



# Five texture zeros in the lepton sector

Luis Muñoz, Alejandro Rico, José Muñoz,  
Alex Tapia, David Vanegas, Richard Benavides

arXiv:2207.04072

Diciembre 1 2022, Villa de Leyva.

# Outline

- **Motivation.**
- **Forms with five texture zeros.**
- **Analysis with-out experiments.**
- **Analysis with DUNE experiment.**
- **Conclussions.**

# Motivation

The Standard Model (SM) of the strong and electroweak interactions has been successful explaining most of the high energy physics observations. However, several unanswered questions remain. Experimentally, neutrino oscillations provide the first evidence the SM is incomplete since neutrinos were assumed to be massless in the first version of the electroweak theory. Assuming only Dirac neutrinos, the masses of the neutral leptons can be obtained in the same way as the ones for the charged leptons in the SM.

The model considered here adds three Right Handed Neutrinos (SMRHN) to the field content of the SM.

The model considered here adds three Right Handed Neutrinos (SMRHN) to the field content of the SM.

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$$

The Dirac Lagrangian mass term for the lepton sector is given by

$$-\mathcal{L} = \bar{\nu}_L \textcolor{blue}{M_n} \nu_R + \bar{l}_L \textcolor{blue}{M_l} l_R + h.c.$$

There are 36 mathematical parameters and in the lepton sector there are 8 constraints: the three charged lepton masses, the three mixing angles, the two mass squared differences, plus an unknown Dirac CP-phase, assuming only Dirac neutrinos.

The model considered here adds three Right Handed Neutrinos (SMRHN) to the field content of the SM.

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$$

The Dirac Lagrangian mass term for the lepton sector is given by

$$-\mathcal{L} = \bar{\nu}_L \textcolor{blue}{M_n} \nu_R + \bar{l}_L \textcolor{blue}{M_l} l_R + h.c.$$

There are 36 mathematical parameters and in the lepton sector there are 8 constraints: the three charged lepton masses, the three mixing angles, the two mass squared differences, plus an unknown Dirac CP-phase, assuming only Dirac neutrinos.

The weak current Lagrangian in the interaction basis is

$$\mathcal{L}_{W^-} = -\frac{g}{\sqrt{2}} W_\mu^- \bar{l}'_L \gamma^\mu \nu'_L + h.c., \quad \mathcal{L}_{W^-} = -\frac{g}{\sqrt{2}} W_\mu^- \bar{l}_L U_l^\dagger U_\nu \gamma^\mu \nu_L + h.c.$$

where,  $U_{PMNS} = U_l^\dagger U_\nu$  called Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS).

where,  $U_{PMNS} = U_l^\dagger U_\nu$  called Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS).

Polar theorem:  $C = H U$

WBT:

$$M_n \rightarrow M_n^R = U M_n U^\dagger$$

$$M_l \rightarrow M_l^R = U M_l U^\dagger,$$

$$U_{PMNS} = U_l^\dagger U_\nu = U_l^\dagger U U^\dagger U_\nu = U_l^{R\dagger} U_\nu^R = U_{PMNS}^R$$

We consider as reference the RRR-forms for the quark sector applied to the lepton sector, they have in total five texture zeros.

Form	$M_n$	$M_l$
$\text{RRR}_1$	$\begin{pmatrix} 0 &  b_n  e^{i\alpha_1} & 0 \\  b_n  e^{-i\alpha_1} & c_n &  d_n  e^{i\alpha_2} \\ 0 &  d_n  e^{-i\alpha_2} & a_n \end{pmatrix}$	$\begin{pmatrix} 0 &  b_l  e^{i\beta_1} & 0 \\  b_l  e^{-i\beta_1} & c_l & 0 \\ 0 & 0 & a_l \end{pmatrix}$
$\text{RRR}_3$	$\begin{pmatrix} 0 &  b_n  e^{i\alpha_1} & 0 \\  b_n  e^{-i\alpha_1} & c_n &  d_n  e^{i\alpha_2} \\ 0 &  d_n  e^{-i\alpha_2} & a_n \end{pmatrix}$	$\begin{pmatrix} 0 &  c_l  e^{i\beta_1} & 0 \\  c_l  e^{-i\beta_1} & 0 &  b_l  e^{i\beta_2} \\ 0 &  b_l  e^{-i\beta_2} & a_l \end{pmatrix}$
$\text{RRR}_4$	$\begin{pmatrix} 0 & 0 &  b_n  e^{i\alpha_1} \\ 0 & c_n &  d_n  e^{i\alpha_2} \\  b_n  e^{-i\alpha_1} &  d_n  e^{-i\alpha_2} & a_n \end{pmatrix}$	$\begin{pmatrix} 0 &  b_l  e^{i\beta_1} & 0 \\  b_l  e^{-i\beta_1} & c_l & 0 \\ 0 & 0 & a_l \end{pmatrix}$
$T_1$	$\begin{pmatrix} 0 & 0 &  b_n  e^{i\alpha_1} \\ 0 & c_n &  d_n  e^{i\alpha_2} \\  b_n  e^{-i\alpha_1} &  d_n  e^{-i\alpha_2} & a_n \end{pmatrix}$	$\begin{pmatrix} 0 &  c_l  e^{i\beta_1} & 0 \\  c_l  e^{-i\beta_1} & 0 &  b_l  e^{i\beta_2} \\ 0 &  b_l  e^{-i\beta_2} & a_l \end{pmatrix}$

Factoring phases:  $\Phi M' \Phi^*$  such that the matrix  $M'$  now is real.

The phase matrix is defined as a general diagonal matrix of the form  $Diag(1, e^{i\phi_1}, e^{i\phi_2})$

where  $\phi_1$  and  $\phi_2$ , are thus function of the  $\alpha$  and  $\beta$  phases.

Factoring phases:  $\Phi M' \Phi^*$  such that the matrix  $M'$  is real.

The phase matrix is defined as a general diagonal matrix of the form  $Diag(1, e^{i\phi_1}, e^{i\phi_2})$

where  $\phi_1$  and  $\phi_2$ , are thus function of the  $\alpha$  and  $\beta$  phases.

With this procedure, the real mass matrices are diagonalized by orthogonal rotation matrices.

$$M'_n = \begin{pmatrix} 0 & 0 & |b_n| \\ 0 & c_n & |d_n| \\ |b_n| & |d_n| & a_n \end{pmatrix}.$$

$$\text{Det}\{M_{(n,l)}^{\text{diag}}\} = \text{Det}\{M_{(n,l)}\},$$

$$\text{Tr}\{M_{(n,l)}^{\text{diag}}\} = \text{Tr}\{M_{(n,l)}\},$$

$$\text{Tr}\{[M_{(n,l)}^{\text{diag}}]^2\} = \text{Tr}\{[M_{(n,l)}]^2\},$$

With  $M_{n,l}^{\text{diag}} = Diag(m_{1,e}, -m_{2,\mu}, m_{3,\tau})$

The lepton mixing matrix can be written as:

$$K = R_l \Phi R_n^T,$$

As a function of the masses of the particles and two phases:  $\phi_1$  and  $\phi_2$

The three mixing angles now are

The lepton mixing matrix can be written as:

$$K = R_l \Phi R_n^T,$$

As a function of the masses of the particles and two phases:  $\phi_1$  and  $\phi_2$

The three mixing angles now are

Standard parametrization, PDG.

$$\tan \theta_{12} = |K_{e,2}| / |K_{e,1}|,$$

$$\sin \theta_{13} = |K_{e,3}|,$$

$$\tan \theta_{23} = |K_{\mu,3}| / |K_{\tau,3}|,$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

The lepton mixing matrix can be written as:

$$K = R_l \Phi R_n^T,$$

As a function of the masses of the particles and two phases:  $\phi_1$  and  $\phi_2$

The three mixing angles now are

Standard parametrization, PDG.

$$\tan \theta_{12} = |K_{e,2}| / |K_{e,1}|,$$

$$\sin \theta_{13} = |K_{e,3}|,$$

$$\tan \theta_{23} = |K_{\mu,3}| / |K_{\tau,3}|,$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

and the Jarlskog invariant

$$J_{CP} = \mathcal{I}\{K_{e1}^* K_{\mu 3}^* K_{e3} K_{\mu 1}\}$$

- Analysis of the forms

In the numerical analyses we use the values of the charged lepton masses and the values of the neutrino oscillation parameters, showed in the next tables:

$m_e$ (MeV)	$m_\mu$ (MeV)	$m_\tau$ (MeV)
0.511	105.658	1776.860

$\sin^2 \theta_{12} \pm \sigma(\sin^2 \theta_{12})$	$\sin^2 \theta_{13} \pm \sigma(\sin^2 \theta_{13})$	$\sin^2 \theta_{23} \pm \sigma(\sin^2 \theta_{23})$	$\Delta m_{21}^2$ (eV <sup>2</sup> )	$\Delta m_{31}^2$ (eV <sup>2</sup> )
$0.320 \pm 0.016$	$0.0220 \pm 0.0007$	$0.574 \pm 0.014$	$7.50 \times 10^{-5}$	$2.55 \times 10^{-3}$

- Analysis of the forms

In the numerical analyses we use the values of the charged lepton masses and the values of the neutrino oscillation parameters, showed in the next tables:

$m_e$ (MeV)	$m_\mu$ (MeV)	$m_\tau$ (MeV)
0.511	105.658	1776.860

$\sin^2 \theta_{12} \pm \sigma(\sin^2 \theta_{12})$	$\sin^2 \theta_{13} \pm \sigma(\sin^2 \theta_{13})$	$\sin^2 \theta_{23} \pm \sigma(\sin^2 \theta_{23})$	$\Delta m_{21}^2$ (eV <sup>2</sup> )	$\Delta m_{31}^2$ (eV <sup>2</sup> )
$0.320 \pm 0.016$	$0.0220 \pm 0.0007$	$0.574 \pm 0.014$	$7.50 \times 10^{-5}$	$2.55 \times 10^{-3}$

Neutrino masses can be written in terms of the neutrino mass-squared differences and the unknown absolute neutrino mass  $m_0$ . For normal neutrino mass ordering

$$m_1 = m_0 ,$$

$$m_2 = \sqrt{m_0^2 + \Delta m_{21}^2} ,$$

$$m_3 = \sqrt{m_0^2 + \Delta m_{31}^2} ,$$

where  $\Delta m_{21}^2$  ( $\Delta m_{31}^2$ ) is the solar (atmospheric) mass-squared difference.

## •Analysis of RRR4-Form

$$c_n = -a_n + m_1 - m_2 + m_3,$$

$$|b_n| = \sqrt{m_1 m_2 m_3 / (-a_n + m_1 - m_2 + m_3)},$$

$$|d_n| = \sqrt{\frac{(a_n - m_1 + m_2)(a_n - m_1 - m_3)(a_n + m_2 - m_3)}{-a_n + m_1 - m_2 + m_3}},$$

$$a_l = m_e - m_\mu,$$

$$|b_l| = m_\tau,$$

$$|c_l| = \sqrt{m_e m_\mu}.$$

## •Analysis of RRR4-Form

$$c_n = -a_n + m_1 - m_2 + m_3,$$

$$|b_n| = \sqrt{m_1 m_2 m_3 / (-a_n + m_1 - m_2 + m_3)},$$

$$|d_n| = \sqrt{\frac{(a_n - m_1 + m_2)(a_n - m_1 - m_3)(a_n + m_2 - m_3)}{-a_n + m_1 - m_2 + m_3}},$$

$$a_l = m_e - m_\mu,$$

$$|b_l| = m_\tau,$$

$$|c_l| = \sqrt{m_e m_\mu}.$$

$$R_l = \begin{pmatrix} \sqrt{\frac{m_\mu}{(m_e+m_\mu)}} & \sqrt{\frac{m_e}{(m_e+m_\mu)}} & 0 \\ -\sqrt{\frac{m_e}{(m_e+m_\mu)}} & \sqrt{\frac{m_\mu}{(m_e+m_\mu)}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_n = \begin{pmatrix} \sqrt{\frac{m_2 m_3 (a_n + m_2 - m_3)}{(m_1+m_2)(m_1-m_3)(-a_n+m_1-m_2+m_3)}} \\ -\sqrt{\frac{m_1 m_3 (-a_n + m_1 + m_3)}{(m_1+m_2)(m_2+m_3)(-a_n+m_1-m_2+m_3)}} \\ \sqrt{\frac{m_1 m_2 (a_n - m_1 + m_2)}{(m_3-m_1)(m_2+m_3)(-a_n+m_1-m_2+m_3)}} \end{pmatrix}$$

$$\begin{aligned} & -\sqrt{\frac{m_1 (a_n - m_1 + m_2) (-a_n + m_1 + m_3)}{(m_1+m_2)(m_3-m_1)(-a_n+m_1-m_2+m_3)}} \\ & -\sqrt{\frac{m_2 (a_n - m_1 + m_2) (a_n + m_2 - m_3)}{(m_1+m_2)(m_2+m_3)(a_n - m_1 + m_2 - m_3)}} \\ & \sqrt{\frac{m_3 (a_n - m_1 - m_3) (a_n + m_2 - m_3)}{(m_1-m_3)(m_2+m_3)(a_n - m_1 + m_2 - m_3)}} \end{aligned} \quad \begin{pmatrix} \sqrt{\frac{m_1 (a_n + m_2 - m_3)}{(m_1+m_2)(m_1-m_3)}} \\ \sqrt{\frac{m_2 (-a_n + m_1 + m_3)}{(m_1+m_2)(m_2+m_3)}} \\ \sqrt{\frac{m_3 (a_n - m_1 + m_2)}{(m_3-m_1)(m_2+m_3)}} \end{pmatrix}$$

To ensure all elements in the matrix  $R_n$  are real, the mathematical restriction must be satisfied.

$m_1 - m_2 < a_n < m_3 - m_2$  Then there are four free parameters, namely  $\vec{\lambda} = \{m_0, a_n, \phi_1, \phi_2\}$ .

In order to constrain the model parameters, the following statistical test was implemented:

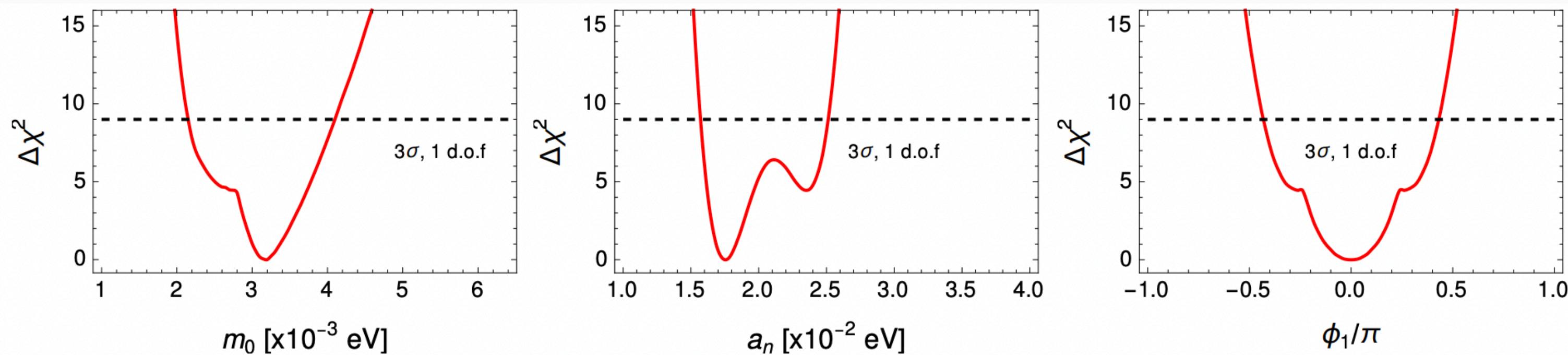
$$\chi^2(\vec{\lambda}) = \sum_{i < j} \left( \frac{\sin^2 \theta_{ij} - \sin^2 \tilde{\theta}_{ij}}{\sigma(\sin^2 \theta_{ij})} \right)^2, \quad \text{with } i, j = 1, 2, 3.$$

where  $\sin^2 \tilde{\theta}_{ij}$  are the mixing angles predicted by the forms, while  $\sin^2 \theta_{ij}$  and  $\sigma(\sin^2 \theta_{ij})$  are the current best fit values and its one-sigma deviations, respectively.

In order to constrain the model parameters, the following statistical test was implemented:

$$\chi^2(\vec{\lambda}) = \sum_{i < j} \left( \frac{\sin^2 \theta_{ij} - \sin^2 \tilde{\theta}_{ij}}{\sigma(\sin^2 \theta_{ij})} \right)^2, \quad \text{with } i, j = 1, 2, 3.$$

where  $\sin^2 \tilde{\theta}_{ij}$  are the mixing angles predicted by the forms, while  $\sin^2 \theta_{ij}$  and  $\sigma(\sin^2 \theta_{ij})$  are the current best fit values and its one-sigma deviations, respectively.



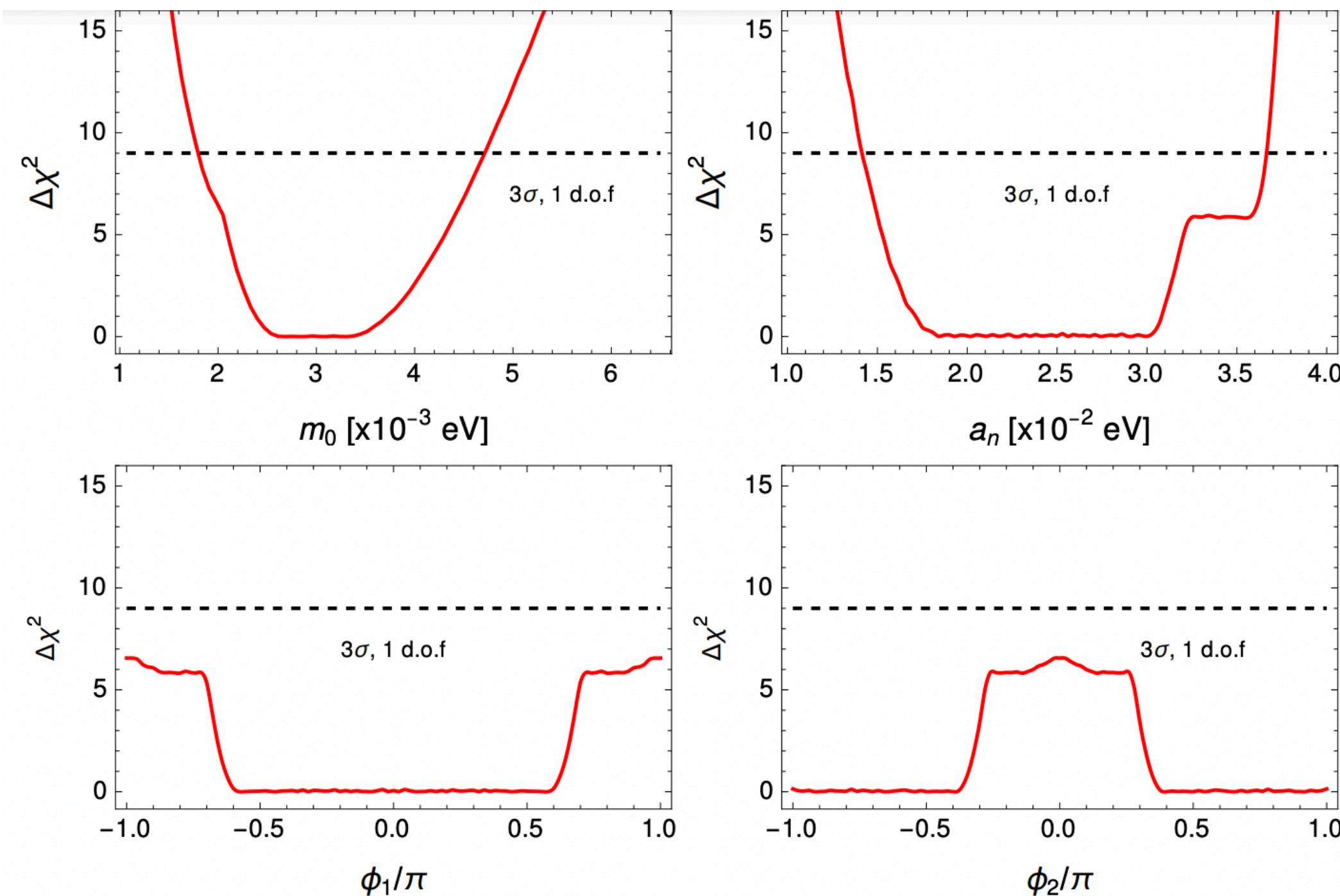
Parameter	Best fit	$3\sigma$ range
$m_0 (\times 10^{-3} \text{ eV})$	3.2	[2.2, 4.1]
$a_n (\times 10^{-2} \text{ eV})$	1.8	[1.6, 2.5]
$\phi_1/\pi$	0	[-0.4, 0.4]
$\phi_2/\pi$	Independent	

Best fit parameters (second column) and three sigma allowed range for 1.d.o.f. (third column). The  $\chi^2$  value at the minimum is  $\chi^2_{\min.} = 1.4$ . Note that in this form,  $(J_{\text{CP}})$  only depends on  $\phi_1$

## • Analysis of the T1-form

The four parameters  $\vec{\lambda} = \{m_0, a_n, \phi_1, \phi_2\}$  contribute to the lepton mixing predicted by this texture.

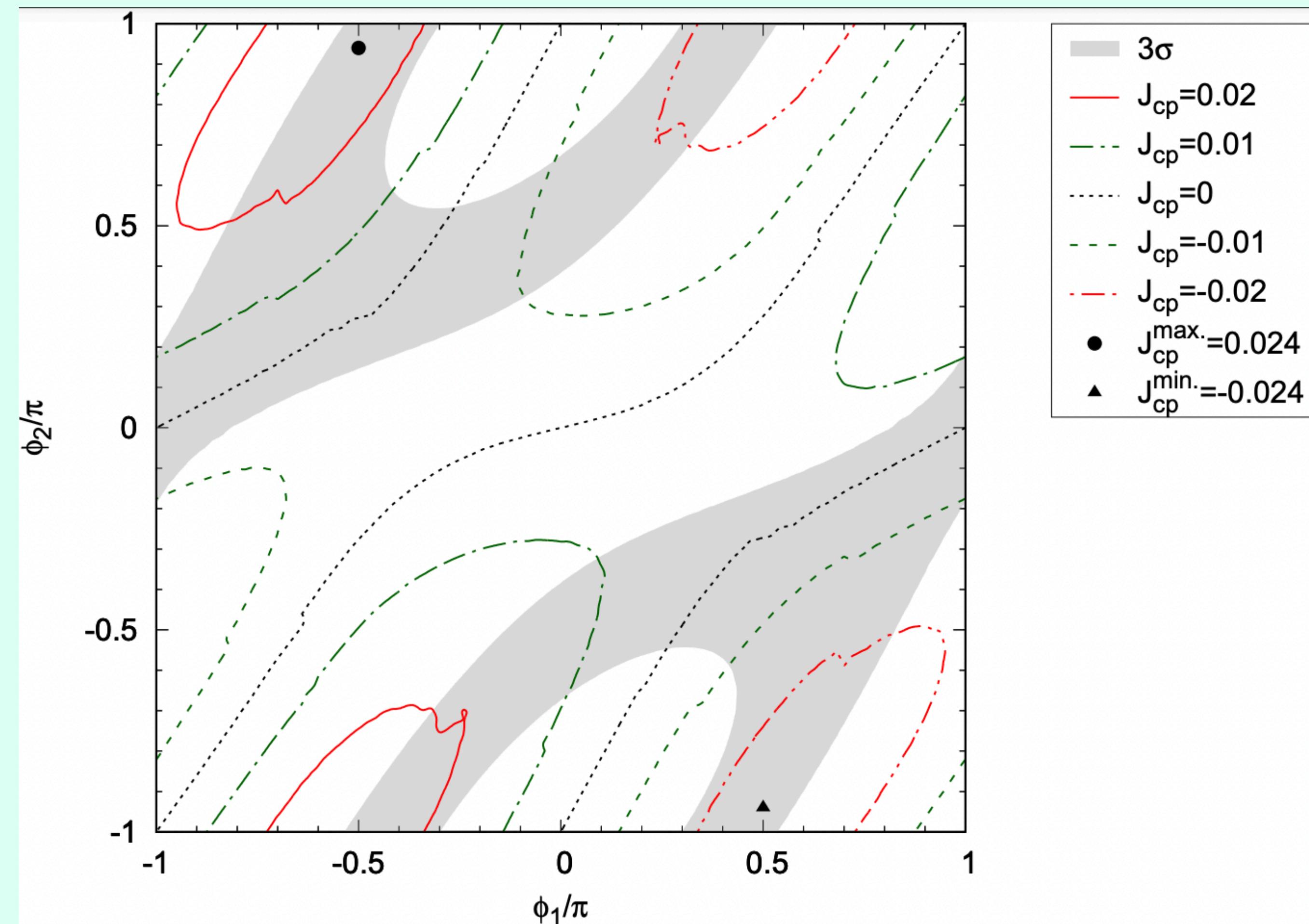
$$\begin{aligned} a_l &= m_e - m_\mu + m_\tau, \\ |b_l| &= \sqrt{(m_e - m_\mu)(m_\mu - m_\tau)(m_e + m_\tau)/(m_e - m_\mu + m_\tau)}, \\ |c_l| &= \sqrt{(m_e m_\mu m_\tau)/(m_e - m_\mu + m_\tau)}, \end{aligned}$$



$$R_l = \begin{pmatrix} -\sqrt{\frac{m_\mu m_\tau (m_\mu - m_\tau)}{(m_e + m_\mu)(m_e - m_\tau)(m_e - m_\mu + m_\tau)}} & -\sqrt{\frac{m_e (m_\mu - m_\tau)}{(m_e + m_\mu)(m_e - m_\tau)}} \\ \sqrt{\frac{m_e m_\tau (m_e + m_\tau)}{(m_e + m_\mu)(m_\mu + m_\tau)(m_e - m_\mu + m_\tau)}} & -\sqrt{\frac{m_\mu (m_e + m_\tau)}{(m_e + m_\mu)(m_\mu + m_\tau)}} \\ \sqrt{\frac{m_e m_\mu (m_e - m_\mu)}{(m_e - m_\tau)(m_\mu + m_\tau)(m_e - m_\mu + m_\tau)}} & \sqrt{\frac{m_\tau (m_e - m_\mu)}{(m_e - m_\tau)(m_\mu + m_\tau)}} \\ \sqrt{\frac{m_e (m_e - m_\mu)(m_e + m_\tau)}{(m_e + m_\mu)(m_e - m_\tau)(m_e - m_\mu + m_\tau)}} & \sqrt{\frac{m_\mu (m_e - m_\mu)(m_\mu - m_\tau)}{(m_e + m_\mu)(m_\mu + m_\tau)(m_e - m_\mu + m_\tau)}} \\ \sqrt{\frac{m_\mu (m_e - m_\mu)(m_\mu - m_\tau)}{(m_e + m_\mu)(m_\mu + m_\tau)(m_e - m_\mu + m_\tau)}} & \sqrt{\frac{m_\tau (m_e + m_\tau)(m_\mu - m_\tau)}{(m_e - m_\tau)(m_\mu + m_\tau)(m_e - m_\mu + m_\tau)}} \end{pmatrix}$$

Parameter	Best fit	$3\sigma$ range
$m_0 (\times 10^{-3} \text{eV})$	3.3	[1.8, 4.7]
$a_n (\times 10^{-2} \text{eV})$	2.3	[1.4, 3.7]
$\phi_1/\pi$	0.4	Unconstrained
$\phi_2/\pi$	0.9	Unconstrained

In this form, the Jarlskog invariant depends on both,  $\phi_1$  and  $\phi_2$ , phases as shown in the next figure



- DUNE sensitivity to the mixing parameters

We calculated the lepton mixing matrix for each form showed before and their were implemented in the GLoBES C-library probability engine.

$$\nu_\mu \rightarrow \nu_\mu, \nu_\mu \rightarrow \nu_e,$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu \text{ and } \bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

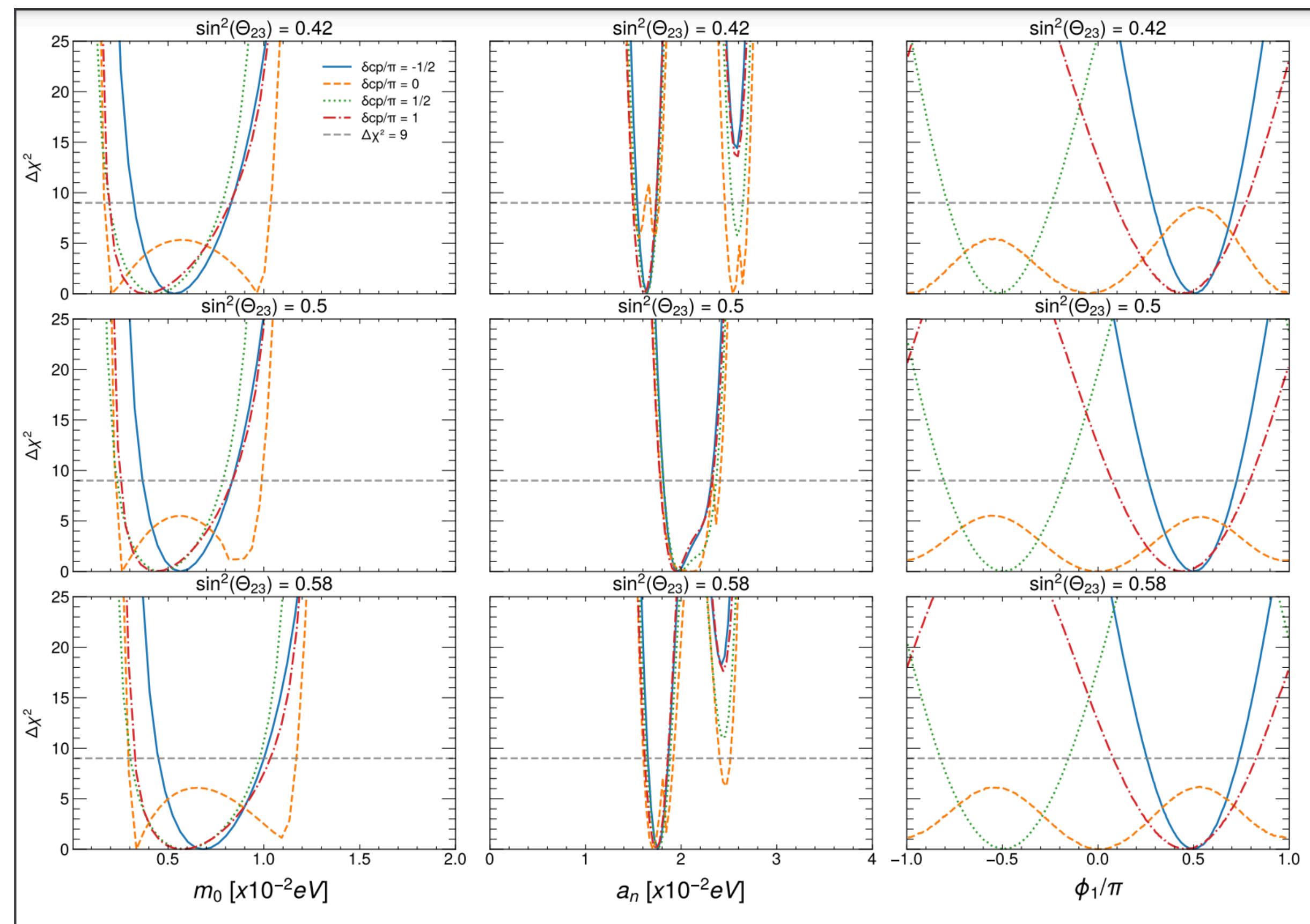
- DUNE sensitivity to the mixing parameters

We calculated the lepton mixing matrix for each form showed before and their were implemented in the GLoBES C-library probability engine.

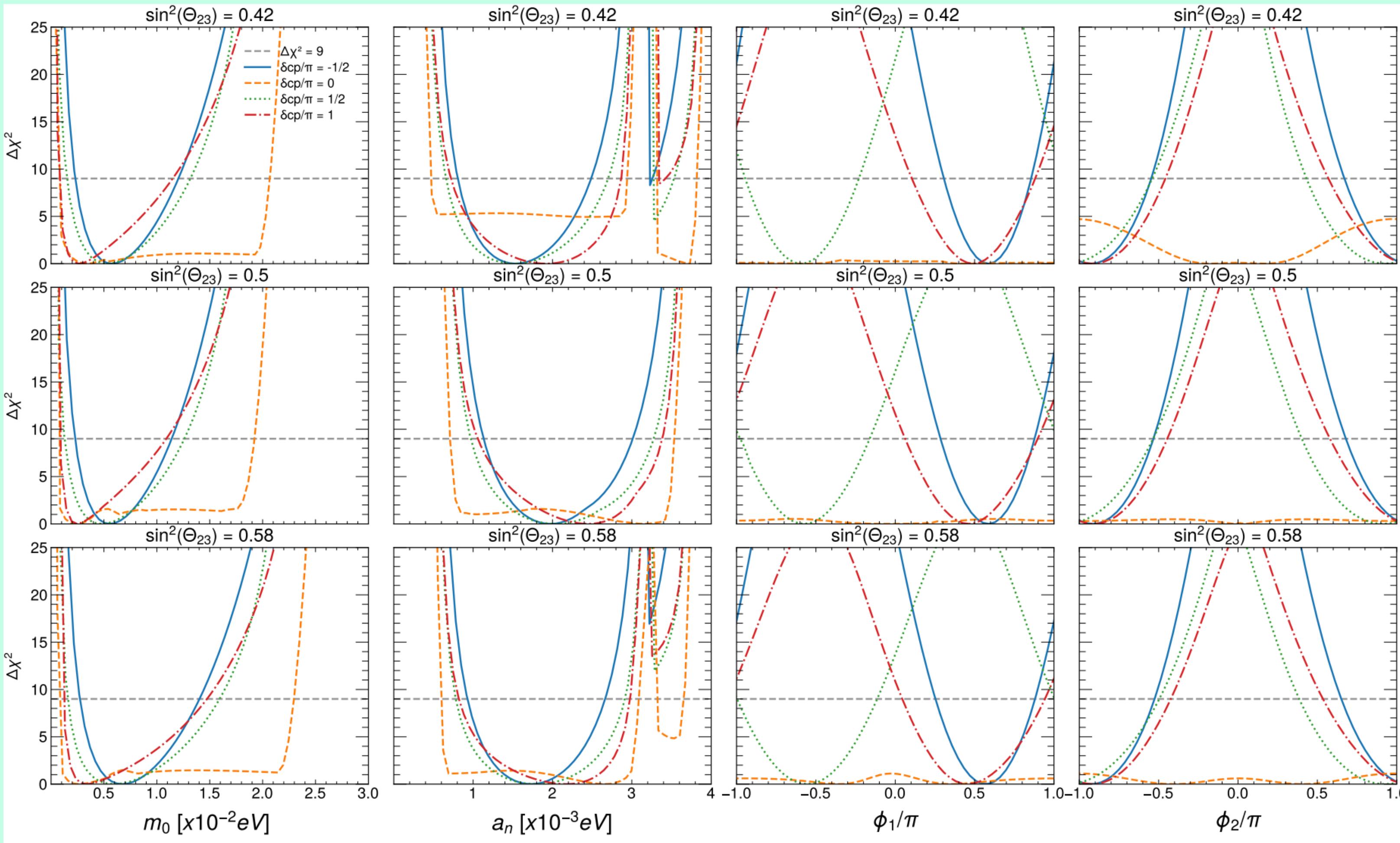
$$\nu_\mu \rightarrow \nu_\mu, \nu_\mu \rightarrow \nu_e,$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu \text{ and } \bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

- For RRR4 form



• T1 form



$\sin^2(\theta_{23})$	$\delta_{cp}/\pi$	$m_0$ ( $\times 10^{-2}$ eV)	$a_n$ ( $\times 10^{-2}$ eV)	$\phi_1/\pi$	$\phi_2/\pi$
0.42	-0.5	[0.3, 1.4]	[0.9, 2.5]	[-1.0, -0.6] $\cup$ [0.7, 1.0]	[-1.0, -0.5] $\cup$ [0.6, 1.0]
0.42	0.0	[0.1, 2.2]	[0.5, 3.0] $\cup$ [3.1, 3.8]	[-1.0, 1.0]	[-1.0, 1.0]
0.42	0.5	[0.3, 1.3]	[0.7, 2.8] $\cup$ [3.1, 3.3]	[-0.9, -2.4]	[-1.0, -0.6] $\cup$ [0.4, 1.0]
0.42	1	[0.2, 1.2]	[0.7, 2.8]	[0.1, 8.72]	[-1.0, -0.5] $\cup$ [0.5, 1.0]
0.5	-0.5	[0.4, 1.3]	[1.2, 3.0]	[0.2 , 0.9]	[-0.5, 0.7]
0.5	0.0	[0.1, 2.0]	[0.6, 3.4]	[-1.0, 1.0]	[-1.0, 1.0]
0.5	0.5	[0.1, 1.3]	[1.0, 3.3]	[-1.0, -0.1]	[-0.5, 0.4]
0.5	1	[0.1, 1.2]	[1.0, 3.2]	[0.0, 0.8]	[-0.4, 0.6]
0.58	-0.5	[0.4, 1.5]	[1.0, 2.5]	[0.3 , 0.9]	[-0.5, 0.6]
0.58	0	[0.1, 2.4]	[0.6, 3.0] $\cup$ [3.3, 3.7]	[-1.0, 1.0]	[-1.0, 1.0]
0.58	0.5	[0.2, 1.6]	[0.9, 2.9]	[-1.0, 0.0]	[-0.5, 0.4 ]
0.58	1	[0.2, 1.5]	[0.9, 2.9]	[0.1, 0.9]	[-0.4, 0.5]

# Conclusions

- The texture zeros diminishing the mathematical parameters in the models.
- The texture zeros give us an alternative to the PDG parametrization, in a function of four parameters.
- The forms showed here predict a neutrino mixing compatible with the current neutrino oscillation phenomenology including the possibility of the CP-symmetry violation encoded in a physical phase. We have shown that DUNE sensitivity to this Dirac CP-phase is comparable to the sensitivity to  $\delta_{cp}$  of the PDG parametrization.

# References

1. This work is in a review process in a journal (PRD), for more details see: [arXiv:2207.04072](https://arxiv.org/abs/2207.04072).
2. P. Ramond, R. G. Roberts, and G. G. Ross, [Nucl. Phys. B \*\*406\*\*, 19 \(1993\)](https://doi.org/10.1016/0550-3213(93)90003-9), [arXiv:hep-ph/9303320](https://arxiv.org/abs/hep-ph/9303320).
3. G. C. Branco, R. G. Felipe, and F. R. Joaquim, [Rev. Mod. Phys. \*\*84\*\*, 515 \(2012\)](https://doi.org/10.1103/RevModPhys.84.515), [arXiv:1111.5332 \[hep-ph\]](https://arxiv.org/abs/1111.5332).
4. G. C. Branco, D. Emmanuel-Costa, and R. Gonzalez Felipe, [Phys. Lett. B \*\*477\*\*, 147 \(2000\)](https://doi.org/10.1016/S0370-2693(00)00900-7), [arXiv:hep-ph/9911418](https://arxiv.org/abs/hep-ph/9911418).
5. B. Abi *et al.* (DUNE), [Eur. Phys. J. C \*\*80\*\*, 978 \(2020\)](https://doi.org/10.1140/epjc/s10050-020-08600-0), [arXiv:2006.16043 \[hep-ex\]](https://arxiv.org/abs/2006.16043).
6. P. F. de Salas, D. V. Forero, S. Gariazzo, P. Martínez-Miravé, O. Mena, C. A. Ternes, M. Tórtola, and J. W. F. Valle, [JHEP \*\*02\*\*, 071 \(2021\)](https://doi.org/10.1007/JHEP02(2021)071), [arXiv:2006.11237 \[hep-ph\]](https://arxiv.org/abs/2006.11237).
7. R. H. Benavides, Y. Giraldo, L. Muñoz, W. A. Ponce, and E. Rojas, [J. Phys. G \*\*47\*\*, 115002 \(2020\)](https://doi.org/10.1088/0954-3899/47/11/115002), [arXiv:2002.01864 \[hep-ph\]](https://arxiv.org/abs/2002.01864).



Gracias.