Introduction to Quantum Chromodynamics (QCD)

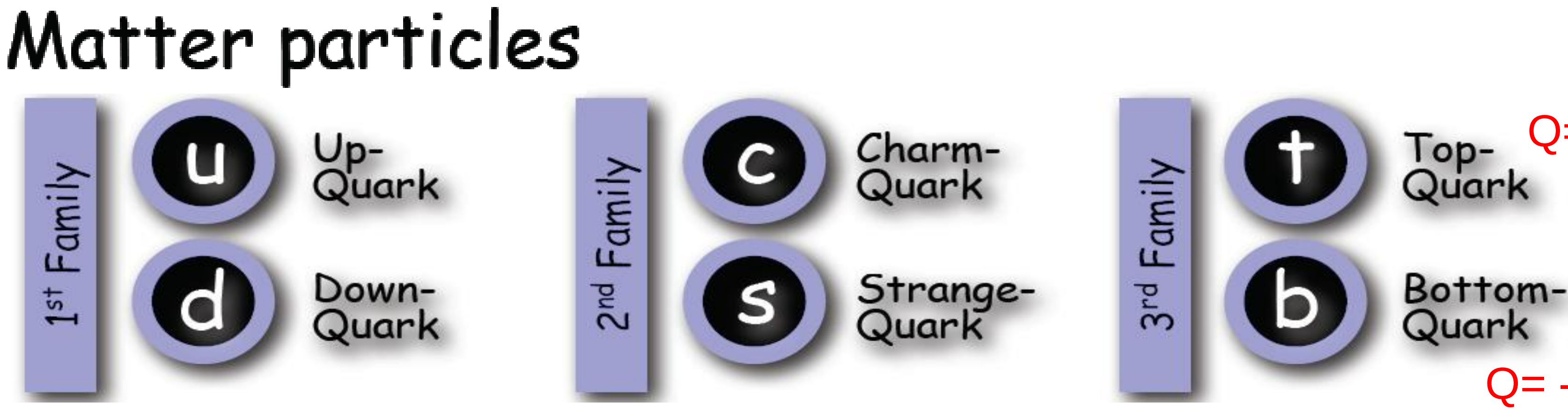
Textbooks: 1) Introduction to Elementary Particles, David Griffiths
2) Quantum Chromodynamics, Walter Greiner, Stefan Schramm, Eckart Stein
3) Quantum Chromodynamics, Andrei Smilga
4) An Introduction to Quantum Field Theory, M. Peskin and D. Schroeder, Addison Wesley (1995)

and some other sources..............

Kazem Azizi/ Doğuş University

Introduction

❖ **QCD** is a theory of the strong interactions, a fundamental force describing interactions of the color carrying quarks and gluons.



Q=2/3 e

Q= -1/3 e

u

$$I(J^P) = \tfrac{1}{2}(\tfrac{1}{2}^+)$$

$m_u = 1.7\text{–}3.3$ MeV Charge $= \frac{2}{3}\, e$ $I_z = +\frac{1}{2}$

d

$$I(J^P) = \tfrac{1}{2}(\tfrac{1}{2}^+)$$

$m_d = 4.1\text{–}5.8$ MeV Charge $= -\frac{1}{3}\, e$ $I_z = -\frac{1}{2}$

s

$I(J^P) = 0(\frac{1}{2}^+)$

$m_s = 101^{+29}_{-21}$ MeV   Charge $= -\frac{1}{3}\ e$ Strangeness $= -1$

c

$I(J^P) = 0(\frac{1}{2}^+)$

$m_c = 1.27^{+0.07}_{-0.09}$ GeV   Charge $= \frac{2}{3}\ e$ Charm $= +1$

b

$I(J^P) = 0(\frac{1}{2}^+)$

Charge $= -\frac{1}{3}\ e$ Bottom $= -1$

$m_b(\overline{\text{MS}}) = 4.19^{+0.18}_{-0.06}$ GeV

t

$I(J^P) = 0(\frac{1}{2}^+)$

$m = 172.0 \pm 0.9 \pm 1.3$ GeV   Charge $= \frac{2}{3}\ e$ Top $= +1$

$\overline{\text{MS}}$ → mass-independent subtraction scheme   at a scale $\mu \approx 2$ GeV

Force carriers

The gluons are massless vector bosons; like the photon, they have spin of 1. However against the photons glouns interact with each other due to color.

The strength of the electromagnetic force is set by the coupling constant $g_e = \sqrt{4\pi\alpha}$ $\quad g_e \rightarrow$ fundamental charge

charge of the positron

"strong" coupling constant

$$g_s = \sqrt{4\pi\alpha_s} \qquad g_s \rightarrow \text{fundamental unit of color}$$

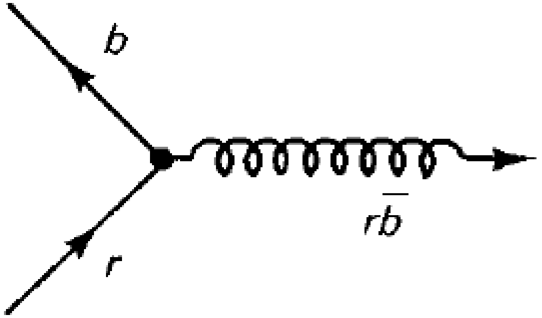
Quarks come in three different colors

$$c = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ for red, } \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ for blue, } \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ for green}$$

Hence a quark state in QCD requires not only the Dirac spinor $u^{(s)}(p)$ describing its momentum and spin but also a color factor c

$$u^{(s)}(p)\, c$$

The quark color changes at a quark gluon vertex and the difference carried off by the gluon



Each gluon carries one unit of color and one of anticolor

It would appear, then, that there should be nine species of gluons—$r\bar{r}$, $r\bar{b}$, $r\bar{g}$, $b\bar{r}$, $b\bar{b}$, $b\bar{g}$, $g\bar{r}$, $g\bar{b}$, $g\bar{g}$. Such a nine-gluon theory is perfectly possible in *principle*, but it would describe a world very different from our own. In terms of color $SU(3)$ symmetry these nine states constitute a $3 \otimes 3 = 8 \oplus 1$

"color octet" and a "color singlet"

"color singlet" → $(r\bar{r} + b\bar{b} + g\bar{g})/\sqrt{3}$

such a state is unaffected by the transformations of color $SU(3)$

It does not exist in the nature. If it exists as a mediator it should appear as free partice since confinement requires that all naturally occuring particles be color singlets.

HW1:

Color $SU(3)$ transformations relabel "red," "blue," and "green" according to the transformation rule

$$c \rightarrow c' = Uc$$

where U is any unitary ($UU^\dagger = 1$) 3×3 matrix of determinant 1, and c is a three-element column vector. For example

$$U = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

would take $r \rightarrow b$, $b \rightarrow g$, $g \rightarrow r$. Show that the "color singlet" is obviously invariant under U.

Color octet

There are many ways of presenting these independent states, which are known as the "color octet". One commonly used list is:

$$|1\rangle = (r\bar{b} + b\bar{r})/\sqrt{2} \qquad |5\rangle = -i(r\bar{g} - g\bar{r})/\sqrt{2}$$

$$|2\rangle = -i(r\bar{b} - b\bar{r})/\sqrt{2} \qquad |6\rangle = (b\bar{g} + g\bar{b})/\sqrt{2}$$

$$|3\rangle = (r\bar{r} - b\bar{b})/\sqrt{2} \qquad |7\rangle = -i(b\bar{g} - g\bar{b})/\sqrt{2}$$

$$|4\rangle = (r\bar{g} + g\bar{r})/\sqrt{2} \qquad |8\rangle = (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$$

These are equivalent to the Gell-Mann matrices.

These matrices are one possible representation of the infinitesimal generators of the special unitary group called SU(3), which is the non abelian gauge group of QCD.

Gell-Mann matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

HW2: Obtain the above matrices using the eight gluon color states from the previouse slide.

HW3: Show that the octet gluons are not invariant under U discussed in HW1.

HW4: Show that: $Tr(\lambda^\alpha \lambda^\beta) = 2\delta^{\alpha\beta}$

$$[\lambda^\alpha, \lambda^\beta] = 2if^{\alpha\beta\gamma}\lambda^\gamma$$

γ—from 1 to 8

structure constants

completely antisymmetric, $f^{\beta\alpha\gamma} = f^{\alpha\gamma\beta} = -f^{\alpha\beta\gamma}$

each index runs from 1 to 8 → $8 \times 8 \times 8 = 512$ structure constants

HW5: Show that the nonzero structure constants are:

$$f^{123} = 1, \qquad f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \tfrac{1}{2},$$
$$f^{458} = f^{678} = \sqrt{3}/2$$

And those obtained by antisymmetry from the above set.

Properties of QCD

❖ **Asymptotic freedom:** in very high energies, quarks and gluons interact very weakly (discovered by David Politzer, Frank Wilczek and David Gross: 2004 Nobel Prize). (we will come back later)

❖ **Confinement:** the force between quarks does not diminish as they are separated. It would take an infinite amount of energy to separate two quarks, so they are forever bound into hadrons. Although analytically unproven, confinement is widely believed to be true because it explains the consistent failure of free quark searches, and it is easy to demonstrate in lattice QCD. All experimental searches for free quarks since 1977 have had negative results. (we will come back later)

QCD Lagrangian

The dynamics of the quarks and gluons are controlled by the quantum chromodynamics Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_q \bar{\psi}_q (i \not{D} - m_q) \psi_q,$$

$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f^{abc} G^b_\mu G^c_\nu$, gluon field-strength tensor

$G^a_\mu(x)$ ⇒ The gluon field

ψ_q ⇒ quark fields

$$\not{D} = \gamma^\mu \mathcal{D}_\mu$$

$$\mathcal{D}_\mu = \partial_\mu + i \frac{g}{2} \lambda^a G^a_\mu$$

HW6: Using QCD lagrangian find equations of motions describing dynamics of quarks.

❖ In principle, this Lagrangian is also responsible for all properties of the hadrons* and hadronic processes. However, a direct use of it is possible only within the limited framework of the perturbation theory.

❖ At high energies or small distances, due to the asymptotic freedom, the quarks move almost freely. Hence, we can use perturbation theory in this regime. Although limited in scope, the perturbation theory has resulted in the most precise tests of QCD to date.

❖ In low energies and hadronic scales, it is difficult to get reliable theoretical results using the perturbation theory since the effective strong coupling constant between quarks and gluons becomes large in such scales and perturbation theory fails. Therefore, in such scales, we need some non-perturbative approaches to describe the non-perturbative phenomena. (we will introduce some nonperturbative approaches later)

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* Hadrons are objects formed from quarks in low energies. The famous hadrons are mesons (containing one quark and one antiquark) and baryons (containing 3 quarks). We will briefly describe the hadrons in our last lecture.

Feynman rules for evaluating the tree-level diagrams in QCD

External Lines

Quark

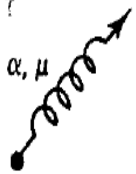
- incoming (): $u^{(s)}(p)c$
- outgoing (): $\bar{u}^{(s)}(p)c^{\dagger}$

Antiquark

- incoming (): $\bar{v}^{(s)}(p)c^{\dagger}$
- outgoing (): $v^{(s)}(p)c$

Gluon

- incoming (): $\epsilon_{\mu}(p)a^{\alpha}$

- outgoing (): $\epsilon^{*}_{\mu}(p)a^{\alpha *}$

c → color

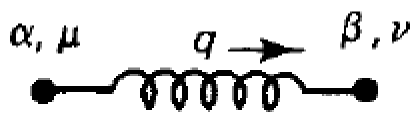
ϵ_{μ} → polarization vector

a^{α} → gluon color

Propagators

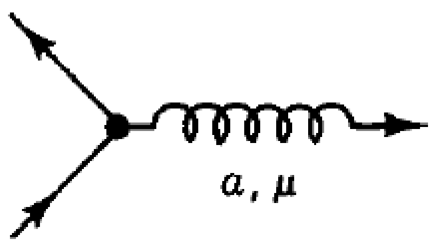
Quark-antiquark (

): $\frac{i(\not{q} + mc)}{q^2 - m^2c^2}$

Gluon (

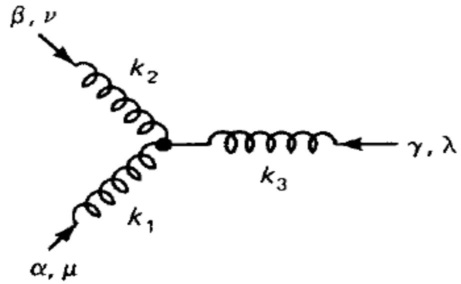
): $\frac{-ig_{\mu\nu}\,\delta^{\alpha\beta}}{q^2}$

Vertices

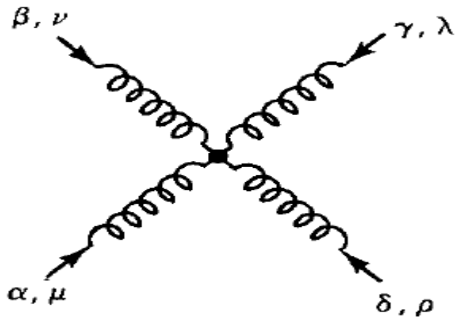
Quark-gluon (

): $\frac{-ig_s}{2}\lambda^{\alpha}\gamma^{\mu}$

Three and four gluon vertices

Three gluon ( **):**

$$-g_s f^{\alpha\beta\gamma}[g_{\mu\nu}(k_1 - k_2)_\lambda + g_{\nu\lambda}(k_2 - k_3)_\mu + g_{\lambda\mu}(k_3 - k_1)_\nu]$$

Four gluon ( **):**

$$-ig_s^2[f^{\alpha\beta\eta}f^{\gamma\delta\eta}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) + f^{\alpha\delta\eta}f^{\beta\gamma\eta}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho}) + f^{\alpha\gamma\eta}f^{\delta\beta\eta}(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\lambda\rho})]$$

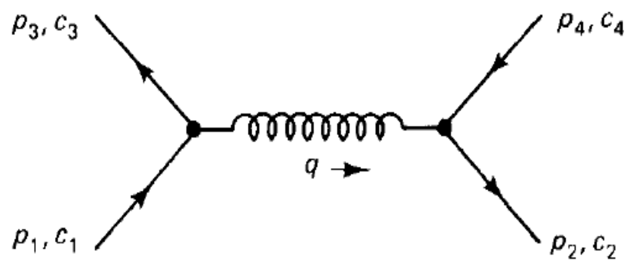
summation over η implied

HW7: Using the techniques you learned in QFT courses obtain the above expressions for propagators and vertices (Do this HW after coming back to your institutes).

Quark-antiquark interaction, Comparison with QED

we cannot observe quark-quark or quark-antiquark scattering directly in the laboratory, but we observe hadron-hadron scattering as an indirect manifestation

The quark-antiquark interaction ➡



$$-i\mathcal{M} = [\bar{u}(3)c_3^\dagger]\left[-i\frac{g_s}{2}\lambda^\alpha\gamma^\mu\right][u(1)c_1]\left[\frac{-ig_{\mu\nu}\,\delta^{\alpha\beta}}{q^2}\right][\bar{v}(2)c_2^\dagger]\left[-i\frac{g_s}{2}\lambda^\beta\gamma^\nu\right][v(4)c_4]$$

$$\mathcal{M} = \frac{-g_s^2}{4}\frac{1}{q^2}[\bar{u}(3)\gamma^\mu u(1)][\bar{v}(2)\gamma_\mu v(4)](c_3^\dagger\lambda^\alpha c_1)(c_2^\dagger\lambda^\alpha c_4)$$

This is the same as electron-positron in QED with g_e ➡ g_s

we have in addition the "color factor"

$$f = \tfrac{1}{4}(c_3^\dagger\lambda^\alpha c_1)(c_2^\dagger\lambda^\alpha c_4)$$

Calculation of color factor in quark-antiquark interaction

In this interaction also the possible color configurations are $3 \otimes \bar{3} = 1 \oplus 8$ (a color octet and a color singlet) .

A typical octet state is $r\bar{b}$ (all members yield the same f).
Here the incoming quark is red, and the incoming antiquark is antiblue. Because color is conserved, the outgoing quark must also be red and the antiquark antiblue. Thus

$$c_1 = c_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad c_2 = c_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

and hence $f = \frac{1}{4}\left[(1\ 0\ 0)\lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]\left[(0\ 1\ 0)\lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right] = \frac{1}{4}\lambda^\alpha_{11}\lambda^\alpha_{22}$

A glance at the λ matrices reveals that the only ones with entries in the 11 and 22 positions are λ^3 and λ^8. So

$$f = \tfrac{1}{4}(\lambda^3_{11}\lambda^3_{22} + \lambda^8_{11}\lambda^8_{22}) = \tfrac{1}{4}[(1)(-1) + (1/\sqrt{3})(1/\sqrt{3})] = -\tfrac{1}{6}$$

Color Factor for Singlet Configuration

singlet state → $(1/\sqrt{3})(r\bar{r} + b\bar{b} + g\bar{g})$

If the incoming quarks are in the singlet state (as they would be for a meson, say) the color factor is a sum of three terms:

$$f = \frac{1}{4}\cdot\frac{1}{\sqrt{3}}\left\{\left[c_3^\dagger\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right][(1\ 0\ 0)\lambda^\alpha c_4] + \left[c_3^\dagger\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right][(0\ 1\ 0)\lambda^\alpha c_4]\right.$$
$$\left. + \left[c_3^\dagger\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right][(001)\lambda^\alpha c_4]\right\}$$

The outgoing quarks are necessarily also in the singlet state, and we get *nine* terms in all, which can be written compactly as follows:

HW8

$$f = \frac{1}{4} \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} (\lambda_{ij}^{\alpha} \lambda_{ji}^{\alpha}) = \frac{1}{12} Tr(\lambda^{\alpha} \lambda^{\alpha})$$

summation over i and j, from 1 to 3, implied

$$Tr(\lambda^{\alpha} \lambda^{\beta}) = 2\delta^{\alpha\beta} \qquad Tr(\lambda^{\alpha} \lambda^{\alpha}) = 16 \qquad f = \frac{4}{3}$$

The *potential* describing the $q\bar{q}$ interaction

$$V_{q\bar{q}}(r) = -f \frac{(\alpha_s \hbar c)}{r}$$

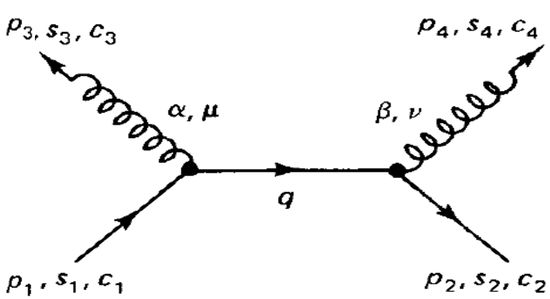
the same as Coulomb potential only with α replaced by $f\alpha_s$

$$V_{q\bar{q}}(r) = -\frac{4}{3}\frac{(\alpha_s \hbar c)}{r} \quad \text{(color singlet)} \Rightarrow \textit{attractive}$$

$$V_{q\bar{q}}(r) = \frac{1}{6}\frac{(\alpha_s \hbar c)}{r} \quad \text{(color octet)} \Rightarrow \textit{repulsive}$$

This helps to explain why quark-antiquark binding (to form mesons) occurs in the singlet configuration but not in the octet (which would have produced colored mesons).

PAIR ANNIHILATION IN QCD



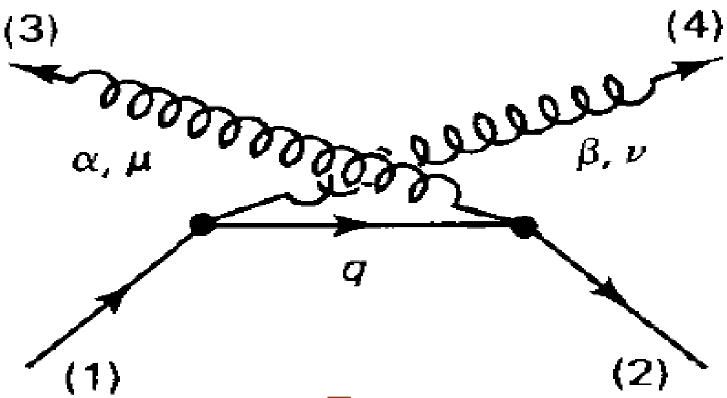
$$-i\mathcal{M}_1 = \bar{v}(2)c_2^\dagger\left[-i\frac{g_s}{2}\lambda^\beta\gamma^\nu\right][\epsilon^*_{4\nu}a_4^{\beta *}]\left[\frac{i(\not{q}+mc)}{q^2-m^2c^2}\right] \times\left[-i\frac{g_s}{2}\lambda^\alpha\gamma^\mu\right][\epsilon^*_{3\mu}a_3^{\alpha *}]u(1)c_1$$

diagram 1

$q = p_1 - p_3 \Rightarrow q^2 - m^2c^2 = p_1^2 - 2p_1 \cdot p_3 + p_3^2 - m^2c^2 = -2p_1 \cdot p_3$

$$\mathcal{M}_1 = \frac{-g_s^2}{8} \frac{1}{p_1 \cdot p_3} \bar{v}(2)[\not{\epsilon}_4(\not{p}_1 - \not{p}_3 + mc)\not{\epsilon}_3]u(1) \times a_3^\alpha a_4^\beta (c_2^\dagger \lambda^\beta \lambda^\alpha c_1)$$

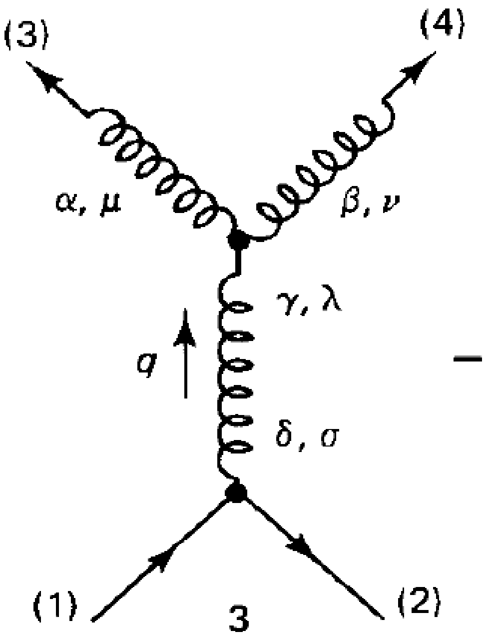
diagram 2



Similarly

$$\mathcal{M}_2 = \frac{-g_s^2}{8} \frac{1}{p_1 \cdot p_4} \bar{v}(2)[\not{\epsilon}_3(\not{p}_1 - \not{p}_4 + mc)\not{\epsilon}_4]u(1)\ a_3^\alpha a_4^\beta (c_2^\dagger \lambda^\alpha \lambda^\beta c_1)$$

diagram 3



$$-i\mathcal{M}_3 = \bar{v}(2)c_2^\dagger\left[-i\frac{g_s}{2}\lambda^\delta\gamma_\sigma\right]u(1)c_1\left[-i\frac{g^{\sigma\lambda}\,\delta^{\delta\gamma}}{q^2}\right]\cdot\{-g_s f^{\alpha\beta\gamma}[g_{\mu\nu}(-p_3+p_4)_\lambda$$

$$+\, g_{\nu\lambda}(-p_4-q)_\mu + g_{\lambda\mu}(q+p_3)_\nu]\}[\epsilon_3^\mu a_3^\alpha][\epsilon_4^\nu a_4^\beta]$$

$$q = p_3 + p_4$$

$$q^2 = 2p_3\cdot p_4$$

$$\epsilon_3\cdot p_3 = 0$$

$$\epsilon_4\cdot p_4 = 0$$

$$\mathcal{M}_3 = i\frac{g_s^2}{4}\frac{1}{p_3\cdot p_4}\bar{v}(2)[(\epsilon_3\cdot\epsilon_4)(\not{p}_4-\not{p}_3)+2(p_3\cdot\epsilon_4)\epsilon_3-2(p_4\cdot\epsilon_3)\epsilon_4]u(1)$$

$$\times f^{\alpha\beta\gamma}a_3^\alpha a_4^\beta(c_2^\dagger\lambda^\gamma c_1)$$

assume that the initial particles are at rest:

$$p_1 = p_2 = (mc, \mathbf{0}), \qquad p_3 = (mc, \mathbf{p}), \qquad p_4 = (mc, -\mathbf{p})$$

Then

$$p_1 \cdot p_3 = p_1 \cdot p_4 = (mc)^2 \quad \text{and} \quad p_3 \cdot p_4 = 2(mc)^2$$

$\epsilon^\mu p_\mu = 0$ (Lorentz condition)

$\epsilon^0 = 0, \quad$ so that $\epsilon \cdot \mathbf{p} = 0$

Coulomb gauge

$$\Rightarrow \quad p_3 \cdot \epsilon_4 = -\mathbf{p} \cdot \epsilon_4 = -p_4 \cdot \epsilon_4 = 0$$

$$p_4 \cdot \epsilon_3 = 0$$

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3$$

$$\mathcal{M} = -\frac{g_s^2}{8(mc)^2} a_3^\alpha a_4^\beta \bar{v}(2) c_2^\dagger [\not\epsilon_3 \not\epsilon_4 \not p_4 \lambda^\alpha \lambda^\beta + \not\epsilon_4 \not\epsilon_3 \not p_3 \lambda^\beta \lambda^\alpha - i(\epsilon_3 \cdot \epsilon_4)(\not p_4 - \not p_3) f^{\alpha\beta\gamma} \lambda^\gamma] c_1 u(1)$$

We orient our coordinates so that the z axis lies along $\mathbf{p}$; then

HW9 (for Monday)

$$\not{p}_3 = mc(\gamma^0 - \gamma^3), \qquad \not{p}_4 = mc(\gamma^0 + \gamma^3), \qquad \not{p}_4 - \not{p}_3 = 2mc\gamma^3$$

HW10: Using the equations,

$$\not{\epsilon} = -\boldsymbol{\epsilon} \cdot \boldsymbol{\gamma} = -\begin{pmatrix} 0 & \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \\ -\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} & 0 \end{pmatrix} \quad \boldsymbol{\Sigma} \equiv \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}$$

$$\not{\epsilon}_3 \not{\epsilon}_4 = \begin{pmatrix} 0 & \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_3 \\ -\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_3 & 0 \end{pmatrix} \begin{pmatrix} 0 & \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_4 \\ -\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_4 & 0 \end{pmatrix}$$

$$= -\begin{pmatrix} (\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_3)(\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_4) & 0 \\ 0 & (\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_3)(\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_4) \end{pmatrix}$$

$$(\boldsymbol{\sigma} \cdot \mathbf{a})(\boldsymbol{\sigma} \cdot \mathbf{b}) = \mathbf{a} \cdot \mathbf{b} + i\boldsymbol{\sigma} \cdot (\mathbf{a} \times \mathbf{b})$$

Show that

$$\not{\epsilon}_3 \not{\epsilon}_4 = -(\boldsymbol{\epsilon}_3 \cdot \boldsymbol{\epsilon}_4) - i(\boldsymbol{\epsilon}_3 \times \boldsymbol{\epsilon}_4) \cdot \boldsymbol{\Sigma}, \qquad \not{\epsilon}_4 \not{\epsilon}_3 = -(\boldsymbol{\epsilon}_3 \cdot \boldsymbol{\epsilon}_4) + i(\boldsymbol{\epsilon}_3 \times \boldsymbol{\epsilon}_4) \cdot \boldsymbol{\Sigma}$$

and the amplitude becoms:

$$\mathcal{M} = \frac{g_s^2}{8mc} a_3^\alpha a_4^\beta \, \bar{v}(2) c_2^\dagger ((\epsilon_3 \cdot \epsilon_4)\{\lambda^\alpha, \lambda^\beta\}\gamma^0$$
$$+ i(\epsilon_3 \times \epsilon_4) \cdot \Sigma([\lambda^\alpha, \lambda^\beta]\gamma^0 + \{\lambda^\alpha, \lambda^\beta\}\gamma^3)) c_1 u(1)$$

HW11: Suppose we put the quarks into a spin 0 (singlet) state, show that the amplitude reduces to

$$\mathcal{M} = -i\sqrt{2}\,\frac{g_s^2}{4}(\epsilon_3 \times \epsilon_4)_z \, a_3^\alpha a_4^\beta (c_2^\dagger \{\lambda^\alpha, \lambda^\beta\} c_1) \qquad \text{(spin singlet)}$$

HW12: If the quarks occupy the color singlet state calculate the color factor

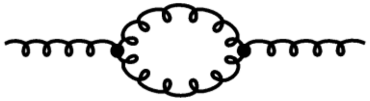
$$f = \tfrac{1}{8} a_3^\alpha a_4^\beta (c_2^\dagger \{\lambda^\alpha, \lambda^\beta\} c_1)$$

HW13: Show that in the spin singlet, color singlet configuration with the quarks at rest for $q + \bar{q} \longrightarrow g + g$,the amplitude is: $\mathcal{M} = -4\sqrt{2/3}\,g_s^2$

Strong coupling constant
Asymptotic freedom

Feynman rules for non physical particles or ghosts

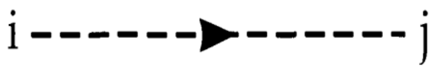
In QCD against the QED, such loop diagrams are also appear in calculations



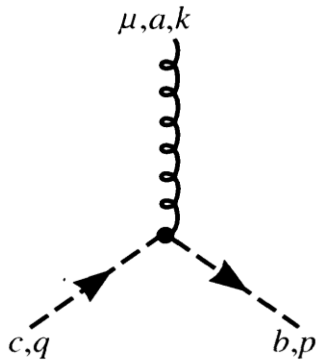
and besides physical transeversal polarization states, non-physical components of the gluons contribute to closed loops. Therefore one has for every inner line to subtract the nonphysical gluonic contributions. This can be achieved by introducing an artificial particle without any physical meaning. The couplings and the propagator of this particle are chosen in such a way that graphs containing this particle cancel the nonphysical gluonic contributions. The corresponding graph is

Because of their nonphysical nature these particles are called "ghosts" or "ghost fields".

Feynman rules for ghosts



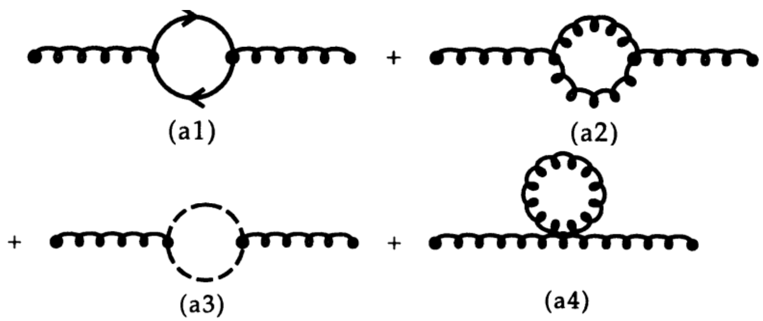
$$\frac{\delta_{ij}}{p^2}$$



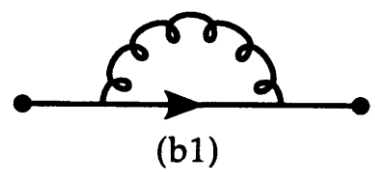
$$-ig\, f_{abc}\, p_\mu (2\pi)^4\, \delta^4(p-k-q)$$

The diagrams contributing to the strong copling constants are:

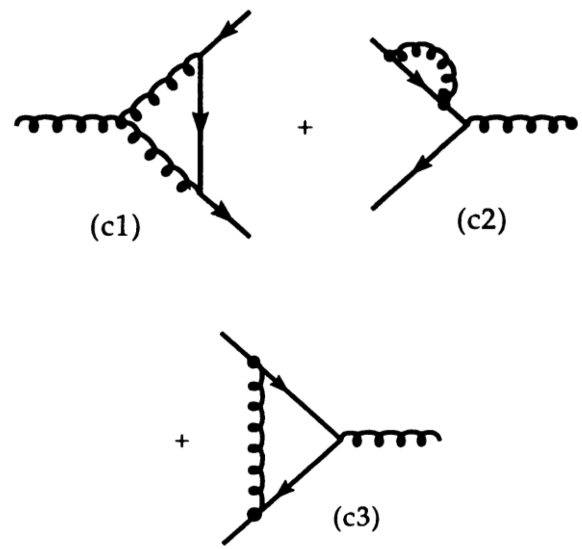
a) Vacuum polarization diagrams



b) Self energy diagram



c) Vertex correction diagrams



We will not discuss the detail of calculation since it is out of the scope of our school. After renormalization of the coupling (you learned in QFT cources) one obtains

$$\alpha_s(|q^2|) = \frac{\alpha_s(\mu^2)}{1 + (\alpha_s(\mu^2)/12\pi)(11n - 2f)\ln(|q^2|/\mu^2)} \qquad (|q^2| \gg \mu^2) \qquad \text{(\#1)}$$

μ → A parameter coms from dimension regulization. Some times An integral diverges in 4 dimention. In such cases we go to d dimension but multiply the integrand by μ^{4-d} to keep the total dimension of the expression unchanged. After doing calculations, we set d=4-2ε then expand the result interms of ε and finally set ε---->0

n → number of colors (3, in the Standard Model)

f → number of flavors (6, in the Standard Model)

Now we introduce new variable Λ such that

$$\ln \Lambda^2 = \ln \mu^2 - 12\pi/[(11n - 2f)\alpha_s(\mu^2)] \quad \text{(\#2)}$$

The running coupling constant can be expressed in terms of a single parameter Λ as:

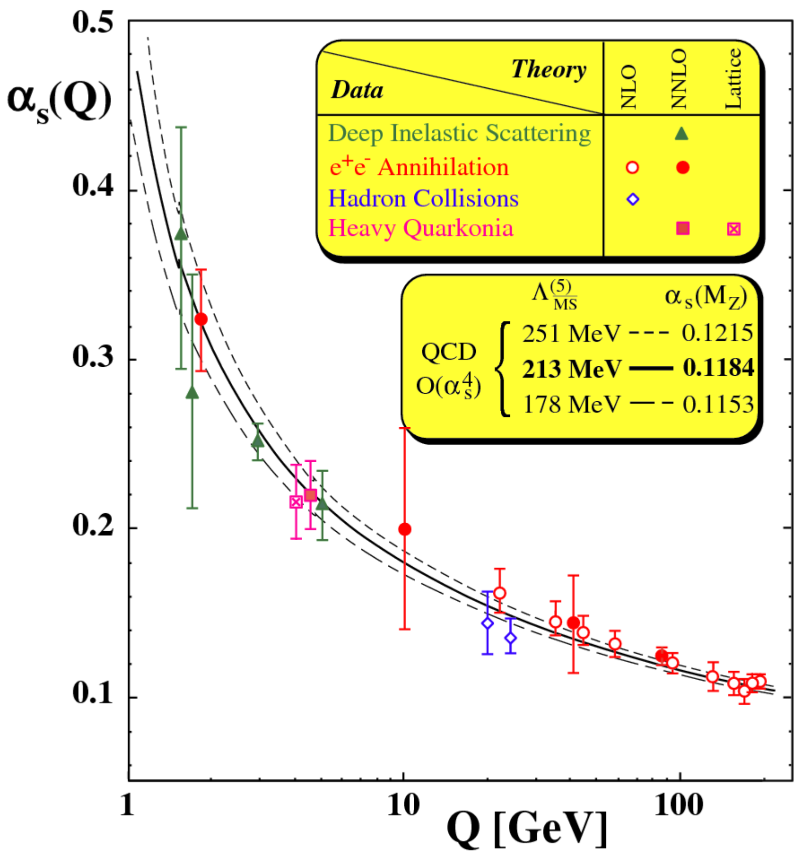
$$\alpha_s(|q^2|) = \frac{12\pi}{(11n - 2f)\ln(|q^2|/\Lambda^2)} \qquad (|q^2| \gg \Lambda^2) \quad \text{(\#3)}$$

HW14: Using the equation (#2) from the above and equation (#1) from the previouse slide, derive the equation (#3).

Λ → QCD scale: the energy scale where the interaction strength reaches the value 1

Λ_{QCD} ~ 200 MeV ~ 1 fm^{-1} Hadronic scale

S. Bethke, J Phys G 26, R27 (2000)



at short distances the "strong" force becomes relatively weak
is the basis of *asymptotic freedom*

Confinement-Hadronization

- As we previousely mentioned color charged particles (such as quarks) cannot be isolated singularly, and therefore cannot be directly observed. The reasons for quark confinement are somewhat complicated; no analytic proof exists that QCD should be confining, but intuitively, confinement is due to the force-carrying gluons having color charge. As two *quarks* separate, the gluon fields form narrow tubes (or strings) of color charge, which tend to bring the quarks together as though they were some kind of rubber band.

- When quarks are produced in particle accelerators, instead of seeing the individual quarks in detectors, scientists see "jets" of many color-neutral particles (mesons and baryons), clustered together. This process is called **hadronization**.

Hadrons

Hadrons
- Baryons : composed of three quarks
- Mesons : composed of one quark and one antiquark

Light Baryons

To construct the light baryons, we consider the SU(3) flavor symmetry with quarks $q^1 = u$, $q^2 = d$ and $q^3 = s$.

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8' \oplus 1$$

$$\begin{aligned}
q^i \times q^j \times q^k &= \frac{1}{6}(q^i q^j q^k + q^j q^i q^k + q^i q^k q^j + q^j q^k q^i + q^k q^j q^i + q^k q^i q^j) \\
&+ \frac{1}{6}(2q^i q^j q^k + 2q^j q^i q^k - q^i q^k q^j - q^j q^k q^i - q^k q^j q^i - q^k q^i q^j) \\
&+ \frac{1}{6}(2q^i q^j q^k - 2q^j q^i q^k + q^i q^k q^j - q^j q^k q^i + q^k q^j q^i - q^k q^i q^j) \\
&+ \frac{1}{6}(q^i q^j q^k - q^j q^i q^k - q^i q^k q^j + q^j q^k q^i - q^k q^j q^i + q^k q^i q^j) \\
&= T^{\{ijk\}} + T^{\{ij\}k} + T^{[ij]k} + T^{[ijk]},
\end{aligned}$$

i, j and k goes from 1 to 3.

decuplet baryons Spin 3/2

$s = 0$	Δ^-		Δ^0		Δ^+		Δ^{++}
$s = -1$		Σ^{*-}		Σ^{*0}		Σ^{*0}	
$s = -2$			Ξ^{*-}		Ξ^{*0}		
$s = -3$				Ω^{*-}			
	$I_3 = -\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$

quark contents for decuplet baryons are

$$ddd \quad udd \quad uud \quad uuu$$

$$sdd \quad sud \quad suu$$

$$ssd \quad ssu$$

$$sss$$

octet baryons spin 1/2

$s = 0$		n		p	
$s = -1$	Σ^-		(Σ^0, Λ)		Σ^+
$s = -2$		Ξ^-		Ξ^0	
	$I_3 = -1$	$-\frac{1}{2}$	0	$\frac{1}{2}$	1

or in terms of the quark contents:

$$udd \quad uud$$

$$sdd \quad sud \quad suu$$

$$ssd \quad ssu$$

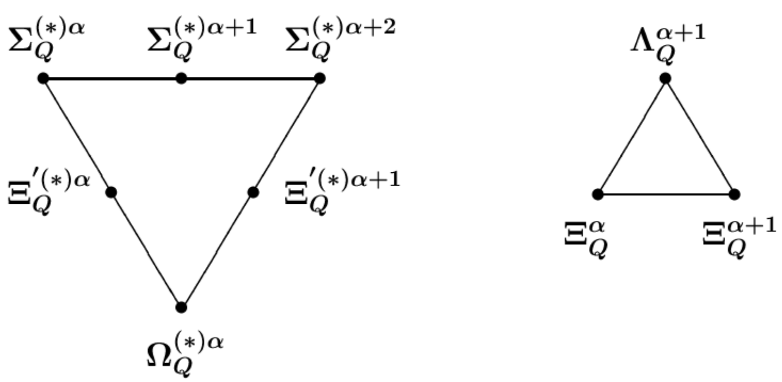
Light singlet (spin 1/2): $\Lambda(uds)$

Heavy Baryons: contain one-two or three heavy "b" or "c" quarks

The baryons containing single heavy quark can be classified according to the spin of the light degrees of freedom in the heavy quark limit, $m_Q \to \infty$.

$$3 \otimes 3 = \bar{3}_F \oplus 6_F$$

The spin of the light diquark is either $S = 1$ for 6_F, or $S = 0$ for $\bar{3}_F$. The ground state will have angular momentum $l = 0$. Therefore, the spin of the ground state is $1/2$ for $\bar{3}_F$, while it can be both $3/2$ or $1/2$ for 6_F



$\mathbf{6_F}$ $\quad$ $\mathbf{\bar{3}_F}$

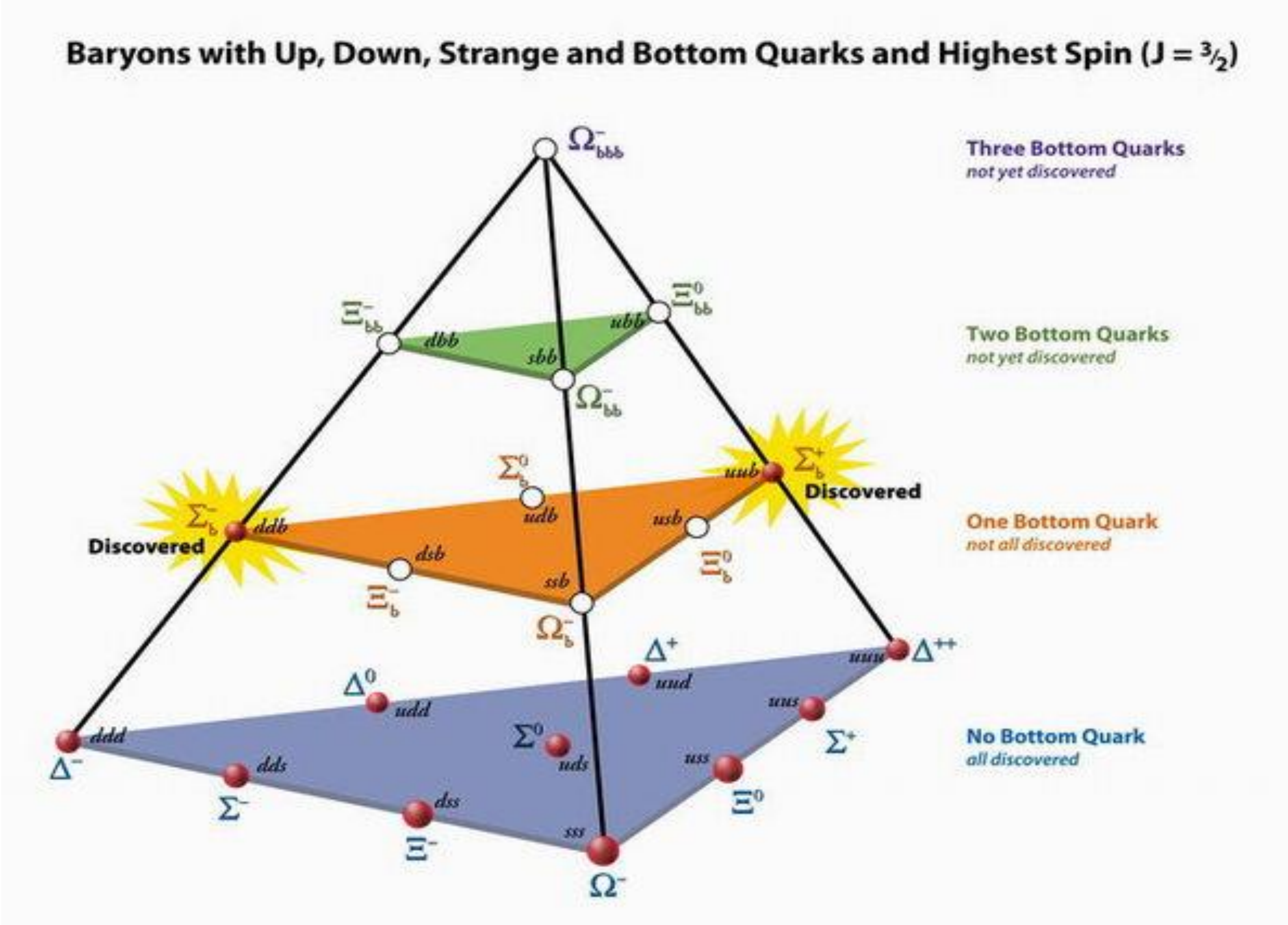
α, $\alpha+1$, $\alpha+2$ determine the charges of baryons ($\alpha = -1$ or 0), and the asterix $(*)$ denote $J^P = \frac{3}{2}^+$ states.

Spin 3/2

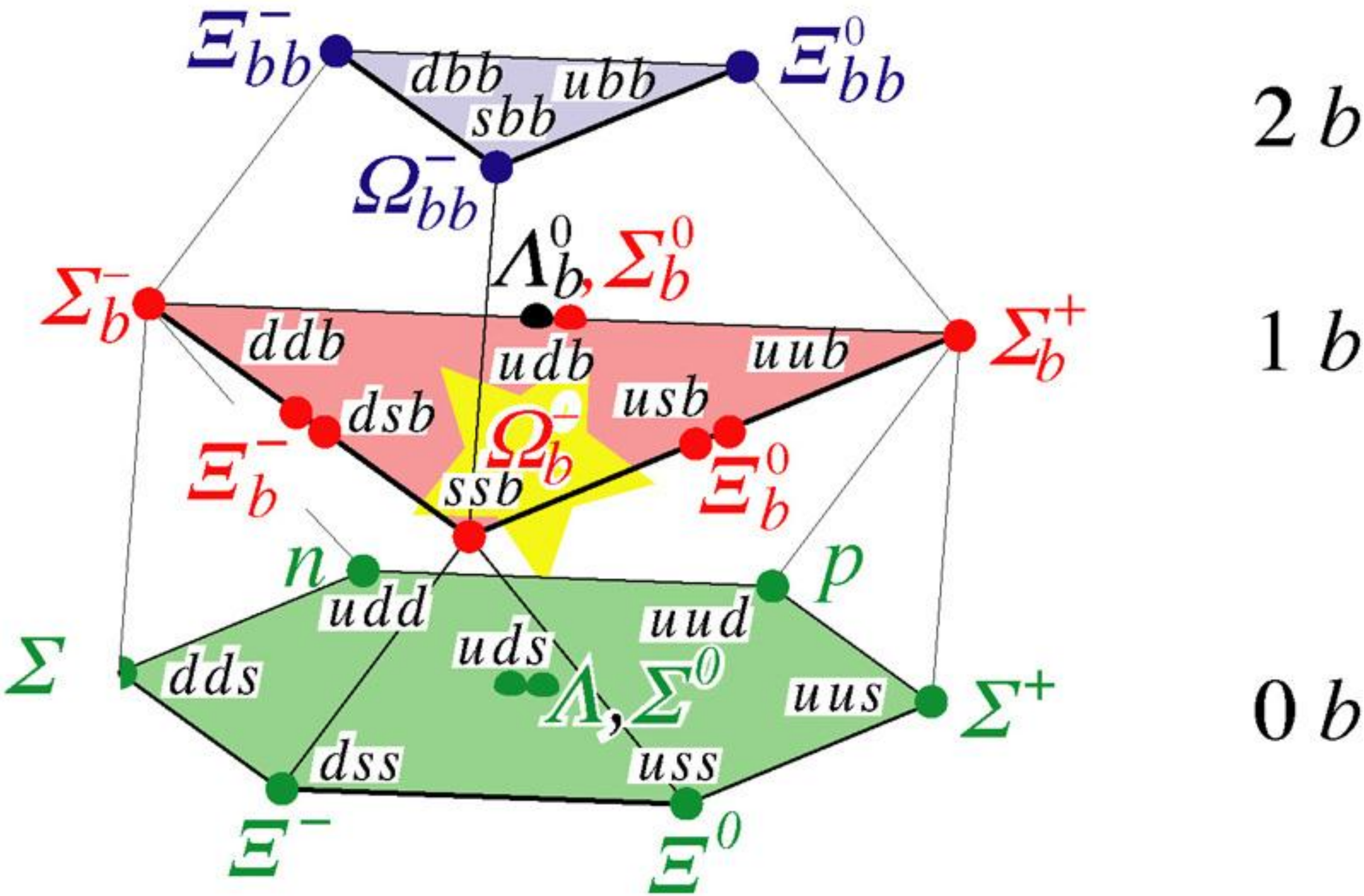
	q_1	q_2
$\Sigma_{b(c)}^{*+(++)}$	u	u
$\Sigma_{b(c)}^{*0(+)}$	u	d
$\Sigma_{b(c)}^{*-(0)}$	d	d
$\Xi_{b(c)}^{*0(+)}$	s	u
$\Xi_{b(c)}^{*-(0)}$	s	d
$\Omega_{b(c)}^{*-(0)}$	s	s

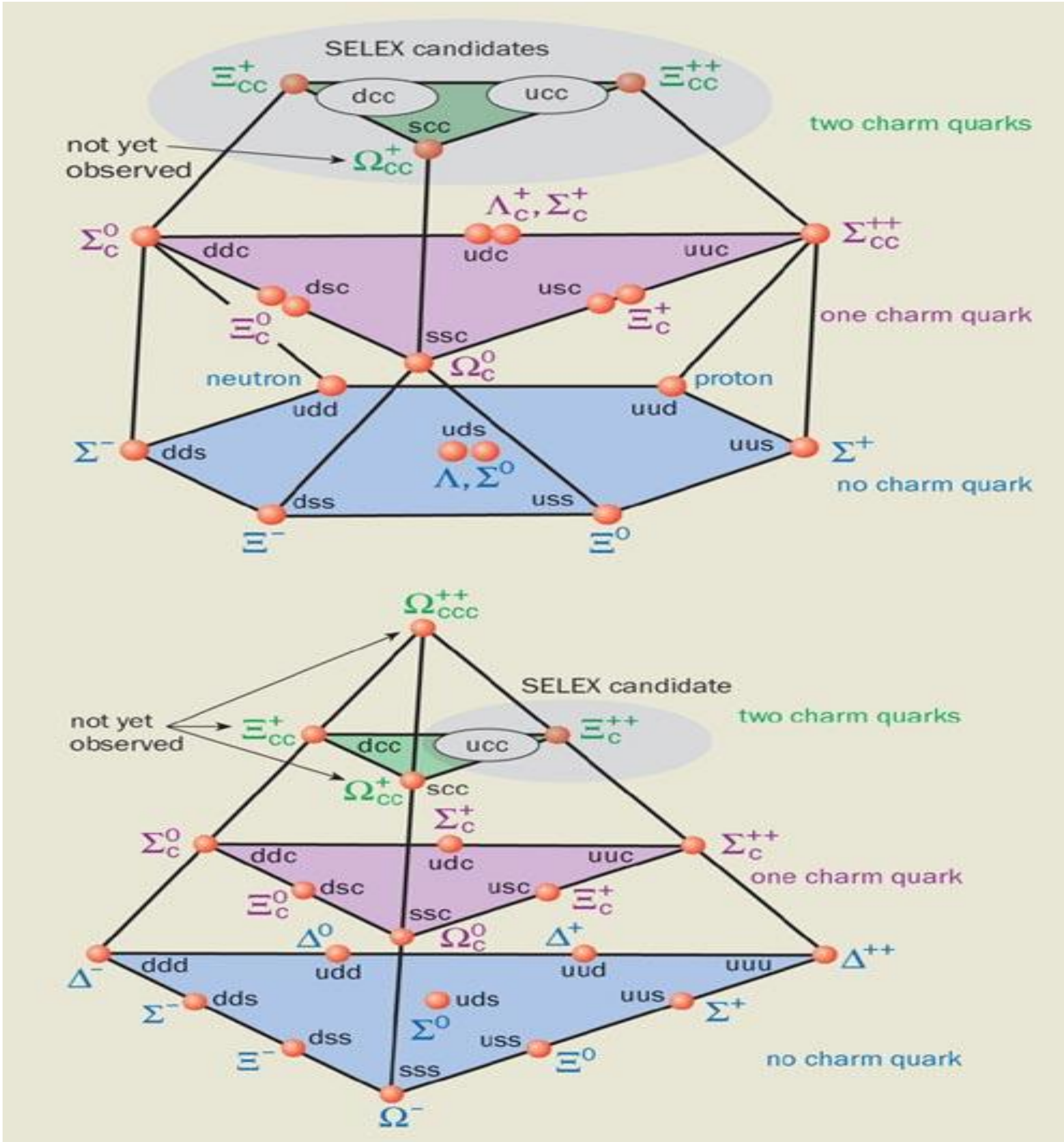
spin 1/2

	q_1	q_2
$\Sigma_{b(c)}^{+(++)}$	u	u
$\Sigma_{b(c)}^{0(+)}$	u	d
$\Sigma_{b(c)}^{-(0)}$	d	d
$\Xi_{b(c)}^{0(+)}$	s	u
$\Xi_{b(c)}^{-(0)}$	s	d
$\Lambda_{b(c)}^{0(+)}$	u	d

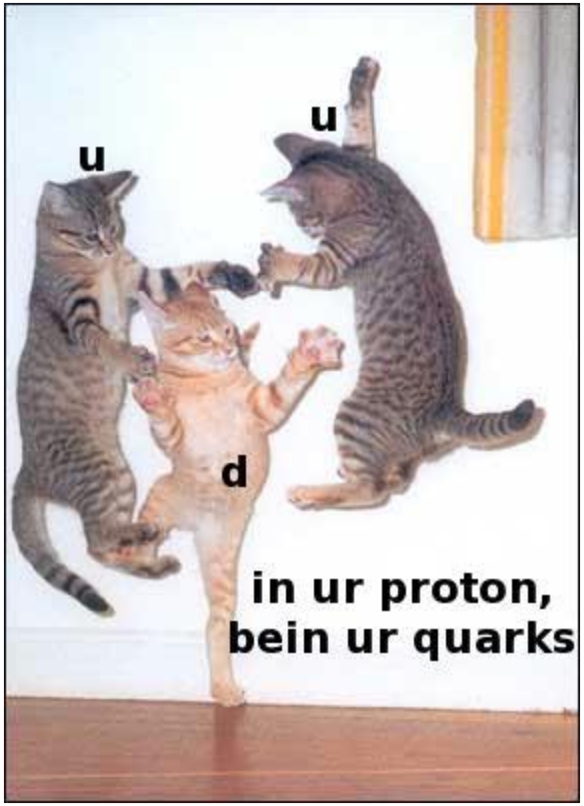

Baryons with Up, Down, Strange and Bottom Quarks and Highest Spin (J = 3/2)
Ω^-_{bbb}
Three Bottom Quarks
not yet discovered
Ξ^-_{bb}
Ξ^0_{bb}
dbb
ubb
sbb
Ω^-_{bb}
Two Bottom Quarks
not yet discovered
Σ^0_b
Σ^+_b
uub
Discovered
Σ^-_b
ddb
udb
usb
Discovered
One Bottom Quark
not all discovered
dsb
ssb
Ξ^-_b
Ξ^0_b
Ω^-_b
Δ^+
Δ^{++}
uuu
Δ^0
uud
uus
udd
ddd
Σ^0
Σ^+
No Bottom Quark
all discovered
Δ^-
dds
uds
uss
Σ^-
dss
Ξ^0
sss
Ξ^-
Ω^-

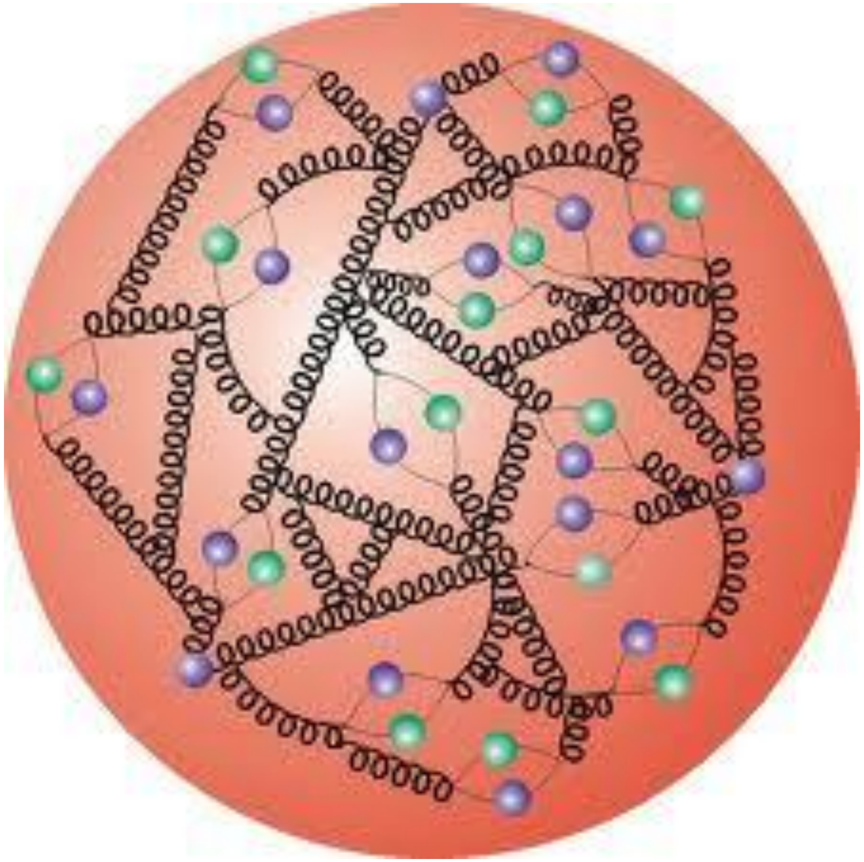
$J = 1/2$ b Baryons




SELEX candidates
Ξ_{cc}^{+}
Ξ_{cc}^{++}
dcc
ucc
scc
two charm quarks
not yet observed
Ω_{cc}^{+}
Λ_c^+, Σ_c^+
Σ_c^0
Σ_{cc}^{++}
ddc
udc
uuc
dsc
usc
Ξ_c^0
Ξ_c^+
one charm quark
ssc
Ω_c^0
neutron
proton
udd
uud
uds
Σ^-
dds
uus
Σ^+
Λ, Σ^0
no charm quark
dss
uss
Ξ^-
Ξ^0
Ω_{ccc}^{++}
SELEX candidate
two charm quarks
not yet observed
Ξ_{cc}^{+}
Ξ_c^{++}
dcc
ucc
Ω_{cc}^{+}
scc
Σ_c^+
Σ_c^0
Σ_c^{++}
ddc
udc
uuc
one charm quark
Ξ_c^0
dsc
usc
Ξ_c^+
ssc
Ω_c^0
Δ^0
Δ^+
Δ^-
ddd
udd
uud
uuu
Δ^{++}
Σ^-
dds
uds
uus
Σ^+
Σ^0
Ξ^-
dss
uss
Ξ^0
no charm quark
sss
Ω^-

Proton (uud)





Mesons

Mesons are classified into types according to their spin configurations. Some specific configurations are given special names based on the mathematical properties of their spin configuration.

Types of mesons

	S	L	P	J	J^P
Scalar	1	1	+	0	0^+
Pseudoscalar	0	0	−	0	0^-
Vector	1	0	−	1	1^-
Axial-vector	0	1	+	1	1^+
Tensor	1	1	+	2	2^+
Pseudotensor	1	1	-	2	2^-

$\bar{s}$

Light mesons: composed of light “u”, “d” or “s” quark and their antiquarks

Light scalar mesons

i) confirmed: $K_0^*(1430)$: $d\bar{s}$

ii) candidates: $K_0^*(800)$ or kappa, $f_0(600)$ or sigma, $f_0(980)$, $a_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $a_0(1450)$

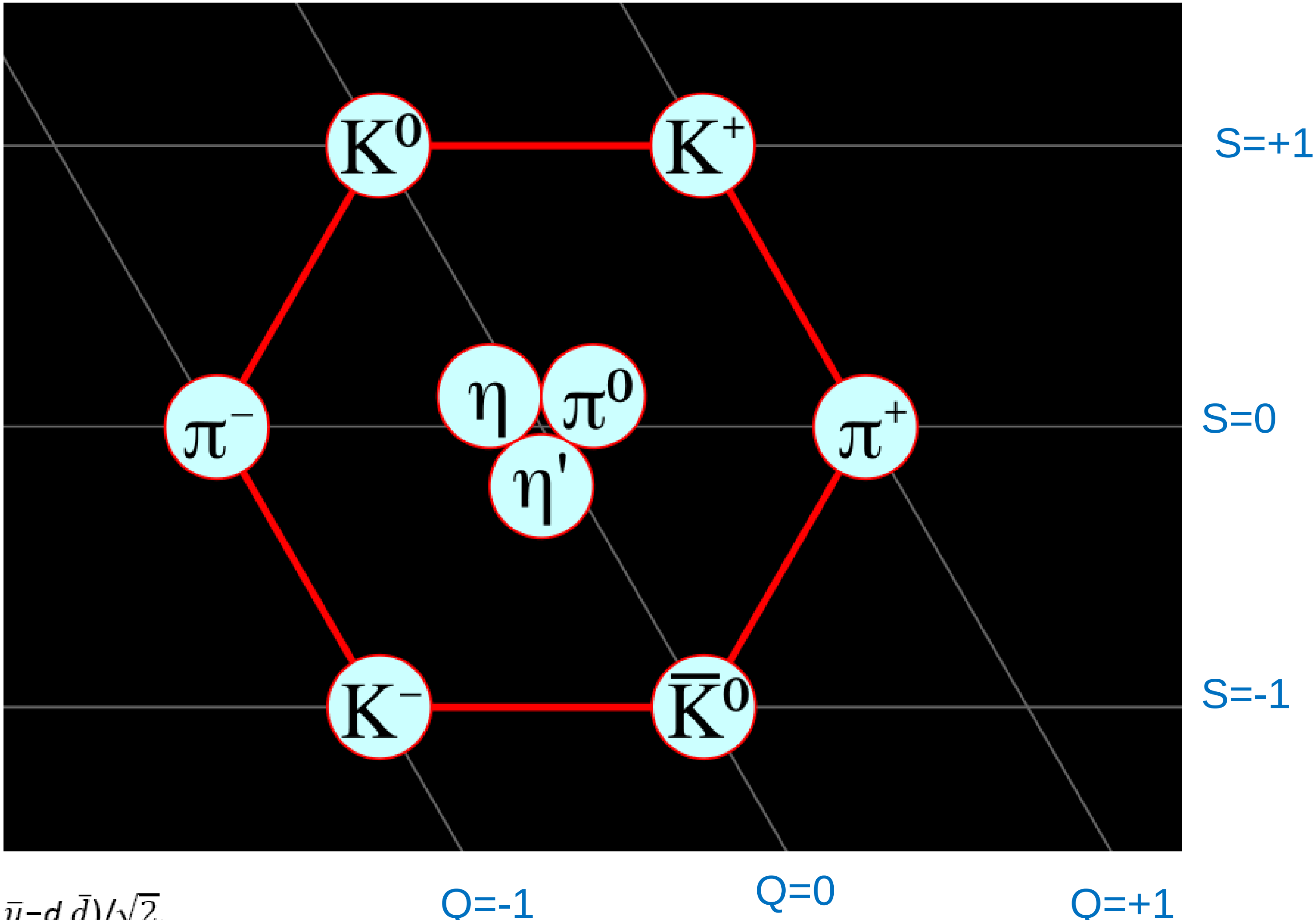
- with + charge $u\bar{d}$
- neutral $(u\bar{u}-d\bar{d})/\sqrt{2}$,
- with - charge $d\bar{u}$

iii) unconfirmed resonances: $f_0(1200\text{-}1600)$, f_0 (1790)

These mesons are most often observed in proton-antiproton annihilation, radiative decays of vector mesons, and meson-meson scattering.

To understand the internal structure of scalar mesons has been a prominent topic in the last 30-40 years. Although the scalar mesons have been investigated for several decades, many properties of them are not so clear yet and identifying the scalar mesons is difficult, experimentally.

Light pseudoscalar mesons

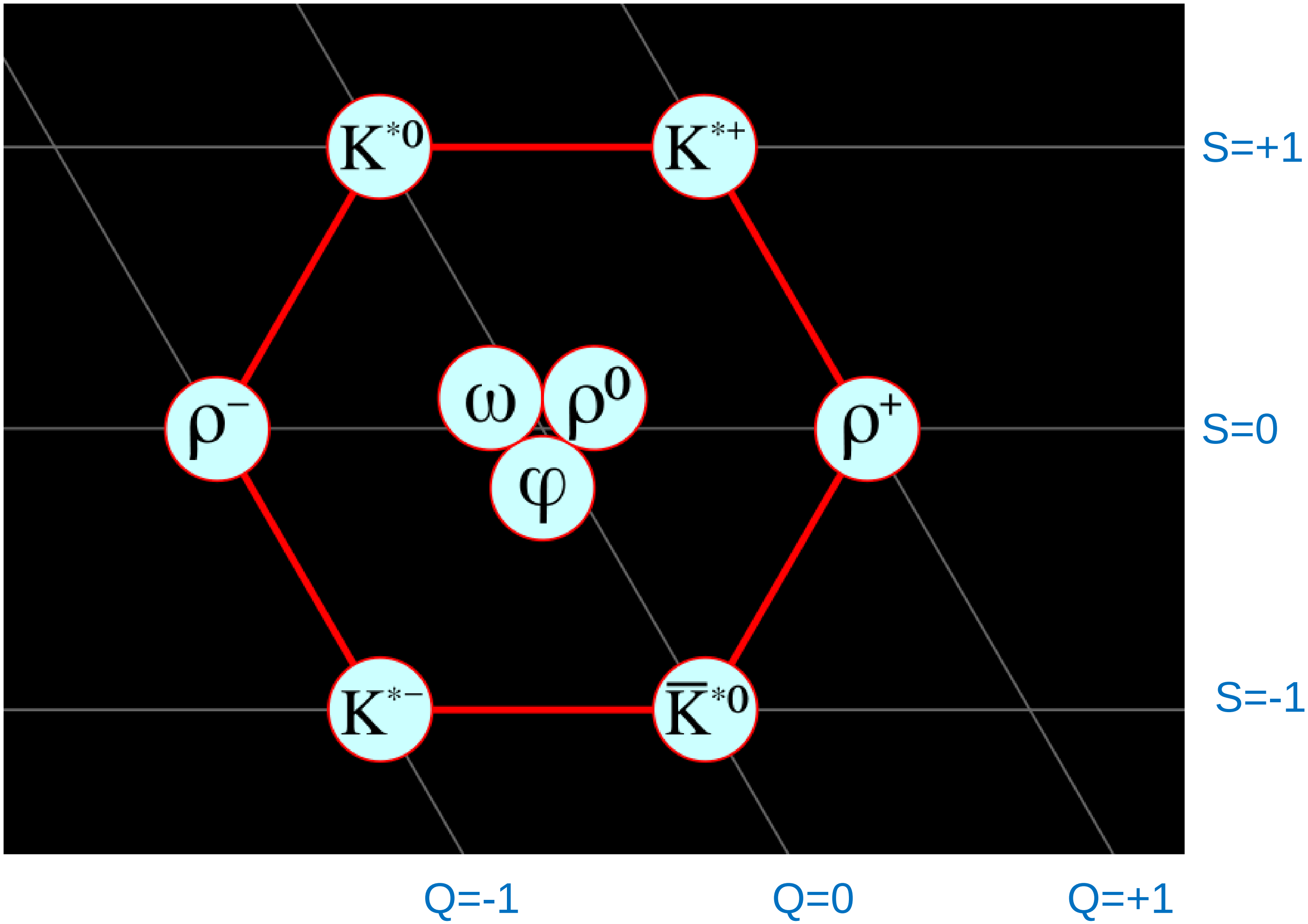


π^0 ⇒ $(u\,\bar{u} - d\,\bar{d})/\sqrt{2}$,

η ⇒ $\frac{1}{\sqrt{6}}[\bar{u}u + \bar{d}d - 2\bar{s}s]$

For quark contents and more information see Particle Data Group: http://pdglive.lbl.gov

Light vector mesons



Light axial-vector mesons

$h_1(1170)$, $b_1(1235)$, $h_1(1380)$

$f_1(1285)$, $a_1(1260)$, $f_1(1420)$

$K_1(1270)$, $K_1(1400)$

Light tensor mesons

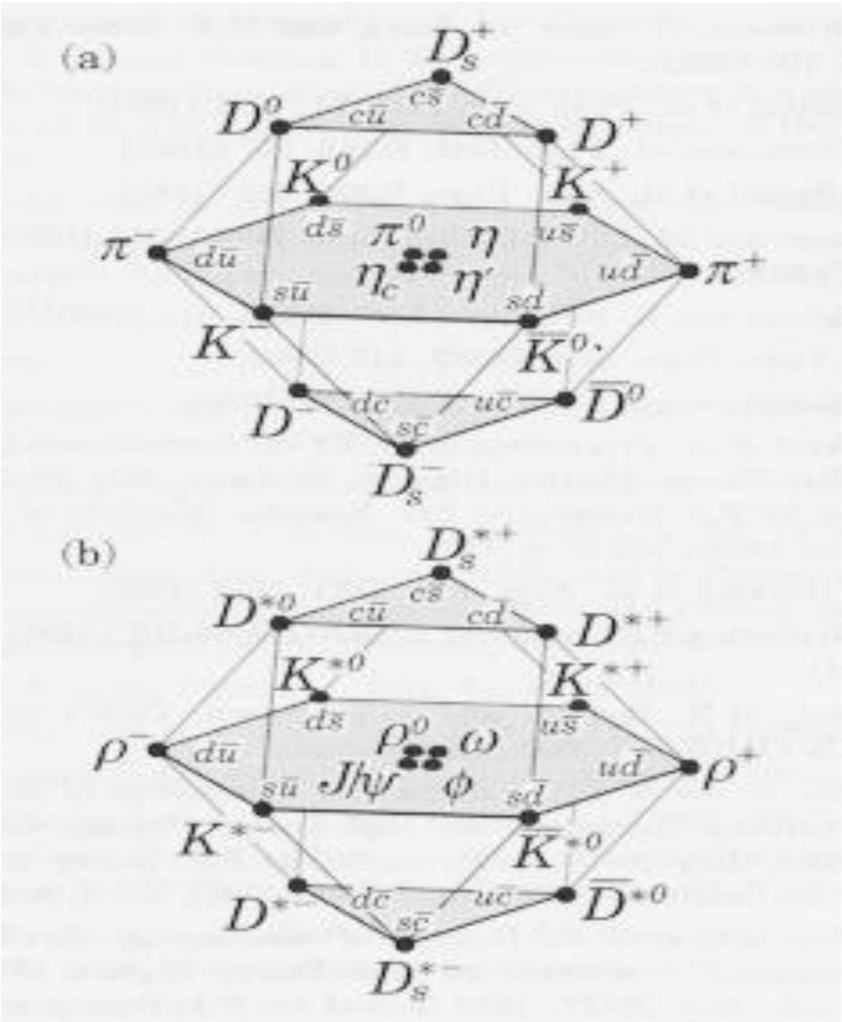
$f_2(1270)$ $K_2^*(1430)$

Heavy mesons:

one “b” or “c” quark-one light antiquark,
one light quark-one heavy “b” or “c” antiquark
one “b” or “c” quark-one “b” or “c” antiquark

Some heavy mesons: B, D, B_c, B^*, D^*, J_ψ, χ_{b0}, χ_{b2}, Ds1(2460) ,.........

PS V S T AV


(a)
D_s^+
$c\bar{s}$
D^0
$c\bar{u}$
$c\bar{d}$
D^+
K^0
K^+
$d\bar{s}$
$u\bar{s}$
π^-
π^0
η
π^+
$d\bar{u}$
η_c
η'
$u\bar{d}$
$s\bar{u}$
$s\bar{d}$
K^-
$\bar{K}^0$
$d\bar{c}$
$u\bar{c}$
D^-
$s\bar{c}$
$\bar{D}^0$
D_s^-
(b)
D_s^{*+}
$c\bar{s}$
D^{*0}
$c\bar{u}$
$c\bar{d}$
D^{*+}
K^{*0}
K^{*+}
$d\bar{s}$
$u\bar{s}$
ρ^-
ρ^0
ω
ρ^+
$d\bar{u}$
J/ψ
ϕ
$u\bar{d}$
$s\bar{u}$
$s\bar{d}$
K^{*-}
$\bar{K}^{*0}$
$d\bar{c}$
$u\bar{c}$
D^{*-}
$s\bar{c}$
$\bar{D}^{*0}$
D_s^{*-}

Nonperturbative methods

Hadrons are formed in low energies very far from perturbative regime. Therefore to calculate their parameters (properties) such as: mass, decay constant or residue lifetime, width as well as their decay properties (strong, weak and electromagnetic decays), we need some non-perturbarive methods.

Some non-perturbative methods are:

- QCD sum rules
- Lattice QCD
- Heavy quark effective theory (HQET)
- Chiral perturbation theory,
- Soft-collinear effective theory
- Nambu-Jona-Lasinio model.
- Different relativistic and non-relativistic quark models

....

In most of non-perturbative approaches, hadrons are represented by their interpolating currents.

interpolating currents of mesons

scalar: $J^{S}(x) = \overline{q}_1(x) q_2(x)$

Pseudoscalar: $J^{PS}(x) = \overline{q}_1(x) \gamma_5 q_2(x)$

Vector: $J^{V}_{\mu}(x) = \overline{q}_1(x) \gamma_\mu q_2(x)$

Axial-Vector: $J^{AV}_{\mu}(x) = \overline{q}_1(x) \gamma_\mu \gamma_5 q_2(x)$

Tensor: $J^{T}_{\mu\nu}(x) = \overline{q}_1(x) \gamma_\mu \gamma_\nu q_2(x)$

interpolating current of each particle can create that particle from the vacuum with the same quantum numbers as the interpolating current.

Interpolating currents for baryons

We start to derive the interpolating field for nucleon (proton). The proton consists of two "u" quarks and one "d" quark and has the quantum numbers I=1/2, I_3=1/2 and $J^P=(1/2)^+$. The simplest way to obtain the approciate isospin is to consider the proton to be an up-down "diquark" with I=0 with an up quark (I_3=1/2) attached. Therefore, one must first consider the construction of an interpolating field for diquark.

The interpolating current for a diquark is expected to be similar to that of a meson.

$$J_{meson} = \bar{q}\Gamma q$$

Γ → a Dirac matrix: $\{I, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$

How does one construct the diquark? It is possible by replacing antiquark with its charge-conjugation analog:

$$q = C\bar{q}^T \quad \xrightarrow{\text{HW15}} \quad \bar{q} = q^T C$$

$C = \gamma_0\gamma_2 \qquad C^T = C^{-1} = C^{+} = -C \qquad \gamma_0 C\, \gamma_0 = -C$

$\Rightarrow$ $J_{diquark}=q^T C\Gamma q$

Now add the other "u" quark. The current must be color singlet so,

$$\eta \sim \varepsilon_{abc}(u^{aT} C\Gamma d^b)\Gamma' u^c$$

$\Gamma \rightarrow \{I, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$

one should determine Γ and Γ'. The form of Γcan be constrained through isospin considerations. Since the "u" quark quark attached to the "ud" diquark has I=1/2 and I_3=1/2, one can most readily guarantee that the proton has these same quantum numbers by insisting that the diquark has I=0.

Consider an infinitesimal isospin rotation. Under this transformation, the "u" quark obtaines a small "d" quark component and vice versa. If the diquark has vanishing isospin, then its contribution to the nucleon interpolating field must remain invariant under this transformation, i.e., one must have

$\varepsilon_{abc} u^{aT} C\Gamma u^b = 0$ (1) and analogous relation for "d" quark

A simple way te determine those values of Γ that satisfy the Eq. (1) is to consider the transpose of both sides of this equation. One must first develop a simple theorem. If A=BC, where A, B and C are matrices whose elements are Grassmann numbers (**anticommuting numbers** or **anticommuting c-numbers**), then $A^T=-C^TB^T$

Proof:

$$(A^T)_{ki} = A_{ik} = B_{ij} C_{jk} = -C_{jk} B_{ij} = -(C^T)_{kj} (B^T)_{ji} = -(C^TB^T)_{ki}$$

Now consider the transpose of the left-hand side of Eq. (1):

$$(\varepsilon_{abc} u^{aT} C\Gamma u^b)^T = -\varepsilon_{abc} u^{bT} \Gamma^T C^T u^a$$

Using $C^T=C^{-1}=C^+=-C$ one obtains

$$(\varepsilon_{abc} u^{aT} C\Gamma u^b)^T = \varepsilon_{abc} u^{bT} C(C\Gamma^T C^{-1}) u^a$$

Where,

$$C\Gamma^T C^{-1} = \begin{cases} \Gamma & \text{for } \Gamma = I, \gamma_5, \gamma_5\gamma_\mu \\ -\Gamma & \text{for } \Gamma = \gamma_\mu, \sigma_{\mu\nu} \end{cases}$$

Switching color dummy indicies, one obtains

$$(\varepsilon_{abc}u^{aT}C\Gamma u^{b})^{T} = \begin{cases} -\varepsilon_{abc}u^{aT}C\Gamma u^{b} & \text{for } \Gamma = I,\ \gamma_5,\ \gamma_5\gamma_\mu \\ \varepsilon_{abc}u^{aT}C\Gamma u^{b} & \text{for } \Gamma = \gamma_\mu,\ \sigma_{\mu\nu} \end{cases}$$

The transpose of a 1 bye1 matrix, such as $\varepsilon_{abc}u^{aT}C\Gamma u^{b}$ İs equal to itself. Therefor,

$$\varepsilon_{abc}u^{aT}C\Gamma u^{b} = 0 \quad \text{for } \Gamma = I,\ \gamma_5,\ \gamma_5\gamma_\mu$$

Thus isospin considerations impose the constraint $\Gamma = I$, γ_5 or $\gamma_5\gamma_\mu$. Now consider the spin. The “u” quark attached to the diquark has J=1/2 and $J_3 = \pm$ ½. In constracting the interpolating field for the proton, it is simplest to take the proton to have the same total spin and spin projection as the “u” quark. Thus the spin of the diquark is zero . This implies $\Gamma = I$, γ_5; therefore, the two possible forms of the proton interpolating field can be written as:

$$\begin{cases} \eta_1=\varepsilon_{abc}(u^{aT}Cd^b)\Gamma_1' u^c \\ \eta_2=\varepsilon_{abc}(u^{aT}C\,\gamma_5\, d^b)\Gamma_2' u^c \end{cases}$$

The values of Γ_1' and Γ_2' are determined through considerations involving Lorentz structure and parity. Since η_1 and η_2 are Lorentz scalars, one must have

$$\Gamma_1' ,\ \Gamma_2' = I \text{ or } \gamma_5$$

Under the parity transformation the spinor ψ(x) becomes

$$\Psi'(x') = \gamma_0\ \psi(x)$$

Applying this transformation to the individual quark fields in η_1 and η_2 one obtains

$$\begin{cases} \eta'_1=\varepsilon_{abc}[(\gamma_0 u^a)^T C\gamma_0 d^b]\Gamma_1'\ \gamma_0 u^c=-\ \varepsilon_{abc}(u^{aT}Cd^b)\Gamma_1'\ \gamma_0 u^c \\ \eta'_2=\varepsilon_{abc}[(\gamma_0 u^a)^T C\ \gamma_5\ \gamma_0 d^b]\Gamma_2'\ \gamma_0 u^c=\ \varepsilon_{abc}(u^{aT}C\ \gamma_5\ d^b)\Gamma_2'\ \gamma_0\ u^c \end{cases}$$

On the other hand, applying the parity transformation to the interpolating fields as a whole gives

$$\begin{cases} \eta'_1=\varepsilon_{abc}\ (u^{aT}Cd^b)\ \gamma_0\Gamma_1' u^c \\ \eta'_2=\varepsilon_{abc}\ (u^{aT}C\ \gamma_5\ d^b)\ \gamma_0\Gamma_2' u^c \end{cases}$$

Thus $-\gamma_0\Gamma_1'=\Gamma_1'\ \gamma_0$ and $\Gamma_2'\gamma_0=\gamma_0\Gamma_2'$, which implies

$\Gamma_1'=\gamma_5$ $\Gamma_2'=I$

Therefore, the two possible interpolating fields are given by

$$\begin{cases} \eta_1=\varepsilon_{abc}\ (u^{aT}Cd^b)\ \gamma_5\ u^c \\ \eta_2=\varepsilon_{abc}\ (u^{aT}C\ \gamma_5\ d^b)\ u^c \end{cases}$$

These two fields can be combined in an arbitrary linear combination. A useful form is

$\eta(t)=2\ \varepsilon_{abc}[(u^{aT}Cd^b)\ \gamma_5\ u^c+\ t(u^{aT}C\ \gamma_5\ d^b)\ u^c]$ t is an arbitrary real parameters.

From similar manner one can obtain the following interpolating currents for the light and heavy baryons.

light spin ½ (octet) baryons:

$$\eta = A\varepsilon^{abc}\left\{(q_1^{aT}Cq_2^b)\gamma_5 q_3^c - (q_2^{aT}Cq_3^b)\gamma_5 q_1^c + \beta(q_1^{aT}C\gamma_5 q_2^b)q_3^c - \beta(q_2^{aT}C\gamma_5 q_3^b)q_1^c\right\}$$

	A	q_1	q_2	q_3
Σ^0	$-\sqrt{1/2}$	u	s	d
Σ^+	$1/2$	u	s	u
Σ^-	$1/2$	d	s	d
p	$-1/2$	u	d	u
n	$-1/2$	d	u	d
Ξ^0	$1/2$	s	u	s
Ξ^-	$1/2$	s	d	s

$$2\eta^{\Sigma^0}(d\to s) + \eta^{\Sigma^0} = -\sqrt{3}\eta^{\Lambda} \ ,$$
$$2\eta^{\Sigma^0}(u\to s) + \eta^{\Sigma^0} = \sqrt{3}\eta^{\Lambda} \ .$$

light spin 3/2 (Decuplet) baryons

$$\eta_\mu = A' \varepsilon^{abc} \{ (q_1^{aT} C \gamma_\mu q_2^b) q_3^c + (q_2^{aT} C \gamma_\mu q_3^b) q_1^c + (q_3^{aT} C \gamma_\mu q_1^b) q_2^c \}$$

	A'	q_1	q_2	q_3
Σ^{*0}	$\sqrt{2/3}$	u	d	s
Σ^{*+}	$\sqrt{1/3}$	u	u	s
Σ^{*-}	$\sqrt{1/3}$	d	d	s
Δ^{++}	$1/3$	u	u	u
Δ^{+}	$\sqrt{1/3}$	u	u	d
Δ^{0}	$\sqrt{1/3}$	d	d	u
Δ^{-}	$1/3$	d	d	d
Ξ^{*0}	$\sqrt{1/3}$	s	s	u
Ξ^{*-}	$\sqrt{1/3}$	s	s	d
Ω^{-}	$1/3$	s	s	s

Heavy spin ½ baryons

$$\eta_Q^{(s)} = -\frac{1}{\sqrt{2}}\epsilon^{abc}\Big\{\Big(q_1^{aT}CQ^b\Big)\gamma_5 q_2^c + \beta\Big(q_1^{aT}C\gamma_5 Q^b\Big)q_2^c - \Big[\Big(Q^{aT}Cq_2^b\Big)\gamma_5 q_1^c + \beta\Big(Q^{aT}C\gamma_5 q_2^b\Big)q_1^c\Big]\Big\} ,$$

$$\eta_Q^{(anti-t)} = \frac{1}{\sqrt{6}}\epsilon^{abc}\Big\{2\Big(q_1^{aT}Cq_2^b\Big)\gamma_5 Q^c + 2\beta\Big(q_1^{aT}C\gamma_5 q_2^b\Big)Q^c + \Big(q_1^{aT}CQ^b\Big)\gamma_5 q_2^c + \beta\Big(q_1^{aT}C\gamma_5 Q^b\Big)q_2^c$$
$$+ \Big(Q^{aT}Cq_2^b\Big)\gamma_5 q_1^c + \beta\Big(Q^{aT}C\gamma_5 q_2^b\Big)q_1^c\Big\}$$

Q=b or c

	$\Sigma_{b(c)}^{+(++)}$	$\Sigma_{b(c)}^{0(+)}$	$\Sigma_{b(c)}^{-(0)}$	$\Xi_{b(c)}^{-(0)\prime}$	$\Xi_{b(c)}^{0(+)\prime}$	$\Omega_{b(c)}^{-(0)}$	$\Lambda_{b(c)}^{0(+)}$	$\Xi_{b(c)}^{-(0)}$	$\Xi_{b(c)}^{0(+)}$
q_1	u	u	d	d	u	s	u	d	u
q_2	u	d	d	s	s	s	d	s	s

Heavy spin 3/2 baryons

$$\eta_\mu = A\epsilon^{abc}\Big\{(q_1^a C\gamma_\mu q_2^b)Q^c + (q_2^a C\gamma_\mu Q^b)q_1^c + (Q^a C\gamma_\mu q_1^b)q_2^c\Big\}$$

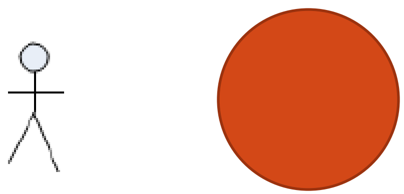
	$\Sigma_{b(c)}^{*+(++)}$	$\Sigma_{b(c)}^{*0(+)}$	$\Sigma_{b(c)}^{*-(0)}$	$\Xi_{b(c)}^{*0(+)}$	$\Xi_{b(c)}^{*-(0)}$	$\Omega_{b(c)}^{*-(0)}$
q_1	u	u	d	u	d	s
q_2	u	d	d	s	s	s
A	$\sqrt{1/3}$	$\sqrt{2/3}$	$\sqrt{1/3}$	$\sqrt{2/3}$	$\sqrt{2/3}$	$\sqrt{1/3}$

QCD sum rules method is one of the most powerfull and applicable non-perturbative tolls to hadron physics. It is based on QCD lagrangian and does not containe any model dependent parameter. This method developed about thirty years ago by Shifman, Vainshtein and Zakharov and it is also called the SVZ sum rule.

In this approach, we relate the hadronic parameters such as mass, decay constant, and etc... to the fundamental QCD parameters such as quark mass and ...... by the help of dispersion relation. In other language, we see a hadron once from outside and another from inside, then we equate these two windows to get QCD sum rules for physical quantities.

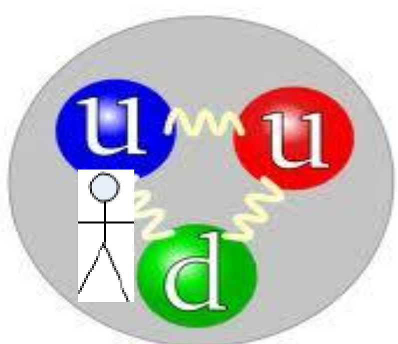
hadron

Window1:



We see a hadron which has a mass, Decay constant, lifetime and other ha-Dronic parameters

Window2:



We see quarks and gluons and their interaction

Setting Window1= Window2 → QCD sum rules for physical quantities

Thank you

Scalar

Pseudoscalar