An overview of the resonances arising from the coupling of the Earth's oblateness with the solar radiation pressure effect*

Roberto Paoli

Faculty of Mathematics, Universitatea Alexandru Ioan Cuza
Bulevardul Carol I 11, Iasi, 100506, Romania
roberto.paoli@uaic.ro

The study of the dynamics associated to the solar radiation pressure (SRP) effect has become increasingly important in the last few decades, both in terms of the prediction of the dynamics of space debris in the Earth's space environment and the design of space mission around the planet with spacecrafts equipped with a solar sail. In both cases it is fundamental to understand how SRP affects the orbital evolution of these bodies, especially when coupled with the gravitational effect of the oblateness of the Earth, encoded in the harmonic coefficient J_2 which appears in the classic Kaula expansion of the geopotential in spherical harmonics, [1].

The main parameter of interest in this context is the *area-to-mass ratio*, i.e. the ratio between the cross section of the body and its mass. The magnitude of the SRP effect is directly proportional to the area-to-mass of a given body and it is known to affect its orbital eccentricity. Many components of modern spacecrafts, such as solar panels, rocket stages and the aforementioned solar sails, are characterized by a large area-to-mass ratio. The SRP effect is modeled using the *cannonball approximation*, which is equivalent to assuming that the surface of the body is always perpendicular to sunlight. This assumption is not restrictive for a small debris or in the case of a solar sail which can be oriented to satisfy this requirement. [2] describes the dynamics of a geosynchronous space debris with high area-to-mass ratio, highlighting periodic variations in its eccentricity and long term variations in inclination. In particular, in this scenario SRP can be considered as the dominant perturbation to the Kepler problem.

The aim of this work is to study the resonances resulting from the coupling of the effects due to J_2 and SRP in the upper Low Earth Orbit (LEO) region and in the Medium Earth Orbit (MEO) region. In particular we are going to focus on resonant terms whose associated resonant angle involves a linear combination of the argument of perigee, right ascension of the ascending node and mean anomaly of the Sun. Resonances involving semi-fast angles such as the Sun's mean anomaly are called semi-secular in [3]. The approximate location of the Solar and SRP semi-secular resonances is computed using the J_2-induced rates of changes of the orbital elements. In order to assess when it is possible to neglect

* The authors of this paper would like to acknowledge the funding received from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 813644.

the third-body gravitational effects, namely the lunar and solar perturbations, we first study the third-body and SRP semi-secular resonances separately using singly averaged Hamiltonian toy models. This approach is similar to the one presented in [4]. We remark that the associated canonical equations of motion are singular for circular orbits.

As a first result we obtain bifurcation diagrams which show that for moderate to high values of the area-to-mass ratio it is possible to neglect the third-body perturbation as they do not qualitative change the phase spaces associated to this kind of dynamics. We then develop analytical estimates for the location of the resonant equilibria, their associated resonance width, and the period of both circulating and librating solutions, thus complementing the description of the phase space associated to the SRP effect presented in [5]. These formulas are accurate for orbits with moderate eccentricities, i.e. greater than 0.2, up to the critical value of the eccentricity (which lead to the object colliding with the Earth) , provided that the semi-major axis and the area-to-mass satisfy a certain smallness condition.

The case of small eccentricities is treated separately, using the Extended Fundamental Model (EFM) of resonances, presented in [6] as an extension of the Second Fundamental Model (SFM) of resonances first described in [7]. The aforementioned singularities for circular orbits are removed by adopting Poincaré-like variables.

Finally we validate the proposed approximations by comparing them to the solutions obtained by propagating the full model which include the higher-degree terms in the geopotential and the lunisolar perturbations in their Cartesian forms. The results from this research show that the SRP effect can be effectively exploited to vary significantly the eccentricity, and therefore the perigee, of a satellite of the Earth over a timespan which can be as small as a few years for an object with moderate area-to-mass ratio, without the need of propulsive maneuvers, contributing to reduce the cost of future space missions.

References

1. Kaula, W.: Theory of Satellite Geodesy: Applications of Satellites to Geodesy. Blaisdell Publishing Company, (1966)
2. Valk, S. and Lemaitre, A. and Anselmo, L.: Analytical and semi-analytical investigations of geosynchronous space debris with high area-to-mass ratios, Advances in Space Research, **41**(7), 1077–1090 (2008)
3. Celletti, A. and Gales, C. and Lhotka, C. (2019). Resonances in the Earth's Space Environment. Communications in Nonlinear Science and Numerical Simulation, **84**, (2020) 105185
4. Breiter, S: Lunisolar Apsidal Resonances at low Satellite Orbits, Celestial Mechanics & Dynamical Astronomy, **74**, 253–274 (1999)
5. Colombo, C. and Lücking, C. and McInnes, C.R.: Orbital dynamics of high area-to-mass ratio spacecraft with J_2 and solar radiation pressure for novel Earth observation and communication services, Acta Astronautica, **81**(1), 137-150 (2012)
6. Breiter, S: Extended Fundamental Model of Resonance, Celestial Mechanics and Dynamical Astronomy, **85**, 209–218 (2003)

7. Henrard, J. and Lemaitre, A.: A second fundamental model for resonance, Celestial Mechanics, **30**, 197–218 (1983)