Fate of the critical end point(s) in the large N_c limit

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Excited QCD 2022
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Francesco Giacosa, UJK Kielce

- Introduction: Large N_c limit
- Effective model at hand: ELSM
- The Phase diagram at large N_c
- Summary

Historically (G. 't Hooft 1974, Witten, ...), $1/N_c$ could be an expansion parameter when N_c is large. (Hope to having only planar diagrams.)

In the $N_c \to \infty$ limit eg.:

- Stable, noninteracting mesons and glue-balls (infinite number with fixed qn.) in the hadronic phase with $m \propto N_c^0$ masses.
- Baryon masses diverges as $m_B \propto N_c^1$.
- Hadronic phase built from noninteracting mesons and glueballs, energy density scales as $\propto N_c^0$
- Phase boundary to quark-gluon plasma at a temperature $\propto N_c^0$
- Energy density of quark-gluon phase N_c^2.
  $\Rightarrow$ First or second order phase transition expected.
- Quark loops are suppressed: the thermodynamics expected to became similar to Yang-Mills.
- Confined, quarkyonic phase may appears for large density

McLerran, Pisarski: **Nucl. Phys. A 796, 83-100 (2007)**

McLerran, Redlich, Sasaki: **Nucl. Phys. A 824, 86-100 (2009)**

In nature $\mathbf{N_c = 3}$ is realized, does it count as **large or not**?

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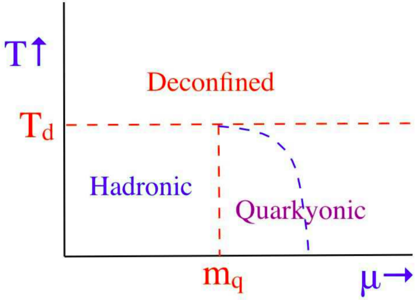

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In nature $\mathbf{N_c = 3}$ is realized, does it count as **large or not**?

Vector and axial vector meson **E**xtended Polyakov **L**inear **S**igma **M**odel. Effective model to study the phase diagram of strongly interacting matter at finite T and μ.

Phys. Rev. D 93, no. 11, 114014 (2016)

- **L**inear **S**igma **M**odel: "simple" quark-meson model, $N_f = 2+1$
- **E**xtended: Vector and Axial vector nonets
  (besides to Scalar and Pseudoscalar)
  Isospin symmetric case: 16 mesonic degrees of freedom.
- Polyakov: Polyakov loop variables give 2 order parameters Φ, $\bar{\Phi}$.
- Starting from the Lagrangian $\mathcal{L}_{\mathrm{LSM}} = \mathcal{L}_m + \mathcal{L}_Y$

Lagrangian with four nonets of meson fields eg: PS — π, K, f_0^L, f_0^H

Yukawa-type fermion-meson interaction for (pseudo)scalars

- $\mathcal{L}_m$ contains the dynamical, the symmetry breaking, and the meson-meson interaction terms.
    - $U(1)_A$ anomaly and explicit breaking of the chiral symmetry.
    - Each meson-meson terms upto 4th order that are allowed by the chiral symmetry.
- Constituent quarks ($N_f = 2 + 1$) in Yukawa Lagrangian

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu \partial_\mu - g_F(S - i\gamma_5 P)\right)\psi \qquad (1)$$

- SSB with nonzero vev. for scalar-isoscalar sector $\phi_N,\ \phi_S$.
  $\Rightarrow m_{u,d} = \frac{g_F}{2}\phi_N,\ m_s = \frac{g_F}{\sqrt{2}}\phi_S$ fermion masses in $\mathcal{L}_Y$.
- After $U(1)_A$ anomaly, ESB and SSB

$$SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A \rightarrow SU(2)_I \times U(1)_V$$

- Mean field level effective potential $\rightarrow$ the meson masses and the thermodynamics are calculated from this.

Thermodynamics: **Mean field level** effective potential:

- Classical potential.
- Fermionic one-loop correction with vanishing fluctuating mesonic fields.

$$\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - \mathrm{diag}(m_u, m_d, m_s)\right)\psi$$

  Functional integration over the fermionic fields.
  The momentum integrals are renormalized.

- Polyakov loop potential.

$$\Omega(T,\mu_q) = U_{Cl} + \Omega_{\bar{q}q}(T,\mu_q) + U_{\mathrm{Pol}}(T,\mu_q) \quad (2)$$

$$\Omega^{\mathrm{v}}_{\bar{q}q} = -2N_c \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} E_f(p),$$

$$\Omega^{\mathrm{T}}_{\bar{q}q}(T,\mu_q) = -2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}_c \Big[\ln\left(1 + L^{\dagger} e^{-\beta(E_f(p)-\mu_q)}\right) + \ln\left(1 + L e^{-\beta(E_f(p)+\mu_q)}\right) \Big]$$

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Field equations (FE):

$$\frac{\partial\Omega}{\partial\bar{\Phi}} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S} = 0 \tag{3}$$

Curvature meson masses:

$$M^2_{ab} = \left.\frac{\partial^2\Omega}{\partial\varphi_a\partial\varphi_b}\right|_{\{\varphi_i\}=0} \tag{4}$$

The scaling of the quark-meson model can be studied by rescaling the model parameters. Parameters $(N_c = 3)$ from

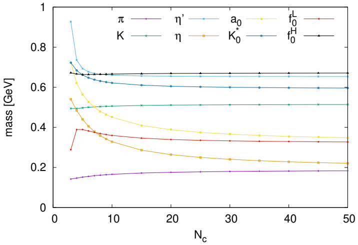
Phys. Rev. D 105, no.10, 103014 (2022)

By theoretical considerations

$m_0^2,\ m_1^2,\ \delta_S$	N_c^0
$g_1,\ g_2,\ g_f$	$1/\sqrt{N_c}$
$\lambda_2,\ h_2,\ h_3$	N_c^{-1}
$\lambda_1,\ h_1$	N_c^{-2}
c_1	$N_c^{-3/2}$
$h_{N/S}$	$\sqrt{N_c}$
g_F	$1/\sqrt{N_c}$

The meson condensates $(\phi_N,\ \phi_S)$ and other thermodyn. quantities are calculated via the FEs

The vacuum meson masses $\propto N_c^0$



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Usually used Potentials works for $N_c = 3$.

Model proposed in Lo, Redlich, Sasaki: **Phys. Rev. D 103, 074026 (2021)**

$$U_{\text{Pol}} = U_{\text{conf}} + U_{\text{glue}} \tag{5}$$

The terms favoring the confining and the deconfined minima, resp.

$$U_{\text{conf}} = -\frac{b}{2} T \ln H, \qquad U_{\text{glue}} = n_{\text{glue}} T \int \frac{d^3p}{(2\pi)^3} \operatorname{Tr} \ln \left(\mathbb{1}_A - L_A e^{-\beta E_A(p)} \right) \tag{6}$$

There are too many parameters at large N_c

- Polyakov-loop operator: $L \in SU(N_c)$, and parameters: $\Phi_k = \frac{1}{N_c}\mathrm{Tr}(L^k)$
- At $N_c = 2k(+1)$ one has $N_c - 1$ independent Polyakov-loop parameters $\Phi_1, \ldots, \Phi_k, \bar{\Phi}_1, \ldots, \bar{\Phi}_{k-1}, (\bar{\Phi}_k)$ $\Rightarrow$ $\mathbf{N_c + 1}$ **Field equations**

Uniform eigenvalue ansatz

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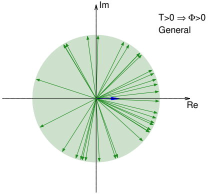
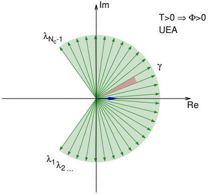
Way to reduce the number of d.o.f.: **Uniform eigenvalue ansatz**

$$L = \begin{pmatrix} \lambda_1 & 0 & & \\ 0 & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_{N_c} \end{pmatrix}$$





Exact at $N_c \leq 3$ for $\mu_q = 0$.

Dumitru, Guo, Hidaka: **Phys. Rev. D 86, 105017 (2012)**

Lo, Redlich, Sasaki: **Phys. Rev. D 103, 074026 (2021)**

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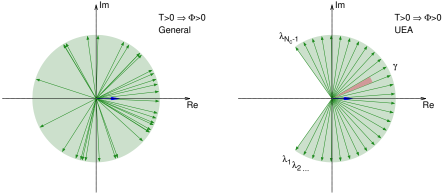

One can use γ as the only d.o.f.

Dumitru, Guo, Hidaka: **Phys. Rev. D 86, 105017 (2012)**

Lo, Redlich, Sasaki: **Phys. Rev. D 103, 074026 (2021)**

For **T=0**

- $\Phi \equiv 0 \quad \Rightarrow \quad$ One can use only the quark-meson model.
- Only two FEs for the meson condensates ϕ_N and ϕ_S
- The first order transition along μ_q smoothed to crossover already at $N_c = 4$

For **T>0**

- Need for the Polyakov-loop
- Three FEs for ϕ_N, ϕ_S and γ i.e. Φ when using UEA
- (Would be $N_c + 1$ without the UEA)

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$$\Omega|_{\mathrm{Pol}} = -\frac{b}{2} T \;\ln H \; + n_{\mathrm{glue}} T \int \frac{d^3p}{(2\pi)^3} \ln g_A \; - 2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \Big[\ln g_f^+ \; + \; \ln g_f^- \Big]$$

For **T=0**

$$\Omega(T=0,\mu_q) = U_{Cl} + \Omega_{\bar{q}q}(T=0,\mu_q)$$

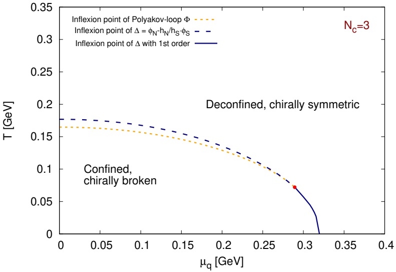
$\propto N_c^0$ $\qquad\qquad$ $\propto N_c^1$

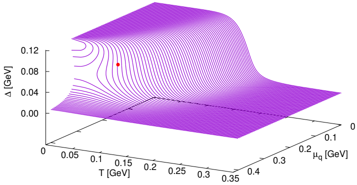
For **T>0**

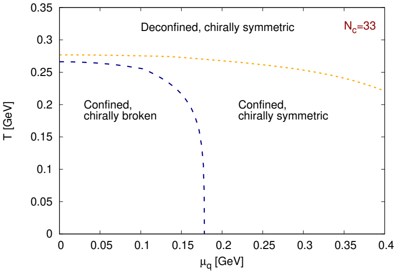
$\ln \mathrm{Det}(U_A) \propto N_c^2$ $\qquad\qquad$ $\ln \mathrm{Det}(U_F) \propto N_c^1$

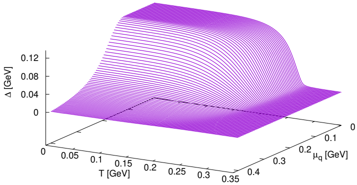
$$\Omega|_{\mathrm{Pol}} = -\frac{b}{2} T \ln g'_A + n_{\mathrm{glue}} T \int \frac{d^3p}{(2\pi)^3} \ln g_A - 2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left[\ln g_f^+ + \ln g_f^- \right]$$

Phase diagram for growing N_c









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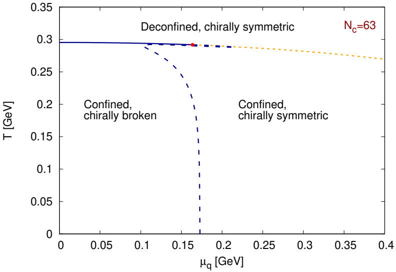
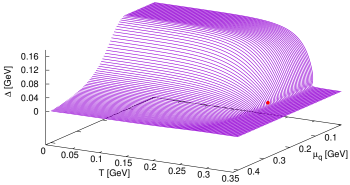
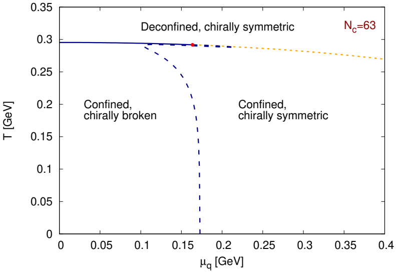
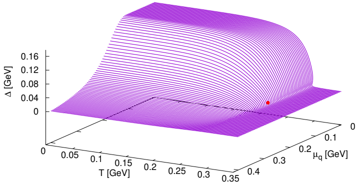
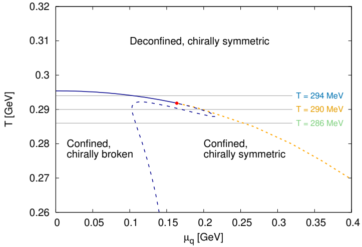
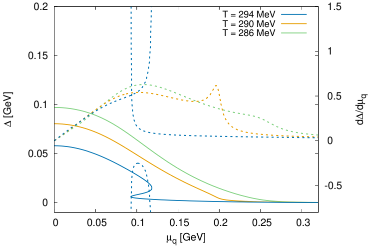




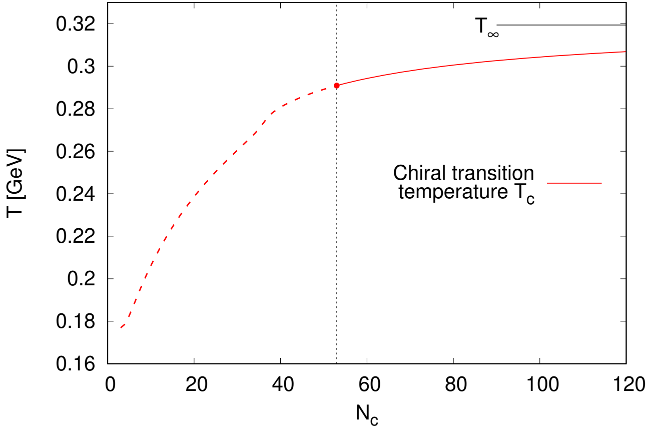

Deconfined, chirally symmetric
N_c=63
Confined,
chirally broken
Confined,
chirally symmetric
T [GeV]
μ_q [GeV]
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4


Δ [GeV]
0.16
0.12
0.08
0.04
0
T [GeV]
0.05
0.1
0.15
0.2
0.25
0.3
0.35
μ_q [GeV]
0
0.1
0.2
0.3
0.4


Deconfined, chirally symmetric
T = 294 MeV
T = 290 MeV
T = 286 MeV
Confined,
chirally broken
Confined,
chirally symmetric
T [GeV]
μ_q [GeV]
0.32
0.31
0.3
0.29
0.28
0.27
0.26
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

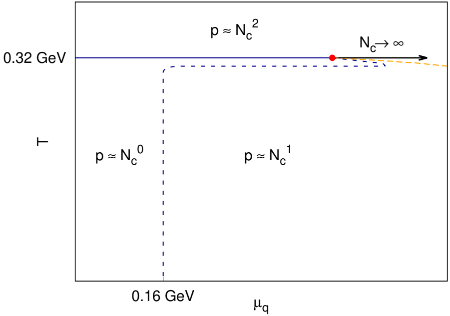

T = 294 MeV
T = 290 MeV
T = 286 MeV
Δ [GeV]
dΔ/dμ_q
μ_q [GeV]
0.2
0.15
0.1
0.05
0
1.5
1
0.5
0
-0.5
0
0.05
0.1
0.15
0.2
0.25
0.3

Saturation of $T_c(\mu_q = 0, N_c \to \infty)$ (below) and $\mu_{q,c}(T = 0, N_c \to \infty)$



Defining the pressure as

$$p(T,\mu_q) = -\left(\Omega(T,\mu_q,\phi_{N/S}(T,\mu_q),\gamma(T,\mu_q)) - \Omega(T,\mu_q,\phi_{N/S}(0,0)\gamma(0,0))\right)$$



Scaling by dominant d.o.f.:

- mesons $\propto N_c^0$
- quarks $\propto N_c^1$
- gluons $\propto N_c^2$

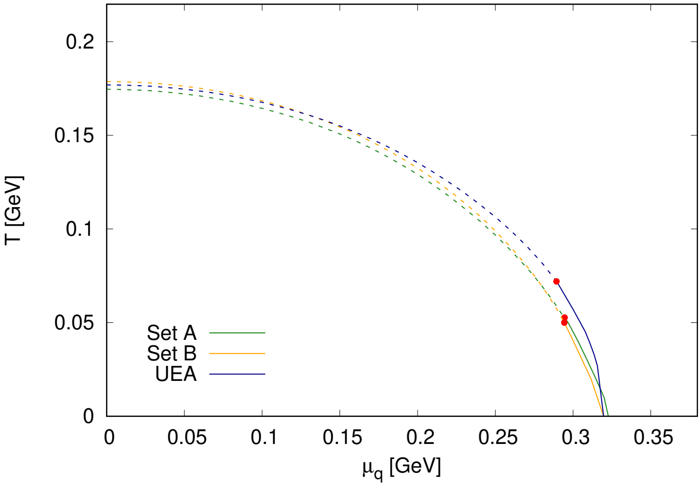
A confined, chir. restored and $p \propto N_c^1$ quarkyonic-like phase appears

Summary

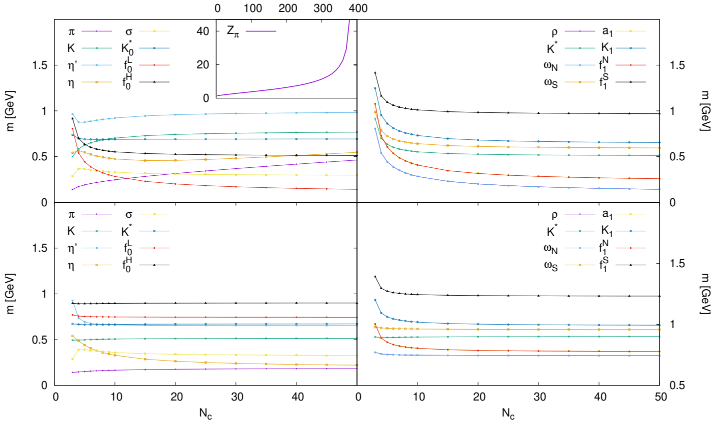
- We investigated the Large N_c limit of an extended Polyakov quark-meson model.
- The $N_c = 3$ critical endpoint rapidly disappears, leaving a crossover on the whole phase boundary.
- At $N_c = 53$ a new CEP appears along the temperature axis giving rise
  to a first order line.
- The crossover of the chiral restoration and the deconfinement separates, leaving
  a confined, but chirally symmetric phase.
- For Large N_c three phases can be separated:
  - A confined and chirally broken phase, dominated by meson, therefore having $p \propto N_c^0$
  - A deconfined and chirally symmetric phase with $p \propto N_c^2$
  - A confined but chirally restored, quarkonia-like phase with $p \propto N_c^0$
- Publication on the way, arXiv:2209.09568

Thank you!

Backup: Phase diagram $N_c = 3$ comparison



Backup: masses



$N_c - 1$ group angles of the Cartan subgroup of $SU(N_c)$

$$\vec{q} \equiv (q_1, \dots, q_{N_c}) = \sum_{j=1}^{N_c-1} \gamma_j \vec{v}_j \ ,$$

Keeping only $\gamma_1 (\equiv \gamma) \neq 0$

$$L = \mathrm{diag}\left(e^{-i\gamma}, e^{-i\left(1-\frac{2}{N_c-1}\right)\gamma}, e^{-i\left(1-2\frac{2}{N_c-1}\right)\gamma}, \dots, \right.$$
$$\left. (e^0), \dots, e^{i\left(1-2\frac{2}{N_c-1}\right)\gamma}, e^{i\left(1-\frac{2}{N_c-1}\right)\gamma}, e^{i\gamma} \right),$$

e^0 only for odd N_c. For example, for $N_c = 6$ and 7

$$L = \mathrm{diag}\left(e^{-i\gamma}, e^{-i3\gamma/5}, e^{-i\gamma/5}, e^{i\gamma/5}, e^{i3\gamma/5}, e^{i\gamma} \right),$$
$$L = \mathrm{diag}\left(e^{-i\gamma}, e^{-i2\gamma/3}, e^{-i\gamma/3}, e^{0}, e^{i\gamma/3}, e^{i2\gamma/3}, e^{i\gamma} \right),$$

Backup: Fermion determinant

The thermal/matter part of the fermion determinant reads as

$$\Omega_{\bar{q}q}^{(0)\mathrm{T}}(T,\mu_q) = -2T\sum_f \int \frac{d^3p}{(2\pi)^3}\left[\ln g_f^+(p) + \ln g_f^-(p)\right]$$

where one can expand

$$\begin{aligned}
g^+ = \mathrm{Det}_c\left[\mathbf{1}_{N_c} + L^\dagger e^{-(E-\mu)/T}\right] =& 1 + e^{-N_c(E_p-\mu)/T} \\
&+ N_c\left[\bar{\Phi}e^{-(E_p-\mu)/T} + \Phi e^{-(N_c-1)(E_p-\mu)/T}\right] \\
&+ \frac{1}{2}\left(N_c^2\bar{\Phi}^2 - N_c\bar{\Phi}_2\right)e^{-2(E-\mu)/T} \\
&+ \frac{1}{2}\left(N_c^2\Phi^2 - N_c\Phi_2\right)e^{-(N_c-2)(E-\mu)/T} \\
&+ [\text{terms with 3 to Nc-3 phases}],
\end{aligned}$$

and g^- differs only in changing $\bar{\Phi} \leftrightarrow \Phi$ and $-\mu \to +\mu$.