TBC

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Institute for Theoretical Particle Physics and Cosmology
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Workshop on Automatic Phenomenology

Institut Henri Poincaré, Paris
9th of June 2022

TBC (not Tuberculosis)

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Automatic Masses and Decays

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Contents

Automatic Phenomenology

Automatic

- get results with little effort by user
- high flexibility of possible models
- high-speed evaluation

mainly technical issues:

- different fields (defined by spin)
- construction of all diagrams
- generic couplings and charges
- renormalization
- flexible integral reduction
- ⋮

Phenomenology

- consistent definition of observables
- high-precision predictions
- uncertainty estimate

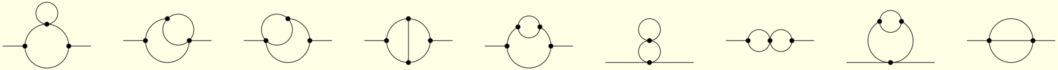
mainly physics issues:

- experimentally accessible
- theoretically well-defined
- more loops, more legs
- compare schemes
- assess missing terms
- ⋮

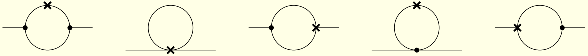
Contents

Feynman diagrams contributing to Σ_S at two-loop order using `FeynArts`:

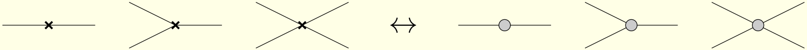
genuine two loop



one loop with counterterm insertion

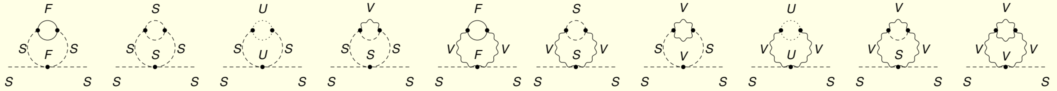


insertions computed from divergences of one-loop diagrams



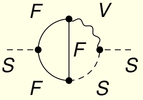
Generic Feynman diagrams

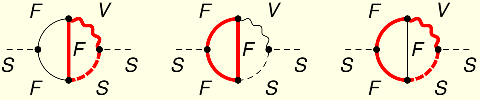
populate genuine two-loop topologies with all possible field types, for example



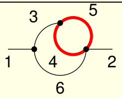
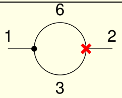
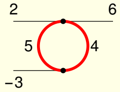
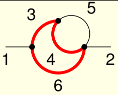
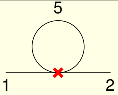
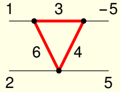
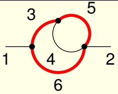
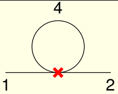
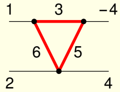
in total: 121 different cases out of the box (93 after using further symmetries)

each diagram contains sub-loops that potentially generate UV divergences with logarithms in the coefficient, for example

diagram 

has sub-loops 

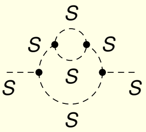
but: for each sub-loop exists a diagram with appropriate counterterm insertion
in order to cancel logarithmic coefficients of UV divergences

UV divergence of each sub-loop renormalized by	corresponding one-loop diagram with	appropriate counterterm insertion
		
		
		

- unique identifier for each diagram necessary in order to
  - select the correct topology
  - insert the correct fields
- UV divergence of each individual one-loop diagram necessary for all (most) two-, three- and four-point functions

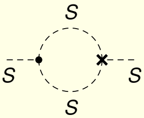
Computation of diagrams

- calculate all 121 genuine two-loop diagrams using `TwoCalc`, for example



$$= -\frac{1}{2}\frac{1}{(16\,\pi^2)^2}\, SSS_{(i1,-i3,i4,1)}\, SSS_{(i2,i3,i7,1)}\, SSS_{(-i4,i5,i6,1)}\, SSS_{(-i5,-i6,-i7,1)}\, T_{11234}\Big(m_{S(i4)}, m_{S(i7)}, m_{S(i3)}, m_{S(i6)}, m_{S(i5)}\Big)$$

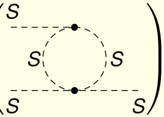
- calculate all 24 one-loop diagrams with counterterm insertion using `OneCalc`, e. g.



$$= \frac{1}{2}\frac{1}{16\,\pi^2}\, SSS_{(i1,-i3,-i4,1)}\, \delta SSS_{(i2,i3,i4,1)}\, B_0\Big(p, m_{S(i3)}, m_{S(i4)}\Big)$$

- calculate all one-loop divergences using `FormCalc`, for example

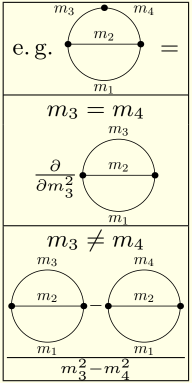
$$\delta SSS_{(i1,i2,i3)}\left(\quad\right)$$



$$= \frac{1}{\epsilon}\frac{1}{2}\frac{1}{16\,\pi^2}\, SSS_{(i1,-i4,-i5,1)}\, SSSS_{(i2,i3,i4,i5,1)}$$

- store each diagram together with its unique label re-implementation of two-loop corrections in **FeynHiggs** [see SP, arXiv:2204.12839]:

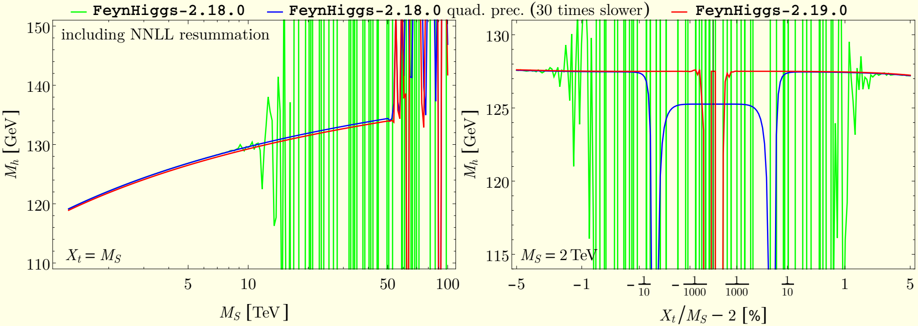
- delayed integral reduction → improved numerical stability:



e. g. in $\tilde{t}$ sector: kinematical situation a priori unclear; if single SUSY scale M_S:

$M_S \gg M_{\text{EW}}$: mixing negligible → degeneracies

$X_t \sim 2\,M_S$: $m_{\tilde{t}_{1,2}} = M_S \pm m_t$ → at threshold

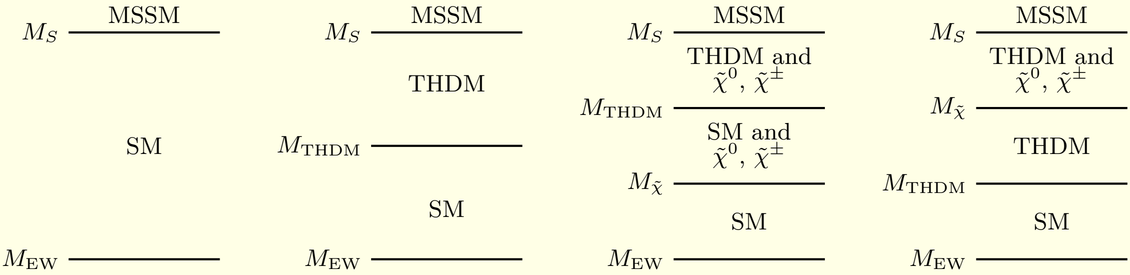


- sum over generalized couplings
  - → reduced algebraic size
  - → improved readability
  - → easier implementation of different schemes

FeynHiggs-2.18.0	FeynHiggs-2.19.0
$\mathcal{O}\left(\alpha_t\,\alpha_s\right)$: 2.9 MB	$\mathcal{O}\left(\alpha_{t,b}\,\alpha_s\right)$: 224 KB
$\mathcal{O}\left(\alpha_t^2\right)$: 364 KB	$\mathcal{O}\left(\left(\alpha_t + \alpha_b\right)^2\right)$: 364 KB

single scale hierarchy:

multi-scale hierarchies: required if large hierarchies among MSSM masses



EFTs in `FeynHiggs`: ($\tilde{\chi}^0$: neutralinos, $\tilde{\chi}^\pm$: charginos, $\tilde{g}$: gluinos)

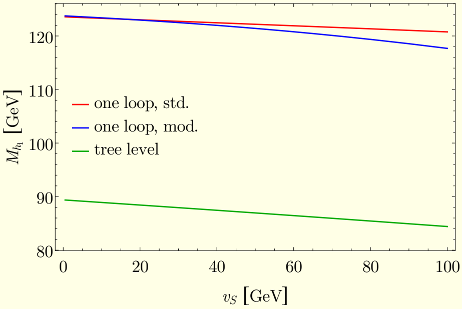
- SM (resums $\ln M_S/M_{\text{EW}}$)
- SM and $\tilde{\chi}^0$, $\tilde{\chi}^\pm$ (resums $\ln M_S/M_{\tilde{\chi}}$)
- SM and $\tilde{g}$ (resums $\ln M_S/M_{\tilde{g}}$)
- SM and $\tilde{\chi}^0$, $\tilde{\chi}^\pm$, $\tilde{g}$
- THDM (resums $\ln M_S/M_{\text{THDM}}$)
- THDM and $\tilde{\chi}^0$, $\tilde{\chi}^\pm$
- THDM and $\tilde{\chi}^0$, $\tilde{\chi}^\pm$, $\tilde{g}$

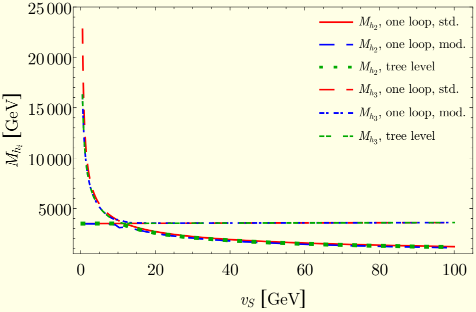
[Bahl, Hollik, arXiv:1805.00867] [Bahl, Sobolev, arXiv:2010.01989] [Bahl, Murphy, Rzehak, arXiv:2010.04711]

Contents

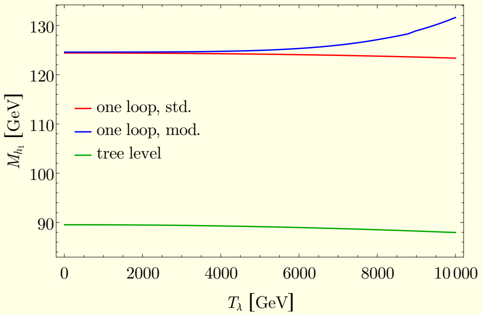
Martin scheme: 'std.'	Fleischer–Jegerlehner scheme: 'mod.'
expectation value fixed to all orders	different expectation value at each order
no tadpole diagrams	explicit tadpole diagrams
gauge invariance manifestly broken	gauge-invariant results
$\Delta M_h \supset -\frac{1}{v}\, t_h$	$\Delta M_h \supset -\frac{6\,\lambda\, v}{m_h^2}\, t_h$
$\Rightarrow$ expect problems for small v	$\Rightarrow$ expect problems for small m_h

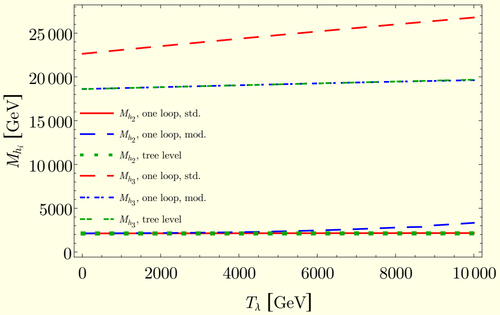
consider singlet extension with v_S (here: NMSSM)





Tadpole schemes





at large trilinear coupling T_λ: enhanced contribution in modified scheme from

$$\hat{\Sigma}_{ij}(p^2) \supset \frac{1}{16\,\pi^2}\,\frac{1}{2\,m_k^2}\,a_{ijk}\,a_{kll}\,A_0(m_l^2)$$

probably better setup: electroweak vev in standard scheme,

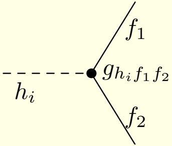
BSM vev in modified scheme

Contents

Two-body Higgs decays

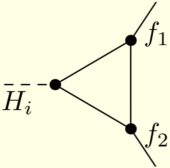
- tree-level decay of h_i into fields f_1 and f_2: $\mathcal{L} \ni g_{h_i f_1 f_2}\, h_i\, f_1\, f_2$
  widths are given by $\Gamma\big[h_i \to f_1\, f_2\big] = c_{\text{kin}} \sum\limits_{\text{dof}} \big|\mathcal{A}\big[h_i \to f_1\, f_2\big]\big|^2$ with amplitude

$$\mathcal{A}\big[h_i \to f_1\, f_2\big] = \mathcal{A}^{\text{tree}}\big[h_i \to f_1\, f_2\big] \equiv$$

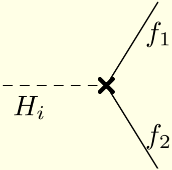


- loop-order decays: symbolically, look for width $\Gamma\big[H_i \to f_1\, f_2\big]$ with amplitude

$$\mathcal{A}^{1\text{L}}\big[H_i \to f_1\, f_2\big] \sim$$



$+$



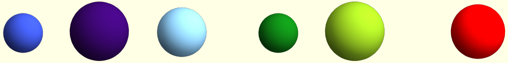
but loop-corrected fields $H_i \notin \mathcal{L}$ $\to$ how are couplings defined?

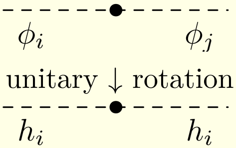
Loop-corrected fields H_i

for simplicity assume CP-conserving NMSSM:

- tree-level mass eigenstates = mixed gauge eigenstates:

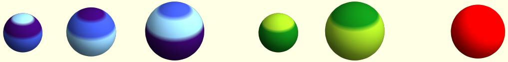
	CP-even			CP-odd		charged
h_i:	h	H	S	A	a	$H^{\pm}$

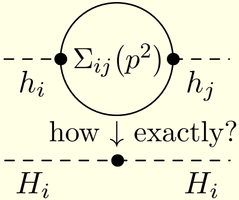




- off-diagonal entries in $\hat{\mathbf{\Sigma}}_{ij}(p^2)$ induce loop-mixing

	CP-even			CP-odd		charged
H_i:	H_1	H_2	H_3	H_4	H_5	$H^{\pm}$





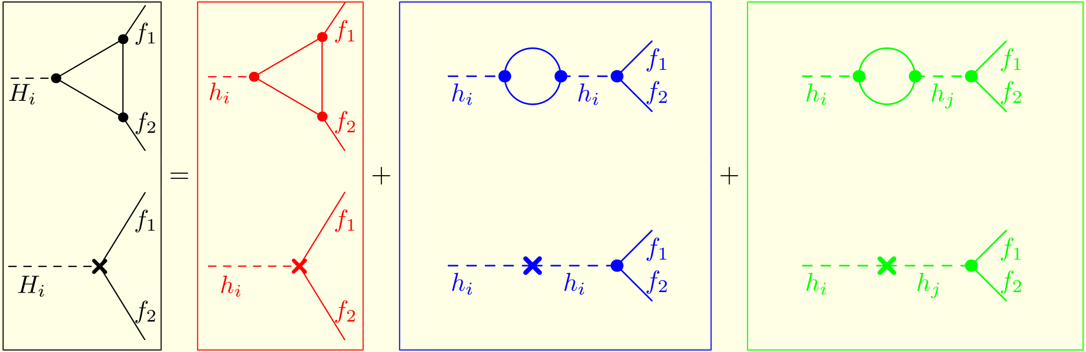
- with CP-mixing: H_1  , etc.

in agreement with LSZ reduction formula:

$$\Gamma\left[H_i \to f_1\, f_2\right] = \Gamma^{\text{tree}}\left[h_i \to f_1\, f_2\right] + c_{\text{kin}} \sum_{\text{dof}} 2\, \Re\left[\left(\mathcal{A}^{\text{tree}}\left[h_i \to f_1\, f_2\right]\right)^* \mathcal{A}^{1\text{L}}\left[H_i \to f_1\, f_2\right]\right]$$

$$\mathcal{A}^{1\text{L}}\left[H_i \to f_1\, f_2\right] = \mathcal{A}^{\text{vert}}\left[h_i \to f_1\, f_2\right] + \mathcal{A}^{\text{mix}}\left[h_i \to f_1\, f_2\right]$$

$$= \mathcal{A}^{\text{vert}}\left[h_i \to f_1\, f_2\right] - \frac{d\hat{\Sigma}_{h_i h_i}}{dp^2}\left(m_{h_i}^2\right) \mathcal{A}^{\text{tree}}\left[h_i \to f_1\, f_2\right] - \sum_{j \neq i} \frac{\hat{\Sigma}_{h_i h_j}\left(m_{h_i}^2\right)}{m_{h_i}^2 - m_{h_j}^2} \mathcal{A}^{\text{tree}}\left[h_j \to f_1\, f_2\right]$$



- if $\left|m^2_{h_i} - m^2_{h_j}\right| \sim \left|\hat{\Sigma}_{h_i h_j}(m^2_{h_i})\right|$, then for degenerate subspace

$$\left(M^2_{h_k}, H_k = S^*_{ki}\, h_i\right) \text{ eigensystem of } M^{2\,\text{eff}} = \operatorname{diag} m^2_{h_i} - \hat{\Sigma}^{\text{eff}}_{h_i h_j}\,,$$

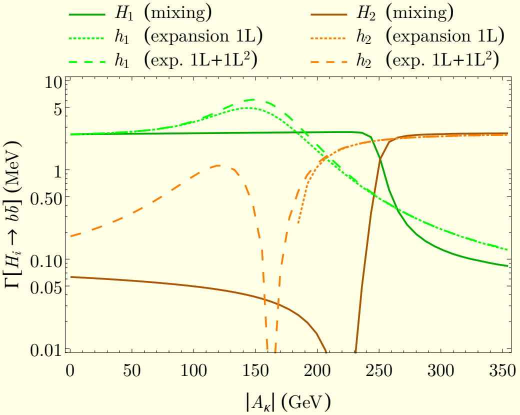
$$\hat{\Sigma}^{\text{eff}}_{h_i h_j} = \hat{\Sigma}_{h_i h_j}\left(\tfrac{1}{2}\left(m^2_{h_i} + m^2_{h_j}\right)\right)$$

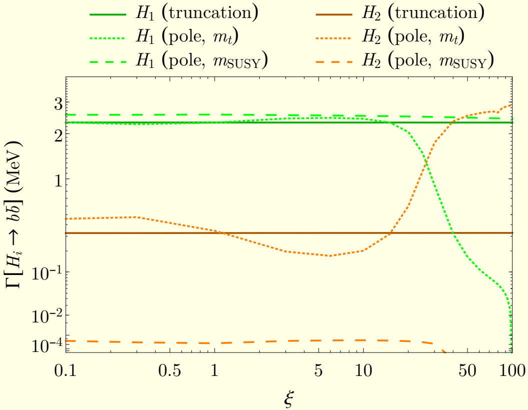
- decays of $H_k \in$ degenerate subspace:

$$\mathcal{A}^{1\text{L}}\left[H_k \to f_1\, f_2\right] = S_{ki}\left\{\mathcal{A}^{\text{vert}}\left[h_i \to f_1\, f_2\right] - \frac{d\hat{\Sigma}_{h_i h_i}}{dp^2}(m^2_{h_i})\, \mathcal{A}^{\text{tree}}\left[h_i \to f_1\, f_2\right] - \sum_{j \neq i} \frac{\hat{\Sigma}_{h_i h_j}(m^2_{h_i}) - \hat{\Sigma}^{\text{eff}}_{h_i h_j}}{m^2_{h_i} - m^2_{h_j}}\, \mathcal{A}^{\text{tree}}\left[h_j \to f_1\, f_2\right]\right\}$$

with $\hat{\Sigma}^{\text{eff}}_{h_i h_j} = 0$ if i, j **not** near-degenerate
(i. e. leading wrong one-loop term in mixing is replaced by correct one;
remaining wrong higher-order terms in S_{ki})

near-degenerate example in NMSSM





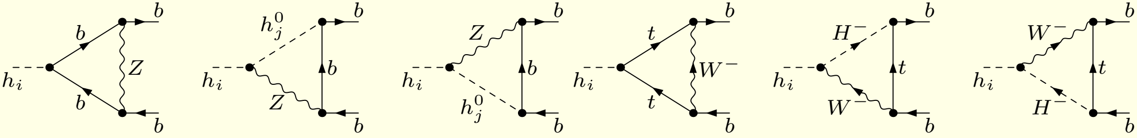
- $H_1 \to b\,\bar{b}$, $H_2 \to b\,\bar{b}$ } near-degenerate method
- $h_1 \to b\,\bar{b}$, $h_2 \to b\,\bar{b}$ } non-degenerate method

- $H_1 \to b\,\bar{b}$, $H_2 \to b\,\bar{b}$ } perturbative expansion (our method)
- $h_1 \to b\,\bar{b}$, $h_2 \to b\,\bar{b}$ } iterative method ($\mathbf{Z}$ matrix)

Contents

Heavy-Higgs decays into fermions

- consider decays of heavy-doublet Higgs states
- analyse two-body decays into fermion pair
- absorb QCD/QED corrections in IR-finite corrected Born width, i. e. radiated gluons/photons in final state are included
- resum $\tan\beta$-enhanced SUSY effects
- add the electroweak one-loop corrections
- identify large contributions, e. g. for $h_i \to b\bar{b}$



Two-body Sudakov double logarithms

- large electroweak contributions due to IR-enhanced squared logarithms
- e. g. for $h_i \to b\bar{b}$:

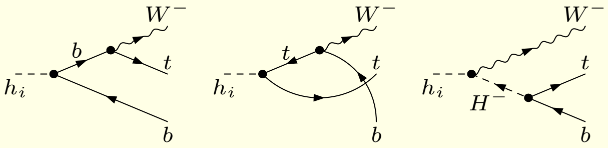
$$\frac{\Delta\Gamma^{\mathrm{DL}}}{\Gamma^{\mathrm{Born}}}\left[h_i \to b\bar{b}\,\right] \simeq -\frac{1}{48\,\pi^2}\left[\frac{5}{12}\,g_1^2 + \frac{9}{4}\,g_2^2 - \frac{2}{3}\,e^2\right] \ln^2 \frac{M_V^2}{M_{h_i}^2}$$

(terms $\propto e^2$ due to factorized QED)

- increasingly important for heavier Higgs (hierarchy with electroweak scale)
- destructive interference with tree level
- resummation possible as $\frac{\Gamma^{\mathrm{EW}}}{\Gamma^{\mathrm{Born}}} = \exp\left[\frac{\Delta\Gamma^{\mathrm{DL}}}{\Gamma^{\mathrm{Born}}}\right]$
- further large single logarithms
- however, not the end of the story . . .

Three-body decays

- tree-level three body same order as one-loop two body,
  i. e. take into account radiation of bosons at electroweak scale (W, Z, h^{SM})
  (in principle, distinguishable at experiments
  in contrast to radiated gluons/photons,
  but maybe difficult, see back-up)
- subtract on-shell contributions (already accounted for by two-body decays)
- identify large contributions, e. g. for radiation of W^-



Three-body Sudakov double logarithms

- large electroweak contributions in phase-space integrals
- e. g. $b\bar{b}$ final state:

$$\frac{\Gamma^{\mathrm{DL}}\left[h_i \to Zb\bar{b}\right]}{\Gamma^{\mathrm{Born}}\left[h_i \to b\bar{b}\right]} \simeq \frac{1}{48\,\pi^2}\left[\frac{5}{12}\,g_1^2 + \frac{3}{4}\,g_2^2 - \frac{2}{3}\,e^2\right]\ln^2\frac{M_V^2}{M_{h_i}^2}\,,$$

$$\frac{\Gamma_b^{\mathrm{DL}}\left[h_i \to W^- t\bar{b}\right]}{\Gamma^{\mathrm{Born}}\left[h_i \to b\bar{b}\right]} \simeq \frac{g_2^2}{64\,\pi^2}\ln^2\frac{M_V^2}{M_{h_i}^2} \qquad \Big|\times 2 \quad (\text{radiation of } W^+)$$

$\Rightarrow$ **sum** with two-body one-loop channel **free** of large logarithms

- resummation possible as $\frac{\Gamma^{\mathrm{EW,rad}}}{\Gamma^{\mathrm{Born}}} = 1 - \exp\left[\frac{\Delta\Gamma^{\mathrm{DL}}}{\Gamma^{\mathrm{Born}}}\right]$
- further large single logarithms

$\Rightarrow$ recover expected RGEs in sum with two-body single logarithms

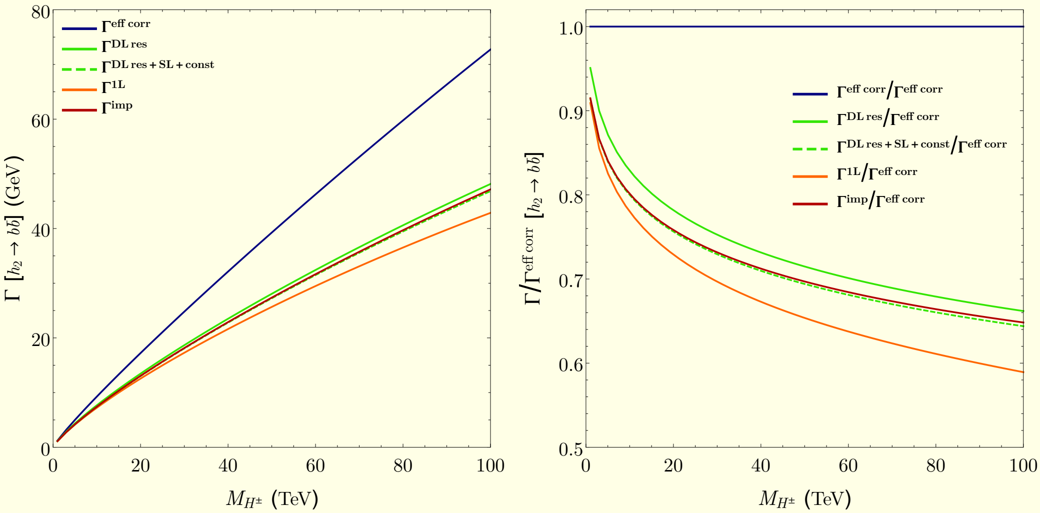
- exclusive widths: enhancement of both two-body and three-body widths

inclusive or total widths: free of such effects

- improved width

$$\Gamma^{\mathrm{imp}} = \frac{\Gamma^{\mathrm{QCD+QED}}}{\Gamma^{\mathrm{Born}}}\left\{\Gamma^{\mathrm{FO}} + \Gamma^{\mathrm{eff}}\left(\exp\left[\frac{\Delta\Gamma^{\mathrm{DL}}}{\Gamma^{\mathrm{Born}}}\right] - \left[1 + \frac{\Delta\Gamma^{\mathrm{DL}}}{\Gamma^{\mathrm{Born}}}\right]\right)\right\}$$

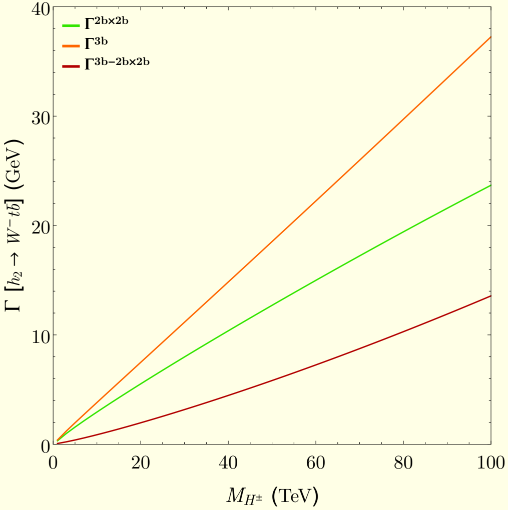
Two-body width example $h_i \to b\bar{b}$

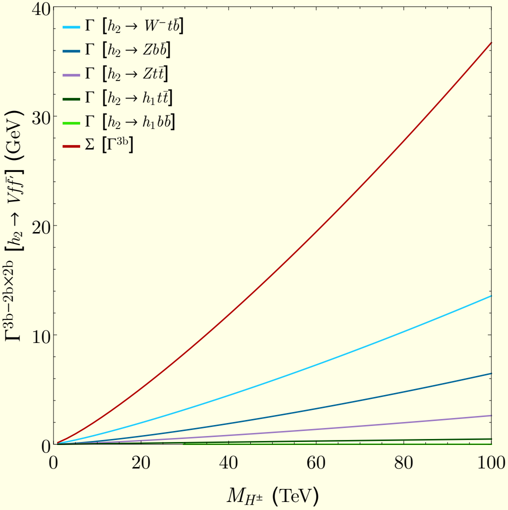


— QCD/QED and decoupling SUSY effects | — resummed double logs

— full one loop | — improved one loop

Three-body widths example h_i



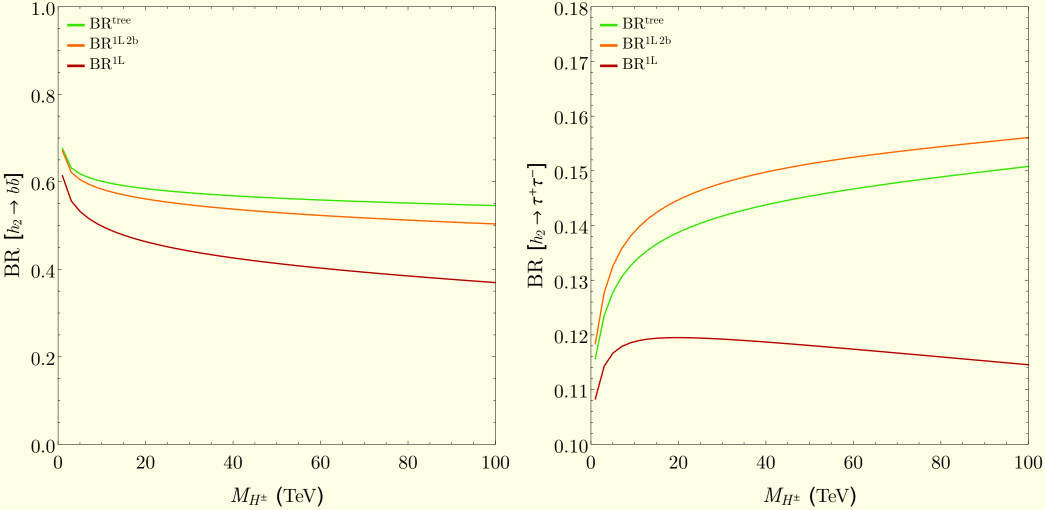


— full three body — on-shell two body

— remaining three body

various off-shell three-body contributions

Branching ratio $h_2 \to b\bar{b}$, $h_2 \to \tau^+\tau^-$

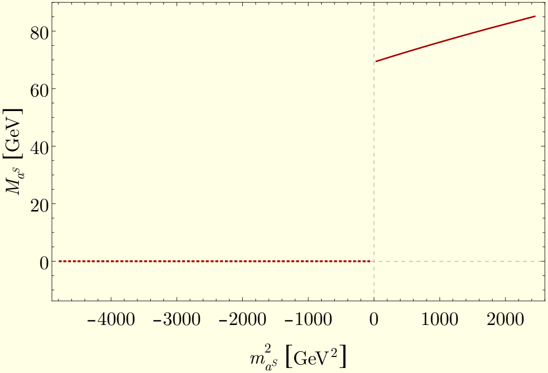


— tree level — one-loop two body

— one-loop two body and three body

Contents

light pseudo-scalar a^S in NMSSM with tree-level mass m_{a^S} and loop-corrected mass M_{a^S}



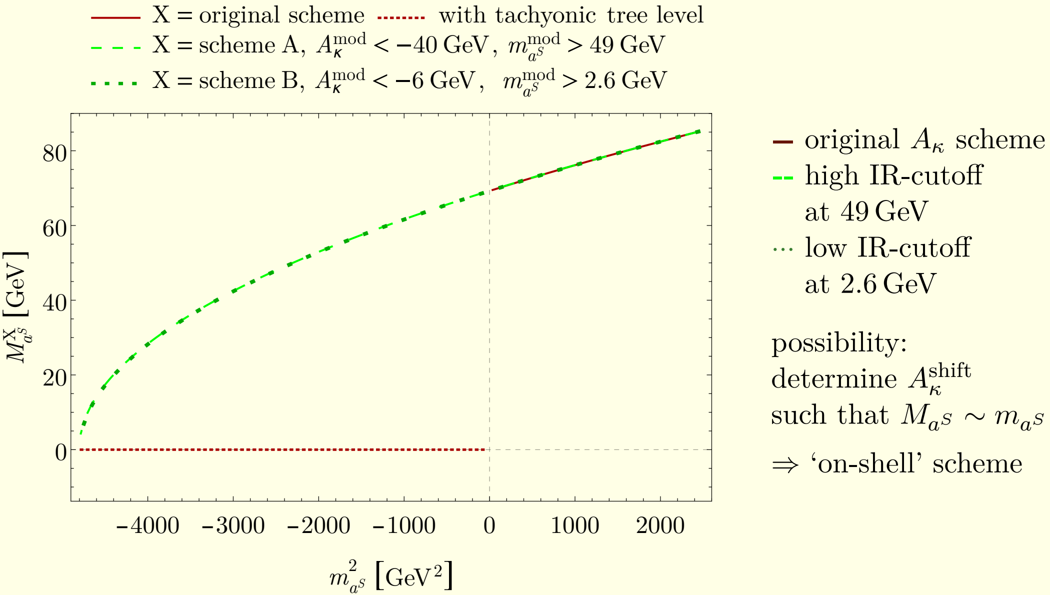
[see parameters in arXiv:2109.00585 [hep-ph]]

in case of free parameter: reparametrization, e. g.

$$A_\kappa + \delta A_\kappa \longrightarrow \underbrace{\left(A_\kappa + A_\kappa^{\text{shift}}\right)}_{\equiv A_\kappa^{\text{mod}}} + \underbrace{\left(\delta A_\kappa - A_\kappa^{\text{shift}}\right)}_{\equiv \delta A_\kappa^{\text{mod}}}$$

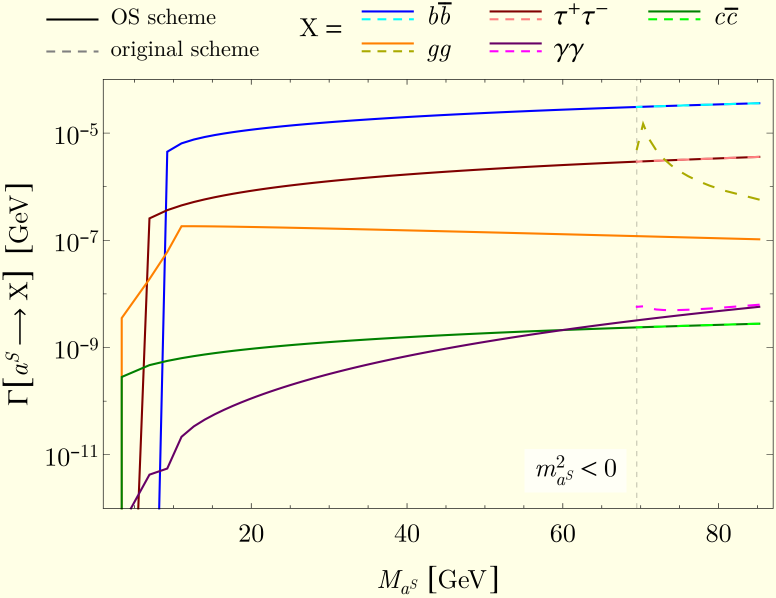
determine A_κ^{shift} such that $m^2_{a^S} > 0$

Modified scheme



possibility:
determine A_κ^{shift}
such that $M_{a^S} \sim m_{a^S}$

$\Rightarrow$ 'on-shell' scheme



dashed lines:
failure of
original scheme at
'tachyonic' boundary

solid lines:
on-shell scheme
works down to
$M_{a^S} \sim 0$

preferably put other light states with large mass corrections also on-shell, e. g. SM-like Higgs, but no free parameter

possible solution: deform Higgs potential (e. g. in MSSM)

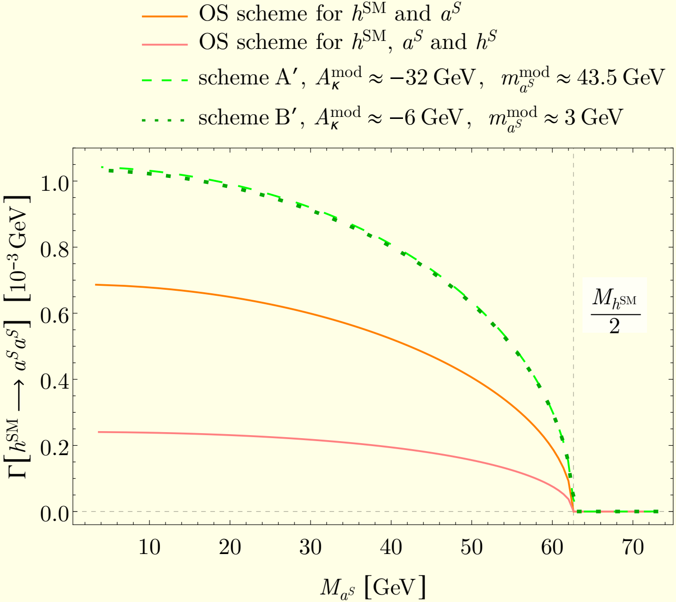
$$\begin{aligned}\mathcal{V}_{\text{THDM}} = {} & m_{H_d}^2\,|H_d|^2 + m_{H_u}^2\,|H_u|^2 + \left[m_{12}^2\,e^{\imath\,\varphi_{12}}\,H_u\cdot H_d + \text{h. c.}\right] \\ & + \tfrac{1}{2}\,\lambda_1\,|H_d|^4 + \tfrac{1}{2}\,\lambda_2\,|H_u|^4 + \lambda_3\,|H_u|^2\,|H_d|^2 + \lambda_4\,|H_u\cdot H_d|^2 \\ & + \left[\tfrac{1}{2}\,\lambda_5\,e^{\imath\,\varphi_5}\,(H_u\cdot H_d)^2 + \lambda_6\,e^{\imath\,\varphi_6}|H_d|^2\,H_u\cdot H_d + \lambda_7\,e^{\imath\,\varphi_7}|H_u|^2\,H_u\cdot H_d + \text{h. c.}\right],\end{aligned}$$

$$\lambda_1^{\text{MSSM}} = \frac{M_Z^2}{2\,v^2} = \lambda_2^{\text{MSSM}} = -\left(\lambda_3^{\text{MSSM}} + \lambda_4^{\text{MSSM}}\right)\,, \quad \lambda_4^{\text{MSSM}} = -\frac{M_W^2}{v^2}\,, \quad \lambda_{5,6,7}^{\text{MSSM}} = 0\,.$$

reparametrize $\lambda_i = \lambda_i^{\text{MSSM}} + \ell_i$,

purpose: define Higgs self-energies that are gauge-independent and only differ by a shift of higher (two-loop) order from those in the MSSM,

some freedom in assignment of ℓ_i, since under-constrained



modified A_κ scheme to control m_{a^S}:
- fix mass to 43.5 GeV
- fix mass to 3 GeV

modified A_κ scheme
and non-SUSY couplings:
- non-zero ℓ_2 to control $m_{h^{\text{SM}}}$
- non-zero ℓ_2 and $\hat{\kappa}$ to control $m_{h^{\text{SM}}}$ and m_{h^S}

some prediction possible,
but $\mathcal{O}(100\%)$ uncertainty
$\Rightarrow$ higher order required

at low mass: $\Gamma \sim 1$ MeV
$\Rightarrow$ probably excluded

Next

- gauge-boson/fermion self-energies, scalar–vector mixing
- λ_{hhh} at $p_i^2 = 0$
- G_F (muon decay)

$\vdots$