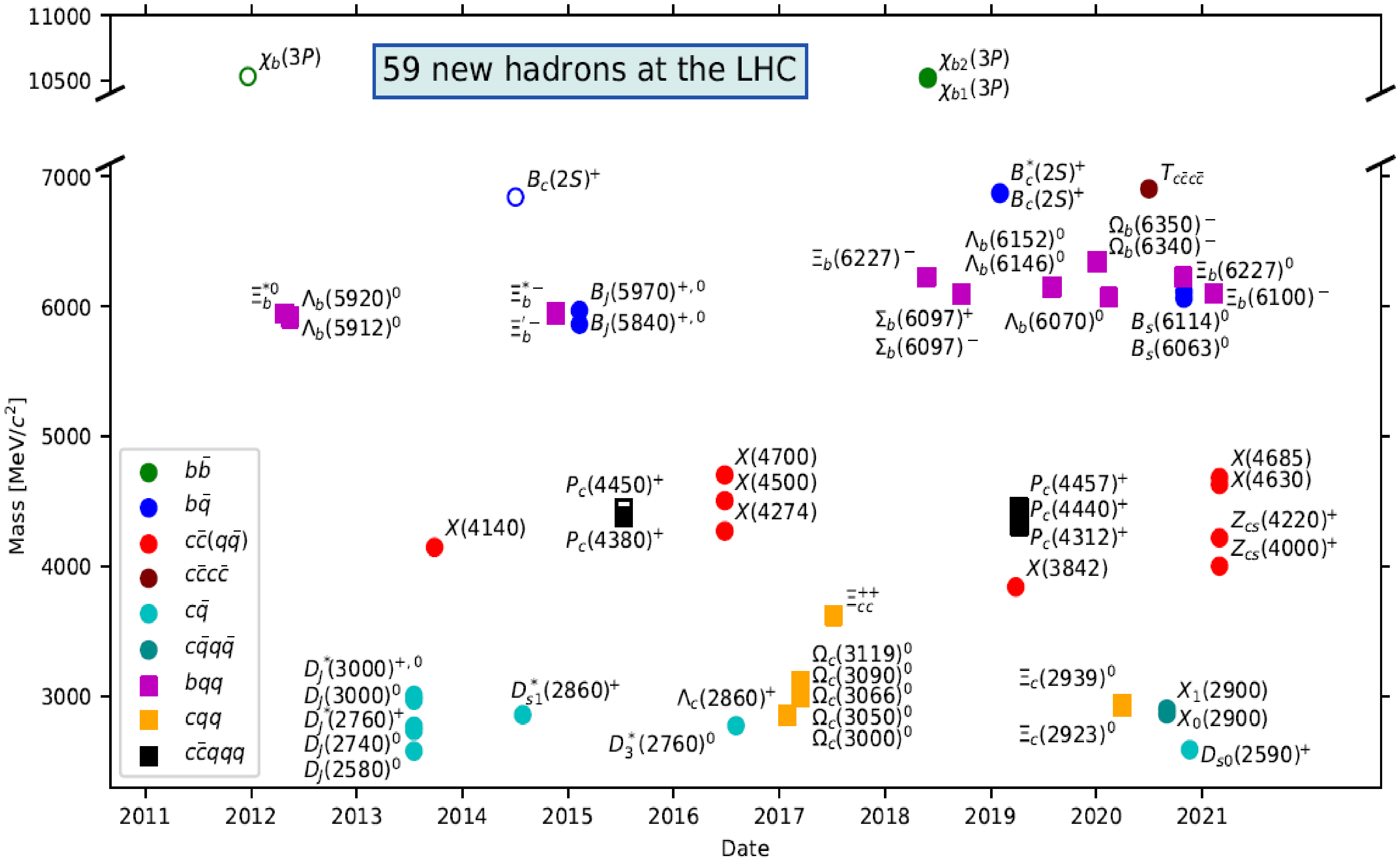
# Confinement and Coulomb Gauge LQCD

Wyatt Smith, Sebastian Dawid, Adam Szczepaniak, César Fernández Ramírez

# Hadron spectrum

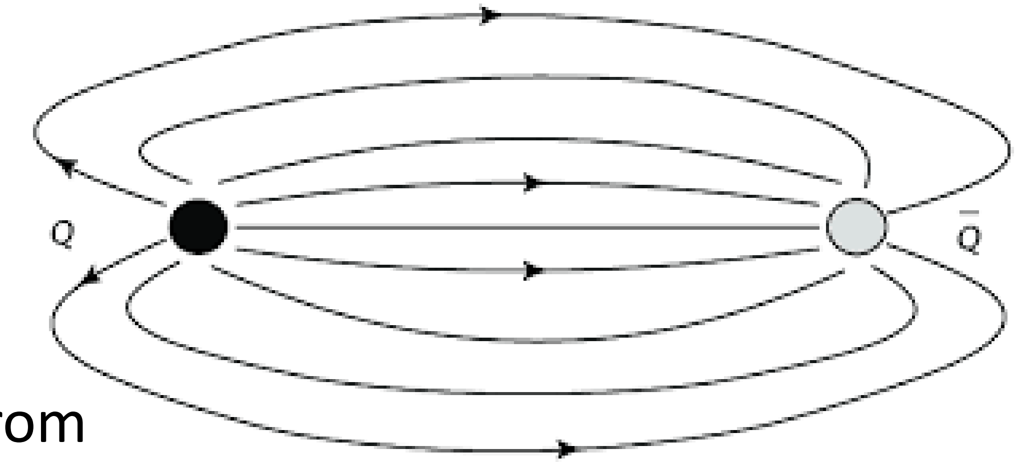



Plot from https://home.cern/news/news/physics/59-new-hadrons-and-counting

# Confinement

- Observed hadrons are color singlets
- The potential between static quark-antiquark pairs must be linear at large distances (before string-breaking)
- Wilson potential (from LQCD)

$$V(r) = A + \frac{B}{r} + \sigma r$$




- Many models for charmonium, bottomonium come from this potential
- Problem: This gives no information about *why* quarks are confined!

# Why Coulomb Gauge Lattice QCD?

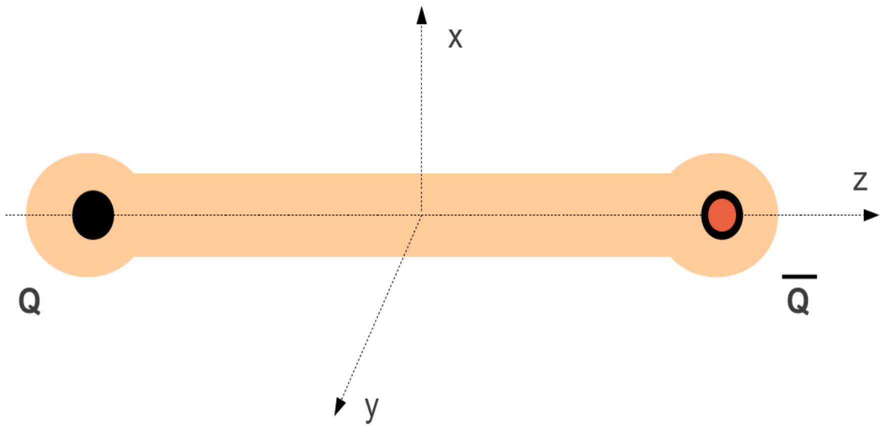
- LQCD is the only way to probe quark-level interaction currently

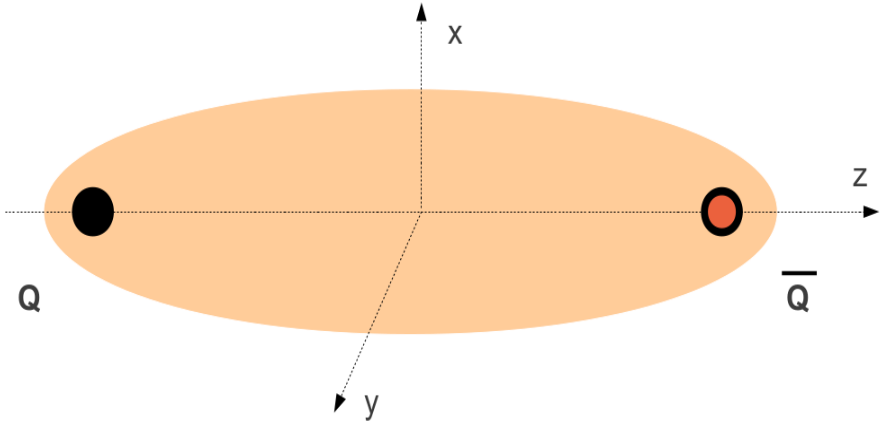
- Need to fix the gauge to employ physical intuition/car understand QCD through analogy to QED in Coulomb Gauge

- Some questions remain about specifics of origin of Cornell potential, and flux tubes [1 2 3] on the Lattice

$$V(r) = A + \frac{B}{r} + \sigma r$$

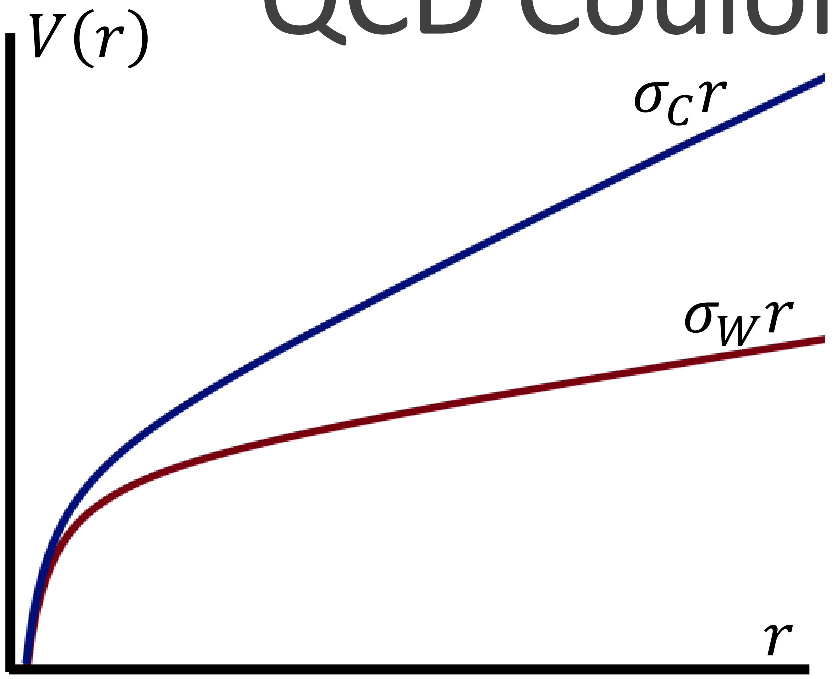
- Remarkable feature of Coulomb Gauge: $gA_0$ in Coulomb gauge is a renormalization-group invariant [4]

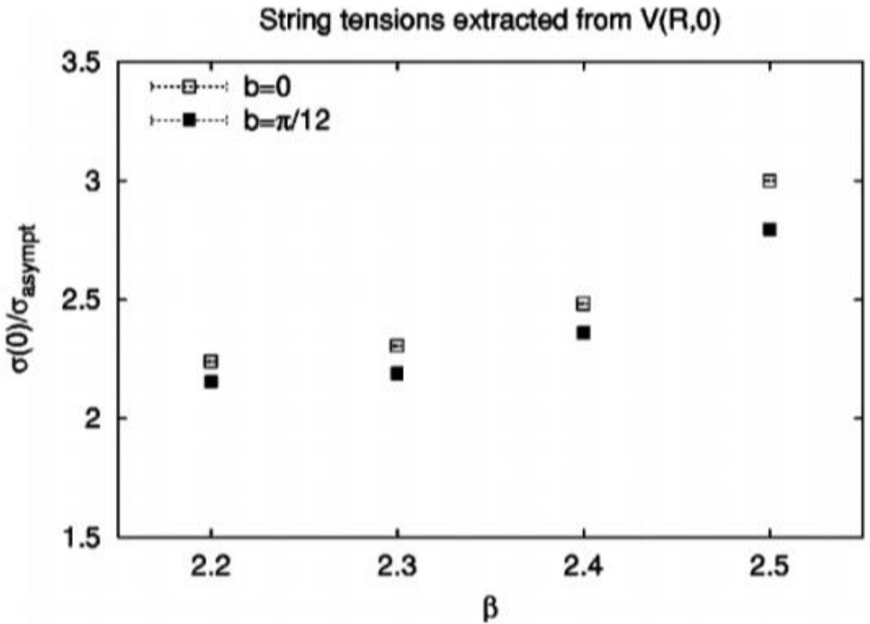



[1] P. O. Bowman and A. P. Szczepaniak, Phys. Rev.D70, 016002 (2004), arXiv:hep-ph/0403075[hep-ph].
[2] K. Chung and J. Greensite, Phys. Rev.D96, 034512 (2017), arXiv:1704.08995 [hep-lat].
[3] S. Dawid and A. P. Szczepaniak , Phys. Rev.D100, 074508 (2019)
[4] D Zwanziger, Nucl.Phys.B 518 (1998) 237-272

# QCD Coulomb Potential vs Wilson Potential




- Wilson potential = potential of static quark antiquark pair in ground state
- Coulomb potential = potential of static quark antiquark pair interacting *instantaneously* in Coulomb gauge
- Both potentials parameterized by Cornell potential

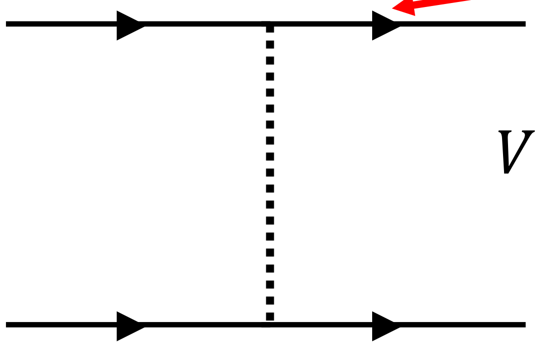
$$V(r) = A + \frac{B}{r} + \sigma r$$

- Confining behavior of Coulomb potential is *necessary* for Wilson confinement[5]

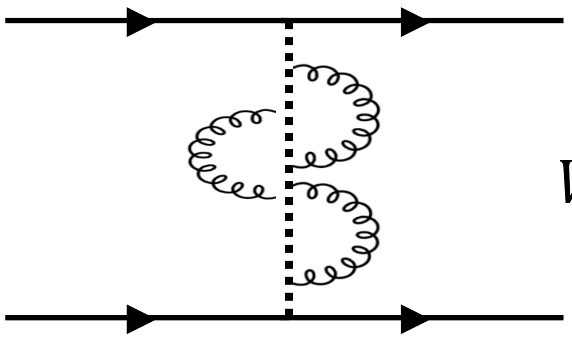
[5] D. Zwanziger, Phys. Rev. Lett.90, 102001 (2003), arXiv:hep-lat/0209105 [hep-lat].
Plot from: J. Greensite and A. P. Szczepaniak,Phys. Rev. D91, 034503(2015).

# Coulomb Gauge Hamiltonian:

$$H_{QCD} = H_q + H_g + H_{qg} + H_C$$



$$V \to \frac{1}{r}$$



$$V \to \sigma_C r$$

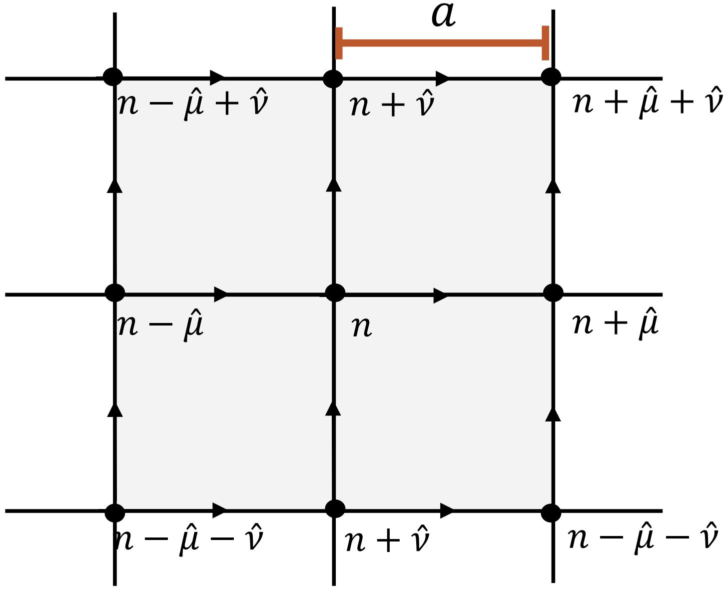
$$\langle q\bar{q}|H_{QCD}|q\bar{q}\rangle = \sigma_C r$$

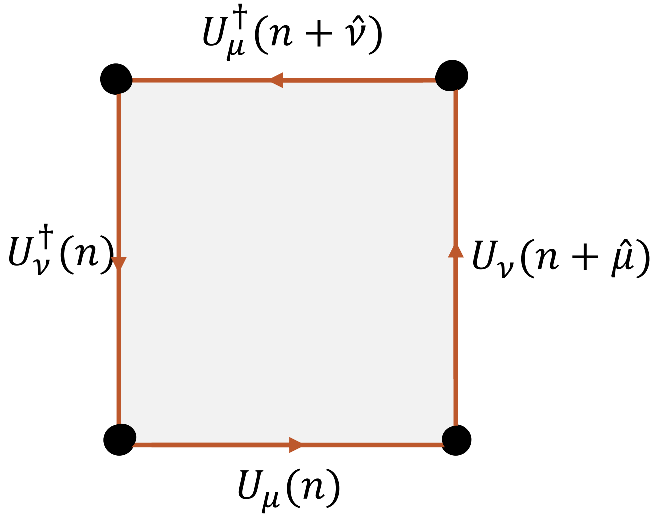
- The static quark-antiquark state which produces the coulomb potential is *not* the ground state!

$$H_{QCD}|q\bar{q}_{true}\rangle = \sigma_W r|q\bar{q}_{true}\rangle$$

$$|q\bar{q}_{true}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}gg\rangle + \cdots$$

# SU(N) Lattice QCD



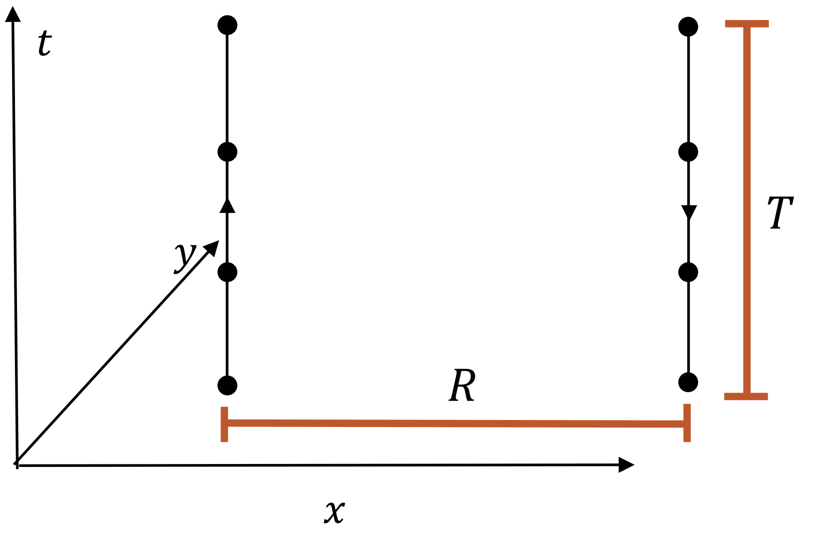

- Links are SU(N) matrices representing gauge transporters between lattice sites

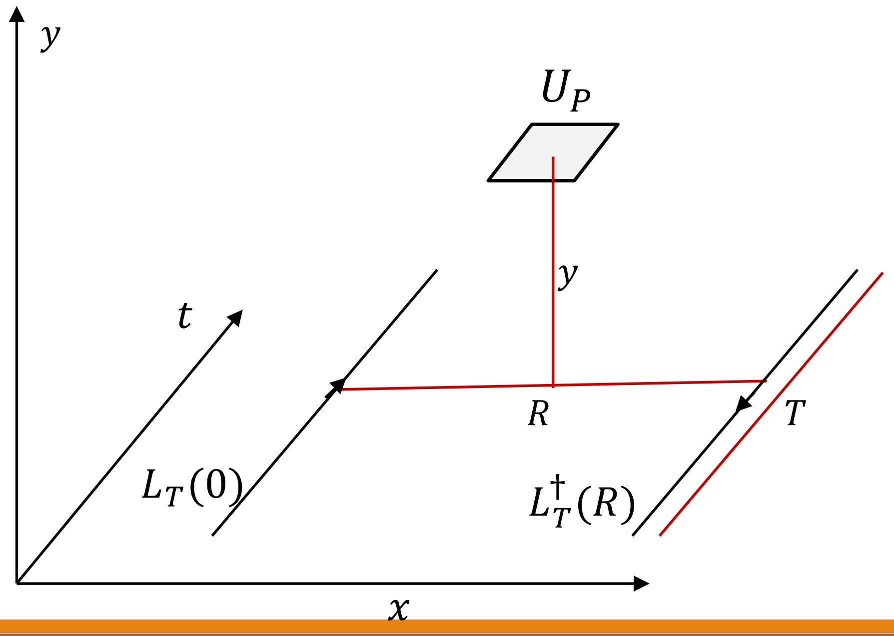
$$U_\mu(n) = e^{iaA_\mu(n)}$$

- Wilson action for SU(N) LQCD:

$$S = \frac{\beta}{N_C} \sum_n \sum_{\mu<\nu} \mathrm{Re}\,\mathrm{Tr}\left[1 - U_{\mu\nu}(n)\right] \qquad \beta = 2N_C/g^2$$

$$U_{\mu\nu}(n) = U_\mu(n) U_\nu(n + \hat{\mu}) U_\mu^\dagger(n + \hat{\nu})\, U_\nu^\dagger(n)$$




- In Coulomb gauge ($\partial_i A^i = 0$), calculate the potential from correlation of two time-like Wilson lines

$$V_C(r) = -\frac{1}{a}\log\left\langle \frac{1}{N}\mathrm{Tr}\left[U_0(0,\mathbf{0})U_0^\dagger(0,\boldsymbol{R}\,)\right]\right\rangle$$

$$V_C(r) = A + \frac{B}{r} + \sigma_C r$$

- $T \to \infty$ should recover the (minimal) Wilson Potential.
- $T \to 0$ gives the lattice version of the Coulomb potential
- Can calculate energy density by inserting “probe” above the Wilson lines

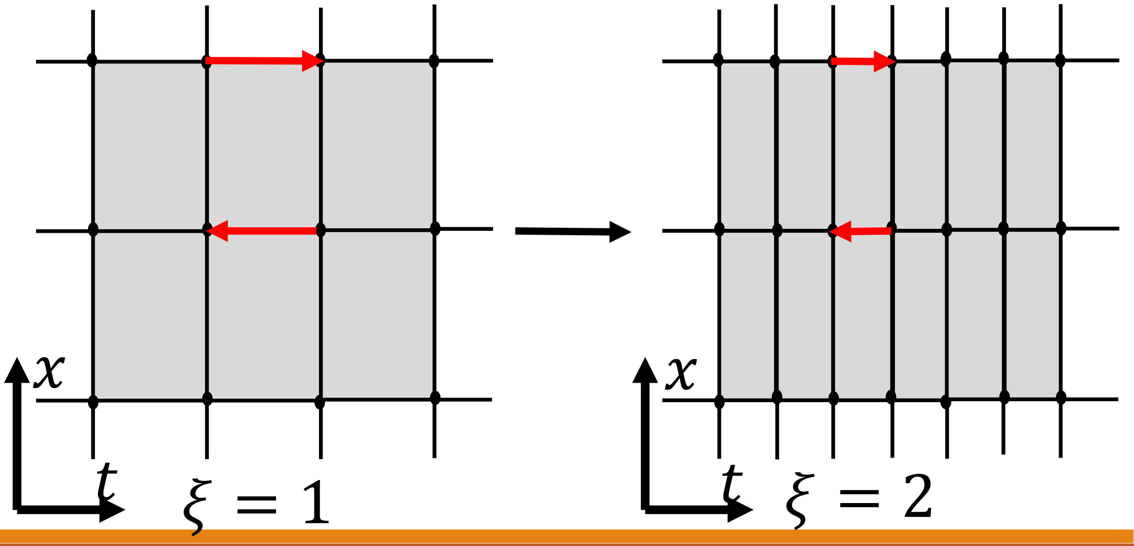
# Lattice Setup

- Forced to use an anisotropic lattice to access $T \to 0$. Must introduce $\beta_s, \beta_t$: different couplings for spatial/time directions

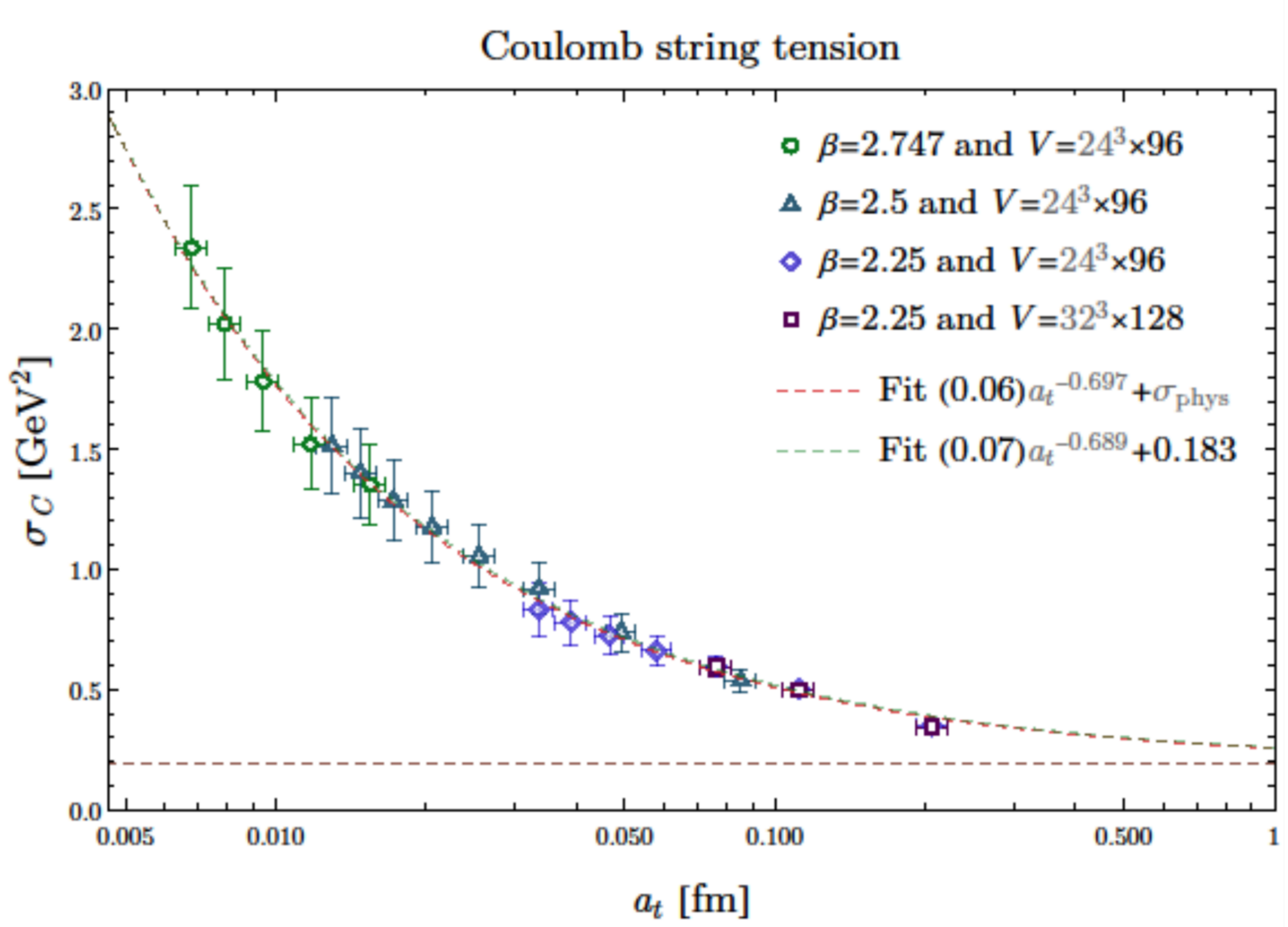
$$S = \sum_n \left[ \beta_s \sum_{j>i=1}^{3} \left( 1 - \frac{1}{2} \mathrm{Tr}\, U_{ij}(n) \right) + \beta_t \sum_{i=1}^{3} \left( 1 - \frac{1}{2} \mathrm{Tr}\, U_{0i}(n) \right) \right]$$

- Quenched Lattice QCD: $N_f = 0$, no fermion determinant (pure gluodynamics, infinitely heavy quarks)

- SU(2) Lattices:
  $\beta = 2.25, 2.5, 2.7, 3.249,\ \xi = 1, \ldots, 8$ ,
  $N^3 \times T = 24^3 \times 96$ , $32^3 \times 128$



# Preliminary Results: SU(2)



# Why is the string tension blowing up?

- Symmetry of switching spatial and temporal directions on the lattice [6]:

$$\langle W(r,t)\rangle = \langle W(t,r)\rangle \qquad \langle W(r,t)\rangle \propto e^{-V(t)r}$$

$$V(r,t) = \sigma_{eff}(t)r, \quad \sigma_{eff}(t) = V'(t)$$

$$\Rightarrow \sigma_{eff}(t) = \sigma + \frac{B}{t^2}$$

[5] G. Bali Phys.Rept.343:1-136,2001

# Why is the string tension blowing up?

- Improvements in defining the lattice observable :

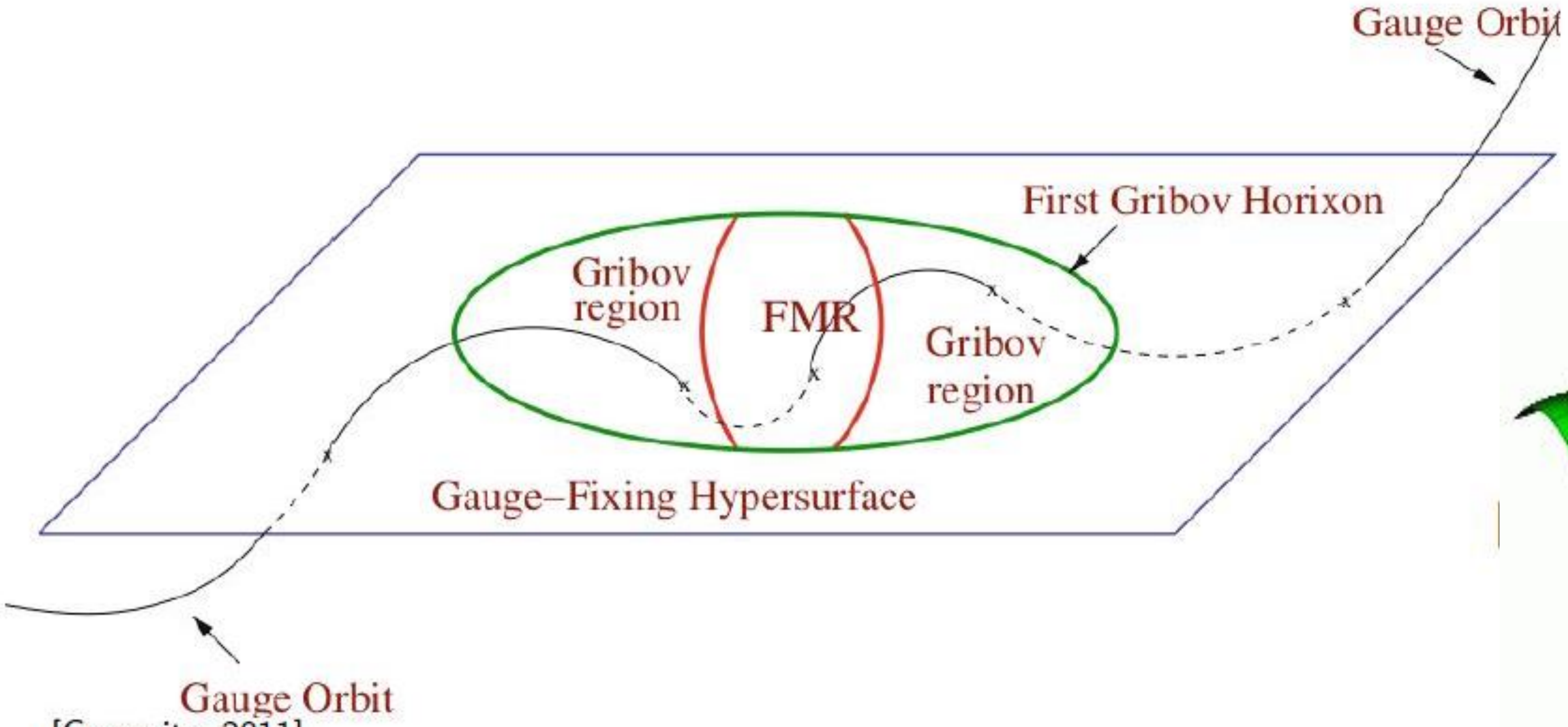
$$V_C(r) = -\frac{1}{a}\log\left\langle\frac{1}{N}\mathrm{Tr}\left[U_0(1,\mathbf{0})U_0^\dagger(1,\boldsymbol{R}\,)\right]\right\rangle$$

$$\Downarrow$$

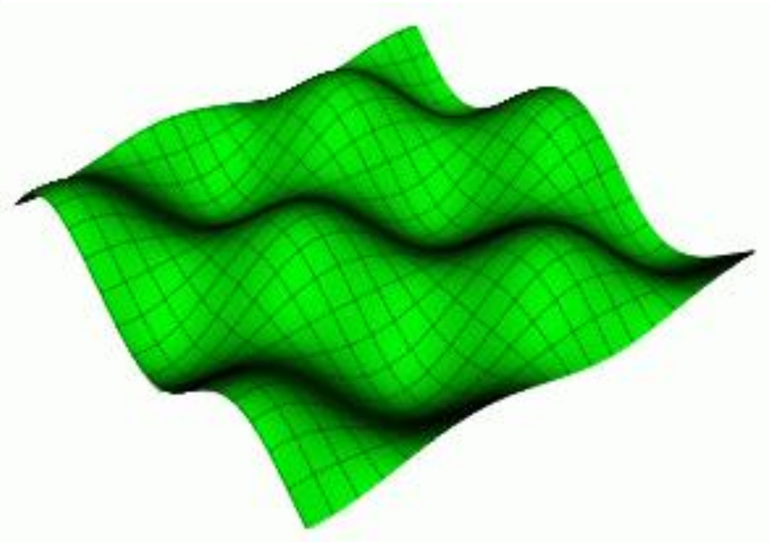
$$V_C(r) = -\frac{1}{a}\log\frac{\left\langle\frac{1}{N}\mathrm{Tr}\left[U_0(1,\mathbf{0})U_0^\dagger(1,\boldsymbol{R}\,)\right]\right\rangle}{\left\langle\frac{1}{N}\mathrm{Tr}\left[U_0(2,\mathbf{0})U_0^\dagger(2,\boldsymbol{R}\,)\right]\right\rangle}$$

# Why is the string tension blowing up?

- Possible gauge fixing issues:




[Greensite, 2011]



# Summary

- Coulomb Gauge Physics is important for understanding hadron spectrum, confinement

- We are making improvements in methods and algorithms for Coulomb Gauge LQCD to get a more precise measurement of the string tension and description of flux tube