Diquark properties from lattice QCD

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Excited QCD 2022
(virtual contribution)

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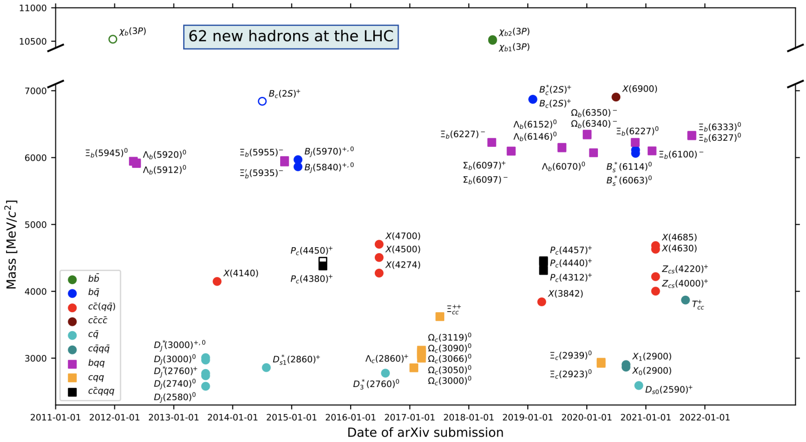
based on: [2203.16583][2203.03230]
JHEP 05 (2022) 062 [2106.09080]

Heavy spectrum - a success story turned challenge to theory

- Pre B-factories: Heavy spectrum is success story
  - predicted/measured masses agree, potential model works, OZI-rule applies w/o exceptions
- In 2003: $X(3872) \rightsquigarrow d\bar{u}c\bar{c}$ discovered at Belle
- Since then many exotic states and especially $\mathcal{O}(12)$ heavy 4-/5-quark states observed

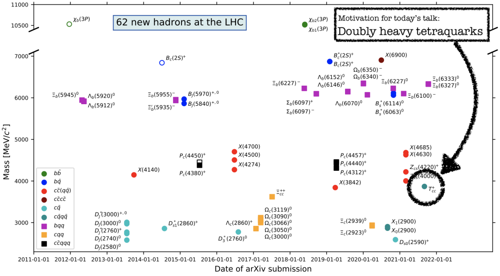


(LHCb collaboration, LHCb-FIGURE-2021- 001, 2021)

Heavy spectrum - a precision tool

Many new and exotic hadrons observed, e.g. 62 at the LHC

- 4-/5-quark states not expected in quark models.
- Many predicted quark model states not found.



(LHCb collaboration, LHCb-FIGURE-2021- 001, 2021)

… many not explained in theory

QCD often approximated in models

⇝ many extensions possible

⇝ many interpretations

⇝ often contradictory statements

model	building blocks
"plain"	$q_{(i,c)}$, $\bar{q}_{(i,c)}$
diquark	$[qq]_{(i,j,c)}$ & $q/\bar{q}$
triquark	$[qq\bar{q}]_{(i,j,k,c)}$ & $q/\bar{q}$
hydro-onium	$[Q\bar{Q}]_{(i,j)}$, $[q\bar{q}]_{(i,j)}$, $[qqq]_{(i,j,k)}$
molecular	$[Q\bar{q}]_{(i,j)}$, $[q\bar{Q}]_{(i,j)}$, $[qqQ]_{(i,j,k)}$, …

One goal: Non-perturbative insights into exotic hadrons in full QCD

A new family of tetraquarks? - observation of T_{cc} at LHCb

Narrow state observed in $D^0 D^0 \pi^+$

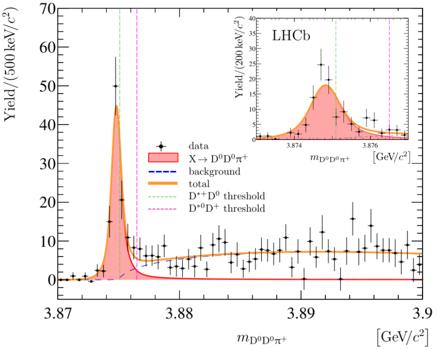
- Fitted to P-wave BW
- $\delta m = -273 \pm 61 \pm 5^{+11}_{-14} keV/c^2$ below $D^0 D^{*+}$ threshold
- $\Gamma = 410 \pm 165 \pm 43^{+18}_{-38} keV$

consistent with $cc\bar{u}\bar{d}$ tetraquark

- Possible family of states: $bc\bar{u}\bar{d}$, $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$, ...
- QN: $I(J^P) = 0(1^+)$
- Recent discussion in theory, both in pheno and lattice
  ⇝ predictions, binding mechanism

$B_{T_{cc}} = 0.3$MeV

⇝ LHCb-PAPER-2021-031



In the following:

- new work on diquarks as possible effective d.o.f's in QCD
- was motivated by our studies of doubly heavy tetraquarks

Phys.Rev.D 102 (2020) 114506 [2006.14294]
Phys.Rev.D 99 (2019) 5, 054505 [1810.10550]
Phys.Rev.Lett. 118 (2017) 14, 142001 [1607.05214]

Diquarks - attractive building blocks for ordinary and exotic hadrons

Diquarks - an attractive concept

> *"The concept of diquarks is almost as old as the quark model, and actually predates QCD [1]"* ⇝ arXiv:2203.16583; [1] PR 155, 1601 (1967)

- Successful for low-lying baryons and exotic hadrons.
  - Well founded in QCD with many predictions.
  - But, experimental evidence has been elusive.
- Light diquarks:
  - special "good" $(\bar{3}_F, \bar{3}_c, J^P = 0^+)$ configuration
  - quarks on "good" diquarks attract each other
  - large mass splitting in good, bad and not-even-bad
  - non-vanishing size or compact?
- HQSS-limit: A diquark acts as an antiquark $[QQ] \leftrightarrow \bar{Q}$.

⇝ currently one motivation for T_{QQ}-type hadrons, next slide

3 types of diquark:
good, bad and not-even bad

Diquark operator:

$$D_\Gamma = q^c C \Gamma q'$$

⇝ c, C =charge conjugation

⇝ Γ acts on Dirac space

J^P	C	F	Op: Γ
0^+	$\bar{3}$	$\bar{3}$	γ_5, $\gamma_0\gamma_5$
1^+	$\bar{3}$	6	γ_i, σ_{i0}
0^-	$\bar{3}$	6	$\mathbb{1}$, γ_0
1^-	$\bar{3}$	$\bar{3}$	$\gamma_i\gamma_5$, σ_{ij}

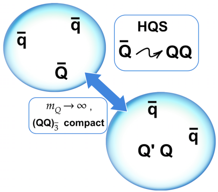
The case for doubly heavy tetraquarks - Diquarks and $qq'\bar{Q}\bar{Q}'$

Revisit ideas for stable multiquarks based on diquarks

$\rightsquigarrow$ Ader et al. ('82); Manohar, Wise ('93); ...

- Effective $q - q$ interaction in "good" diquarks
- HQS ($Q \sim b$) relates $[\bar{Q}\bar{Q}]_3 \leftrightarrow Q$
- $[\bar{Q}\bar{Q}]_3^{m_Q \to \infty}$ becomes compact
- Combine (HH)+(ll) diquarks into tetraquarks:

$$\boxed{\{qq'\}[\bar{Q}\bar{Q}'] = (qC\gamma_5 q')(\bar{Q}C\gamma_i\bar{Q}') := T_{QQ'}}$$

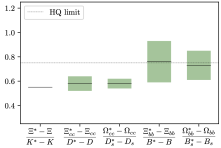


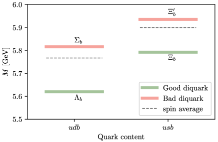
Model expectations:

- Wave-fct, prefer $J^P(T_{QQ'}) = 1^+$
- HQS, prefer $\bar{b}\bar{b}$
  $\Rightarrow$ heavier $[\bar{Q}\bar{Q}']$ more binding
- Diquark, prefer $\{ud\}$ type
  $\Rightarrow$ lighter $\{qq'\}$ more binding

Binding opportunity in model

- PDG mesons/baryons provide constraints





Doubly heavy tetraquarks - deeply bound $J^P = 1^+$ T_{bb} and $T_{bb}^{\ell s}$

HQ spin symmetry - good diquark (HQS-GDQ) picture predictions:

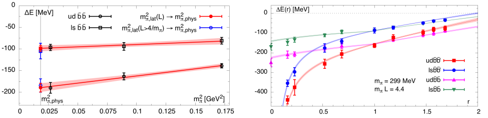
- $J^P = 1^+$ *ground state* tetraquark
- Deeper binding with:
  $\rightarrow$ heavier Q in $[\bar{Q}'\bar{Q}]$
  $\rightarrow$ lighter q in $\{qq'\}$

All observed on the Lattice!

$bb\bar{q}\bar{q}'$ **are a focal point** $\rightarrow$ All efforts observe deeply bound $bb\bar{u}\bar{d}$

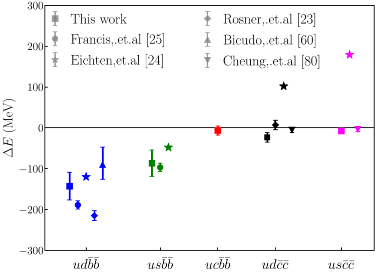
.
· Junnarkar, Mathur, Padmanath ('18)
· Leskovec, Meinel, Plaumer, Wagner ('19)
· HadronSpectrum Coll. ('17)
· Mohanta, Basak ('20)
· Colquhoun, AF, Hudspith, Lewis, Maltman ('17, '18, '20)

Lattice energies give robust predictions



⇝AF et al. ('17, '18)

Community overview



⇝Mathur et al. ('19)

Diquarks - Towards understanding $T^{qq'}_{QQ'}$ and other hadrons

HQS-GDQ predictions all observed on the Lattice!

Many open questions:

- Binding mechanism? Flavor dependence?
- Role of diquarks?
- Structure of $T^{qq'}_{QQ'}$?
- **Consequences for other hadrons?**

HQS-GDQ picture in $T^{qq'}_{QQ'}$ is just one example where diquarks play a crucial role in understanding the hadron spectrum. ⇝ [2203.16583][2203.03230]

Need for fully non-perturbative insight

Towards a clearer understanding and footing in QCD using lattice calculations

1. **diquark formalism:** Find gauge invariant probe
2. **diquark spectrum:** Fundamental properties
3. **diquark structure:** Probe $q - q$ interaction

Surveyed $T^{qq'}_{QQ'}$ candidates

observed (>1 group)
no deep binding
observed (1 group)
not confirmed (>1 group)

channel	deeply bound
$J^P = 1^+$	$bb\bar{u}\bar{d}$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $bc\bar{\ell}\bar{s}$
	$bs\bar{u}\bar{d}$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $bb\bar{s}\bar{c}$
	$cc\bar{u}\bar{d}$ $cc\bar{\ell}\bar{s}$
	$bb\bar{b}\bar{b}$
$J^P = 0^+$	$bb\bar{u}\bar{u}$ $cc\bar{u}\bar{u}$
	$bb\bar{u}\bar{d}$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $bc\bar{\ell}\bar{s}$
	$bb\bar{s}\bar{s}$ $cc\bar{s}\bar{s}$
	$bs\bar{u}\bar{d}$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $bb\bar{s}\bar{c}$
	$bb\bar{c}\bar{c}$ $cc\bar{u}\bar{d}$
	$bb\bar{b}\bar{b}$

Diquarks on the lattice

Diquarks on the lattice - a gauge invariant probe

- A problem for the lattice is that diquarks are colored, i.e. not-gauge invariant.
  - Could fix a gauge, but then properties are gauge-dependent (masses, sizes,...)

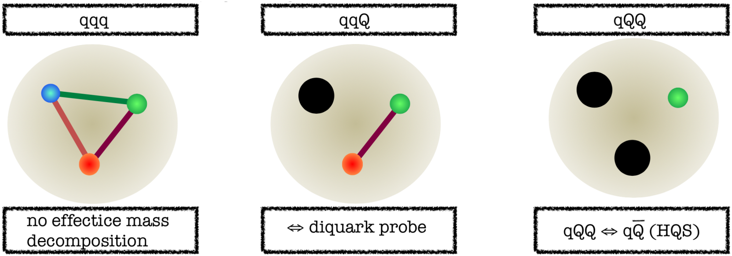
⇝ lattice and Dyson-Schwinger, see e.g. [15-20] in 2106.09080

- **Alternative:** Static spectator quark Q ($m_Q \to \infty$) cancels in mass differences.
  - Diquark properties exposed in a gauge-invariant way.

⇝ hep-lat/0510082, hep-lat/0509113, hep-lat/0609004, arxiv:1012.2353

$$\boxed{C_\Gamma(t) \sim \exp\left[-t\left(m_{D_\Gamma} + m_Q + \mathcal{O}(m_Q^{-1})\right)\right]}$$

⇝ $t \to$ large, $m_Q \to$ large



⇝ picture of baryons from Hosaka, 2013

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$\rightsquigarrow$ $t \to$ large, $m_Q \to$ large

- **Lattice correlator:** Diquark embedded in a static-light-light baryon

$$C_\Gamma(t) = \sum_{\vec{x}} \left\langle [D_\Gamma Q](\vec{x}, t)\ [D_\Gamma Q]^\dagger(\vec{0}, 0)\right\rangle$$

$\rightsquigarrow$ static quark=Q and $D_\Gamma = q^c C\Gamma q$

$\rightsquigarrow$ flavor combinations ud, ℓs, ss'

$\rightsquigarrow$ static-light mesons $[\bar{Q}\Gamma q]$

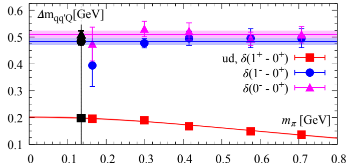
Clearer understanding by studying the diquark ...

1. **spectrum:** *[diquark] mass differences are fundamental characteristics of QCD* `(Jaffe '05, arXiv:hep-ph/0409065)`
2. **spatial correlations:** *study attraction and special status of the "good" diquark*
3. **structure:** *estimate size and shape of the "good" diquark*

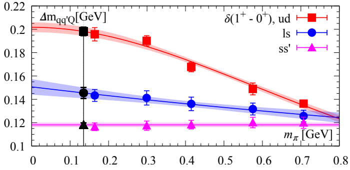
Diquark spectroscopy

Lattice spectroscopy - diquark-diquark differences

ud 0^+ versus 1^+, 0^- and 1^-



$(1^+ - 0^+)_{qq'}$ splitting



We consider mass differences of $qq'Q$ baryons:

$$C_{\Gamma}^{qq'Q}(t) - C_{\gamma_5}^{qq'Q}(t)$$

$\rightsquigarrow$ Q drops out

$\rightsquigarrow$ measures diquark-diquark mass difference

Bad-good diquark splitting:

- Special status of good diquark observed
- Good 0^+ ud diquark lies lowest in the spectrum
- Bad 1^+ ud diquark 100-200 MeV above
- 0^- and 1^- ud diquarks ~ 0.5 GeV above
- Pattern repeated in ℓs and ss'

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- Chiral limit: $\sim$ const
- Heavy-quark limit: decreases $\sim 1/(m_{q_1} m_{q_2})$, with $m_\pi \sim (m_{q_1} + m_{q_2})$

$$\delta(1^+ - \ 0^+)_{q_1 q_2} = A / \left[1 + \left(m_\pi/B\right)^{n \in 0,1,2}\right]$$

Lattice spectroscopy - diquark-quark differences

We consider mass differences of a $qq'Q$ baryon and a light-static meson:

$$C^{qq'Q}_{\Gamma=\gamma_5}(t) - C^{q'\bar{Q}}_{\gamma_5}(t)$$

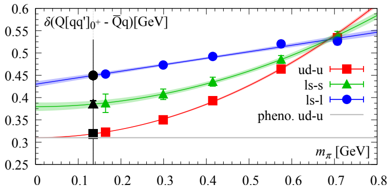
$\rightsquigarrow$ Q drops out
$\rightsquigarrow$ diquark-quark mass difference

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- Chiral vs. heavy-quark limiting behaviours, as before

$$\delta(Q[q_1 q_2]_{0^+} - \bar{Q}q_2) = C\left[1 + (m_\pi/D)^{n\in 0,1,2}\right]$$

$Qqq' - \bar{Q}q'$ splittings



Diquark-quark splitting:

- Established mass differences between a good diquark and an [anti]quark
- May prove useful in identifying favourable tetra-, pentaquark channels
- Omits possible distortions through additional light quarks, Pauli-blocking, spin-spin interactions ...

Diquark spectroscopy - comparing results

- We want to compare our results with phenomenology

$\rightsquigarrow$ more details in extra info slides

  - Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
  - For pheno estimates combine charm and bottom hadron masses such that leading $\mathcal{O}(1/m_Q)$ ($Q = c, b$) cancel

- The main spectroscopy results are summarised as:

All in [MeV]	$\delta E_{\text{lat}}(m_\pi^{\text{phys}})$	δE_{pheno}	$\delta E_{\text{pheno}}^{\text{bottom}}$	$\delta E_{\text{pheno}}^{\text{charm}}$
$\delta(1^+ - 0^+)_{ud}$	198(4)	206(4)	206	210
$\delta(1^+ - 0^+)_{\ell s}$	145(5)	145(3)	145	148
$\delta(1^+ - 0^+)_{ss'}$	118(2)			
$\delta(Q[ud]_{0^+} - \bar{Q}u)$	319(1)	306(7)	306	313
$\delta(Q[\ell s]_{0^+} - \bar{Q}s)$	385(9)	397(1)	397	398
$\delta(Q[\ell s]_{0^+} - \bar{Q}\ell)$	450(6)			

$\rightsquigarrow$ use the bottom estimate for static

$\rightsquigarrow$ use charm-bottom difference as estimate for deviation from static

$\Rightarrow \lesssim \mathcal{O}(7)$MeV deviation

- Overall, very good agreement observed.

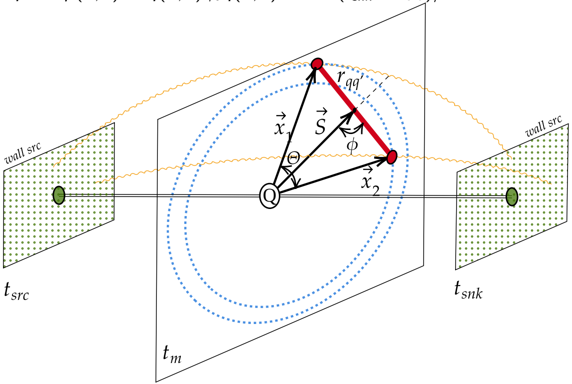
Diquark structure

Diquarks - spatial correlations

We access (good) diquark structure information through density-density correlations:

$$C_\Gamma^{dd}(\vec{x}_1, \vec{x}_2, t) = \left\langle \mathcal{O}_\Gamma(\vec{0}, 2t)\ \rho(\vec{x}_1, t)\rho(\vec{x}_2, t)\ \mathcal{O}_\Gamma^\dagger(\vec{0}, 0) \right\rangle := \rho_2(r_{ud}, S, \phi; \Gamma)$$

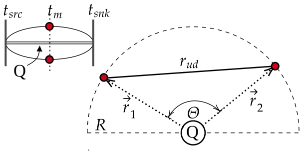
$\rightsquigarrow$ $\mathcal{O}_\Gamma = q^c C\Gamma q$ and $\rho(\vec{x}, t) = \bar{q}(\vec{x}, t)\gamma_0 q(\vec{x}, t)$, $t_m = (t_{snk} + t_{src})/2$ to minimize excited states



Main tool: Correlations between two light quarks' relative positions to the static quark.
S, r_{ud} fixed: Distance between static quark Q and closer of the two light quarks q, q' is

- Minimized for $\phi = \pi$, possible disruption due to Q is largest
- Maximized for $\phi = \pi/2$, possible disruption due to Q is smallest

Good diquark attraction



Setting $\phi = \pi/2$:

- $|\vec{x}_1| = |\vec{x}_2| = R$, use R, Θ:

$$\rho_2^\perp(R,\Theta) = \rho_2(r_{ud}, S, \pi/2)$$

- Attraction visible through increase in $\rho_2^\perp$ for small Θ at any fixed R

Two limiting cases for the two quarks:

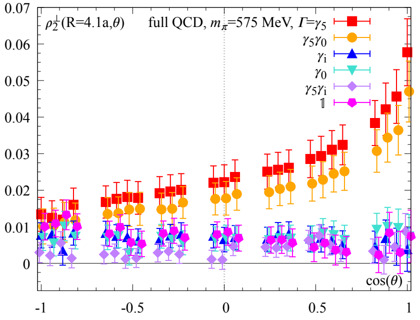
- $\cos(\Theta) = 1$ on top of each other
- $\cos(\Theta) = -1$ opposite each other

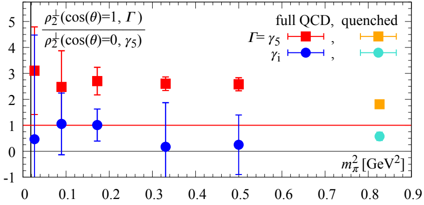
"Lift" as qualitative criterion:

$$\frac{\rho_2^\perp(R,\Theta=0,\Gamma)}{\rho_2^\perp(R,\Theta=\pi/2,\gamma_5)}$$

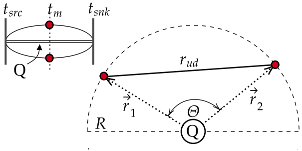
Increase observed in good diquark only

Spatial correlation over Θ





Good diquark size



- Distance between quarks:

  $r_{ud} = R\sqrt{2\,(1-\cos(\Theta))}$

  $\rightsquigarrow$ different visualisation

- $\rho_2^{\perp}(R, r_{ud}) \sim \exp(-r_{ud}/r_0)$

  $\rightsquigarrow$ "characteristic size" r_0

- Need to control:
  - interference from Q

    $\rightsquigarrow$ we limit analysis to $r_{ud} < R$
  - periodicity effects

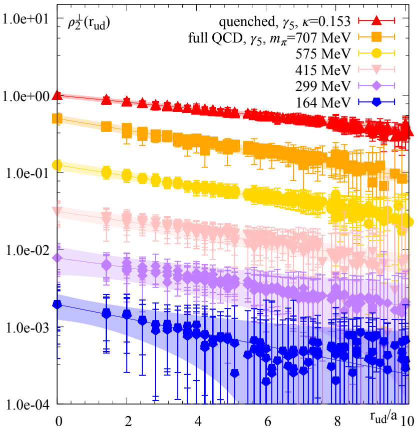
    $\rightsquigarrow$ in practice we find $L = 5r_0$

- Further checks:

  $A(R, r_{ud} = 0) \sim \exp(-R/R_0)$

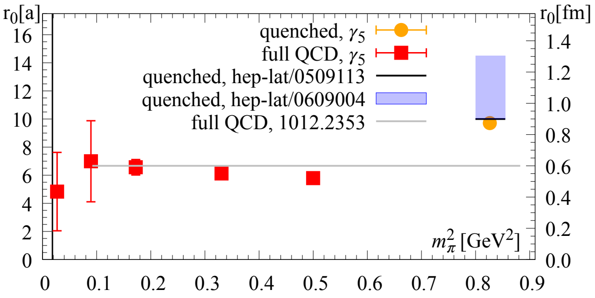
Data well described by (single) exponential Ansatz

Spatial correlation over r_{ud}



- $r_{ud} = 0$ normalised, offset for each m_π
- all R shown simultaneously
- combined fits over $\forall R$ with shared r_0

Size dependence $r_0(m_\pi)$



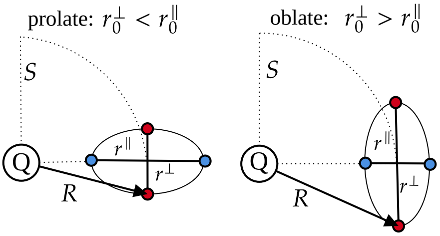
Good diquark size:

- Agreement w/ prev. quenched and dynamical
- Refinement through our results
- $r_0 \simeq \mathcal{O}(0.6)$fm weak m_π dependence
  $\rightsquigarrow \sim r_{\text{meson, baryon}}$, arXiv:1604.02891

$r_0(m_\pi)$ dependence:

- $m_{q,q'} \uparrow$ should produce more compact object
- But, diquark attraction$\downarrow$ works opposite
- Former effect dominates at large m_π?
- But, in quenched diquarks definitely larger...

Shape of good diquarks - studying wavefunction "oblateness"



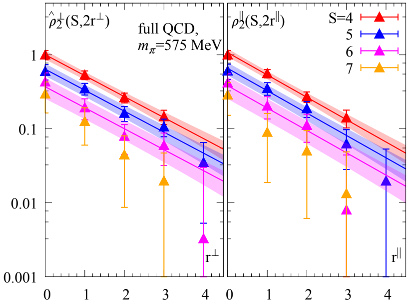
Tangential and radial spatial correlation decay

As opposed to before $R \neq$fixed:

- $\phi = \pi$: radial correlation, size $\rightsquigarrow r_0^\parallel$
- $\phi = \pi/2$: tangential correlation, size $\rightsquigarrow r_0^\perp$
- $r_0^\perp / r_0^\parallel$ gives information on shape:
  - $= 1$, spherical
  - $\neq 1$, prolate/oblate

- Probe $J = 0$ nature of good diquark (spherical, S-wave expectation)
- Diquark polarisation through static quark?

Oblateness results at $m_\pi = 575$MeV



- Goal:
  - $r_0^{\perp}$, $r_0^{||}$ at fixed S

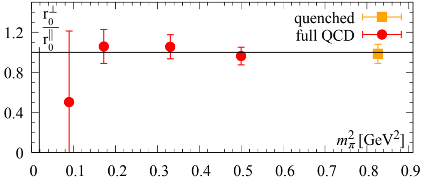
Technical issue:

  - ($||$) as before: $R = S$
  - ($\perp$) different: $R = \sqrt{(r^{\perp})^2 + S^2}$

Solution:

  - Introduce "nuisance" parameter R_0
  - Adjusted in figure
  - Parallel lines $\rightsquigarrow$ $r_0^{\perp} = r_0^{||}$

Shape dependence $r_0^{\perp}/r_0^{||}(m_\pi)$



- $r_0^{\perp}/r_0^{||}(m_\pi)$ dependence:
  - Ratio $\simeq 1$ for all m_π $\Rightarrow$ spherical
  - Consistent w/ scalar, $J = 0$, shape
  - No diquark polarisation through Q observed

Summary - Diquarks on the lattice

Gauge invariant approach to diquarks in $n_f = 2 + 1$ lattice QCD

- Lattice setup with short chiral extrapolations, continuum limit still required

Diquark spectroscopy

- Special status of "good" diquark confirmed, attraction of 198(4)MeV over "bad"
- Chiral and flavor dependence modelled through simple Ansatz
- Very good agreement with phenomenological estimates

Diquark structure

- $q - q$ attraction in good diquark induces compact spatial correlation
- Good diquark size $r_0 \simeq \mathcal{O}(0.6)\text{fm} \sim r_{\text{meson, baryon}}$, weakly m_π dependent
- Good diquark shape appears nearly spherical

Outlook

- Results provide clear, quantitative support for the good diquark picture
- Hope to refine diquark model parameters
- Insights for studies of exotic tetraquarks (esp. doubly heavy), heavy-baryons, etc.
- Refinement towards diquarks in light baryons? Tetraquark diquark content? ...

Thank you for your attention.

Further material

A gauge invariant probe - lattice calculation details

- **Lattice correlator:** Diquark embedded in a static-light-light baryon

$$C_\Gamma(t) = \sum_{\vec{x}} \Big\langle [D_\Gamma Q](\vec{x},t)\ [D_\Gamma Q]^\dagger(\vec{0},0) \Big\rangle$$

$\rightsquigarrow$ static quark$=Q$ and $D_\Gamma = q^c C\Gamma q$
$\rightsquigarrow$ flavor combinations ud, ℓs, ss'
$\rightsquigarrow$ static-light mesons $[\bar{Q}\Gamma q]$

setting up on the lattice - we recycle

- $n_f = 2+1$ full QCD, $32^3 \times 64$, $a = 0.090$fm, $a^{-1} = 2.194$GeV (PACS-CS gauges)
- $\boxed{m_\pi = 164, 299, 415, 575, 707\,\mathrm{MeV}}$, $m_s \simeq m_s^{\mathrm{phys}}$, propagators re-used from before
- Quenched gauge $a \simeq 0.1$fm, $\boxed{m_\pi^{\mathrm{valence}} = 909\,\mathrm{MeV}}$, to match `hep-lat/0509113`

Diquark spectroscopy - phenomenological estimates

We want to compare our results with phenomenology

- Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
- For pheno estimates use charm and bottom hadron masses where leading $\mathcal{O}(1/m_Q)$ ($Q = c, b$) can be cancelled

Four estimates considered:

- $\delta(1^+ - 0^+)_{ud}$: $\boxed{\frac{1}{3}\left(2M(\Sigma_Q^*) + M(\Sigma_Q)\right) - M(\Lambda_Q)}$
- $\delta(1^+ - 0^+)_{us}$: $\boxed{\frac{2}{3}\left(M(\Xi_Q^*) + M(\Sigma_Q) + M(\Omega_Q)\right) - M(\Xi_Q) - M(\Xi_Q')}$
- $\delta(Q[ud]_{0^+} - \bar{Q}u)$: $\boxed{M(\Lambda_Q) - \frac{1}{4}\left(M(P_{Qu}) + 3M(V_{Qu})\right)}$

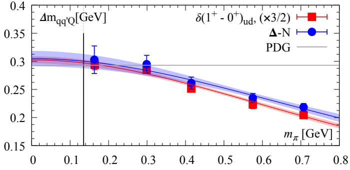
  $\rightsquigarrow$ P_{Qu},V_{Qu} are the ground-state, heavy-light mesons
- $\delta(Q[us]_{0^+} - \bar{Q}s)$:

$$\boxed{M(\Xi_Q) + M(\Xi_Q') - \frac{1}{2}(M(\Sigma_Q) + M(\Omega_Q)) - \frac{1}{4}(M(P_{Qs}) + 3M(V_{Qs}))}$$

$\rightsquigarrow$ P_{Qs},V_{Qs} are the ground-state, heavy-strange mesons

Δ-Nucleon mass difference

$[\Delta - N](m_\pi)$

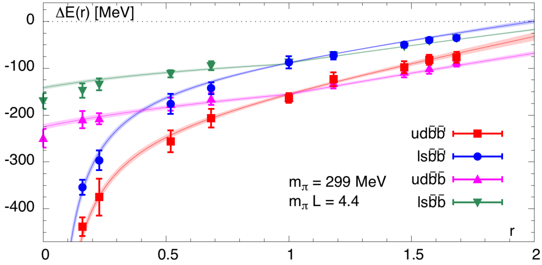


Measured the mass difference of $\Delta - N$

- Prediction: $\delta(\Delta - N) = 3/2 \times \delta(1^+ - 0^+)_{ud}$
- Same Ansatz as before
- Prediction holds well, even at fairly large m_π

A tunable system - opportunity together with pheno

AF et al. ('18) *5 parameter pheno-Ansatz in Appendix



◦ E.g. scans in $m_{b'}$ map out the heavy quark mass dependence.

◦ Away from physical masses the binding mechanism can be probed.
→ Mass dependence can be confronted with model predictions.
→ System can be tuned continuously from the bound to the resonant or non-interacting regimes.

→ Requires robust control of finite volume spectrum.

Review of doubly heavy tetraquarks in lattice QCD

Confirm and predict doubly heavy tetraquarks non-perturbatively

Tetraquarks as ground states? What would their binding mechanism/properties be?

HQS-GDQ picture, consequences for $qq'\bar{Q}'\bar{Q}$ tetraquarks:

- ◦ $J^P = 1^+$ *ground state* tetraquark below meson-meson threshold
- ◦ Deeper binding with heavier quarks in the $\bar{Q}'\bar{Q}$ diquark
- ◦ Deeper binding for lighter quarks in the qq' diquark

Ideal for lattice: Diquark dynamics and HQS could enable $J^P = 1^+$ *ground state doubly heavy tetraquarks* with flavor content $qq'\bar{Q}\bar{Q}'$.

Goal: $\Delta E = E_{\rm tetra} - E_{\rm meson-meson}$, e.g. in $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$ and others
$\Rightarrow$ Verify, quantify predictions of binding mechanism in mind.

Lattice point of view

- Hidden flavor $qQ\bar{q}'\bar{Q}$ are tetraquark candidates as excitations of $Q\bar{Q}'$.
  $\rightsquigarrow$ technical difficulty for lattice calculations, need to resolve many f.vol states.
  $\rightsquigarrow$ $qq'\bar{Q}\bar{Q}'$, i.e. ground state candidates would be better to handle.

In the following

- ◦ Tetraquarks with two heavy (c, b) and two light (ℓ, s) quarks.
- ◦ Lattice evidence for $bb\bar{u}\bar{d}$, $bb\bar{\ell}\bar{s}$.
- ◦ Recent updates on systematics.
- ◦ Survey of candidates status.

Lattice tetraquarks - 4 main approaches

1. Static quarks ($m_Q = \infty$)
Fitted potentials used to predict bound states and resonances.
◦ Allows for potential formulation.
◦ Ansatz fitted to lattice data.
◦ Plug into Schrödinger Eq. for E_n.

$\rightsquigarrow$ $bb\bar{u}\bar{d}$, Bicudo et al. ('17,'19)

2. HAL QCD method
Lattice potentials studied for scattering properties.
◦ Expansion of energy dependent potential (systematics?).
◦ Method under debate, best motivated for heavy systems.

$\rightsquigarrow$HAL QCD ('16,'18)

3. Finite volume energy levels
Lattice energies equated to (un)observed states.
◦ Operator matrix (GEVP) gives $\lambda_i \propto E_i$ $\Rightarrow$ Finite volume states.
◦ Binding? Get $\Delta E = E_0 - E_{thresh}$.
◦ Mechanism? Vary quark masses.

$\rightsquigarrow$AF et al. ('17,'18, '20), Hughes et al. ('17), Junnarkar et al. ('18), Leskovec et al. ('19), Mohanta et al. ('20)

4. Scattering analysis
Lattice energies studied in terms of scattering phase shifts.
◦ Excited state energies via GEVP.
◦ Analyse fvol spectrum $\Rightarrow$ Resonant, bound, virtual bound, free.

$\rightsquigarrow$Hadron Spectrum Coll. ('18,'20)

Lattice tetraquarks - 4 step recipe

The main tool is to adopt a variational approach

Lattice GEVP gives access to finite volume energy states (masses, overlaps).

Beware: Operator overlaps do not necessarily connect to the naively expected structures. Be careful when equating lattice correlators with trial-wave functions.

Step I: Set up a basis of operators, here $J^P = 1^+$

Diquark-Antidiquark:

$$D = \Big((q_a)^T (C\gamma_5) q'_b \Big) \times \Big[\bar{Q}_a (C\gamma_i)(\bar{Q}'_b)^T - a \leftrightarrow b \Big]$$

Dimeson:
$$M = (\bar{b}_a \gamma_5 u_a)\,(\bar{b}_b \gamma_i d_b) - (\bar{b}_a \gamma_5 d_a)\,(\bar{b}_b \gamma_i u_b)$$

Step II: Solve the GEVP and fit the energies

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}, \quad F(t)\nu = \lambda(t) F(t_0)\nu \,,$$

$$G_{\mathcal{O}_1\mathcal{O}_2} = \frac{C_{\mathcal{O}_1\mathcal{O}_2}(t)}{C_{PP}(t)C_{VV}(t)} \,, \; \lambda(t) = A e^{-\Delta E (t-t_0)} \,.$$

$\rightsquigarrow \Delta E = E_{\text{tetra}} - E_{\text{thresh}}$ in case of binding correlator $(C_{\mathcal{O}_1\mathcal{O}_2}(t))/(C_{PP}(t)C_{VV}(t))$.

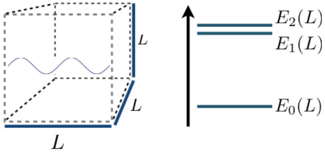
Most use these operators, but a larger basis has been worked out.
$\Rightarrow$ Need to be used by more groups. $\rightsquigarrow$ HadronSpectrum Coll. ('17)

Step III: Finite volume corrections

Large energy shifts are possible due to the finite lattice volume.

Scenario I: Scattering state
The finite volume energy belongs to a scattering state, the corrections go as

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + \frac{a}{L^3} + \mathcal{O}(\frac{1}{L^4})\right]$$



⇝ M. Hansen

Scenario II: Stable state
The corrections are exponentially suppressed with $\kappa = \sqrt{E_{b,\infty}^2 + p^2}$

$$E_{b,L} \sim E_{b,\infty} \cdot \left[1 + Ae^{-\kappa L}\right]$$

With a single volume available:

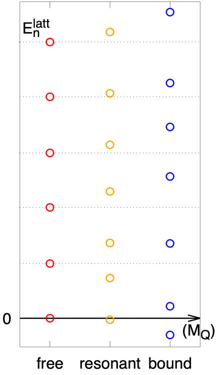
- In a bound state corrections are $\sim$ exp(binding momentum) ⇝ strong supp. m_{had} =heavy
- In a scattering state expect large deviation around threshold

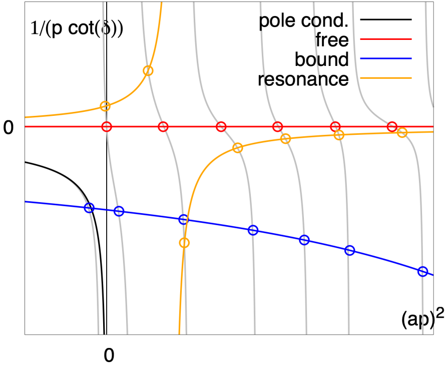
With multiple volumes available:

- Track mass dependence ⇝ decide bound/scatt. state
- Power law corrections might be too small to resolve

Limitation: Small GEVP without f.vol analysis ok for deeply bound states.
Insufficient to tell apart free, resonant or virtual bd. states.

Extension: Connect energies to scattering phase shifts via finite volume quantisation conditions (Lüscher-formalism).



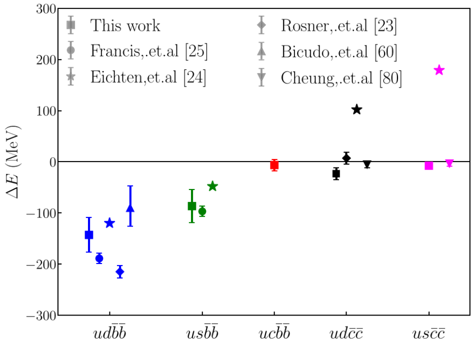


- connect (many) f.vol states to scattering parameters (sketch: BW)
- resonance: extra state(s) appear, lowest state close to threshold

What we know: A review of recent lattice studies

What we know: Deeply bound $J^P = 1^+$ $bb\bar{u}\bar{d}$ and $bb\bar{\ell}\bar{s}$ tetraquarks

Community overview



⇝Mathur et al. ('19)

Qualitative agreement with pheno

○ All three predictions met:
→ $J^P = 1^+$ bound ground state.
→ deeper binding with $m_Q \uparrow$.
→ deeper binding with $m_q \downarrow$.

○ $bb\bar{q}\bar{q}'$ are a focal point → All efforts observe deeply bound $bb\bar{u}\bar{d}$
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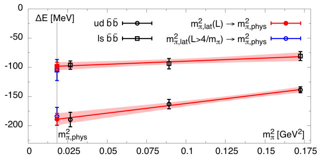
AF et al. ('17)



· Junnarkar, Mathur, Padmanath ('18)
· Leskovec, Meinel, Plaumer, Wagner ('19)
· HadronSpectrum Coll. ('17)
· Mohanta, Basak ('20)
· Colquhoun, AF, Hudspith, Lewis, Maltman ('17, '18, '20)

Overview -possible doubly heavy tetraquark candidates

Surveying candidates

observed (>1 group)
no deep binding
observed (1 group)
not confirmed (>1 group)

channel	deeply bound
$J^P = 1^+$	$bb\bar{u}\bar{d}$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $bc\bar{\ell}\bar{s}$
	$bs\bar{u}\bar{d}$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $bb\bar{s}\bar{c}$
	$cc\bar{u}\bar{d}$ $cc\bar{\ell}\bar{s}$
	$bb\bar{b}\bar{b}$
$J^P = 0^+$	$bb\bar{u}\bar{u}$ $cc\bar{u}\bar{u}$
	$bb\bar{u}\bar{d}$ $bc\bar{u}\bar{d}$
	$bb\bar{\ell}\bar{s}$ $bc\bar{\ell}\bar{s}$
	$bb\bar{s}\bar{s}$ $cc\bar{s}\bar{s}$
	$bs\bar{u}\bar{d}$ $cs\bar{u}\bar{d}$
	$bb\bar{u}\bar{c}$ $bb\bar{s}\bar{c}$
	$bb\bar{c}\bar{c}$ $cc\bar{u}\bar{d}$
	$bb\bar{b}\bar{b}$

Deeply bound states

Focus: strong interaction stable

→ $bb\bar{u}\bar{d}$ and $bb\bar{\ell}\bar{s}$ in $J^P = 1^+$.
→ $cc\bar{q}\bar{q}'$ not deep.
→ $bc\bar{q}\bar{q}'$ not clear.
→ further candidates not observed.
→ none observed in $J^P = 0^+$.

⇝Bicudo et al. ('17), AF et al. ('17,'18, '20), HadSpec Coll. ('18), Hughes et al. ('17), Junnarkar et al. ('18), Leskovec et al. ('19), Mohanta et al. ('20)

States above threshold, resonances?

→ $bb\bar{u}\bar{d}$ in $J^P = 1^+$ /w static quarks find a resonance just above threshold. ⇝Bicudo et al. ('19)
→ No results from other approaches.
→ What about $cs\bar{u}\bar{d}$?
⇝ under investigation Hudspith, AF et al.('20), HadSpec ('20)

Shallow binding?

∘ $cc\bar{u}\bar{d}$ now observed by LHCb, robust lattice post-diction?
→ Work to remove current limitations.