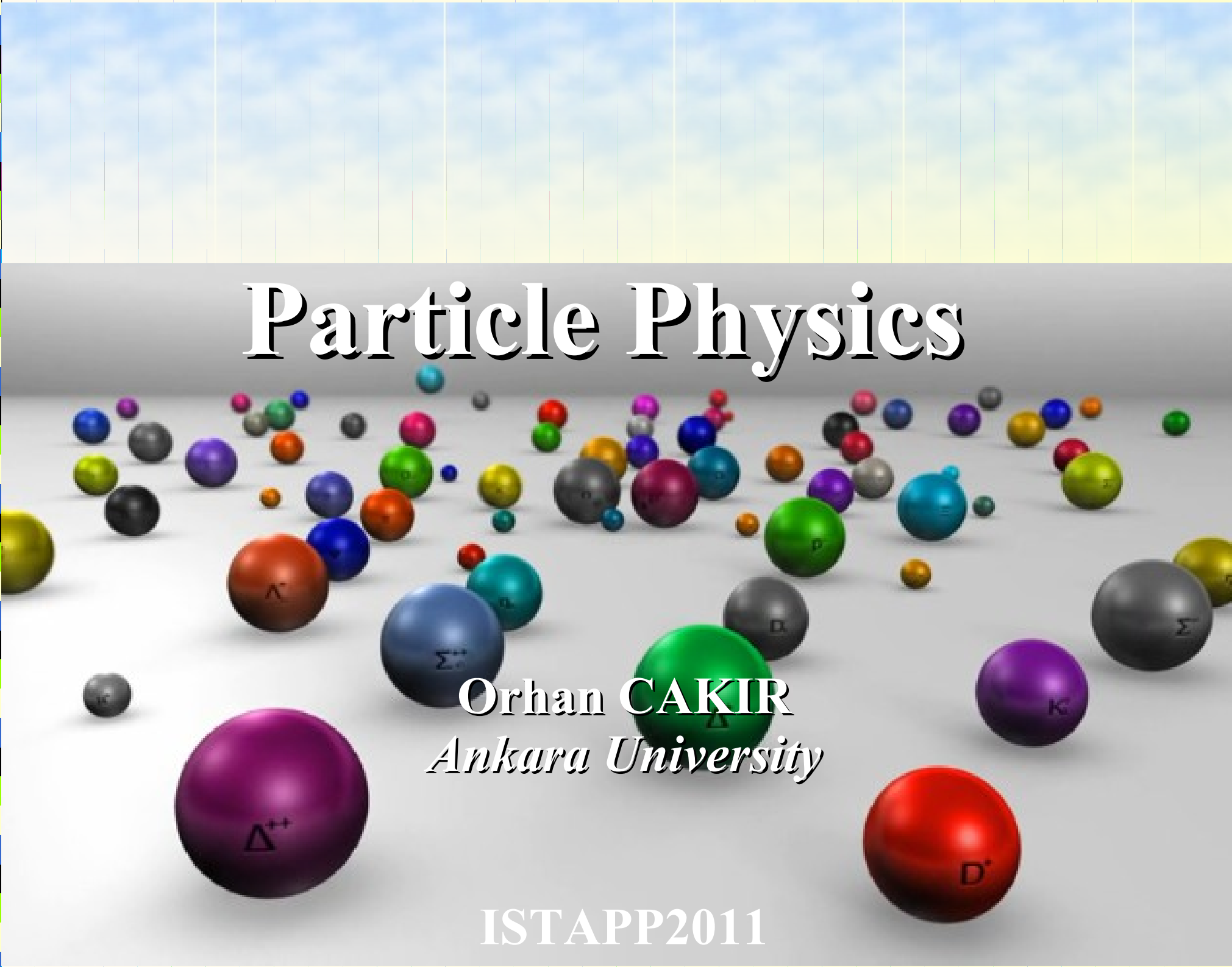


# Particle Physics

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ISTAPP2011



# Overview

- units in particle physics
- historical introduction
- elementary particles
- four forces
- symmetries
- hadrons
- particle decays
- interaction cross sections
- standard model and beyond

# Units in Particle Physics

- S.I. Units: kg, m, s are a natural choice for “everyday” objects, but they are inconveniently large in particle physics.
- Atomic physicists introduced the **electron volt** – the energy of an electron when accelerated through a potential difference of 1 volt:  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ joules}$ .
- However, in particle physics we use **Natural Units**
  - From Quantum Mechanics – the unit of action:  $\hbar$
  - From Relativity – speed of light:  $c$
  - From Particle Physics – unit of energy: **GeV**  
(proton rest-mass energy  $\sim 938 \text{ MeV}/c^2 = 1.67 \times 10^{-24} \text{ g}$ )
- Natural units are used throughout these lectures.

# Units in Particle Physics - 2

- Units become (with the dimensions)
  - Energy:  $\text{GeV}$       Time:  $(\text{GeV}/\hbar)^{-1}$
  - Momentum:  $\text{GeV}/c$       Length:  $(\text{GeV}/\hbar c)^{-1}$
  - Mass:  $\text{GeV}/c^2$       Area:  $(\text{GeV}/\hbar c)^{-2}$
- Algebra can be simplified by setting  $\hbar=c=1$ . Now all quantities are given in powers of  $\text{GeV}$ .
- To convert back to S.I. Units, one need to keep missing factors of  $\hbar$  and  $c$ .
- In Heaviside-Lorentz units we set  $\hbar=c=\epsilon_0=\mu_0=1$ , and Coulomb's law takes the form
$$F = \frac{1}{4\pi} \frac{q^2}{r^2}$$
- Electric charge  $q$  has dimensions:  $(FL^2)^{1/2}=(EL)^{1/2}=(\hbar c)^{1/2}$

# Historical Introduction

People have long asked:

- What is the world made of?
- What holds it together?
- Why do so many things in this world share the same characteristics?



People have come to realize that the matter of the world is made from **a few fundamental building blocks** (simple and structureless objects-not made of anything smaller) of nature.

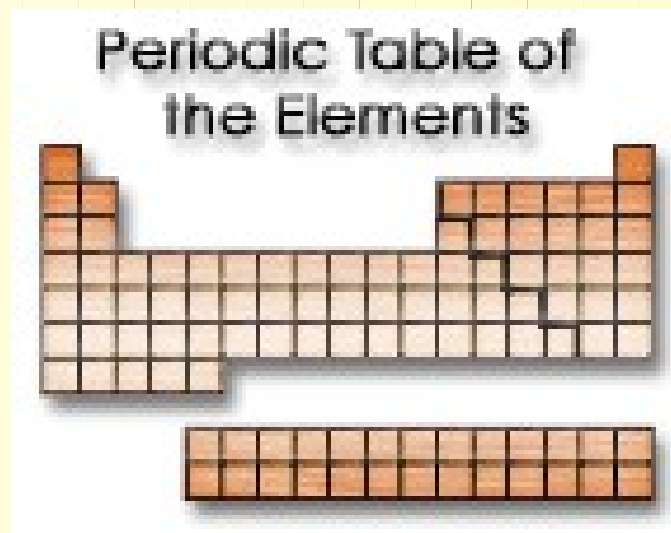
*"...in reality there are atoms and space." (Democritus 400 BC).*

# Historical Introduction - 2

The study (by Mendeleev, 1869) to categorize atoms into groups that shared similar chemical properties (Periodic Table of the Elements).

*Due to Moseley's work, the modern periodic table is based on the atomic numbers of the elements rather than atomic mass.*

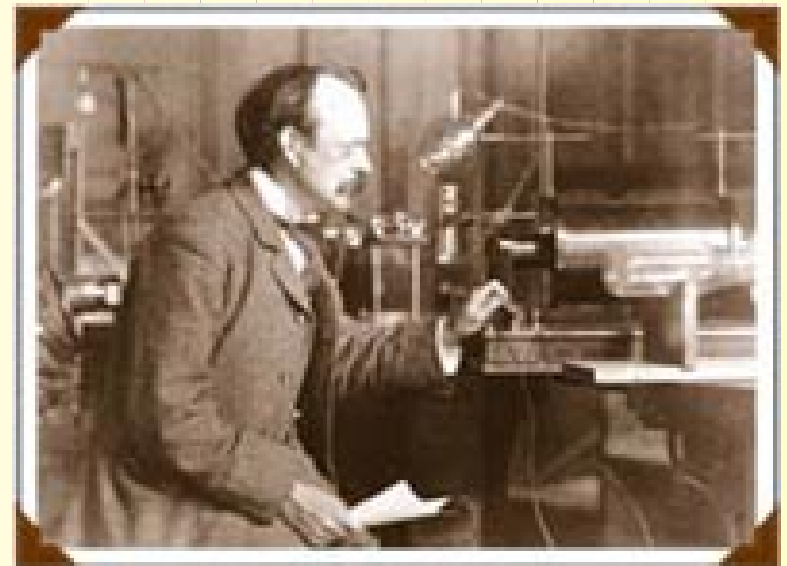
Moreover, the experiments helped scientists determine that atoms have a tiny but dense, positive nucleus ( $N^+$ ) and a cloud of negative electrons ( $e^-$ ).



# Historical Introduction - 3

Elementary particle physics was born in 1897 with J.J. Thomson's discovery of the electron (“corpuscles”).

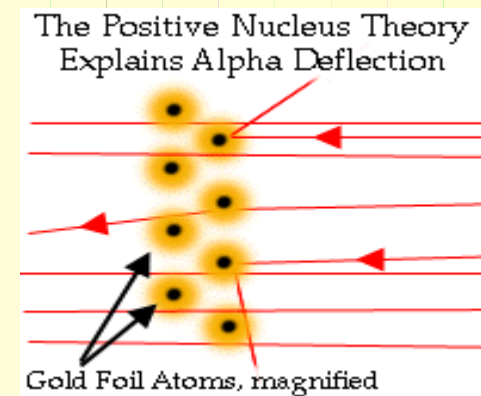
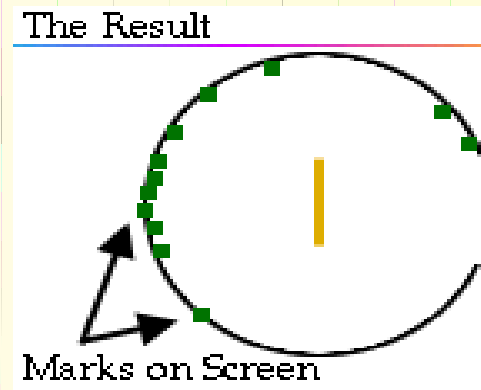
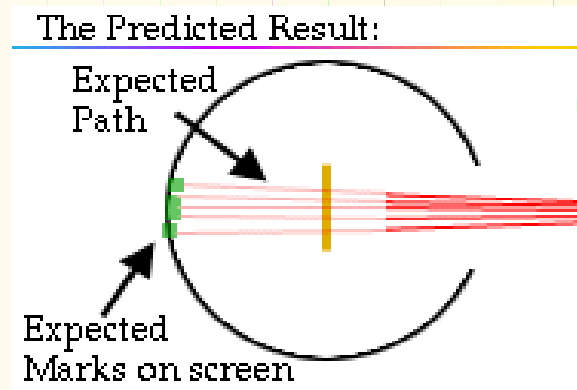
- The cathode rays (beam of particles) emitted by a hot filament could be deflected by a magnet, and this suggested they carried electric charge, and the direction of the curvature required that the charge be negative.
- *The word 'electron' first used by G. Johnstone Stoney in 1891 to denote the unit of charge in an electro-chemistry experiment.*





# Historical Introduction - 4

In 1909, Ernest Rutherford set up an experiment to test the validity of the prevailing theory. In doing so he established a way that for the first time physicists could "look into" tiny particles they couldn't see with microscopes.



*Some of the alpha particles were deflected at large angles to the foil; some even hit the screen in front of the foil! Obviously some other explanation was needed.*

- Rutherford concluded that there must be something inside an atom for the alpha particles to bounce off of that is small, dense, and positively charged: **the nucleus.**



# Historical Introduction - 5

- **Photon (1900-1924)**
  - ✓ M. Planck explains blackbody radiation (1900), radiation is quantized.
  - ✓ A. Einstein proposes a quantum of light which behaves like a particle (1905), photoelectric effect ( $E \leq h\nu - w$ ), equivalence of mass and energy, special relativity.
- A. H. Compton found that the light scattered from a particle at rest is shifted in wavelength (1923), according to  $\lambda' = \lambda + \lambda_c(1 - \cos\theta)$ , where  $\lambda_c$  is the Compton wavelength of the target particle.
- ✓ The name photon is suggested by the chemist Gilbert Lewis (1926).

# Historical Introduction - 6

- **Mesons (1934-1947)**

- Yukawa theory for strong force,  $m \sim 300m_e$
- ➔ meson: middle-weight, baryons: heavy-weight, lepton: light-weight.
- Powel *et al.* (1947) discovers two middle-weight particles, pion and muon.
- I.I.Rabi (1946) comments "who ordered that?" for the muon.

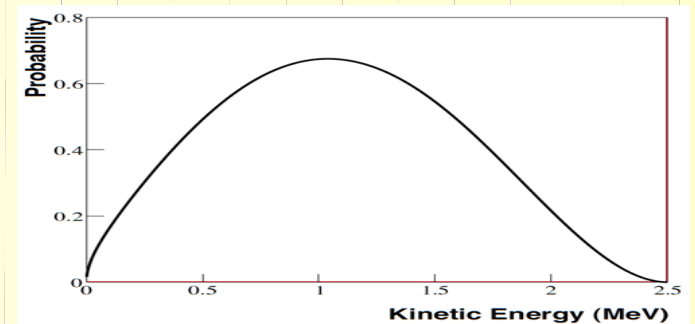
- **Anti-particles (1930-1956)**

- Dirac equation gives positive and negative energy solutions (particle / antiparticle) - Feynman-Stuckelberg formulation (1940).
- Anderson discovers positron (1932).
- Anti-proton and anti-neutron was discovered at Berkeley (1955-56).

# Historical Introduction - 7

- **Neutrinos (1930-1962)**

- W.Pauli (1930) suggest neutral particle to explain continuous electron spectrum in beta decay.



- In nuclear beta decay, a neutral missing particle, E.Fermi called it “neutrino” (1933) and introduction of weak interaction theory.

- J.Chadwick discovers neutron (1931)

- H.Yukawa (1934) meson (pion) theory of nuclear forces

- **Strange Particles (1947-1960)**

- ➔ Pions in cosmic rays, calculation of electromagnetic processes, introduction of Feynman diagrams, The Berkeley synchro-cyclotron produces the first lab. pions.

# Historical Introduction - 8

## Strange Particles (1947-1960)

- Discovery of  $K^+$  meson (1949) via its decay.
- Neutral pion is discovered in 1950.
- Discovery of particles called “delta” baryon ( $s=3/2$ ).
- C.N.Yang and R.Mills develop a new class of theories called "gauge theories" in 1954.

## Unification/Classification Ideas (1957)

- J.Schwinger (1957) writes a paper proposing unification of weak and electromagnetic interactions.
- The group  $SU(3)$  to classify and organize particles (1961)
- Experiments verify two types of neutrinos,  $\nu_e$  and  $\nu_m$  (1962).

# Historical Introduction - 9

## Quark Model (1964)

- M.Gell-Mann and G.Zweig (1964) suggest that mesons and baryons are composed of quarks (flavors u,d,s)
- Omega particles discovered (1964, BNL) , as Gell-Mann predicted within baryon decuplet.
- O.W.Greenberg, M.Y.Han and Y.Nambu introduce the quark property of color charge (1965). All observed hadrons are color neutral.

## EW Theory (1967-70)

- S.Weinberg and A.Salam (1967) propose a theory that unifies electromagnetic and weak interactions.
- J.Bjorken and R.Feynman (1969) analyze the SLC data in terms of a model of constituent particles inside the proton (provided evidence for quarks!)

# Historical Introduction - 10

## EW Theory (1970-74)

- S.Glashow, J.Iliopoulos, L.Maiani (1970) recognize the critical importance of a fourth type of quark in the context of a standard model,  $Z^0$  has no flavor-changing weak interactions.
- H.Fritzsch and M.Gell-Mann suggested the strong interaction theory QCD (1973)
- D.Politzer, D.Gross, F.Wilczek discover that the color theory of the strong interaction has a special property, now called "asymptotic freedom"
- In 1974, J.Iliopoulos presents the view of particle physics called the Standard Model (SM).



# Historical Introduction - 11

- **New Members (1974-78)**

- S.Ting et al. at BNL and B.Richter et al. at SLAC announce on the same day that they discovered the same new particle. Since the discoveries are given equal weight, the particle is commonly known as the  $J/\Psi (c\bar{c})$  meson.

- G.Goldhaber, F.Pierre find the  $D_0$  meson (anti-up and charm quarks) (76)

- The tau lepton (lepton of third generation) is discovered by M.Pperl et al. at SLAC (76).

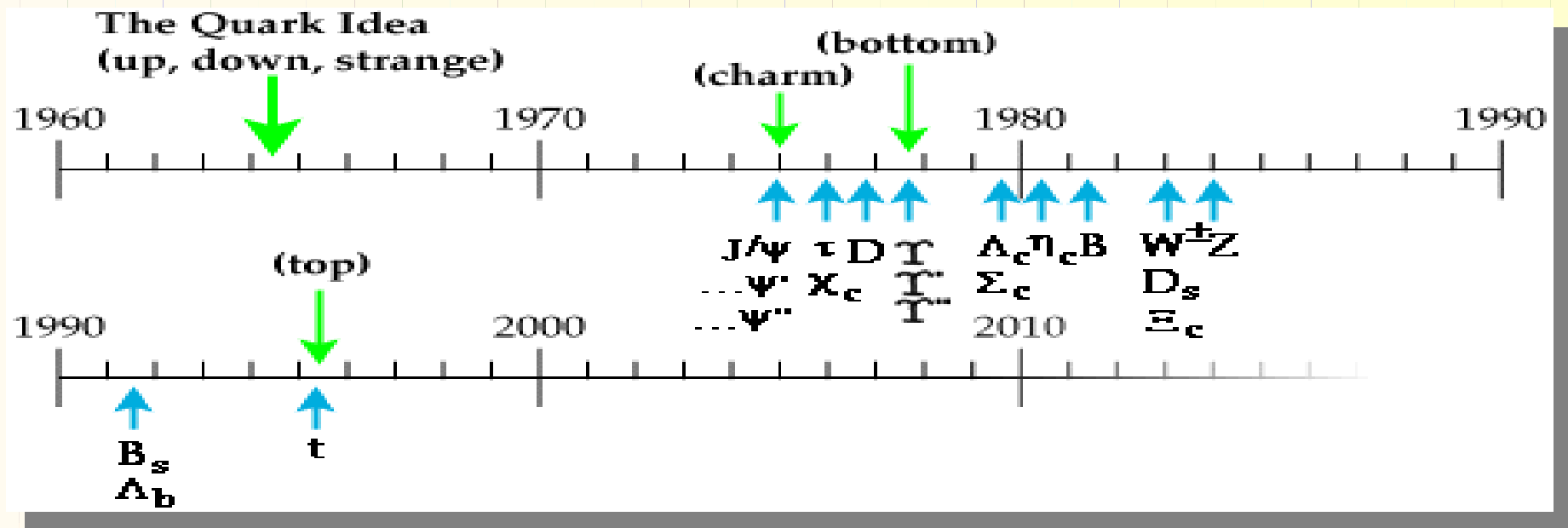
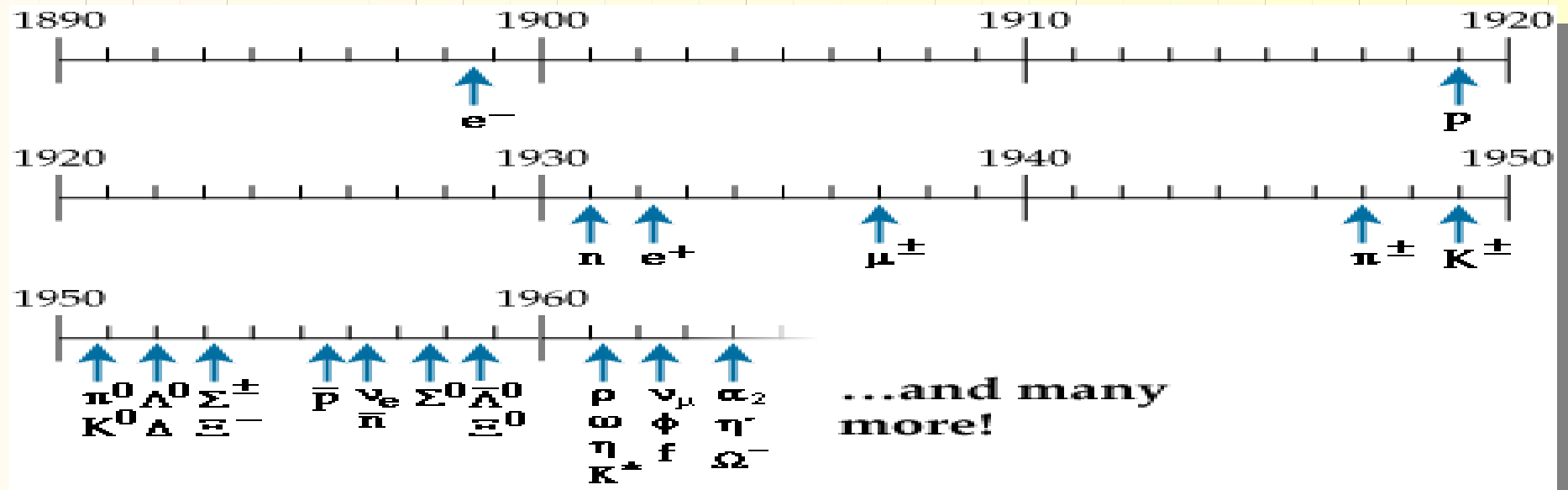
- L.Lederman et al. at Fermilab discover yet another quark (and its antiquark) (77), the "bottom" quark → this leads to the search for sixth quark – "top".

# Historical Introduction - 12

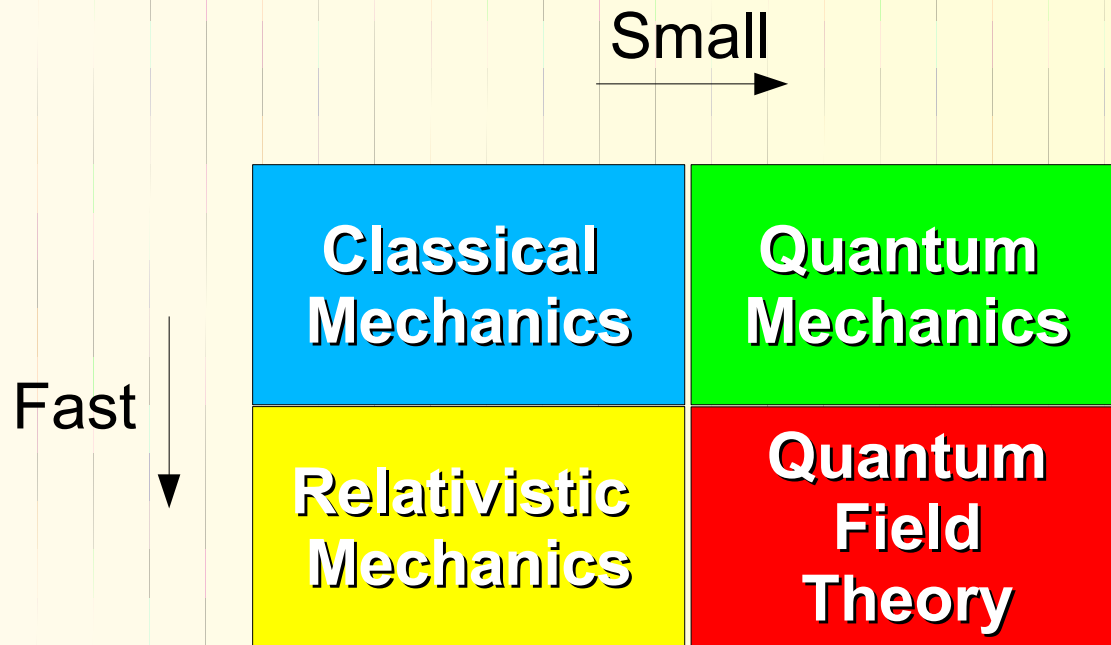
## Standard Model (1978-)

- C.Prescott, R.Taylor (1978) observe a violation of parity conservation, as predicted by the SM.
- Strong evidence for a gluon (1979) radiated by the initial quark or antiquark at the DESY.
- In 1983, the  $W^{\pm}$  and  $Z^0$  intermediate bosons demanded by the electroweak theory are observed at CERN SpS by C.Rubbia et al.
- SLAC and CERN experiments suggest (1989) three generations of fundamental particles (inferring from  $Z^0$ -boson lifetime) with three very light neutrinos.
- In 1995, CDF and D0 experiments at Fermilab discover the top quark (175 GeV).

# ... and the timeline summary



# Low Energy - High Energy

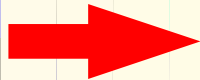
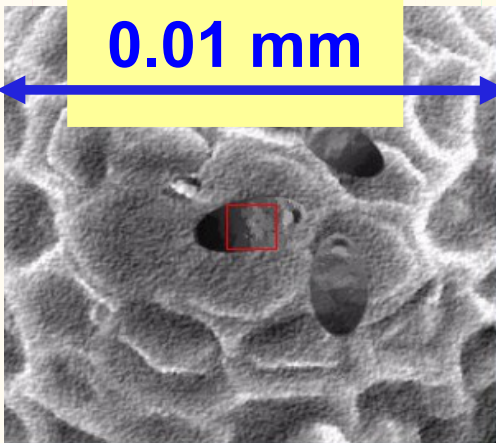


- For things that are both fast and small (particles), we require a theory that incorporates relativity and quantum principles: Quantum Field Theory (QFT).
- Most of our experimental information comes from three main sources: 1) scattering events, 2) decays, 3) bound states.

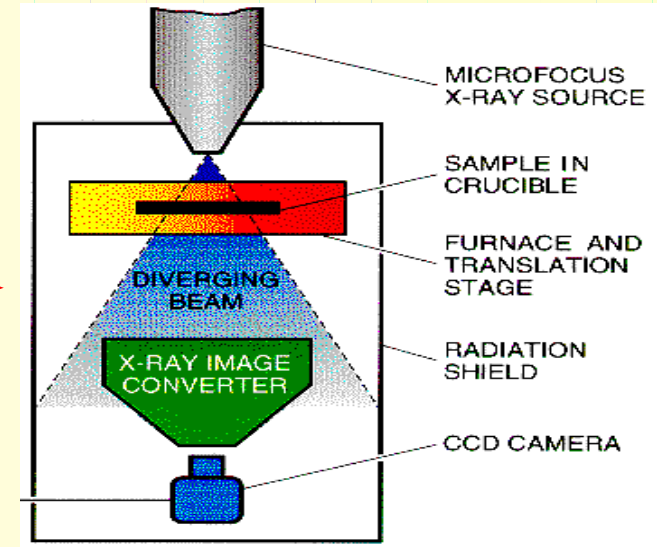
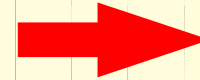
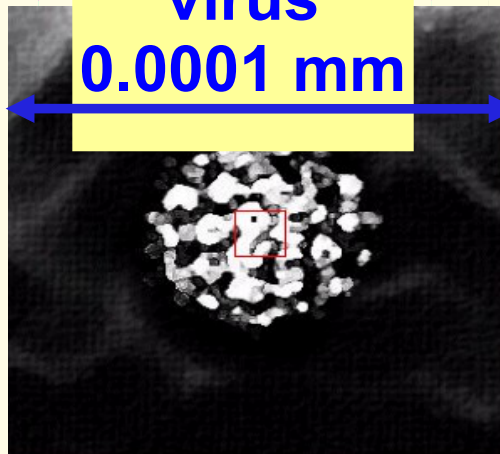
# Our curiosity and tools for research



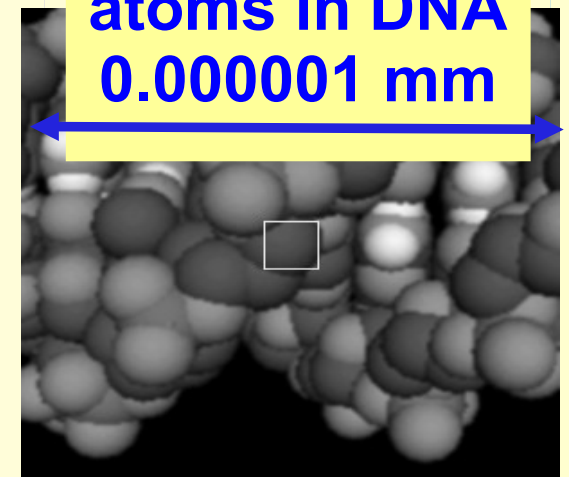
bacteria  
0.01 mm



virus  
0.0001 mm



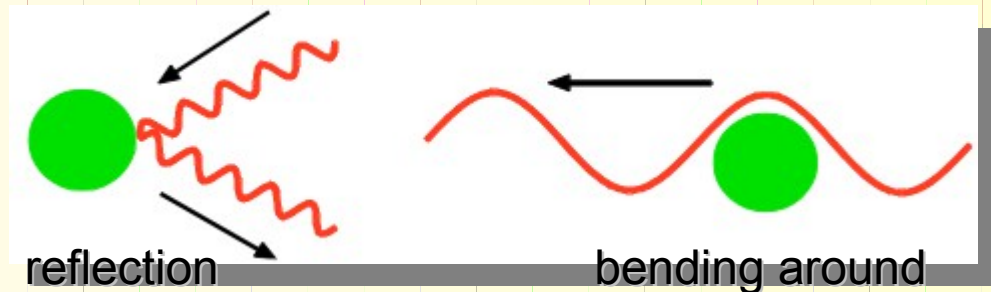
atoms in DNA  
0.000001 mm



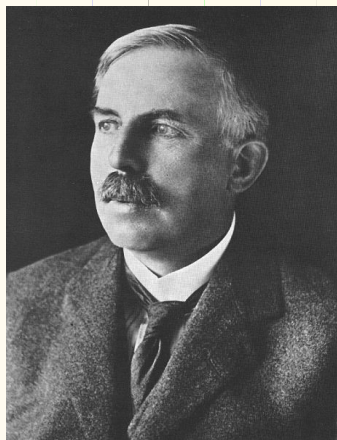
- **We need particle accelerators to look into the deepest structure of matter!**

# Probing the elementary building blocks of nature

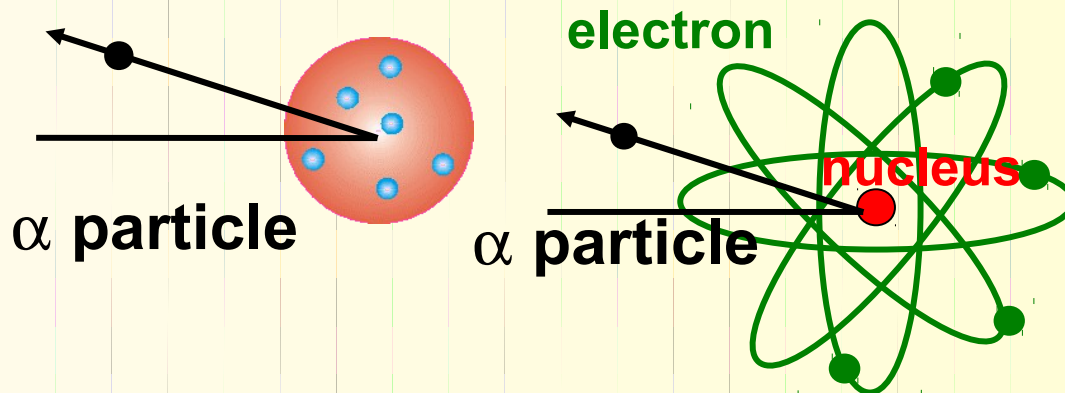
$$E = \frac{hc}{\lambda}$$



Higher Beam Energy → Shorter Wave Length → Better resolution



Rutherford's Experiment, 1909

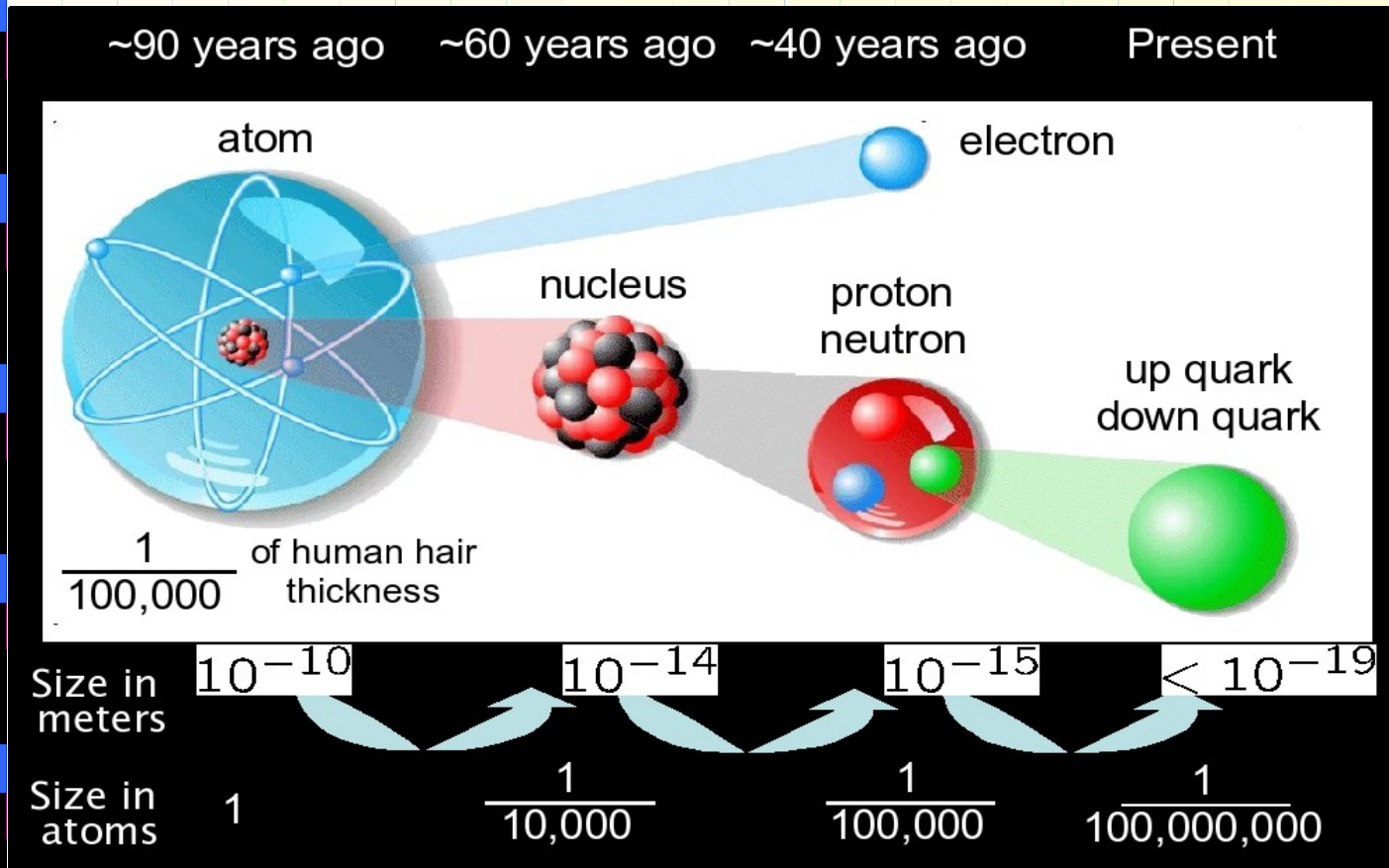


$$\lambda = hc/E = 1.24 \times 10^{-6} \text{ eV m} / E$$
$$E = 2.8 \times 10^6 \text{ eV} \implies \lambda = 4.4 \times 10^{-13} \text{ m}$$



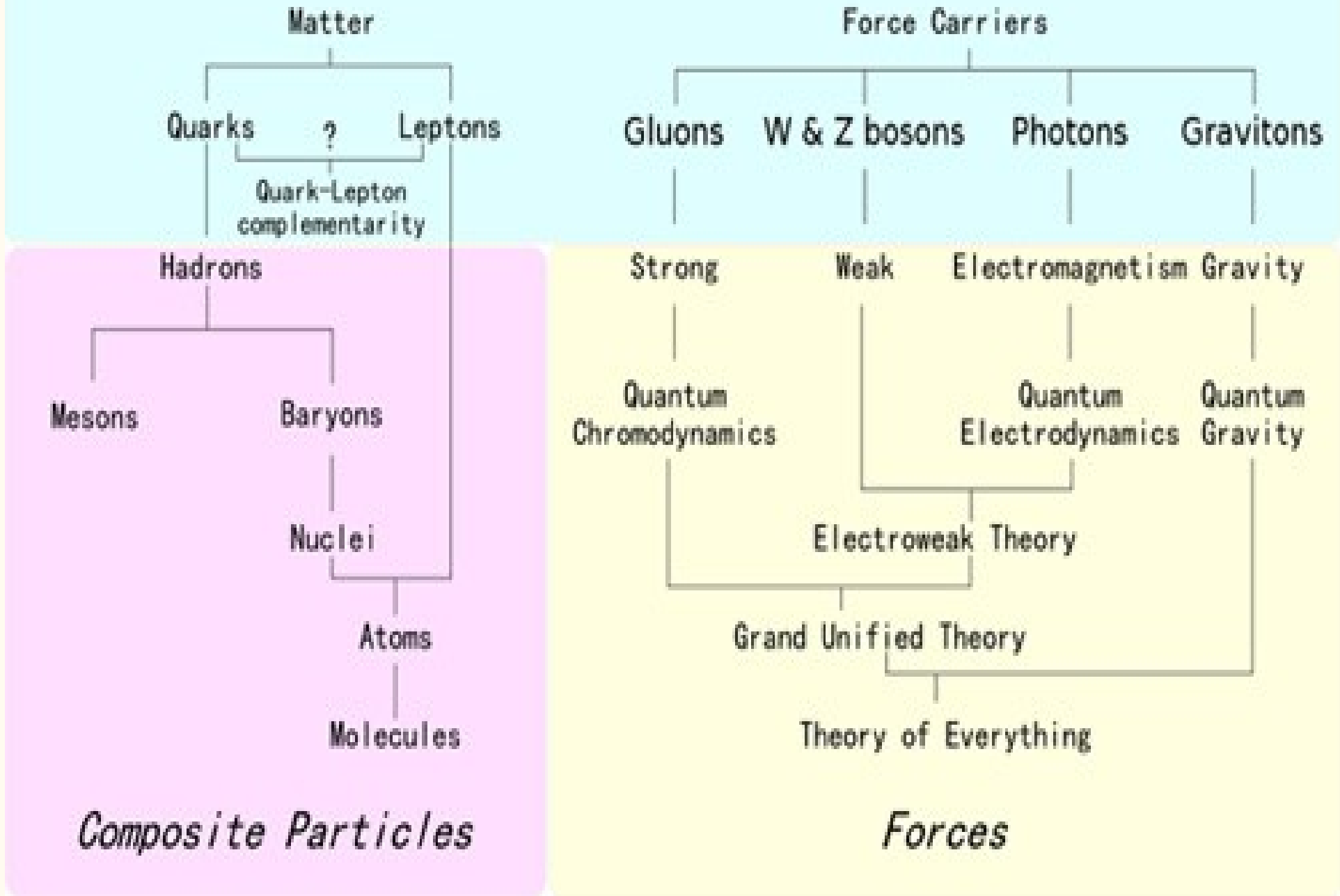
# The Particle Physics is the study of

- Matter (spin-1/2 particles): the fundamental constituents of the universe - *the elementary particles*
- Force (spin-1 particles): the fundamental forces of nature - *the interactions between the elementary particles*



- Try to categorize the **Particles** and **Forces** in as simple and fundamental manner possible.

# Particle Physics

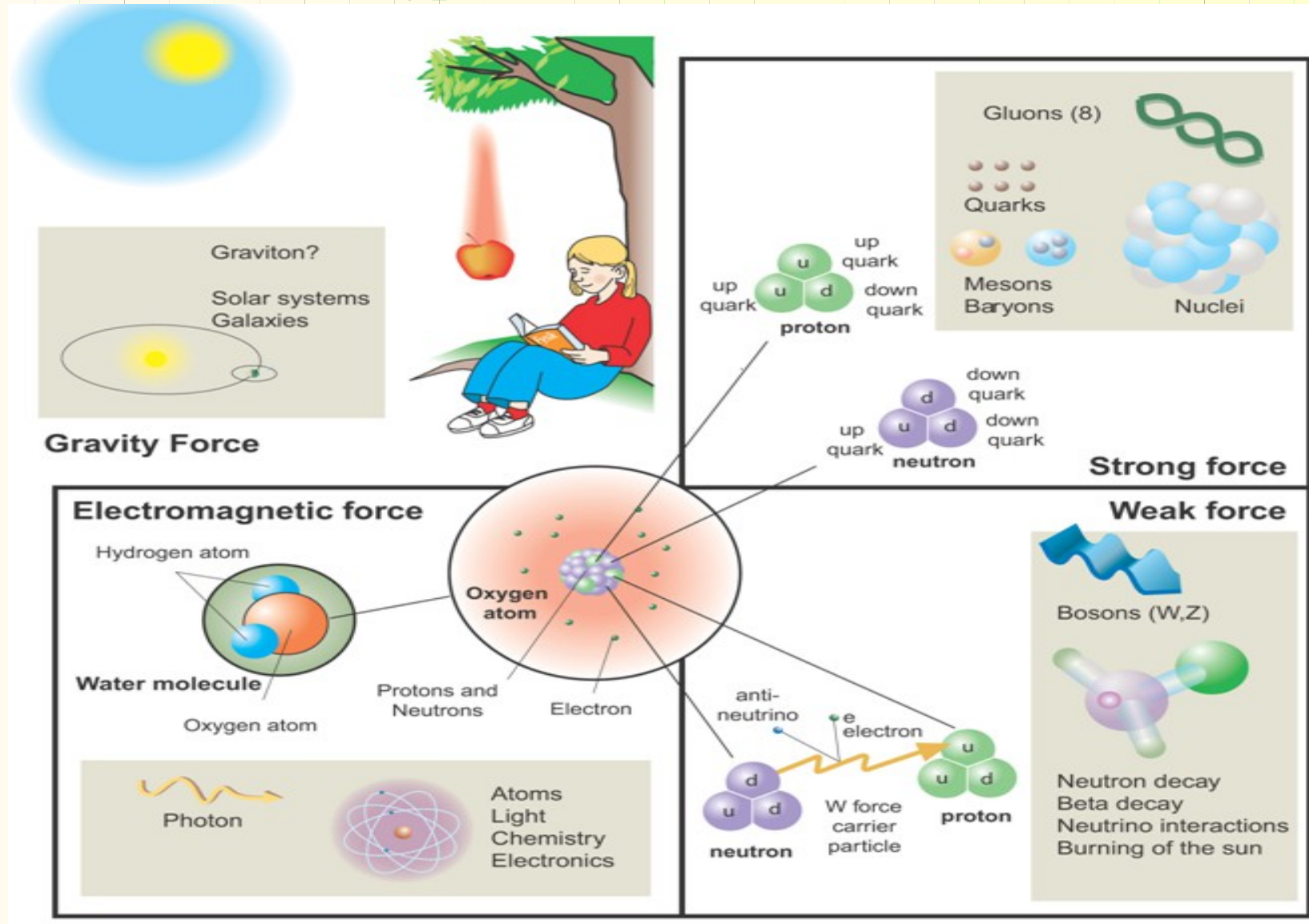


# ELEMENTARY PARTICLES

	CONTEXT	MASS	CHARGE	SPIN	STRENGTH	RANGE	OBSERVED?
<b>BOSONS (forces)</b>							
GRAVITON	gravity	0	0	2	$10^{-38}$	infinite	no
PHOTON	electromagnetism	0	0	1	$10^{-27}$	infinite	yes
GLUON	strong force	0	0	1	$10^{-20}$	$10^{-13}$	indirectly
<b>WEAK GAUGE BOSONS</b>							
W <sup>+</sup>	weak force	80,000	1	1	$10^{-13}$	$10^{-16}$	yes
W <sup>-</sup>	weak force	80,000	-1	1	$10^{-13}$	$10^{-16}$	yes
Z <sup>0</sup>	weak force	91,000	0	1	$10^{-13}$	$10^{-16}$	yes
HIGGS BOSON	weak force	>114,000	0	0	[ ? ]	[ ? ]	no
<b>FERMIONS (matter)</b>							
<i>LEPTONS, FAMILY 1:</i>							
ELECTRON	radioactive decay	0.51	-1	1/2	n/a	n/a	yes
ELECTRON NEUTRINO	atomic structure	0?	0	1/2	n/a	n/a	yes
<i>QUARKS, FAMILY 1:</i>							
UP	atomic nuclei	5	2/3	1/2	n/a	n/a	indirectly
DOWN	atomic nuclei	9	-1/3	1/2	n/a	n/a	indirectly
<i>LEPTONS, FAMILY 2:</i>							
MUON		106	-1	1/2	n/a	n/a	yes
MUON NEUTRINO		~0	0	1/2	n/a	n/a	yes
<i>QUARKS, FAMILY 2:</i>							
CHARM		1,400	2/3	1/2	n/a	n/a	indirectly
STRANGE		170	-1/3	1/2	n/a	n/a	indirectly
<i>LEPTONS, FAMILY 3:</i>							
TAU		1,784	-1	1/2	n/a	n/a	yes
TAU NEUTRINO		>35	0	1/2	n/a	n/a	yes
<i>QUARKS, FAMILY 3:</i>							
TOP		174,000	2/3	1/2	n/a	n/a	indirectly
BOTTOM		4,400	-1/3	1/2	n/a	n/a	indirectly

# Four Forces of Nature

Forces can be explained as the boson exchange between fermions, and the type of boson defines the force.

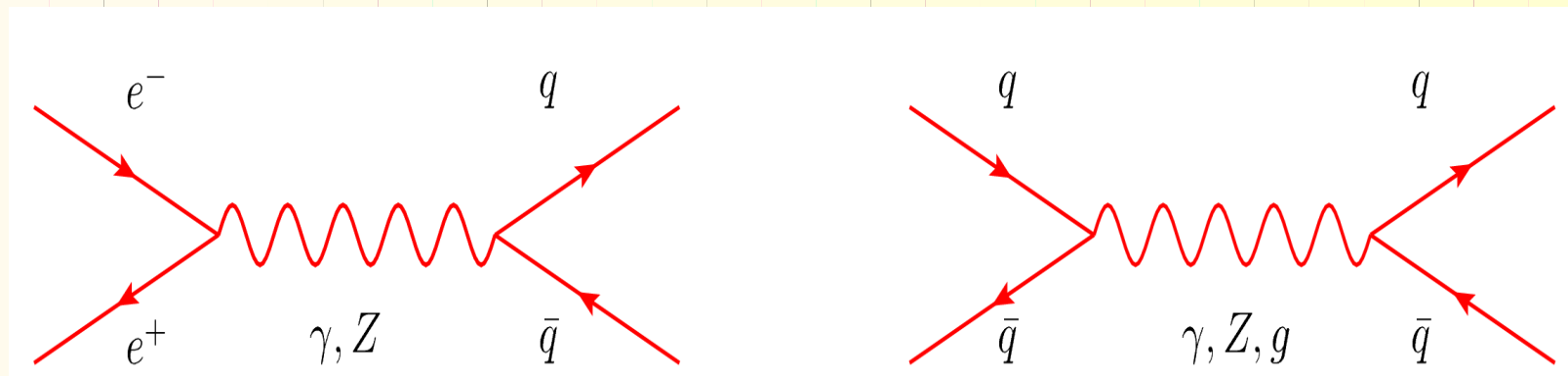


# Dynamics of Particle Physics

Quantum Electrodynamics (QED), Weak Interactions (WI), Quantum Chromodynamics (QCD)

Forces	Electromagnetic	Weak	Strong
Bosons	Photon ( $\gamma$ )	$W^\pm, Z$	Gluon ( $g$ )
Matter involved	$f^\pm$ , quarks	$f^\pm, \nu$ , quarks	quarks

Feynman diagrams: creation and annihilation of matter



# Symmetry/Conservation

The laws of physics are symmetrical with respect to translation in time (they work the same today as they did yesterday): Noether's (1971) theorem relates this invariance to conservation of energy. The symmetries are associated with conservation laws.

Symmetry		Conservation law
Translation in time	$\leftrightarrow$	Energy
Translation in space	$\leftrightarrow$	Momentum
Rotation	$\leftrightarrow$	Angular momentum
Gauge transformation	$\leftrightarrow$	Charge

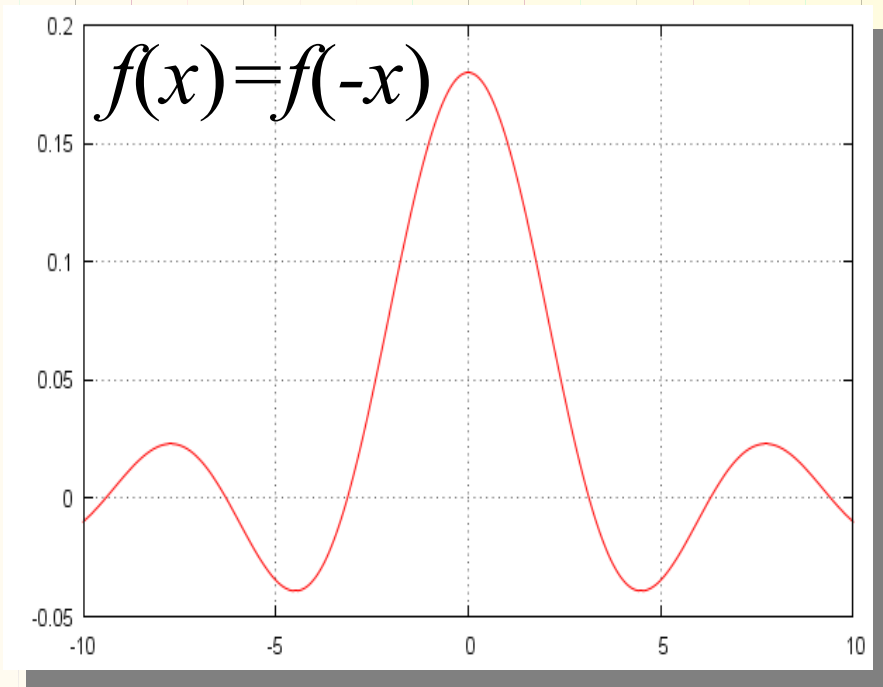


# Systematic Study of Symmetries

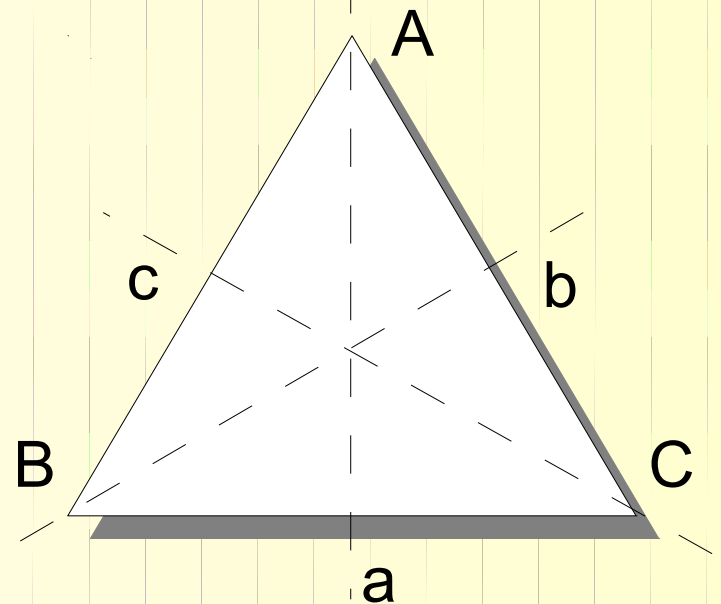
Symmetry operations has the following properties:

- If  $S_i$  and  $S_j$  are in the set, then the product  $S_i S_j = S_k$ .
- There is an element  $I$  such that  $I S_i = S_i I = S_i$  for all  $S_i$ .
- For every element  $S_i$  there is an inverse  $S_i^{-1}$ , such that  $S_i S_i^{-1} = S_i^{-1} S_i = I$ .
- There is associativity such that  $S_i (S_j S_k) = (S_i S_j) S_k$ .

$S: x \rightarrow -x$



$S_+$ : rotation  $120^\circ$   
 $S_a$ : flips around a



# Systematic Study of Symmetries - 2

- Translations in space and time form an Abelian group ( $S_i S_j = S_j S_i$ ), while rotations in 3D do not ( $S_i S_j \neq S_j S_i$ ).
- If the elements depend on continuous parameters (angle of rotation)  $\rightarrow$  continuous groups
- If the elements are labelled by an index that takes only integer values  $\rightarrow$  discrete groups
  - In **elementary particle physics**, the most common groups are of the type  $U(n)$ : a collection of unitary ( $U^{-1} = \tilde{U}^*$ )  $n \times n$  matrices. Unitary matrices with “determinant 1” called  **$SU(n)$** : special unitary. For real unitary matrices ( $O^{-1} = \tilde{O}$ ), the group is  **$O(n)$** .  **$SO(n)$**  is the group of special orthogonal  $n \times n$  matrices.

# Symmetries in Particle Physics

- One of the aim of the particle physics is to discover the fundamental symmetries of our universe.
- Introduce the ideas of the SU(2) and SU(3) symmetry groups which play a major role in particle physics.
  - Suppose physics is invariant under the transformation  $\psi \rightarrow \psi' = \hat{U}\psi$   
(like the rotation of coordinate axes)

Conserving the probability normalization require

$$\langle \psi | \psi \rangle = \langle \psi' | \psi' \rangle = \langle \hat{U}\psi | \hat{U}\psi \rangle = \langle \psi | \hat{U}^\dagger \hat{U} | \psi \rangle \rightarrow \hat{U}^\dagger \hat{U} = 1$$

$\hat{U}$  has to be unitary

# Symmetries in Particle Physics - 2

- For physical predictions to be unchanged by the symmetry transformation, also require all QM matrix elements unchanged

$$\langle \psi | \hat{H} | \psi \rangle = \langle \psi' | \hat{H} | \psi' \rangle = \langle \psi | \hat{U}^\dagger \hat{H} \hat{U} | \psi \rangle$$

$$\hat{U} \hat{U}^\dagger \hat{H} \hat{U} = \hat{U} \hat{H} \longrightarrow \hat{H} \hat{U} = \hat{U} \hat{H} \longrightarrow [\hat{H}, \hat{U}] = 0$$

Consider the infinitesimal transformation ( $\epsilon$  small)  $\hat{U} = 1 + i\epsilon \hat{G}$

$$\hat{U} \hat{U}^\dagger = (1 + i\epsilon \hat{G})(1 - i\epsilon \hat{G}^\dagger) = 1 + i\epsilon(\hat{G} - \hat{G}^\dagger) + O(\epsilon^2)$$

here unitarity requires  $\hat{G} = \hat{G}^\dagger$

$$[\hat{H}, 1 + i\epsilon \hat{G}] = 0 \longrightarrow \hat{G} \text{ is a conserved quantity.}$$

Well, “doing nothing” ( $\mathbf{1}$ ) is also a symmetry operation!

# Discrete Symmetries

## Charge Conjugation (C)

- Classical electrodynamics is invariant under C, potentials and fields reverse sign under C, but there is a compensating charge factor so the force remain the same.
- $C|p\rangle = |\bar{p}\rangle = \pm|p\rangle$ , it changes the sign of all “internal quantum numbers” such as charge, baryon number, lepton number, strangeness, etc.— while leaving mass, energy, momentum and spin untouched. It has limited eigenstates (photon, rho, eta etc.).
- It is not a symmetry of the weak interactions (no  $\bar{\nu}_L$ !)
- An extended transformation “G-parity”,  $G=CR_2$  where  $R_2=e^{i\pi I(2)}$ . Ex: all pions are eigenstates of G.

# Discrete Symmetries - 2

## Parity (P)

- Lee and Yang (56) propose a test for parity in weak interactions. Beta decay in  $^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e + \nu_e$  where most electrons are emitted opposite to nuclear spin. It is a symmetry of strong and electromagnetic interactions, but it is violated in weak interactions.

Scalar	$P(s) = s$
Pseudoscalar	$P(p) = -p$
Vector	$P(v) = -v$
Pseudovector (or axial vector)	$P(a) = a$

- neutrinos are left-handed; antineutrinos are right-handed.

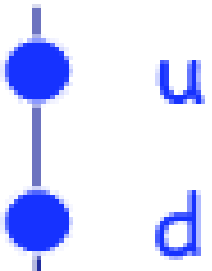
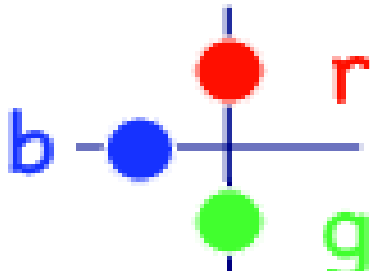
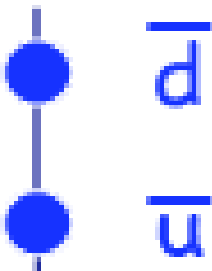
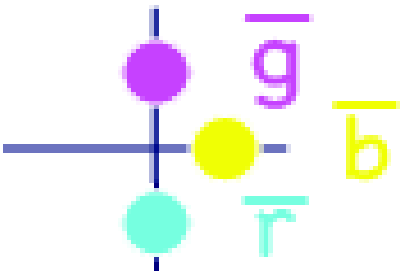


# Discrete Symmetries - 3

## Time Reversal (T) and the TCP

- Like “running the movie backward”, say  $n+p \rightarrow d+\gamma$  process and its reverse order process  $d+\gamma \rightarrow n+p$ . No experiment has shown direct evidence of T violation. The most sensitive experimental tests are the upper limits on the electric dipole moments of neutron ( $d_n < 6 \times 10^{-26}$  cm e) and electron ( $d_e < 1.6 \times 10^{-27}$  cm e).
- TCP theorem states that combined operation (TCP) is an exact symmetry of any interaction.

# Gauge Groups in Particle Physics

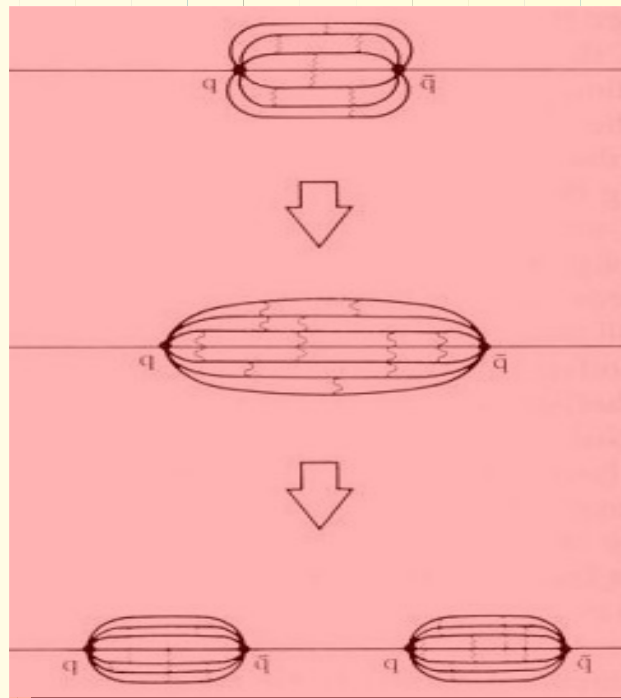
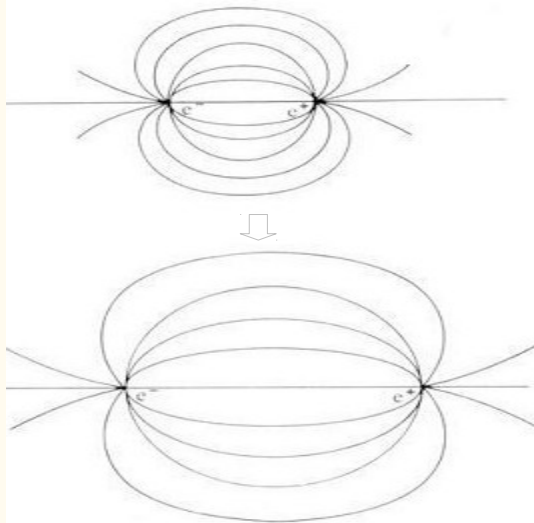
	U(1)	SU(2)	SU(3)
charge	+		
anti-charge	-		
	complex number	three 2x2 Matrices (Pauli)	eight 3x3 matrices (Gell-Mann)

# Hadrons

Hadrons are color singlet bound states of quarks.

- Mesons are  $q_i \bar{q}_j$  states of quarks and antiquarks.
- Baryons are  $q_i q_j q_k$  states of quarks.
- Quarks are confined into hadrons.

*Electric field lines are spread out as the charges are separated.*



Color force lines collimated into a tube as the quarks are separated, it will break into two when the force applied as required.

# Mesons

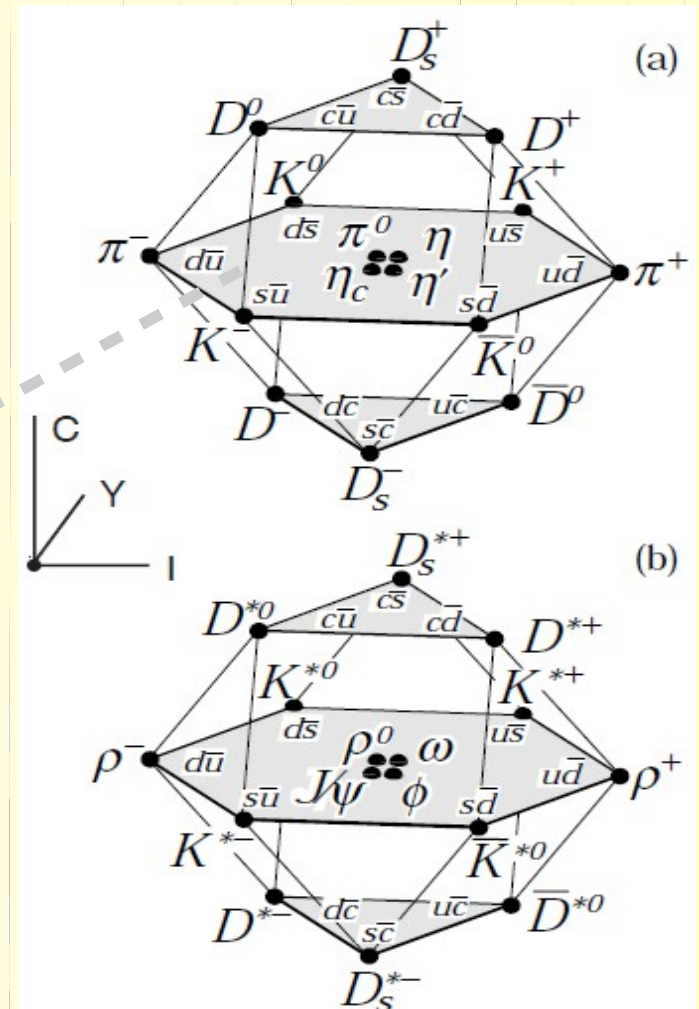
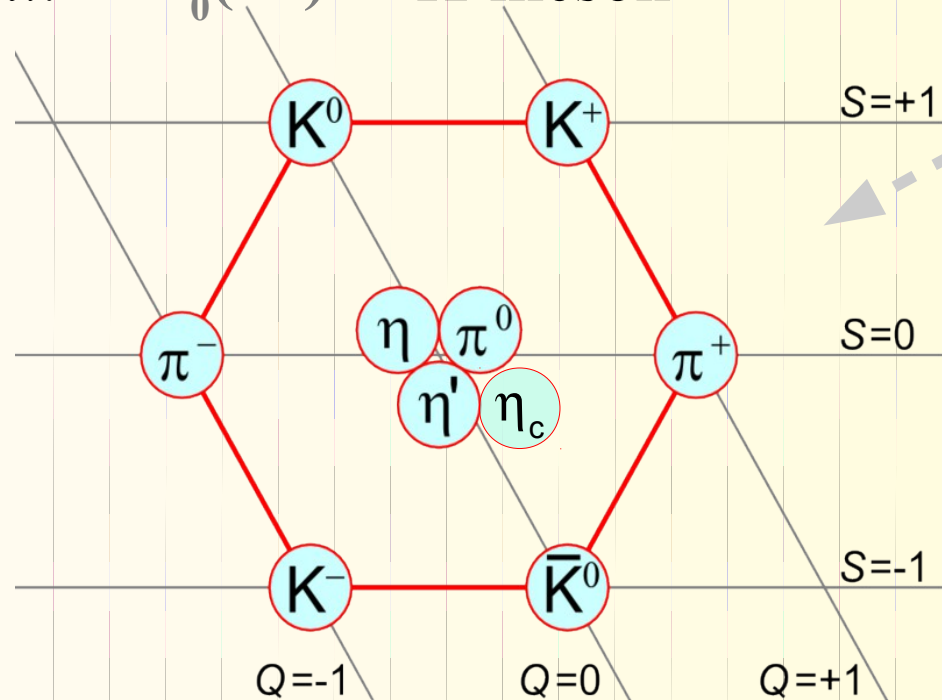
If the orbital angular momentum of the states is  $l$ , then the parity  $P=(-1)^{l+1}$ . For  $qq$  mesons the charge conjugation parity  $C=(-1)^{l+s}$  and the G-parity  $(-1)^{l+l+s}$ . In an  $SU(4)$  classification we have

$4 \times \bar{4} = 15 + 1$ . Spectrum:  $n^{2s+1}l_j (J^{PC})$ ,

$l=0$ : pseudoscalars ( $0^-$ ) and vectors ( $1^-$ )

$l=1$ : scalars ( $0^{++}$ ), axial vectors ( $1^{++}$ ) and ( $1^+$ ), tensors ( $2^{++}$ ).

ex:  $1^1S_0(0^-) \rightarrow K\text{-meson}$

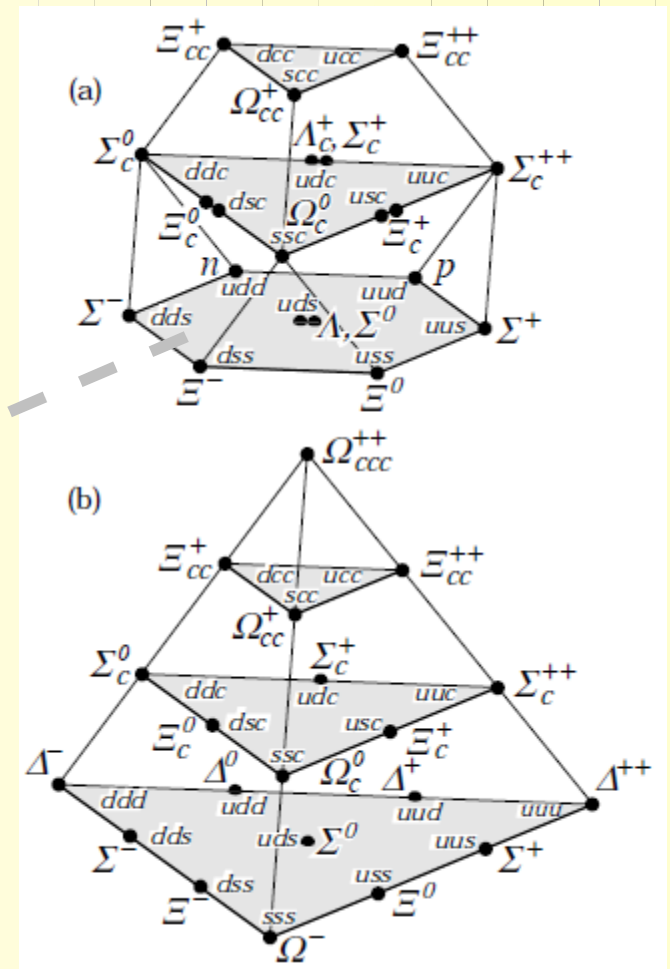
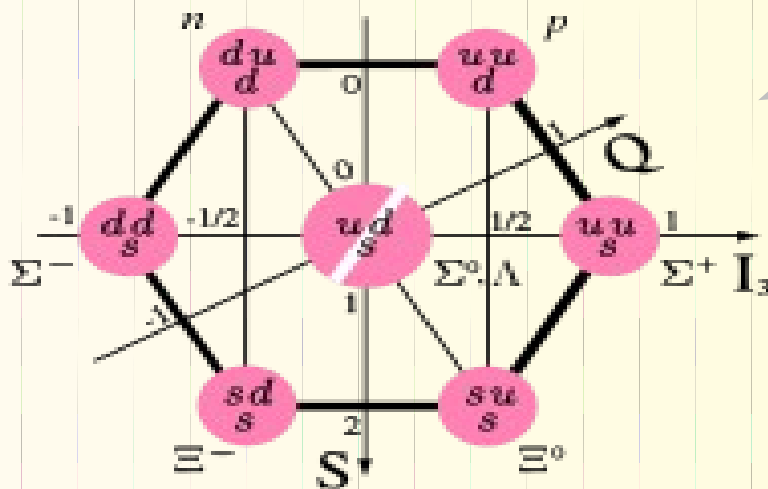


# Baryons

Baryons are fermions with baryon number  $B=1$ , and they are in color singlet state.

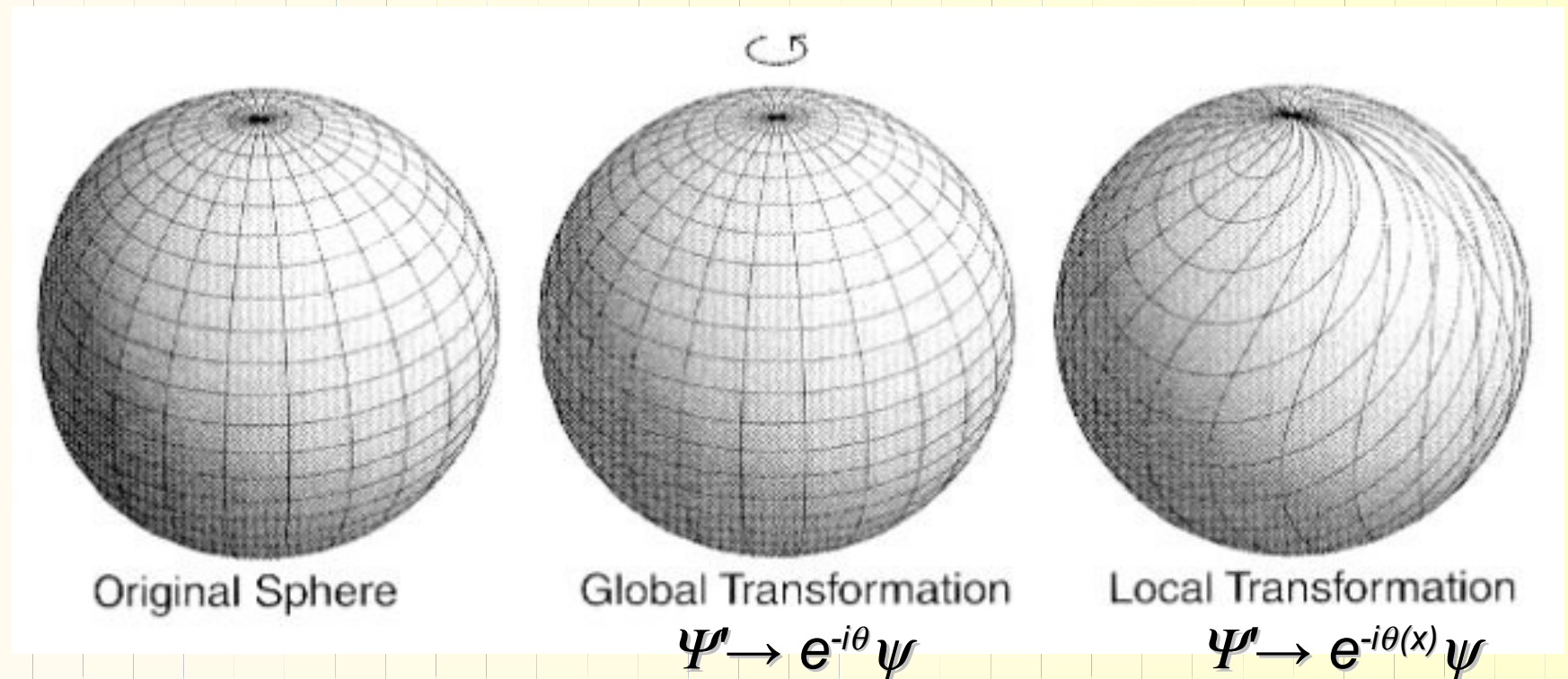
$$|qqq\rangle_A = |\text{color}\rangle_A \times |\text{space, spin, flavor}\rangle_S$$

It is useful to classify baryons into bands having the same quantum number of excitation.



# Symmetries in Particle Physics - 3

- Symmetries play a crucial role in our life! In the QFT they play very special role:
  - ➔ every force can be derived from the principle of the internal symmetry – *local gauge invariance predicts gauge bosons*





# Four Vectors

Four vector notation

$$p^\mu = \{E, \vec{p}\} = \{E, p_x, p_y, p_z\} \quad \text{(contravariant)}$$

$$p_\mu = g_{\mu\nu} p^\mu = \{E, -\vec{p}\} = \{E, -p_x, -p_y, -p_z\} \quad \text{(covariant)}$$

In particle physics we deal with the relativistic particles  $\rightarrow$  all calculations should be Lorentz invariant. These calculations can be performed via four-vector production.

$$p^\mu p_\mu = E^2 - p^2 = m^2 \quad \text{Invariant mass}$$

$$x^\mu p_\mu = Et - \vec{p} \cdot \vec{r} \quad \text{Phase}$$

Lorentz transformation

$$\begin{aligned} \begin{pmatrix} E' \\ p'_z \end{pmatrix} &= \begin{pmatrix} \gamma_0 & -\gamma_0 \beta_0 \\ -\gamma_0 \beta_0 & \gamma_0 \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix} \\ &= \begin{pmatrix} \cosh y_0 & -\sinh y_0 \\ -\sinh y_0 & \cosh y_0 \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix} \end{aligned}$$

$$p'_T = p_T$$

# Ex: Top Quark Invariant Mass

Energy-momentum conservation gives

$$p_t = p_b + p_l + p_\nu$$

$$m_t^2 \equiv m_{bl\nu}^2 = (E_b + E_l + E_\nu)^2 - (\vec{p}_b + \vec{p}_l + \vec{p}_\nu)^2$$

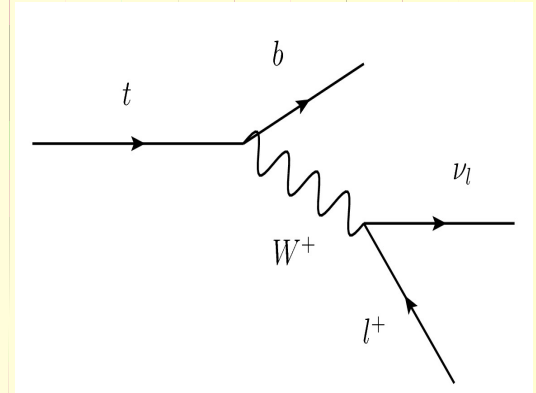
here we want to calculate  $p_z$  of neutrino via  $W$ -boson mass constraint

$$p_W = p_l + p_\nu; \quad m_{l\nu}^2 = (E_l + E_\nu)^2 - (\vec{p}_l + \vec{p}_\nu)^2$$

$$m_W^2 \approx 2(E_l E_\nu - |\vec{p}_l| |\vec{p}_\nu| \cos \alpha)$$

$$m_W^2 \approx 2(|\vec{p}_l| |\vec{p}_\nu| - \vec{p}_{lT} \cdot \vec{p}_{\nu T} - p_{lz} p_{\nu z})$$

$$m_W^2 \approx 2(\sqrt{p_{lT}^2 + p_{lz}^2} \sqrt{p_{\nu T}^2 + p_{\nu z}^2} - \vec{p}_{lT} \cdot \vec{p}_{\nu T} - p_{lz} p_{\nu z})$$



- opening angle

$$\cos \alpha = \frac{\vec{p}_l \cdot \vec{p}_\nu}{|\vec{p}_l| |\vec{p}_\nu|}$$

# $W$ -boson mass constraint

$$m_W^2 \approx 2(\sqrt{p_{lT}^2 + p_{lz}^2} \sqrt{p_{\nu T}^2 + p_{\nu z}^2} - \vec{p}_{lT} \cdot \vec{p}_{\nu T} - p_{lz} p_{\nu z})$$

$$p_{\nu z} = \frac{\chi p_{lz} \pm [p_l^2 (\chi^2 - 4 p_{lT}^2 p_{\nu T}^2)]^{1/2}}{2 p_{lT}^2}$$

$$\chi = m_W^2 + 2 \vec{p}_{lT} \cdot \vec{p}_{\nu T}$$

where we choose the solution well reconstruct the top mass

$$m_t^2 = ((\vec{p}_b^2 + m_b^2)^{1/2} + |\vec{p}_l| + |\vec{p}_\nu|)^2 - (\vec{p}_b + \vec{p}_l + \vec{p}_\nu)^2$$

$$m_t^2 = [(\vec{p}_b^2 + m_b^2)^{1/2} + (\vec{p}_{lT}^2 + p_{lz}^2)^{1/2} + (\vec{p}_{\nu T}^2 + p_{\nu z}^2)^{1/2}]^2 - [\vec{p}_b^2 + \vec{p}_l^2 + (\vec{p}_{\nu T}^2 + p_{\nu z}^2) + 2 \vec{p}_b \cdot \vec{p}_l + 2(\vec{p}_{bT} \cdot \vec{p}_{\nu T} + p_{bT} p_{\nu z}) - 2(\vec{p}_{lT} \cdot \vec{p}_{\nu T} + p_{lz} p_{\nu z})]$$

# Interaction Lagrangians

## → Vertex Factors

- *QED interaction term*

$$L_{QED, \bar{\psi}\psi A} = -g_e \bar{\psi} \gamma^\mu \psi A_\mu$$

where three fields – incoming fermion – outgoing fermion – photon ( $\psi, \bar{\psi}, A$ ) interacts at one point, and the vertex is defined. After removing the fields the remain part will give vertex factor  $-ig_e q \gamma^\mu$ .

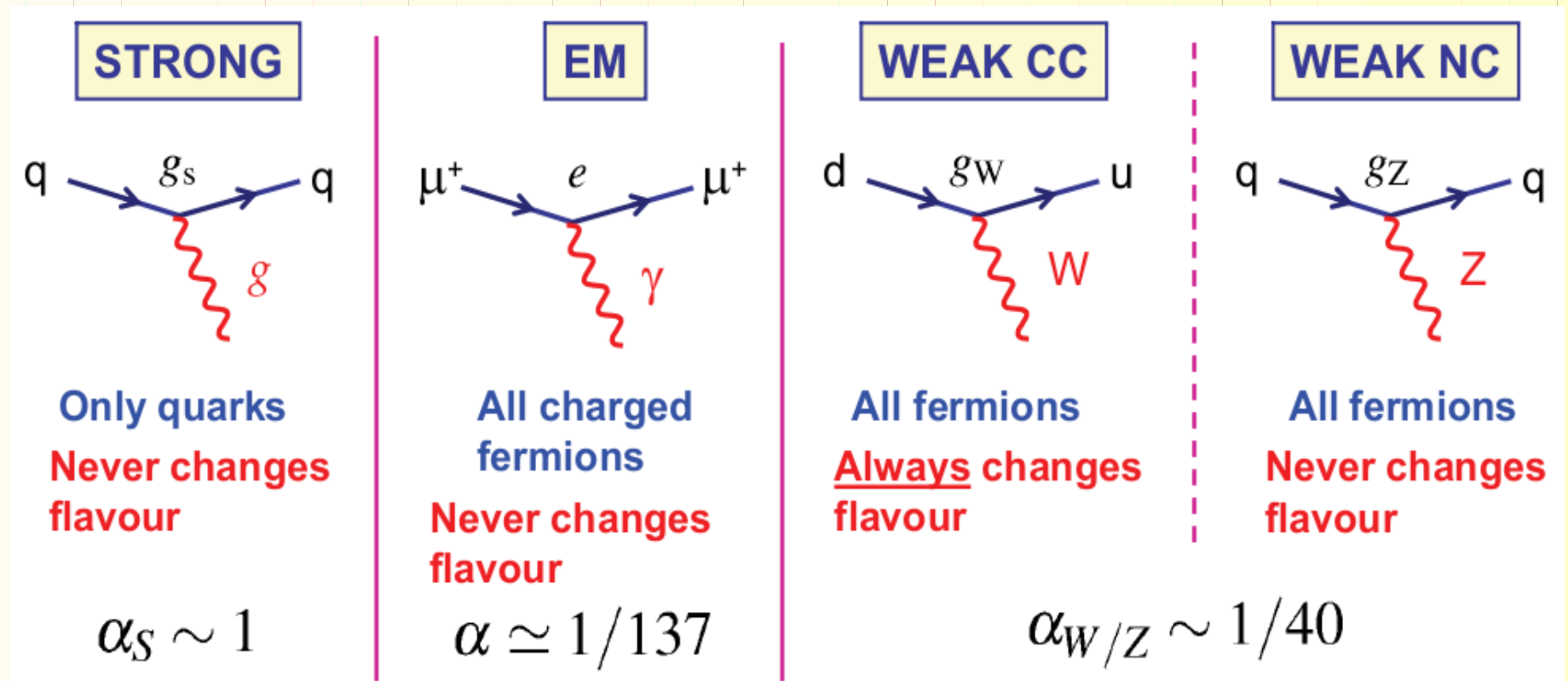
- In a similar way, incoming quark – outgoing quark – gluon ( $q, \bar{q}, g$ ) interacts at one point via *QCD interaction term*

$$L_{QCD, \bar{\psi}\psi g} = -g_s \bar{\psi} \gamma^\mu \frac{\lambda^a}{2} \psi G_\mu^a$$

vertex factor is  $-ig_s \lambda/2 \gamma^\mu$ .

# Interactions

Interactions of gauge bosons with fermions are described by the vertices. Type of gauge bosons and nature of interactions determine the properties of interactions.



# Interaction $\rightarrow$ Amplitude

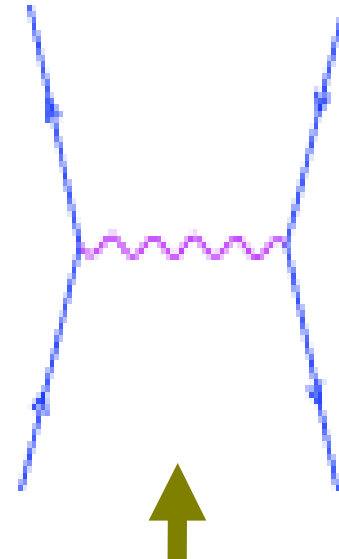
$$J^\mu(x) \frac{e^2}{|x-y|} J_\mu(y)$$

$\longleftrightarrow$   
Fourier trans-  
formation

$$J^\mu \frac{e^2}{q^2} J_\mu$$

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

elastic  
scattering





# Feynman Rules

- Free Lagrangian  
→ propagator

$$\frac{i}{p^2 - m^2} \quad (\text{spin} - 0)$$

$$\frac{i(\not{p} + m)}{p^2 - m^2} \quad (\text{spin} - 1/2)$$

$$\frac{-i}{p^2 - m^2} \left[ g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2} \right] \quad (\text{spin} - 1)$$

- Interaction terms  
→ vertex factors

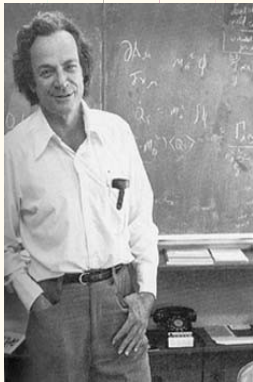
$$-ig_e q \gamma^\mu \quad (\text{QED})$$

$$\frac{-ig_s}{2} \lambda^a \gamma^\mu \quad (\text{QCD})$$

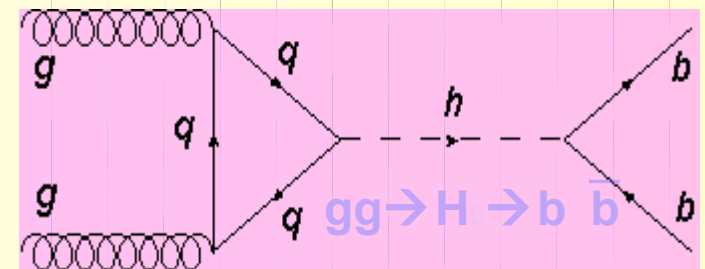
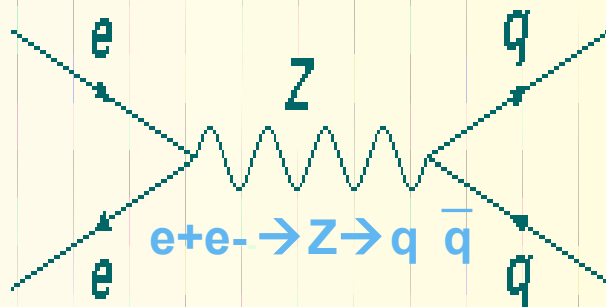
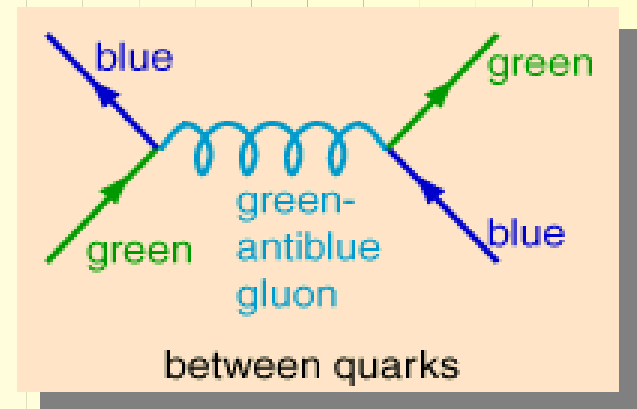
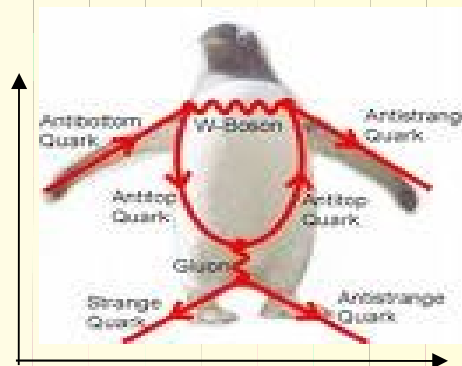
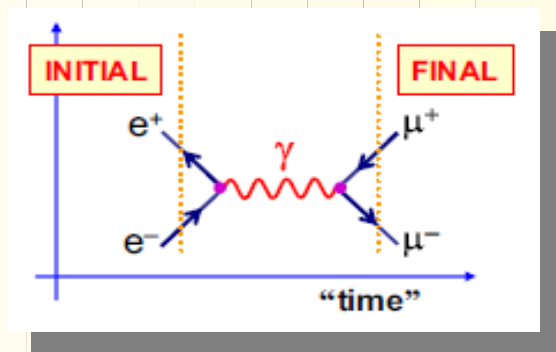
$$\frac{-ig_w}{2\sqrt{2}} \gamma^\mu (1 - \gamma^5) V_{ij} \quad (\text{EW/W})$$

$$\frac{-ig_z}{2} \gamma^\mu (c_V^f - c_A^f \gamma^5) \quad (\text{EW/Z})$$

# Feynman Diagrams



Processes of high energy physics are complex, they include radiation, loops, etc. However, the LO processes can be considered as a first approximation to the interactions between elementary particles (leptons, quarks and gauge bosons). Feynman diagrams are graphical presentations of the physics processes.

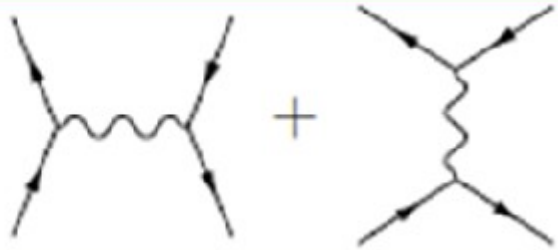


# Electroweak Processes

Lagrangian for fermion fields after the symmetry breaking

$$\begin{aligned}
 \mathcal{L}_F = & \sum_i \bar{\psi}_i \left( i \not{\partial} - m_i - \frac{gm_i H}{2M_W} \right) \psi_i \\
 & - \frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i \gamma^\mu (1 - \gamma^5) (T^+ W_\mu^+ + T^- W_\mu^-) \psi_i \\
 & - e \sum_i q_i \bar{\psi}_i \gamma^\mu \psi_i A_\mu \\
 & - \frac{g}{2 \cos \theta_W} \sum_i \bar{\psi}_i \gamma^\mu (g_V^i - g_A^i \gamma^5) \psi_i Z_\mu .
 \end{aligned}$$

For a certain type of process the amplitude and the differential cross section via diagrams

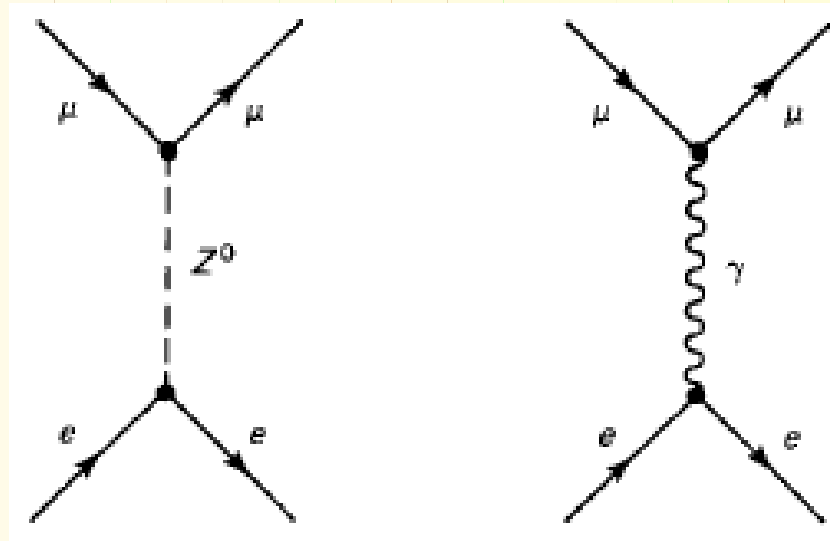
Amplitude = 

The diagram shows two fermion lines interacting via a wavy line representing a photon or Z boson. The first diagram shows the fermions exchanging the boson in the s-channel, and the second diagram shows them exchanging in the t-channel.

$\frac{d\sigma}{d\Omega} \propto \left| \text{diagram} + \text{diagram} \right|^2$

The diagram shows the same two Feynman diagrams as in the previous block, enclosed in large vertical bars with a superscript 2, representing the squared magnitude of the sum of the two amplitudes.

# Example: $e^+e^- \rightarrow \gamma / Z^0 \rightarrow \mu^+\mu^-$ process



**Sembojik Calculation:**  
**REDUCE, FORM, Mathematica**  
**etc.**

**Total amplitude:**  $M = M_\gamma + M_Z$

$$M_\gamma = -\frac{g_e^2}{q_\gamma^2} [\bar{u}(p_4) \gamma^\mu v(p_3)] g_{\mu\nu} [\bar{v}(p_2) \gamma^\nu u(p_1)]$$

$$M_Z = -\frac{g_z^2}{4(q_Z^2 - m_Z^2 + im_Z\Gamma_Z)} [\bar{u}(p_4) \gamma^\mu (c_V^\mu - c_A^\mu \gamma^5) v(p_3)] (g_{\mu\nu} - q_{Z\mu} q_{Z\nu} / m_Z^2) [\bar{v}(p_2) \gamma^\nu (c_V^e - c_A^e \gamma^5) u(p_1)]$$

# Interaction Cross Section

The differential cross section is given by

$$d\sigma = \frac{(2\pi)^4}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} |M_{fi}|^2 d\Phi_n(p_1 + p_2; p_3, p_4, \dots, p_{n+2})$$

in the center of mass frame

$$\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} = p_{1\text{cm}} \sqrt{s}$$

and it is useful to define the Mandelstam variables

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

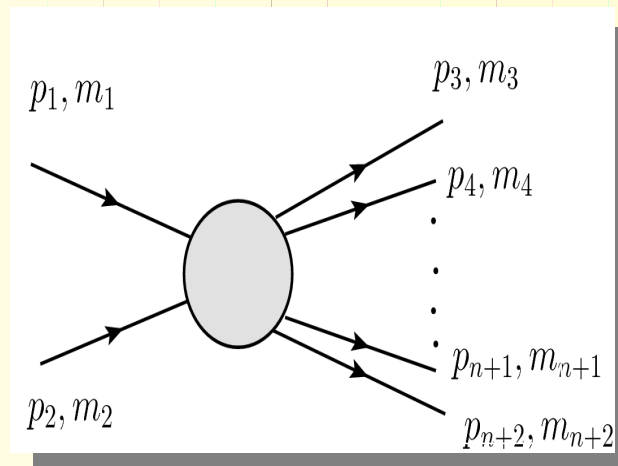
$$= m_1^2 + m_2^2 + 2E_1 E_2 - 2\vec{p}_1 \cdot \vec{p}_2$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$

$$= m_1^2 + m_3^2 - 2E_1 E_3 - \vec{p}_1 \cdot \vec{p}_3$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

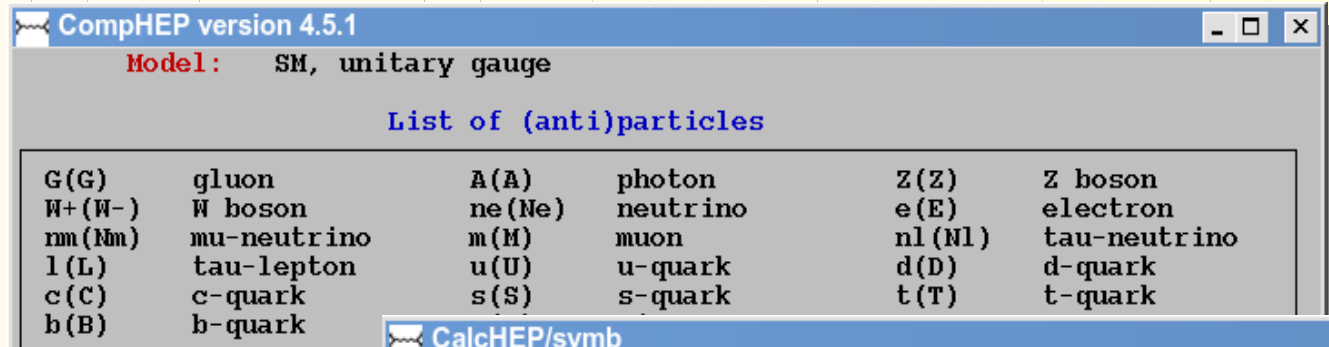
$$= m_1^2 + m_4^2 - 2E_1 E_4 + 2\vec{p}_1 \cdot \vec{p}_4$$



- the two-body cross section is given by

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s} \frac{|M|^2}{|p_{1\text{cm}}^{\vec{}}|^2}$$

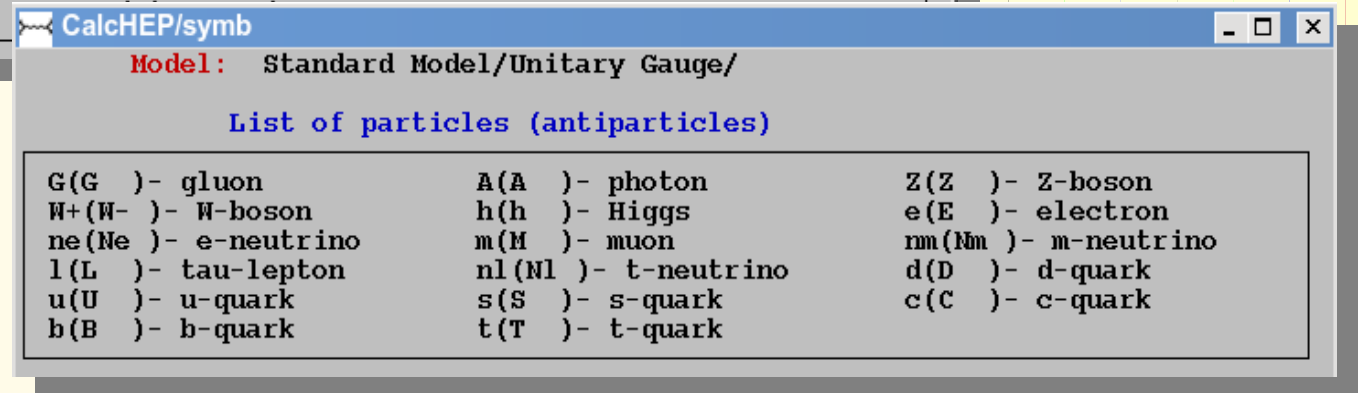
# Tools for Calculations



CompHEP version 4.5.1  
Model: SM, unitary gauge

List of (anti)particles

G(G)	gluon	A(A)	photon	Z(Z)	Z boson
W+(W-)	W boson	ne(Ne)	neutrino	e(E)	electron
nm(Nm)	mu-neutrino	m(M)	muon	nl(Nl)	tau-neutrino
l(L)	tau-lepton	u(U)	u-quark	d(D)	d-quark
c(C)	c-quark	s(S)	s-quark	t(T)	t-quark
b(B)	b-quark				



CalcHEP/symb  
Model: Standard Model/Unitary Gauge/

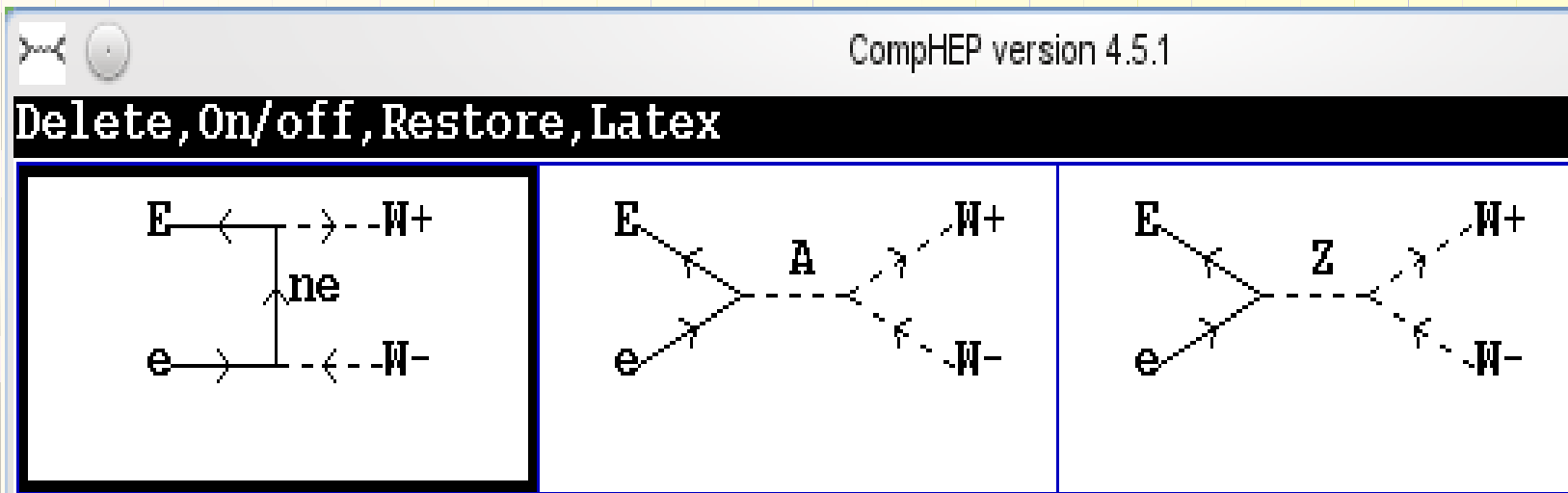
List of particles (antiparticles)

G(G )- gluon	A(A )- photon	Z(Z )- Z-boson
W+(W- )- W-boson	h(h )- Higgs	e(E )- electron
ne(Ne )- e-neutrino	m(M )- muon	nm(Nm )- m-neutrino
l(L )- tau-lepton	nl(Nl )- t-neutrino	d(D )- d-quark
u(U )- u-quark	s(S )- s-quark	c(C )- c-quark
b(B )- b-quark	t(T )- t-quark	

CompHEP/CalcHEP[\*] program packages are used for calculation of decay and collision processes of elementary particles in the tree level approximation. They make available passing from lagrangian to the final distributions with the high level of automatization. There are also some other packages for different purposes.

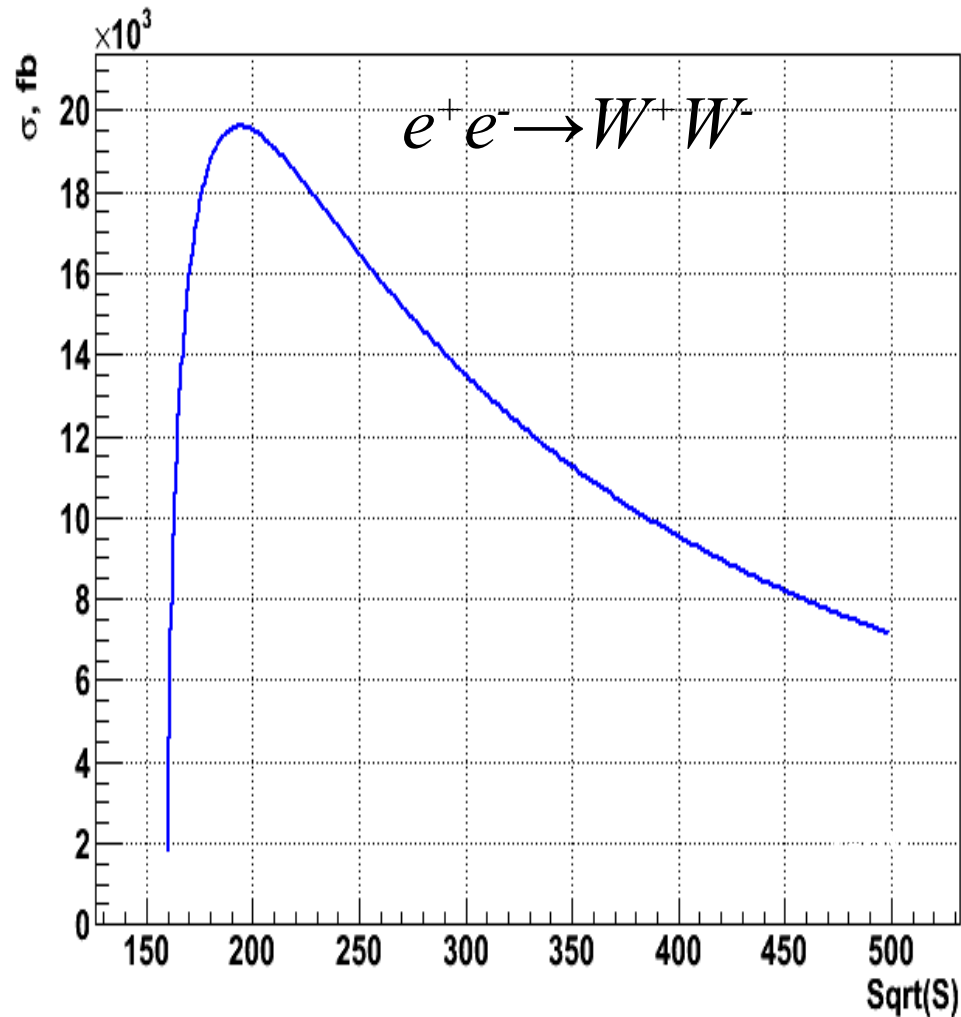


# Example: $e^+e^- \rightarrow W^+W^-$ process



CompHEP, calculates  $|\text{amplitude}|^2$  expressions via symbolic session, and gives the output for Reduce/Mathematica/Reduce input codes. Numerical session is used for cross section calculations, decay width calculations, and event generations. The distributions can be analyzed further by using Root program. Event information can be in the LHE format.

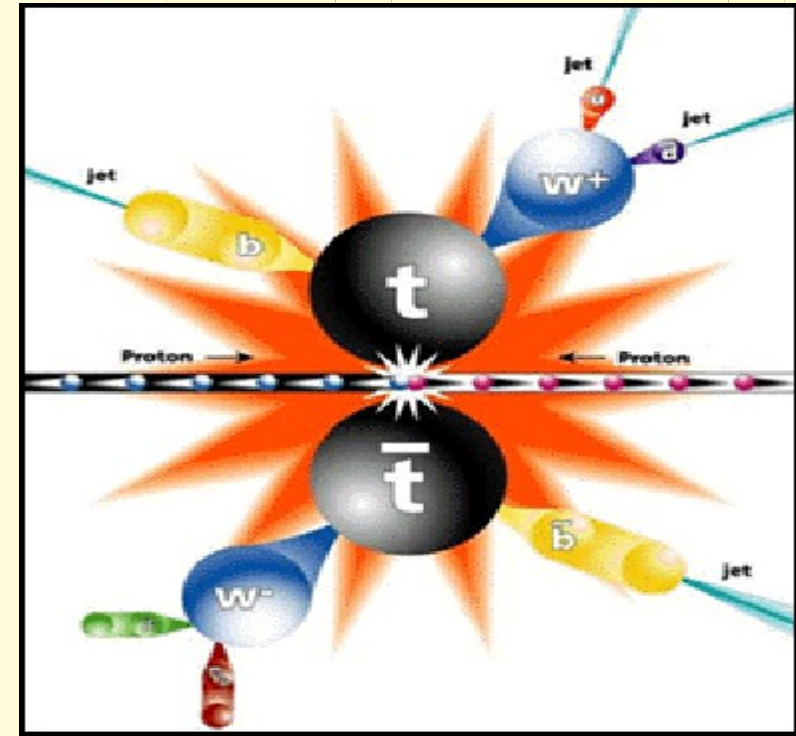
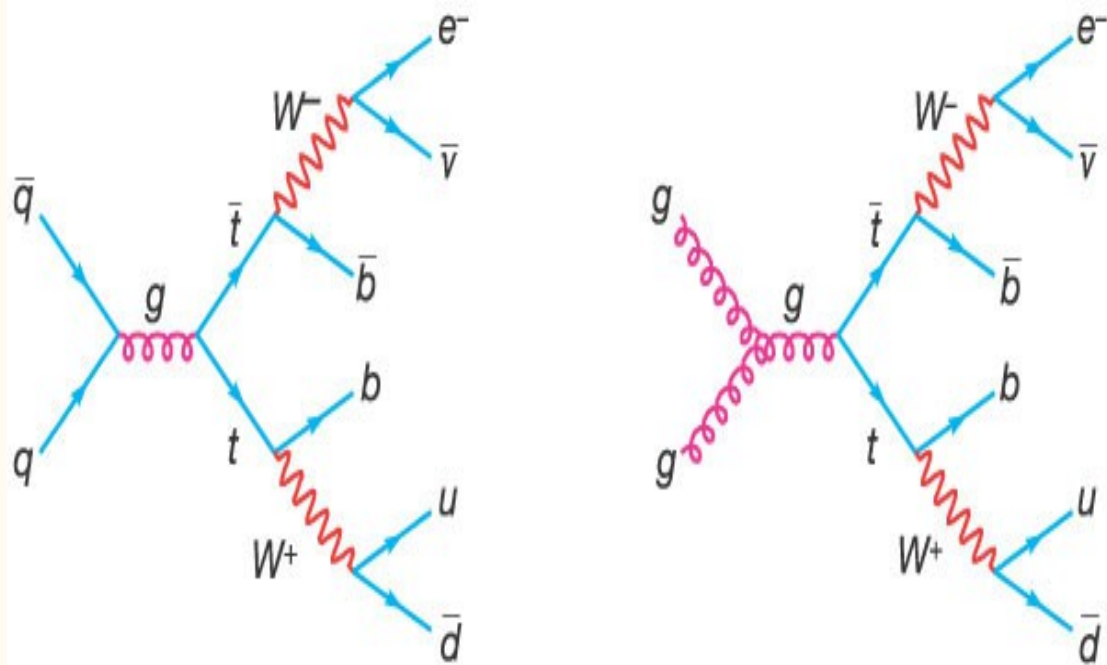
# Kinematical Distributions



The threshold energy scan can be performed for some energy range. After a threshold energy ( $\sim 160$  GeV) the cross section increases and reaches maximum at  $\sqrt{s} \sim 190$  GeV. Increasing the energy will not be more beneficial with the cross section for this process.

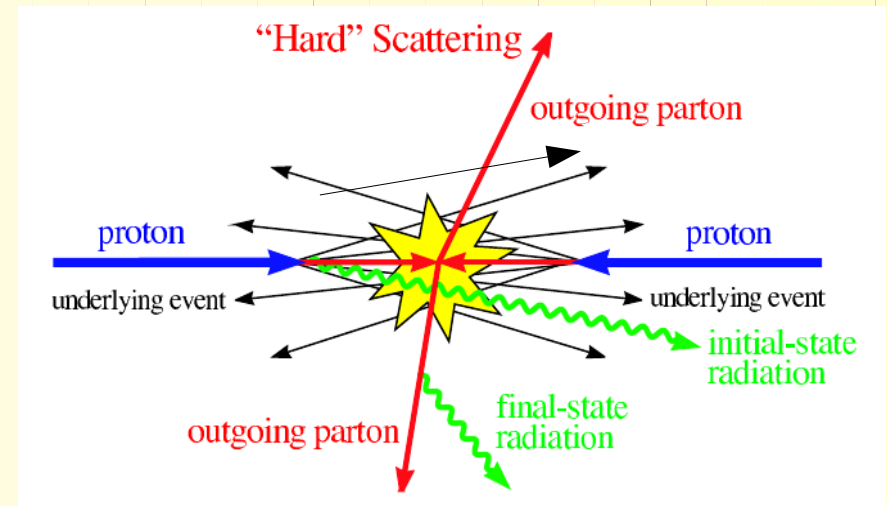
# Pair Production at Colliders

At the colliders, one can study matter and forces, also create the new and heavy matter, via the forces, converting the energy of colliding particles into mass.



# Missing Transverse Energy (MET)

- An important portion of incoming hadron energy goes into beam pipe. For the particles (neutrinos) undetectable directly, transverse momentum component in the perpendicular (to the beam direction) plane can be calculated.

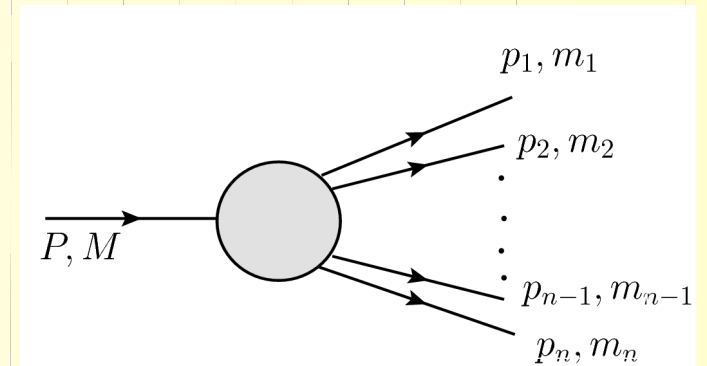


$$E_T^{\text{miss}} = - \sum_i p_T(i)$$

Sum over transverse momentum of all final state detectable particles.

# Particle Decays

The decay rate of a particle of mass  $M$  into  $n$  bodies in its rest frame is given by



$$d\Gamma = \frac{(2\pi)^4}{2M} |M_{fi}|^2 d\Phi_n(P; p_1, p_2, \dots, p_n)$$

with

$$d\Phi_n(P; p_1, p_2, \dots, p_n) = \delta^4(P - \sum_{i=1}^n p_i) \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

$M_{fi}$  is the Lorentz invariant amplitude specific to the process for the initial to final state transition.

# Standard Model of Particle Physics

Standard Model includes quantum field theories, Quantum ElectroDynamics (QED), Quantum ChromoDynamics (QCD) and Weak interactions (WI). These are based on the relativistic quantum mechanics.

In quantum mechanics, an  $s$  state of a physical system is defined by  $\psi_s$  wave function (Schrodinger) or ket  $|s\rangle$  (Dirac). Observables  $\langle O \rangle = \int d^n x \psi^* O \psi$  remain invariant under global (independent from position) phase transformation.

# The Standard Model (SM)

The gauge group of standard model is

$$SU(3)_C \times SU(2)_W \times U(1)_Y$$

where  $C$  denotes color,  $W$  for weak isospin, and  $Y$  for hypercharge. The corresponding gauge fields are  $G_\mu^a (a=1,8)$ ,  $W_\mu^i (i=1,3)$  and  $B_\mu$ .

An important feature of weak interactions is the violation of the “parity”. This suggests that it is natural to study with two-component spinors ( $1 \times 2$  or  $2 \times 1$ ). These spinors also comprise the basics of the four-component spinor representation of the Lorentz group.



# SM Summary

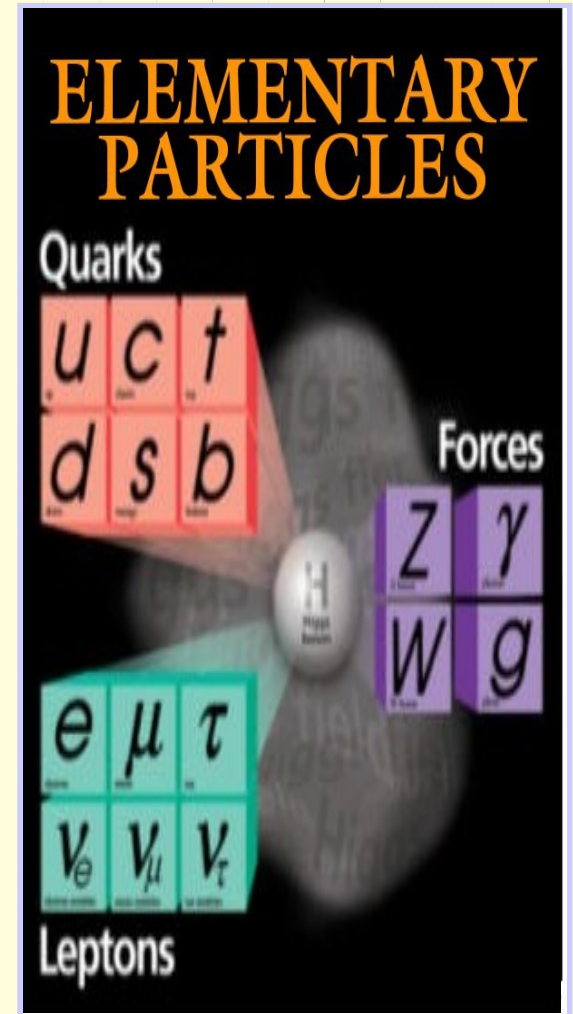
\*Standard Model is based on the gauge symmetry:  $SU(3)_c \times SU(2)_w \times U(1)_y$ .

However, this symmetry is broken spontaneously to  $SU(3)_c \times U(1)_{em}$ .

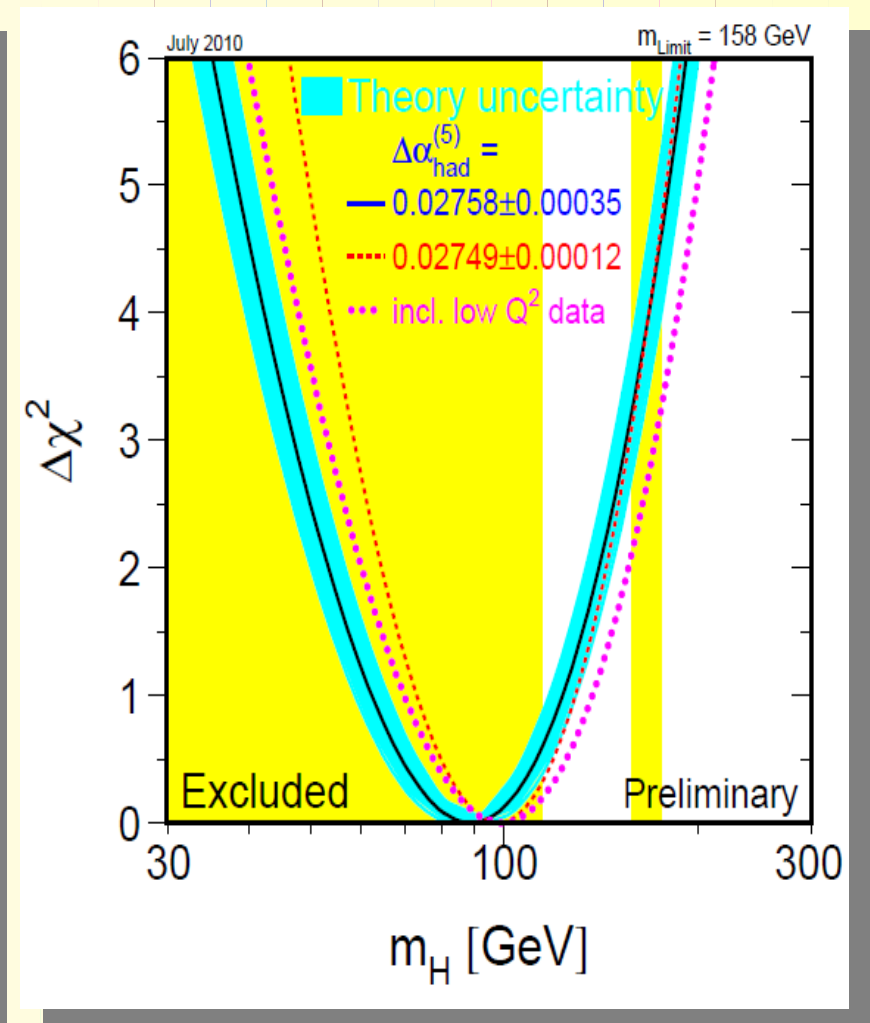
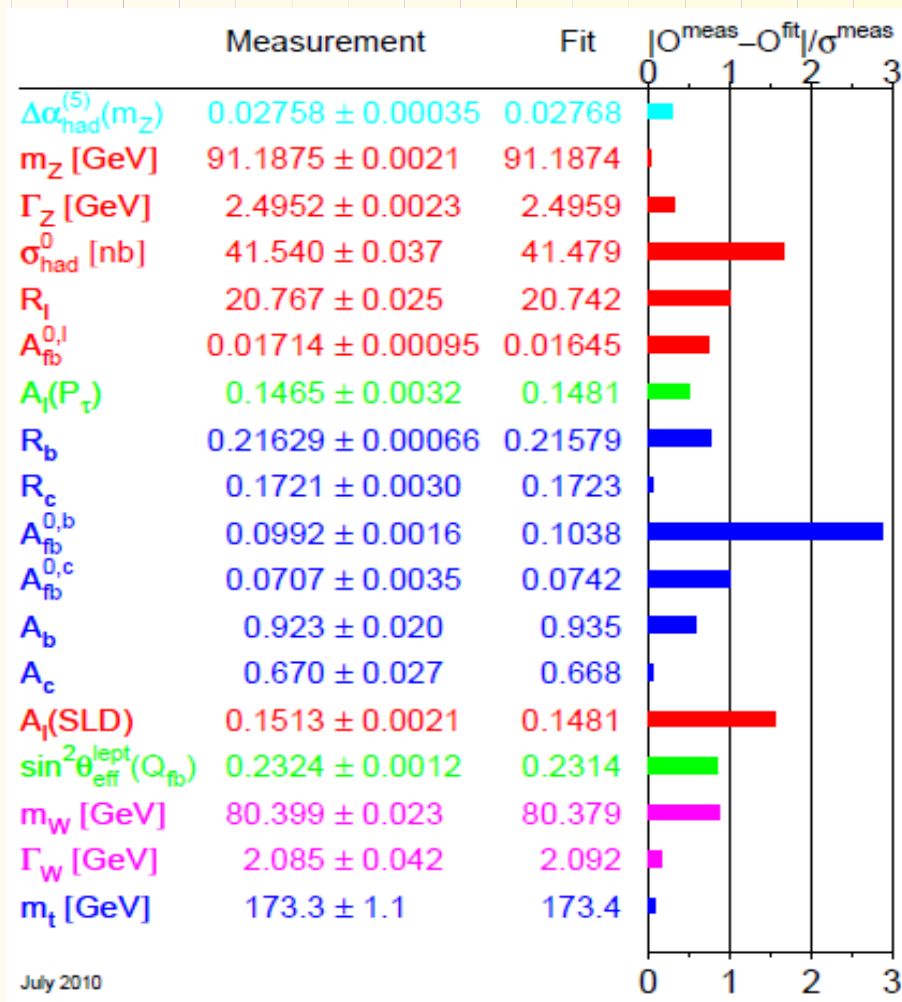
\*Matter: there are three leptons and quarks families.

\*Higgs field have an important rol:

- one Higgs doublet interacts the other fields
- gain a vacuum expectation value ( $\sim 246$  GeV)
- Quarks and leptons, W/Z bosons and Higgs itself get their masses from this mechanism.



# Status of Precision Measurements



EW fit - <http://lepewwg.web.cern.ch/LEPEWWG/>

# Quantum numbers of Particles

- Quantum numbers of elementary particles according to  $SU(3) \times SU(2) \times U(1)$

$L_L$ $E_R$	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$ $e_R^-, \mu_R^-, \tau_R^-$	$(1, 2, -1)$ $(1, 1, -2)$
$Q_L$ $U_R$ $D_R$	$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$ $u_R, c_R, t_R$ $d_R, s_R, b_R$	$(3, 2, +1/3)$ $(3, 1, +2/3)$ $(3, 1, -1/3)$

- Lagrangian:**
  - gauge interactions
  - Matter fermions
  - Yukawa interactions
  - Higgs potential

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F_{\mu\nu}^a F^{a\ \mu\nu} \\ & + i\bar{\psi} \not{D}\psi + h.c. \\ & + \psi_i y_{ij} \psi_j \phi + h.c. \\ & + |D_\mu \phi|^2 - V(\phi) \end{aligned}$$

# Field Theories

- Lagrangian in classical mechanics is given by  $L(q, \dot{q}, t)$  and  $L=T-V$ . Equation of motion is given by

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0$$

conjugate momentum corresponding to the coordinates which is not present explicitly in the Lagrangian is conserved.

→ Newton's laws

- In field theory we study with a field  $\phi(x, y, z, t)$  as the function of space and time. In the relativistic theory (4D space-time) Euler-Lagrange equation

$$\frac{\partial}{\partial x^\mu} \left( \frac{\partial \mathcal{L}}{\partial (\partial \phi / \partial x^\mu)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

→ **spin-0**: Klein-Gordon equation;  
**spin-1/2**: Dirac equation; **spin-1**: Proca equation.

# Global ve Local Phase Transformations

Free Dirac lagrangian

$$L = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

## Global phase transformation

$$\psi(x) \rightarrow e^{-iq\alpha}\psi(x)$$

- Dirac lagrangian remains invariant under this transformation
- Absolute phase of wave function is not measurable (arbitrary)
- The relative phases in interference are not affected by phase transformation.
- Symmetry  $\rightarrow$  charge conservation

## Local phase transformation

$$\psi(x) \rightarrow e^{-iq\alpha(x)}\psi(x)$$

Derivation of wave function

$$\partial_\mu[e^{-iq\alpha(x)}\psi(x)] \rightarrow e^{-iq\alpha(x)}[\partial_\mu\psi(x) - iq(\partial_\mu\alpha(x))\psi(x)]$$

leads to an extra term  $iq\partial_\mu\alpha(x)$

The Lagrangian reads

$$L \rightarrow L + q\bar{\psi}\gamma^\mu\psi(\partial_\mu\alpha(x))$$

Total lagrangian should remain invariant under this transformation, then we should add both the kinetic term and interaction term with the gauge boson to the free Dirac Lagrangian.

**This will automatically generate the gauge boson of the interaction.**

# U(1) Gauge Symmetry

Electromagnetic Lagrangian remains invariant under local U(1) gauge transformation.

$$L = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - q\bar{\psi}\gamma^\mu\psi A_\mu$$

where the vector field transforms and the covariant derivative becomes

$$A_\mu \rightarrow A_\mu + \partial_\mu\alpha(x)$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$D_\mu = \partial_\mu + iqA_\mu$$

The type of the interaction is obtained from the local phase transformation. Quantum electrodynamics is a gauge theory with U(1) phase symmetry.

# QED Lagrangian

The image shows a handwritten QED Lagrangian on a piece of paper, with four blue boxes pointing to specific terms:

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - e \bar{\psi} \not{A} \psi - m \bar{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- Fermion kinetic term**:  $\bar{\psi} i \not{\partial} \psi$
- Interaction term**:  $- e \bar{\psi} \not{A} \psi$
- Mass term**:  $- m \bar{\psi} \psi$
- Vector boson kinetic term**:  $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$



# Yang-Mills Teori

Yang and Mills extended the local symmetry to non-abelian case. Transformation matrix  $S$  has determinant 1. Lagrangian is invariant under  $SU(2)$  global phase transformation. There will be extra terms for local transformation, in order to remove these terms we should add extra fields and interaction terms.

$$\psi \rightarrow S\psi, \quad S = e^{-iq\tau \cdot \lambda(x)}$$

Covariant derivative can be read

$$\partial_\mu \psi \rightarrow S(\partial_\mu \psi) + (\partial_\mu S)\psi \quad D_\mu = \partial_\mu + iq\tau \cdot A_\mu$$

scalar product transforms as

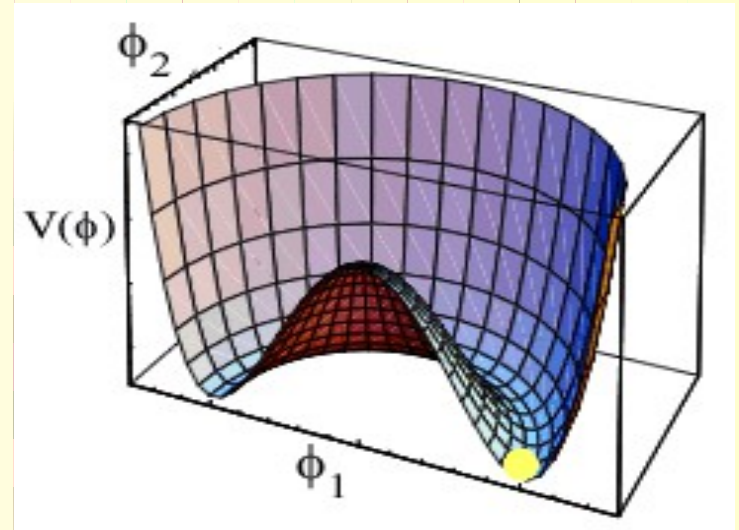
$$\tau \cdot A'_\mu = S(\tau \cdot A_\mu)S^{-1} + (i/q)(\partial_\mu S)S^{-1}$$

Lagrangian invariant under local  $SU(2)$  gauge transformation

$$L = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu} \cdot F_{\mu\nu} - (q\bar{\psi}\gamma^\mu \tau \psi) \cdot A_\mu$$

# Broken Symmetry

- Some symmetries are not exact - *they are broken! very important feature!*
- Spontaneous symmetry breaking predicts masses for fermions and W, Z gauge bosons
- Predict one more massive particle: the Higgs boson, responsible for mass generation - *the only missing and the most wanted Standard Model particle!*



# Spontaneous Symmetry Breaking

- Lagrangian for scalar field,

$$L = \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

For  $\phi \rightarrow -\phi$ , Lagrangian remains invariant. Potential have minimum at  $\phi = \pm \mu/\lambda$ . New variable in terms of the deviation from minimum is  $\eta = \phi \pm \mu/\lambda$ . In this case the Lagrangian



$$L = \frac{1}{2}(\partial^\mu\eta)(\partial_\mu\eta) - \mu^2\eta^2 \pm \mu\lambda\eta^3 - \frac{1}{4}\lambda^2\eta^4 + \frac{1}{4}(\mu^2/\lambda)^2$$

- The new Lagrangian is not symmetrical for  $\eta \rightarrow -\eta$ ; the symmetry is spontaneously broken (SSB).

# Higgs Mechanism

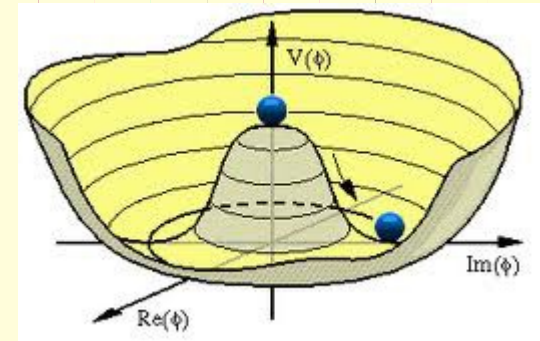
- Complex field

$$\phi = \phi_1 + i\phi_2$$

and the Lagrangian

$$L = \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi)^* + \frac{1}{2}\mu^2(\phi\phi^*) - \frac{1}{4}\lambda^2(\phi\phi^*)^2$$

We want it to be invariant under local gauge transformation, make a transformation that the system will be at minimum energy state



$$L(\phi_1, \phi_2, A) \rightarrow L'(\phi'_1, \phi'_2, A')$$

*Before SSB*                      *After SSB*

*Due to choice  $\phi'_1 = \phi_1$  and  $\phi'_2 = \phi_1 - \mu/\lambda$  massless Goldstone boson field  $\phi'_1$  disappears and gives the mass to boson  $A'$ .  $\phi'_2$  field (Higgs boson) has mass.*

# Gauge Boson Masses

Gauge boson masses are obtained from the term  $|D_\mu \phi|^2$ . Covariant derivative

$$D_\mu \phi = \left[ \partial_\mu - ig \frac{\sigma^i}{2} W_\mu^i - ig' \frac{Y}{2} B_\mu \right] \phi$$

scalar field

$$\phi = U(x) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

and gauge fields mass eigenstates

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}},$$

$$Z_\mu = W_\mu^3 \cos \theta_W - B_\mu \sin \theta_W,$$

$$A_\mu = W_\mu^3 \sin \theta_W + B_\mu \cos \theta_W$$

mass terms,

$$\rightarrow \frac{1}{4} g^2 v^2 W_\mu^+ W^{\mu-} + \frac{1}{8} v^2 (g W_\mu^3 - g' B_\mu)^2$$

$$m_W = gv/2, \quad m_Z = m_W / \cos \theta_W$$

# Fermion masses

Fermion masses is obtained from the couplings of left-handed fermion ( $f_L$ ) and right-handed fermion ( $f_R$ ) with scalar field ( $\phi$ )

$$\begin{aligned} L_Y &= -y_f (\bar{f}_R \phi^\dagger f_L - \bar{f}_L \phi f_R) = -\frac{y_f(v+h)}{\sqrt{2}} (\bar{f}_R f_L + \bar{f}_L f_R) \\ &= -\frac{y_f}{\sqrt{2}} (v+h) \bar{f} f \rightarrow -m_f \bar{f} f - \frac{g m_f}{2m_W} h \bar{f} f \end{aligned}$$

fermion masses are given in terms of Yukawa coupling and the vacuum expectation value ( $v=246$  GeV)

$$m_f = y_f v / \sqrt{2}$$

*for top quark,  
 $y_t = \sqrt{2} m_t / v \approx 1.$*

*For an explanation of neutrino masses and mixings one needs to go beyond the Standard Model!*

# Neutrinos

In the SM formulated in 1970's neutrinos are assumed to be massless, then there is only one helicity state (left-handed) for neutrinos. In 1960's Pontecorvo, Maki, Nakagaya and Sakata (PMNS) suggested that neutrinos can be produced and annihilated in flavor eigenstates  $(\nu_e, \nu_\mu, \nu_\tau)$ , in the processes, and they can propagate in space in the mass eigenstates  $(\nu_1, \nu_2, \nu_3)$ .

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



# Neutrino Mixing

Considering the mixing of  $\nu_\mu$  and  $\nu_\tau$  in terms of  $\nu_2$  and  $\nu_3$  (atmospheric neutrinos) with one mixing angle  $\theta$ , the wave amplitudes

$$\begin{aligned}\nu_\mu &= \nu_2 \cos \theta + \nu_3 \sin \theta \\ \nu_\tau &= -\nu_2 \sin \theta + \nu_3 \cos \theta\end{aligned}$$

where  $E$  denotes neutrino energy, mass eigenstates depend on time

$$\begin{aligned}\nu_2(t) &= \nu_2(0) \exp(-i E_2 t) \\ \nu_3(t) &= \nu_3(0) \exp(-i E_3 t)\end{aligned}$$

# Neutrino Mixing - 2

Study on an example of muon-type neutrinos in the initial state

$$\nu_2(0) = \nu_\mu(0) \cos \theta$$

$$\nu_3(0) = \nu_\mu(0) \sin \theta$$

time dependency  $\nu_\mu(t) = \nu_2(t) \cos \theta + \nu_3(t) \sin \theta$

Amplitude for muon-neutrinos

$$A_\mu(t) = \nu_\mu(t) / \nu_\mu(0) = \cos^2 \theta \exp(-iE_2 t) + \sin^2 \theta \exp(-iE_3 t)$$

and intensity

$$I_\mu(t) / I_\mu(0) = 1 - \sin^2 2\theta \sin^2 [(E_3 - E_2) t / 2]$$

# Neutrino Masses

If neutrinos are **Dirac particles**:

- neutrino and anti-neutrino are distinct particles
- left-handed state and massless

If **Majorana particles**:

- Particle and anti-particle are the same  $\nu = \nu^c$ .

In general, **lepton masses** can originate from both Dirac and Majorana mass terms.

$$\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

Here  $m_L$  and  $m_R$ , Majorana masses for left-handed and right-handed states, respectively.  $m_D$  denotes Dirac mass.

# Neutrino Masses - 2

- We can diagonalize the mass matrix, in this case eigenvalues

$$m_{1,2} = [ (m_R + m_L) \pm \sqrt{(m_R - m_L)^2 + 4 m_D^2} ] / 2$$

here we assume that  $m_L$  is much smaller; and  $m_R = M$  is much greater than Dirac scale and it is around GUT scale. Physical neutrino mass is given by

$$m_1 \approx \frac{m_D^2}{M}, \quad m_2 \approx M$$

This mechanism (see-saw), taking right-handed neutrino mass very large, gives left-handed Majorana neutrino mass very small.

# SM Parameters

- 3 gauge couplings ( $g_1, g_2, g_3$ )
  - 2 Higgs parameters ( $\mu, \lambda$ )
  - 6 quark masses
  - 3 quark mixing angle + 1 phase
  - 3 (+3) lepton masses
  - (3 lepton mixing angles + 1 phase)
- ( )=Dirac neutrino mass case

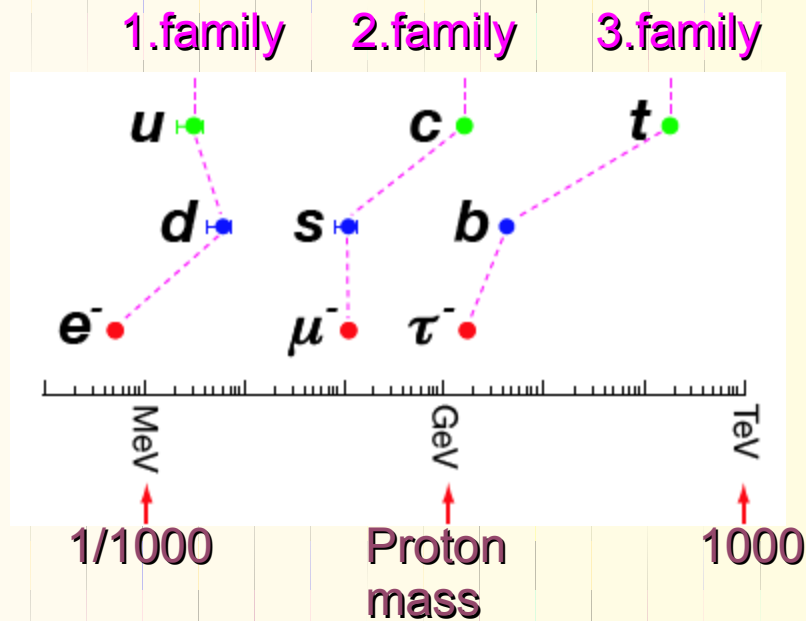
CKM  
matrix

PMNS  
matrix

FLAVOR  
PARAMETERS

# Flavor Problem

- Mass hierarchy

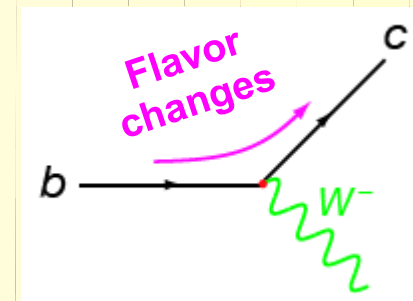


*Electroweak symmetry breaking may explain how they get their masses, but it does not explain what the value of their masses are.*

*Heavy quarks forming hadrons are **b** and **c** quarks. The hadrons (mesons/baryons) from these can be detected effectively.*

*Charged weak current leads to flavor mixing.*

	$d$	$s$	$b$
$u$	■	■	■
$c$	■	■	■
$t$	■	■	■



# Heavy Flavor Physics

<ul style="list-style-type: none"> <li>• <math>m_u \approx 3 \text{ MeV}</math></li> <li>• <math>m_d \approx 5 \text{ MeV}</math></li> <li>• <math>m_s \approx 100 \text{ MeV}</math></li> </ul>	Light quarks ( $m \leq \Lambda_{\text{QCD}}$ )	<ul style="list-style-type: none"> <li>• <math>m_{\nu_1} \leq 10^{-6} \text{ MeV}</math></li> <li>• <math>m_{\nu_2} \leq 10^{-5} \text{ MeV}</math></li> <li>• <math>m_{\nu_3} \leq 10^{-4} \text{ MeV}</math></li> </ul>	Light neutrinos (Neutrino-ph)
<ul style="list-style-type: none"> <li>• <math>m_c \approx 1270 \text{ MeV}</math></li> <li>• <math>m_b \approx 4200 \text{ MeV}</math></li> </ul>	Very heavy quark	<ul style="list-style-type: none"> <li>• <math>M_e \approx 0.5 \text{ MeV}</math></li> <li>• <math>m_\mu \approx 100 \text{ MeV}</math></li> </ul>	Light leptons (EDM/MDM)
<ul style="list-style-type: none"> <li>• <math>m_t \approx 172000 \text{ MeV}</math></li> </ul>		<ul style="list-style-type: none"> <li>• <math>m_\tau \approx 1800 \text{ MeV}</math></li> </ul>	Tau lepton



# CP Violation in SM

Complex coupling constants in the Lagrangian terms,

$$\mathcal{L} = \sum_i a_i \mathcal{O}_i + h.c. \quad (CP) \mathcal{O}_i (CP)^\dagger = \mathcal{O}_i^\dagger$$

Except the charged current couplings, all couplings in the mass basis of the SM (3 families and 1 Higgs doublet) can be made real. An important property is

$$V_{CKM} \neq V_{CKM}^*$$

In the SM, 1 phase in the mixing matrix is responsible from the CP violation in weak interactions.

# CKM

- Three Euler angles  $\theta_{ij}$

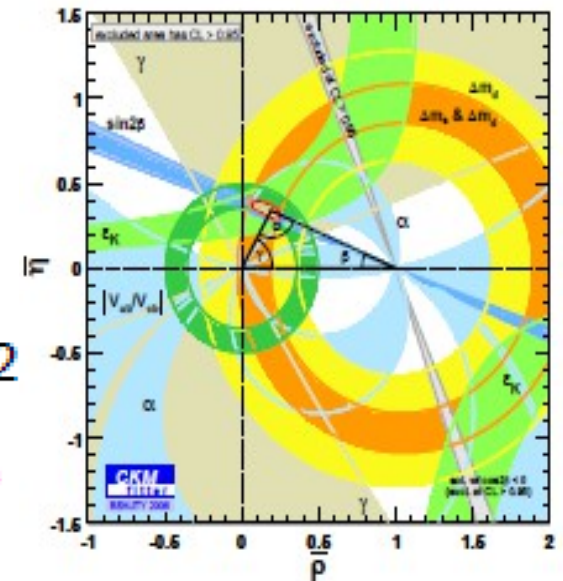
$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix}, \quad U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}$$

- one phase  $\delta$ : 
$$U_{\delta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix}.$$

- PDG CKM Parametrization

$$V_{\text{CKM}} = U_{23} U_{\delta}^{\dagger} U_{13} U_{\delta} U_{12}$$

- large phases  $V_{ub} = |V_{ub}| e^{-i\gamma} = s_{13} e^{-i\delta_{13}}$   
 $V_{td} = |V_{td}| e^{i\beta}$



**CP violation is small in the SM**  
**( $\delta_{\text{exp}} = 0.0001$ ), flavor physics and CP violation re-**  
**quires precision calculations/measurements.**

# SM Parameters

Mixing matrix for the charged current  $W^{+/-}$  interactions coupling with quarks  $u_L$  and  $d_L$

$$V_{CKM} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Standard choice of parameters

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Magnitude of the elements:  $|V_{ud}| \approx 0.97425$ ,  $|V_{us}| \approx 0.2252$ ,  $|V_{ub}| \approx 0.00389$ ,  $|V_{cd}| \approx 0.230$ ,  $|V_{cs}| \approx 1.023$ ,  $|V_{cb}| \approx 0.0406$ ,  $|V_{td}| \approx 0.0084$ ,  $|V_{ts}| \approx 0.0387$ ,  $|V_{tb}| \approx 0.88$ .

# CP ve BAU

- Baryon asymmetry in the universe (BAU) can be calculated from KM CP case:

$$(n_B - n_{\bar{B}})/n_\gamma \approx n_B/n_\gamma \sim JP_u P_d / M^{12}$$

- Jarlskog parameter ( $J \sim O(10^{-5})$ ) is a parametrization of CP violation in quark sector.

- Mass parameter at electroweak scale  $O(100 \text{ GeV})$ , calculated asymmetry  $O(10^{-17})$  is well below the observed value  $O(10^{-10})$ .
- Therefore we need more sources for CP violation.

$$J = \cos(\theta_{12}) \cos(\theta_{23}) \cos^2(\theta_{13}) \sin(\theta_{12}) \sin(\theta_{23}) \sin(\theta_{13}) \sin(\delta)$$

$$P_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$

$$P_d = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$



# Homework

- 1) Classify elementary particles - for each case - according to their (a) colors, (b) electric charges, (c) isospins, (d) hypercharges, (e) masses, (f) spins.
- 2) For the leptonic decay of W-boson (a) define the invariant mass of the leptonic system, (b) calculate the transverse mass of the system in terms of their momentum components.
- 3) A  $\pi^0$  meson (rest mass 135 MeV/c) with momentum 270 GeV/c decays into two photons. If the mean lifetime of  $\pi^0$  is  $8.5 \times 10^{-17}$  sec, calculate to 10% accuracy how far the pion will travel prior to decay. What will be the minimum value of opening angle of its two decay photons in the laboratory.