Theory & Analysis in Particle Physics

[31 January - 11 February 2011, Istanbul, Turkey]

This is a 12 day school on Theory & Analysis in Particle Physics. The school is to be held in English with a maximum of 50 students. The first part will focus more on the theory side and the second part, separated by a social activity day, on the analysis side. The target audience is the 4th year undergrad and early graduate students.

The theory week will focus on introduction to Particle Physics, Quantum Field Theory and the Standard Model. The analysis week will cover the analysis tools, programming and simulation techniques followed by a black box analysis game in the "LHC Olympics" style. The knowledge acquired during lectures will be solidified through nightly homeworks followed by discussions on subsequent mornings.

Lectures on Flavour Physics

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quark flavour, CKM mechanism

indirect search for new physics

neutrino masses

Tobias Hurth

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

[Application]

Deadline: 25 December 2010
http://physics.dogus.edu.tr/ISTAPP2011

[Lecturers]

Juan Antonio Aguilar-Saavedra (Granada U., Spain)
Kazem Azizi (Dogus U., Turkey)
Orhan Çakır (Ankara U., Turkey)
Durmuş Ali Demir (IYTE, Turkey)
Daniel Froidevaux (CERN, Switzerland)
Tobias Hurt (Mainz U., Germany)
Can Kozçaz (CERN, Switzerland)
Wolfgang Lerche (CERN, Switzerland)
Erkcan Özcan (UCL, UK & Boğaziçi U., Turkey)
Altuğ Özpineci (METU, Turkey)
Sara Pasquetti (CERN, Switzerland)
Joerg Stelzer (DESY, Germany)
Nefer Şenoğuz (Doğuş U., Turkey)
Gökhan Ünel (UCI, USA)

The lectures cover a selected numbers of topics in flavour physics, reflecting the flavour of the lecturer. The focus will be on the fundamental concepts.

Focus: * neutrino physics * B meson physics

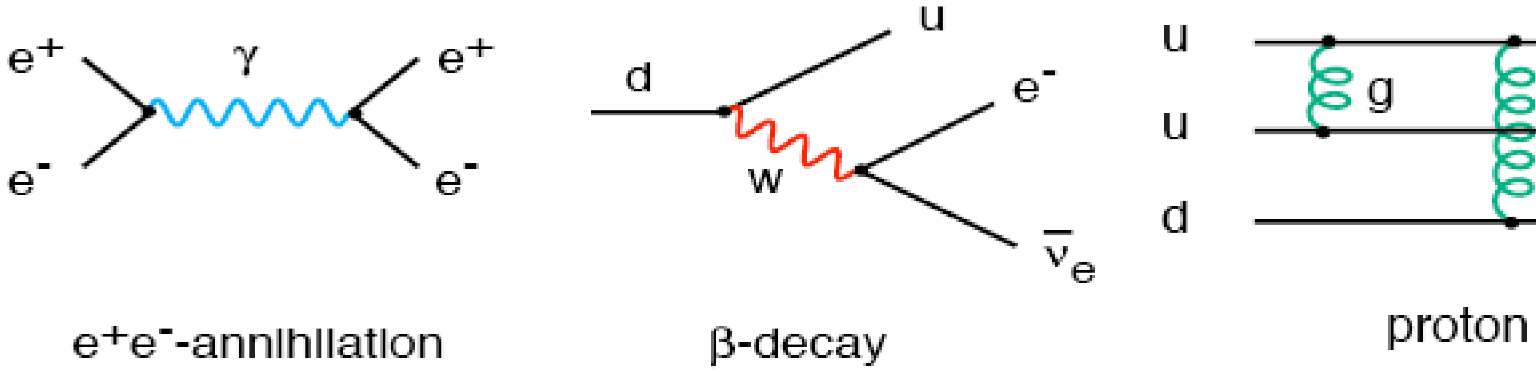
A complete coverage of the field can be found in recent books, reviews, reports and published lectures:

$\Rightarrow$ Reading list

Prologue Standard Model of Elementary Particle Physics (SM)

- Fundamental forces in nature ⇔ Local gauge principle $U(1) \times SU(2)_L \times SU(3)$

  Electromagnetism (QED) Weak interactions Strong interactions (QCD) Gravity



- Building blocks of matter:
  fundamental leptons and quarks (left-handed doublets, right-handed singlets):

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L, \qquad u_R,\ d_R,\ c_R,\ s_R,\ t_R,\ b_R$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \qquad e_R^-,\ \mu_R^-,\ \tau_R^-,\ \nu_{eR},\ \nu_{\mu R},\ \nu_{\tau R}.$$

- Flavour physics is that part of the SM which differentiates between the three families of fundamental fermions.

Main successes of SM:

- All gauge bosons ($J = 1$) and fundamental fermions ($J = \frac{1}{2}$) experimentally verified
- Electroweak precison measurements at LEP (CERN), SLC (SLAC), Tevatron (Fermilab) confirmed SM predictions in the gauge sector : 0.1% accuracy !

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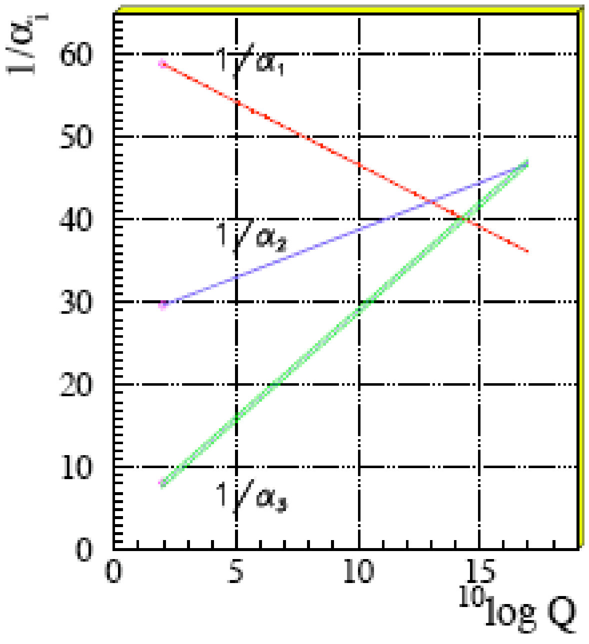
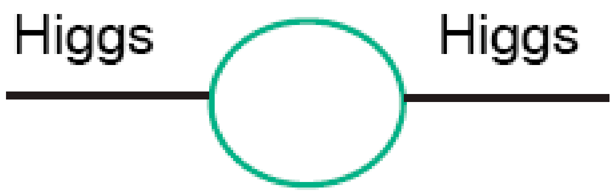
Weaknesses of SM:

- Higgs boson not observed yet, mechanism of mass generation not confirmed yet (unitarity problem has to be solved)
- Many free parameters, mainly in the flavour sector of SM (hierarchy of masses and mixing parameters)
- Gravity not involved in unification (Planck scale)
- Unification of electromagnetic, weak and strong force.

  Indications:
  - quarks, leptons compatible with higher gauge symmetry: $U(1) \times SU(2)_L \times SU(3) \rightarrow SU(5)$ or $SU(10)$
  - unification of coupling constants at higher scale

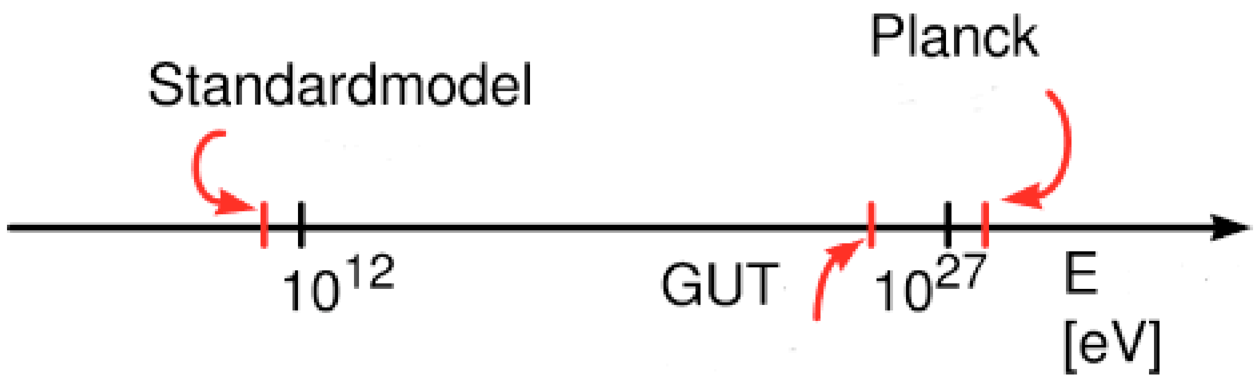


Hierarchy problem: Quantum corrections to Higgs boson mass:



$$m_H^2 \approx (m_H^2)_{tree} + 1/(16\pi^2)\Lambda_{NP}^2$$

$\Rightarrow$ Quadratic sensitivity to highest scale in the theory



After inclusion in larger theory: No stabilisation of the Higgs boson mass at the SM scale

Comparison:
Photon and quark masses protected by gauge symmetry and chiral symmetry, respectively

Many solutions to the hierarchy problem on the market:
Little Higgs Models, Extra Dimensions, Supersymmetry, ...

- Supersymmetry offers most elegant solution for the hierarchy problem

$$\delta m_{\mathrm{H}}^2 \sim \Lambda_{\mathrm{NP}}^2 \Rightarrow \delta m_{\mathrm{H}}^2 \approx log(\mathrm{M_{stop}/M_{top}});\ \mathrm{M_{SUSY}} \leq 1\mathrm{TeV}$$

- Generally to avoid fine-tuning of the Higgs mass (working hypothesis of LHC):

$$m_{\mathrm{H}}^2 \approx (m_{\mathrm{H}}^2)_{\mathrm{tree}} + 1/(16\pi^2)\Lambda_{\mathrm{NP}}^2 \Rightarrow \Lambda_{\mathrm{NP}} \leq 4\pi m_{\mathrm{W}} \approx 1\mathrm{TeV}$$

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- However, electroweak precision measurements (LEP,SLC,Tevatron) naturally indicate a higher new-physics scale (parametrized by higher-dimensional operators):

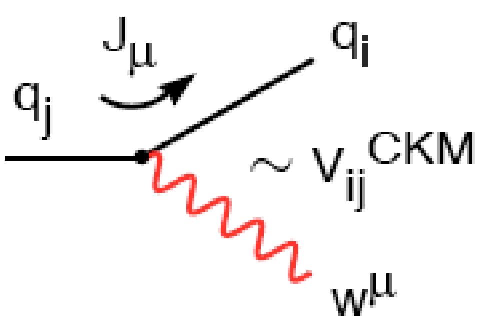
  Little hierarchy problem $\Lambda_{\mathrm{NP}} \approx 3-10\mathrm{TeV}$

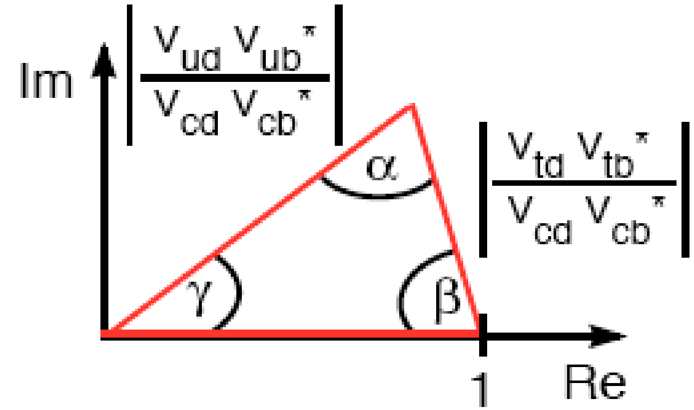
  Highly nontrivial constraint on the possible new physics in the LHC reach!

- There is yet another indirect way to look for new-physics beyond SM ....

Flavour in the SM

CKM mechanism of flavour mixing and CP violation: $V_{\rm CKM}$, $J_{\rm CKM}$





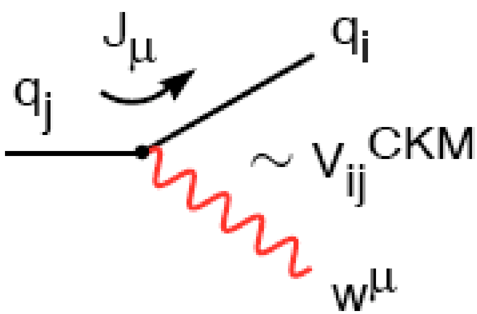
$$Im[V_{ij}\, V_{kl}\, V^*_{il}\, V^*_{kj}] = J_{\rm CKM} \sum_{m,n=1}^{3} \epsilon_{ikm}\, \epsilon_{jln}$$

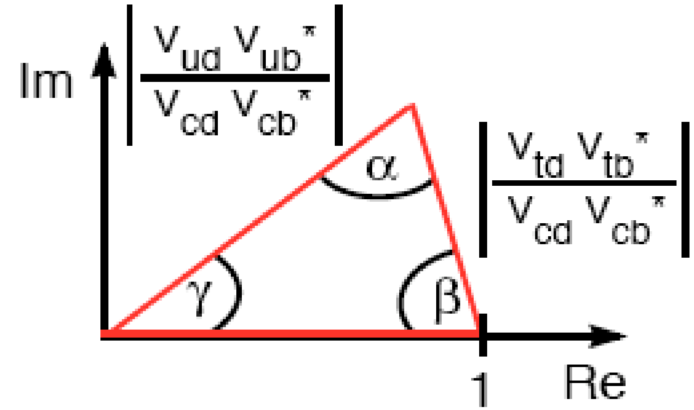
$$J_{\rm CKM} \sim \mathcal{O}(10^{-5})$$

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$

First status report   Flavour in the SM

CKM mechanism of flavour mixing and CP violation: V_{CKM}, J_{CKM}





$$Im[V_{ij} V_{kl} V_{il}^* V_{kj}^*] = J_{\text{CKM}} \sum_{m,n=1}^{3} \epsilon_{ikm} \epsilon_{jln} \qquad J_{\text{CKM}} \sim \mathcal{O}(10^{-5})$$

All present measurements (BaBar, Belle, CLEO, CDF, D0,....)
of rare decays ($\Delta F = 1$),
of mixing phenomena ($\Delta F = 2$) and
of all CP violating observables at tree and loop level
are consistent with the CKM theory.

Impressing success of SM and CKM theory !!

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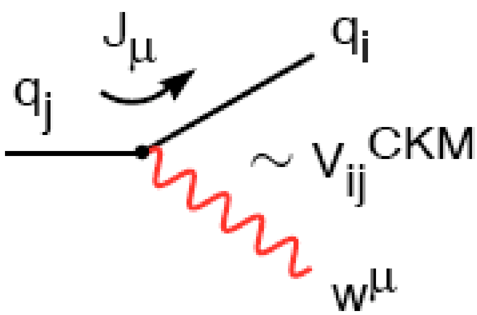
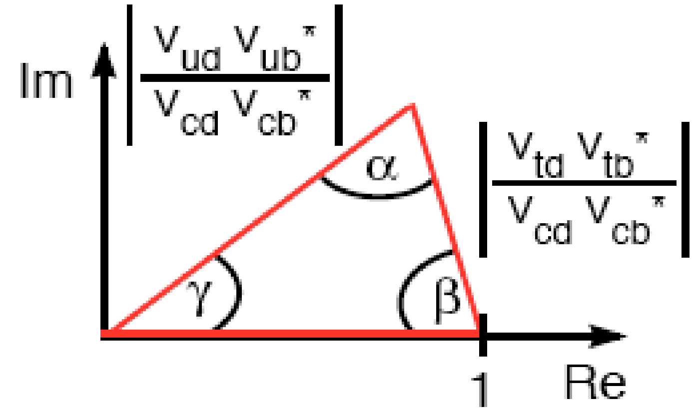
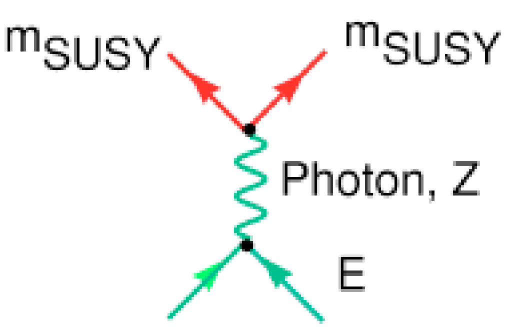
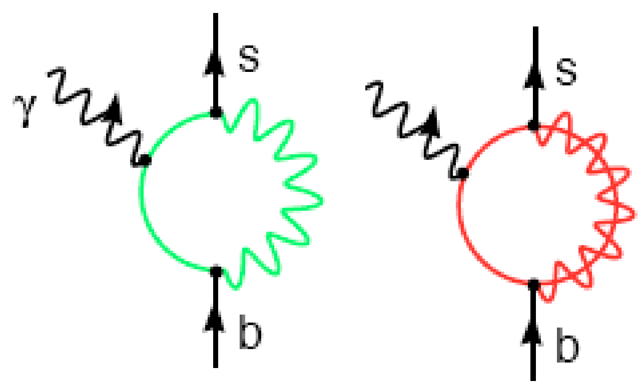




This success is somehow unexpected !!

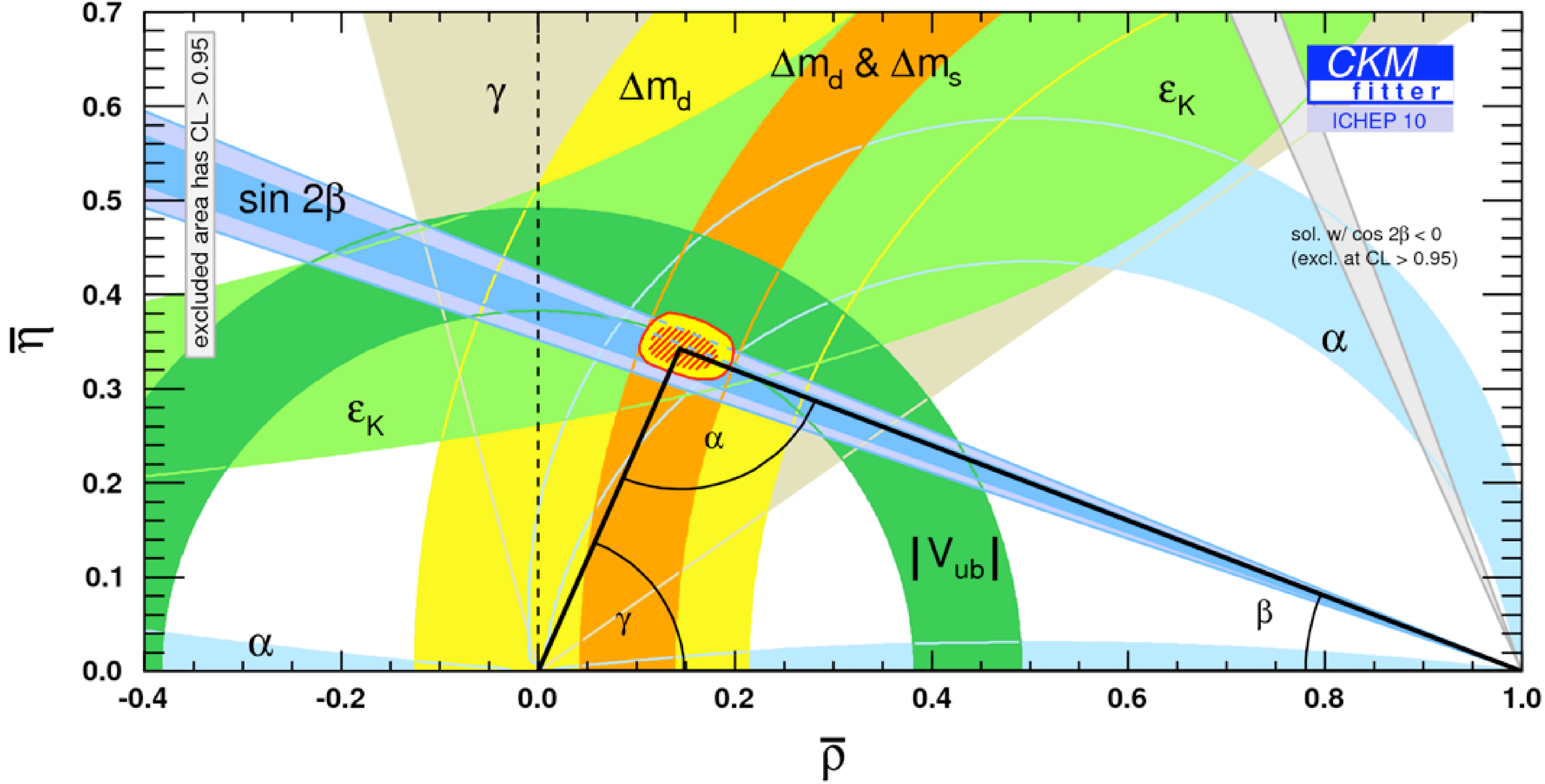




Flavour-changing-neutral-currents as loop-induced processes are highly-sensitive probes for possible new degrees of freedom

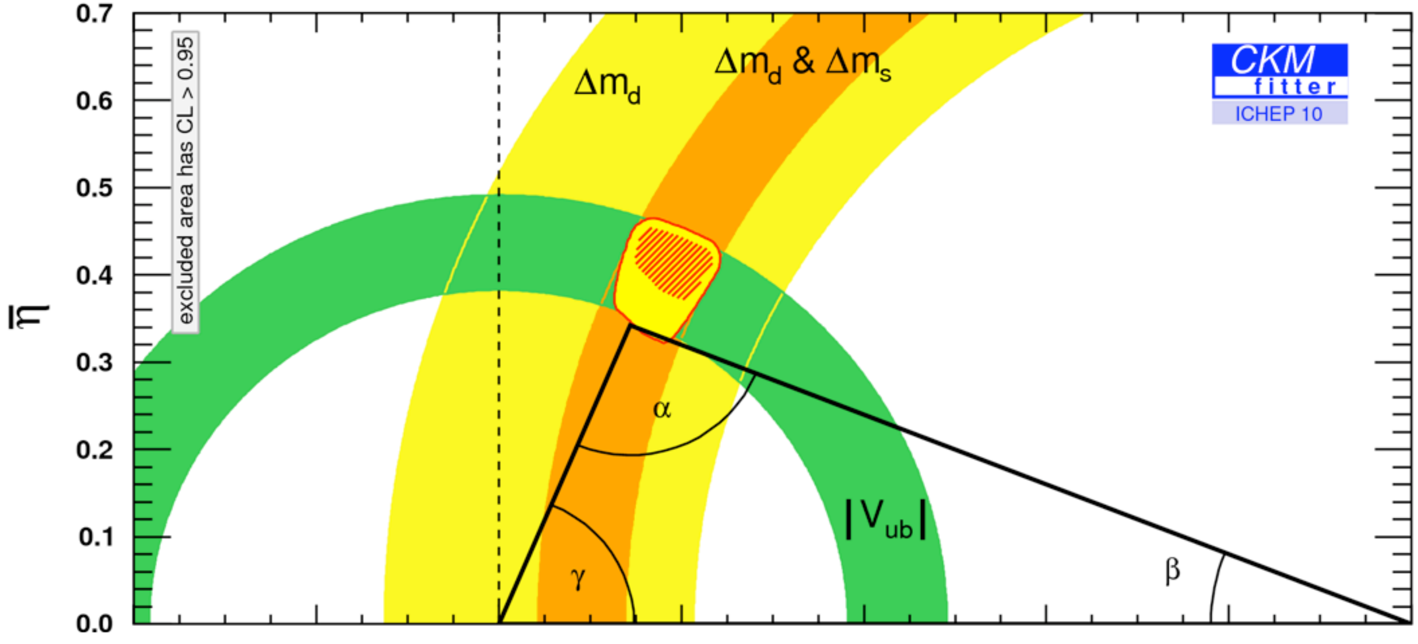
Impressing success of SM and CKM theory !!

Global fit, consistency check of the CKM theory.



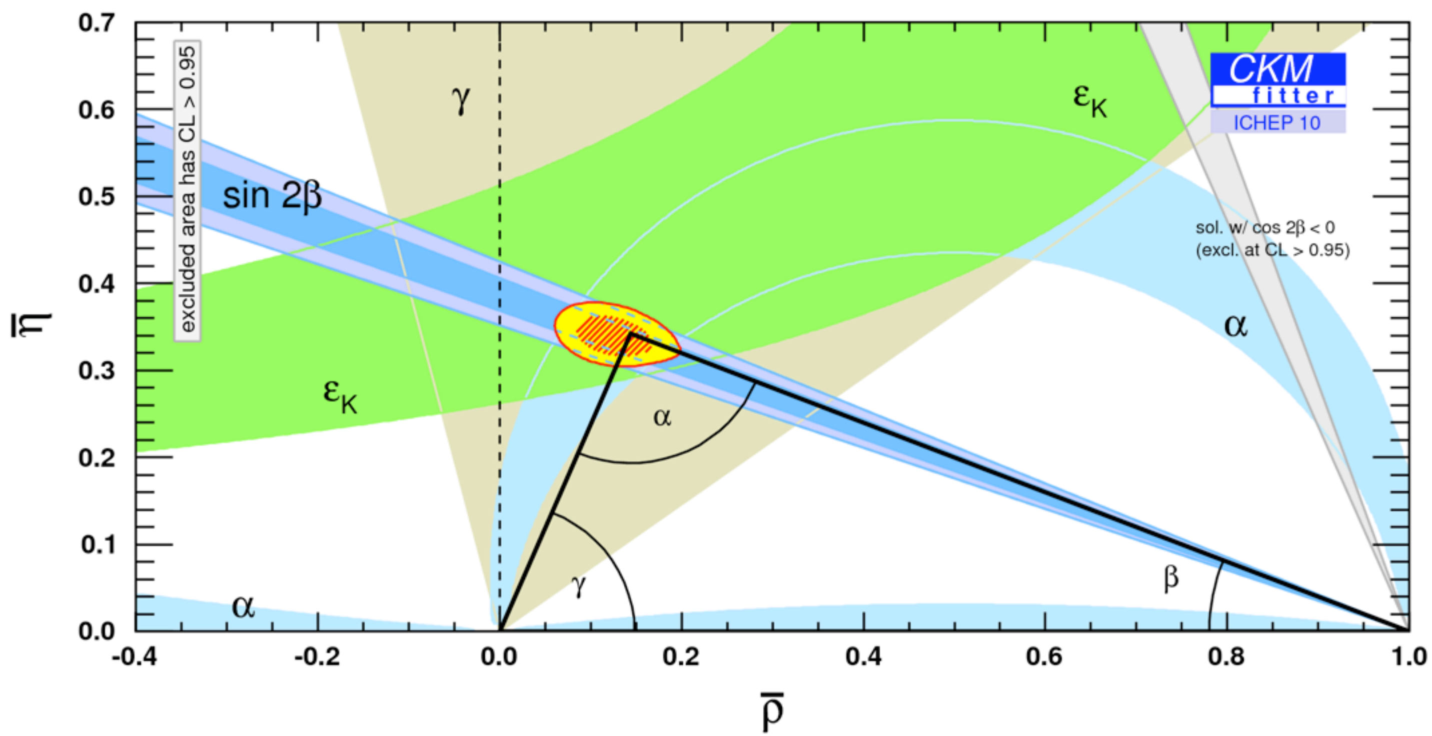
Closer Look:

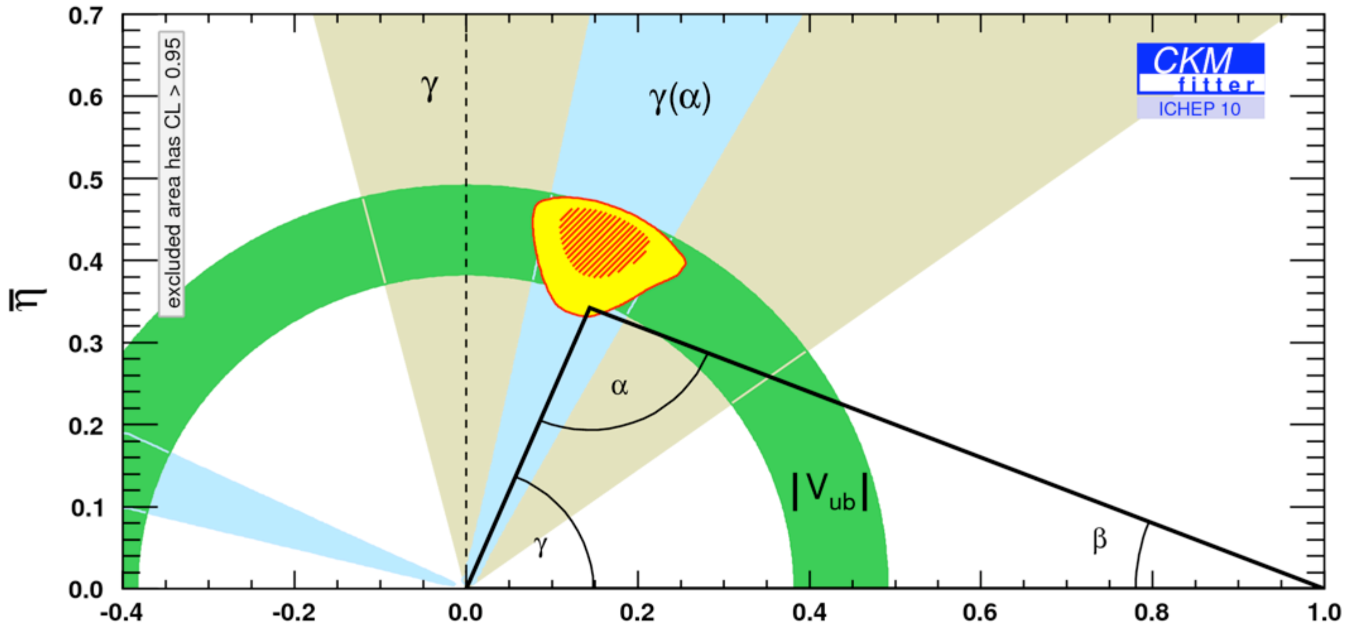
CP conserving



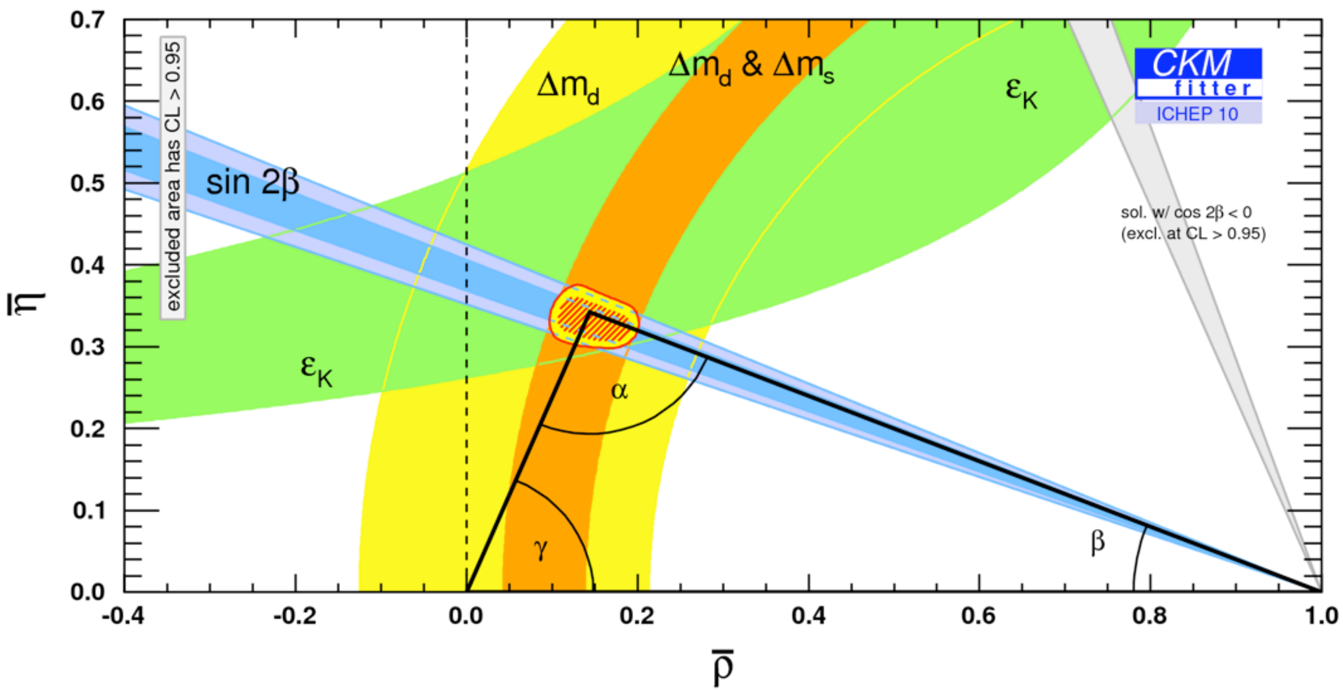
CP violating

observables



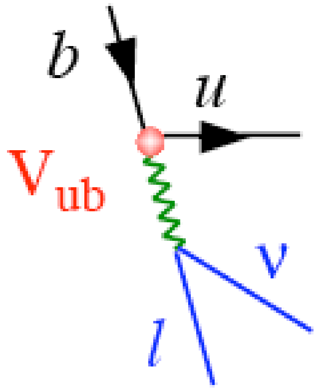


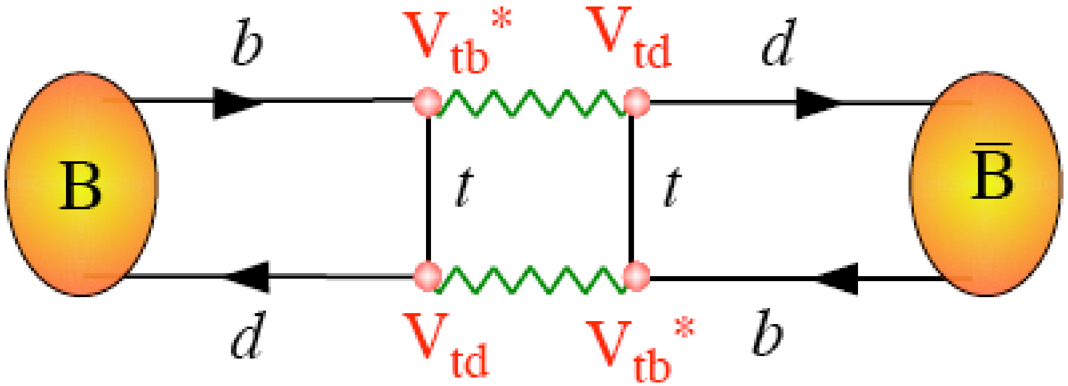
Tree processes



Loop processes

Most surprising is the consistency between the tree-level and loop-induced observables





Semileptonic tree-decays versus Neutral-meson mixing $\Delta F = 2$

SM-dominated

Potentially more sensitive to New Physics

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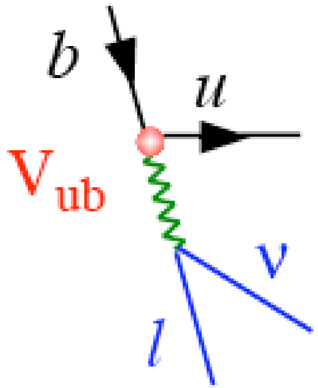

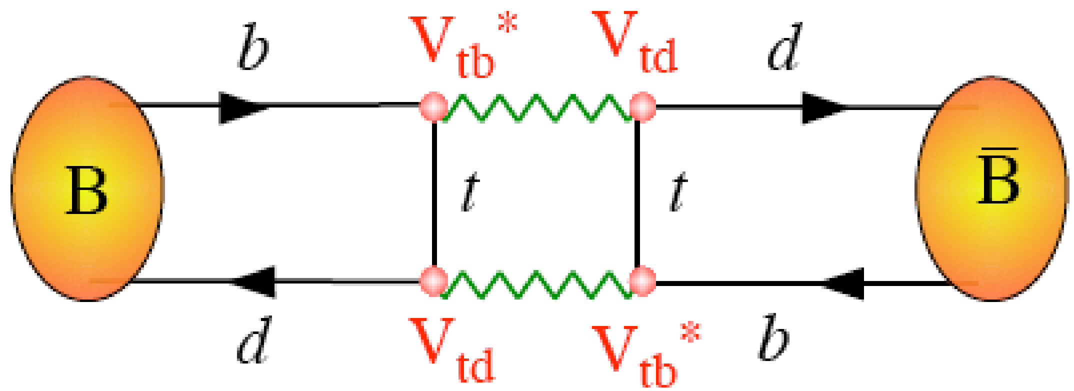

Semileptonic tree-decays versus Neutral-meson mixing $\Delta F = 2$

SM-dominated

Potentially more sensitive to New Physics

There is much more data not shown in the unitarity fits which confirms the SM pedictions of flavour mixing like rare decays ($\Delta F = 1$)

From left, Yoichiro Nambu, Makoto Kobayashi and Toshihide Masukawa, who shared the Nobel Prize in Physics on Tuesday

Nobel Prize 2008

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Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

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In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.

When we apply the renormalizable theory of weak interaction[1] to the hadron system, we have some limitations on the hadron model. It is well known that there exists, in the case of the triplet model, a difficulty of the strangeness changing neutral current and that the quartet model is free from this difficulty. Furthermore, Maki and one of the present authors (T.M.) have shown[2] that, in the latter case, the strong interaction must be chiral $SU(4)\times SU(4)$ invariant as precisely as the conservation of the third component of the iso-spin I_z. In addition to these arguments, for the theory to be realistic, CP-violating interactions should be incorporated in a gauge invariant way. This requirement will impose further limitations on the hadron model and the CP-violating interaction itself. The purpose of the present paper is to investigate this problem. In the following, it will be shown that in the case of the above-mentioned quartet model, we cannot make a CP-violating interaction without introducing any other new fields when we require the following conditions: a) The mass of the fourth member of the quartet, which we will call ζ, is sufficiently large, b) the model should be consistent with our well-established knowledge of the semi-leptonic processes. After that some possible ways of bringing CP-violation into the theory will be discussed.

We consider the quartet model with a charge assignment of Q, $Q-1$, $Q-1$ and Q for p, n, λ and ζ, respectively, and we take the same underlying gauge group $SU_{weak}(2)\times U(1)$ and the scalar doublet field φ as those of Weinberg's original model.[1] Then, hadronic parts of the Lagrangian can be divided in the following way:

$$\mathcal{L}_{had}=\mathcal{L}_{kin}+\mathcal{L}_{mass}+\mathcal{L}_{strong}+\mathcal{L}',$$

where $\mathcal{L}_{kin}$ is the gauge-invariant kinetic part of the quartet field, q, so that it contains interactions with the gauge fields. $\mathcal{L}_{mass}$ is a generalized mass term of q, which includes Yukawa couplings to φ since they contribute to the mass of q through the spontaneous breaking of gauge symmetry. $\mathcal{L}_{strong}$ is a strong-inter-

action part which conserves I_z and therefore chiral $SU(4)\times SU(4)$ invariant.[2] We assume C- and P-invariance of $\mathcal{L}_{strong}$. The last term denotes residual interaction parts if they exist. Since $\mathcal{L}_{mass}$ includes couplings with φ, it has possibilities of violating CP-conservation. As is known as Higgs phenomena,[3] three massless components of φ can be absorbed into the massive gauge fields and eliminated from the Lagrangian. Even after this has been done, both scalar and pseudoscalar parts remain in $\mathcal{L}_{mass}$. For the mass term, however, we can eliminate such pseudoscalar parts by applying an appropriate constant gauge transformation on q, which does not affect on $\mathcal{L}_{strong}$ due to gauge invariance.

Now we consider possible ways of assigning the quartet field to representations of the $SU_{weak}(2)$. Since this group is commutative with the Lorentz transformation, the left and right components of the quartet field, which are respectively defined as $q_L=\frac{1}{2}(1+\gamma_5)q$ and $q_R=\frac{1}{2}(1-\gamma_5)q$, do not mix each other under the gauge transformation. Then, each component has three possibilities:

A) $\mathbf{4}=\mathbf{2}+\mathbf{2}$,

B) $\mathbf{4}=\mathbf{2}+\mathbf{1}+\mathbf{1}$,

C) $\mathbf{4}=\mathbf{1}+\mathbf{1}+\mathbf{1}+\mathbf{1}$,

where on the r.h.s., $\mathbf{n}$ denotes an n-dimensional representation of $SU(2)$. The present scheme of charge assignment of the quartet does not permit representations of $n\geq 3$. As a result, we have nine possibilities which we will denote by (A, A), (A, B), $\cdots$, where the former (latter) in the parentheses indicates the transformation properties of the left (right) component. Since all members of the quartet should take part in the weak interaction, and also of the strangeness changing neutral current is bounded experimentally to a very small value, the cases of (B, C), (C, B) and (C, C) should be abandoned. The models of (B, A) and (C, A) are equivalent to those of (A, B) and (A, C), respectively, except relative signs between vector and axial vector parts of the weak current. Since g_A/g_V ratios are measured only for composite states, this difference of the relative signs would be reduced to a dynamical problem of the composite system. So, we investigate in detail the cases of (A, A), (A, B), (A, C) and (B, B).

i) Case (A, C)

This is the most natural choice in the quartet model. Let us denote two ($SU_{weak}(2)$) doublets and four singlets by L_1, L_2, R_1^p, R_2^p, R_1^n and R_2^n, where superscript $p(n)$ indicates p-like (n-like) charge states. In this case, $\mathcal{L}_{mass}$ takes, in general, the following form:

$$\mathcal{L}_{mass}=\sum_{i,j=1,2}\left[M_{ij}^p\bar{L}_i\varphi R_j^p+M_{ij}^n\bar{L}_i\varepsilon\varphi^* R_j^n\right]+\text{h.c.},$$

$$\varphi=\begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix},\qquad \varepsilon=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \tag{1}$$

where M_{ij}^p and M_{ij}^n are arbitrary complex numbers. We can eliminate three Goldstone modes χ_i by putting

$$\varphi=e^{i\chi\cdot\tau}\begin{pmatrix}0\\ \lambda+\sigma\end{pmatrix}, \tag{2}$$

where λ is a vacuum expectation value of φ^0 and σ is a massive scalar field. Thereafter, performing a diagonalization of the remaining mass term, we obtain

$$\mathcal{L}_{mass}=\bar{q}mq\left(1+\frac{\sigma}{\lambda}\right),$$

$$m=\begin{pmatrix}m_p & 0 & 0 & 0\\ 0 & m_n & 0 & 0\\ 0 & 0 & m_\lambda & 0\\ 0 & 0 & 0 & m_\zeta\end{pmatrix},\qquad q=\begin{pmatrix}p\\ n\\ \zeta\\ \lambda\end{pmatrix}. \tag{3}$$

Then, the interaction with the gauge field in $\mathcal{L}_{kin}$ is expressed as

$$\sum_{i=1}^{3}A_\mu^i\bar{q}\gamma^\mu A_i\frac{1+\gamma_5}{2}q. \tag{4}$$

Here, A_i is the representation matrix of $SU_{weak}(2)$ for this case and explicitly given by

$$A_+=\frac{A_1+iA_2}{2}=K\begin{pmatrix}0 & U\\ 0 & 0\end{pmatrix}K^{-1},\quad A_3=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix},\quad K=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}, \tag{5}$$

where U is a 2×2 unitary matrix. Here and hereafter we neglect the gauge field corresponding to $U(1)$ which is irrelevant to our discussion. With an appropriate phase convention of the quartet field we can take U as

$$U=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}. \tag{6}$$

Therefore, if $\mathcal{L}'=0$, no CP-violations occur in this case. It should be noted, however, that this argument does not hold when we introduce one more fermion doublet with the same charge assignment. This is because all phases of elements of a 3×3 unitary matrix cannot be absorbed into the phase convention of six fields. This possibility of CP-violation will be discussed later on.

ii) Case (A, B)

This is a rather delicate case. We denote two left doublets, one right doublet and two singlets by L_{1L}, L_{2L}, R_R, $R_1^{(p)}$ and $R_1^{(n)}$, respectively. The general form

of $\mathcal{L}_{mass}$ is given by

$$\mathcal{L}_{mass}=\sum_{\alpha=1,2}\left[m_\alpha\bar{L}_{\alpha L}R_R+M_\alpha^{(p)}\bar{L}_{\alpha L}\varphi R_1^{(p)}+M_\alpha^{(n)}\bar{L}_{\alpha L}\varepsilon\varphi^* R_1^{(n)}\right]+\text{h.c.},$$

where m_α, $M_\alpha^{(p)}$ and $M_\alpha^{(n)}$ are arbitrary complex numbers. After diagonalization of mass terms (in this case, the CP-odd part of coupling with σ does not disappear in general) each multiplet can be expressed as follows:

$$L_{1L}=\frac{1+\gamma_5}{2}\begin{pmatrix}p\\ \cos\theta e^{i\alpha}n+\sin\theta e^{i\beta}\lambda\end{pmatrix},\qquad L_{2L}=\frac{1+\gamma_5}{2}\begin{pmatrix}e^{i\gamma}\zeta\\ -\sin\theta e^{i\alpha}n+\cos\theta e^{i\beta}\lambda\end{pmatrix},$$

$$R_R=\frac{1-\gamma_5}{2}\begin{pmatrix}\sin\xi\cdot p+\cos\xi\cdot\zeta\\ \sin\eta\cdot n+\cos\eta\cdot\lambda\end{pmatrix},\qquad R_1^{(p)}=\frac{1-\gamma_5}{2}(\cos\xi\cdot p-\sin\xi\cdot\zeta),$$

$$R_1^{(n)}=\frac{1-\gamma_5}{2}(\cos\eta\cdot n-\sin\eta\cdot\lambda), \tag{7}$$

where phase factors α, β and γ satisfy two relations with the masses of the quartet:

$$e^{i\gamma}m_\zeta\sin\theta\cos\xi=m_p\cos\theta\sin\xi-e^{i\alpha}m_n\sin\eta,$$

$$e^{i\gamma}m_\zeta\cos\theta\cos\xi=-m_p\sin\theta\cos\xi+e^{i\beta}m_\lambda\cos\eta. \tag{8}$$

Owing to the presence of phase factors, there exists a possibility of CP-violation also through the weak current. However, the strangeness changing neutral current is proportional to $\sin\eta\cos\eta$ and its experimental upper bound is roughly

$$\sin\eta\cos\eta\lesssim 10^{-3\sim-4}. \tag{9}$$

Thus, making an approximation of $\sin\eta\sim 0$ (the other choice $\cos\eta\sim 0$ is less critical) we obtain from Eq. (8)

$$m_\zeta/m_p\sim\cot\theta\cdot\tan\xi,$$

$$m_\zeta/m_n\sim\sin\xi/\sin\theta. \tag{10}$$

We have no low-lying particle with a quantum number corresponding to ζ, so that m_ζ, which is a measure of chiral $SU(4)\times SU(4)$ breaking, should be sufficiently large compared to the masses of the other members. However, the present experimental results on the g_A/g_V ratios of the octet baryon β-decay would not permit $\sin\xi\gg\sin\theta$. Thus, it seems difficult to reconcile the hierarchy of chiral symmetry breaking with the experimental knowledge of the semileptonic processes.

iii) Case (B, B)

As a previous one, in this case also, occurrence of CP-violation is possible, but in order to suppress $|\Delta S|=1$ neutral currents, coefficients of the axial-vector part of $\Delta S=0$ and $|\Delta S|=1$ weak currents must take signs opposite to each other. This contradicts again the experiments on the baryon β-decay.

iv) Case (A, A)

In a similar way, we can show that no CP-violation occurs in this case as far as $\mathcal{L}'=0$. Furthermore this model would reduce to an exactly $U(4)$ symmetric one.

Summarizing the above results, we have no realistic models in the quartet scheme as far as $\mathcal{L}'=0$. Now we consider some examples of CP-violation through $\mathcal{L}'$. Hereafter we will consider only the case of (A, C). The first one is to introduce another scalar doublet field ϕ. Then, we may consider an interaction with this new field

$$\mathcal{L}'=\bar{q}\boldsymbol{\phi}C\frac{1-\gamma_5}{2}q+\text{h.c.}, \tag{11}$$

$$\boldsymbol{\phi}=\begin{pmatrix}\bar{\phi}^0 & \phi^+ & 0 & 0\\ -\phi^- & \phi^0 & 0 & 0\\ 0 & 0 & \bar{\phi}^0 & \phi^+\\ 0 & 0 & -\phi^- & \phi^0\end{pmatrix},\qquad C=\begin{pmatrix}c_{11} & 0 & c_{12} & 0\\ 0 & d_{11} & 0 & d_{12}\\ c_{21} & 0 & c_{22} & 0\\ 0 & d_{21} & 0 & d_{22}\end{pmatrix},$$

where c_{ij} and d_{ij} are arbitrary complex numbers. Since we have already made use of the gauge transformation to get rid of the CP-odd part from the quartet mass term, there remains no such arbitrariness. Furthermore, we note that an arbitrariness of the phase of ϕ cannot absorb all the phases of c_{ij} and d_{ij}. So, this interaction can cause a CP-violation.

Another one is a possibility associated with the strong interaction. Let us consider a scalar (pseudoscalar) field S which mediates the strong interaction. For the interaction to be renormalizable and $SU_{weak}(2)$ invariant, it must belong to a $(4, 4^*)+(4^*, 4)$ representation of chiral $SU(4)\times SU(4)$ and interact with q through scalar and pseudoscalar couplings. It also interacts with φ and possible renormalizable forms are given as follows:

$$\mathrm{tr}\{G_1S^\dagger\boldsymbol{\varphi}\}+\text{h.c.},$$

$$\mathrm{tr}\{G_2S^\dagger\boldsymbol{\varphi}G_2'\boldsymbol{\varphi}^\dagger S\}+\text{h.c.},$$

$$\mathrm{tr}\{G_3S^\dagger\boldsymbol{\varphi}G_3'S^\dagger\boldsymbol{\varphi}\}+\text{h.c.}, \tag{12}$$

with

$$\boldsymbol{\varphi}=\begin{pmatrix}\varphi^0 & \varphi^+ & 0 & 0\\ -\varphi^- & \bar{\varphi}^0 & 0 & 0\\ 0 & 0 & \varphi^0 & \varphi^+\\ 0 & 0 & -\varphi^- & \bar{\varphi}^0\end{pmatrix},$$

where G_i is a 4×4 complex matrix and we have used a 4×4 matrix representation for S. It is easy to see that these interaction terms can violate CP-conservation.

Next we consider a 6-plet model, another interesting model of CP-violation. Suppose that 6-plet with charges $(Q, Q, Q, Q-1, Q-1, Q-1)$ is decomposed into $SU_{weak}(2)$ multiplets as $2+2+2$ and $1+1+1+1+1+1$ for left and right components, respectively. Just as the case of (A, C), we have a similar expression for the charged weak current with a 3×3 instead of 2×2 unitary matrix in Eq. (5). As was pointed out, in this case we cannot absorb all phases of matrix elements into the phase convention and can take, for example, the following expression:

$$\begin{pmatrix}\cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3\\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3e^{i\delta}\\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\cos\theta_3e^{i\delta}\end{pmatrix} \tag{13}$$

Then, we have CP-violating effects through the interference among these different current components. An interesting feature of this model is that the CP-violating effects of lowest order appear only in $\Delta S\neq 0$ non-leptonic processes and in the semi-leptonic decay of neutral strange mesons (we are not concerned with higher states with the new quantum number) and not in the other semi-leptonic, $\Delta S=0$ non-leptonic and pure-leptonic processes.

So far we have considered only the straightforward extensions of the original Weinberg's model. However, other schemes of underlying gauge groups and/or scalar fields are possible. Georgi and Glashow's model[4] is one of them. We can easily see that CP-violation is incorporated into their model without introducing any other fields than (many) new fields which they have introduced already.

References

1) S. Weinberg, Phys. Rev. Letters 19 (1967), 1264; 27 (1971), 1688.
2) Z. Maki and T. Maskawa, RIFP-146 (preprint), April 1972.
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However,...

- CKM mechanism is the dominating effect for CP violation and flavour mixing in the quark sector;

  but there is still room for sizable new effects and new flavour structures (the flavour sector has only be tested at the 10% level in many cases).

- The SM does not describe the flavour phenomena in the lepton sector.

Flavour problem of SM

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge}(A_i, \psi_i) + \mathcal{L}_{Higgs}(\Phi, \psi_i, v)$$

- Gauge principle governs the gauge sector of the SM.

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Compare for example:

$|V_{us}| \approx 0.2, |V_{cb}| \approx 0.04, |V_{ub}| \approx 0.004$ versus $g_s \approx 1, g \approx 0.6, g' \approx 0.3$

Many open fundamental questions of particle physics are related to flavour :

- How many families of fundamental fermions are there ?
- How are neutrino and quark masses and mixing angles are generated ?
- Do there exist new sources of flavour and CP violation ?
- Is there CP violation in the QCD gauge sector ?
- Relations between the flavour structure in the lepton and quark sector ?

What can we learn from decays of B mesons ?

$$B^0_{d,(s)} = \bar{b}d(s),\ \bar{B}^0_{d,(s)} = b\bar{d}(\bar{s}),\ B^+_u = \bar{b}u,\ B^-_u = b\bar{u}$$

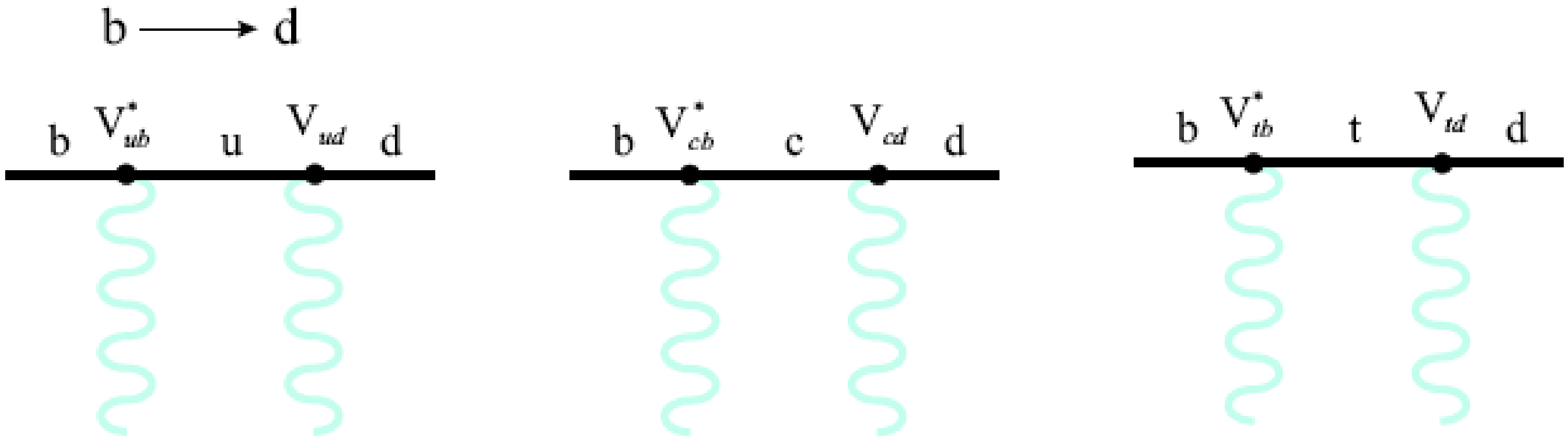
- b quark heaviest quark with pronounced hadronic bound states (QCD tests)

- Many different decay modes ($m_B = 5.27 GeV$) $\rightarrow$ rich CKM phenomenology

- GIM suppression largely relaxed because m_t very large (BR of FCNC in B system $\approx 10^{-5} \leftrightarrow K$ or D system)

- Independent test of the mechanism of CP violation (large effects $\leftrightarrow K$ system)

Large m_{top} overrides GIM suppression



$$A = V^*_{ub} V_{ud} f(m_u) + V^*_{cb} V_{cd} f(m_c) + V^*_{tb} V_{td} f(m_t)$$

$A = 0$ if $m_u = m_c = m_t$

However $m_t \gg m_c, m_u$

$f(m) \approx m^2$ quadratic GIM $\qquad f(m) \approx log(m)$ logarithmic GIM

Central Questions in B Physics

CKM phenomenology

Mechanism of CP violation

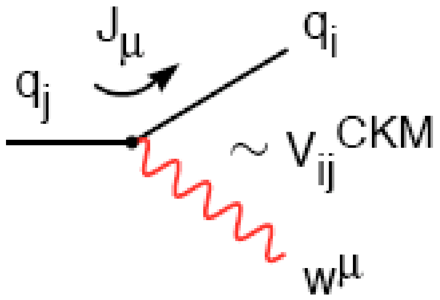
Indirect search for new physics

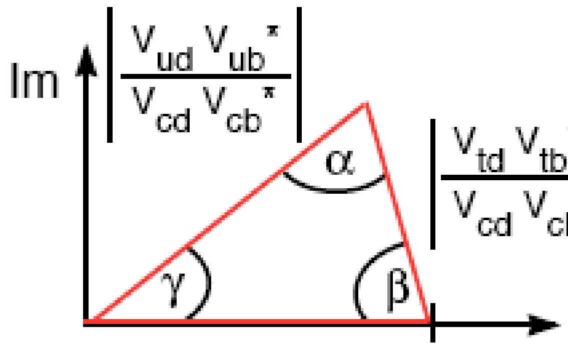
Quantitative understanding of long-distance strong interactions

CKM Phenomenology, Unitarity Triangle

Why ?

- determine fundamental SM parameters (Yukawa-matrices $Y^{u,d}$ $\to$ model building)
- CKM phase: the only source of CP-violation?
- overconstraining the unitarity angle (possible signals for new physics)





$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - \mathrm{i}\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-\mathrm{i}\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Unitarity: $\qquad V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

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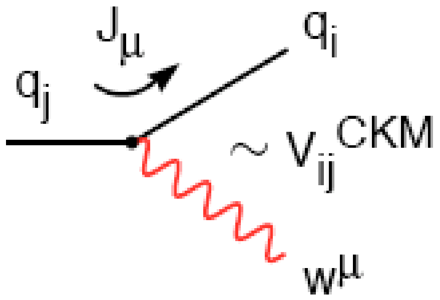

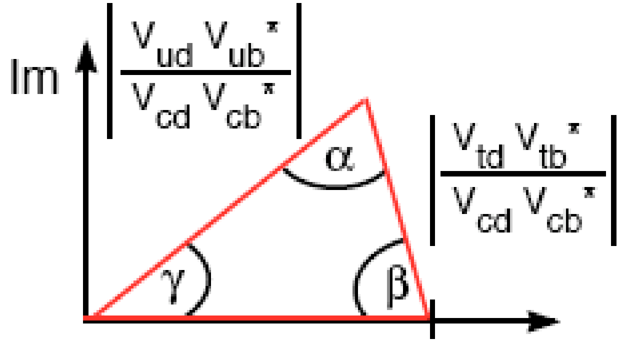

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Caveat: Yukawa couplings $\Leftrightarrow$ CKM matrix

CP Violation

Why is CP violation interesting ?

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- Various extensions of the SM offer new sources of CP violation .

CP violation in the SM

In chiral gauge theories CP is a natural symmetry.

$$\mathcal{L}_{gauge} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \psi_L^\dagger(i\bar{\sigma}D)\psi_L + \psi_R^\dagger(i\bar{\sigma}\partial)\psi_R$$

D is the covariant derivative

$\mathcal{L}$ violates P — *Right-handed fermions do not couple to gauge bosons.*

$\mathcal{L}$ violates C — *Left-handed antifermions do not couple to gauge bosons.*

$\mathcal{L}$ preserves CP — *Both left-handed fermions and right-handed antifermions couple to gauge bosons.*

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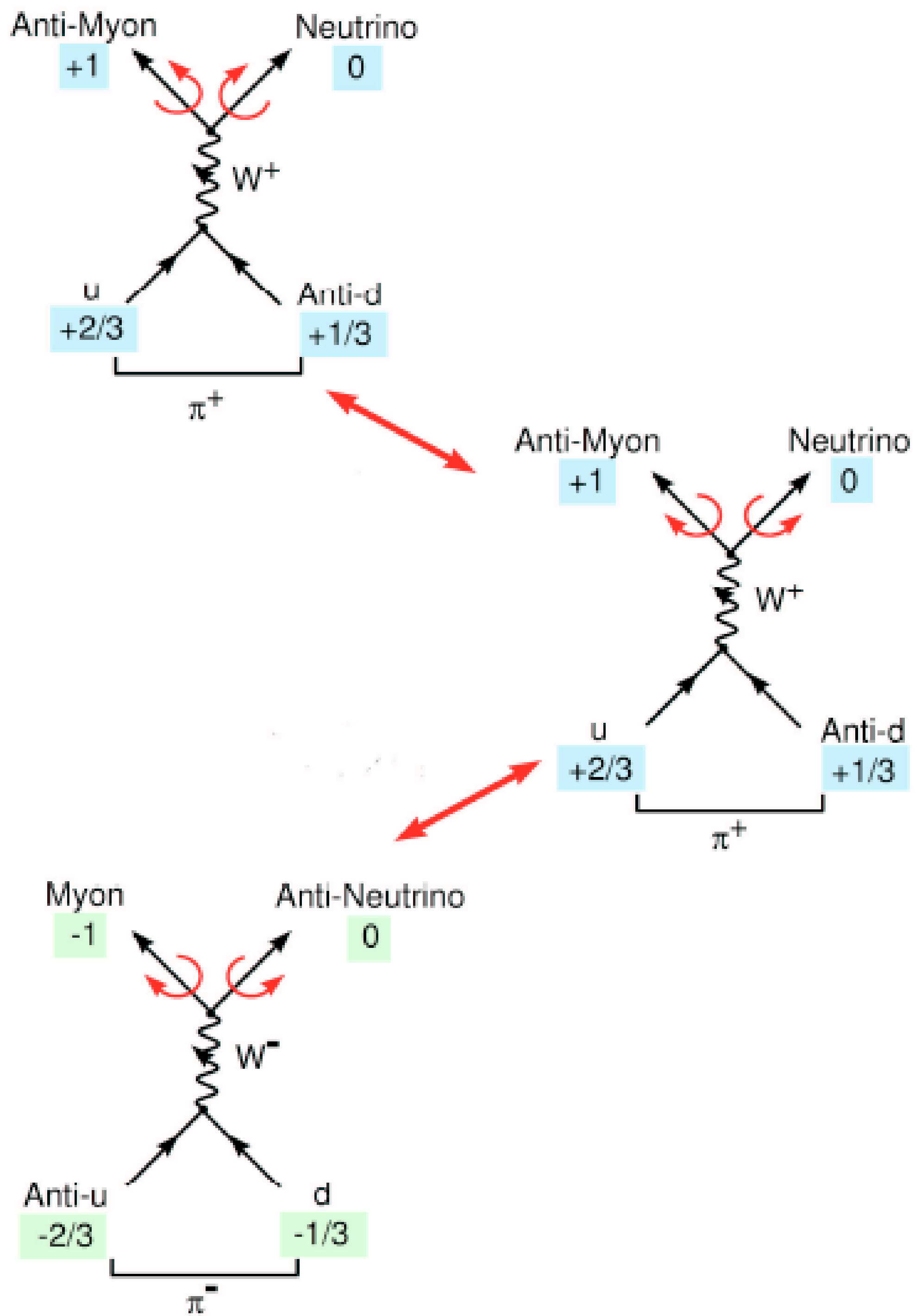
Massless gauge theories are invariant under CP

The weak force breaks C and P maximally



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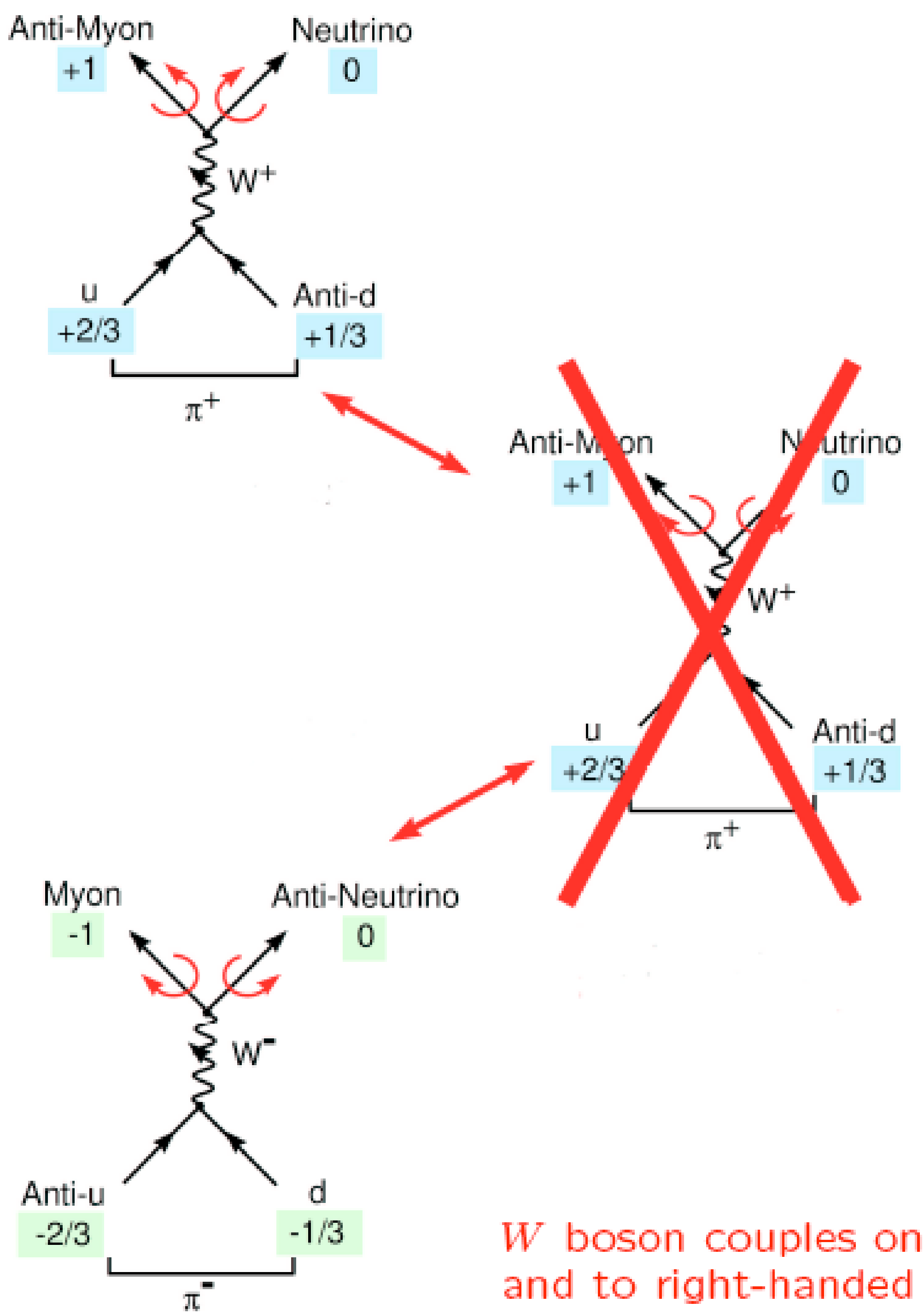
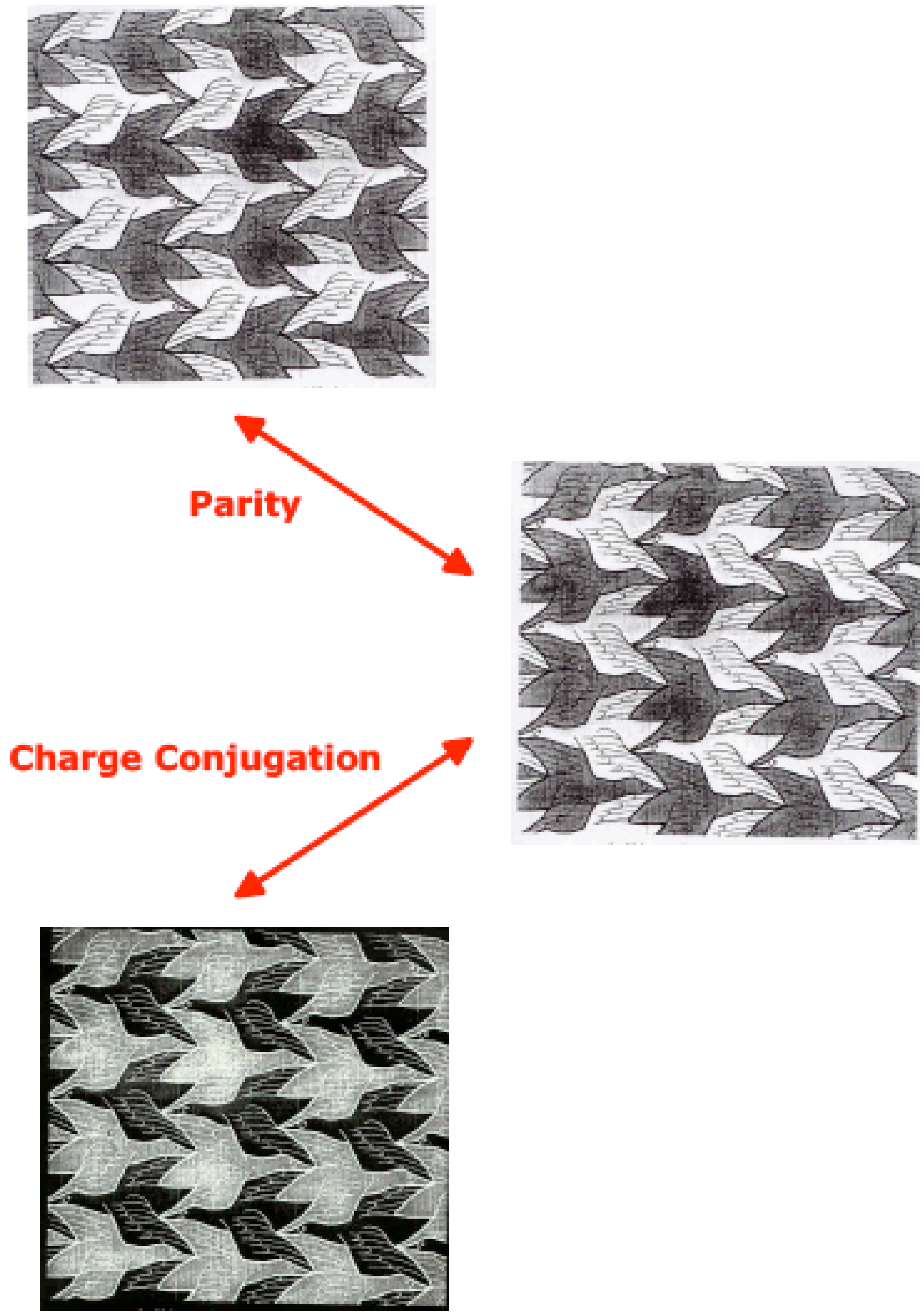


W boson couples only to left-handed fermions and to right-handed anti-fermions

M. C. Escher



SM basics

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- Fermion representations

$$Q^I_{Li}(3,2)_{+1/6},\ \ U^I_{Ri}(3,1)_{+2/3},\ \ D^I_{Ri}(3,1)_{-1/3},\ \ L^I_{Li}(1,2)_{-1/2},\ \ E^I_{Ri}(1,1)_{-1}.$$

Notation: left-handed quarks, Q^I_L: $SU(3)_{\rm C}$, doublets of $SU(2)_{\rm L}$ and carry hypercharge $Y = +1/6$

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I interaction eigenstates

$i = 1, 2, 3$ flavor index

- Spontaneous symmetry breaking

$$\phi(1,2)_{+1/2} \qquad \langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad G_{\rm SM} \to SU(3)_{\rm C} \times U(1)_{\rm EM}$$

$$\mathcal{L}_{\rm gauge}(Q_L) = i\overline{Q^I_{Li}}\gamma_\mu \left(\partial^\mu + \frac{i}{2} g_s G^\mu_a \lambda_a + \frac{i}{2} g W^\mu_b \tau_b + \frac{i}{6} g' B^\mu \right) Q^I_{Li}$$

CP conserving

- $-\mathcal{L}_{\text{Yukawa}}^{\text{quarks}} = Y_{ij}^{d}\,\overline{Q_{Li}^{I}}\phi D_{Rj}^{I} + Y_{ij}^{u}\,\overline{Q_{Li}^{I}}\tilde{\phi} U_{Rj}^{I} + \text{h.c.}$

CP violating if and only if $\text{Im}\left\{\det[Y^{d}Y^{d\dagger}, Y^{u}Y^{u\dagger}]\right\} \neq 0.$

Jarlskog 1985

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- CP violation is related to *complex* Yukawa couplings

  Hermiticity of the Lagrangian $\quad Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj} + Y^*_{ij}\overline{\psi_{Rj}}\phi^\dagger\psi_{Li}$

  A CP transformation $\quad \overline{\psi_{Li}}\phi\psi_{Rj} \leftrightarrow \overline{\psi_{Rj}}\phi^\dagger\psi_{Li}$.

  CP invariance if $Y_{ij} = Y^*_{ij}$.

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Yukawa couplings only source of CP violation in SM

Yukawa couplings only source of flavour violation in SM

- Quark Yukawa couplings break the quark flavour symmetry down to baryon number conservation

$$G^{quark}(Y^f = 0) = U(3)_Q \times U(3)_D \times U(3)_U \;\to\; G^{quark} = U(1)_B$$

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- Number of physical parameters in quark Yukawa couplings

$$(18 \times 2) - (9 \times 3) + 1 = 10$$

'Complex phase in CKM matrix related to CP violation'

- $-\mathcal{L}_M^q = (M_d)_{ij}\overline{D_{Li}^I}D_{Rj}^I + (M_u)_{ij}\overline{U_{Li}^I}U_{Rj}^I + \text{h.c.}$ $\quad M_q = \frac{v}{\sqrt{2}}Y^q$

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- $P_q\ (q = u, d)$ diagonal unitary (phase) matrices

$\tilde{V}_{qL} = P_q V_{qL} \quad \tilde{V}_{qR} = P_q V_{qR} \qquad M_q^{\text{diag}}$ unchanged

'Complex phase in CKM matrix related to CP violation'

- $-\mathcal{L}_M^q = (M_d)_{ij}\overline{D_{Li}^I}D_{Rj}^I + (M_u)_{ij}\overline{U_{Li}^I}U_{Rj}^I + \text{h.c.} \qquad M_q = \frac{v}{\sqrt{2}}Y^q$

$$\text{Re}(\phi^0) \to (v + H^0)/\sqrt{2}$$

- Diagonalization of mass matrices by unitary matrices V_{qL} and V_{qR}

$V_{qL}M_qV_{qR}^\dagger = M_q^{\text{diag}} \qquad q_{Li} = (V_{qL})_{ij}q_{Lj}^I, \quad q_{Ri} = (V_{qR})_{ij}q_{Rj}^I \quad (q = u, d)$

- $-\mathcal{L}_{W^\pm}^q = \frac{g}{\sqrt{2}}\overline{u_{Li}}\gamma^\mu(V_{uL}V_{dL}^\dagger)_{ij}\, d_{Lj}W_\mu^+ + \text{h.c.}$

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- Physical parameters:

6 quark masses + (9 CKM parameters −5 relative phases) = 10

Naive argument:

- The charge current interaction Lagrangian in mass eigenstate basis

$$\mathcal{L}_{W^+} = \frac{g}{\sqrt{2}}\bar{u}_{Li}\gamma^\mu V_{ij}\, d_{Lj}W_\mu^+ + \frac{g}{\sqrt{2}}\bar{d}_{Lj}\gamma^\mu V_{ij}^* u_{Lj}W_\mu^-$$

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- A representation of CP is given via

$$W^{+}_{\mu} \xrightarrow{CP} W^{-}_{\mu} \qquad \bar{\psi}_1\gamma_{\mu}\psi_2 \xrightarrow{CP} \bar{\psi}_2\gamma_{\mu}\psi_1$$

$$\Rightarrow \mathcal{L}^{CP}_{W+} = \frac{g}{\sqrt{2}}\bar{d}_{Lj}\gamma^{\mu}V_{ij}\, u_{Li}W^{-}_{\mu} + \frac{g}{\sqrt{2}}\bar{u}_{Li}\gamma^{\mu}V^{*}_{ij}\, d_{Lj}W^{+}_{\mu}$$

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Argument more involved (not all phases in CKM matrix are physical)!

Physically quantities must be invariant under a rephasing of the fields

- Rephasing invariants:

  1. Moduli of CKM matrix elements $\mid V_{\alpha i} \mid^2$.
  2. Quartets: $Q_{\alpha i \beta j} = V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}$.
  3. Invariants of higher order may in general

     be written as functions of 1 and 2:

     Example: $$V_{\alpha i} V_{\beta j} V_{\gamma k} V^*_{\alpha j} V^*_{\beta k} V_{\gamma i} = \frac{Q_{\alpha i \beta j} Q_{\beta i \gamma k}}{\mid V_{\beta i} \mid^2}$$

     (singular cases if some elements vanish)

- The most general CP transformation which leaves invariant all terms of the Lagrangian, except $\mathcal{L}_{W+}$, is given by

$$
\begin{aligned}
U_{CP} u_\alpha(t, \overrightarrow{r}) U_{CP}^\dagger &= e^{i\xi_\alpha} \gamma^0 C \bar{u}_\alpha^T(t, -\overrightarrow{r}), \\
U_{CP} \bar{u}_\alpha(t, \overrightarrow{r}) U_{CP}^\dagger &= -e^{-i\xi_\alpha} \bar{u}_\alpha^T(t, -\overrightarrow{r}) C^{-1} \gamma^0, \\
U_{CP} d_k(t, \overrightarrow{r}) U_{CP}^\dagger &= e^{i\xi_k} \gamma^0 C \bar{d}_k^T(t, -\overrightarrow{r}), \\
U_{CP} \bar{d}_k(t, \overrightarrow{r}) U_{CP}^\dagger &= -e^{-i\xi_k} d_k^T(t, -\overrightarrow{r}) C^{-1} \gamma^0, \\
U_{CP} W^{+\mu}(t, \overrightarrow{r}) U_{CP}^\dagger &= -e^{-i\xi_W} W_\mu^-(t, -\overrightarrow{r}).
\end{aligned}
$$

- The CP invariance of $\mathcal{L}_{W+}$ constrains V_{CKM} to satisfy

$$
V_{\alpha k}^* = e^{i(\xi_W + \xi_k - \xi_\alpha)} V_{\alpha k}, \quad \mathrm{Im} Q_{\alpha i \beta j} = \mathrm{Im}\,(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*) = 0.
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- The CP invariance requires that all rephasing invariant combinations of CKM matrix elements be real!

(parametrization-independent criterium)

- Parametrization-independent CP violating quantity in V_{CKM}:

$$\mathrm{Im}[V_{ij}V_{kl}V_{il}^*V_{kj}^*] = J_{CKM}\sum_{m,n=1}^{3}\epsilon_{ikm}\epsilon_{jln} \qquad (i,j,k,l = 1,2,3)$$

Jarlskog parameter

All $|\,\mathrm{Im}Q_{ijkl}\,|$ are equal (use unitarity relations)

$$J_{CKM} \simeq \lambda^6 A^2 \eta = \mathcal{O}(10^{-5})$$

Jarlskog Criterion in Weak Interaction Basis

- Start with Lagrangian in its initial form in the weak basis.
  All gauge currents are diagonal and real
- Consider the most general CP transformation which leaves invariant the part of the Lagrangian containing the gauge interactions.
- Check whether the CP transformations thus defined implies any restrictions on the remaining of the Lagrangian.

$\Rightarrow$ Restrictions on $\mathcal{L}_{mass}$

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- CP violation arises as a clash between the CP properties of the gauge interactions and the mass terms. $\mathcal{L}_{gauge} \leftrightarrow \mathcal{L}_{mass}$

- Condition for CP violation in the quark sector of the SM:

$$J_{CKM} \Delta m^2_{tc} \Delta m^2_{cu} \Delta m^2_{bs} \Delta m^2_{bd} \Delta m^2_{sd} \neq 0, \quad \Delta m^2_{ij} \equiv m^2_i - m^2_j.$$

Jarlskog 1985

- Requirements on the SM to violate CP:

(a) within each quark sector, no mass degeneracy allowed

(b) none of the three mixing angles should be zero or $\frac{\pi}{2}$ ($J_{CKM} \sim A$)

(c) the physical phase should not be 0 or π.

- Parametrizations of the CKM matrix

$$\mathbf{V_{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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- Parametrizations of the CKM matrix

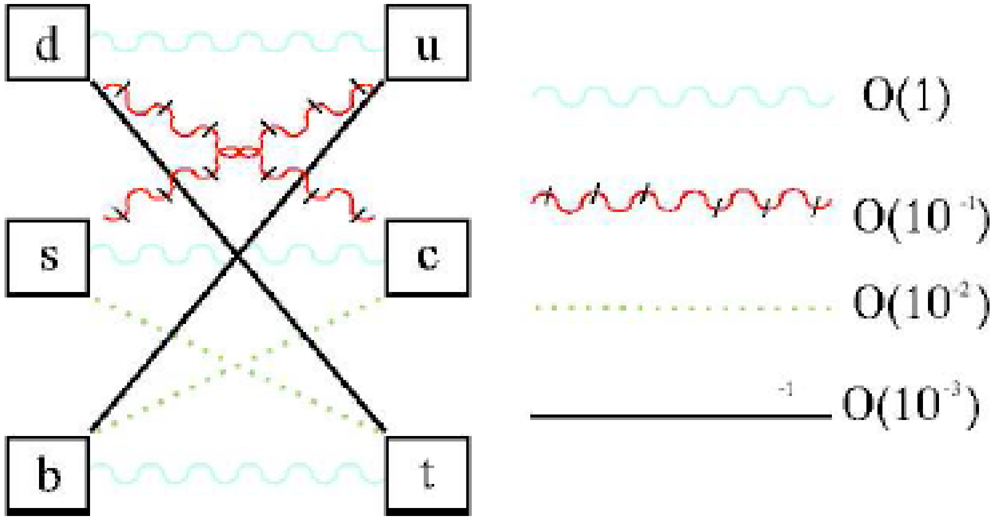
  Standard parametrization:

$$\mathbf{V_{CKM}} = \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13}e^{i\delta} & -C_{12}C_{23} - S_{12}S_{23}S_{13}e^{i\delta} & S_{23}C_{13} \\ S_{12}S_{23} - C_{12}C_{23}S_{13}e^{i\delta} & C_{12}S_{23} - S_{12}C_{23}S_{13}e^{i\delta} & C_{23}C_{13} \end{pmatrix}$$

where $C_{ij} = \cos\theta_{ij}$, $S_{ij} = \sin\theta_{ij}$ $(i.j = 1, 2, 3)$ and δ is the phase necessary for CP violation.

C_{ij} and S_{ij} can all be choose to be positive and δ may vary in the range $0 \leq \delta \leq 2\pi$.

- Hierarchy of charged current processes SM flavour problem



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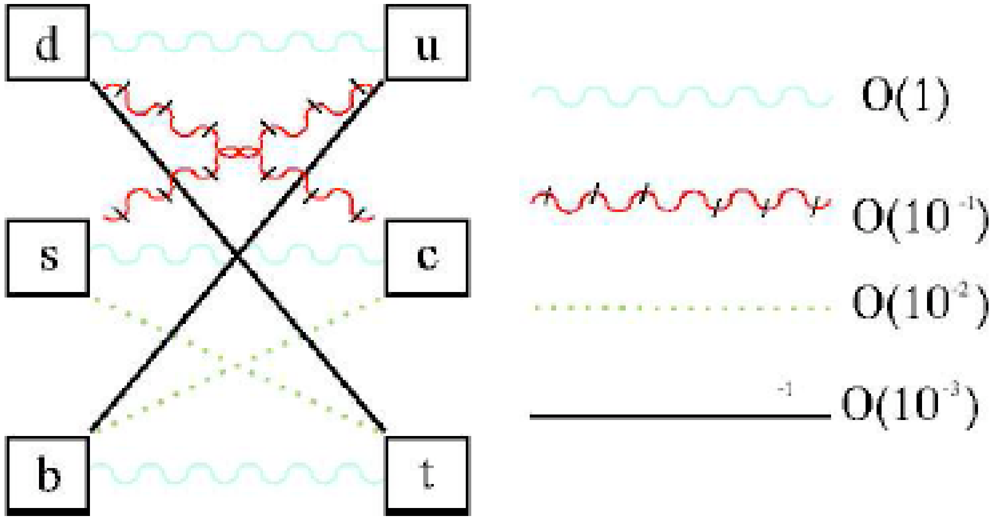

$$S_{12} = 0.22 \gg S_{23} = \mathcal{O}(10^{-2}) \gg S_{13} = \mathcal{O}(10^{-3})$$

- The Wolfenstein parametrization reflects hierarchy manifestly

$$S_{12} = \lambda = 0.22; \qquad S_{23} = A\lambda^2; \qquad S_{13}e^{-i\delta_{13}} = A\lambda^3(\rho - i\eta)$$

$$\mathbf{V_{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ \lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(\rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Hierarchy in unitarity relations

$$\underbrace{V_{ud}V_{us}^*}_{O(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{O(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{O(\lambda^5)} = 0,$$

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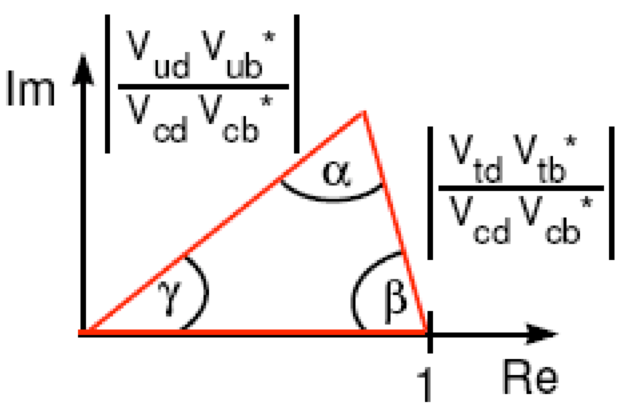
- The angles α, β, γ are rephasing invariants:



$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = arg(-Q_{ubtd}),$$

$$\beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = arg(-Q_{tbcd}),$$

$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = arg(-Q_{cbud}).$$

Addendum   CKM Matrix N quark families

- In general, a unitary $N \times N$ matrix have N^2 parameters.

- Redefinition of quark fields: $(2N-1)$ arbitrary phases

$$u_i \longrightarrow e^{i\phi_i} u_i, \qquad d_j \longrightarrow e^{i\theta_j} d_j, \qquad V_{ij} \longrightarrow e^{(i\theta_j - \phi_i)} V_{ij}.$$

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- No CP violation possible with two families! (1 angle, 0 phases)

Cabbibo matrix (1963)

$$V_c = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

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- Example:

  The electric charge of all (!) fermions within one family has to be zero:

$$Q(l_i^-, \nu_i) = (-1) \times |e|$$

$$Q(u_i) = 3 \times (+\,2/3) \times |e| = +2|e|$$

$$Q(d_i) = 3 \times (-1/3) \times |e| = -1|e|$$

- However:

  The τ lepton - as first evidence for the third lepton family - was found 1975 by Martin Perl (SLAC) after (!) the KM paper. (Nobelprize for Perl 1995)