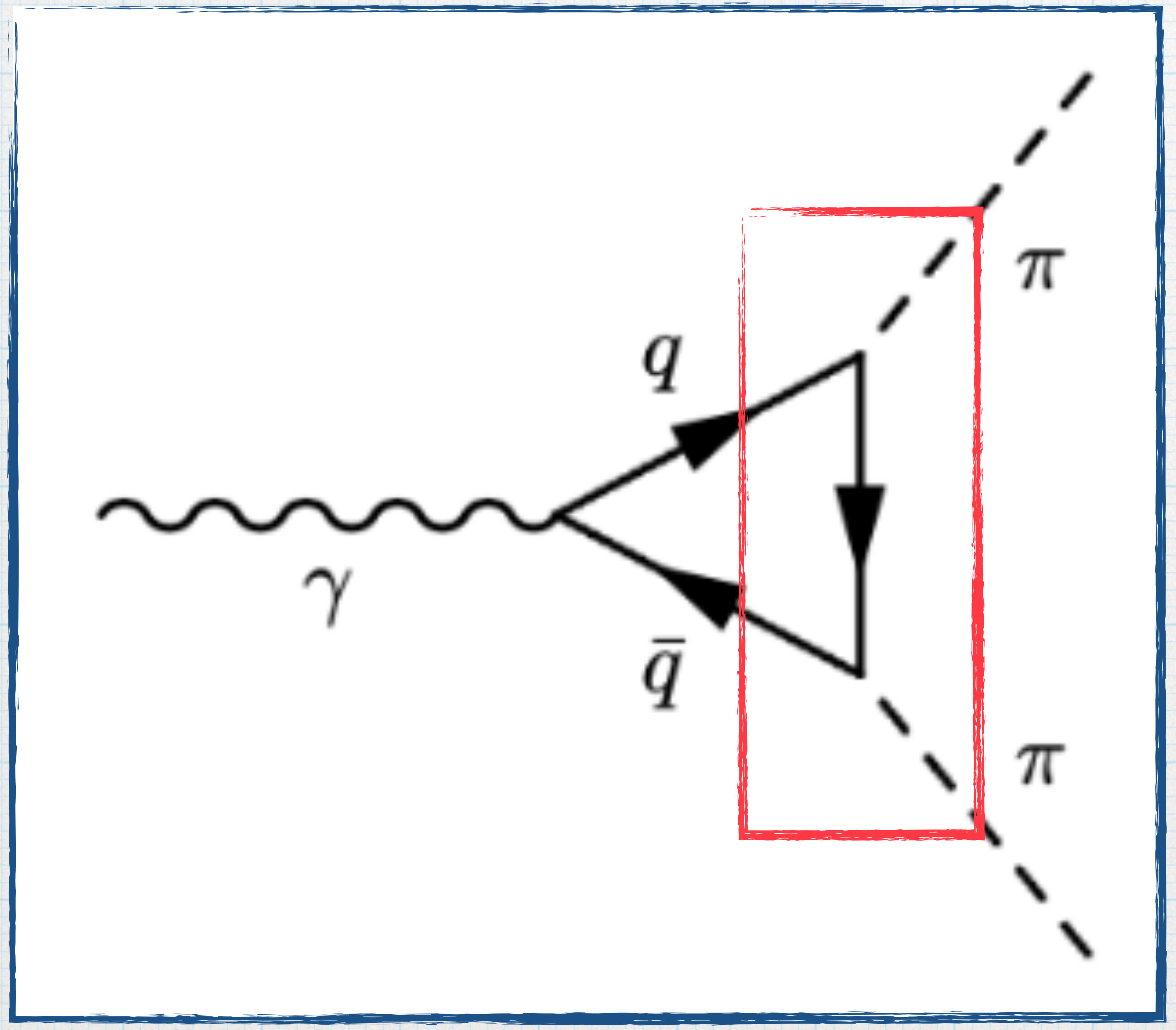
- Form factor dominated by $z \simeq 1$ low momentum transfer region ($z = \cos\theta_s$)

- None of the s dependence of the $q\bar{q}$ pair is transferred to the transverse quark

- Simple modification of the transverse quark should not effect asymptotic behavior of the FF

Effects of Reggiezation

- There is potential to affect the s behavior of the FF by more complex modification of the transverse quark

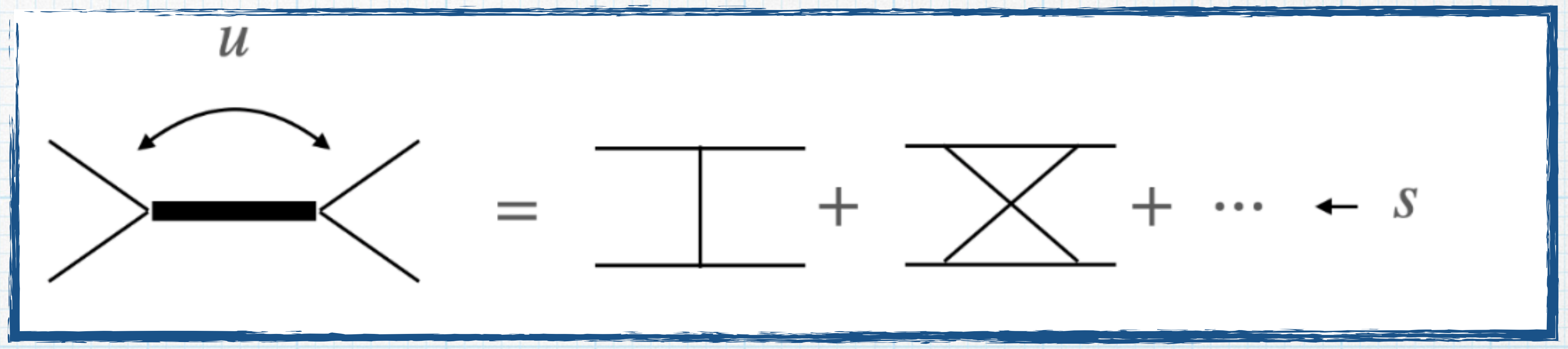


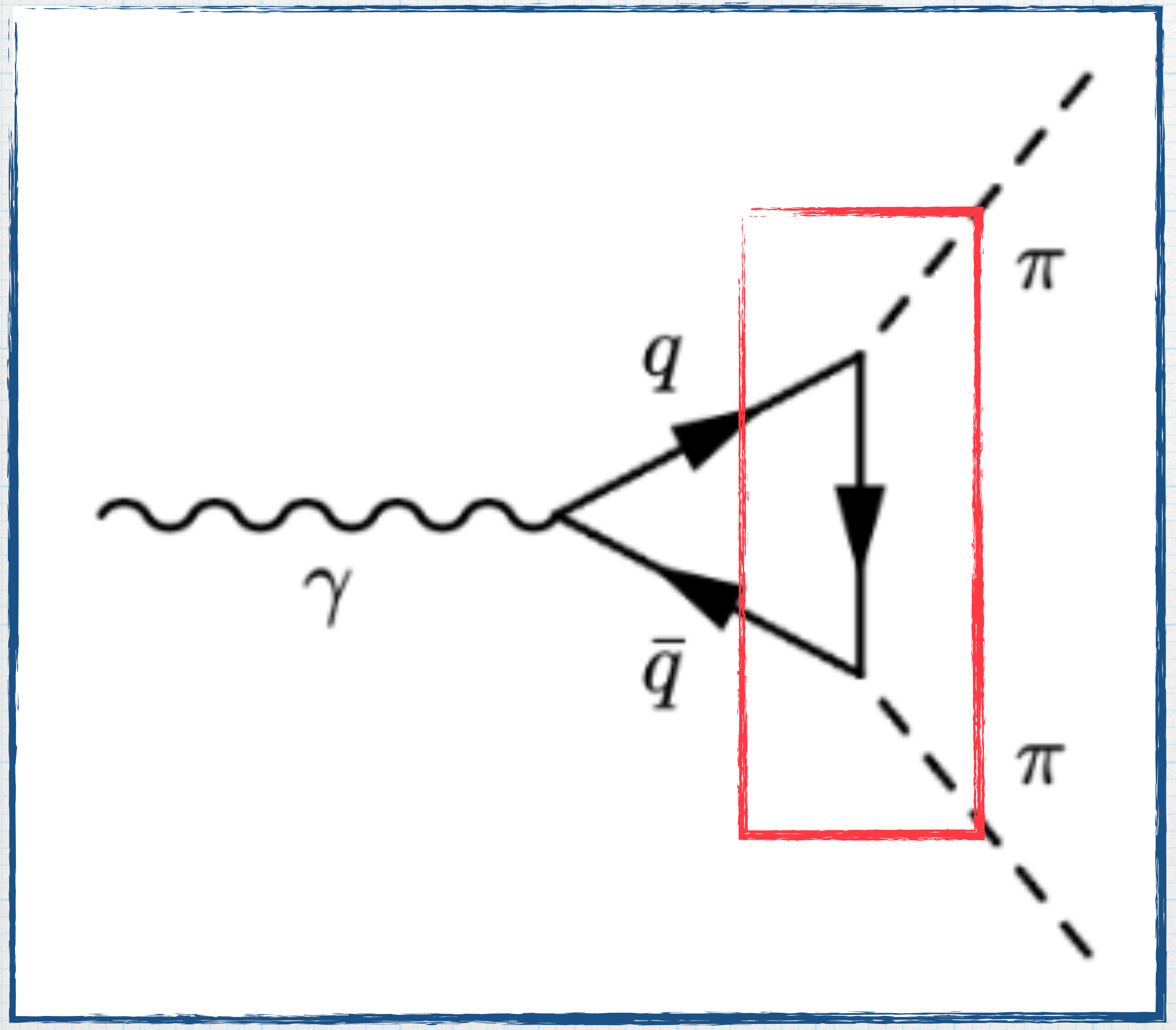
- Generally believed that QCD interactions are described by exchange of Regge poles
- Describe $q\bar{q} \to \pi\pi$ amplitude as a pole in the complex angular momentum plane

$$A_\ell(s,u) = \frac{\beta(s)}{\ell - \alpha(u)}$$

Effects of Reggiezation

- Regge behavior achieved by summing over ladder diagrams





$$A(s,u) \simeq \frac{1}{s}\sum_{n=0}^{\infty}\frac{\left(\ln(s)\,L(u)\right)^n}{n!} = s^{-1+L(u)}$$

- Contributes infinite powers of $\ln(s)$ resulting in

$$A_\ell(s,u) = \frac{\beta(s)}{\ell - \alpha(u)} \rightarrow A(s,u) \sim (s)^{\alpha(u)}$$

Promoting Quarks to Spinors

- Spinor propagators: $\frac{1}{k^2 - m^2} \to \frac{1}{\hat{k} - m}$
- Spinor QED interaction: $\gamma q\bar{q} : 1 \to \gamma^\mu$
- Pion quark interaction: $\pi q\bar{q} : 1 \to \gamma^5, \gamma^\mu\gamma^5$

Same asymptotic behavior in all scenarios discussed in all scalar case, just more complicated numerators

$$F^\mu(s) \simeq \int d^4k \frac{\mathrm{Tr}[\ \gamma^5(\hat{k} + \hat{p}_2 + m_q)\gamma^\mu(\hat{k} - \hat{p}_3 + m_q)\gamma^5(\hat{k} + \mu)\ ]}{(k^2 - \mu^2)((k + p_2)^2 - m_q)^2((k - p_3)^2 - m_q^2)} \sim \frac{\ln^2 s}{s}(p_2 - p_3)^\mu$$

Scalar Quark Trajectory

$$A(s,u) \sim (s)^{\alpha(u)}, \quad A_\ell(s) = \frac{\beta(s)}{\ell - \boxed{\alpha(s)}}$$

scalar quark Regge trajectory

- Start with a scalar quark. Need to model $\alpha(s)$

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$$A(s,u) \sim (s)^{\alpha(u)}, \quad A_\ell(s) = \frac{\beta(s)}{\ell - \alpha(s)}$$

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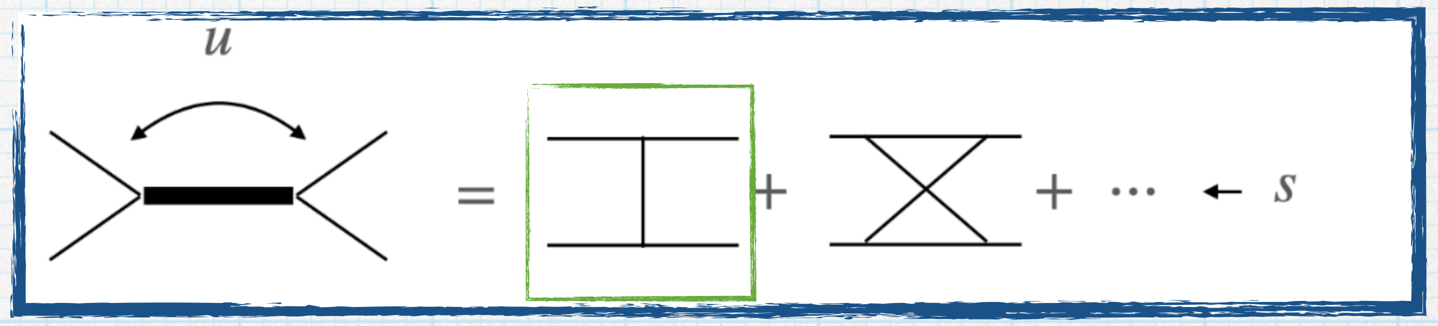
- Start with a scalar quark. Need to model $\alpha(s)$

- Construct amplitude by summing over ladder diagrams

$A_\ell(s) =$



$$= \frac{N_\ell(s)}{1 - \frac{s}{\pi}\int ds' \frac{\rho(s')N_\ell(s')}{s'(s'-s)}}$$

$$A_\ell^{(1)}(s) = g_0^2 N_\ell(s)$$

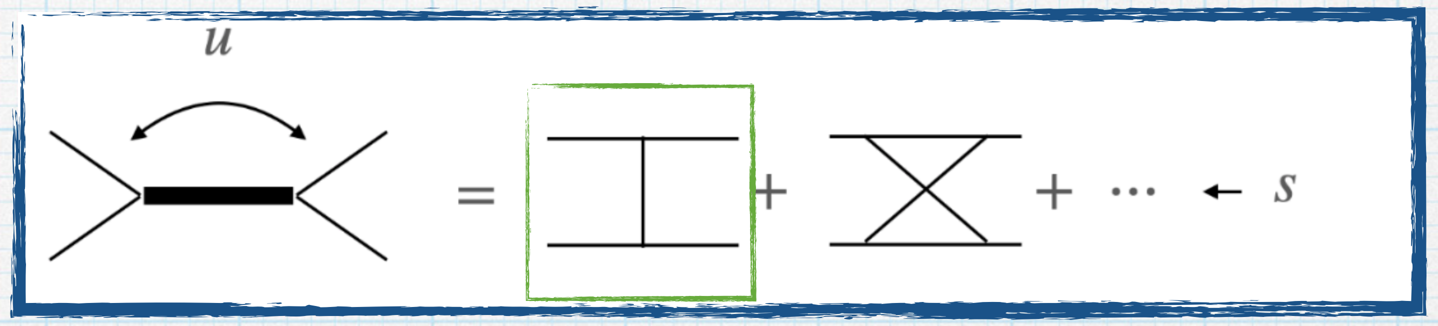
Scalar Quark Trajectory

$$A(s,u) \sim (s)^{\alpha(u)}, \quad A_\ell(s) = \frac{\beta(s)}{\ell - \alpha(s)}$$

scalar quark Regge trajectory

- Start with a scalar quark. Need to model $\alpha(s)$
- Construct amplitude by summing over ladder diagrams
- Multiply and divide by $(\ell + 1)$ to regularize $N_{\ell=-1}(s)$

$$A_\ell(s) =$$



$$= \frac{N_\ell(s)}{1 - \frac{s}{\pi}\int ds' \frac{\rho(s')N_\ell(s')}{s'(s'-s)}}$$

$$A_\ell^{(1)}(s) = g_0^2 N_\ell(s)$$

$$\hat{D}(\ell, s) = -1 + \frac{s}{\pi}\int ds' \frac{\rho(s')\hat{N}_\ell(s')}{s'(s'-s)}$$

$$A_\ell(s) \simeq \frac{\hat{N}_\ell(s)}{\ell - \hat{D}(\ell, s)}, \quad \alpha(s) = \hat{D}(\alpha(s), s), \quad \beta(s) = \frac{\hat{N}(\alpha(s), s)}{1 - \frac{d}{d\ell}\hat{D}(\ell, s)\big|_{\ell=\alpha(s)}}$$

Scalar Quark Trajectory

- Now trajectory can be solved numerically by iteration

$$A_\ell(s) \simeq \frac{\hat{N}_\ell(s)}{\ell - \hat{D}(\ell, s)}, \quad \alpha(s) = \hat{D}(\alpha(s), s), \quad \beta(s) = \frac{\hat{N}(\alpha(s), s)}{1 - \frac{d}{d\ell}\hat{D}(\ell, s)\,|_{\ell=\alpha(s)}}$$

$$\hat{D}(\ell, s) = -1 + \frac{s}{\pi}\int ds' \frac{\rho(s')\hat{N}_\ell(s')}{s'(s'-s)}$$

$$\alpha_{i-1}(u) = \text{guess}$$

$$\hat{N} = \hat{N}(\alpha_{i-1}(s), s)$$

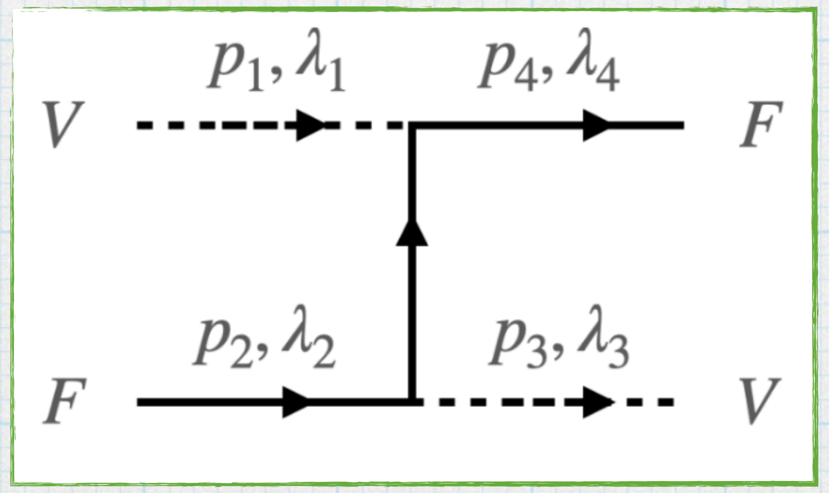
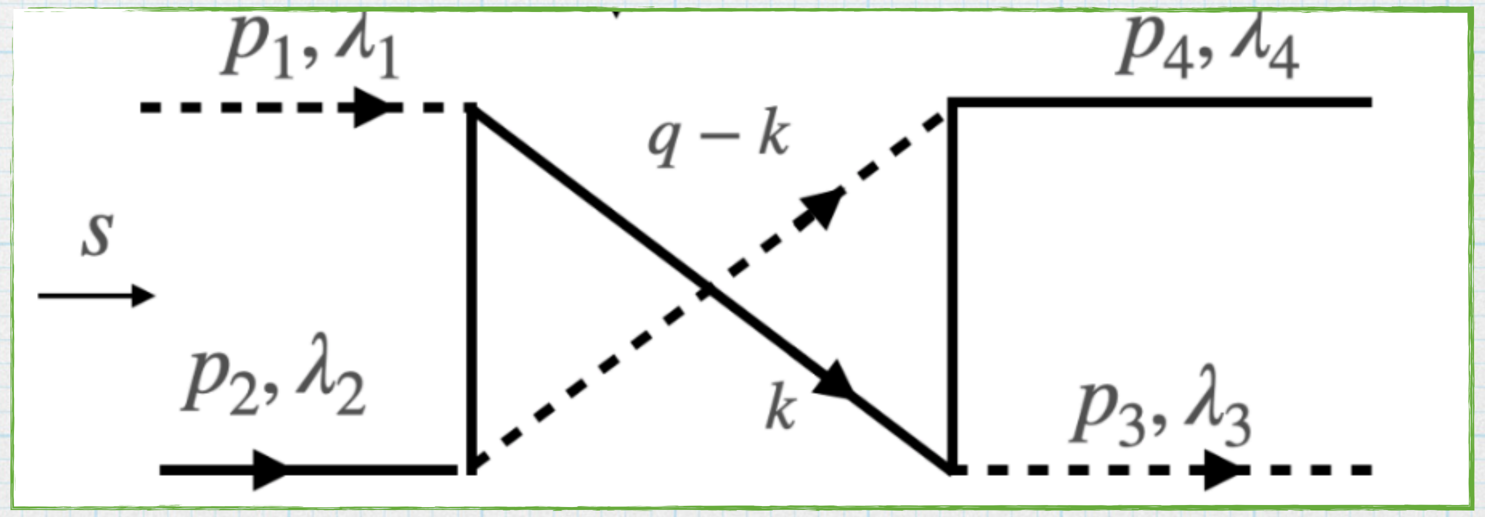
$$\beta = \beta(\alpha_{i-1}(s), s)$$

$$\mathcal{I}m[\alpha(s)]_i = \beta(\alpha_{i-1}(s), s)\rho(s)$$

$$\mathcal{R}e[\alpha(s)]_i = a_0 + \frac{s}{\pi}\int \frac{\mathcal{I}m[\alpha_{i-1}(s)]}{s'(s'-s)}$$

Adding Quark & Gluon Spin

- In principle the procedure for Reggiezing a spin ½ particle is the same
- Now need to consider parity, signature, and helicity
- Need a nonsense amplitude to get structure $\dfrac{1}{\ell - \alpha}$

$A_\ell(s) \rightarrow A_\ell^{\eta\sigma}(s, \lambda_1, \lambda_2, \lambda_3, \lambda_4) =$  $+$  $+ \ldots$

Outlook

- Complete model for quark Regge trajectory, apply Regge amplitude to model of pion FF
- Study asymptotics of pion FF with all non-perturbative modifications and compare to data
- Ideally we need higher energy data in the timelike region to compare our high energy prediction with.