WP9.5: Phase Monitoring

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Phase noise basics (1)

- Phase noise is usually given in terms of dBc/Hz for oscillators and associated devices.
- That is, we are given the power in the components of the noise spectrum relative to the carrier.
- In order to find the total noise of the system, this noise needs to be integrated over the frequencies of interest.

Phase noise basics (2)

- Suppose we have a oscillator outputting some noisy signal:

$$V(t)=\sin(\omega_0 t+\phi_N(t))$$
$$V(t)=\sin(\omega_0 t)\cos(\phi_N(t))+\cos(\omega_0 t)\sin(\phi_N(t))$$
$$V(t)\approx\sin(\omega_0 t)+\cos(\omega_0 t)\phi_N(t)$$

- The phase noise, $\phi_N(t)$, can be rewritten as an integral:

$$\phi_N(t)=\int a_\omega \sin(\omega t+\theta_\omega)\,d\omega$$

- The power in one of these components, compared to the power in the carrier, is just a_ω^2. The noise power spectrum is just these coefficients:

$$N_P(\omega)=10\log_{10}(a_\omega^2)$$

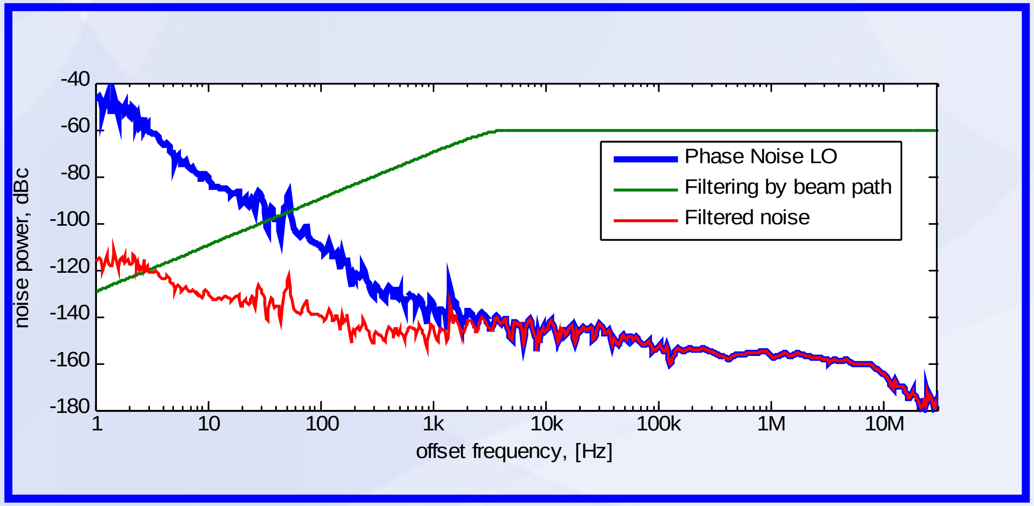
Task 9.5

Phase noise basics (3)

- The upper bound of our for the frequencies we include in our noise calculation will be given by the lowest bandwidth in the oscillator signal path.
- Good oscillators very quickly get down to the thermal noise floor (-174 dBm) where filtering can no longer help.
- The lower frequency bound will be given by how how long we need to keep accurate time for.
- The lower bound is usually much more important as we find higher noise power the closer we get to the carrier frequency.
- Keeping time accurately for a long time is *difficult*!

Task 9.5

Phase noise example



- We see that the phase noise increases significantly towards low frequencies.
- In the CLIC case, we know that we have to keep time for at most 160 µs.
- This will impose a high pass filtering function on the system

Total noise

- To calculate total noise:
- Take our noise function, $N_P(\omega)$, and the system filter function $H^2(\omega)$
- Then calculate $N_{total}=\sqrt{\int_{\omega_L}^{\omega_U} 10^{H^2(\omega)N_P(\omega)/10}\, d\omega}$
- This is the RMS voltage noise, compared to the 1 we get from the carrier
- This is thus directly the phase noise in Radians
- If we would like to know it in terms of time, compute: $\frac{T\, N_{total}}{2\pi}$ where T is the period of the oscillator

How to improve noise performance

- Averaging, averaging, averaging
- Must ensure that the various copies of the signal are added together in phase
- Noise add in squares
- Signal to noise ratio improvement goes with the square-root of the number of devices.

AM-PM conversion

- If we are trying to build a high resolution phase measuring system, it is not just noise that concern us. We must also be able to treat the RF containing phase information with sufficiently low distortion.

Carrier	Phase Modulation (quadrature)	Amplitude Modulation (inline)
$\sin(\omega_0 t)$	$\varphi_0 \sin(\omega_d t)\cos(\omega_0 t)$	$a\cos(\omega_d t)\sin(\omega_0 t)$

$$-\frac{\varphi_0}{2}\{\sin([\omega_0+\omega_d]t)-\sin([\omega_0-\omega_d]t)\}$$

$$\frac{a}{2}\{\sin([\omega_0+\omega_d]t)+\sin([\omega_0-\omega_d]t)\}$$

AM-PM amplitude flatness

- Suppose we have a device that affect the amplitude of the two frequencies differently:

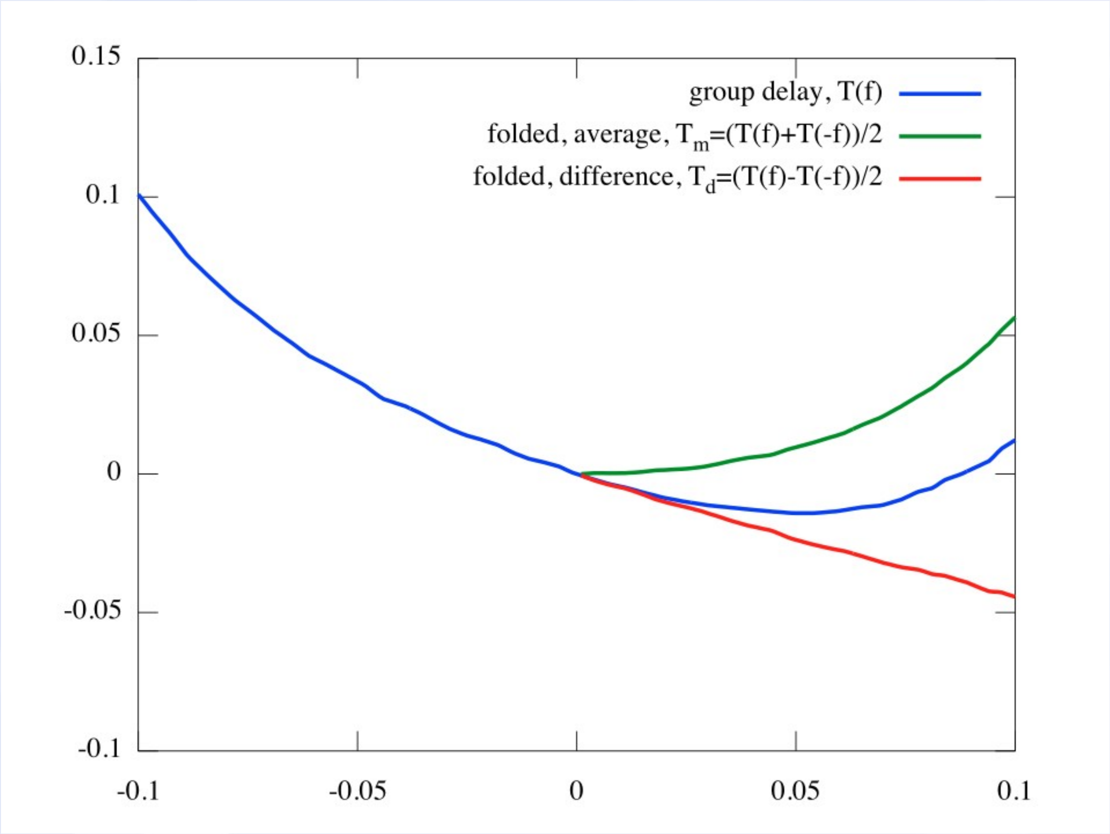
$$a_1 \sin([\omega_0+\omega_d]t)+a_2 \sin([\omega_0-\omega_d]t)$$ Rewrite as sums and diffs

$$\frac{a_1+a_2}{2}\sin([\omega_0+\omega_d]t)+\frac{a_1+a_2}{2}\sin([\omega_0-\omega_d]t)$$ Amplitude Modulation

$$+\frac{a_1-a_2}{2}\sin([\omega_0+\omega_d]t)-\frac{a_1-a_2}{2}\sin([\omega_0-\omega_d]t)$$ Phase Modulation

- Phase modulation thus induced scales with the modulation depth and the defects in device amplitude flatness.
- There is a similar phase to amplitude conversion, which will scale the phase modulation by some amount

AM-PM dispersion



AM-PM dispersion

- Suppose we have a device that is dispersive, so that the frequency components of an amplitude modulation have different transit times

$$\sin(\omega_0 t)+a\{\sin([\omega_0+\omega_d][t+T_m(\omega_d)+T_d(\omega_d)])+\sin([\omega_0-\omega_d][t+T_m(\omega_d)-T_d(\omega_d)])\}$$

- Rewrite

$$\sin(\omega_0 t)$$
$$+a\cos(T_m\omega_0+T_d\omega_d)\cos(T_m\omega_d+T_d\omega_0+\omega_d t)\sin(\omega_0 t)$$ Amplitude Mod.
$$+a\sin(T_m\omega_0+T_d\omega_d)\cos(T_m\omega_d+T_d\omega_0+\omega_d t)\cos(\omega_0 t)$$ Phase Modulation

- The AM-PM conversion is thus $a\sin(T_m\omega_0)$ *with* $\omega_0 \gg \omega_d$

  that is, proportional to the size of the amplitude modulation and the average transit time difference of the two modulation components

AM-PM dispersion (2)

- The resulting modulation term

$$\cos(T_m \omega_d + T_d \omega_0 + \omega_d t) \approx \cos(T_d \omega_0 + \omega_d t)$$

  causes distortion of the signal envelope
- T_d is of course dependent on the modulation frequency ω_d. The linear part of $T_d(\omega_d)$ causes only a time-shift of the modulation, any non-linear part distorts the signal
- The phase modulation term works exactly the same way. That is, some phase modulation will turn into amplitude modulation, and some distortion of the phase signal will occur

AM-PM – some numbers

- Devices can be specified to 0.1dB flatness, but not 0.01dB. (Influence of connectors and cable become to large for reliable measurements to be made)
- With 0.1 dB flatness, we get 0.0033º per percent of amplitude modulation
- 0.1 dB flatness will scale a phase modulation by 0.5%
- At 12 Ghz, with 1% amplitude modulation, we get 0.04º per ps

AM-PM device non-linearities

$$A\sin(\omega_1 t)+B\sin(\omega_2 t)$$

$$\Rightarrow A\sin(\omega_1 t)+a_2A^2\sin^2(\omega_1 t)++a_3A^3\sin^3(\omega_1 t)+\ldots$$

$$+B\sin(\omega_2 t)+b_2B^2\sin^2(\omega_2 t)+b_3B^3\sin^3(\omega_2 t)+\ldots$$

$$+ABc_{1,1}\sin(\omega_1 t)\sin(\omega_2 t)+A^2Bc_{2,1}\sin^2(\omega_1 t)\sin(\omega_2 t)+AB^2c_{1,2}\sin(\omega_1 t)\sin(\omega_2 t)$$

$$\omega_{1,}\omega_{2,}2\omega_{1,}2\omega_{2,}2\omega_1-\omega_{2,}\omega_1-2\omega_{2,}\ldots$$

- By device non-linearities we get a large set of frequencies in the output that were not in the input.
- These grow with the amplitude to the power of term order of the input signals.
- By lowering input power we can have a smaller fraction of the output of these frequencies

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$$+A\,B\,c_{1,1}\sin(\omega_1 t)\sin(\omega_2 t)+A^2 B\,c_{2,1}\sin^2(\omega_1 t)\sin(\omega_2 t)+A\,B^2 c_{1,2}\sin(\omega_1 t)\sin(\omega_2 t)$$

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AM-PM device non-linearities

- In phase detection devices, there are in particular a second order term that mixes to baseband and is indistinguishable from real phase.
- We must thus lower input power until this contribution becomes small enough.
- This reduces our signal to noise ratio.
- Recover better SNR by using multiple devices in parallel and averaging their output.
- We get an improvement of square root of N for N devices.