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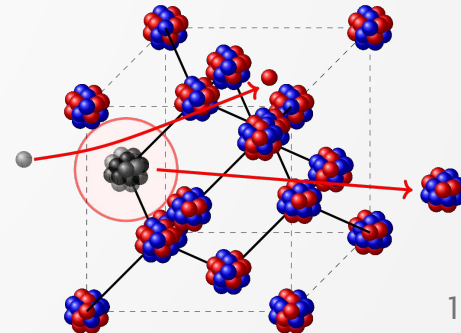


NIEL (non-ionizing energy loss)

Geant4 simulations and OPTICS algorithm towards more complex NIEL concept for radiation damage modelling and prediction



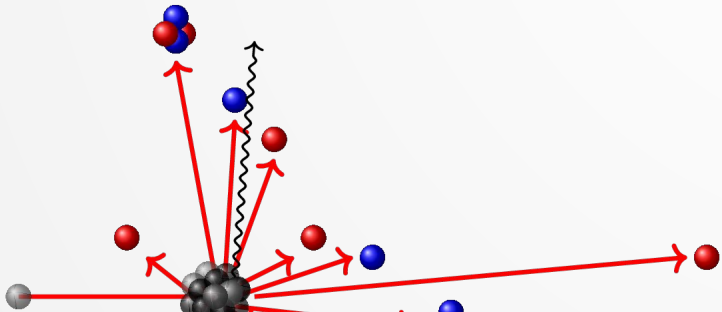
The 40th RD-50 workshop, CERN, 21.06.2022



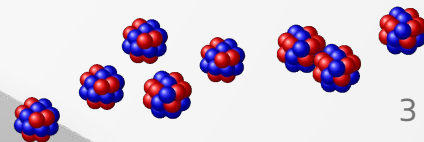
CONTENTS

1. The **NIEL** hypothesis: concept explanation and its shortcomings.
2. **Geant4** simulation framework
3. Primary knocked-on atoms (**PKA**)
4. OPTICS (Ordering points to identify the clustering structure) algorithm
5. Secondary knocked-on atoms (**SKA**)
6. Summary and next steps

The NIEL hypothesis



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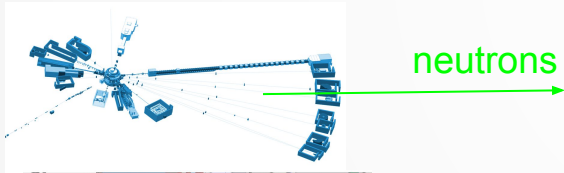
NIEL (non-ionizing energy loss)

- NIEL is a physical quantity describing the non-ionizing energy loss as the particle travels through the medium.
- The amount of NIEL can be correlated to the amount of radiation damage (NIEL scaling model) and therefore to predict the life time of the detectors.
- NIEL is usually expressed as an equivalent to NIEL of 1 MeV neutrons.

- NIEL scaling assumption is used by **most of the LHC experiments**

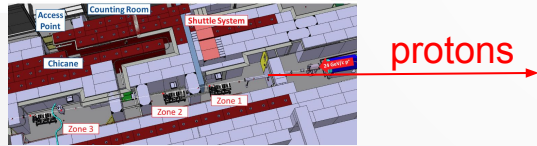
Long term goal: to revisit the damage factor stated by different irradiation facilities

ESS



neutrons

IRRAD



protons

GIF++



gamma-rays

NIEL (non-ionizing energy loss)

$$NIEL(T_0) = \frac{N_A}{A} \sum_i \int_{T_{min}}^{T_{max}} Q(T) T \left(\frac{d\sigma}{dT} \right)_i dT$$

Displacement damage function

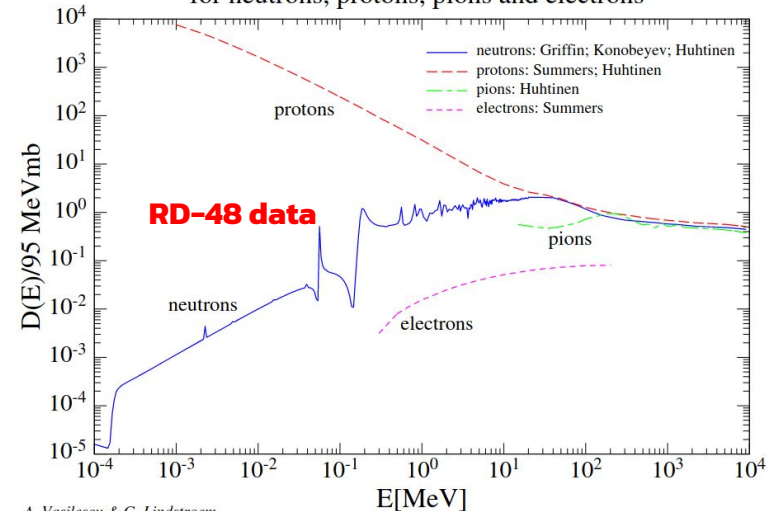
$$NIEL(T_0) = \frac{N_A}{A} D(T)$$

MeV cm²/g

MeV mb

- T_0 : energy of incident particle
- T : energy transferred to the recoil atom
- $(d\sigma/dT)$: differential partial cross section for a particle with energy T_0 to create a recoil atom with energy T in the i -th reaction
- $Q(T)$: partition factor giving the fraction of T that is going into further displacements
- N_A : Avogadro number
- A : atomic mass of target atom

Displacement damage in Silicon
for neutrons, protons, pions and electrons 1,2,3,4,5



A. Vasilescu & G. Lindstroem

- 1) Data from A. Vasilescu (INPE Bucharest) and G. Lindström (Univ. of Hamburg)
- 2) P.J. Griffin et al., SAND92-0094 (Sandia Natl. Lab.93), priv. comm. 1996: $E = 1.025E-10 - 1.995E+01$ MeV
- 3) Konobeyev, Alexander Yu., et al. "Nuclear Data to Study Damage in Materials under Irradiation by Nucleons with Energies up to 25 GeV." Journal of Nuclear Science and Technology, vol. 39, no. sup2, Aug. 2002, pp. 1236-39. Taylor and Francis+NEJM, <https://doi.org/10.1080/00223131.2002.10875327>.
- 4) Huhtinen, M., and P. A. Aarnio. "Pion Induced Displacement Damage in Silicon Devices." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, vol. 335, no. 3, Nov. 1993, pp. 580-82. ScienceDirect, [https://doi.org/10.1016/0168-9002\(93\)91246-J](https://doi.org/10.1016/0168-9002(93)91246-J).
- 5) Summers, G. P., E. A. Burke, P. Shapiro, et al. "Damage Correlations in Semiconductors Exposed to Gamma, Electron and Proton Radiations." IEEE Transactions on Nuclear Science, vol. 40, no. 6, Dec. 1993, pp. 1372-79. IEEE Xplore, <https://doi.org/10.1109/23.273529>.

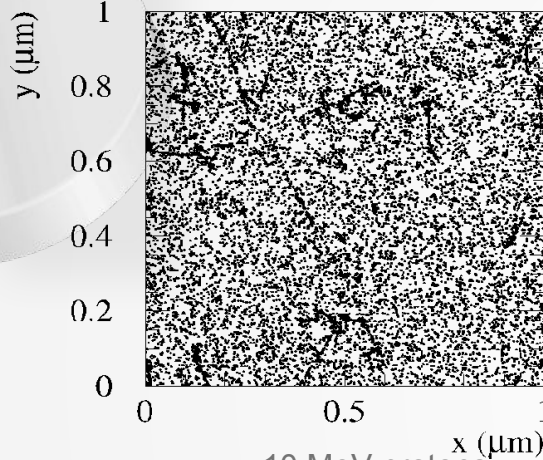
Revisiting NIEL

Simulations of radiation damage by M. Huhtinen⁶.

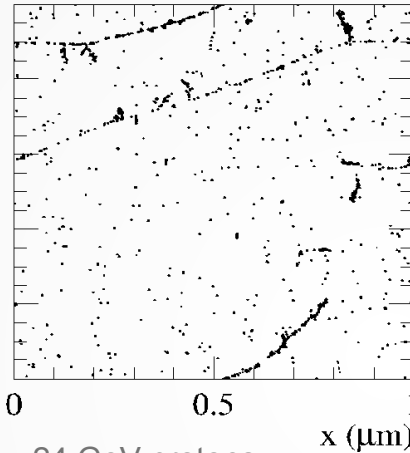
36824 vacancies

4145 vacancies

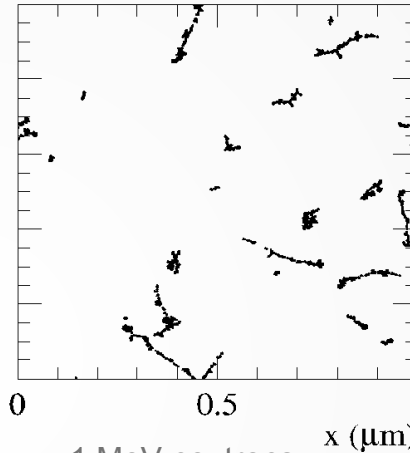
8870 vacancies



10 MeV protons



24 GeV protons



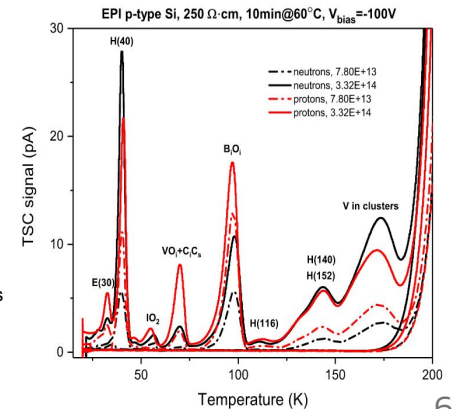
1 MeV neutrons

- NIEL doesn't distinguish between cluster and point displacement, i.e. the same displacement energy has a very different distribution of damage on the microscopic level.
- NIEL scaling violation reported in oxygen enriched silicon samples (CERN RD-48), differences between neutron's and proton's damage

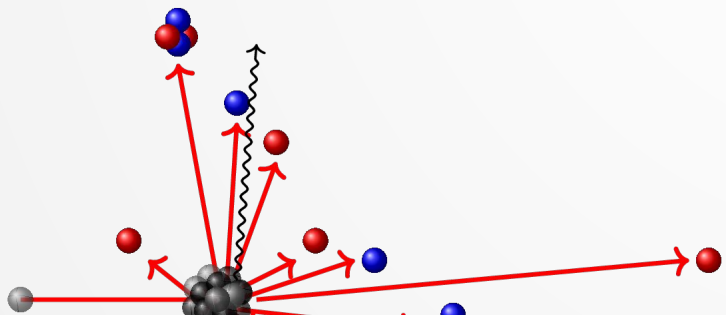
6) Huhtinen, M. "Simulation of Non-Ionising Energy Loss and Defect Formation in Silicon." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, vol. 491, no. 1, Sept. 2002, pp. 194–215. ScienceDirect, [https://doi.org/10.1016/S0168-9002\(02\)01227-5](https://doi.org/10.1016/S0168-9002(02)01227-5).

7) Gurinskaya, Yana, et al. "Radiation Damage in P-Type EPI Silicon Pad Diodes Irradiated with Protons and Neutrons." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, vol. 958, Apr. 2020, p. 162221. ScienceDirect, <https://doi.org/10.1016/j.nima.2019.05.062>.

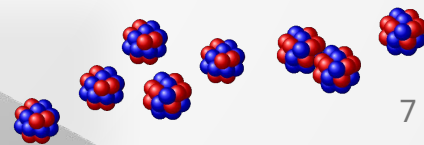
Radiation Damage in P-Type EPI Silicon Pad Diodes Irradiated with Protons and Neutrons⁷.



Geant4 simulations



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Geant4 simulation framework

Geant4^{8,9}(for GEometry ANd Tracking) is a Monte Carlo simulation platform for the passage of particles through matter.

Define a geometry:



For most of the simulations:
1mm x1 mm x100 μm

Define a beam profile:



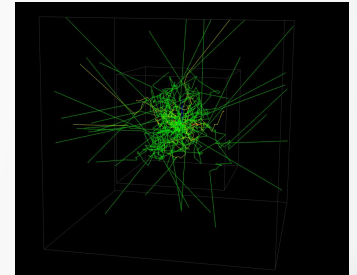
1. For PKA: Monochromatic pencil beam of protons and neutrons of various energies.
2. For subsequent cascade: Single Si particles of various energies being injected

Choose a physics list:

1. For PKA:
 - a. *QGSP_BERT_HP* (Nuclear scattering)
 - b. *QGSP_BERT_HP__SS* (Coulomb scattering)
2. For subsequent cascade:
 - a. *Custom PhysicsList*¹⁰

Launch a simulation:

QGSP_BERT_HP
QGSP_BERT_HP__SS
*Custom Physics*¹⁰



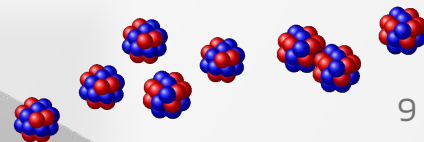
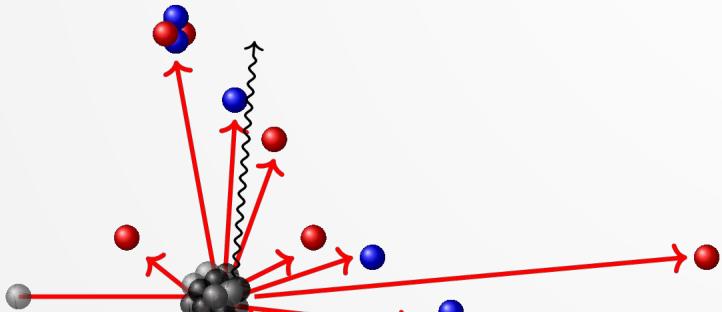
Analyze (c++, python)

8) Agostinelli, S., et al. "Geant4—a Simulation Toolkit." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, D Detectors and Associated Equipment, vol. 506, no. 3, July 2003, pp. 250–303. ScienceDirect, [https://doi.org/10.1016/S0168-9002\(03\)01368-8](https://doi.org/10.1016/S0168-9002(03)01368-8).

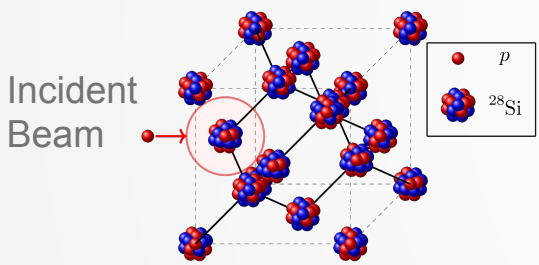
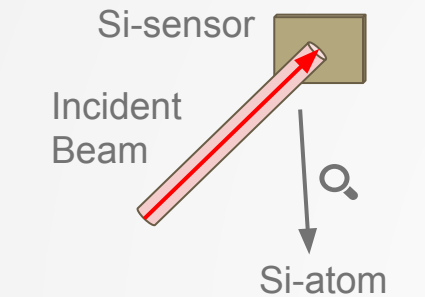
9) Allison, J., K. Amako, J. Apostolakis, H. Araujo, et al. "Geant4 Developments and Applications." IEEE Transactions on Nuclear Science, vol. 53, no. 1, Feb. 2006, pp. 270–78. IEEE Xplore, <https://doi.org/10.1109/TNS.2006.869826>.

10) Raine, Melanie, et al. "Simulation of Single Particle Displacement Damage in Silicon - Part I: Global Approach and Primary Interaction Simulation." IEEE Transactions on Nuclear Science, vol. 64, no. 1, Oct. 2016, pp. 133–40. HAL Archives Ouvertes, <https://doi.org/10.1109/TNS.2016.2615133>.

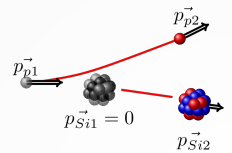
PKA (Primary knocked-on atoms)



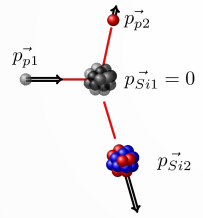
PKA cross section example



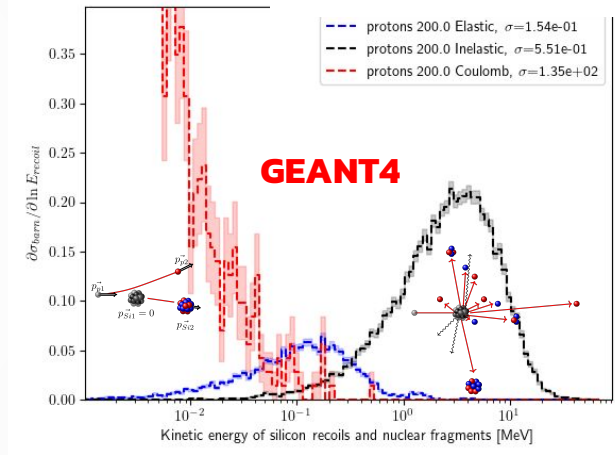
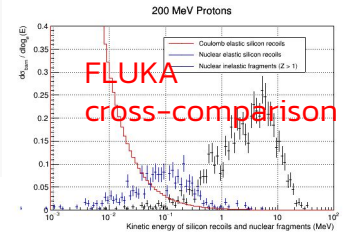
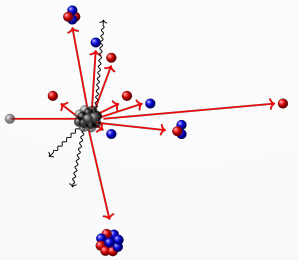
1) Coulomb elastic scattering (only charged particles)



2) Nuclear elastic scattering



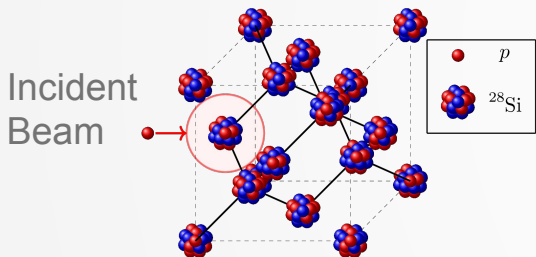
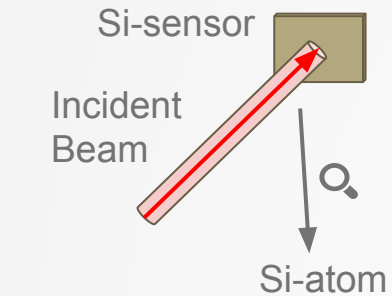
3) Nuclear inelastic scattering



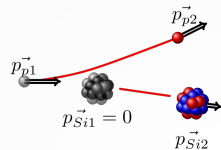
Area below the curve corresponds to the cross section.

$$NIEL(T_0) = \frac{N_A}{A} \sum_i \int_{T_{min}}^{T_{max}} Q(T) T \left(\frac{d\sigma}{dT} \right)_i dT$$

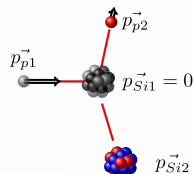
PKA cross section example



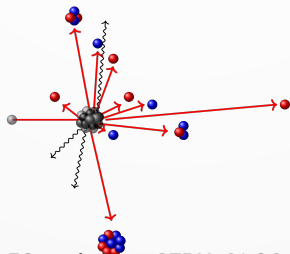
1) Coulomb elastic scattering (only protons)



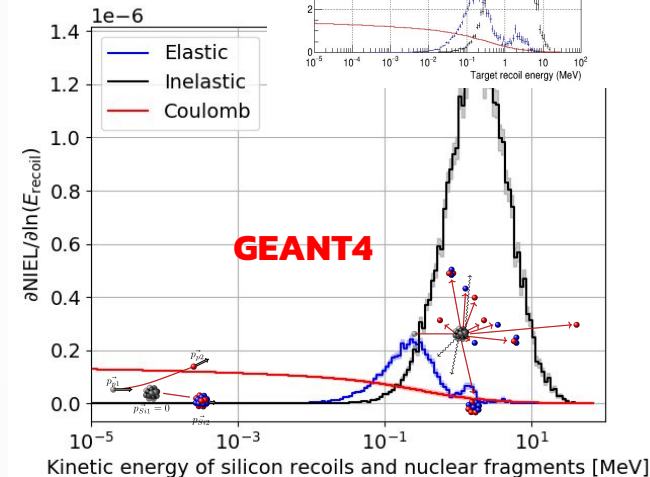
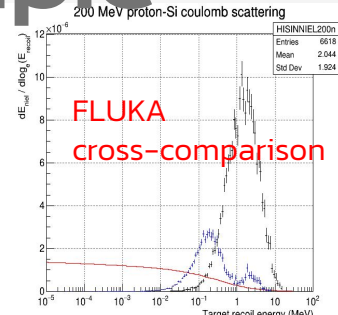
2) Nuclear elastic scattering



3) Nuclear inelastic scattering



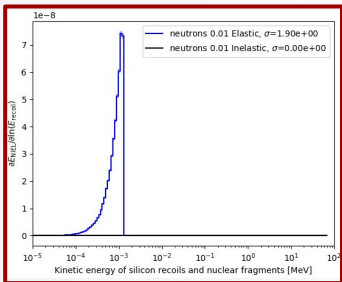
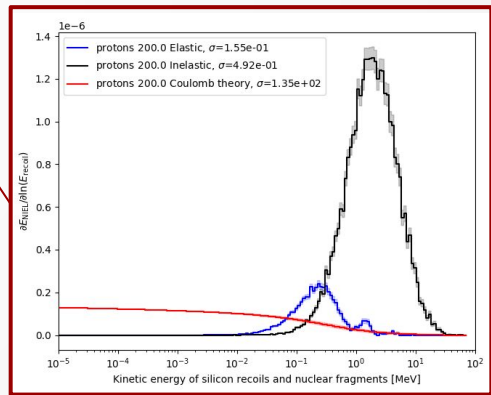
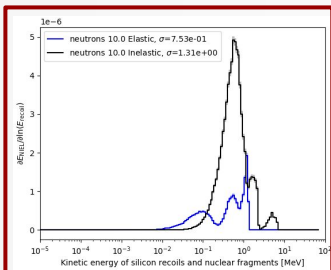
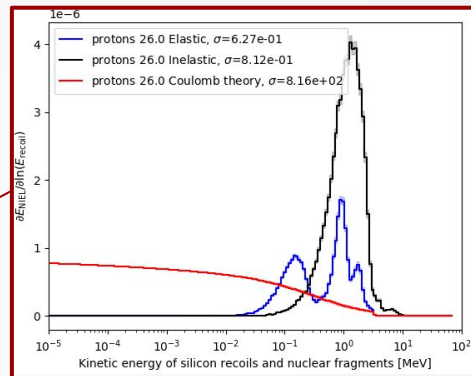
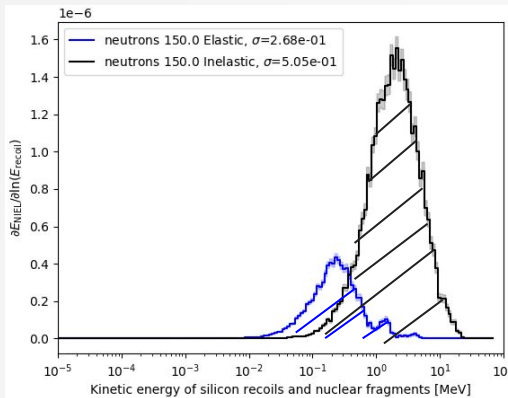
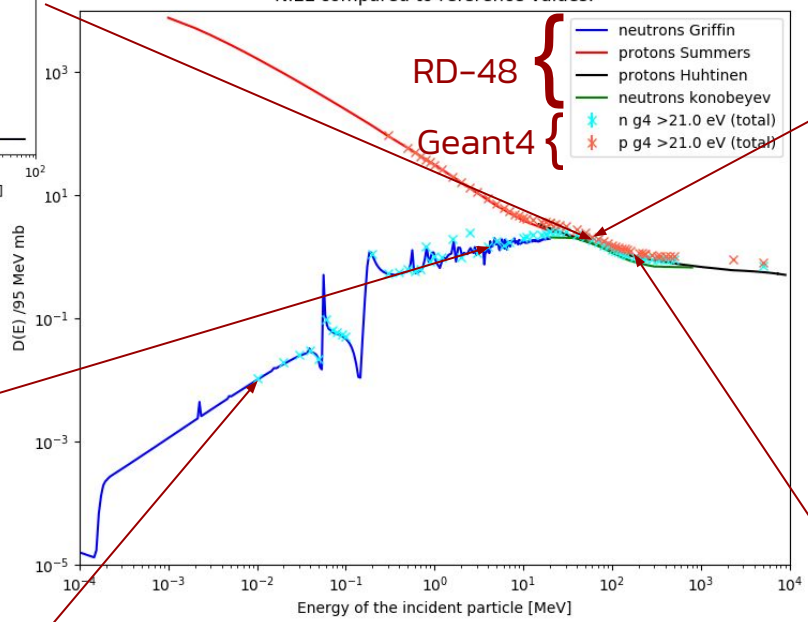
$$NIEL(T_0) = \frac{N_A}{A} \sum_i \int_{T_{min}}^{T_{max}} Q(T) T \left(\frac{d\sigma}{dT} \right)_i dT$$



Area below the curve corresponds to the NIEL for respective reaction.

NIEL

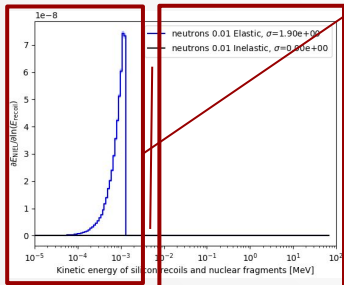
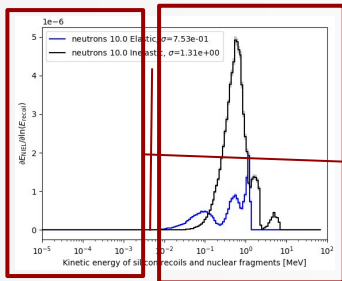
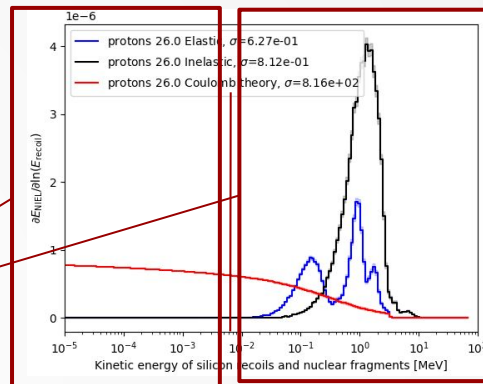
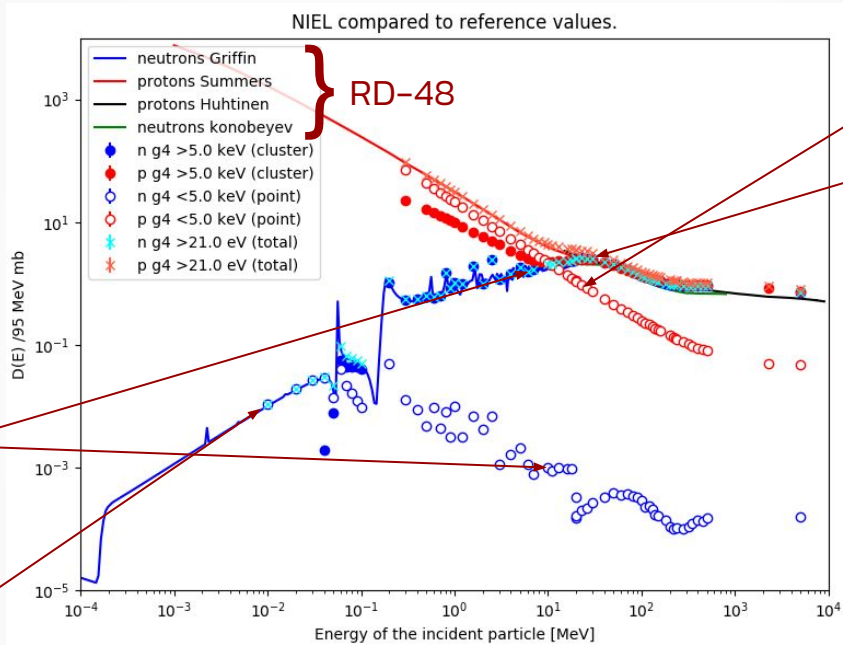
NIEL compared to reference values.



NIEL

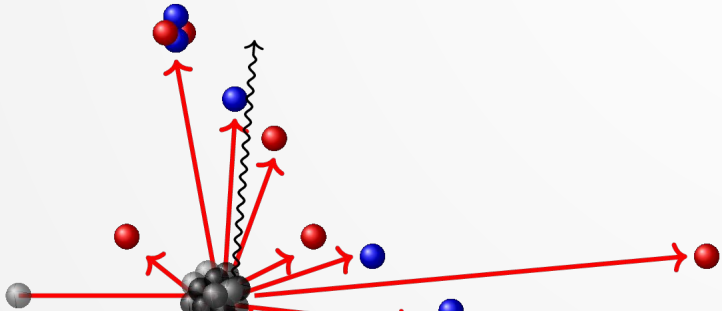
Recoils < 5 keV: point displacements

Recoils > 5 KeV: point displacements+ cluster displacements

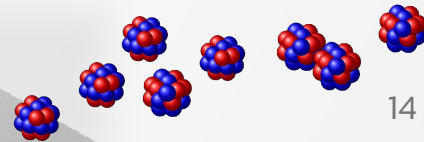


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SKA Secondary knocked-on atoms)

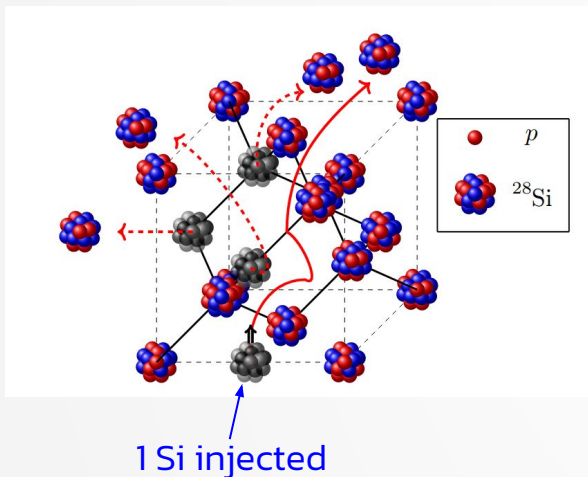


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How to define a cluster? What algorithm to choose?

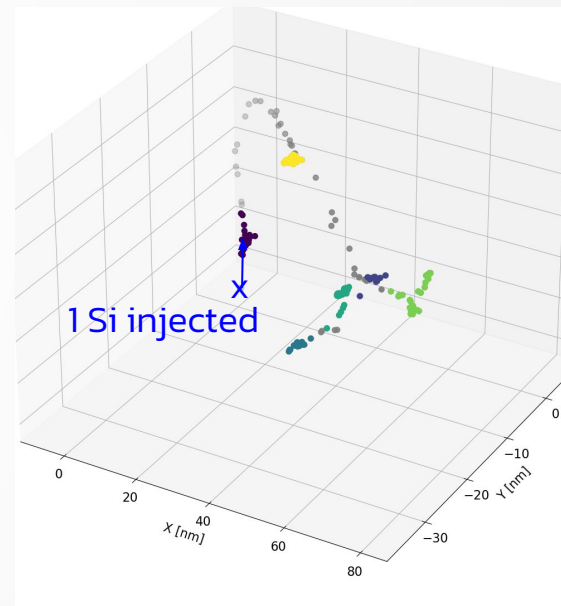
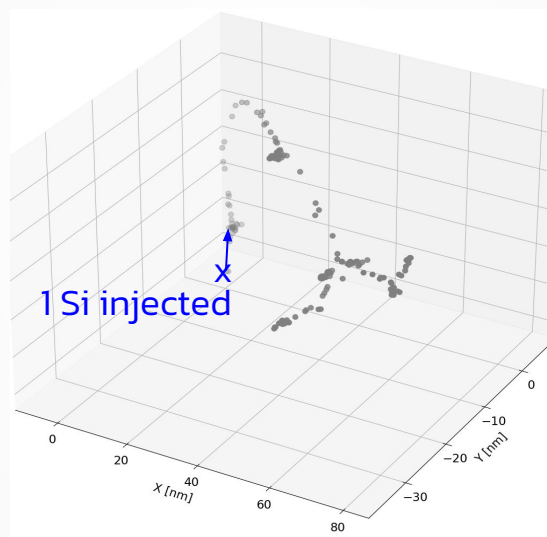
- Geant4 studies with Screened Nuclear Recoil process¹⁰
- focus on the propagation of Si-recoil in Silicon (no incident beam)



Example:

- 100 keV Silicon track
- originating from **the blue cross** (position 0,0,0)
- initial momentum in +z direction

Grey dots: isolated displacements
Colored dots: clustered displacements

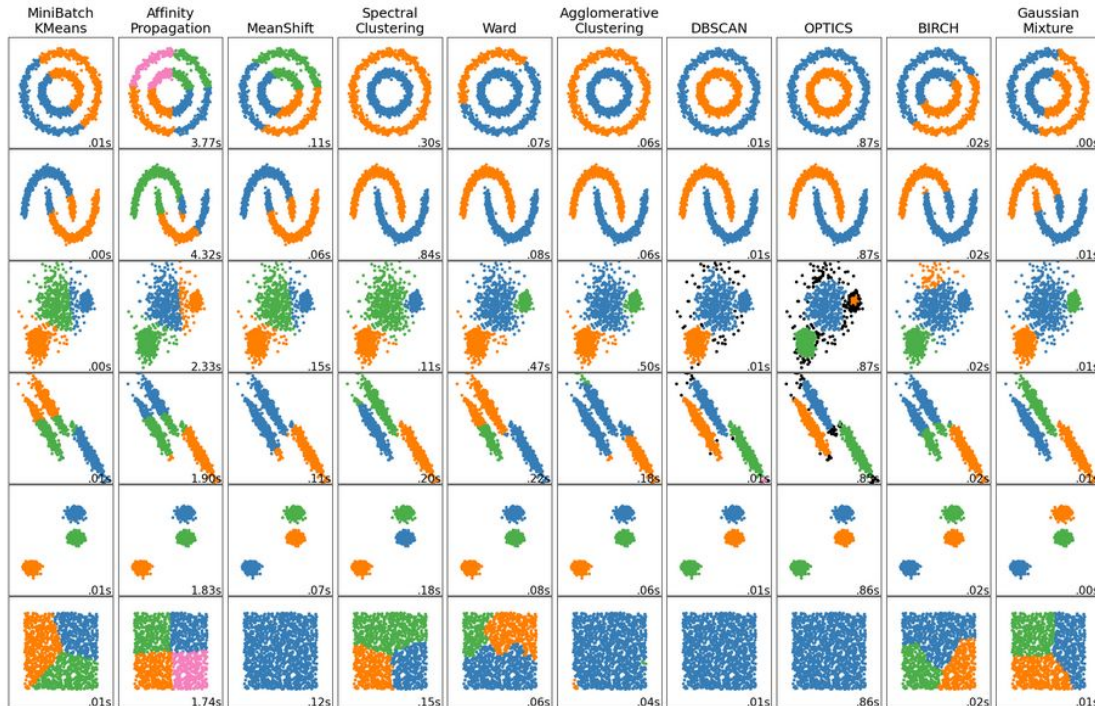


10) Raine, Melanie, et al. "Simulation of Single Particle Displacement Damage in Silicon - Part I: Global Approach and Primary Interaction Simulation." IEEE Transactions on Nuclear Science, vol. 64, no. 1, Oct. 2016, pp. 133–40. HAL Archives Ouvertes, <https://doi.org/10.1109/TNS.2016.2615133>.

Clusters

- Cluster detection is a big topic in machine learning and mathematics
- Depending on its application, different algorithms are the best fit.

Image from^{11,12}



- Various clustering algorithms applied to 6 different sample datasets
- aim : identify clusters
- Algorithm must be able to process:
 - samples with large number of "outliers" (=single displacements for us)
 - samples with clusters of different shapes
 - samples with clusters with various densities

11) Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., ... Duchesnay, E. (2011). Scikit-learn: Machine Learning in Python. *Journal of Machine Learning Research*, 12, 2825–2830.

12) https://scikit-learn.org/stable/auto_examples/cluster/plot_cluster_comparison.html#sphx-glr-auto-examples-cluster-plot-cluster-comparison-py

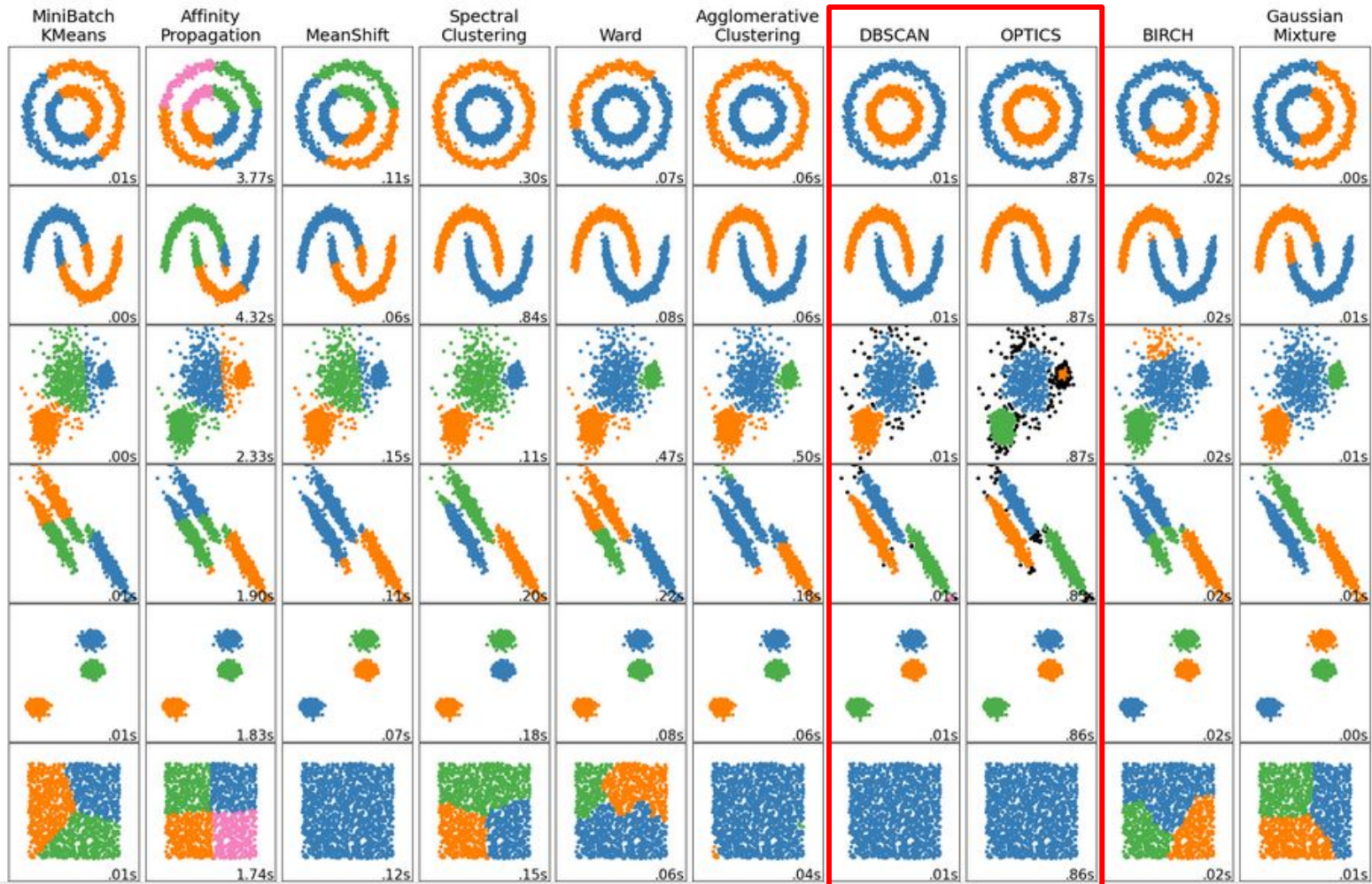


Image from^{11,12}

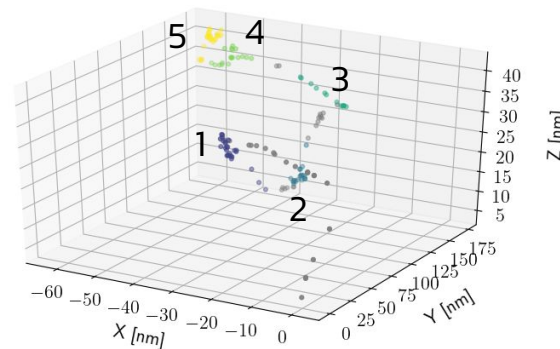
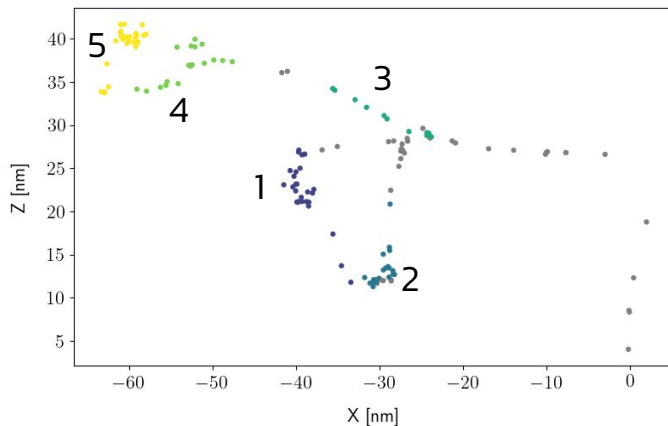
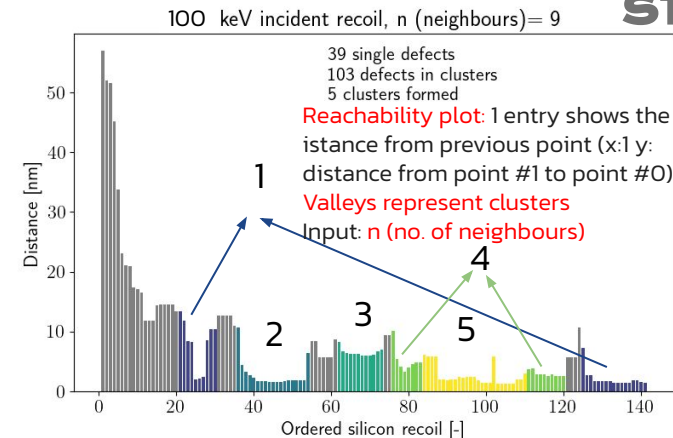
OPTICS performs better than DBSCAN for clusters with **varying densities**¹³.

11) Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., ... Duchesnay, E. (2011). Scikit-learn: Machine Learning in Python. *Journal of Machine Learning Research*, 12, 2825–2830.

12) https://scikit-learn.org/stable/auto_examples/cluster/plot_cluster_comparison.html#sphx-glr-auto-examples-cluster-plot-cluster-comparison-py

13) <https://scikit-learn.org/stable/modules/clustering.html#optics>

OPTICS^{13,14} (Ordering points to identify the clustering structure)



- Algorithm flow explain in the Appendix (Slide 31)
- Basic idea:
 - Ordering points and plotting their distances produces **Reachability plot**
 - Valleys in the reachability plot represent clusters
- Algorithm needs a user input: minimum number of samples to create cluster

13) Ankerst, Mihael, Markus M. Breunig, Hans-Peter Kriegel, and Jörg Sander. "OPTICS: ordering points to identify the clustering structure." ACM SIGMOD Record 28, no. 2 (1999): 49-60.

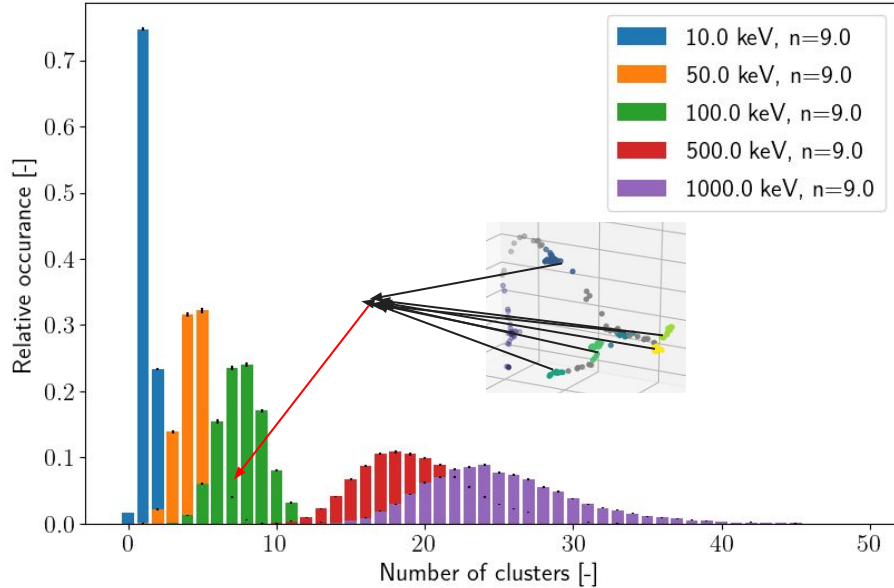
14) Schubert, Erich, Michael Gertz. "Improving the Cluster Structure Extracted from OPTICS Plots." Proc. of the Conference "Lernen, Wissen, Daten, Analysen" (LWDA) (2018): 318-329.

15) <https://www.youtube.com/watch?v=CV0mWaHOTA8&t=133s> tutorial

RESULTS

How many clusters?

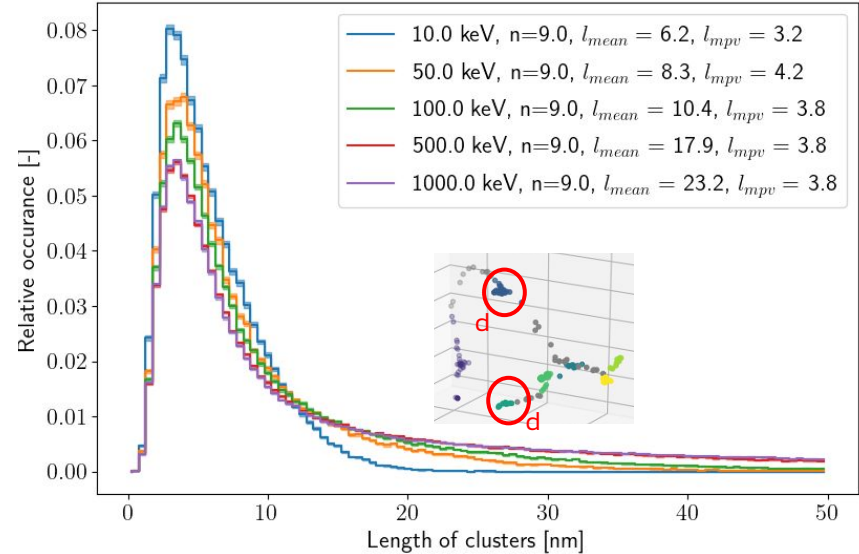
Relative occurrence of a particular number of clusters being created per event.



How large are the clusters?

Length := diameter of the smallest enclosed sphere around the cluster

Relative occurrence of a particular length of clusters being created per event.



- If the cluster is too big, the algorithm prefers to break the cluster down into a smaller subclusters.
- While the cluster size remains similar with energy, the number of clusters increases.

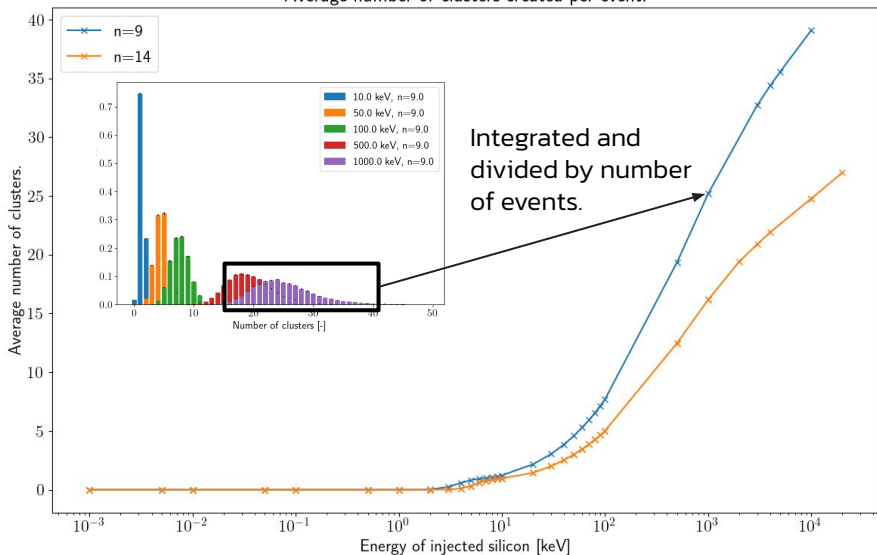
RESULTS

How many clusters?

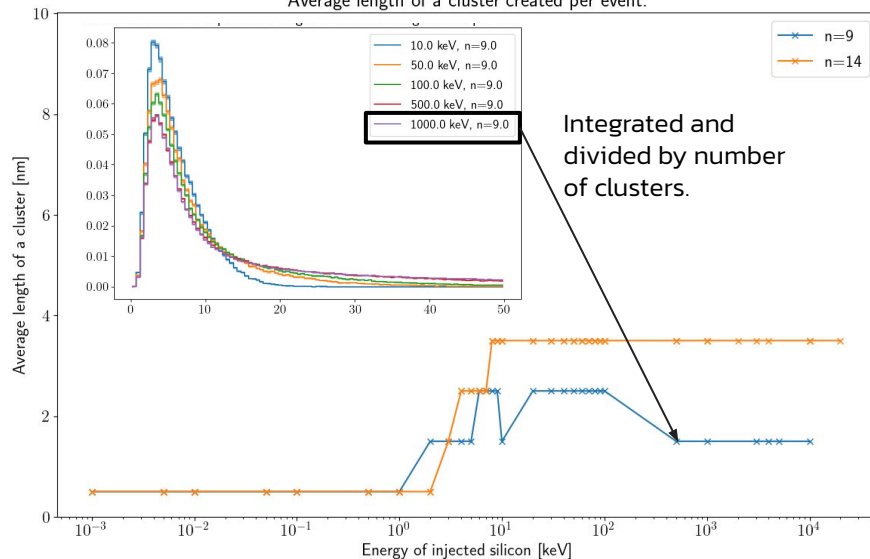
How large are the clusters?

Length := diameter of the smallest enclosed sphere around the cluster

Average number of clusters created per event.



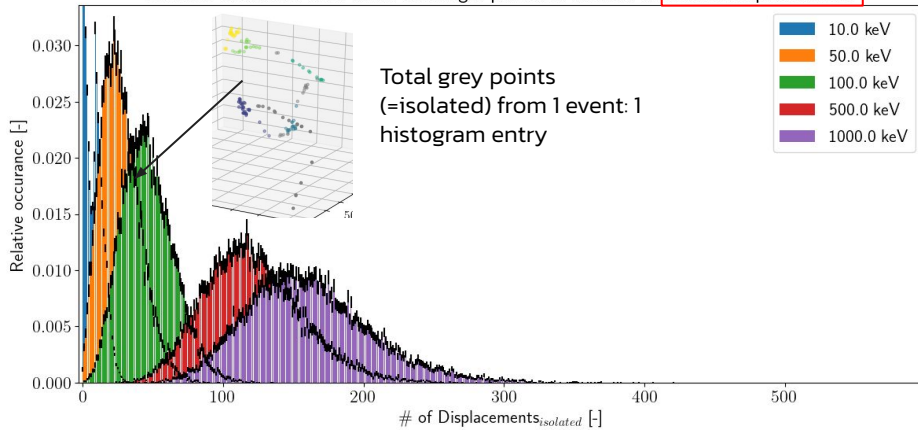
Average length of a cluster created per event.



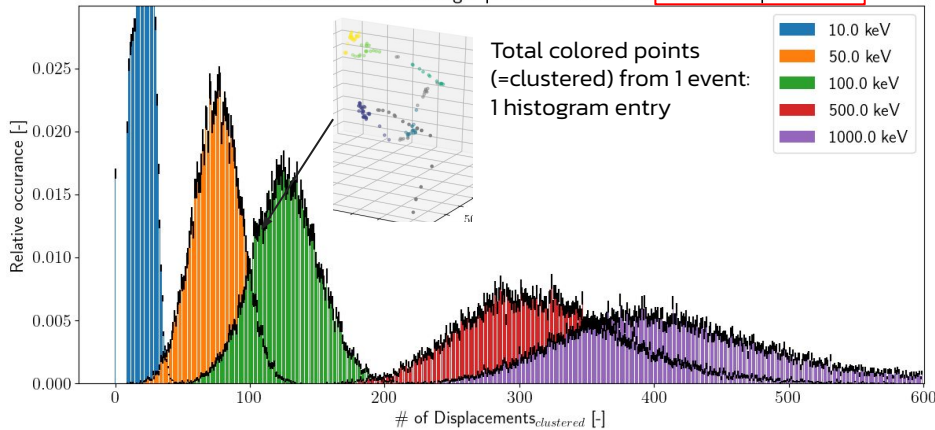
- For Si > 10 keV the length of the cluster for n=14 ~ 3 nm, for Si > 10 keV the length of the cluster for n=9 ~ 1-2 nm.
- For n=9 the algorithm detects higher number of clusters with smaller sizes, for n=14 it is the opposite.

RESULTS

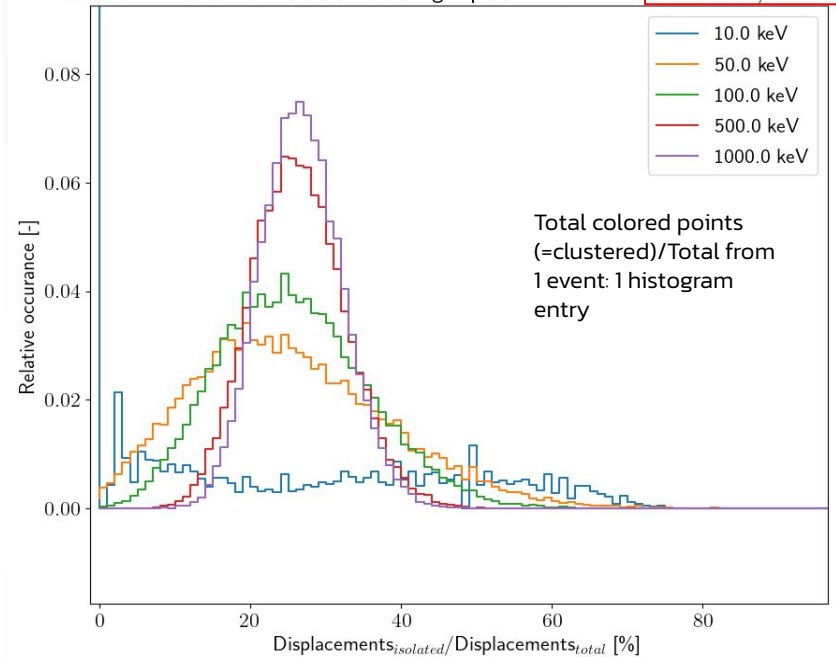
Relative occurrence of Si event creating a particular number of **isolated displacements**.



Relative occurrence of Si event creating a particular number of **clustered displacements**.



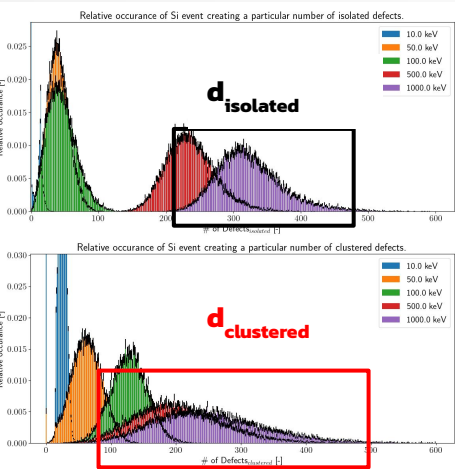
Relative occurrence of Si event creating a particular ratio of **ISOLATED/TOTAL**.



- For $E > 500$ keV, ~25% of the displacements are point-like
- For $E < 500$ keV, the fluctuations are larger. Event can create between 0-60% point like defects.

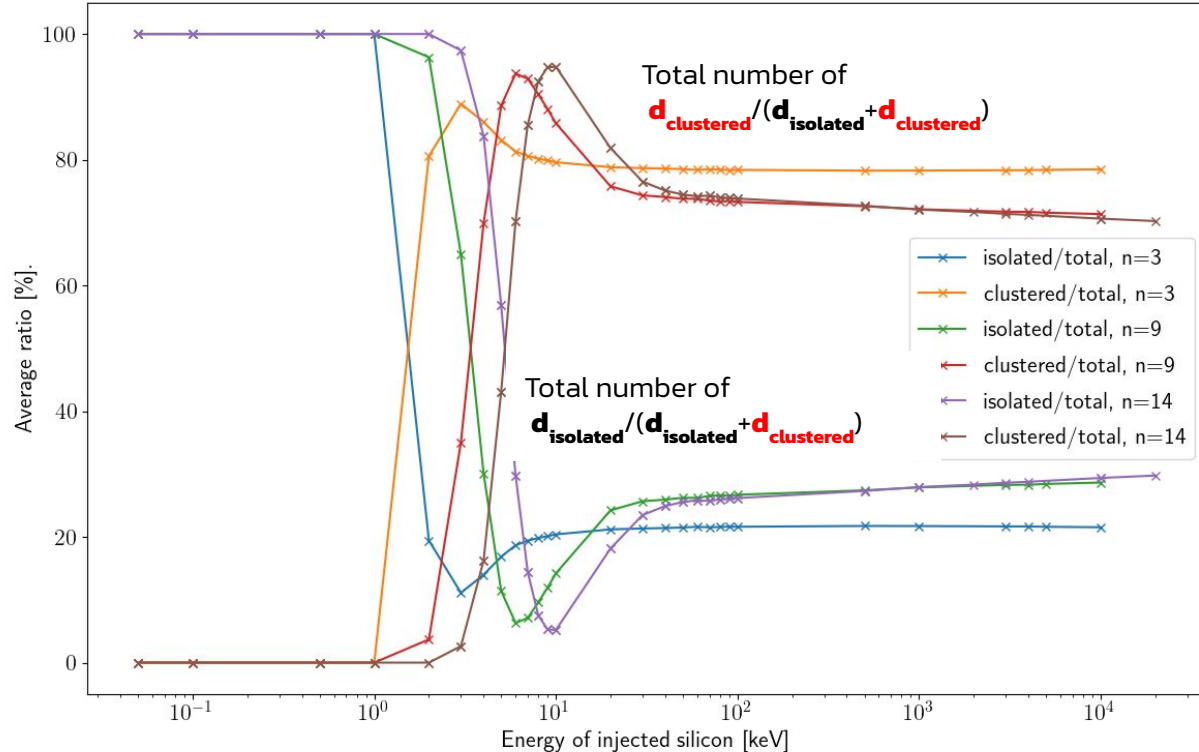
RESULTS

By integrating plots from previous slide for respective energies and dividing by the number of histogram entries, we gain 1 point on the plot on the right plot.

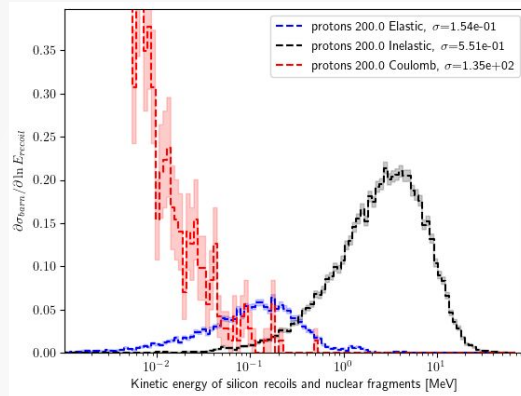
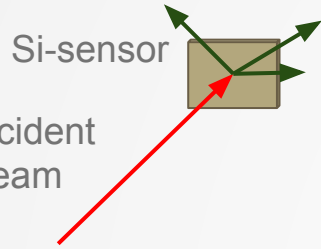


- For $E > 50$ keV stable ratio of point-like and cluster displacements are independent of the choice of n
- The threshold energy for creating stable ratio of point-like and cluster displacements depends on the algorithm parameter.

Average ratio of isolated and clustered displacements created per event.

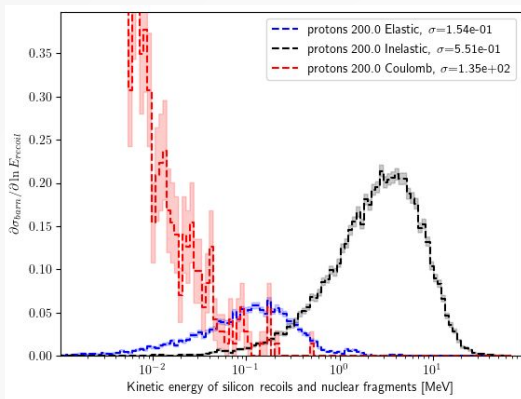
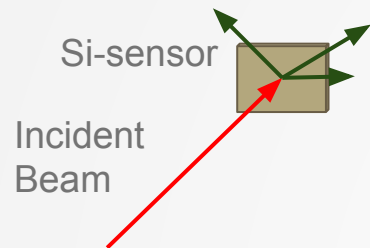


Goal



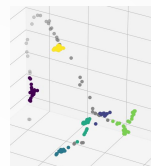
PKA characterized

Goal

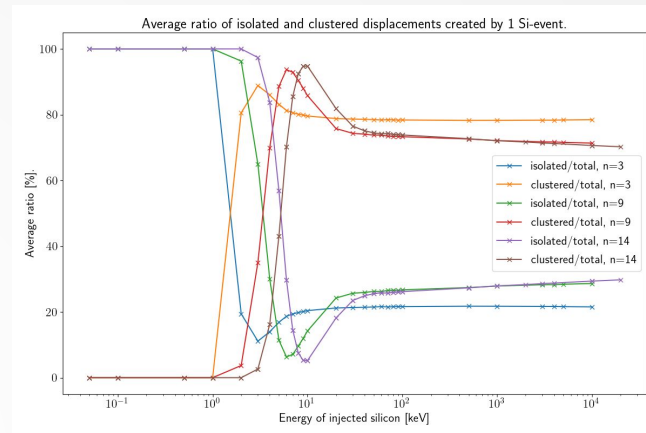


1) PKA characterized

Si-sensor
● Si injection

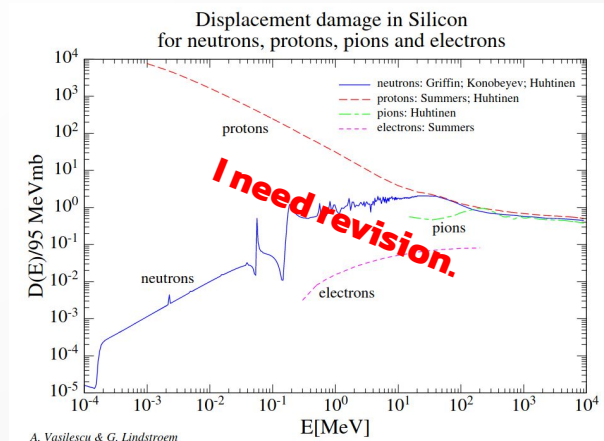


2) Subsequent Si cascade characterized

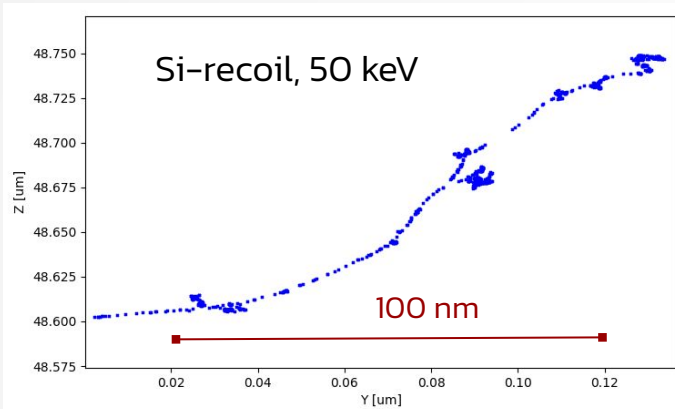


3) Goal:

- take the output of 1) for all projectile energies and apply the output of 2) subsequent Si cascade.
- A revised version of RD48 should show the cluster vs isolated defect contribution.
- At this point benchmark with measurements will be possible.



Outlook & next steps



- A Geant4-based simulations and analysis are being carried out together with FLUKA to revisit the RD-48 NIEL curves.
- NIEL curves in literature with Geant4 and Fluka simulations successfully reproduced
- Algorithm for identifying clustered versus point defect damage implemented.
- For Si recoil of the energy above 50 keV energy threshold the cluster to point defect ration remains constant, in agreement with the literature¹⁴.
- Further developments of algorithm for damage differences between different particles are envisioned.
 - Closure on cluster's parameter definition.
 - Further studies and comparisons with FLUKA.
 - Cluster/Isolated displacements containing update of the RD48 plot for protons and neutrons and extending studies to electrons and gammas.
- Benchmarking with DLTS measurements data.
- Determine fraction of cluster NIEL in irradiation facilities, e.g. CERN PS versus JSI research reactor.

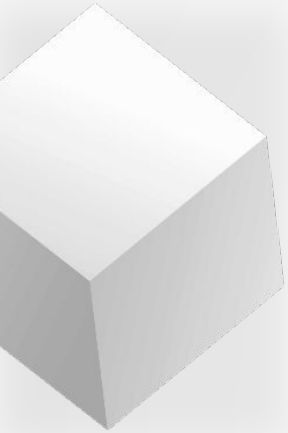
¹⁴) S. R. Srouf and J. W. Palko, "A framework for understanding displacement damage mechanisms in irradiated silicon devices," IEEE Trans. Nucl. Sci., vol. 53, no. 6, pp. 3610–3620, Dec. 2006

THANKS!

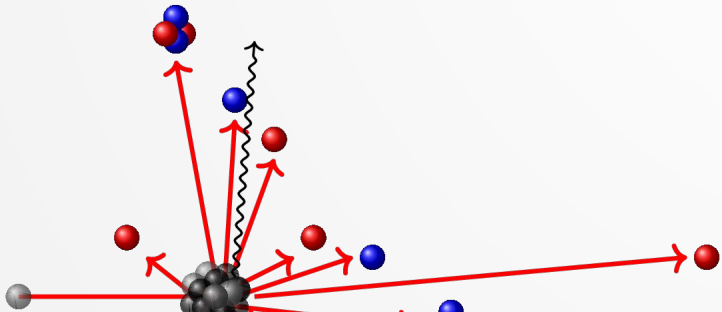


Do you have any questions?

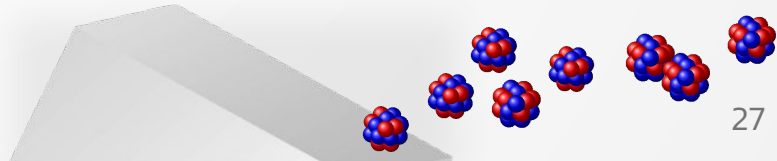
The 40th RD-50 workshop, CERN, 21.06.2022



Extra slides



CERN, 14.06.2022



Geant4 physics list, step functions

Derived from CERN Geant4 User's Workshop November 11th–15th 2002, John Apostolakis, Marc Verderi
Ecole Polytechnique - LLR

For physics list:

- **AtRest functions:** decay, e+ annihilation
- **AlongStep functions:** to describe continuous (inter)actions, occurring along the path of the particle, like ionisation
- **PostStep actions:** For describing point-like (inter)actions, like decay in flight, hard Radiation..

G4VProcess: can implement any combination of **AtRest**, **AlongStep**, **PostStep** action

GetPhysicalInteractionLength():

- Used to limit the step size:
 - either because the process « triggers » an interaction, a decay;
 - Or any other reasons, like fraction of energy loss;
 - geometry boundary;
 - user's limit ..

Geant4 physics list, step

Derived from CERN Geant4 User's Workshop November 11th–15th 2002, John Apostolakis, Marc Verderi
Ecole Polytechnique - LLR

The stepping:

- The stepping treats processes generically:
- The stepping does not know what processes it is Handling
- The stepping imposes on the processes to Cooperate in their **AlongStep** actions;
Compete for PostStep and **AtRest** actions;
- Processes can optionally emit also a «signal» to require particular treatment:
 - notForced: «standard» case;
 - forced: PostStepDolt action is applied anyway;
 - conditionallyForced: PostStepDolt
 - applied if AlongStep has limited the step;

The stepping: Stepping Invocation Sequence of Processes for a particle travelling

1. At the beginning of the step, determine the step length: Consider all processes attached to the current G4Track; Define the step length as the smallest of the lengths among: All
AlongStepGetPhysicalInteractionLength(), All
PostStepGetPhysicalInteractionLength()
2. Apply all AlongStepDolt() actions, « at once »: Changes computed from particle state at the beginning of the step; Accumulated in the G4Step; Then applied to the G4Track, from the G4Step.
3. Apply PostStepDolt() action(s) « sequentially », as long as the particle is alive: Apply PostStepDolt() of process which proposed the smallest step length; apply « forced » and « conditionally forced » actions

Geant4 physics list, step

Derived from CERN Geant4 User's Workshop November 11th–15th 2002, John Apostolakis, Marc Verderi

At rest: Ecole Polytechnique - LLR

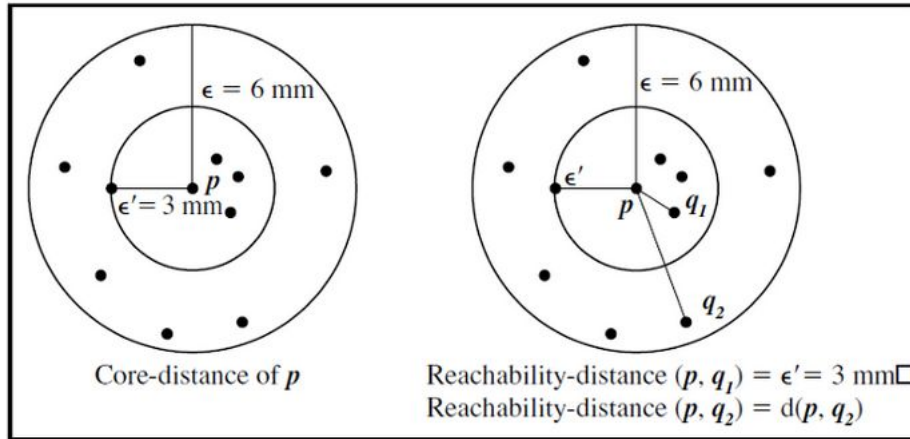
1. If the particle is **at rest**, is stable and can't annihilate, it is killed by the tracking: To be more accurate: if a particle at rest has no « AtRest » actions defined, it is killed.
2. Otherwise determine the lifetime: Take the smallest time among: All AtRestGetPhysicalInteractionLength() Called «physical interaction length» but returns a time!
3. Apply the AtRestDoIt() action of the process which returned the smallest time.

OPTICS^{13,14} (Ordering points to identify the clustering structure)

Important concepts:

- **n (number of neighbours):** user input
- **Core distance:** The minimum distance to make a point a core point, so that it contains number of neighbours n
- **Reachability-distance:**
 - a. If point $<$ the core-distance reachability distance = core-distance
 - b. If point $>$ core-distance, reachability distance = distance between the point and core point

If $n = 4$



CERN, 14.06.2022

13) Ankerst, Mihael, Markus M. Breunig, Hans-Peter Kriegel, and Jörg Sander. "OPTICS: ordering points to identify the clustering structure." ACM SIGMOD Record 28, no. 2 (1999): 49-60.

14) Schubert, Erich, Michael Gertz. "Improving the Cluster Structure Extracted from OPTICS Plots." Proc. of the Conference "Lernen, Wissen, Daten, Analysen" (LWDA) (2018): 318-329.

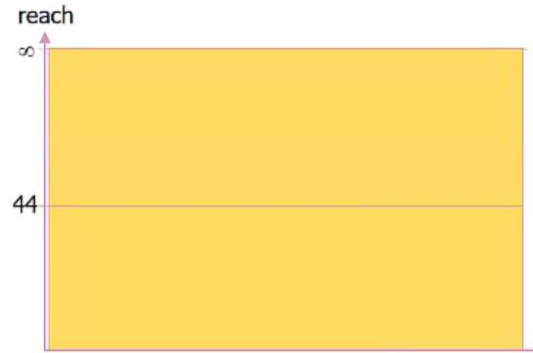
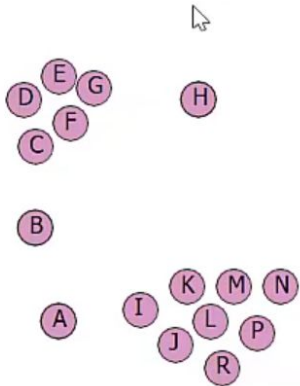
OPTICS

Explanation from:

<https://www.youtube.com/watch?v=CV0mWaHOTA8&t=133s>

Optics algorithm takes the points in a certain order and assigns them properties.

- $\epsilon = 44$, $MinPts = 3$



seedlist:

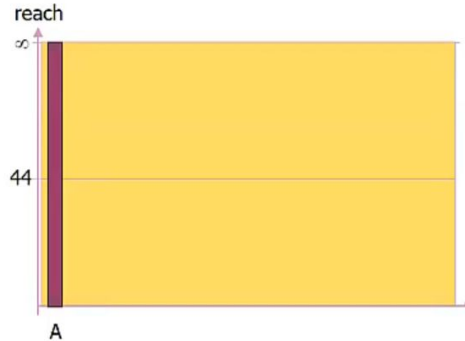
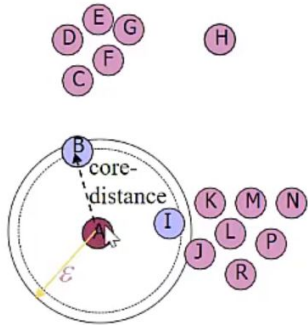
Min neighbours:2

OPTICS

Explanation from:

<https://www.youtube.com/watch?v=CV0mWaHOTA8&t=133s>

- Example Database (2-dimensional, 16 points)
- $\epsilon = 44$, $MinPts = 3$



seedlist: (B,40) (I, 40)

- A is the first point \rightarrow it's reachability is infinite. (How far is the point from the last point?)
- B and C have are 40 units far away from A.

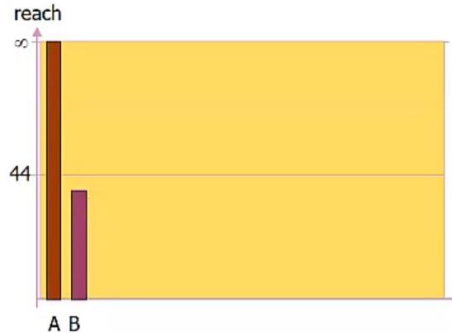
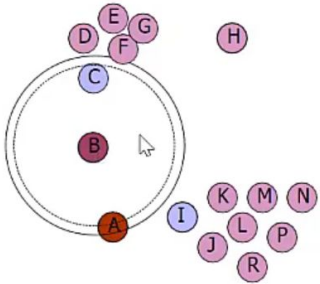
Min neighbours:2

OPTICS

Explanation from:

<https://www.youtube.com/watch?v=CV0mWaHOTA8&t=133s>

- Example Database (2-dimensional, 16 points)
- $\epsilon = 44$, $MinPts = 3$



seedlist: (I, 40) (C, 40)

- Next point: B.
- Seedlist is updated and ordered by reachability.

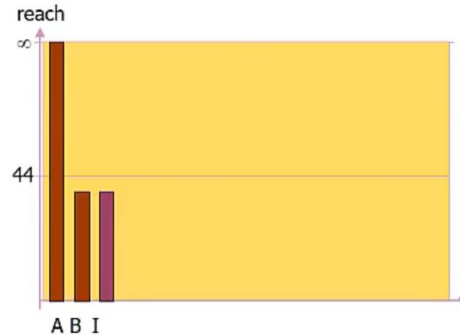
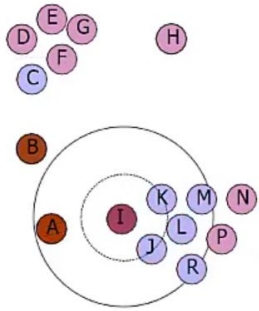
Min neighbours:2

OPTICS

Explanation from:

<https://www.youtube.com/watch?v=CV0mWaHOTA8&t=133s>

- Example Database (2-dimensional, 16 points)
- $\epsilon = 44$, $MinPts = 3$



seedlist: (J, 20) (K, 20) (L, 31) (C, 40) (M, 40) (R, 43)

- Next point I.
- The core distance is much smaller (K and J are close).
- The seedlist is updated and ordered by reachability.

Min neighbours:2

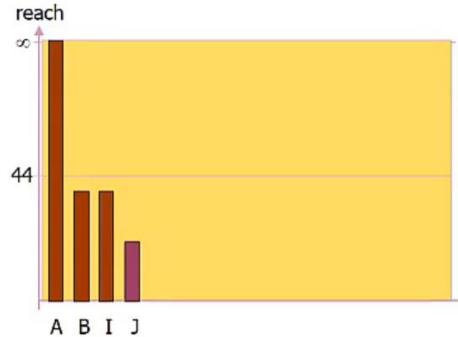
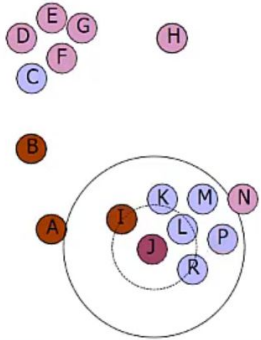
OPTICS

Explanation from:

<https://www.youtube.com/watch?v=CV0mWaHOTA8&t=133s>

- Example Database (2-dimensional, 16 points)

- $\epsilon = 44$, $MinPts = 3$



- Next point J
- The seedlist is updated and ordered by the reachability.

seedlist: (L, 19) (K, 20) (R, 21) (M, 30) (P, 31) (C, 40)

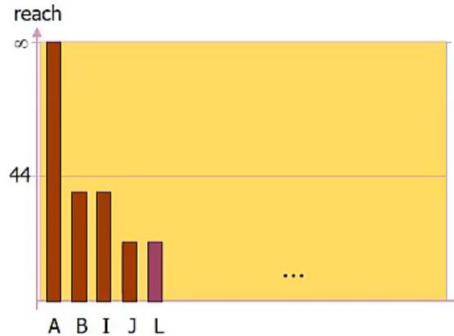
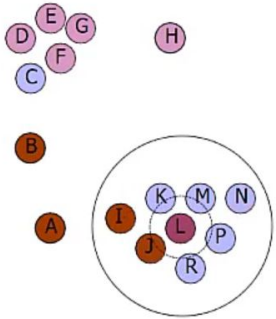
Min neighbours:2

OPTICS

Explanation from:

<https://www.youtube.com/watch?v=CV0mWaHOTA8&t=133s>

- Example Database (2-dimensional, 16 points)
- $\epsilon = 44$, $MinPts = 3$



- Next point L
- The seedlist is updated and ordered by the reachability.

seedlist: (M, 18) (K, 18) (R, 20) (P, 21) (N, 35) (C, 40)

Min neighbours:2

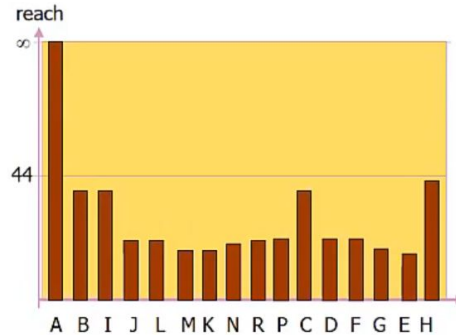
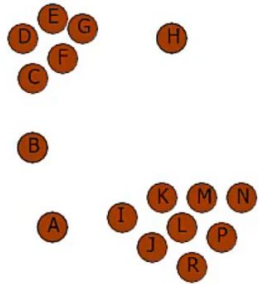
OPTICS

Explanation from:

<https://www.youtube.com/watch?v=CV0mWaHOTA8&t=133s>

- Example Database (2-dimensional, 16 points)

- $\epsilon = 44$, $MinPts = 3$



- The valleys represent the clusters.

seedlist: -

Min neighbours:3

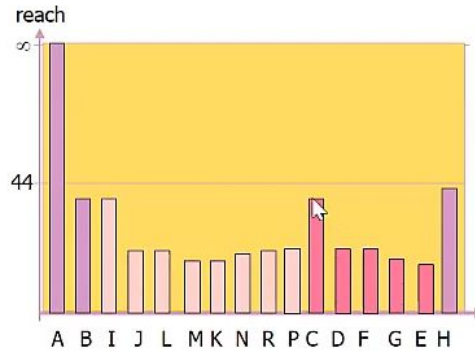
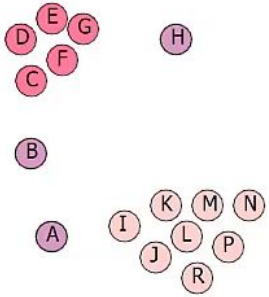
OPTICS

Explanation from:

<https://www.youtube.com/watch?v=CV0mWaHOTA8&t=133s>

- Example Database (2-dimensional, 16 points)

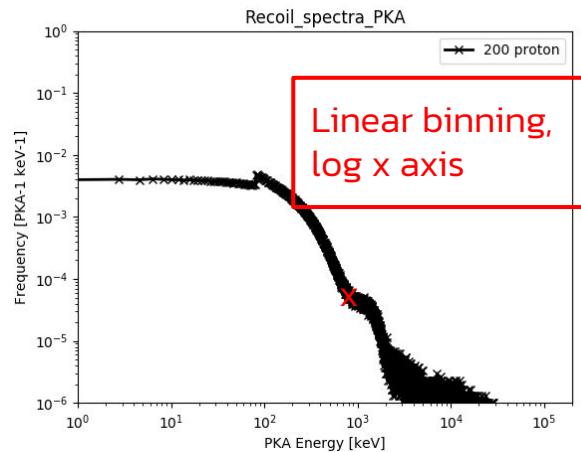
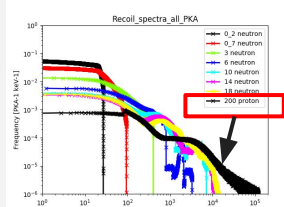
- $\epsilon = 44$, $MinPts = 3$



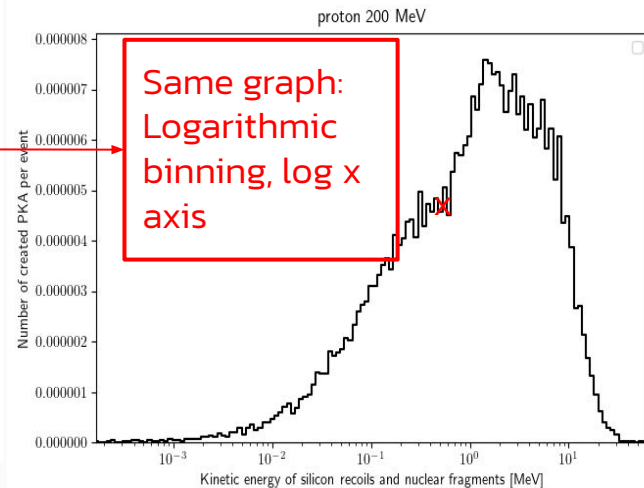
- The valleys represent the clusters.

Min neighbours:2

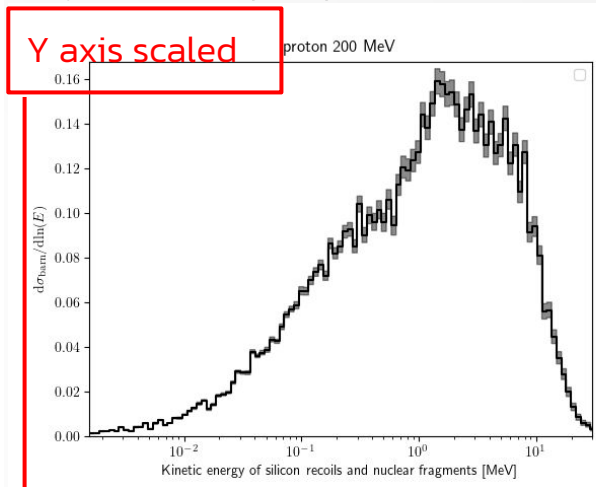
note: representing
y axis as $\partial\sigma_{barn} / \partial \ln(E)$



- 1) PKA are summed and divided by the number of incident particles.



- 2) Logarithmic binning is used instead.



- 3) Each content in a bin is divided by the length of the bin. That makes the y-axis linear. Furthermore the y axis is scaled by:

Conversion from σ to probability:

$$\frac{I_{scattered}}{I_{incident}} = \frac{N_A \rho_{Si} d}{m_{Si}} \sigma = 0.0005 \sigma_{barn}$$

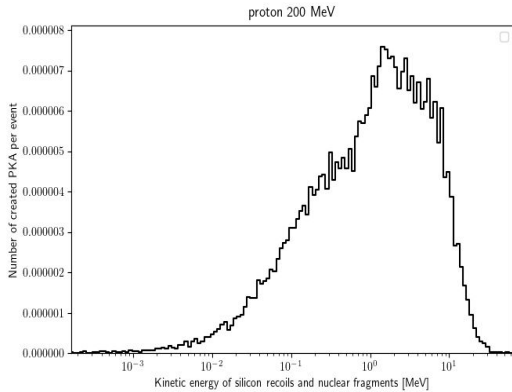
$$\frac{10^{24}}{N_{targ} d} \quad d = 0.01 \text{ cm}$$

$$N_{targ} = \frac{N_A \rho_{Si}}{m_{Si}}$$

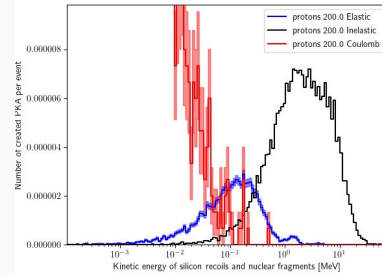
so that the **total area** corresponds to the **total cross section** of creating the PKA.

note: representing

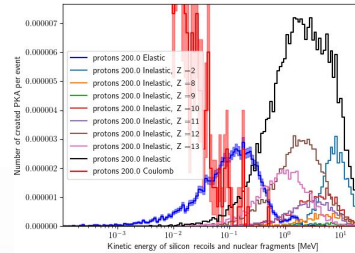
y axis as $\partial E_{NIEL} / \partial \ln(E_{recoil})$



2) PKA are divided into Elastic and Inelastic parts (Coulomb part is added from QGSP_BIC_HP_SS simulation). Inelastic part is further divided into different spectra according to the Z number.



3) Inelastic part is further divided into different spectra according to the Z number.



4) For Coulomb, Elastic and Inelastic Si, Al and Mg recoils a Lindhard formulation is used¹².

For a recoil silicon in a silicon lattice, they read as:

$$E_{dc} = \frac{E_{Si}}{1 + k \times g(\epsilon)}, \quad (2)$$

with $k = 0.1462$, $\epsilon = 1.014 \times 10^{-2} \times Z_{Si}^{-7/3} \times E_{Si} = 2.147 \times 10^{-5} E_{Si}$ and the universal function

$$g(\epsilon) = 3.4008 \times \epsilon^{1/6} + 0.40244 \times \epsilon^{3/4} + \epsilon \quad (3)$$

5) For alphas¹³, Xapsos-Burke values were used to calculate NIEL.

6) Each content in a bin is divided by the length of the bin so that the **total area** corresponds to the **total NIEL**.

1) PKA are summed and divided by the number of incident particles. Logarithmic binning is used instead, that makes the y axis linear.

12) Bergmann, Benedikt, et al. "Ionizing Energy Depositions After Fast Neutron Interactions in Silicon." IEEE Transactions on Nuclear Science, vol. 63, Aug. 2016, pp. 2372–78. NASA ADS, <https://doi.org/10.1109/TNS.2016.2574961>.

13) Xapsos, M.A. & Burke, E.A. & Badavi, F.F. & Townsend, Lawrence & Wilson, John & Jun, I.. (2005). NIEL calculations for high-energy heavy ions. Nuclear Science, IEEE Transactions on. 51. 3250 - 3254. 10.1109/TNS.2004.839136.

