International School on Theory & Analysis in Particle Physics

The Standard Model (SM)

Main textbook: Modern Elementary Particle Physics, Kane

Additional reading: SM: An Introduction, Novaes, arXiv:hep-ph/0001283
Symmetries of SM, Willenbrock, arXiv:hep-ph/0410370

Other advanced undergraduate / beginning graduate level textbooks:
Quantum Field Theory, Ryder
Introduction to Elementary Particles, Griffiths
Quarks & Leptons, Halzen & Martin
An Introduction to the SM of P.P., Cottingham & Greenwood
Gauge Theories in Particle Physics, Aitchison & Hey

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Outline

Lecture 1
- Review: groups, spinors, gauge invariance
- The Standard Model: introduction and particle content

Lecture 2
- QCD: Quarks, confinement, mesons and baryons
- Electroweak Theory: Neutral and charged currents

Lecture 3
- Spontaneous symmetry breaking and Higgs mechanism
- Cross sections and decay widths: Γ_W and Γ_Z

Lecture 4
- Higgs Boson
- Quark mixing and CP violation
- Open questions, grand unification

What is the Standard Model (SM) of Particle Physics?

- SM is the theory describing electromagnetic, weak & strong interactions
- ~40 years old, spectacularly confirmed (except Higgs)
- beyond the SM: neutrino masses, theoretical puzzles
- whatever LHC finds, SM is valid as an effective theory for E < TeV

- SM is a gauge theory based on $SU(3)_C \times SU(2)_L \times U(1)_Y$

  ⇒ review: groups, gauge invariance

- matter content: quarks and leptons ⇒ review: Dirac spinors

Rotation group

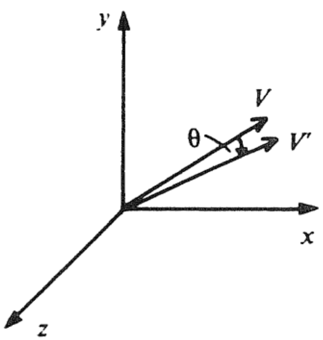
$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = (R)\begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{or } r' = Rr$$

$$x'^2 + y'^2 + z'^2 = x^2 + y^2 + z^2 \;\Rightarrow\; R^{\mathrm{T}} R = 1$$

O(n): [Orthogonal], n(n-1)/2 parameters

SO(n): [Special] determinant = 1

SO(3): 3 parameter group (Euler angles)



$$R_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix}$$

$$R_y(\psi) = \begin{pmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{pmatrix}$$

$$R_x(\phi)R_z(\theta) \neq R_z(\theta)R_x(\phi)$$

non-abelian

Generators of rotation:

$$\boxed{R_z(\delta\theta) = 1 + \mathrm{i}J_z\delta\theta}$$

$$J_z = \frac{1}{\mathrm{i}}\frac{\mathrm{d}R_z(\theta)}{\mathrm{d}\theta}\bigg|_{\theta=0} = \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$J_x = \frac{1}{\mathrm{i}}\frac{\mathrm{d}R_x(\phi)}{\mathrm{d}\phi}\bigg|_{\phi=0} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix}$$

$$J_y = \frac{1}{\mathrm{i}}\frac{\mathrm{d}R_y(\psi)}{\mathrm{d}\psi}\bigg|_{\psi=0} = \begin{pmatrix} 0 & 0 & \mathrm{i} \\ 0 & 0 & 0 \\ -\mathrm{i} & 0 & 0 \end{pmatrix}$$

$$[J_x, J_y] = \mathrm{i}J_z$$

Angular momentum operators J_x, J_y, J_z are the generators of SO(3)

Infinitesimal rotation: $R_z(\delta\theta) = 1 + \mathrm{i}J_z\delta\theta$

Finite rotation:
$$R_z(\theta) = [R_z(\delta\theta)]^N = (1 + \mathrm{i}J_z\delta\theta)^N = \left(1 + \mathrm{i}J_z\frac{\theta}{N}\right)^N = \mathrm{e}^{\mathrm{i}J_z\theta}.$$

generator

$$R(\delta\theta_z) = 1 + iJ_z\delta\theta_z$$

$$R(\theta_z) = \exp(i\theta_z J_z)$$

$$R(\theta_x, \theta_y, \theta_z) = \exp\left(\sum_{k=1}^{3} i\,\theta_k J_k\right)$$

$$[J_k, J_l] = i\epsilon_{klm}J_m$$

Lie algebra

ϵ_{klm} : structure constants

U(n): n x n unitary matrices ($U^\dagger U=1$)

n^2 independent parameters

$$U = \exp\left(\sum_{j=1}^{n^2} i\theta_j H_j\right)$$ where $H_j^\dagger = H_j$

SU(n): n x n unitary matrices ($U^\dagger U=1$) with det U = 1

n^2-1 independent parameters: 3 for SU(2), 8 for SU(3)

generators of SU(n): n^2-1 Hermitian and traceless matrices

$$\det e^{A} = e^{Tr A}$$

SU(2):

$$U = e^{i\vec{\sigma}\cdot\vec{\theta}/2}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

SU(2):

$$U = e^{i\vec{\sigma}\cdot\vec{\theta}/2}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(exercise – optional)

Convince yourself that $U = e^{i\vec{\sigma}\cdot\vec{\theta}/2}$ belongs to SU(2) by showing:

1) that a 2 x 2 unitary matrix with det U = 1 has the general form

$$U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}, \quad |a|^2 + |b|^2 = 1$$

2) that $\mathrm{e}^{\mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{\theta}/2} = \cos\theta/2 + \mathrm{i}\boldsymbol{\sigma}\cdot\mathbf{n}\sin\theta/2$

3) that $U = \mathrm{e}^{\mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{\theta}/2} = \cos\theta/2 + \mathrm{i}\boldsymbol{\sigma}\cdot\mathbf{n}\sin\theta/2$

satisfies $U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}, \quad |a|^2 + |b|^2 = 1$

SU(2) transformation:

$$U = e^{i\boldsymbol{\sigma}\cdot\boldsymbol{\theta}/2} = \cos\theta/2 + i\boldsymbol{\sigma}\cdot\mathbf{n}\sin\theta/2$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\left[\frac{\sigma_k}{2}, \frac{\sigma_l}{2}\right] = i\epsilon_{klm}\frac{\sigma_m}{2}$$

SO(3) transformation:

$$R = e^{i\vec{J}\cdot\vec{\theta}}$$

$$[J_k, J_l] = i\epsilon_{klm}J_m$$

for rotation around the y-axis:

$$U = \begin{pmatrix} \cos\beta/2 & \sin\beta/2 \\ -\sin\beta/2 & \cos\beta/2 \end{pmatrix} \leftrightarrow R = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$

SU(2) transformation acts on spinors:

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}; \quad \xi \to U\xi, \quad \xi^\dagger \to \xi^\dagger U^\dagger \quad \Rightarrow \quad \xi^\dagger\xi = |\xi_1|^2 + |\xi_2|^2$$

under 2π rotation $\xi \to -\xi$

Lorentz Group SO(3,1): Rotations (J_x, J_y, J_z) + Boosts (K_x, K_y, K_z)

Boost:
$$\begin{pmatrix} x^{0\prime} \\ x^{1\prime} \\ x^{2\prime} \\ x^{3\prime} \end{pmatrix} = \begin{pmatrix} \cosh\phi & \sinh\phi & 0 & 0 \\ \sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$R_x(\delta\phi) = 1 + iJ_x\delta\phi$$

$$K_x = \frac{1}{\mathrm{i}}\frac{\partial B}{\partial\phi}\bigg|_{\phi=0} = -\mathrm{i}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$J_x = \frac{1}{\mathrm{i}}\frac{\mathrm{d}R_x(\phi)}{\mathrm{d}\phi}\bigg|_{\phi=0} = -\mathrm{i}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$[K_x, K_y] = -\mathrm{i}J_z$ and cyclic perms.

$[J_x, K_x] = 0$ etc.,

$[J_x, K_y] = \mathrm{i}K_z$ and cyclic perms,

$[J_x, J_y] = \mathrm{i}J_z$ and cyclic permutations.

$$\mathbf{K} = \pm\mathrm{i}\frac{\boldsymbol{\sigma}}{2}$$

$$\mathbf{A} = \tfrac{1}{2}(\mathbf{J} + \mathrm{i}\mathbf{K}) \qquad \mathbf{B} = \tfrac{1}{2}(\mathbf{J} - \mathrm{i}\mathbf{K})$$

$\Rightarrow$

$$\left.\begin{aligned} &[A_x,\, A_y] = \mathrm{i}A_z \text{ and cyclic perms,} \\ &[B_x,\, B_y] = \mathrm{i}B_z \text{ and cyclic perms,} \\ &[A_i,\, B_j] = 0\ (i,\, j = x,\, y,\, z). \end{aligned}\right\} \quad SU(2) \otimes SU(2)$$

under rotation by θ, boost by ϕ:

$$\text{Type I:} \quad (\tfrac{1}{2}, 0): \quad \mathbf{J}^{(1/2)} = \boldsymbol{\sigma}/2, \quad \mathbf{K}^{(1/2)} = -\mathrm{i}\boldsymbol{\sigma}/2. \qquad \xi \to \exp\left[\mathrm{i}\frac{\boldsymbol{\sigma}}{2} \cdot (\boldsymbol{\theta} - \mathrm{i}\boldsymbol{\phi})\right]\xi \equiv M\xi.$$

$$\text{Type II:} \quad (0, \tfrac{1}{2}): \quad \mathbf{J}^{(1/2)} = \boldsymbol{\sigma}/2, \quad \mathbf{K}^{(1/2)} = \mathrm{i}\boldsymbol{\sigma}/2. \qquad \eta \to \exp\left[\mathrm{i}\frac{\boldsymbol{\sigma}}{2} \cdot (\boldsymbol{\theta} + \mathrm{i}\boldsymbol{\phi})\right]\eta \equiv N\eta$$

$$P : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}. \qquad \begin{aligned} \mathbf{v} &\to -\mathbf{v} \\ \mathbf{K} &\to -\mathbf{K} \\ \mathbf{J} &\to +\mathbf{J} \end{aligned}$$

$\Rightarrow$ under parity $\xi \leftrightarrow \eta$.

review: Dirac spinors

$$\psi = \begin{pmatrix} \xi \\ \eta \end{pmatrix} \Rightarrow \psi = \begin{pmatrix} \phi_R \\ \phi_L \end{pmatrix}$$

is the Dirac spinor: $(i\gamma^\mu \partial_\mu - m)\psi = 0$

$$\left(\gamma^\mu p_\mu - m\right)\psi = 0$$

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$$

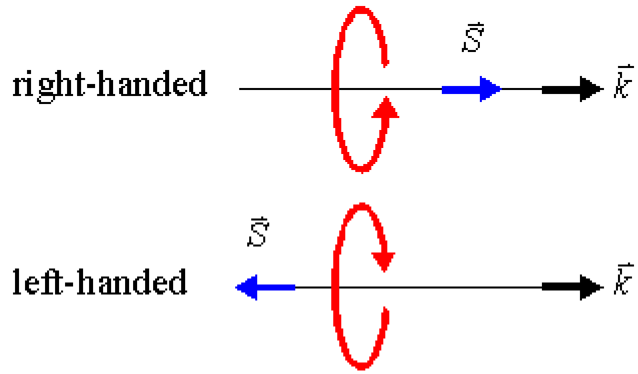
$$\left.\begin{aligned} \gamma^0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \gamma^i &= \begin{pmatrix} 0 & -\sigma_i \\ \sigma_i & 0 \end{pmatrix} \end{aligned}\right\} \begin{pmatrix} -m & p_0 + \boldsymbol{\sigma}\cdot\mathbf{p} \\ p_0 - \boldsymbol{\sigma}\cdot\mathbf{p} & -m \end{pmatrix} \begin{pmatrix} \phi_R(\mathbf{p}) \\ \phi_L(\mathbf{p}) \end{pmatrix} = 0$$

For massless particles:

$$(p_0 + \boldsymbol{\sigma}\cdot\mathbf{p})\phi_L(\mathbf{p}) = 0,$$

$$(p_0 - \boldsymbol{\sigma}\cdot\mathbf{p})\phi_R(\mathbf{p}) = 0.$$

$$\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\,\phi_L = -\phi_L, \quad \boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\,\phi_R = \phi_R.$$



ϕ_R : right-handed fermion
or left-handed antifermion

$$\phi_R \to \exp\left[\frac{i}{2}\boldsymbol{\sigma}\cdot(\boldsymbol{\theta} - i\boldsymbol{\phi})\right]\phi_R, \quad \phi_L \to \exp\left[\frac{i}{2}\boldsymbol{\sigma}\cdot(\boldsymbol{\theta} + i\boldsymbol{\phi})\right]\phi_L$$

$$\phi_R^\dagger \to \phi_R^\dagger \exp\left[\frac{-i}{2}\boldsymbol{\sigma}\cdot(\boldsymbol{\theta} + i\boldsymbol{\phi})\right], \quad \phi_L^\dagger \to \phi_L^\dagger \exp\left[\frac{-i}{2}\boldsymbol{\sigma}\cdot(\theta - i\phi)\right]$$

$$\bar{\psi} = \psi^\dagger\gamma^0 = (\phi_R^\dagger \phi_L^\dagger)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (\phi_L^\dagger \phi_R^\dagger) \Rightarrow \bar{\psi}\psi = \phi_L^\dagger\phi_R + \phi_R^\dagger\phi_L$$

scalar: invariant under Lorentz and P

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow \bar{\psi}\gamma^5\psi = \phi_L^\dagger\phi_R - \phi_R^\dagger\phi_L$$

pseudoscalar: changes sign under P

Bilinear Covariants:

$\bar{\psi}\psi$ scalar,

$\bar{\psi}\gamma_5\psi$ pseudoscalar,

$\bar{\psi}\gamma^\mu\psi$ vector,

$\bar{\psi}\gamma^\mu\gamma^5\psi$ axial vector,

$\bar{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\psi$ antisymmetric tensor.

Chirality operator: $\gamma^5 = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}$ (chirality same as helicity for m = 0)

$$P_L = \frac{1-\gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$
$$P_R = \frac{1+\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_L^2 = P_L, \quad P_R^2 = P_R, \quad P_L + P_R = 1, \quad P_L P_R = 0.$$

$$u_L = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} U_R \\ U_L \end{pmatrix} = \begin{pmatrix} 0 \\ U_L \end{pmatrix}$$
$$u_R = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} U_R \\ U_L \end{pmatrix} = \begin{pmatrix} U_R \\ 0 \end{pmatrix}$$
$$(\psi_L)^c = (\psi^c)_R$$

parity is broken in the SM: e_L and e_R interact differently.
ν_L exists but ν_R does not

HMW1: Show the following:

vector current preserves chirality: $\bar{\psi}\gamma^\mu\psi = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R$

mass term is a chirality flip: $\bar{\psi}\psi = (\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$

L current is same as V – A: $\bar{\psi}_L\gamma^\mu\psi_L = \frac{1}{2}\bar{\psi}\gamma^\mu(1-\gamma^5)\psi$

hint: $P_L\gamma^\mu = \gamma^\mu P_R$ and $P_R\gamma^\mu = \gamma^\mu P_L$, $\{\gamma^\mu, \gamma^5\} = 0$, also note: $\bar{\psi}_L \equiv \overline{(\psi_L)}$

Main symmetries of the SM:

Symmetry		Conservation Law
Poincaré	Translations×SO(3,1)	Energy, Momentum, Angular momentum
Gauge	SU(3)×SU(2)×U(1)	Color charge, Weak isospin, Hypercharge

note: $K_x = x\ E - t\ p_x$

review: gauge invariance

$$\Psi \rightarrow \Psi' = e^{-i\alpha}\Psi$$: global gauge transformation

$$\Psi(\vec{x},t) \rightarrow \Psi'(\vec{x},t) = e^{-i\chi(\vec{x},t)}\Psi(\vec{x},t)$$: local gauge transformation

$$-\frac{1}{2m}\nabla^2\Psi(\vec{x},t) = i\frac{\partial\Psi(\vec{x},t)}{\partial t}$$ not invariant.

$$\frac{1}{2m}\left(-i\nabla + e\vec{A}\right)^2\Psi = \left(i\frac{\partial}{\partial t} + eV\right)\Psi$$ is invariant, if:

$$\left.\begin{aligned} \Psi(\vec{x},t) &\rightarrow \Psi'(\vec{x},t) = e^{-i\chi}\Psi(\vec{x},t) \\ A &\rightarrow A' = A + \tfrac{1}{e}\nabla\chi \\ V &\rightarrow V' = V - \tfrac{1}{e}\frac{\partial\chi}{\partial t} \end{aligned}\right\}$$

Existence of EM field and photon is *required* by invariance under

$$\Psi(\vec{x},t) \rightarrow \Psi'(\vec{x},t) = e^{-i\chi(\vec{x},t)}\Psi(\vec{x},t)$$

Electromagnetism is a U(1) gauge theory (interactions determined from invariance under local U(1) gauge transformation)

Gauge transformation: $\Psi' = U\Psi$

$\mathcal{D}^{\mu} = \partial^{\mu} - igA^{\mu}$ where A^{μ} is an interacting field **imposed** by gauge invariance

$\mathcal{D}^{\mu\,\prime}\Psi' = U\,(\mathcal{D}^{\mu}\Psi)$ Covariant derivative: transforms like Ψ

$$(\partial^{\mu} - igA^{\mu\,\prime})\,U\Psi = U\,(\partial^{\mu} - igA^{\mu})\,\Psi \;\Rightarrow\; A^{\mu\,\prime} = -\frac{i}{g}(\partial^{\mu}U)\,U^{-1} + U\,A^{\mu}U^{-1}$$

(exercise – optional)

Take g = |e| and U=$e^{-i\chi(x,t)}$, and obtain

the Schrödinger equation for charged particle and

the gauge transformations shown in the previous slide.

Dirac Lagrangian for free particle: $\mathcal{L}_\psi = \bar{\psi}(i\,\partial\!\!\!/ - m)\psi$

not invariant under $\psi \to \psi' = \exp\left[-i\alpha(x)\right]\psi$

Introduce gauge field:

$$D_\mu \equiv \partial_\mu + ieA_\mu\ ,\ \ A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha\ .$$

$$\begin{aligned}
\mathcal{L}_\psi \to \mathcal{L}'_\psi &= \bar{\psi}'\left[(i\,\partial\!\!\!/ - e\,A\!\!\!/') - m\right]\psi' \\
&= \bar{\psi}\exp(+i\alpha)\left[i\,\partial\!\!\!/ - e\left(A\!\!\!/ + \frac{1}{e}\,\partial\!\!\!/\alpha\right) - m\right]\exp(-i\alpha)\psi \\
&= \mathcal{L}_\psi - \underbrace{e\bar{\psi}\gamma_\mu\psi A^\mu}.
\end{aligned}$$

Demanding gauge invariance determines the interaction term.

U(1) Gauge Theory $\Psi(\vec{x},t) \rightarrow \Psi'(\vec{x},t) = e^{-i\chi(\vec{x},t)}\Psi(\vec{x},t)$

$$\mathcal{D}^\mu = \partial^\mu - igA^\mu$$

Non-Abelian (Yang-Mills) Gauge Theory

SU(2): $\chi(\vec{x},t)$ ➡ $\vec{\epsilon}\cdot\vec{\tau}$

$$\mathcal{D}^\mu = \partial^\mu - ig_2\frac{\vec{\tau}}{2}\cdot\vec{W^\mu}$$

⬇

3 spin 1 gauge bosons

Purpose of W_i^μ is to make theory SU(2) invariant

if $W_i'^\mu = W_i^\mu + \delta W_i^\mu$, where $\delta W_i^\mu = \frac{1}{g_2}\partial^\mu\epsilon_i - \epsilon_{ijk}\epsilon_j W_k$

$\mathcal{D}^\mu$ transforms covariantly: $D'^\mu\psi' = e^{i\vec{\epsilon}(x)\cdot\vec{\tau}/2}\mathcal{D}^\mu\psi$

U(1)

$$A^\mu \rightarrow A^{\mu\,\prime} = A^\mu - \partial^\mu \chi / e$$

SU(2)

$$\delta W_i^\mu = \frac{1}{g_2}\partial^\mu \epsilon_i - \epsilon_{ijk}\epsilon_j W_k$$

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -\varepsilon & 0 \\ \varepsilon & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} V_x - \varepsilon V_y \\ V_y + \varepsilon V_x \\ V_z \end{pmatrix}$$

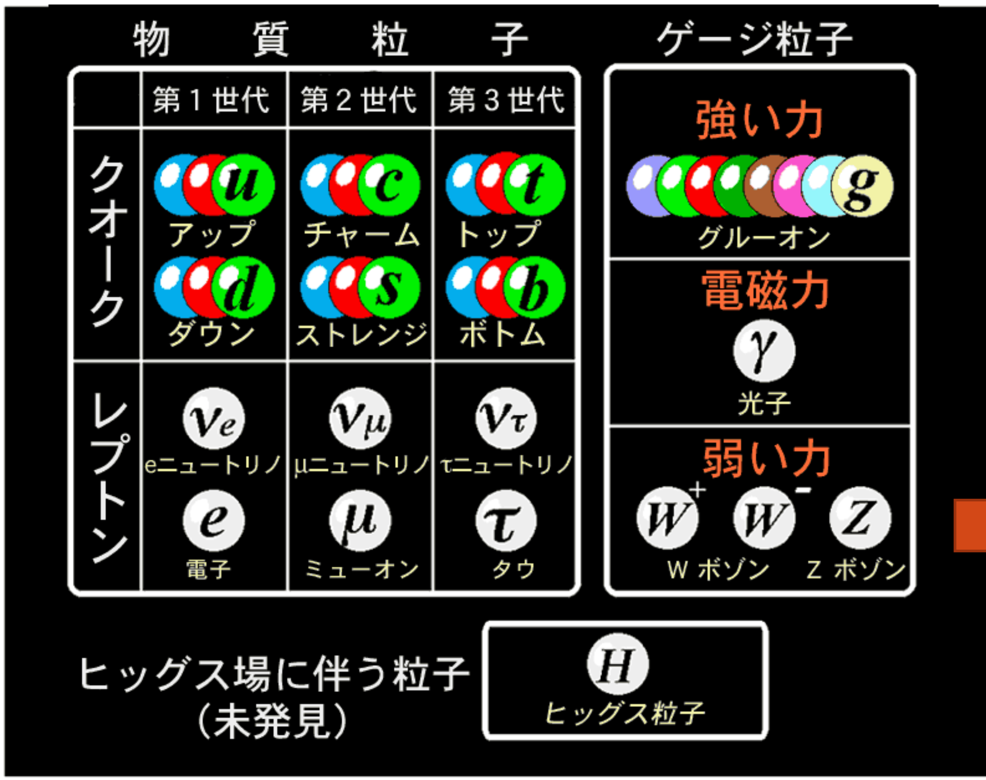
$$\delta V_i = \epsilon_{ijk}\varepsilon_j V_k$$

$$\mathcal{D}^\mu = \partial^\mu - i g_2 \frac{\vec{\tau}}{2} \cdot \vec{W^\mu}$$ acts on doublets

a state with isospin t : 2t + 1 components

$$\mathcal{D}^\mu = \partial^\mu - i g_2 \vec{T} \cdot \vec{W_\mu}$$ where $\vec{T}$ is the $(2t+1) \times (2t+1)$ representation of SU(2) generators

The Standard Model: $SU(3)_C \times SU(2)_L \times U(1)_Y$



SU(2):

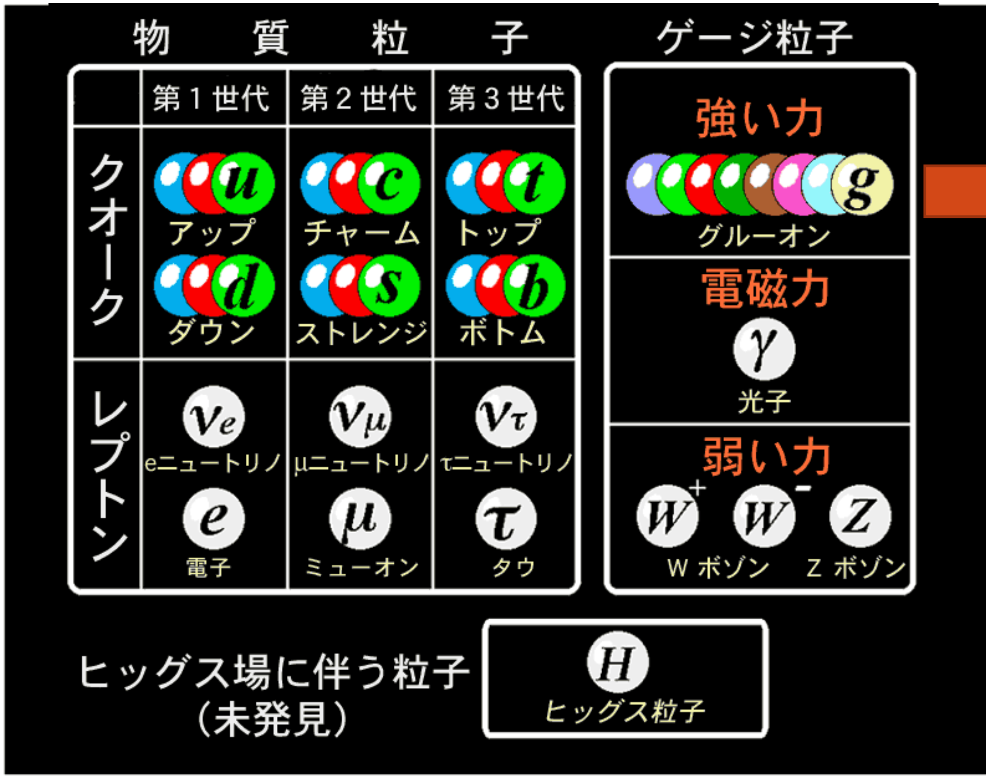
$$U = e^{i\vec{\sigma}\cdot\vec{\theta}/2}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\left[\frac{\sigma_k}{2}, \frac{\sigma_l}{2}\right] = i\epsilon_{klm}\frac{\sigma_m}{2}$$

3 generators for SU(2),

1 generator for U(1)



SU(3):

3^2-1=8 generators

$$U = \exp\left(\sum_{j=1}^{8} i\,\alpha_j \lambda_j\right)$$

$$[\lambda_a,\ \lambda_b] = 2if_{abc}\lambda_c$$

$$\begin{pmatrix} a'_1 \\ a'_2 \\ a'_3 \end{pmatrix} = e^{i\vec{\alpha}\cdot\vec{\lambda}/2} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

Apply $SU(3)_C$ transformation (rotate quarks in $SU(3)_C$ space) and demand invariance.

SU(2)$_L$ representation of leptons & quarks

weak isospin

$$e^-_R = P_R \psi_{e^-} \qquad e^-_R = SU(2) \text{ singlet} \qquad (T=0)$$

$$e^-_L = P_L \psi_{e^-}$$

$$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

T=1/2, T_3=1/2 for ν_{eL}, T_3=–1/2 for e^-_L

connected by W bosons (playing the role of J^+, J^-)

$$Q_{L\alpha} = \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix}_L \qquad d_{R\alpha},\ u_{R\alpha}$$

SU(3)$_C$ representation: leptons are color singlets,

quarks are color triplets (r, g, b)

connected by 8 gluons

$SU(3)_C \times SU(2)_L \times U(1)_Y$ representation		$Q = T_3 + Y/2$
$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$	$(1, 2, -1)$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
e^-_R	$(1, 1, -2)$	-1
$Q_{L\alpha} = \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix}_L$	$\left(3, 2, \frac{1}{3}\right)$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
$u_{R\alpha}$	$\left(3, 1, \frac{4}{3}\right)$	$2/3$
$d_{R\alpha}$	$\left(3, 1, -\frac{2}{3}\right)$	$-1/3$

3 families: $\left.\begin{matrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} \\ \begin{pmatrix} u \\ d \end{pmatrix} \end{matrix}\right\} \longrightarrow \begin{matrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \\ \begin{pmatrix} c \\ s \end{pmatrix} \end{matrix}$ or $\begin{matrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \\ \begin{pmatrix} t \\ b \end{pmatrix} \end{matrix}$

The Standard Model: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{D}^\mu = \partial^\mu - ig_1 \frac{Y}{2} B^\mu - ig_2 \frac{\tau_i}{2} W_i^\mu - ig_3 \frac{\lambda_a}{2} G_a^\mu$$

hypercharge

3 Pauli matrices

8 Gell-Mann matrices

$- ig_2 \frac{\tau_i}{2} W_i^\mu$ is a Lorentz vector,
is a 2 x 2 matrix in SU(2) space (if acting on a SU(2) doublet),
is a singlet ($\propto$ unit matrix) in SU(3) space

The Standard Model: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{D}^\mu = \partial^\mu - ig_1 \frac{Y}{2} B^\mu - ig_2 \frac{\tau_i}{2} W_i^\mu - ig_3 \frac{\lambda_a}{2} G_a^\mu$$

B^μ, W_i^μ, G_a^μ : Gauge bosons

$SU(3)_C \times SU(2)_L \times U(1)_Y$ representation	
B^μ	$(1, 1, 0)$
W_i^μ	$(1, 3, 0)$
G_a^μ	$(8, 1, 0)$

γ is neutral – particles emitting γ just change p, not charge

gluons are colored [charged under SU(3)]

for weak interaction there is both CC (charged current) and NC (neutral current)

The Standard Model: $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{D}^\mu = \partial^\mu - ig_1 \frac{Y}{2} B^\mu - ig_2 \frac{\tau_i}{2} W_i^\mu - ig_3 \frac{\lambda_a}{2} G_a^\mu$$

B^μ, W_i^μ, G_a^μ : Gauge bosons

g_1, g_2, g_3 : Gauge couplings (universal - same for all representations)

measure g_2 for one process (muon decay), use it for any other process

Non-Abelian part: couplings of fermions determined from the gauge symmetry

Abelian part: Y (hypercharge) different for different representations
like the electric charge

The Standard Model Lagrangian: $\mathcal{L}_{SM} = \mathcal{L}_{Gauge} + \mathcal{L}_{Matter} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs}$

$$\overline{\psi}\gamma^\mu\partial_\mu\psi \rightarrow \overline{\psi}\gamma^\mu\mathcal{D}_\mu\psi\ , \qquad \mathcal{D}_\mu = \partial_\mu - ig_1\frac{Y}{2}B_\mu - ig_2\frac{\tau^i}{2}W_\mu^i - ig_3\frac{\lambda^a}{2}G_\mu^a\ .$$

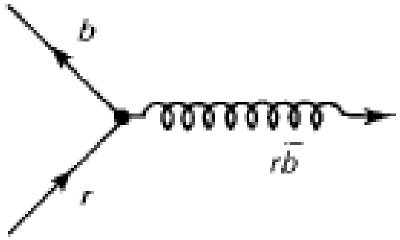
$$\mathcal{L}_{Matter} = \sum_{\substack{f = L,\, e_R,\\ Q_L,\, u_R,\, d_R}} \overline{f}i\gamma^\mu\mathcal{D}_\mu f\ .$$

the g_1, g_2 and g_3 terms only act on non-singlets:

$\tau^i W^i$ gives zero acting on e_R, u_R, d_R

$\lambda^a G^a$ gives zero acting on the leptons (L, e_R)

QCD Lagrangian: $\mathcal{L}_{Matter} = \frac{g_3}{2}\bar{q}_\alpha \gamma^\mu \lambda^a_{\alpha\beta} G^a_\mu q_\beta$



$$q + g \to q', \quad q \to q' + g, \quad g \to q\bar{q}$$

quarks are color triplets (r, g, b),

anti-quarks are color anti-triplets ($\bar{r}, \bar{g}, \bar{b}$)

gluons belong to a color octet [8 linearly independent gluons corresponding to 8 generators of SU(3)]

$$\left\{ \begin{array}{ll} |1\rangle = (r\bar{b} + b\bar{r})/\sqrt{2} & |5\rangle = -i(r\bar{g} - g\bar{r})/\sqrt{2} \\ |2\rangle = -i(r\bar{b} - b\bar{r})/\sqrt{2} & (6) = (b\bar{g} + g\bar{b})/\sqrt{2} \\ |3\rangle = (r\bar{r} - b\bar{b})/\sqrt{2} & |7\rangle = -i(b\bar{g} - g\bar{b})/\sqrt{2} \\ 14) = (r\bar{g} + g\bar{r})/\sqrt{2} & |8\rangle = (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6} \end{array} \right\}$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$|9\rangle = (r\bar{r} + b\bar{b} + g\bar{g})/\sqrt{3}$ would be a color singlet, which does not participate in strong interaction.

brief remarks on QCD

details: QCD lectures by Kazım Azizi on Friday and Saturday

bound states in QED: atoms (including exotic atoms - positronium, muonic hydrogen, ...)

bound states in QCD: hadrons

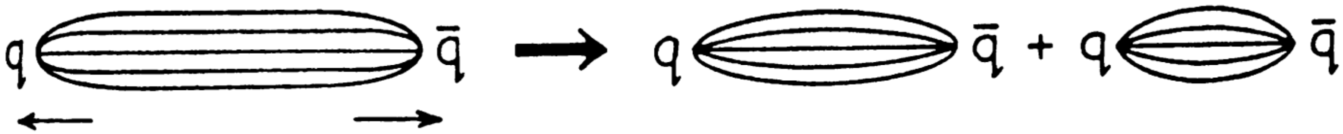
main difference of QCD: potential E of colored particles increases with r

due to gluon self-interaction

↓

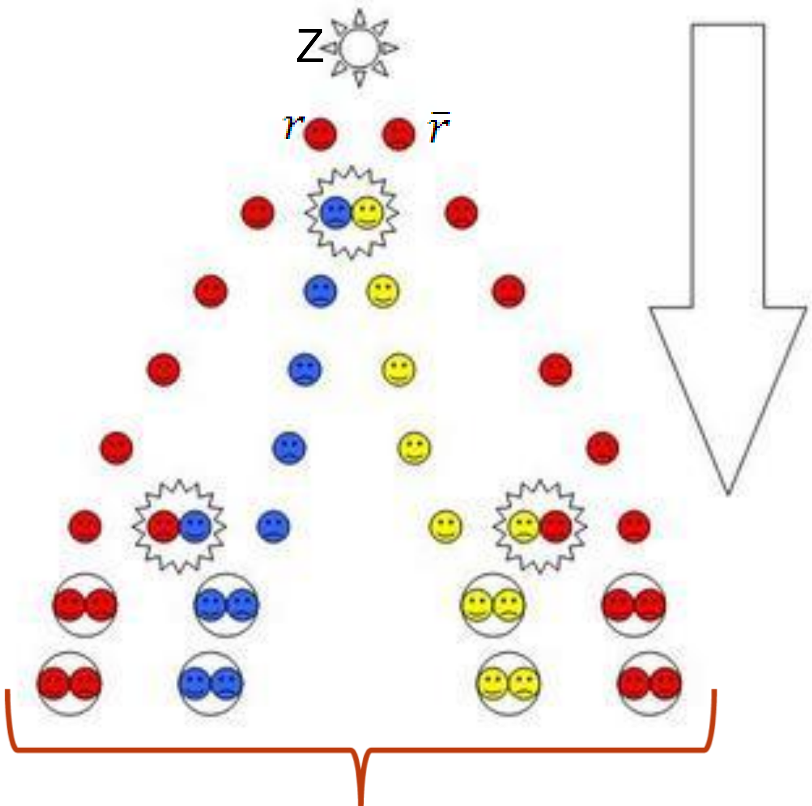
you can never separate off a colored particle

Confinement: only color-singlet combinations of quarks and gluons (hadrons) can be separated to >~ 1 fm and appear in detectors



Colliding hadrons creates more hadrons (mostly pions) in a jet:

the energetic colored particles hadronize as they separate



two jets (collimated stream of hadrons)

Hadrons: two basic ways to make color singlets out of quarks:

1. Mesons: $(r\bar{r} + g\bar{g} + b\bar{b})/\sqrt{3}$

   pions, kaons, ...

2. Baryons: $\epsilon_{ijk} q_i q_j q_k$

   protons, neutrons, ...

Exotic Hadrons:

tetraquarks ($q\bar{q}q\bar{q}$) and pentaquarks ($qqqq\bar{q}$)
also color singlets, but are they stable enough?

gluonic hadrons (glueballs and hybrids)

Electroweak Theory

back to the Standard Model Lagrangian:

$$\mathcal{L}_{Matter} = \sum_{\substack{f = L,\, e_R,\\ Q_L,\, u_R,\, d_R}} \overline{f} i\gamma^\mu \mathcal{D}_\mu f \, . \qquad \mathcal{D}_\mu = \partial_\mu - ig_1 \frac{Y}{2} B_\mu - ig_2 \frac{\tau^i}{2} W_\mu^i - ig_3 \frac{\lambda^a}{2} G_\mu^a \, .$$

We now look at $SU(2)_L$ x $U(1)_Y$ (electroweak) part

$U(1)_Y$ terms:

$$-\mathcal{L}_{\text{ferm}}(U(1)\,,\text{leptons}) = \overline{L} i\gamma^\mu \left(ig_1 \frac{Y_L}{2} B_\mu \right) L + \overline{e}_R i\gamma^\mu \left(ig_1 \frac{Y_R}{2} B_\mu \right) e_R \, .$$

$$\mathcal{L}_{U(1)} = \frac{g_1}{2} J_Y^\mu B_\mu \qquad \text{where} \qquad J_Y^\mu = (\bar{L}\gamma^\mu Y_L L + \bar{e}_R \gamma^\mu Y_R e_R)$$

$$= (\bar{e}_L \gamma^\mu Y_L e_L + \bar{\nu}_L \gamma^\mu Y_L \nu_L + \bar{e}_R \gamma^\mu Y_R e_R)$$

$$\mathcal{L}_{Matter} = \sum_{\substack{f = L,\, e_R,\\ Q_L,\, u_R,\, d_R}} \bar{f} i\gamma^\mu \mathcal{D}_\mu f \,. \qquad \mathcal{D}_\mu = \partial_\mu - ig_1 \frac{Y}{2} B_\mu - ig_2 \frac{\tau^i}{2} W^i_\mu - ig_3 \frac{\lambda^a}{2} G^a_\mu \,.$$

$U(1)_Y$ terms:

$$-\mathcal{L}_{\text{ferm}}(U(1)\,,\text{leptons}) = \bar{L} i\gamma^\mu \left(ig_1 \frac{Y_L}{2} B_\mu \right) L + \bar{e}_R i\gamma^\mu \left(ig_1 \frac{Y_R}{2} B_\mu \right) e_R \,.$$

$$\mathcal{L}_{U(1)} = \frac{g_1}{2} J^\mu_Y B_\mu \qquad \text{where} \qquad J^\mu_Y = (\bar{L}\gamma^\mu Y_L L + \bar{e}_R \gamma^\mu Y_R e_R)$$

$$= (\bar{e}_L \gamma^\mu Y_L e_L + \bar{\nu}_L \gamma^\mu Y_L \nu_L + \bar{e}_R \gamma^\mu Y_R e_R)$$

$SU(2)_L$ terms:

$$-\mathcal{L}_{\text{ferm}}(SU(2)\,,\ \text{leptons}) = \bar{L} i\gamma^\mu \left[ig_2 \frac{\tau^i}{2} W^i_\mu \right] L$$

$$= -\frac{g_2}{2} (\bar{\nu}_L \quad \bar{e}_L) \gamma^\mu \begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\mathcal{L}_{SU(2),\text{neutral}} = g_2 \bar{L} \gamma^\mu \frac{\tau^3}{2} W^3_\mu L = g_2 J^\mu_3 W^3_\mu$$

$$J^\mu_3 = \bar{L} \gamma^\mu \frac{\tau^3}{2} L = \frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma^\mu e_L$$

$SU(2)_L \times U(1)_Y$ terms involving neutral gauge bosons:

$$\frac{g_1}{2} J_Y^\mu B_\mu + g_2 J_3^\mu W_\mu^3 \quad ,$$

$$J_Y^\mu = (\bar{e}_L \gamma^\mu Y_L e_L + \bar{\nu}_L \gamma^\mu Y_L \nu_L + \bar{e}_R \gamma^\mu Y_R e_R)$$

$$J_3^\mu = \underline{\frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L} - \frac{1}{2} \bar{e}_L \gamma^\mu e_L$$

Hypercharge normalization: $Y_L = -1$

$$J_Y^\mu = (-\bar{e}_L \gamma^\mu e_L \underline{- \bar{\nu}_L \gamma^\mu \nu_L} + \bar{e}_R \gamma^\mu Y_R e_R)$$

$Y_R \rightarrow -2$

Compare with electromagnetism:

$$\mathcal{L}_{EM} = -e(\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R) A_\mu = e J_{EM}^\mu A_\mu$$

$$J_{EM}^\mu = -(\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R)$$

Higgs mechanism: $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

$$J_{EM}^\mu = J_3^\mu + \frac{J_Y^\mu}{2} \Longrightarrow Q = T_3 + \frac{Y}{2}$$

$SU(2)_L \times U(1)_Y$ terms involving neutral gauge bosons:

$$\frac{g_1}{2} J_Y^\mu B_\mu + g_2 J_3^\mu W_\mu^3 \ ,$$

$$J_Y^\mu = (-\bar{e}_L \gamma^\mu e_L - \bar{\nu}_L \gamma^\mu \nu_L - 2\bar{e}_R \gamma^\mu e_R)$$

$$J_3^\mu = \frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma^\mu e_L$$

Higgs mechanism: (a combination of B_μ and $W_\mu{}^3$) $\rightarrow A_\mu$

neutrino terms:

$$\underbrace{\left(-\frac{g_1}{2} B_\mu + \frac{g_2}{2} W_\mu^3\right)}_{\perp A_\mu} \bar{\nu}_L \gamma^\mu \nu_L$$

$$\Rightarrow \quad Z_\mu = \frac{-g_1 B_\mu + g_2 W_\mu^3}{\sqrt{g_1^2 + g_2^2}}, \quad A_\mu = \frac{g_2 B_\mu + g_1 W_\mu^3}{\sqrt{g_1^2 + g_2^2}}$$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

$$\cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$$

where $\sin^2\theta_w = 0.23$

θ_w : weak (Weinberg) mixing angle

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \Rightarrow \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

$SU(2)_L$ x $U(1)_Y$ terms involving neutral gauge bosons:

$$\frac{g_1}{2} J_Y^\mu B_\mu + g_2 J_3^\mu W_\mu^3$$

where

$$J_Y^\mu = (-\bar{e}_L \gamma^\mu e_L - \bar{\nu}_L \gamma^\mu \nu_L - 2\bar{e}_R \gamma^\mu e_R)$$

$$J_3^\mu = \frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L - \frac{1}{2} \bar{e}_L \gamma^\mu e_L$$

Write these in terms of A_μ and Z_μ. For A_μ show that:

$$-g_2 \sin\theta_W \, (\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R) A_\mu$$

Which means: $e = g_2 \sin\theta_W = g_1 \cos\theta_W$

Fine structure constant: $\alpha \equiv \frac{e^2}{\hbar c (4\pi\epsilon_0)} \cong \frac{1}{137}$

Natural units: $\hbar = 1,\quad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 1$, $e = \sqrt{4\pi\alpha} \cong 0.30$ in units $\epsilon_0 = \mu_0 = 1$

Different units can be used. In α you can trust.

$$\mathcal{L} = \frac{g_1}{2} J_Y^\mu B_\mu + g_2 J_3^\mu W_\mu^3$$

$$\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix}$$

$$J_Y^\mu = (-\bar{e}_L \gamma^\mu e_L - \bar{\nu}_L \gamma^\mu \nu_L - 2\bar{e}_R \gamma^\mu e_R)$$

$$J_3^\mu = \frac{1}{2}\bar{\nu}_L \gamma^\mu \nu_L - \frac{1}{2}\bar{e}_L \gamma^\mu e_L$$

$$e = g_2 \sin\theta_W = g_1 \cos\theta_W$$

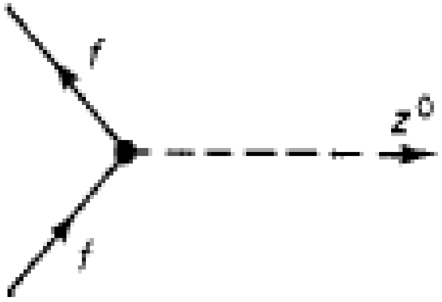
Prediction of SM: Neutral current mediated by Z_μ

$$\mathcal{L}_{NC} = \left(-\frac{g_1}{2} J_Y^\mu \sin\theta_W + g_2 J_3^\mu \cos\theta_W\right) Z_\mu = \frac{g_2}{\cos\theta_W}\left[J_3^\mu - \sin^2\theta_W \left(J_3^\mu + \frac{J_Y^\mu}{2}\right)\right] Z_\mu$$

$$= \frac{g_2}{\cos\theta_W}[J_3^\mu - J_{EM}^\mu \sin^2\theta_W] Z_\mu$$

$$J_{EM}^\mu = \sum_{L,R} Q_f \bar{f}\gamma^\mu f, \qquad J_3^\mu = \sum_{L,R} T_3^f \bar{f}\gamma^\mu f$$

$$\mathcal{L}_{NC} = \frac{g_2}{\cos\theta_W} \sum_{L,R} (T_3^f - Q_f \sin^2\theta_W)\, \bar{f}\gamma^\mu f Z_\mu$$



examples: ν_L (1/2), u_R ($-2\sin^2\theta_W$ / 3)

d_L ($-1/2 + \sin^2\theta_W$ / 3), ...

HMW2: Show that $\mathcal{L}_{NC} = \frac{g_2}{\cos\theta_W} \sum_{L,R} \left(T_3^f - Q_f \sin^2\theta_W\right) \bar{f}\gamma^\mu f Z_\mu$

can also be expressed as:

$$\mathcal{L}_{NC} = \frac{g_2}{2\cos\theta_W} \sum \bar{f}\gamma^\mu \left(g_V^f - g_A^f \gamma_5\right) f Z_\mu$$

where the sum is over fermions (e, ν, ...), $g_V^f = T_3^f - 2Q_f \sin^2\theta_W$

$$g_A^f = T_3^f$$

$$-\mathcal{L}_{\rm ferm}(SU(2)\,,\,{\rm leptons}) = \overline{L} i\gamma^\mu \left[i g_2 \frac{\tau^i}{2} W^i_\mu \right] L$$

$$= -\frac{g_2}{2} \begin{pmatrix} \overline{\nu}_L & \overline{e}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= -\frac{g_2}{2} \begin{pmatrix} \overline{\nu}_L & \overline{e}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} W^0_\mu & -\sqrt{2} W^+_\mu \\ -\sqrt{2} W^-_\mu & -W^0_\mu \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= -\frac{g_2}{2} \begin{pmatrix} \overline{\nu}_L & \overline{e}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} W^0_\mu \nu_L - \sqrt{2} W^+_\mu e_L \\ -\sqrt{2} W^-_\mu \nu_L - W^0_\mu e_L \end{pmatrix}$$

$$= -\frac{g_2}{2} \left[\overline{\nu}_L \gamma^\mu \nu_L W^0_\mu - \sqrt{2}\, \overline{\nu}_L \gamma^\mu e_L W^+_\mu - \sqrt{2}\, \overline{e}_L \gamma^\mu \nu_L W^-_\mu - \overline{e}_L \gamma^\mu e_L W^0_\mu \right]$$

where $W^{\pm}_\mu = (-W^1_\mu \pm iW^1_\mu) \,/\, \sqrt{2}$

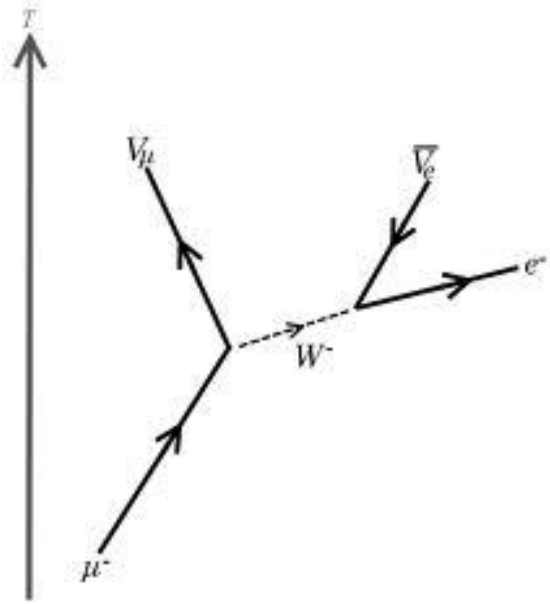
Charged current: $\mathcal{L}_{\rm ferm} = \frac{g_2}{\sqrt{2}} \left(\overline{\nu}_L \gamma^\mu e_L W^+_\mu + \overline{e}_L \gamma^\mu \nu_L W^-_\mu \right).$

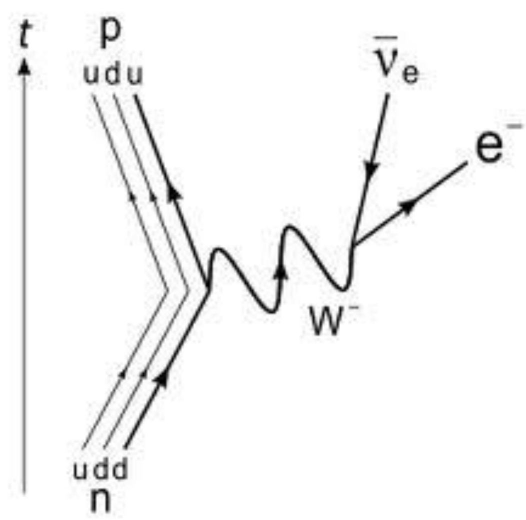
V – A

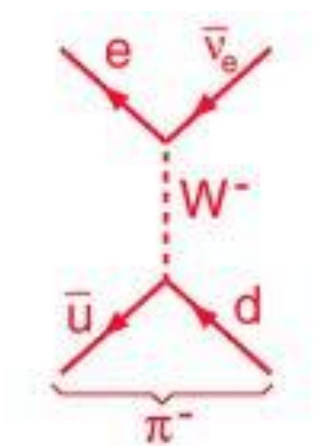
$$\overline{\nu}_L \gamma^\mu e_L = \frac{1}{2} \overline{\nu} \gamma^\mu (1 - \gamma_5) e$$

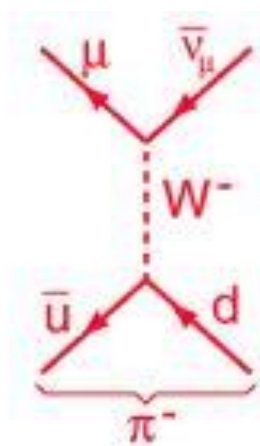
HMW3: **Explain why detecting the process $\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-$ would automatically demonstrate the existence of neutral currents, while detecting $\overline{\nu}_e + e^- \to \overline{\nu}_e + e^-$ would not.**

Examples of charged current processes: muon decay, beta decay, pion decay









The Standard Model Lagrangian: $\mathcal{L}_{SM} = \mathcal{L}_{Gauge} + \mathcal{L}_{Matter} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs}$

$$\mathcal{L}_{Matter} = \sum_{\substack{f = L,\, e_R,\\ Q_L,\, u_R,\, d_R}} \overline{f} i \gamma^\mu \mathcal{D}_\mu f \, . \qquad \mathcal{D}_\mu = \partial_\mu - i g_1 \frac{Y}{2} B_\mu - i g_2 \frac{\tau^i}{2} W^i_\mu - i g_3 \frac{\lambda^a}{2} G^a_\mu \, .$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\left(G^a_{\mu\nu} G^{\mu\nu}_a + W^i_{\mu\nu} W^{\mu\nu}_i + B_{\mu\nu} B^{\mu\nu}\right)$$

where field strength tensors are defined as:

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W^i_{\mu\nu} \equiv \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_2 \epsilon_{ijk} W^j_\mu W^k_\nu$$

$$G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_3 f_{abc} G^b_\mu G^c_\nu$$

kinetic terms $+ \boxed{W^+W^-A} + \boxed{W^+W^-Z}$

$+ \boxed{W^+W^-AA} + \boxed{W^+W^-ZZ} + \boxed{W^+W^-AZ} + \boxed{W^+W^-W^+W^-}$

So far we have: 4 massless gauge fields, massless fermions

Experiments: weak bosons and fermions are massive.

Mass terms such as

$$\begin{aligned} m\overline{\psi}\psi &= m\overline{\psi}(P_L + P_R)\psi \\ &= m\overline{\psi}P_L P_L\psi + m\overline{\psi}P_R P_R\psi \\ &= m\left(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R\right). \end{aligned}$$

or

$$\frac{1}{2}m_B^2 B^\mu B_\mu$$

are not gauge invariant (adding explicit mass term spoils predictivity of SM)

Higgs mechanism:

- makes W and Z bosons massive without spoiling predictivity
- keeps photon massless
- predicts masses of W and Z bosons in terms of measured quantities
- allows fermion masses
- is the most fragile part of SM

note: to be politically correct we should call it Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism, and don't forget Anderson too..

Higgs as the origin of mass? Yes, for W, Z, quarks and leptons. But:

→ baryonic mass is mostly QCD binding energy

→ baryonic mass accounts for 4% of cosmic energy budget

mass of a nucleon ~ 940 MeV

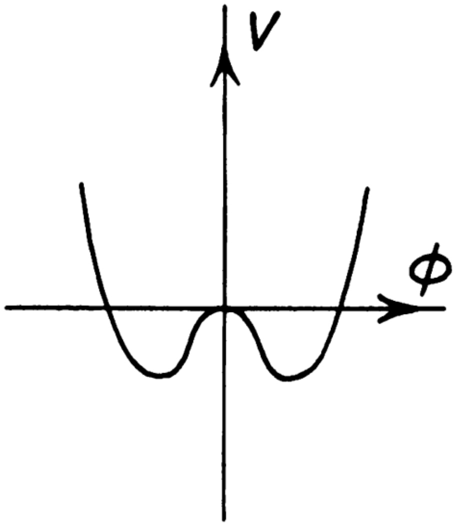
it consists of 3 u or d quarks having bare mass ~ 5 MeV each
(thanks to Higgs)

almost the entire mass of a nucleon is due to: QCD binding energy
(strong interactions with quarks & gluons)

problem: SU(2) x U(1) must be broken to allow mass terms,
but adding explicit mass terms spoils predictivity of SM

solution: Spontaneous symmetry breaking
Lagrangian is symmetric but the ground state (vacuum) is not

Toy example: real scalar with reflection symmetry



$$\mathcal{L} = T - V = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right) \quad , \quad \mu^2 < 0$$

$\phi = \pm\sqrt{\frac{-\mu^2}{\lambda}} \equiv v$, choose a ground state: $\phi(x) = v + \eta(x)$

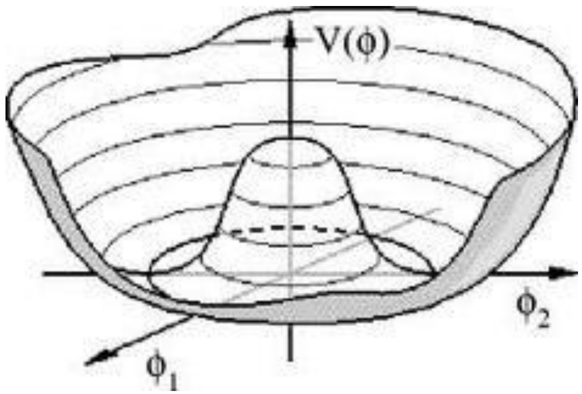
reflection symmetry is spontaneously broken:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta\partial^\mu\eta) - \left(\lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4\right) + \text{constant}$$

Complex scalar field with global U(1) symmetry:

$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$ invariant under $\phi \to \phi' = e^{i\chi} \phi$

$\phi = (\phi_1 + i\phi_2)/\sqrt{2},$ $$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 - \frac{1}{2}\mu^2 (\phi_1^2 + \phi_2^2) - \frac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2 .$$



minimum: $\phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda} = v^2.$

choose a point in the circle: $\phi_1 = v,\ \phi_2 = 0,$

$$\phi = \frac{(v + \eta(x) + i\rho(x))}{\sqrt{2}} .$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}(\partial_\mu \eta)^2 + \mu^2 \eta^2 - \lambda v (\eta \rho^2 + \eta^3) - \frac{\lambda}{2}\eta^2 \rho^2 - \frac{\lambda}{4}\eta^4 - \frac{\lambda}{4}\rho^4 + \text{constant} .$$

ρ: Goldstone Boson associated with SSB of global U(1)

Abelian Higgs Mechanism

Consider $\mathcal{L}$ invariant under local U(1): $\mathcal{L} = (\mathcal{D}_\mu \phi)^* (\mathcal{D}^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.$

$$\phi(x) \rightarrow \phi'(x) = e^{i\chi(x)} \phi(x) .$$

$$A_\mu \rightarrow A_\mu{}' = A_\mu - \frac{1}{g} \partial_\mu \chi(x)$$

$$\partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu - igA_\mu$$

$\mu^2 > 0$: charge particle interacting with photon

4 degrees of freedom

$\mu^2 < 0$:

$\phi(x) = \eta(x) e^{-i\rho(x)}$ in general, but we can choose $\chi(x)=\rho(x)$: $\phi(x) = \frac{(v + h(x))}{\sqrt{2}}$

$$\begin{aligned}
\mathcal{L} =& \frac{1}{2} [(\partial^\mu + igA^\mu)(v+h)] [(\partial_\mu - igA_\mu)(v+h)] \\
& - \frac{\mu^2}{2} (v+h)^2 - \frac{\lambda}{4} (v+h)^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
=& \frac{1}{2} (\partial_\mu h)(\partial^\mu h) + \underline{\frac{1}{2} g^2 v^2 A_\mu A^\mu} - \lambda v^2 h^2 - \lambda v h^3 \\
& - \frac{\lambda}{4} h^4 + g^2 v h A^\mu A_\mu + \frac{1}{2} g^2 h^2 A_\mu A^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}.
\end{aligned}$$

$\mathcal{L}$ is still gauge invariant

but we have a massive A_μ

Before SSB: $\mathcal{L} = (\mathcal{D}_\mu\phi)^* (\mathcal{D}^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$

complex scalar + photon (4 degrees of freedom)

After SSB:

$$\frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \underline{\frac{1}{2}g^2v^2A_\mu A^\mu - \lambda v^2h^2} - \lambda v h^3$$
$$-\frac{\lambda}{4}h^4 + g^2 v h A^\mu A_\mu + \frac{1}{2}g^2h^2A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

real massive scalar (Higgs boson) + massive A_μ (1 + 3 = 4)

Compare with global U(1):

$$\mathcal{L} = (\partial_\mu\phi)^* (\partial^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 \Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu\rho)^2 + \frac{1}{2}(\partial_\mu\eta)^2 + \mu^2\eta^2$$
$$-\lambda v(\eta\rho^2 + \eta^3) - \frac{\lambda}{2}\eta^2\rho^2 - \frac{\lambda}{4}\eta^4 - \frac{\lambda}{4}\rho^4$$
$$+ \text{constant}.$$

What happens to the goldstone boson ρ? It is "eaten" by the gauge boson (it gets replaced by the longitudinal polarization state of A_μ so that A_μ becomes massive)

Higgs Mechanism in the SM

Higgs field is now an SU(2) doublet: $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ where $\phi^+ = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$, $\phi^0 = \frac{\phi_3 + i\phi_4}{\sqrt{2}}$.

$$\mathcal{L}_\phi = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2$$

where $\phi^\dagger \phi = (\phi^{+*} \phi^{0*}) \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \phi^{+*}\phi^+ + \phi^{0*}\phi^0 = \frac{(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)}{2}$.

$V(\phi)$ is invariant under $SU(2)_L \times U(1)_Y$: $\phi(x) \rightarrow \phi'(x) = e^{i \vec{\alpha}(x) \cdot \vec{\tau}/2} \phi(x)$

(must introduce B_μ and W^i_μ) $\phi(x) \longrightarrow \phi'(x) = e^{i\alpha(x)(Y/2)} \phi(x)$

break $SU(2)_L \times U(1)_Y$ (4 generators) to $U(1)_{EM}$ (1 generator)

3 goldstone bosons eaten by 3 gauge bosons: W^+, W^-, Z^0

A_μ remains massless

$$\mathcal{L}_\phi = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

V(ϕ) is minimum at $\phi^\dagger \phi = \frac{-\mu^2}{2\lambda} = \frac{v^2}{2}$ where $\phi^\dagger \phi = \frac{(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)}{2}$.

Choose a vacuum: $\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ and expand around it: $\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$

Why this vacuum? It must respect $U(1)_{EM}$: $\phi_0 \to \phi_0{}' = e^{i\alpha(x)Q} \phi_0 = \phi_0$.

remember $Q = T_3 + Y/2$. If Y = 1 for Higgs doublet $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$,

$$\delta \phi_0 = i\alpha Q \phi_0 = \frac{i\alpha}{2}(1 + \tau_3)\phi_0 = 0$$

Also note: $\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$ corresponds to a specific gauge choice (unitary gauge) where disappearance of Goldstone bosons is explicit

$$\mathcal{L}_\phi = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \quad , \quad \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

substitute $\mathcal{D}_\mu = \partial_\mu - i g_1 \frac{Y}{2} B_\mu - i g_2 \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu$ in $\mathcal{L}$:

$$\phi^\dagger \left(i g_1 \frac{Y}{2} B_\mu + i g_2 \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu \right)^\dagger \left(i g_1 \frac{Y}{2} B^\mu + i g_2 \frac{\vec{\tau}}{2} \cdot \vec{W}^\mu \right) \phi \, .$$

First, look at terms not involving H:

$$\frac{1}{8} \left| \begin{pmatrix} g_1 B_\mu + g_2 W_\mu^3 & g_2 (W_\mu^1 - i W_\mu^2) \\ g_2 (W_\mu^1 + i W_\mu^2) & g_1 B_\mu - g_2 W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$
$$= \frac{1}{8} v^2 g_2^2 \left((W_\mu^1)^2 + (W_\mu^2)^2 \right) + \frac{1}{8} v^2 (g_1 B_\mu - g_2 W_\mu^3)^2 \, .$$

first term: $\left(\frac{1}{2} v g_2\right)^2 W_\mu^+ W^{-\mu}$, since $W_\mu^\pm = (-W_\mu^1 \pm i W_\mu^1) / \sqrt{2}$ ➡ $M_W = v g_2 / 2$

second term: $(M_Z{}^2 Z_\mu Z^\mu)/2$ since $Z_\mu = \dfrac{-g_1 B_\mu + g_2 W_\mu^3}{\sqrt{g_1^2 + g_2^2}}$ ➡ $M_Z = \frac{1}{2} v \sqrt{g_1^2 + g_2^2}$

$$M_W = vg_2/2$$

$$M_Z = \frac{1}{2}v\sqrt{g_1^2 + g_2^2}$$

$$M_W/M_Z = \cos\theta_w$$

Define $\rho = M_W/M_Z \cos\theta_W$.

SM predicts ρ = 1 (at tree level)

Homework4: Using the expressions below, obtain interactions of H with W and Z, and also the self interactions of H.

$$\mathcal{L}_\phi = (\partial_\mu\phi)^\dagger(\partial^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2$$

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

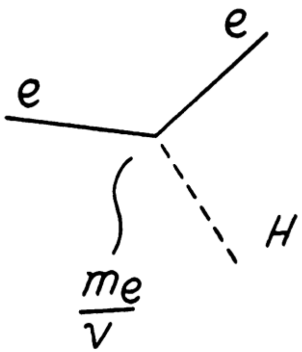
$$\mathcal{D}_\mu = \partial_\mu - ig_1\frac{Y}{2}B_\mu - ig_2\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu$$

Fermion Masses

With a Higgs doublet, we can add SU(2) invariant mass terms:

$$\mathcal{L}_{\text{int}} = g_e \left(\overline{L} \phi e^-_R + \phi^\dagger \overline{e^-_R} L \right)$$

Yukawa coupling. Value arbitrary in SM.



$$\phi \to \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} \Rightarrow \mathcal{L}_{\text{int}} = \frac{g_e v}{\sqrt{2}} \left(\overline{e^-_L} e^-_R + \overline{e^-_R} e^-_L \right) + \frac{g_e}{\sqrt{2}} \left(\overline{e^-_L} e^-_R + \overline{e^-_R} e^-_L \right) H$$

$g_e = \sqrt{2} m_e / v$ where $v = 2M_W / g_2 \approx 246$ GeV

$$\mathcal{L}_{\text{int}} = m_e \bar{e} e + \frac{m_e}{v} \bar{e} e H \,.$$

That's all for lepton masses (no ν_R in SM).

But u_R as well as d_R has mass.

We don't need a new Higgs, just use the charge conjugate:

$$\phi_c = i\sigma_2 \phi_c^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$$ is also SU(2) doublet, with opposite Y = -1

$$\mathcal{L}_{\text{int}} = g_d \overline{Q}_L \phi d_R + g_u \overline{Q}_L \phi_c u_R + \text{Herm. conjugate}$$

$$\phi \longrightarrow \begin{pmatrix} 0 \\ (v+H)/\sqrt{2} \end{pmatrix}, \qquad \phi_c \longrightarrow \begin{pmatrix} (v+H)/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\mathcal{L}_{\text{int}} = m_d \overline{d} d + m_u \overline{u} u + \frac{m_d}{v} \overline{d} d H + \frac{m_u}{v} \overline{u} u H$$

note that H couples most strongly to heaviest fermions.

$$
\begin{aligned}
&\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalar}} = \\
- \;& \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}W^{+}_{\mu\nu}W^{-\,\mu\nu} + M_W^2 W^{+}_{\mu}W^{-\,\mu} \\
- \;& \frac{1}{4}Z_{\mu\nu}Z^{\,\mu\nu} + M_Z^2 Z_\mu Z^\mu + \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2}M_H^2 H^2 \\
+ \;& \boxed{W^+W^-A} + \boxed{W^+W^-Z} \\
+ \;& \boxed{W^+W^-AA} + \boxed{W^+W^-ZZ} + \boxed{W^+W^-AZ} + \boxed{W^+W^-W^+W^-} \\
+ \;& \boxed{HHH} + \boxed{HHHH} \\
+ \;& \boxed{W^+W^-H} + \boxed{W^+W^-HH} + \boxed{ZZH} + \boxed{ZZHH}\,.
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{L}_{\text{leptons}} + \mathcal{L}^{\ell}_{\text{yuk}} = \\
&\sum_{\ell=e,\mu,\tau} \bar{\ell}(i\,\partial\!\!\!/ - m_\ell)\ell + \sum_{\nu_\ell=\nu_e,\nu_\mu,\nu_\tau} \bar{\nu}_\ell(i\,\partial\!\!\!/)\nu_\ell \\
+ \;& \boxed{\bar{\ell}\,\ell\,A} + \boxed{\bar{\nu}_\ell\,\ell\,W^+} + \boxed{\bar{\ell}\,\nu_\ell\,W^-} + \boxed{\bar{\ell}\,\ell\,Z} + \boxed{\bar{\nu}_\ell\,\nu_\ell\,Z} \\
+ \;& \boxed{\bar{\ell}\,\ell\,H}\,.
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{L}_{\text{quarks}} + \mathcal{L}^{q}_{\text{Yuk}} = \\
&\sum_{q=u,\cdots,t} \bar{q}(i\,\partial\!\!\!/ - m_q)q \\
+ \;& \boxed{\bar{q}\,q\,A} \\
+ \;& \boxed{\bar{u}\,d'\,W^+} + \boxed{\bar{d}'\,u\,W^-} + \boxed{\bar{q}\,q\,Z} \\
+ \;& \boxed{\bar{q}\,q\,H}\,.
\end{aligned}
$$

HMW5: Draw Feynman diagrams for H → γγ by combining the above terms.

Draw the most relevant Feynman diagram for gg → H.

Cross Sections and Decay Widths

so far: description of SM.   now: calculate (estimate) observables

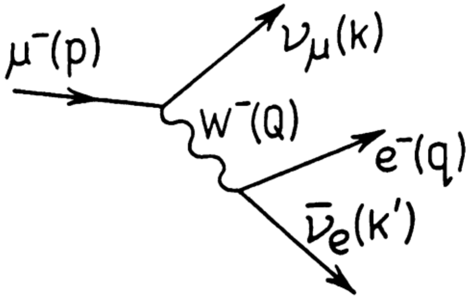
Fermi's Golden Rule:

transition rate = | matrix element $|^2$ · phase space

dynamics – use Feynman rules   kinematics

Muon decay

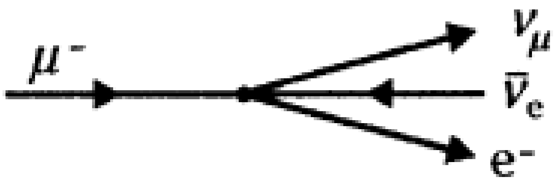
$$\mu^- \to \nu_\mu e^- \overline{\nu}_e$$



$$M = \frac{g_2^2}{2}\left(\overline{\nu}_\mu \gamma^\lambda P_L \mu\right) \frac{1}{Q^2 - M_W^2} \left(\overline{e}\gamma_\lambda P_L \nu_e\right)$$

since $Q^2 << M_W^2$,

$$M \simeq \frac{g_2^2}{8M_W^2}\left(\overline{\nu}_\mu \gamma^\lambda (1-\gamma_5)\,\mu\right)\left(\overline{e}\gamma_\lambda (1-\gamma_5)\,\nu_e\right)$$



Effective 4-fermion interaction (Fermi theory)

Fermi coupling $G_F/\sqrt{2} = g_2^2/8M_W^2$ can be determined from muon lifetime τ_μ

Using Fermi's golden rule, $\Gamma_\mu \propto G_F^2$. Dimensional analysis: $\Gamma_\mu \propto G_F^2 m_\mu^5$

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \Rightarrow G_F \cong 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

With α and G_F known, measurement of $\sin\theta_W$ yields M_W :

$$M_W^2 = \frac{\sqrt{2}g_2^2}{8G_F} = \frac{\pi\alpha}{\sqrt{2}\,G_F \sin^2\theta_W},\ M_W \cong 78\ \mathrm{GeV}$$

(the actual value is 80.4 GeV due to radiative corrections)

Fermi's Golden Rule:

transition rate = | matrix element |^2 · phase space

$$d\Gamma = \frac{V}{\prod_i 2E_i V}(2\pi)^4 \underbrace{\delta^4(P_f - P_i)}_{\text{total E and P conserved}} \overline{|M_{fi}|^2} \prod_f \frac{d^3 p_f}{(2\pi)^3 2E_f}$$

$$\frac{d^3 p}{2E} = \int \delta(E^2 - \vec{p}^2 - m^2)\theta(E) d^4 p$$

$\overline{|M_{fi}|^2}$: summed or averaged over unobserved degrees of freedom (spin, color etc.).

each term in $\delta^4(P_f - P_i)\overline{|M_{fi}|^2}\prod_f \frac{d^3 p_f}{(2\pi)^3 2E_f}$ is Lorentz invariant

Compare with non-relativistic version: $\Gamma_{fi} = \frac{2\pi}{\hbar}|\langle\psi_f|V_{fi}|\psi_i\rangle|^2 \rho(E_f)$

$2E_i$ and $2E_f$ terms come from relativistic normalization: under boost $dV \to dV/\gamma$

to keep number density $|\psi|^2 dV$ invariant, $\psi \to \sqrt{2E}\psi$

$\prod 2E_i \prod 2E_f$ is inside |matrix element|^2 with relativistically normalized ψ

$$d\Gamma = \frac{V}{\prod_i 2E_i V}(2\pi)^4\delta^4(P_f - P_i)\overline{|M_{fi}|^2}\prod_f \frac{d^3p_f}{(2\pi)^3 2E_f}$$

2 body decay: A $\rightarrow$ B + C

CM frame: $\sqrt{s} = E_A = m_A,\ \ \vec{p}_A = 0$

$$\Gamma = \frac{1}{32\pi^2 m_A}\int |M|^2 \frac{d^3\vec{p}_B}{E_B}\frac{d^3\vec{p}_C}{E_C}\delta^3(\vec{p}_B + \vec{p}_C)\delta(E_A - E_B - E_C)$$

$$= \frac{1}{32\pi^2 m_A}\int |M|^2 \frac{p_f^2\, dp_f\, d\Omega}{E_B E_C}\delta\left(\sqrt{s} - E_B - E_C\right)$$

, either use $\delta\left(g(p_f)\right)$

or integrate over $\sqrt{s}$

$$\sqrt{s} = E_B + E_C = \sqrt{m_B^2 + p_f^2} + \sqrt{m_C^2 + p_f^2}\ ,\qquad \frac{d\sqrt{s}}{dp_f} = p_f\left(\frac{1}{E_B} + \frac{1}{E_C}\right) = p_f\left(\frac{E_B + E_C}{E_B E_C}\right)$$

$$\Gamma = \frac{1}{32\pi^2 m_A}\int |M|^2 \frac{p_f\, d\Omega\, d\sqrt{s}}{E_B + E_C}\delta\left(\sqrt{s} - E_B - E_C\right) \Rightarrow \boxed{\frac{d\Gamma}{d\Omega} = \frac{|M|^2 p_f}{32\pi^2 m_A \sqrt{s}}}$$

$$d\Gamma = \frac{V}{\prod_i 2E_i V}(2\pi)^4\delta^4(P_f - P_i)\overline{|M_{fi}|^2}\prod_f \frac{d^3p_f}{(2\pi)^3 2E_f}$$

2 body cross section: A + B → C + D

$$d\Gamma = d\sigma \cdot F \text{ where flux } F = \frac{\text{\# of particles}}{A \cdot t} = \frac{|\vec{v}_A - \vec{v}_B|}{V} \text{ for a collinear collision}$$

$$d\sigma = \frac{|M|^2\delta^4(P_f - P_i)}{64\pi^2|\vec{v}_A - \vec{v}_B|E_A E_B}\frac{d^3\vec{p}_C}{E_C}\frac{d^3\vec{p}_D}{E_D}$$

CM frame: $|\vec{p}_A| = |\vec{p}_B| = p_i$,
$|\vec{p}_C| = |\vec{p}_D| = p_f$, $\sqrt{s} = E_A + E_B$

$$|\vec{v}_A - \vec{v}_B|E_A E_B = (|\vec{p}_A|E_B + |\vec{p}_B|E_A) = p_i\sqrt{s}$$

$$d\sigma = \frac{|M|^2}{64\pi^2 p_i\sqrt{s}}\frac{d^3\vec{p}_C}{E_C}\frac{d^3\vec{p}_D}{E_D}\delta^3(\vec{p}_C + \vec{p}_D)\delta\left(\sqrt{s} - E_C - E_D\right)$$

the rest is
same as before.

$$\frac{d\sigma}{d\Omega} = \frac{|M|^2 p_f}{64\pi^2 s p_i}$$

Meaning of Decay Width

$\Gamma = 1 / \tau$ (in natural units), but why width?

$\psi(t) = \psi(0)\exp(-iEt/\hbar)$, and for a decaying particle $|\psi(t)|^2 = |\psi(0)|^2 e^{-t/\tau}$

so: $E = E_0 - i\Gamma/2$

Fourier transform:

$$\begin{aligned}\tilde{\psi}(E) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt\, e^{iEt/\hbar}\psi(t) \\ &= \frac{1}{\sqrt{2\pi}}\psi(0)\int_{0}^{\infty} dt\, e^{i(E-E_0)t/\hbar - \Gamma t/2\hbar} \\ &= \frac{i\hbar\psi(0)}{\sqrt{2\pi}}\frac{1}{(E-E_0)+i\Gamma/2}\end{aligned}$$

($\psi = 0$ before $t = 0$)

$$|\tilde{\psi}(E)|^2 = \frac{\hbar^2|\psi(0)|^2}{2\pi}\frac{1}{(E-E_0)^2+\Gamma^2/4}\,.$$

large Γ corresponds to E being spread out (Heisenberg uncertainty principle)

Breit – Wigner Resonance

peak in σ due to particle production and decay ($A + B \rightarrow R \rightarrow C + D$)

Resonance in QM: $\sigma_{EL} = \frac{\pi}{\kappa^2} \sum_l (2l+1)\,|e^{2i\delta_l} - 1|^2 = \frac{4\pi}{\kappa^2} \sum_l (2l+1) \sin^2 \delta_l$ $\quad (\kappa = |\vec{p}|)$

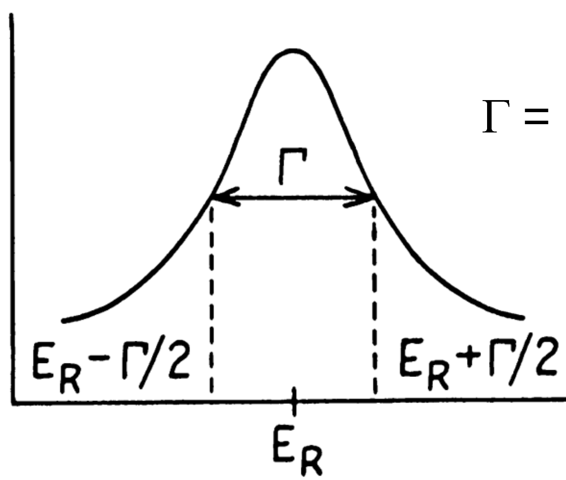
$\delta_l = \pi/2$ corresponds to resonance of partial wave with l

$$e^{2i\delta_l} - 1 = 2ie^{i\delta_l} \sin \delta_l = 2i \sin \delta_l / (\cos \delta_l - i \sin \delta_l) = 2i/(\cot \delta_l - i)\,.$$

Near the resonance: $\cot \delta_l \approx \cot \delta_l (E_R) + (E - E_R)\,(d\cot \delta_l / dE)|_{E=E_R}\,.$

Define Γ: $(d\cot \delta / dE)|_{E=E_R} = -2/\Gamma$

$$\sigma_{EL} \simeq \frac{4\pi}{\kappa^2}(2l+1)\frac{\Gamma^2/4}{(E-E_R)^2 + \Gamma^2/4}$$



Γ = FWHM

QM: $$\sigma_{EL} \simeq \frac{4\pi}{\kappa^2}(2l+1)\frac{\Gamma^2/4}{(E-E_R)^2+\Gamma^2/4}$$

$$A + B \to R \to C + D$$

spin: sum over final states (2s + 1 if spin s, 2 for gluon)

do the same for color (1 for singlets, 3 for quarks, 8 for gluons)

average over initial states:

consider 2 spin ½ initial particle (4 states – singlet + triplet)

3/4 participate in s = 1 R only, 1/4 participate in s = 0 R only

$$\sigma(A+B \to R \to C+D+E\ldots) \approx \frac{\pi}{\kappa^2}\left[\frac{(2S+1)\,c_R}{(2S_A+1)(2S_B+1)\,c_A c_B}\right]\frac{\Gamma^R_{AB}\Gamma^R_f}{(E_R-E)^2+\Gamma_R^2/4}\,.$$

Γ_R : total width, Γ^R_{AB} : partial width for $AB \to R$

Γ^R_f : partial width for $R \to C+D+E+\cdots$

$$\sigma(A+B\to R\to C+D+E\ldots) \approx \frac{\pi}{\kappa^2}\left[\frac{(2S+1)\,c_R}{(2S_A+1)\,(2S_B+1)\,c_A c_B}\right]\frac{\Gamma^R_{AB}\Gamma^R_f}{(E_R-E)^2+\Gamma_R^2/4}\,.$$

In CM ref. frame: $s=(p_A+p_B)^2=E^2,\qquad E_R=m_R$

$$E_R+E\cong 2m_R\cong 2\sqrt{s}$$

$$\left[(E_R-E)^2+\frac{\Gamma_R^2}{4}\right]\frac{(E_R+E)^2}{(E_R+E)^2}=\frac{(s-m_R^2)^2+\Gamma_R^2 m_R^2}{4s}$$

$$\sigma(A+B\to R\to C+D+E\cdots) \approx \frac{4\pi s}{\kappa^2}\left[\frac{(2S+1)\,c_R}{(2S_A+1)\,(2S_B+1)\,c_A c_B}\right]\frac{\Gamma^R_{AB}\Gamma^R_f}{(s-m_R^2)^2+m_R^2\Gamma_R^2}$$

Estimating Γ_W

kinematically forbidden

$$\frac{g_2}{\sqrt{2}}\left[\left(\overline{u}_L\gamma^\mu d_L + \overline{\nu}_{eL}\gamma^\mu e_L\right) W_\mu^+ + \text{h.c.}\right]$$

$W^+ \to e^+\nu_e,\ u\bar{d}$, including other families $W^+ \to \mu^+\nu_\mu$, $\tau^+\nu_\tau$, $c\bar{s}$, $t\bar{b}$

$$M = \frac{g_2}{\sqrt{2}}\epsilon_\mu \bar{e}\gamma^\mu P_L \nu \quad \text{for } W^+ \to e^+\nu_e$$

Calculating $\overline{|M|^2}$ involves some Diracology.

$\overline{|M|^2}$ can be estimated (up to factors of order 1) with simple approximations:

Sum over final states: 1 (only left-handed)

$\bar{u}u$ has mass dimension 1, $\overline{|M|^2}$ has mass dimension 2.

The only mass here is M_W (m_e is negligible): $\overline{|M|^2} \simeq g_2^2 M_W^2/2$

(correct answer is $g_2^2 M_W^2/3$)

$$\frac{d\Gamma}{d\Omega} = \frac{|M|^2 p_f}{32\pi^2 m_A \sqrt{s}} \quad , \quad d\Gamma = \frac{p_f\, d\Omega}{32\pi^2 m_A^2} |M|^2$$

$$d\Gamma \simeq \frac{P_e}{M_W^2} \frac{d\Omega_e}{32\pi^2} \frac{1}{3} g_2^2 M_W^2 . \quad \text{for } W^+ \to e^+ \nu_e$$ in the W rest frame.

Since matrix element has no angular dependence, $\int d\Omega \to 4\pi$

Neglecting mass of positron, $P_e \approx P_{0e} \simeq M_W/2$

$$\Gamma_W^{e\nu} = \frac{\alpha_2 M_W}{12} \quad \text{where } \alpha_2 = g_2^2/4\pi.$$

$W^+ \to e^+\nu_e$	1
$u\bar{d}$	3
$\mu^+\nu_\mu$	1
$c\bar{s}$	3
$\tau^+\nu_\tau$	1
	9

$$\mathrm{BR}(W^\pm \to e^\pm \nu) = \mathrm{BR}(W^\pm \to \mu^\pm \nu) \simeq 1/9$$

$$\mathrm{BR}(W^\pm \to u\bar{d}) = \mathrm{BR}(W^\pm \to c\bar{s}) \simeq 1/3$$

Numerically, $\alpha_2 \simeq 1/30$ and $M_W \simeq 80$ GeV

$$\Gamma_W^{\mathrm{TOT}} = \frac{3}{4} \alpha_2 M_W .$$

$\Gamma_W^{\mathrm{TOT}} \simeq 2$ GeV

$\Gamma_W^{e\nu} \simeq 0.23$ GeV

$\Gamma_W^{u\bar{d}} \simeq 0.68$ GeV

Estimating Γ_Z

$$Z^0 \to e^+e^-$$
$$\to \nu_e \overline{\nu}_e$$
$$\to u\overline{u}$$
$$\to d\overline{d}$$

$$\mathcal{L}_{NC} = \frac{g_2}{\cos\theta_W} \sum_{L,R} \left(T_3^f - Q_f \sin^2\theta_W\right) \bar{f} \gamma^\mu f Z_\mu$$

$$M_{e^+e^-} = \frac{g_2 \epsilon_\mu}{\cos\theta_w} \left[\overline{e^-_L} \gamma^\mu e^-_L \left(-\frac{1}{2} + \sin^2\theta_w\right) + \overline{e^-_R} \gamma^\mu e^-_R \left(0 + \sin^2\theta_w\right) \right]$$

Neglecting fermion masses, each term has the same form as W decay.

take $\Gamma_W^{e\nu}$ and replace: $\left(\frac{g_2}{\sqrt{2}}\right)^2 \to \left(\frac{g_2}{\cos\theta_w}\right)^2 \left[\left(-\frac{1}{2} + \sin^2\theta_w\right)^2 + \left(0 + \sin^2\theta_w\right)^2 \right]$

using $\sin^2\theta_w = 0.23$, $\Gamma_Z^{\mathrm{TOT}} = 2.36 \text{ GeV}$

Invisible Z decays: $\Gamma^{\overline{\nu}_e \nu_e} = 0.16 \text{ GeV}$ for each family

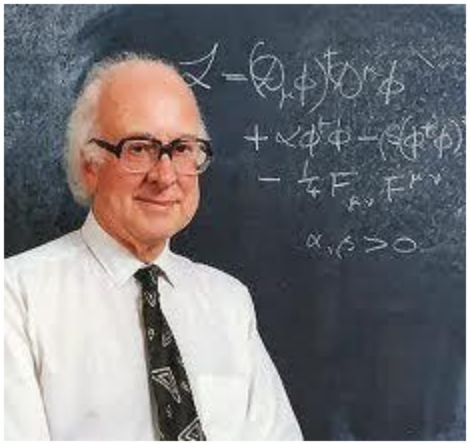
Measured invisible decay width implies 3 families of neutrinos with m < M_Z / 2

Homework:

1. Estimate the cross-section for colliding ν_μ with e_R, assuming $m_Z^2 >> s >> m_e^2$.

2. Estimate the top quark lifetime. Convert your answer to seconds.

Higgs boson is the only undetected SM particle



Higgs

Higgs boson

$$\mathcal{L}_\phi = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2 \quad , \quad \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad ,$$

Higgs vev: $v = \sqrt{\frac{(-\mu^2)}{\lambda}} = \sqrt{\sqrt{2} G_F} \cong 246 \text{ GeV}$

$$V = \mu^2 \frac{(v+H)^2}{2} + \lambda \frac{(v+H)^4}{4} \Rightarrow M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda} v = ?$$

bounds on SM Higgs

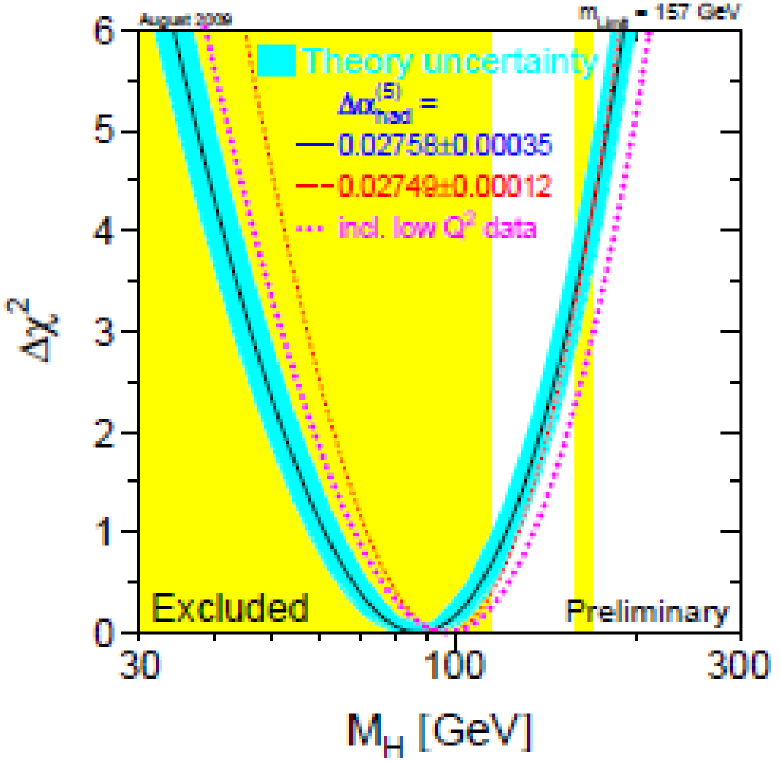
m_H < 114 GeV excluded by LEP

162 GeV < m_H < 166 GeV
excluded by Tevatron

Electroweak precision data
prefer light Higgs (<~ 200 GeV)

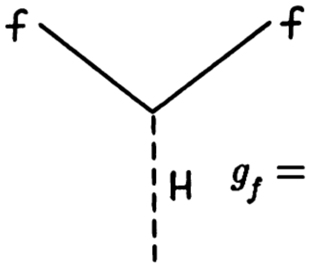
Theoretical upper bound from
quantum corrections:
~ 180 GeV if SM is valid all the way
up to Planck scale
~ 600 GeV if SM valid up to TeV

Theoretical upper bound from unitarity: ~ 1 TeV

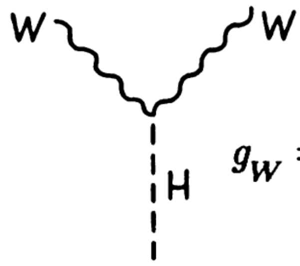


see e.g. Hollik, http://arxiv.org/abs/1012.3883 for details and references

Higgs couplings



$g_f = \frac{g_2 M_f}{2 M_W}$



$g_W = g_2 M_W$

no ggH or γγH vertex at tree level

Higgs decays to fermions

$$M = g_f \bar{u} u \;\Rightarrow\; \overline{|M|^2} \simeq 4 g_f^{\,2} M_H^2$$, assuming $M_H \gg M_f$

$$d\Gamma = \frac{p_f\, d\Omega}{32\pi^2 m_A^2} |M|^2 \;\Rightarrow\; \Gamma_{H f \bar{f}} \simeq \frac{\overline{|M|^2}}{16\pi M_H} = \frac{1}{4\pi} M_H \frac{g_2^2 M_f^2}{4 M_W^2} = \frac{\alpha_2}{4} \frac{M_f^2}{M_W^2} M_H \,.$$

The correct answer is 3/2 of this for a quark, 1/2 of this for a lepton.

Higgs decays to W bosons: $H \rightarrow WW$

assume $M_H >> 2M_W$. naive estimate: $\Gamma \sim \alpha_2 M_H$

remember: W became massive by eating the goldstone boson

the longitudinal part is special: it causes Γ to be larger than naive estimate

Matrix element: $M = g_2 M_W \epsilon \cdot \epsilon'$

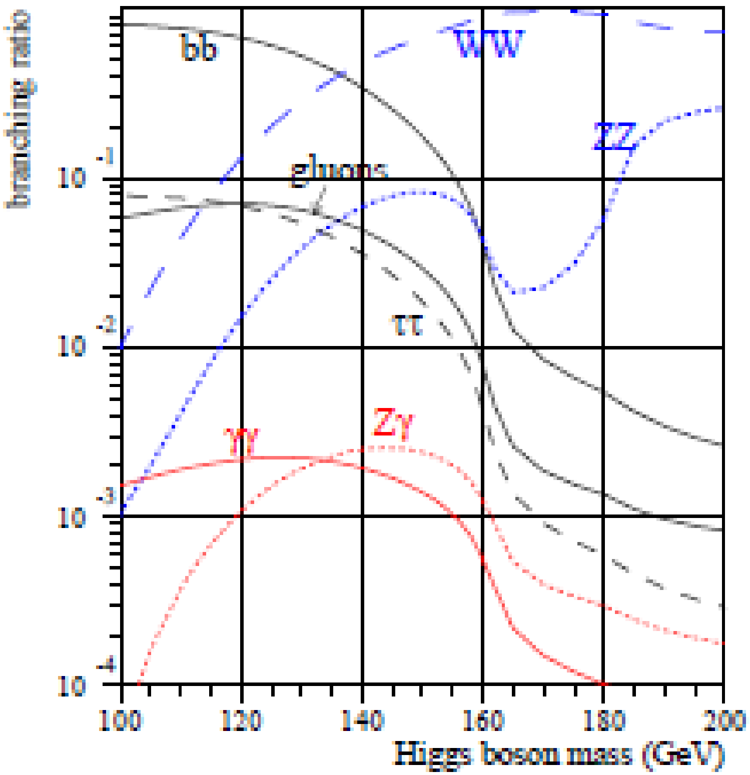
$\epsilon_\mu^{(z)} = (0,0,0,1)$ at rest frame. Boost: $\epsilon_\mu^{(z)} = (\gamma\beta, 0,0,\gamma)$

$$\gamma = \frac{p_0}{M_W}, \qquad \gamma\beta \cong \frac{p_0}{M_W} \quad \Rightarrow \quad \epsilon_\mu^{(z)} = \frac{p_0}{M_W}(1,0,0,1) = \frac{p_\mu}{M_W} \quad , \qquad \epsilon \cdot \epsilon' \cong \frac{p \cdot p'}{M_W^2}$$

$$M_H^2 = (p+p')^2 = 2M_W^2 + 2p \cdot p', \;\; p \cdot p' \cong \frac{M_H^2}{2}$$

$$\overline{|M|^2} \simeq \frac{g_2^2 M_H^4}{4M_W^2}$$ and using $$d\Gamma = \frac{p_f \, d\Omega}{32\pi^2 m_A^2} |M|^2, \quad \Gamma_{HWW} \simeq \frac{1}{16\pi M_H} \frac{g_2^2 M_H^4}{4M_W^2} = \frac{\alpha_2 M_H^3}{16 M_W^2}$$

(this Γ is for $H \rightarrow W^+ W^-$, and is twice the Γ for $H \rightarrow ZZ$)



bb channel is unaccessible
due to QCD background

M_H >~ 140 GeV:

H $\rightarrow$ WW, ZZ $\rightarrow$ 4f

M_H <~ 140 GeV:

H $\rightarrow \gamma\gamma$

Light Higgs harder to discover

needs ~ 10 fb^{-1} at 14 TeV (~ 2014)

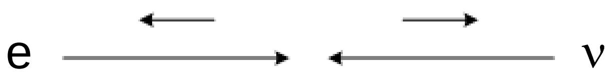
need more (> 30 fb^{-1})

to check couplings (> 2014)

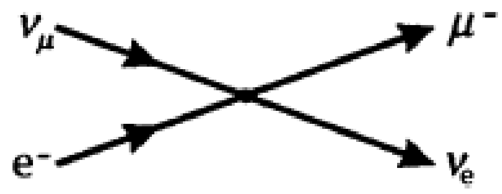
in any case: if there is SM Higgs we will know in a few years.

Unitarity bound $\sigma_{EL} \simeq \frac{4\pi}{\kappa^2}(2l+1)\frac{\Gamma^2/4}{(E-E_R)^2+\Gamma^2/4} \leq \frac{4\pi}{\kappa^2}(2l+1)$

Example: Fermi theory



s-wave (l = 0)



$$\sigma = \frac{G_F^2 s}{\pi}$$

Unitarity bound: $\kappa = |\vec{p}| = \frac{\sqrt{s}}{2}, \quad \sigma \leq \frac{4\pi}{\kappa^2} = \frac{16\pi}{s}$,

for V – A coupling: $\sigma_{LL,LL} \leq \frac{4\pi}{s}$

$$\frac{G_F^2 s}{\pi} \leq \frac{4\pi}{s}, \qquad s \leq \frac{2\pi}{G_F} \cong (700\ \mathrm{GeV})^2$$

Fermi theory violates unitarity bound when each particle has 350 GeV.

With W boson this unitarity problem solved

but unitarity still violated in WW scattering unless:

there is Higgs with m_H < 1 TeV or other *new physics* mimicking Higgs

THERE MUST BE SOMETHING AT TEV SCALE!!! → LHC WILL SHOW consider $W^+ + W^- \to H \to Z + Z$

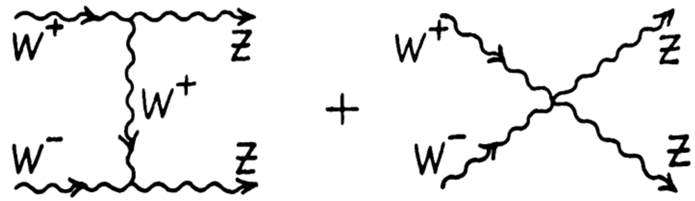
$$M = \frac{g_2^2 M_W^2 (\epsilon_W \cdot \epsilon_W')(\epsilon_Z \cdot \epsilon_Z')}{M_H^2 - \hat{s}}$$

$$\begin{cases} \epsilon_W \cdot \epsilon_W' = p \cdot p'/M_W^2 \\ \epsilon_Z \cdot \epsilon_Z' = k \cdot k'/M_W^2 \end{cases}$$

$$\begin{cases} p \cdot p' \simeq \hat{s}/2 \simeq k \cdot k' \\ \text{for} \quad \hat{s} \gg M_W^2 \end{cases}$$

$M \simeq \frac{g_2^2}{4M_W^2} \frac{\hat{s}^2}{M_H^2 - \hat{s}}$ implies $\sigma(WW \to WW) = |M|^2/16\pi\hat{s}$ grows with ŝ

Including other diagrams,



$$M = \frac{g_2^2}{4M_W^2} \frac{\hat{s}^2}{M_H^2 - \hat{s}} + f(\hat{s}, M_W^2)$$

At high energy, $f(\hat{s}, M_W^2)$ must also grow with ŝ, cancelling first term and keeping σ well behaved

If M_H is too large (> TeV) first term is suppressed, only WW → WW term remains: unitarity is violated due to WW → WW scattering

Quark mixing and CP violation

Quark masses:

$$\mathcal{L}_{\text{int}} = g_d \overline{Q}_L \phi d_R + g_u \overline{Q}_L \phi_c u_R + \text{Herm. conjugate}$$

For three families of quarks the Yukawa couplings are 3 x 3 complex matrices

there is no reason why these should be diagonal

You could diagonalize them by unitary transformations on quark fields, but then

$\frac{g_2}{\sqrt{2}}\left(\bar{u}_L \gamma^\mu d_L W_\mu^+ + \bar{d}_L \gamma^\mu u_L W_\mu^-\right)$ would no longer be diagonal:

a weak interaction eigenstate is not a mass eigenstate

it is a linear combination of mass eigenstates

} Quark Mixing

↓

s and b can decay

Consider charged current for first two families.

$$J^{\mu}_{\rm ch} = (\overline{u} \quad \overline{c})\,\gamma^{\mu} P_L \begin{pmatrix} d \\ s \end{pmatrix} = \overline{u}\gamma^{\mu} P_L d + \overline{c}\gamma^{\mu} P_L s$$

u, c, d, s : mass eigenstates

weak interaction eigenstates q´: $\begin{pmatrix} d' \\ s' \end{pmatrix}_L = V \begin{pmatrix} d \\ s \end{pmatrix}_L$ where V is unitary

$$V = \begin{pmatrix} \cos\theta e^{i\alpha} & \sin\theta e^{i\gamma} \\ -\sin\theta e^{-i\gamma} & \cos\theta e^{-i\alpha} \end{pmatrix} \Rightarrow$$

$$\begin{aligned} d' &= \cos\theta e^{i\alpha} d + \sin\theta e^{i\gamma} s \\ &= e^{i\alpha}\left(d\cos\theta + s\sin\theta e^{i(\gamma-\alpha)}\right) \\ s' &= -\sin\theta e^{-i\gamma} d + \cos\theta e^{-i\alpha} s \\ &= e^{-i\gamma}\left(-d\sin\theta + s\cos\theta e^{-i(\alpha-\gamma)}\right). \end{aligned}$$

to get rid of phases multiply d' by $e^{-i\alpha}$, s' by $e^{i\gamma}$, and s by $e^{-i(\gamma-\alpha)}$

do the same for RH fields: SM Lagrangian remains invariant

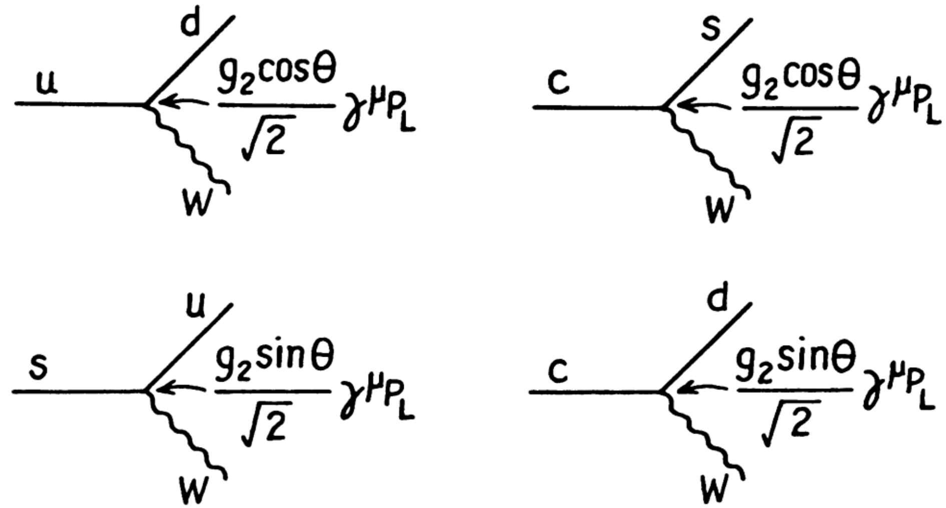
$$\left.\begin{aligned} d' &= d\cos\theta + s\sin\theta, \\ s' &= -d\sin\theta + s\cos\theta \end{aligned}\right\} \quad V = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$V = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Cabibbo angle:

$$\theta \approx 13^\circ$$

$$J^\mu_{\text{ch}} = (\bar{u} \quad \bar{c})\, \gamma^\mu P_L \begin{pmatrix} d' \\ s' \end{pmatrix}$$
$$= (\bar{u} \quad \bar{c})\, \gamma^\mu P_L V \begin{pmatrix} d \\ s \end{pmatrix}$$
$$= \bar{u}\gamma^\mu P_L d \cos\theta + \bar{u}\gamma^\mu P_L s \sin\theta$$
$$- \bar{c}\gamma^\mu P_L d \sin\theta + \bar{c}\gamma^\mu P_L s \cos\theta .$$



note: we can write a similar expression for up type quarks: $(\bar{u}\ \bar{c})\, \gamma^\mu P_L V^\dagger_{\text{up}} V_{\text{down}} \begin{pmatrix} d \\ s \end{pmatrix}$

but $V^\dagger_{\text{up}} V_{\text{down}}$ is just another rotation matrix so we don't get anything new:

it is convention to rotate just down type quarks and take q = q´ for up type quarks.

The neutral current remains diagonal in mass eigenstate.

$$J^{\mu}_{\text{neu}} = \sum_{f=u,c,d,s} \left(\bar{f}_L \gamma^{\mu} \left[T_3^L - Q \sin^2\theta_w \right] f_L + \bar{f}_R \gamma^{\mu} \left[-Q \sin^2\theta_w \right] f_R \right)$$

$$\begin{pmatrix} d \\ s \end{pmatrix} \to V \begin{pmatrix} d \\ s \end{pmatrix}, \qquad (\bar{d} \;\; \bar{s}) \to (\bar{d} \;\; \bar{s}) V^{\dagger}, \;\; V^{\dagger} V = 1$$

no tree level Flavor Changing Neutral Currents (FCNC) in the SM

Homework 1: Draw Feynman diagrams for the following processes:

$$K^+ (u\bar{s}) \to \mu^+ + \nu_{\mu},$$

$$D^+ (c\bar{d}) \to \overline{K^0} (\bar{d}s) + e^+ + \nu_e,$$

$$b \to s + \gamma, \quad b \to s + e^+ + e^-.$$

Three families:

$$J^{\mu}_{\text{ch}} = (\,\overline{u} \quad \overline{c} \quad \overline{t}\,)\,\gamma^{\mu} P_L V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

a 3 x 3 unitary matrix has 9 independent real parameters.

Remember how we got rid of phases in V by absorbing them in quark fields:

$$V = \begin{pmatrix} \cos\theta e^{i\alpha} & \sin\theta e^{i\gamma} \\ -\sin\theta e^{-i\gamma} & \cos\theta e^{-i\alpha} \end{pmatrix} \Rightarrow V = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

For n quarks we can absorb n - 1 phases. (overall phase does not matter)

3 families, 6 quarks: We are left with 9 – 5 = 4 parameters: 3 angles, 1 phase

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

phase δ is responsible for all CP violating phenomena in SM review:

P (parity): $P\psi(\vec{x}) = \pm\psi(\vec{x})$ if system is invariant

parity of a state is parity under space reflection x intrinsic parities

Helicity $\vec{\sigma} \cdot \vec{p}$ changes sign under parity

V – A

$$\overline{\nu}_L \gamma^\mu e_L = \frac{1}{2}\overline{\nu}\gamma^\mu (1-\gamma_5) e$$: parity violated

C (charge conjugation): symmetry under C means particles and antiparticles have same interactions

C also clearly violated in SM: no interaction for left-handed antineutrinos

CP: $\nu_L \rightarrow \bar{\nu}_R$

CP violation is more subtle

CPT is sacred in QFT: CP violation implies T violation

T invariance: $\langle\psi'|\,H\,|\psi\rangle = \langle T\psi|\,THT^{-1}\,|T\psi'\rangle$

$T = UK$ ⇒ T (and therefore CP) can be violated if H is complex

$$J^{\mu}_{\rm ch} = (\,\overline{u}\quad \overline{c}\quad \overline{t}\,)\,\gamma^{\mu} P_L V \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \qquad V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

CP violation first observed in neutral Kaons $K^0(d\bar{s})$ and $\bar{K}^0(\bar{d}s)$

Suppose CP symmetry valid. CP eigenstates: $K_S = K^0 + \bar{K}^0$ (CP even)

$K_L = K^0 - \bar{K}^0$ (CP odd)

p odd under P, so for L = 0, $\pi\pi$ is CP even: $K_S \to \pi\pi$

$\pi\pi\pi$ is CP odd: $K_L \to \pi\pi\pi$ (kinematically suppressed)

CP violation observed (1964): $\dfrac{K_L \to \pi\pi}{K_S \to \pi\pi} \sim 2 \times 10^{-3}$

CPV observed in B physics as well as kaons.
So far everything is consistent with CPV coming from CKM phase.

Rare decays (such as $b \to s\,\gamma$) and CPV observables are crucial for probing new physics. They will be discussed in lectures by Tobias Hurth

Magnitudes of CKM (Cabibbo-Kobayashi-Maskawa) matrix elements:

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{bmatrix}.$$

see http://pdg.lbl.gov/

$$J^{\mu}_{\rm ch} = (\overline{u} \quad \overline{c} \quad \overline{t})\,\gamma^{\mu} P_L V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$: $b \to u + W^- \propto |V_{ub}|^2$, $t \to d + W^+ \propto |V_{td}|^2$

Homework 2: Estimate b-quark decay width (hint: $\Gamma_{\mu} = \frac{G_F^2 m_{\mu}^5}{192\pi^3}$)

Parameters of the Standard Model

Symbol	Description	Renormalization scheme (point)	Value
m_e	Electron mass		511 keV
m_μ	Muon mass		105.7 MeV
m_τ	Tau mass		1.78 GeV
m_u	Up quark mass	$\mu_{MS} = 2$ GeV	1.9 MeV
m_d	Down quark mass	$\mu_{MS} = 2$ GeV	4.4 MeV
m_s	Strange quark mass	$\mu_{MS} = 2$ GeV	87 MeV
m_c	Charm quark mass	$\mu_{MS} = m_c$	1.32 GeV
m_b	Bottom quark mass	$\mu_{MS} = m_b$	4.24 GeV
m_t	Top quark mass	On-shell scheme	172.7 GeV
θ_{12}	CKM 12-mixing angle		13.1°
θ_{23}	CKM 23-mixing angle		2.4°
θ_{13}	CKM 13-mixing angle		0.2°
δ	CKM CP-violating Phase		1.2
g_1	U(1) gauge coupling	$\mu_{MS} = m_Z$	0.357
g_2	SU(2) gauge coupling	$\mu_{MS} = m_Z$	0.652
g_3	SU(3) gauge coupling	$\mu_{MS} = m_Z$	1.221
θ_{QCD}	QCD vacuum angle		~0
μ	Higgs quadratic coupling		Unknown
λ	Higgs self-coupling strength		Unknown

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If there is a Higgs boson,
SM model does not explain
why it is so much lighter than M_P
(hierarchy problem)

electroweak symmetry breaking
more natural in extended models
such as MSSM, which also predicts
an extended Higgs sector
(5 Higgs bosons)

among other possibilities:
dynamical symmetry breaking:
no elementary scalar fields,
Higgs is composite

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Strong CP problem

QCD Lagrangian includes a term

$\propto \theta G_{\mu\nu}\tilde{G}^{\mu\nu}$ which is CP violating

similar to E·B.

From neutron EDM, $\theta \leq 10^{-10}$.

Why so small?

Peccei-Quinn mechanism:
add a global symmetry
predicts a light particle
(pseudo-Goldstone boson)
with suppressed couplings: axion
introduces a new scale (> 10^9 GeV)

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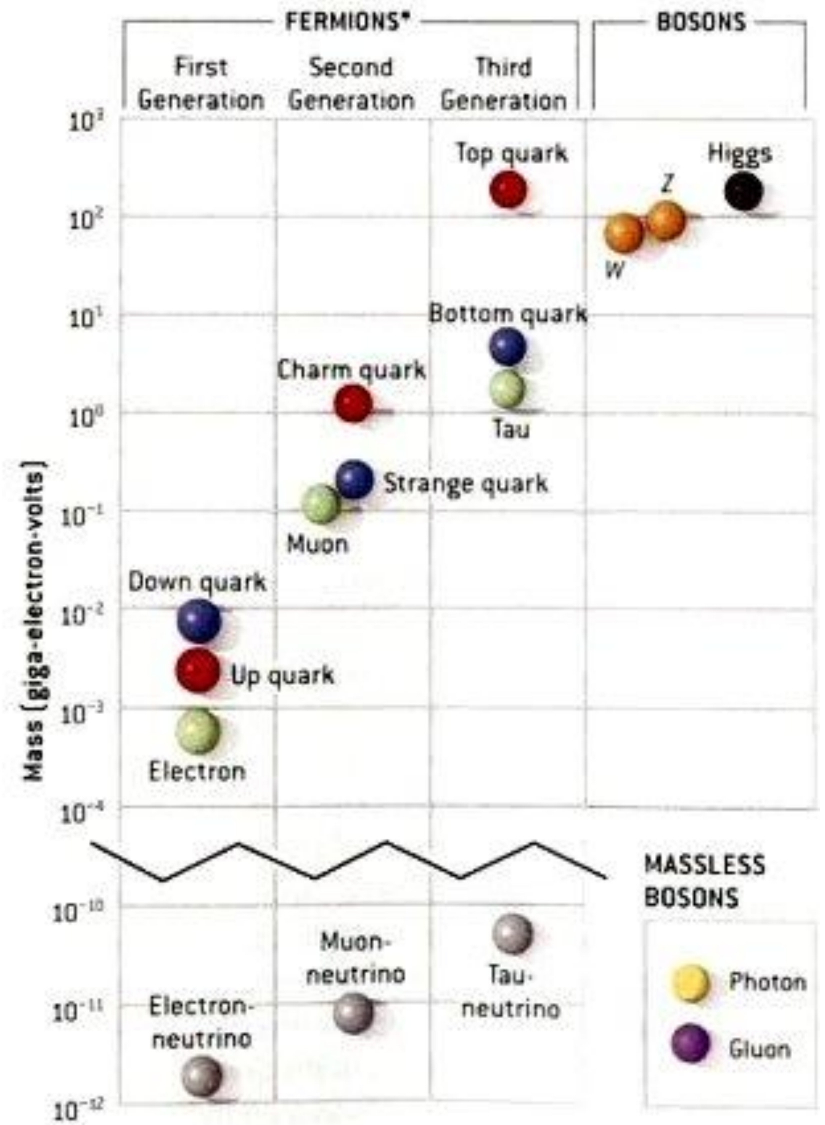

Hierarchical mass pattern: flavor symmetries?

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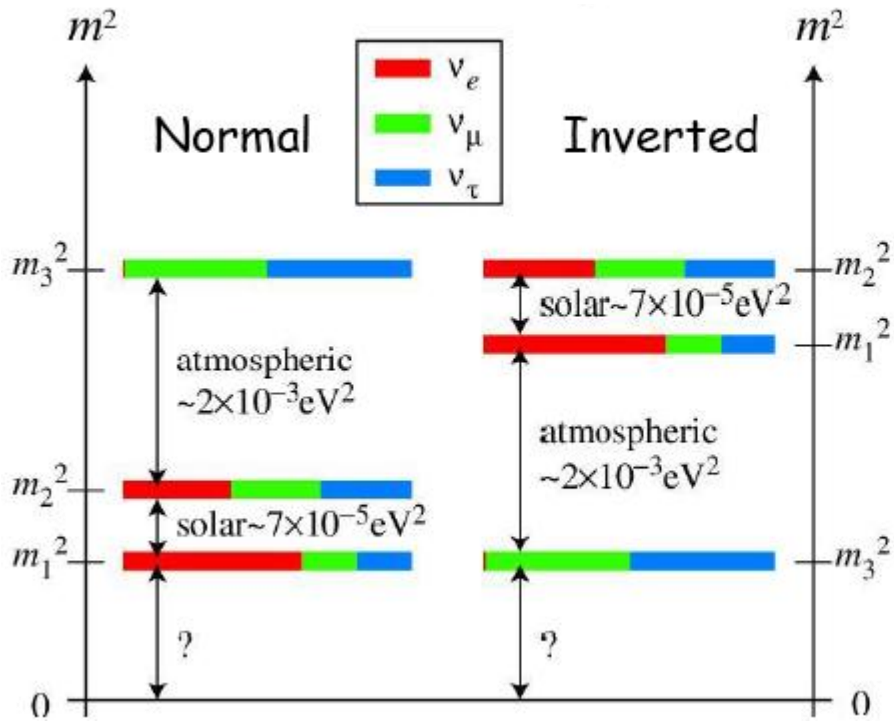
m_t	Top quark mass	On-shell scheme	172.7 GeV
θ_{12}	CKM 12-mixing angle		13.1°
θ_{23}	CKM 23-mixing angle		2.4°
θ_{13}	CKM 13-mixing angle		0.2°
δ	CKM CP-violating Phase		1.2
g_1	U(1) gauge coupling	$\mu_{MS} = m_Z$	0.357
g_2	SU(2) gauge coupling	$\mu_{MS} = m_Z$	0.652
g_3	SU(3) gauge coupling	$\mu_{MS} = m_Z$	1.221
θ_{QCD}	QCD vacuum angle		~0
μ	Higgs quadratic coupling		Unknown
λ	Higgs self-coupling strength		Unknown

Neutrino masses and mixings



Seesaw mechanism:
Introduce RH neutrinos with M_N<~10^{14} GeV

$m_\nu \approx M_W^2 / M_N$

Matter-antimatter asymmetry:
CP violation in SM not enough – RH neutrinos can help

Parameters of the Standard Model

Symbol	Description	Renormalization scheme (point)	Value
m_e	Electron mass		511 keV
m_μ	Muon mass		105.7 MeV
m_τ	Tau mass		1.78 GeV
m_u	Up quark mass	μ_{MS} = 2 GeV	1.9 MeV
m_d	Down quark mass	μ_{MS} = 2 GeV	4.4 MeV
m_s	Strange quark mass	μ_{MS} = 2 GeV	87 MeV
m_c	Charm quark mass	$\mu_{MS} = m_c$	1.32 GeV
m_b	Bottom quark mass	$\mu_{MS} = m_b$	4.24 GeV
m_t	Top quark mass	On-shell scheme	172.7 GeV
θ_{12}	CKM 12-mixing angle		13.1°
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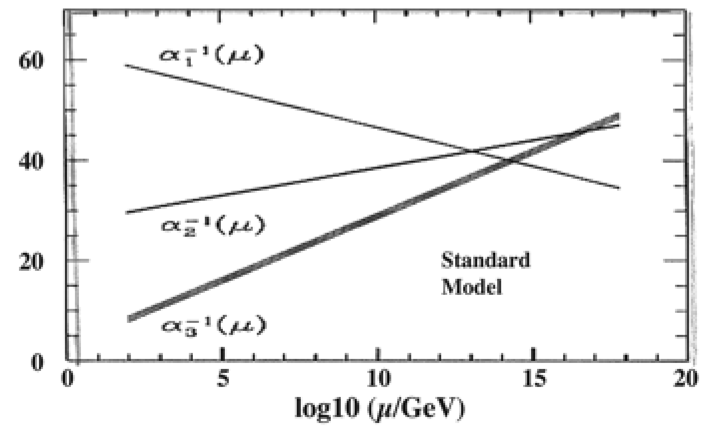
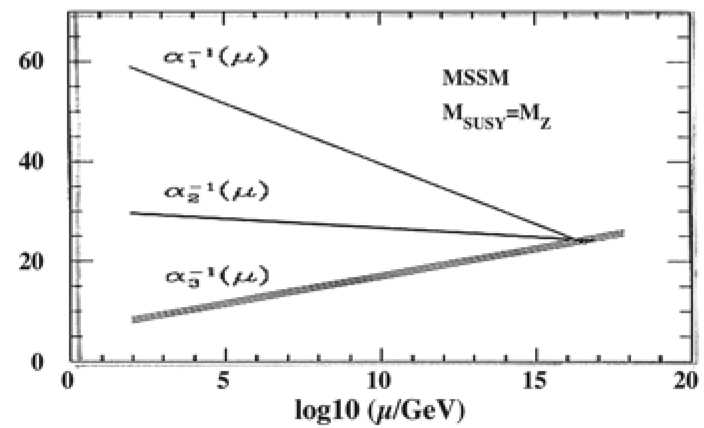
θ_{QCD}	QCD vacuum angle		~0
μ	Higgs quadratic coupling		Unknown
λ	Higgs self-coupling strength		Unknown

Gauge coupling unification:

SM: almost

MSSM: yes!





Grand Unification – an example why it is fascinating to go beyond the SM

Simplest possibility: SU(5)

[SO(10) combines
$10+\bar{5}+\nu_R$ into 16]

$$\bar{5}_\alpha = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ -e \\ \nu \end{pmatrix}_L, \quad 10^{\alpha\beta} = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & u_3^c & -u_2^c & u^1 & d^1 \\ -u_3^c & 0 & u_1^c & u^2 & d^2 \\ u_2^c & -u_1^c & 0 & u^3 & d^3 \\ \hline -u^1 & -u^2 & -u^3 & 0 & e^c \\ -d^1 & -d^2 & -d^3 & -e^c & 0 \end{array}\right)_L$$

Pauli matrices for SU(2) and Gell-Mann matrices for SU(3), we have
24 traceless, Hermitian matrices as generators of SU(5).

Generators of SU(5) contain those of $SU(3)_C$ x $SU(2)_L$ x $U(1)_Y$

Charge operator $Q = T_3 + Y/2$ also traceless – sum of eigenvalues is zero

$$Q(\nu_e) + Q(e^-) + 3Q(\bar{d}) = 0 \quad \Rightarrow$$ d quark has charge -1/3

The SU(5) embedding of SM particles nicely explain

charge quantization, as well as equal number of quark & lepton families

Calculating Weinberg Angle

In SM θ_W was a free parameter, but in grand unification it can be calculated.

Write Q in terms of SU(5) generators: $Q = T_3 + cT_1$

c is a constant to be fixed such that cT_1 corresponds to $Y/2$

SU(5) covariant derivative: $\partial^\mu - ig_5 T_a V_a^\mu = \partial^\mu - ig_5(T_3 W_3^\mu + T_1 B^\mu + \ldots)$

$$B^\mu = A^\mu \cos\theta_w + Z^\mu \sin\theta_w \;,$$
$$W_3^\mu = -A^\mu \sin\theta_w + Z^\mu \cos\theta_w \;;$$

A^μ terms: $-g_5 T_3 \sin\theta_w + g_5 T_1 \cos\theta_w$

$$= -g_5 \sin\theta_w (T_3 - \cot\theta_w\, T_1) = eQ$$

Therefore, $e = g_5 \sin\theta_w$ and $c = -\cot\theta_w$ ⇒ $\sin^2\theta_W = \dfrac{1}{1+c^2}$

Calculating Weinberg Angle

$$Q = T_3 + cT_1 \quad , \quad e = g_5 \sin\theta_w \quad , \quad \sin^2\theta_W = \frac{1}{1+c^2}$$

value of c follows from $\mathrm{Tr}_R T_a T_b = N_R \delta_{ab}$

for any representation R of a simple group, Tr T^2 must be same for each generator

Example: $\mathrm{Tr}\, J_x^2 = \mathrm{Tr}\, J_y^2 = \mathrm{Tr}\, J_z^2$ = ½ for J = ½

$$\mathrm{Tr}\, Q^2 = \mathrm{Tr}(T_3 + cT_1)^2 = \mathrm{Tr}\, T_3^2 + c^2 \mathrm{Tr}\, T_1^2$$

$$\mathrm{Tr}\, T_1^2 = \mathrm{Tr}\, T_3^2$$

$$\Rightarrow \quad 1 + c^2 = \frac{\mathrm{Tr}\, Q^2}{\mathrm{Tr}\, T_3^2}$$

$$\begin{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \\ \begin{pmatrix} \bar{d}_r \\ \bar{d}_g \\ \bar{d}_b \end{pmatrix} \end{pmatrix}_L$$

$$\mathrm{Tr}\, Q^2 = 0 + 1 + 3\left(\frac{1}{9}\right) = \frac{4}{3},$$

$$\mathrm{Tr}\, T_3^2 = \frac{1}{4} + \frac{1}{4} + 0 + 0 + 0 = \frac{1}{2}$$

$$\Rightarrow \quad 1 + c^2 = \frac{8}{3} \quad , \quad \sin^2\theta_w = 3/8$$

Calculating Weinberg Angle

$$Q = T_3 + cT_1 \quad , \quad e = g_5 \sin\theta_w \quad , \quad 1 + c^2 = \frac{8}{3}$$

SU(5) prediction: $\sin^2\theta_w = 3/8$ at M_{GUT}

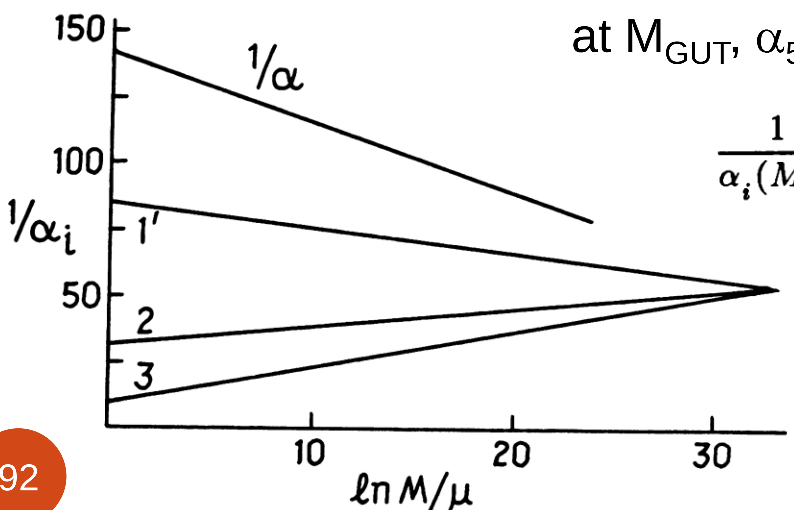
to compare with experiment we need to run couplings from M_{GUT} to M_W

at M_{GUT}, $\alpha_5 = \alpha_3 = \alpha_2 = c^2\,\alpha_1$

$$\partial^\mu - ig_5 T_a V_a^\mu = \partial^\mu - ig_5(T_3 W_3^\mu + T_1 B^\mu + \ldots)$$

$$\mathcal{D}_\mu = \partial_\mu - ig_1\frac{Y}{2}B_\mu - ig_2\frac{\tau^i}{2}W_\mu^i - ig_3\frac{\lambda^a}{2}G_\mu^a \,.$$

$$g_5\,T_1 = g_1\,\frac{Y}{2} = g_1\,c\,T_1$$



at M_{GUT}, $\alpha_5 = \alpha_3 = \alpha_2 = c^2\,\alpha_1$

$$\frac{1}{\alpha_i(M^2)} = \frac{1}{\alpha_i(\mu^2)} + \frac{b_i}{4\pi}\ln\frac{M^2}{\mu^2}$$

M: p transfer scale

μ: where *g* are measured, M_W

b_i: depends on gauge group and number of fermions with $M_F \lesssim M$

Observed θ_W explained by SUSY GUT


That's all Folks