Electromagnetic form factors of hadrons from spectral Dyson-Schwinger equations

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Outline

- Theory for production processes

- Integral representations for QCD(SM) Green's functions

- spectral Dyson-Schwinger equation – quark gap equation in Minkowski space

- First applications- pion TF, pion EFF, hadron vacuum polarizations

Intro

QCD theory of hadrons for E GeV, albeit it has 1 coupling and five masses relevant for hardon physics, is NP.

PT is out of the game: npt tools : lattice, AdS/CFT, effective models with more phenomenology, (CHPT, CHPT+VM, Regge, nonrelativistic QM with lattice potentials...), SDEs

production processes -timelike scales, resonances. more phenomelogy required, attempts of analytical continuations from lattice or SDEs

and only very recently- SDEs for spectral functions (SDEs in Minkowski space) marriage of Integral representations for Green's functions and Dyson-Schwinger equations in suited scheme can explain resonant structure

1] Integral representations

2] Calculation framework- coupled set DSEs for QCD/QED vertices

Integral representations in QCD

-selfconsistent NP generalization of Nakinishi's Perturbation Theory Integral representation

N. Nakanishi- Graph Theory and Feynman Integrals, 1971

- NPTIR scalar propagators –> QCD propagators

$$G(k) = \int_0^\infty dx \frac{g(x)}{k^2 - x + i\varepsilon} - - > S_f(p) = \int_{-\infty}^{\infty} dx \frac{\rho_f(x)}{(\not{p} - x)} \tag{1}$$

The inverse propagator (selfenergy, polarization, etc..) DR

$$\begin{aligned} G^{-1} &= k^2 - m^2 + \Sigma(k) \\ \Sigma(k) &= \int_0^\infty dx \frac{g_\sigma(x)}{k^2 - x + i\varepsilon} - - > easy :) \end{aligned}$$

IR higher vertices are not easy, only Gauge Technique was known before 2020...

$qq\gamma$ non-amputated vertex (known thanks to gauge invariance)

$$G^{\mu}(q;Q) \quad = \quad \int_{\Gamma} dx \frac{1}{\not{q}_{-} - x} \left(\gamma^{\mu}\rho(x)\right) \frac{1}{\not{q}_{+} - x}$$

NPTIR for 3 legs scalar theory diagram

$$\Gamma(p_1,p_2,p_3) = \int\limits_0^\infty d\alpha \prod_{i=1}^{3} \int\limits_0^1 dz_i\, \delta(1-\sum_{i=1}^{3} z_i)\, \frac{\rho_3(\alpha,\vec{z})}{\alpha - \left(z_1 p_1^2 + z_2 p_2^2 + z_3 p_3^2\right) - i\epsilon}\,,$$

generalizes for gauge theory (proved by DSEs in 2020

V.S. Phys. Rev. D (106, 3) 2022 for $qq\gamma$

8+4 , 8 transverse components of proper v.

$$\Gamma^{\mu}_{iT}(q,Q) = \int_{-1}^{1} dz \int_0^\infty doda \frac{\rho_i(o,z,a) T^{\mu}_i}{q^2 + zq.Q + aQ^2/4 - o + \varepsilon} \qquad (2)$$

NIR + DSEs ,Numerical solution

IR emerges as a selfconsitent solution of DSEs

2] add DSEs kernels - Yang-Mills theory- ghosts, gluons, quarks

DSEs truncation used for purpose of form factor calculations:

are various and different :)

NIR + DSEs ,Numerical solution

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2] add DSEs kernels - Yang-Mills theory- ghosts, gluons, higher vertices Papavasiliou/Sauli PRD 2022

DSES truncation used for purpose of form factor calculations

LRA DSE for quarks with gluon obtained via solution of YM DSE Confinement within the use Minkowski integral representation; Sauli PRD 2022

with complicated Gauge Technique + transverse structure of vertices (by J. Papavasilliou)

very agreement with lattice (in Landau gauge)

extrapolation from Landau gauge was made...

NIR + DSE/BSE formalism: properties

- solving spectral SDEs -put IR to SDEs, integrate analyticaly in E space, make continuation and solve sytem for σ's

  For strong coupling QFT the system of spectral DSEs turns out to be underconstrained and infinite numerically stable solutions exist. They are not automatically consistent with all assumptions made, even if forced!

- Choice of the correct solution must be made-

  choose the real parts and search the phases (search for absorptive parts at the timelike fixed point) which provides consistent analytical picture. Scanning the imaginary part at given renormalization point (not always) works. Gluodynamics at Landau gauge (for QFT model with small coupling it is not necessary). The first results for QCD appear at 2017 , published FBS 2019, PRD 2022, PRD 2022,...

Properties of solutions:

To identify individual solutions with a given flavor, we solve also meson BSE in complex mom . space. Those which consistent meson properties show up confinement!: Quark propagator has not on mass shell pole, it has only a single cut. Spectral function starts (almost) smoothly from beginning of cpx. plane of p^2.

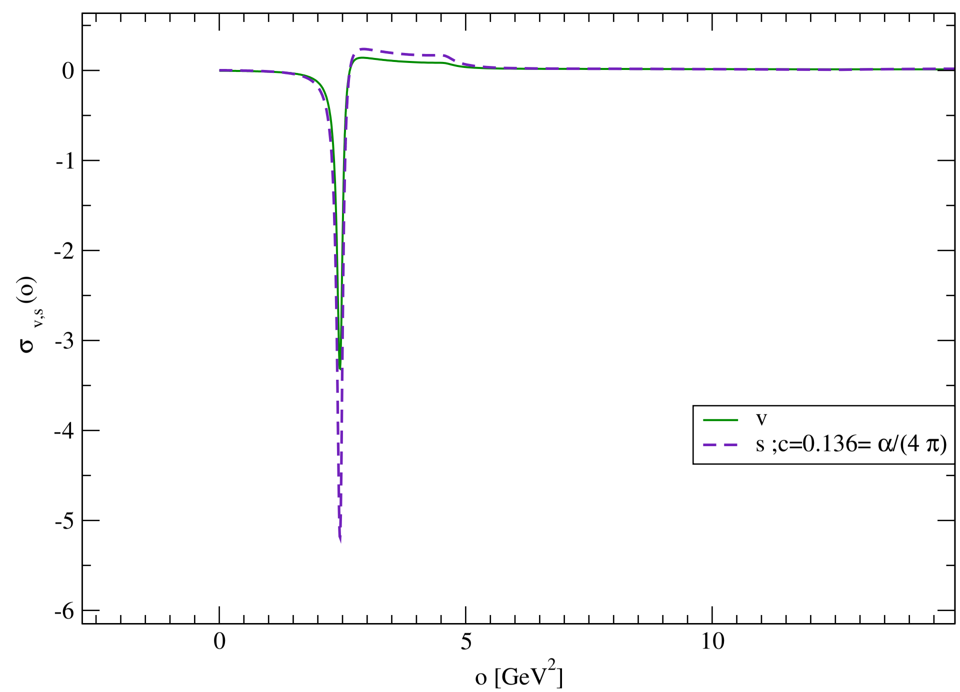

Figure 1: charm quark

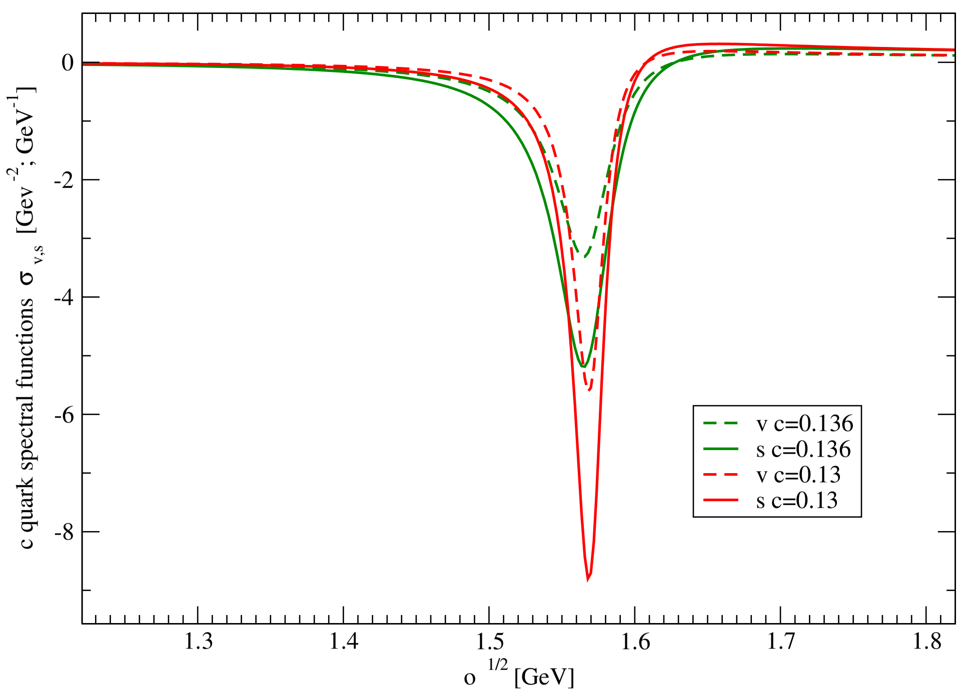

Figure 2: charm quark

(no sign for deviations, complex conjugated poles are not seen, almost unlimited accuracy) light flavour are similar - no pole but bump. positioned now at $M \simeq 265MeV$ for u,d, quarks.

Evaluation of continuous hadronic form factors at the timelike domain of momentum is possible!

(calculation scheme exists in DSE formalim- skeleton expansions)

Hadron Vacuum Polarization

$$\Pi_h^{\mu\nu}(s) = -ie^2 N_c \sum_q e_q Tr \int \frac{d^4k}{(2\pi)^4} \Gamma_q^{\mu}(k-q,k) S_q(k) \gamma^{\nu} S_q(k-q)$$

$$= T^{\mu\nu} \Pi_h(q^2)$$

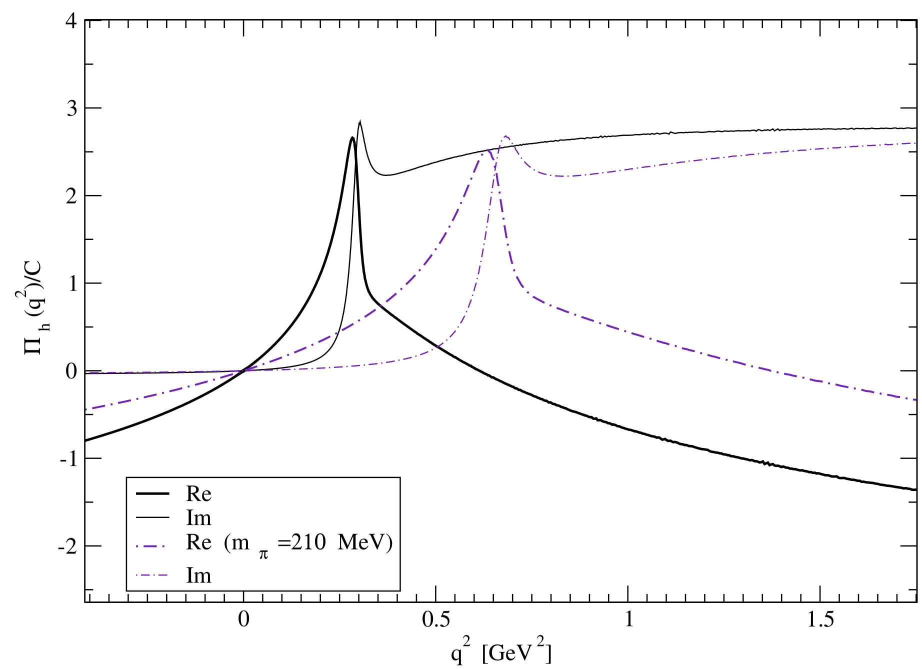

Figure 3: Π_h/C obtained via Gauge Technique with the constant C defined as $C = -40\alpha/(9\pi)$ (FBS 2020)

spectral DSE in QCD- Pion Transition Form Factor

$\gamma^*\gamma^* \to \pi^o$

$$T_{\mu\nu}(k,l) = \frac{e^2}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}k^\alpha l^\beta G(k^2,l^2)\,, \tag{3}$$

$$M_{\mu\nu}(k,l) = i\int \frac{d^4q}{(2\pi)^4}\Gamma_\pi(q_1,q_2)G_{\mu\nu}(q_1,q_2;k,l)\,, \tag{4}$$

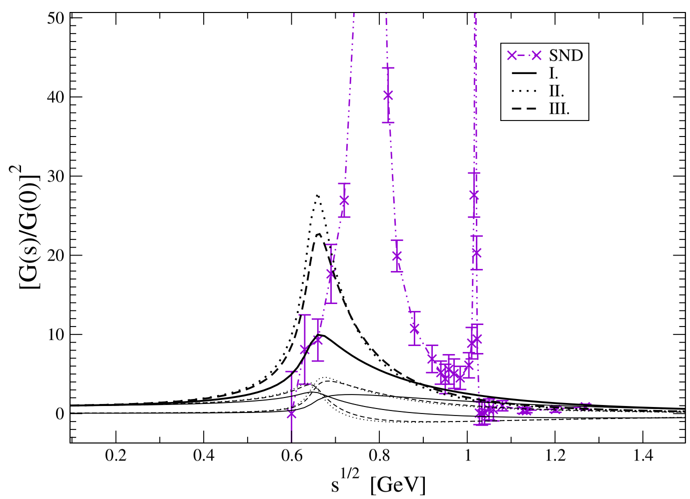

Figure 4: Pion transition form factor $G(q^2) = G(q^2, 0)$ for the timelike argument $s = q^2$: the quantity $(q^2 G(q^2)/G(0))$ is shown. Each line labeled by I II and III use a different interpolator for the pion vertex function (V.S. PRD2020

spectral DSE in QCD- Pion EMG form factor

$$\gamma^* \to \pi^+ \pi^-$$

$$\begin{aligned}
\mathcal{J}^\mu(p,Q) &= eF_\pi(Q^2)p^\mu \\
&= \frac{2N_c}{3} ie \int \frac{d^4k}{(2\pi)^4} tr\left[G^\mu_{EM,u}(k+Q/2,k-Q/2)\Gamma_\pi(k_{r\pi_-},p+Q/2)S_d(k+p)\tilde{\Gamma}_\pi(k_{r\pi_+},Q\right. \\
&+ \quad ...
\end{aligned}$$

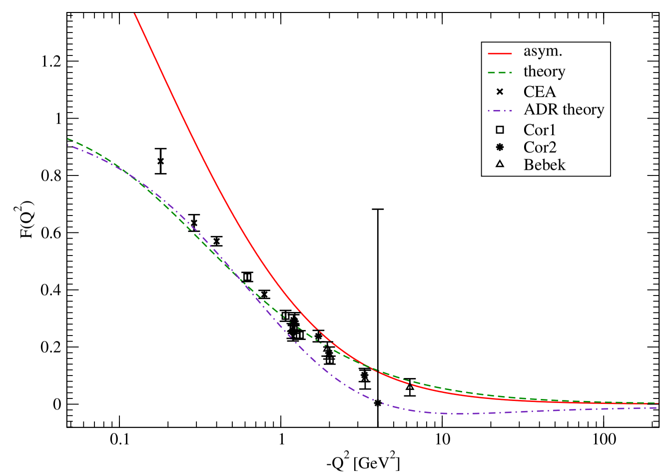

Figure 5: F for $Q^2 < 0$ and comparison with experiment and asymptotic prediction.

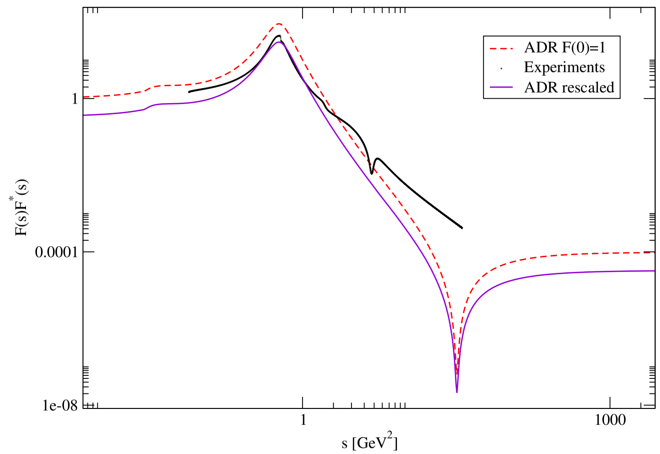

Figure 6: Calculated magnitude of the pion electromagnetic form factor for $Q^2 > 0$ and comparison with experiments. The error bars are not shown, they are within the visible size of the line and are much smaller then the deviation of presented calculations. The solid line is rescaled by a constant as described in the text.

Conclusions,Achievements:

Exact proof of dispersion relation for PEFF, VHP (TPF is more delicate)

$$F(q^2) = \int dx \frac{\rho_F(x)}{q^2 - x + \varepsilon} \tag{6}$$

$$\rho_F = \int \sigma_v(1)\sigma_v(2)\rho_\pi(3)\rho_\pi(4)K(1,2,3,4,5) + \tag{7}$$

Agreement with experiment is within large systematical uncertainties due to approximations made. A: dominant vertices, Only G.T. Term, further integral reduction.

Calling for BSE solution in terms of NIR (done for constituent model only Tobias Frederico, Giovani Salme, Vladimir Sauli), numerical solutions for transverse vertices,

main advantage: To express F's one does not need to calculate vertices, just spectral and weight functions are needed.

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main disadvantage: To get F's one need to integrate over many calculated spectral and weight functions.

Open questions: Thresholds, Unitarity relations of S-matrix, comparison with complex structure known from effective theories, supplementary material

quark interaction kernels of BSE/DSE

$$
\begin{aligned}
V(l) &= \gamma_\mu \times \gamma_\nu \left(g^{\mu\nu} V_V(l) - \frac{4g^2}{3} \xi \frac{L^{\mu\nu}(l)}{l^2} \right) \; , \\
V_V(l) &= \frac{c_{V1}}{(l^2 - m_g^2 + i\varepsilon)} - \frac{c_{V2}}{(l^2 - \Lambda_g^2 + i\varepsilon)} \; , \\
L^{\mu\nu}(l) &= l^\mu l^\nu / l^2 \; ,
\end{aligned}
$$

V is the fit of the gluon propagator solution in PT-BFM. Overall prefactor is adjusted in order to get correct meson properties in given gauge.