Physical constraints for wormholes existence in the frame of the general theory of relativity

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Wormholes and energy conditions

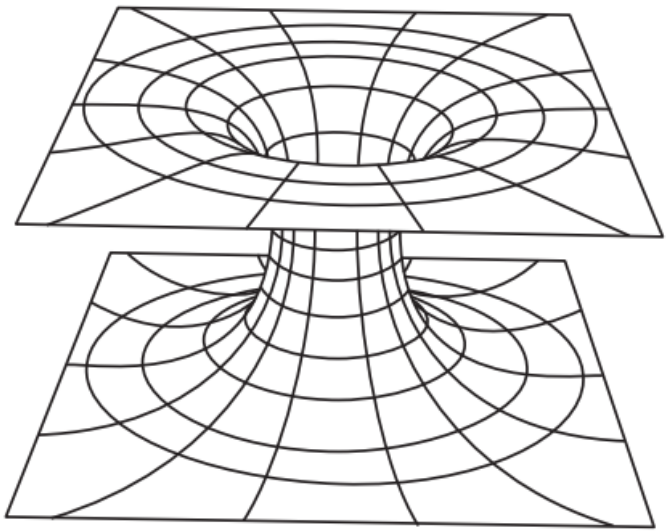

Figure 1: Wormhole scheme

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}.$$

Energy conditions

- WEC: $T_{\mu\nu}t^\mu t^\nu \geq 0. \Leftrightarrow \rho > 0$
- DEC:

  $T_{\mu\nu}t^\mu t^\nu \geq 0$ and $T_{\mu\nu}t^\nu$ it's not spacelike

  $\rightarrow \rho > 0$ (locally), energy flux is timelike or null.

Einstein-Rosen bridge

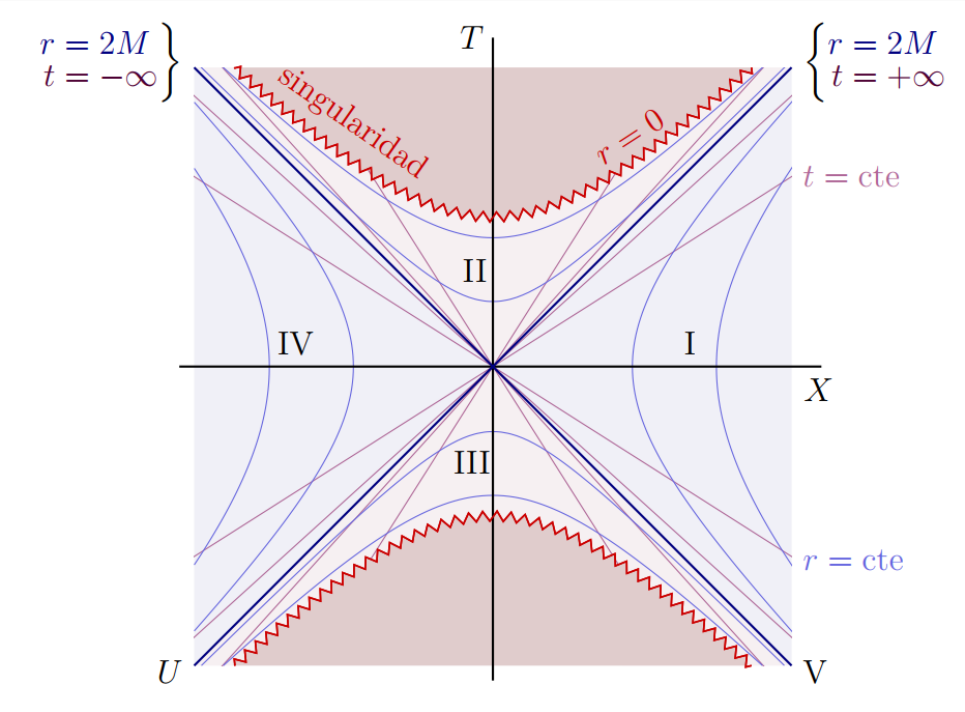

Figure 2: Kruskal diagram illustrating the wormhole connecting regions I and IV.

- It does not violate energy conditions: solution in empty spacetime.
- not traversable wormhole solution.

Solución de Visser

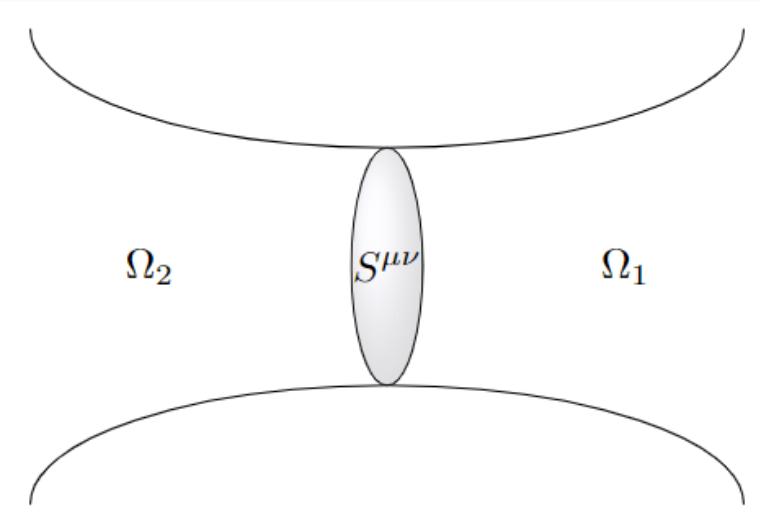

Figure 3: Surface stress-energy at the junction

- Two connected Schwarzschild manifolds without event horizon.
- Einstein equations in terms of surface stress-energy $S^{\mu\nu}$ and extrinsic curvature κ^i_j

$$S^i_{\ j} = -\frac{1}{8\pi G}[\kappa^i_{\ j} - \delta^i_j \kappa^k_k]. \qquad (1)$$

surface energy density turns out to be negative!!!

$$\sigma = -\frac{1}{2\pi a} \cdot \sqrt{1 - \frac{2MG}{a}} \qquad (2)$$

Analysis and conclusion

- Since surface energy density is negative, the solution violates WEC and DEC.
- It is concluded that Visser's solution is not physically possible in the frame of general relativity.

References

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