Fast and reliable computation of the instantaneous orbital collision probability

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Abstract. This study deals with the computation of the instantaneous collision probability between two Earth-orbiting objects of spherical shape. It is expressed as a three-dimensional integral of a Gaussian density on a Euclidean ball of given radius. We propose a new algorithm to efficiently evaluate this integral. This is an extension of a previously studied 3-D method based on Laplace transform and power series evaluation, combined with a new technique using a saddle-point method. A preliminary comparison study with both a Monte-Carlo method and the previous state-of-the-art highlights the potential of this evaluation method, regarding both its efficiency and reliability.

Keywords: Orbital Risk · Instantaneous Collision Probability · Saddle-point method

1 Introduction

Due to increasing congestion, satellite conjunction analysis has become a fundamental task for space agencies and operators of the field. To prevent potential collisions on an active satellite, one solution consists in performing evasive maneuvers when the predicted risk is too high. This is usually evaluated in a probabilistic manner because of the imperfect knowledge of the objects' positions and velocities, which are often modeled as Gaussian random variables. The collision probability can be modeled as cumulative (the overall probability during a time range) or instantaneous (at a single time instant). Many previous works dealt with the cumulative case (both for the so-called short-term and long-term encounters [3]), as this formulation better captures the overall risk on a time range. Alternatively, the instantaneous probability can be for instance used to analyze and validate a maneuver plan, by checking that the collision risk stays low at any given time. Its evaluation is thus required to be both very efficient and reliable at each time instant.

Chan [3] formulated the instantaneous collision probability computation as the integral of a three-dimensional (3-D) Gaussian density –representing the objects' relative position uncertainty $X_r \sim \mathcal{N}(\mu, \Sigma)$– over a Euclidean ball with given hard-body radius R (see also Figure 1(a)):

$$\mathcal{P} = \frac{1}{(2\pi)^{3/2}|\Sigma|^{1/2}} \int\limits_{\mathcal{B}(0,R)} \exp\left(-\frac{1}{2}(r-\mu)^T \Sigma^{-1}(r-\mu)\right) \mathrm{d}r. \quad (1)$$

He proposed several approximate evaluation methods, mainly based on the notion of equivalent area or equivalent volume. Recently, another equivalent volume formulation was proposed in [6], using a single cuboid to approximate the hard body shape.

A different approach was introduced in [5]. It is a 3-D extension to the 2-D method presented in [4], which proposes an analytic formula based on a convergent power series, derived by use of Laplace transform and properties of so-called D-finite functions. This approach is now extended by combining it with a new technique based on a saddle-point method for ill-conditioned conjunction data.

2 Brief description and analysis of the new algorithm

The convergent series-based method of [5] sometimes requires employing a very large number of terms in the series. This is a known issue in the evaluation of certain power series. A complementary point of view is to directly consider (for these difficult cases) the inverse Laplace transform formula, which is a complex-variable integral that can be evaluated by a so-called saddle-point method [2, Chap. 6]. This allows for a more efficient evaluation, based on very few terms. Thus, the overall strategy consists in switching between the convergent-series method [5] and the saddle-point alternative based on a simple estimation of a conservative number of terms required for the first method available in [5].

This algorithm was successfully tested on several cases, including the Alfano [1] Case 4, which was also analyzed by [6]. As seen in Fig. 1 our method produces very accurate results similar to the Monte-Carlo simulation, while the previous state-of-the-art [6] suffers from a higher approximation error.

3 Conclusion

This work proposes a semi-analytical approach (by extending the 2-D work of [4]) to compute satellite instantaneous collision probability. Preliminary numerical tests show that our method is more accurate than the current state-of-the-art, while alleviating the difficult cases which required a high-number of terms in [5]. Providing truncation error bounds for the saddle-point alternative is however quite difficult and part of an ongoing work. Furthermore, extensive tests of this method in the framework of Gaussian mixtures are also planned. Finally, the use of this computation in an optimization framework for designing maneuvers plans is also under study.

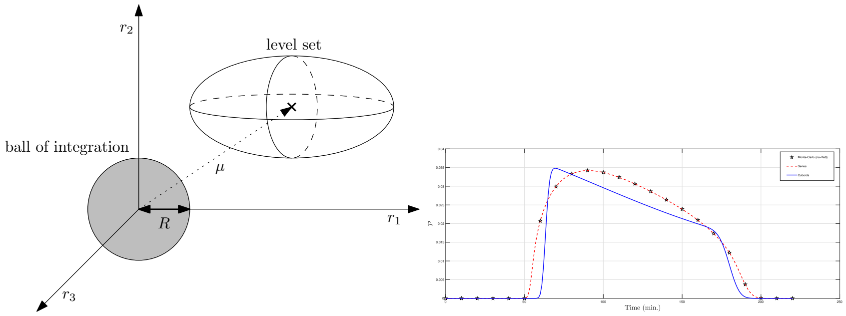

Fig. 1. Instantaneous collision probability. Left: geometrical illustration of the integration. Right: results obtained for various methods (Zhang's cuboids-based [6], Monte-Carlo and ours) on Alfano's [1] test case 4.

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