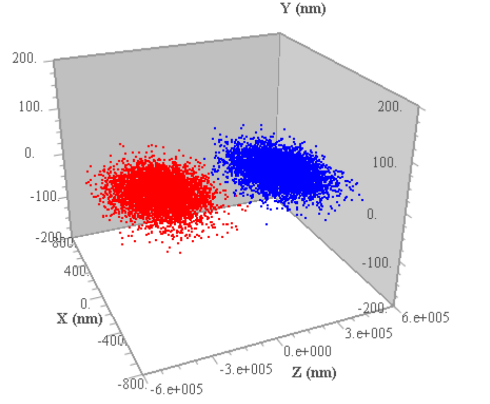

Introduction to Accelerator Physics

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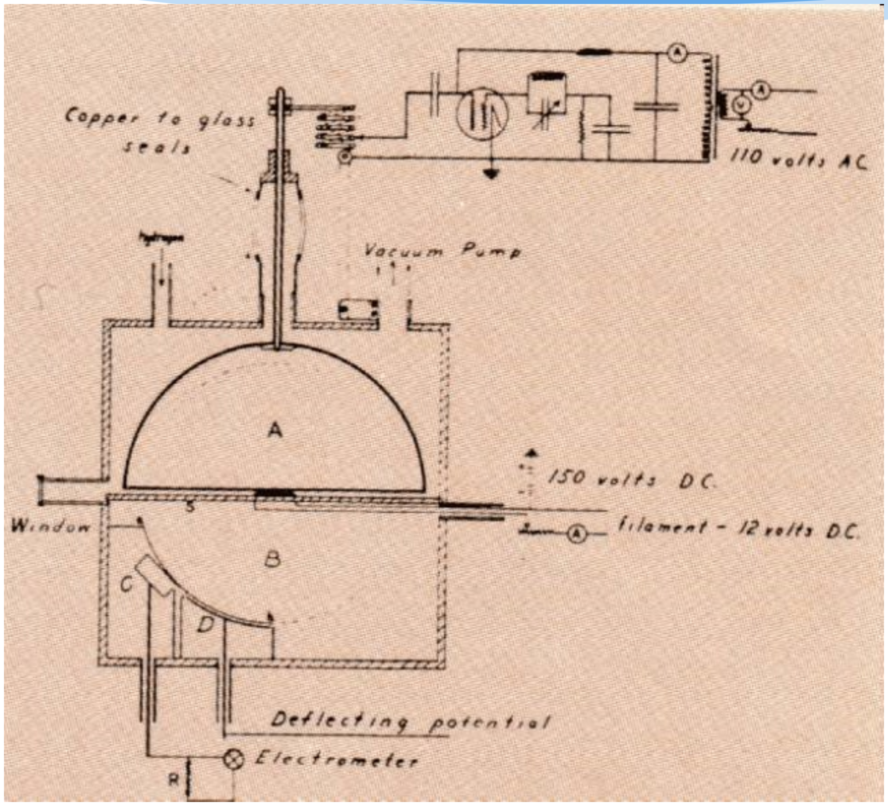
ISTAPP2011, 31 Jan-11 Feb 2011, Dogus Univ. Istanbul

Contents

First circular accelerator !

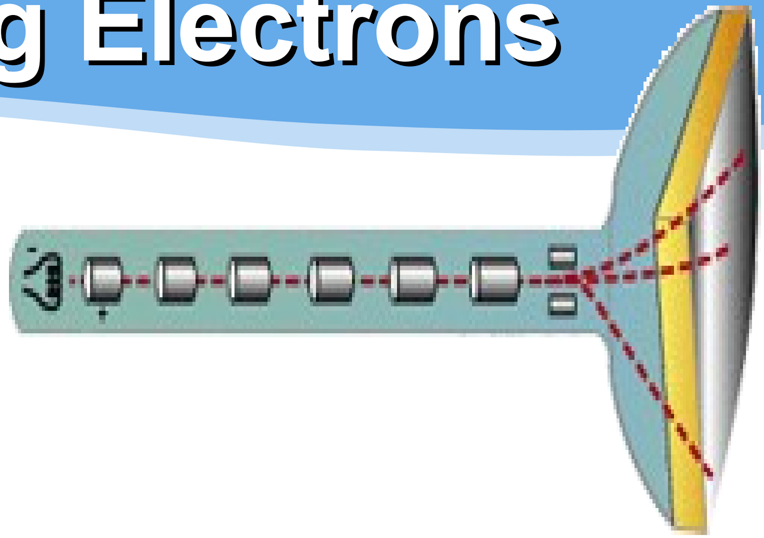
Ernest Lawrence (1939)



First cyclotron plot by
E. Lawrence and M.S.Livingston,
It has a diameter D ~13 cm.

Accelerating Electrons

Electron gun in TV/CRT and x-ray machines (low energy accelerators)



Long length and built in underground (high energy accelerators).

Accelerators

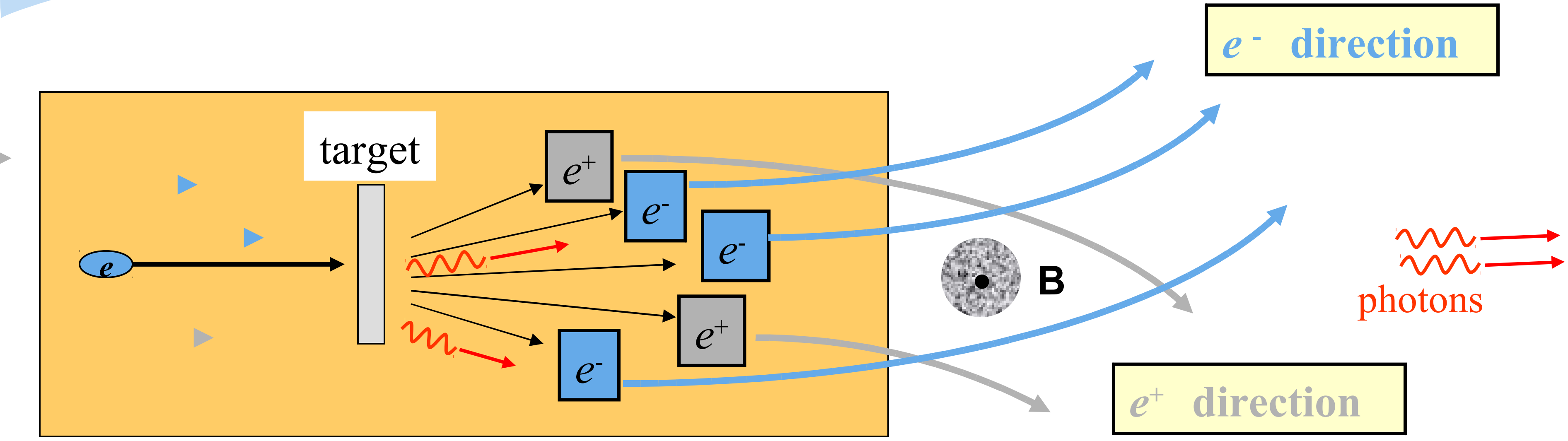
Accelerator is a machine using electric field to accelerate charged particles like electron, proton, and keep them together in a beam using magnetic field. High energy accelerators are tools commonly used to search for the structure of matter and elementary interactions.

- **Today:** stable particles (& anti-particles) are accelerated efficiently

  Protons and electrons

  (pion, neutrino etc. secondary beams)
- In general, circular accelerators → long lifetime for the beam
- **Proton:** limited by bending power of dipoles in the ring.
  - Powerful magnets (superconductors) → reach to higher energies, LHC (7 TeV in 2010, 14 TeV in next years).
- **Electron:** limited by sinkrotron radiation (=energy loses in circular path).
  - Loss per turn ~ 88.5 E^4/R keV (E (GeV), R(m))
  - LEP was 27.5 km → max 100 GeV beams
- For **Leptons** in future
  - Linear accelerators → beams up to ~ 3 TeV (CLIC)
  - Muon accelerators? Energy loss per turn $(m_e/m_\mu)^4$.

Anti-particle (positron) Production

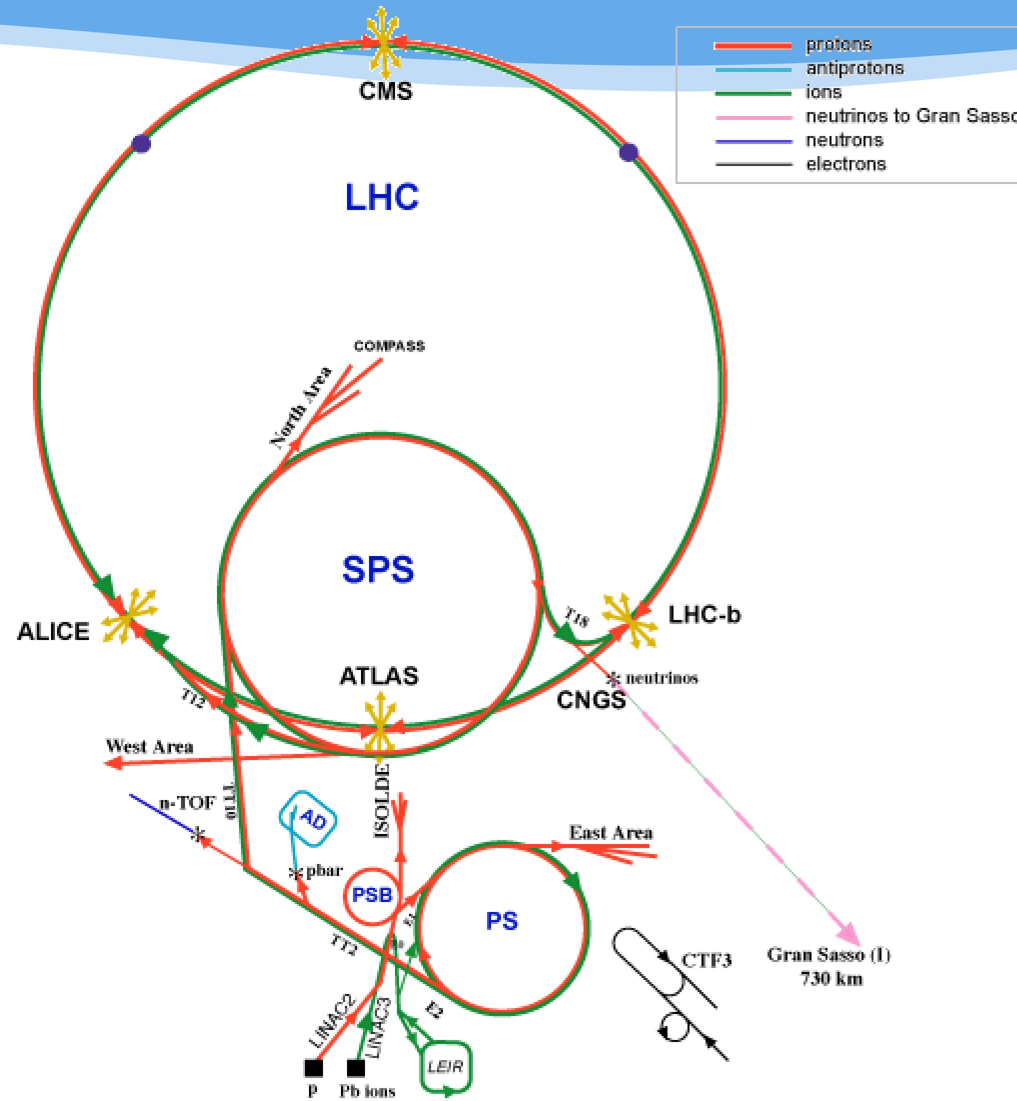


Using magnets, electrons (-e) and positrons (+e) are deflected in different directions. Separating positrons with this method they can be focused and accelerated.

Accelerators at CERN

Accelerators and detectors requires advanced and edge technologies. For this purpose, CERN is in close relationship with the industry. Related technological spin-offs enter quickly to daily use.

Medical and industrial imaging, radiation applications, electronics, measurement tools, new production and materials, w w w etc. are developed in the basic particle physics research at CERN.



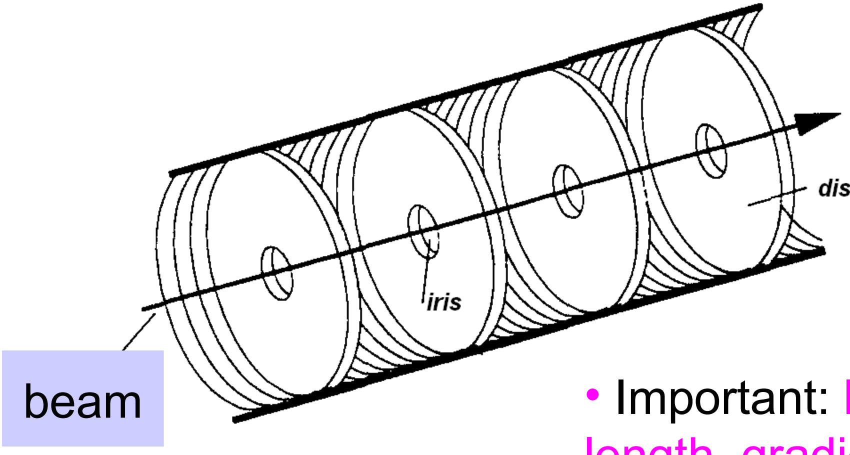
Linear Accelerator

• Charged particles are accelerated through a straight line in a linear accelerator by static and RF fields. The magnetic flux changing in time induces an electric field around it, as known from the Maxwell equations. Electric force acting on the charged particle is given by

$$F = eE = \frac{dmc\,\beta\gamma}{dt}$$

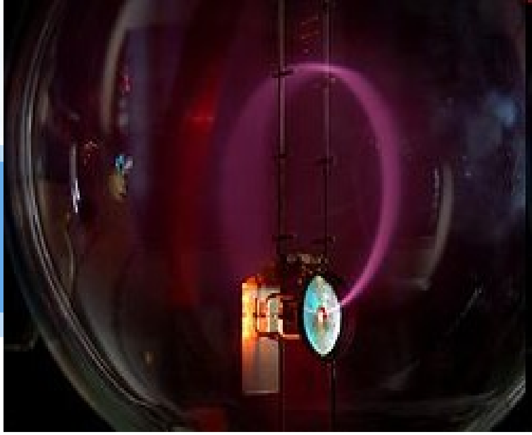
$$\Delta p = mc(\gamma\beta - \gamma_0\beta_0) = e\int E dt$$

where the energy $\Delta E_{kin} = e\int \vec{E}\cdot d\vec{s}$



• Important: linac length, gradient

Circular Accelerator



- Kinematics of circular motion, $\left|\frac{d\vec{v}}{dt}\right| = \frac{v^2}{\rho}$
- Relativistic dynamics, $\vec{p} = m\gamma\vec{v}$

$$\vec{F} = \frac{d\vec{p}}{dt} = m\frac{d}{dt}(\gamma\vec{v}) \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

  - Magnetic part of Lorentz force $\quad \vec{F}_B = q\vec{v}\times\vec{B} \qquad F = evB$

  - Assume " $a_L < a_T$ "

$$\left(\frac{d}{dt}\gamma\right)v << \gamma\frac{d}{dt}v$$

$$m\frac{d}{dt}(\gamma v) \sim m\gamma\frac{d}{dt}v$$

$$F = m\gamma\left|\frac{d\vec{v}}{dt}\right| = m\gamma\frac{v^2}{\rho} \Rightarrow eB = m\gamma\frac{v}{\rho} = \frac{p}{\rho} \qquad p = eB\rho$$

Energy, Magnetic Field and Orbit

- "Momentum-magnetic field-orbit radius" relation,
- Four-momentum conservation

$$p = eB\rho$$

$$E^2 - p^2c^2 = m^2c^4, \qquad E = \sqrt{m^2c^4 + p^2c^2}$$

- Ultra-relativistic case

$$pc >> mc^2, \qquad E \sim pc \Rightarrow E = ceB\rho$$

beam energy

Important in design!

- Practical units (c=1):
  - 1 eV=1.602× 10^{-19} J
  - 1 e= 1.602× 10^{-19} C
  - B(Tesla)

$$E[GeV] = 0.3 \times B[T] \times \rho[m]$$

Example for LHC:
$2\pi\rho$=27 km,
E=7000 GeV→B≈ 8 Tesla

Transverse Motion of Beam

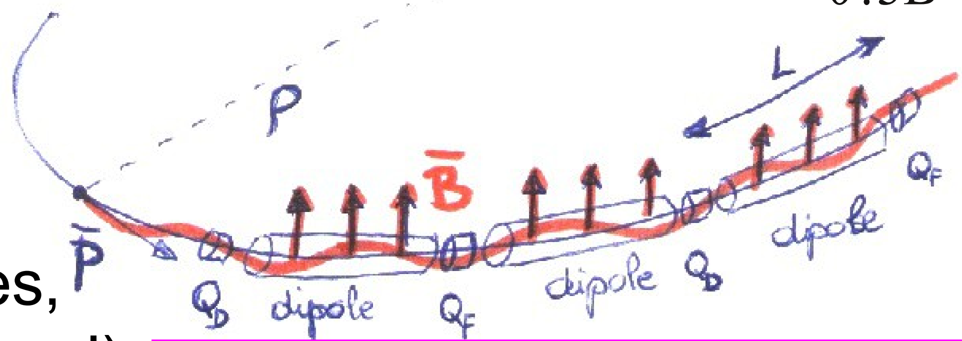
Equations of transverse motion

$$\frac{d^2x}{ds^2}+K_1(s)x=0$$

$$K_1=\frac{1}{B\rho}\frac{\partial B_y}{\partial x}=\frac{G}{B\rho}$$

$$\frac{d^2y}{ds^2}-K_1(s)y=0$$

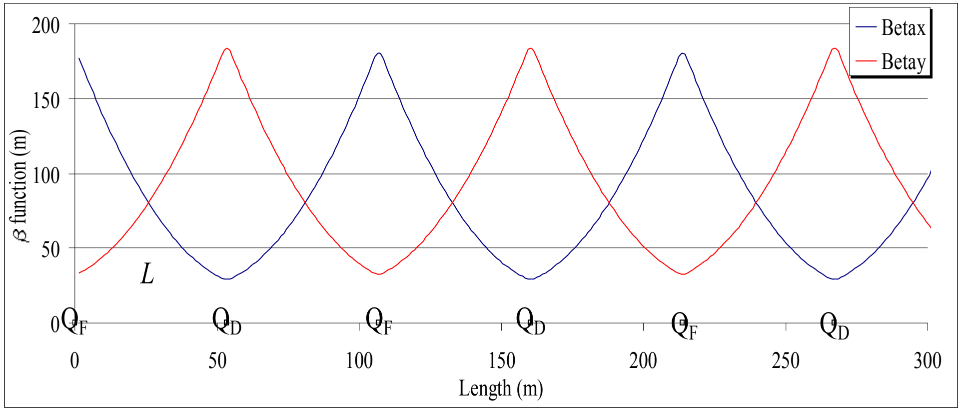
$$L_d=2\pi\rho=2\pi\frac{E}{0.3B}$$



here *K*-term is zero in dipoles, and $K_1=|G|/B\rho$ in FO(quadrupol) structure, $K_1=-|G|/B\rho$ in DO structure.

Example for LHC:
L=50 m, β_F=170 m, β_D =30 m

- Beam optics quantity is determined from the magnet configuration of the accelerator. It oscillates through the ring, reaching minimum and maximum at quadrupoles.



Accelerator Design Steps

- Define basic parameters
  - Accelerator design figure and optics
  - Analysis of local and general properties, performance measurements
  - Beam stability
  - Geometry and setup

Computer Application Needed

- **Design of accelerator elements**
  - Source, Magnets, RF cavities, etc.
  - Vacuum components, cryogenics, etc.
  - Optics calculations, parameter tune, etc.

- **Beam dynamics studies**
  - Simulation and design of accelerators
  - Beam tracking
  - Control and run

Interaction of Beams → Simulation

Beams interact machine elements

- Interact with wakefields
- Impedances
- Electron cloud
- Intersecting elements

Beams interact with other beams

- **Other beam behaves as a nonlinear lens**
- **Incoherent beam-beam effects**
- **Coherent beam-beam effects (particle group effects)**

Beams interact themselves

- Particles in one beam interact with the other particles in the same beam.
- Space charge effects
- Intra-beam scattering
- Multi-bunch effects

Accelerator Simulation

- Every element ***M*** of an accelerator has an effect on the beam, these can be the magnet, RF cavity, vacuum chamber etc.
- These elements can be defined suitably for computational purposes such that
  - $\mathbf{z_1,z_2}$: *denotes the quantities (coordiantes, beam sizes, etc) before and after the element*
  - *an element for **M** is an mathematical object*
    - ***M* defines an element**
    - *General :* $z_2 = \mathbf{M} \circ z_1$
- Sequential element sets link the simulated parts, and build the beamlines.

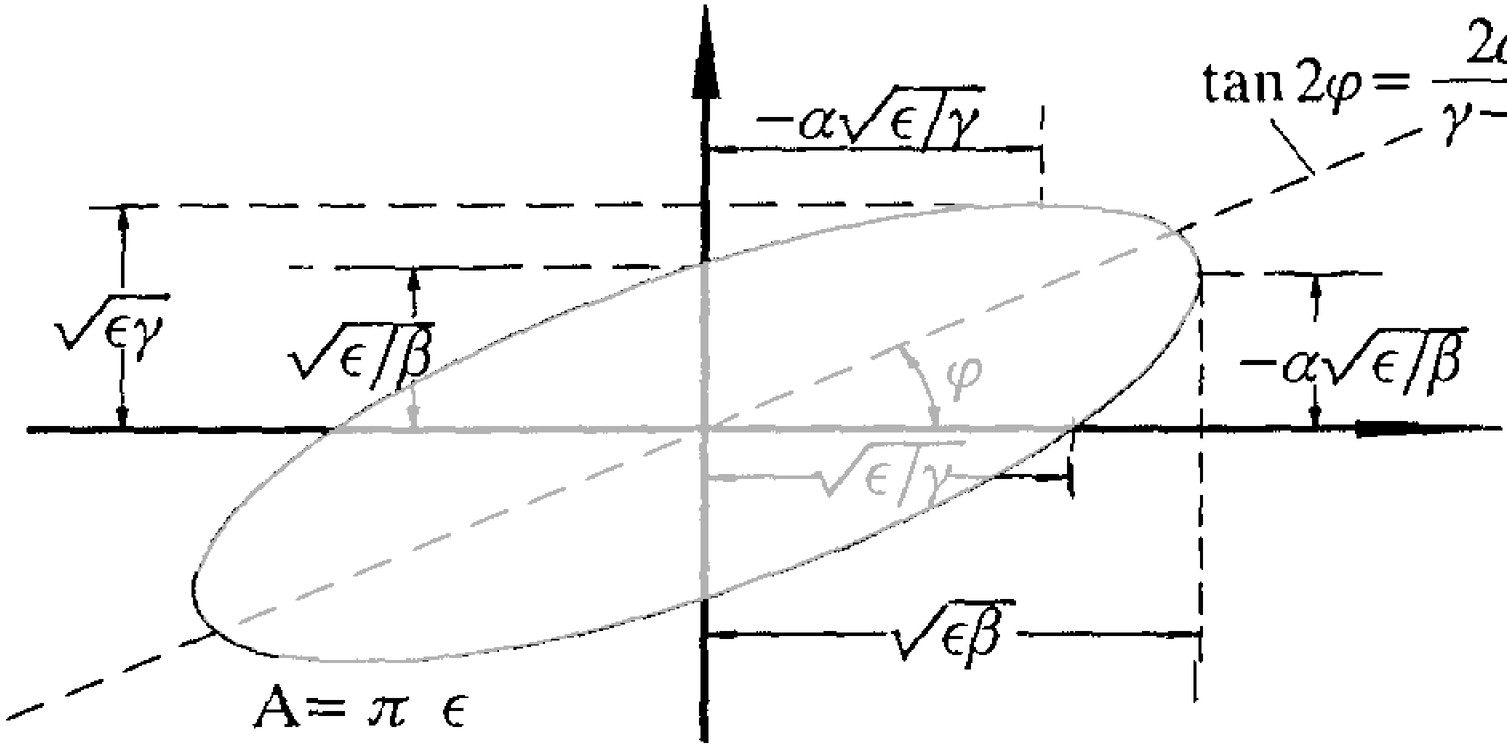
In general, $z_2 \neq z_1$

$$M = M_1 \circ M_2 \circ M_3 \circ \ldots \circ M_n$$

M defines properties of machine element

- Linear matrix or transformation
- High level integral algorthm
- Program, altprogram, etc.
- Any definition for going from z_1 to z_2 .

Particles in Phase Space



$$\text{beam emittance } (\varepsilon): \int_{ellips} dx\,dx' = \pi\varepsilon$$

Accelerator Design Tools (a partial list)

- **EGUN** (program for design of electron source)
- **PARMELA** (beam dynamics simulation program. This program reads field distributions obtained from **FISH** for RF problems and **POISSON** for magnet problems).
- **MAD-X** (FODO structures at circular accelerators, injector design, used for determination of betatron function and stable run region).
- **PLACET** (beam dynamics simulation, studying wakefields in linac, single and multi-bunch effects, beam output for Guinea-Pig input format)
- ...

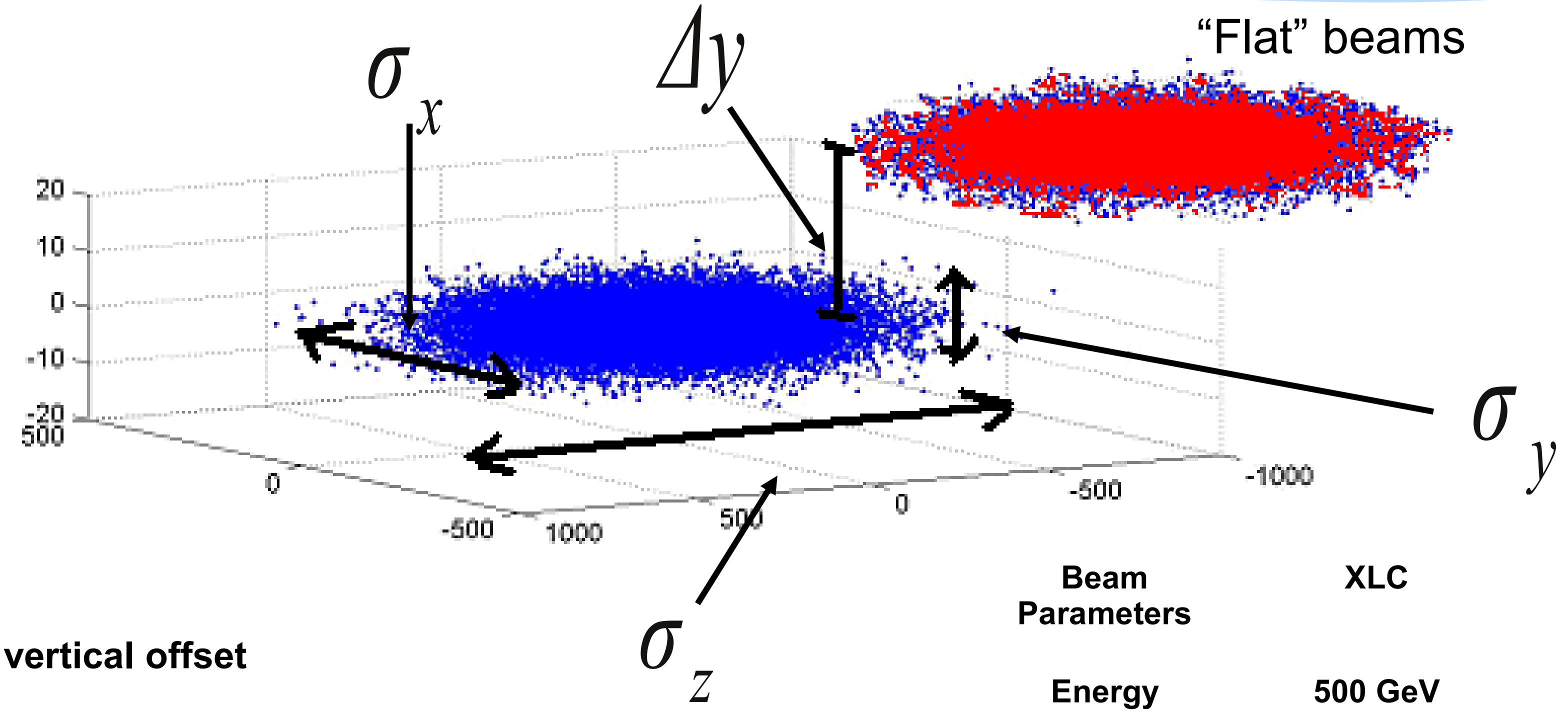
Explanation for Some Accelerator Terms

- **Bunch** (*paketçik*): a group of particles entering some region in the phase space
- **RF** (*radyo frekans*): frequency of accelerating electric field in the RF region
- **Transverse Emittance** (*enine yayınım*): the area filled by particle beam in phase space (mm-mrad)
- **Injection** (*enjeksiyon*): setting particle beam into an accelerator
- **Linac** (*linak*): linear accelerator
- **Beamline** (*demet hattı*): a vacuum line and magnet system that takes particle beam from a part of accelerator to another part
- **Cavity** (*kovuk*): a volume covered by conducting surface, an accelerator unit resonating at a frequency equal to beam turn frequency.

Collision

- The collision point is inside the detector. The *beam sizes* σ_x and σ_y at collision point can be expressed in terms of *transverse emittance* (ε) and *amplitude function* (β), such that $\sigma \sim (\varepsilon\beta)^{1/2}$.
  - ε (**emittance**) beam quality quantity, reflects the bunch preperation process, mostly depend on syncrothron radiation. Normalized emittance is defined as ε / γ.
  - β (**beta function**) beam optics quantity, determined by accelerator magnet configuration.
  - To reach a high luminosity, the requirement: large number of particles in a bunch, low emittance and high collision frequency, their optimal values are determined from the simulation

Beam Sizes



Δy - **vertical offset**

σ_x - **beam profile x (V)**

σ_y - **beam profile y (H)**

σ_z - **bunch length**

Beam Parameters	XLC
Energy	500 GeV
Number of particles	7.5×10^9
σ_x	243 nm
σ_y	3 nm
σ_z	110 µm

Collider Parameters

In the collisions of two particles of masses m_1 and m_2 and four-momenta p_1 and p_2, the total energy squared in the center of momentum (CM) frame is given by a Lorentz invariant Mandelstam variable

$$s \equiv (p_1 + p_2)^2 = \begin{cases} (E_1 + E_2)^2 & \text{in the c.m. frame } \vec{p}_1 + \vec{p}_2 = 0 \\ m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2). & \end{cases}$$

The total center of mass energy of two-particle system is given by

$$E_{CM} \equiv \sqrt{s} \approx \begin{cases} 2E_1 \approx 2E_2 & \text{in the c.m. frame } \vec{p}_1 + \vec{p}_2 = 0, \\ \sqrt{2E_1 m_2} & \text{in the fixed target frame } \vec{\mathrm{p}}_2 = 0 \end{cases}$$

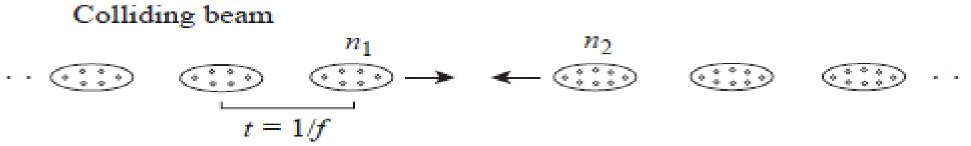
The kinetic energy of the system is $T \sim E_1$ in the fixed target frame $\mathbf{p}_2 = \mathbf{0}$ and $T=0$ in the center of momentum frame $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{0}$. We see that only in the COM frame there will be no kinetic motion of the system, and the beam energies are maximally converted to reach higher threshold.

Collider Parameters - 2

In e^+e^- collision process, **center of mass energy** may be fully converted to reraching the physics threshold.

However, in hadron collisions only a fraction of the total energy center of mass energy is used by the partons. Example: Tevatron ($\sqrt{s} \approx 2$ TeV) can reach a few hundred GeV at parton level; while the LHC will enhance this to multi-TeV.

Another important parameter for a collider is the instantaneous **luminosity**, it is the number of particles interacting at unit transverse area and time. In fact, particle beams usually come in bunches. There are n_1 particles in each bunch in beam 1, and n_2 particles in aech bunch in beam 2, then collider luminosity $L \sim f n_1 n_2 / A$, where f is beam collision frequency and A is the transverse profile of the beam ($4\pi\sigma_x\sigma_y$).



$$L = f \frac{n_1 n_2}{4\pi\sigma_x\sigma_y}$$

Collider Parameters - 3

Reaction rate (*R*) – the number of scattering events per unit time, which is directly proportional to the luminosity, cross section and efficiency for detection: $R=\sigma.L.\varepsilon$. The unit of the cross section taken as fb ($1fb=10^{-39}$ cm^2), and the integrated luminosity of fb^{-1}/yıl ($1fb^{-1}$/yıl$=10^{32}$ $cm^{-2}s^{-1}$).

The instantaneous luminosity has some spread around the peak energy ($\sqrt{s}$), written as ***dL/dτ*** with $\tau=\hat{s}/s$. The rate *R(s)*

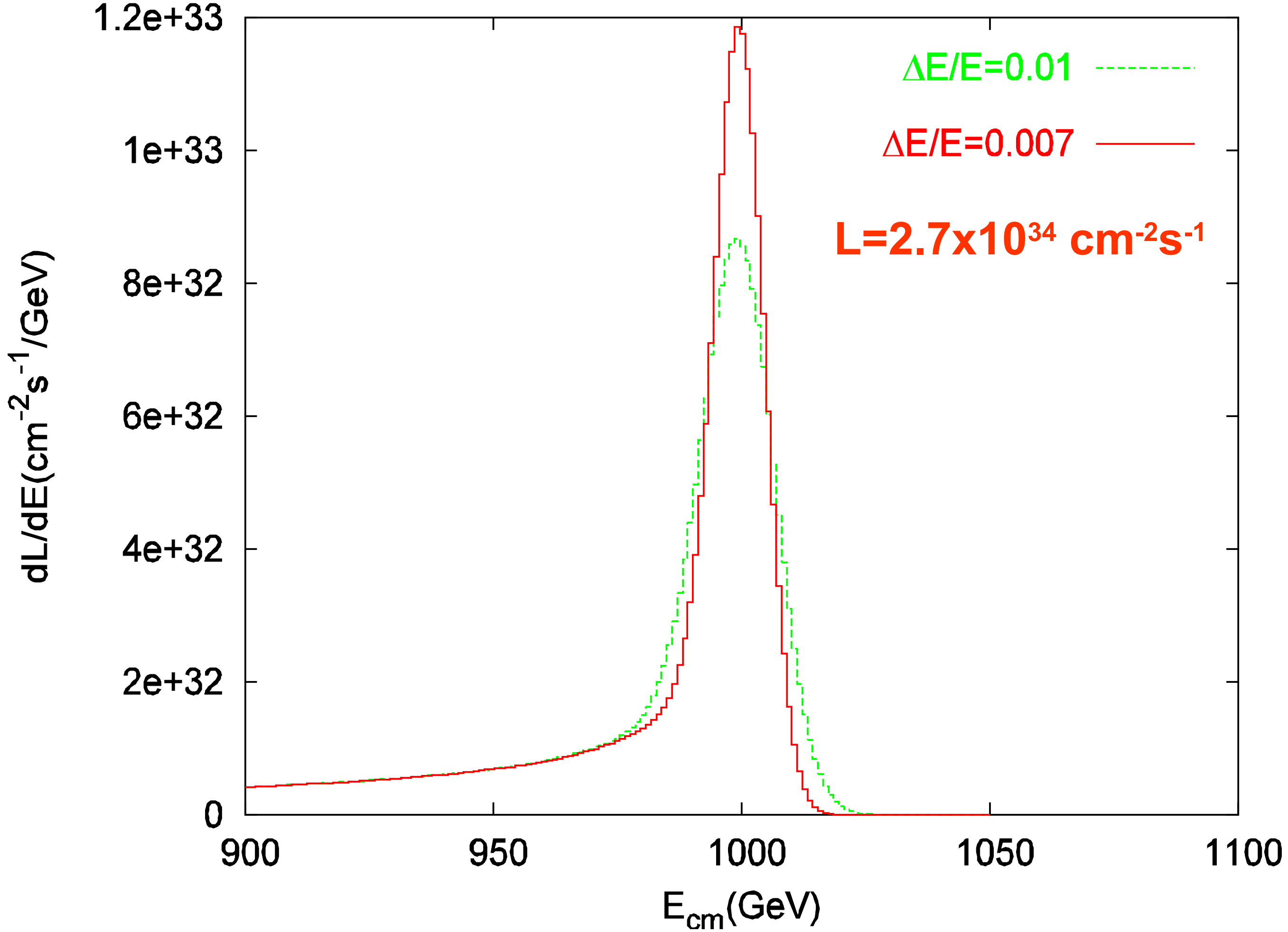
$$R(s)=L_0\int d\tau \frac{dL}{d\tau}\sigma(\tau s)$$

where L_0 is the peak instantaneous luminosity. Normalization $\int (dL/d\tau)d\tau=1$ and the energy spectrum can be parametrized by a Gaussian distribution with energy spread δE ($\sim\sqrt{s}-\sqrt{\hat{s}}$).

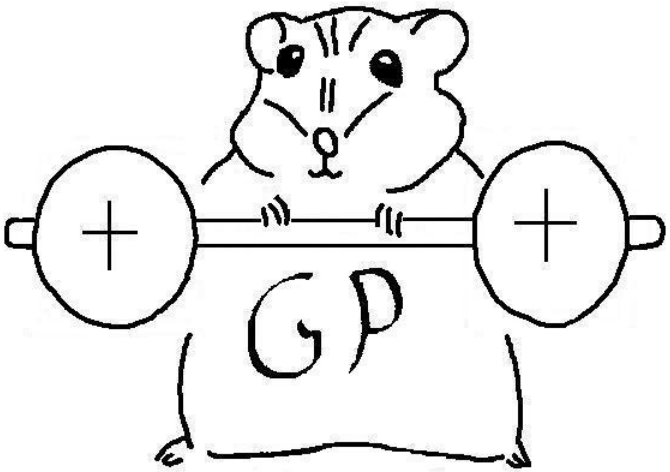
Collider Parameters - 4

Collider type	$\sqrt{s}$ (TeV)	L ($cm^{-2}s^{-1}$)	$\delta E/E$ (%)	f (MHz)	N (10^{10})	2π R or L (km)
Tevatron ($p\bar{p}$)	1.96	2.1×10^{32}	0.009	2.5	p:27, $\bar{p}$:7.5	6.28
HERA (ep)	0.314	1.4×10^{31}	0.1, 0.02	10	e:3, p:7	6.34
LHC (pp)	14	10^{34}	0.01	40	10.5	27
ILC (e^+e^-)	0.5-1	2.5×10^{34}	0.1	3	2 Pol: 60%,80%	14-33
CLIC (e^+e^-)	0.5-5	10^{35}	0.35	1500	0.4 Pol: 60%,80%	33-53

Luminosity Spectrum



Collision Point Simulations (partial list)



- **GUINEA-PIG** (luminosity/energy distributions e+e- and options, including beamstrahlung)
  - Beam parameters can be given directly to get luminosity
  - Collision can be performed with input beam file
- **CAIN** (e+e- and e-e-, eγ, $\gamma\gamma$ options, luminosity spectrum)
  - Luminosity spectrum can be obtained by giving the beam parameters
- **CIRCE** (beam radiation spectrum)
  - It can be used by calling from other event generators
- ...

What we have learned ?

- Types of accelerators, motion of a particle in the electric and magnetic field, beam properties
- Bunches, emittance, beta amplitude function
- Interaction point, center of mass energy, event rate, luminosity
- Initial state radiation (ISR), beamstrahlung (BS), beam energy spread
- Spectrum calculators

Homework

1. An electron - positron collider has the beam arameters as given on the right.

   a) Calculate the center of mass energy of the collider

   b) What is the boost parameter for a resonance production.

   c) Calculate the geometric instantenous luminosity as well as integrated luminosity for one year.

   d) What will be the normalized emittances of beams at the interaction point.

$$E_{e^-} = 8\,\mathrm{GeV},\quad E_{e^+} = 3.5\,\mathrm{GeV}$$

$$\sigma_x / \sigma_y = 110/1.9\ \mu\mathrm{m},\quad \sigma_z = 0.65\ \mathrm{cm}$$

$$e^- : \beta_x / \beta_y (\mathrm{IP}) = 56/0.59\ \mathrm{cm}$$

$$e^+ : \beta_x / \beta_y (\mathrm{IP}) = 59/0.65\ \mathrm{cm}$$

$$N = 3.3 \times 10^{10},\quad T = 5.9\,ns$$