

Scalar potential analysis of the \mathbb{Z}_5 multi-component dark matter model¹

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Outline

- The \mathbb{Z}_5 model.
- Experimental constraints.
- Theoretical bounds.
- Final Remarks.

The \mathbb{Z}_5 DM model

The \mathbb{Z}_5 group contains the five 5th roots of 1,

$$\mathbb{Z}_5 = \{\omega_5^\alpha = \exp(i2\pi\alpha/5), \text{ with } \alpha = 0, 1, \dots, 4\}.$$

Then we can introduce two new complex scalar fields that transform as

$$\phi_1 \sim \omega_5, \quad \phi_2 \sim \omega_5^2; \quad \omega_5 = e^{i2\pi/5}.$$

After of the spontaneous electroweak symmetry breaking, the scalar potential of the model reads

$$\begin{aligned}
 \mathcal{V}_{\mathbb{Z}_5} = & \frac{1}{2} M_h^2 h^2 + \lambda_H v_H h^3 + \frac{1}{4} \lambda_H h^4 \\
 & + \sum_{i=1}^2 \left(M_i^2 |\phi_i|^2 + \lambda_{4i} |\phi_i|^4 + \lambda_{Si} v_H h |\phi_i|^2 + \frac{1}{2} \lambda_{Si} h^2 |\phi_i|^2 \right) \\
 & + \lambda_{412} |\phi_1|^2 |\phi_2|^2 \\
 & + \frac{1}{2} (\mu_{S1} \phi_1^2 \phi_2^* + \mu_{S2} \phi_2^2 \phi_1 + \lambda_{31} \phi_1^3 \phi_2 + \lambda_{32} \phi_1 \phi_2^{*3} + \text{h.c.}) ,
 \end{aligned}$$

where $\phi_{1,2}$ do not acquire vev, and $M_1 < M_2 < 2M_1$ so that both are stable. They are singlets under \mathcal{G}_{SM} and the SM particles are singlets under the \mathbb{Z}_5 symmetry.

Parameter space

Set of free parameters

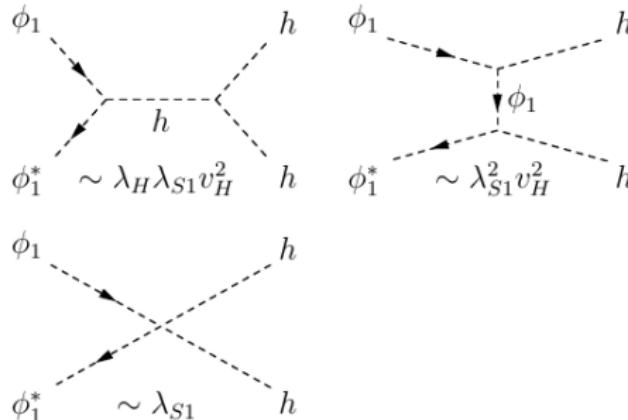
$$M_i, \lambda_{4i}, \lambda_{Si}, \lambda_{412}, \mu_{Si}, \lambda_{3i}.$$

- $M_i \Rightarrow \phi_i$ mass
- $\lambda_{4i} \Rightarrow \phi_i$ self-coupling
- $\lambda_{412} \Rightarrow \phi_1\text{-}\phi_2$ coupling
- $\lambda_{Si} \Rightarrow \phi_i\text{-}h$ coupling

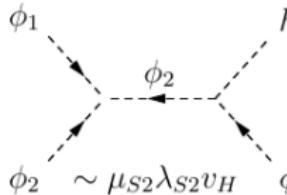
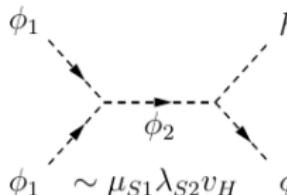
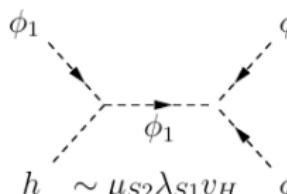
New processes

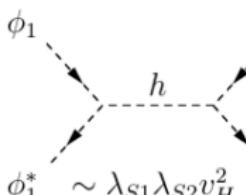
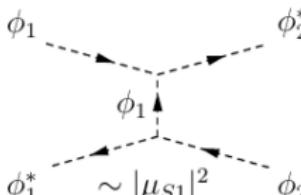
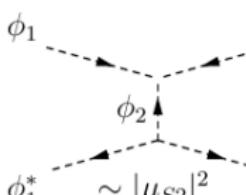
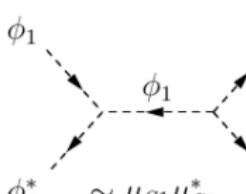
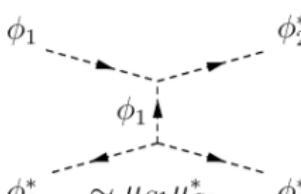
- $\mu_{Si} \Rightarrow \phi_1\text{-}\phi_2$ trilinear-coupling (semi-annihilations)
- $\lambda_{3i} \Rightarrow \phi_1\text{-}\phi_2$ quartic-coupling (conversions)

Processes

Type	Process	Channels
Annihilation	$\phi_1 + \phi_1^* \rightarrow h + h$ (1100)	

Semi-annihilation

Type	Process	Channels
	$\phi_1 + \phi_2 \rightarrow \phi_2 + h$ (1220)	 $\phi_2 \sim \mu_{S2} \lambda_{S2} v_H$
	$\phi_1 + \phi_1 \rightarrow \phi_2^* + h$ (1120)	 $\phi_1 \sim \mu_{S1} \lambda_{S2} v_H$
	$\phi_1 + h \rightarrow \phi_2 + \phi_2$ (1022)	 $h \sim \mu_{S2} \lambda_{S1} v_H$

Type	Process	Channels
Conversion	$\phi_1 + \phi_1^* \rightarrow \phi_2 + \phi_2^*$ (1122)	 $\sim \lambda_{S1}\lambda_{S2}v_H^2$
		 $\sim \mu_{S1} ^2$
		 $\sim \mu_{S2} ^2$
	$\phi_1 + \phi_2^* \rightarrow \phi_2^* + \phi_2^*$ (1222)	 $\sim \mu_{S1}\mu_{S2}^*$
		 $\sim \mu_{S1}\mu_{S2}^*$

Constraints on the parameter space

Theoretical bounds

- Perturbative unitarity.
- Positivity (bounded from below).
- Potential Stability (EW , \mathbb{Z}_5).
- RGEs stability at high energy.

Experimental constraints

- Relic abundance.
- Direct detection experiments
 - XENON-1T.
 - PANDAX.
 - LUX-ZEPLIN.
 - DARWIN.

Scan

The scan on the parameter space was performed in the following ranges.

$$40 \text{ GeV} \leq M_1 \leq 2 \text{ TeV},$$

$$M_1 < M_2 < 2M_1,$$

$$10^{-4} \leq \lambda_{4i}, |\lambda_{412, Si, 3i}| \leq \sqrt{4\pi},$$

$$100 \text{ GeV} \leq |\mu_{Si}| \leq 10 \text{ TeV}.$$

Experimental constraints

Bèlanger, Pukhov, Yaguna and Zapata (2020) showed that the model fulfills the direct detection and relic abundance constraints for a wide set of points in the parameter space.

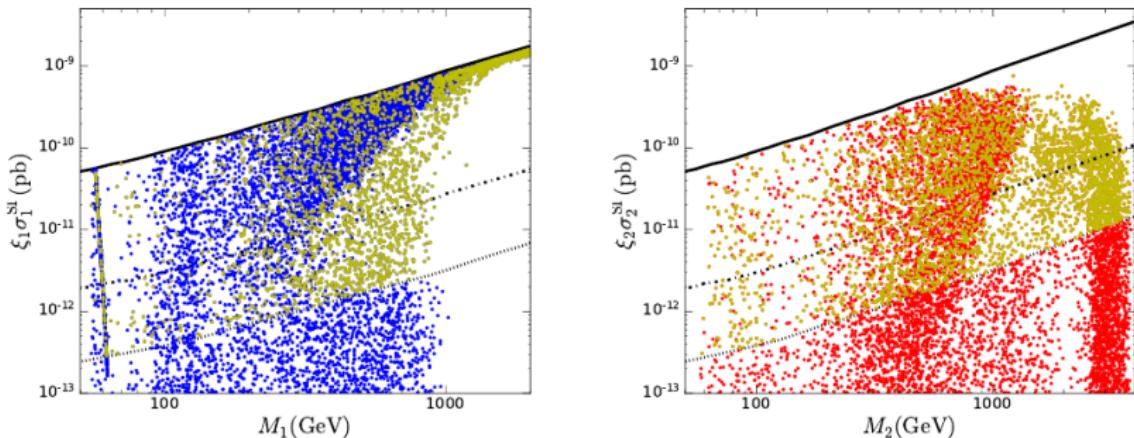


Figure: arXiv:2006.14922

- **Perturbative unitarity**

We must guarantee the probability conservation, maintaining the S -matrix unitary. This is

$$\text{Re}(a_0^i) \leq \frac{1}{2},$$

where a_0^i is the i th eigenvalue of the S -matrix.

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The coupling matrix remains *copositive*

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The scalar potential has 12 minima, but the SM minimum ($\text{EW}^\pm, \mathbb{Z}_5$) must be global.

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- **RGEs stability**

The above constraints must be fulfilled at high energy scales. In addition, the parameter space must remain real.

Some of the obtained constraints...

For $s \rightarrow \infty$, the eigenvalues are analytical and read

$$\begin{aligned} & \lambda_{Si} < 8\pi, \\ & \left| 2\lambda_{4i} + \lambda_{412} \pm \sqrt{18\lambda_{3i}^2 + (2\lambda_{4i} - \lambda_{412})^2} \right| < 16\pi, \\ & |\alpha_{1,2,3}| \leq 1/2, \end{aligned} \quad (1)$$

where α_i are the roots of the polynomial $c_3x^3 + c_2x^2 + c_1x + c_0$ with

$$\begin{aligned} c_0 &= 2v_H^2 \left(-3\lambda_{412}^2\lambda_H + \lambda_{41} (48\lambda_{42}\lambda_H - 4\lambda_{S2}^2) \right. \\ &\quad \left. - 4\lambda_{42}\lambda_{S1}^2 + 2\lambda_{412}\lambda_{S1}\lambda_{S2} \right), \\ c_1 &= 6\pi v_H^2 \left(24(\lambda_{41} + \lambda_{42})\lambda_H - \lambda_{412}^2 + 16\lambda_{41}\lambda_{42} \right. \\ &\quad \left. - 2(\lambda_{S1}^2 + \lambda_{S2}^2) \right), \\ c_2 &= 512\pi^2 v_H^2 (3\lambda_H + 2(\lambda_{41} + \lambda_{42})), \\ c_3 &= 4096\pi^3 v_H^2. \end{aligned} \quad (2)$$

Some of the obtained constraints...

$$\lambda_{4i} > 0, D > 0 \wedge (Q > 0 \vee R > 0),$$

$$\begin{aligned} D = & -27\lambda_{42}^2|\lambda_{31}|^4 - 4|\lambda_{32}|^3|\lambda_{31}|^3 + 18|\lambda_{32}|\lambda_{42}\lambda_{412}|\lambda_{31}|^3 \\ & - 4\lambda_{42}\lambda_{412}^3|\lambda_{31}|^2 + |\lambda_{32}|^2\lambda_{412}^2|\lambda_{31}|^2 - 6|\lambda_{32}|^2\lambda_{41}\lambda_{42}|\lambda_{31}|^2 \\ & + 144\lambda_{41}\lambda_{42}^2\lambda_{412}|\lambda_{31}|^2 - 192|\lambda_{32}|\lambda_{41}^2\lambda_{42}^2|\lambda_{31}| \\ & - 80|\lambda_{32}|\lambda_{41}\lambda_{42}\lambda_{412}^2|\lambda_{31}| + 18|\lambda_{32}|^3\lambda_{41}\lambda_{412}|\lambda_{31}| + 16\lambda_{41}\lambda_{42}\lambda_{412}^4 \\ & + 256\lambda_{41}^3\lambda_{42}^3 - 4|\lambda_{32}|^2\lambda_{41}\lambda_{412}^3 - 27|\lambda_{32}|^4\lambda_{41}^2 - 128\lambda_{41}^2\lambda_{42}^2\lambda_{412}^2 \\ & + 144|\lambda_{32}|^2\lambda_{41}^2\lambda_{42}\lambda_{412}, \end{aligned}$$

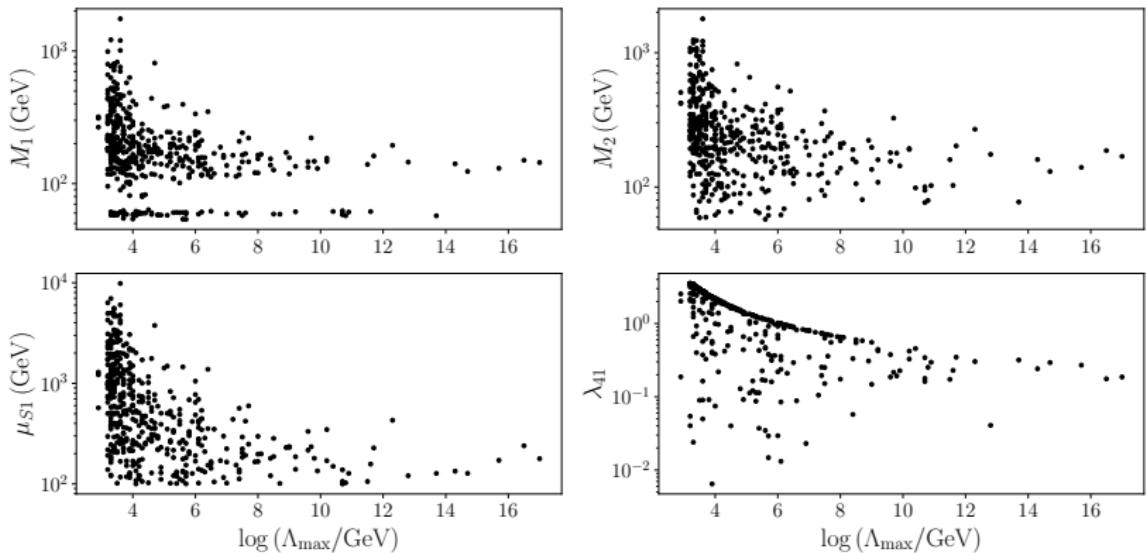
$$Q = 8\lambda_{41}\lambda_{412} - 3|\lambda_{31}|^2,$$

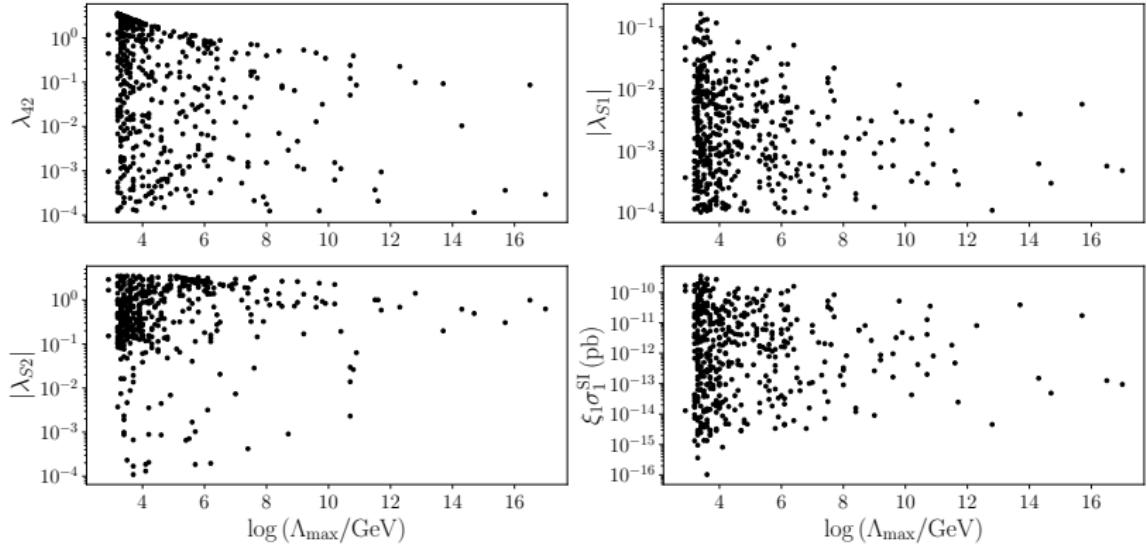
$$\begin{aligned} R = & -3|\lambda_{31}|^4 + 16\lambda_{41}\lambda_{412}|\lambda_{31}|^2 + 64\lambda_{41}^3\lambda_{42} \\ & - 16\lambda_{41}^2(\lambda_{412}^2 + |\lambda_{31}||\lambda_{32}|). \end{aligned}$$

Allowing λ_{Si} to take negative values, further conditions arise. They are not shown here by their analytical extension.

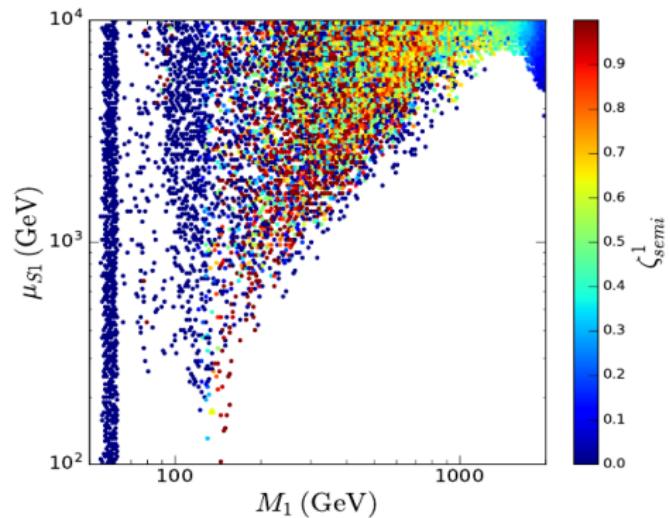
Theoretical bounds (combination of constraints)

Taking $\lambda_{412} = \lambda_{3i} = \mu_{S2} = 0$, we obtained the following behaviors for each parameter.



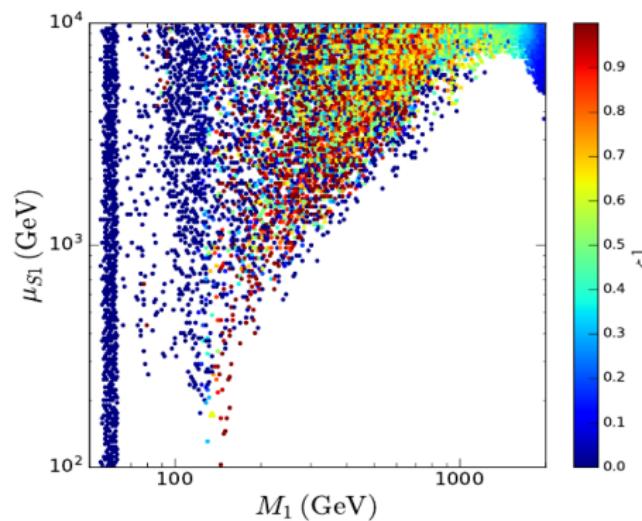


Requiring $\log(\Lambda_{\max}/\text{GeV}) > 4$

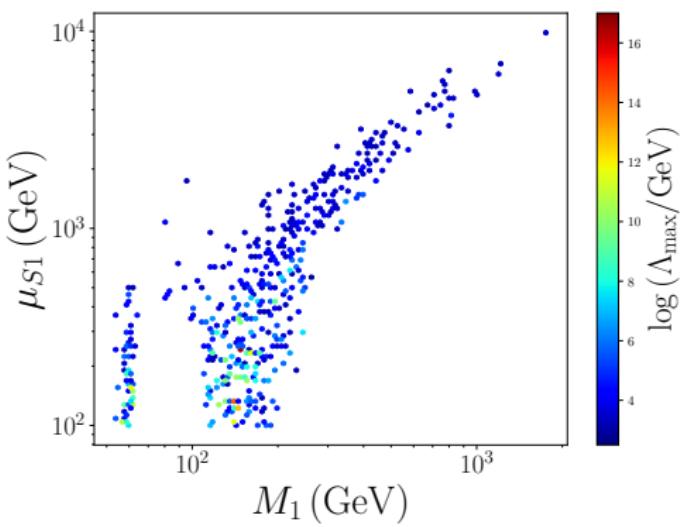


arXiv:2006.14922

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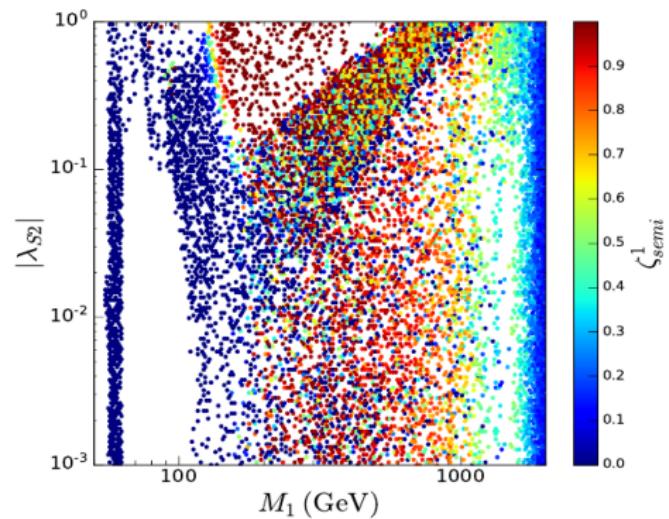


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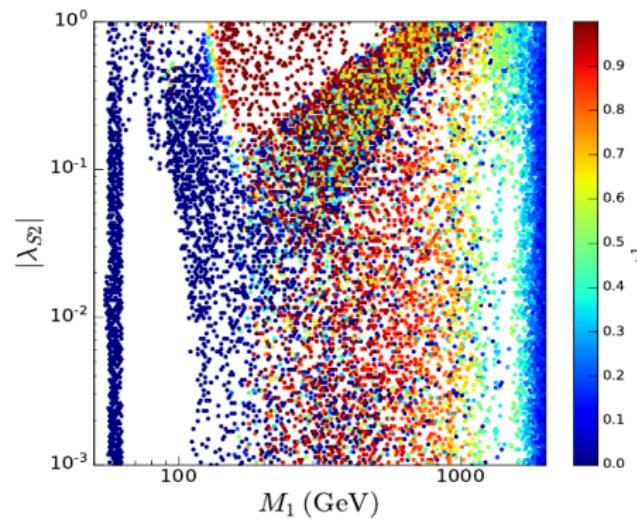
Combination of constraints

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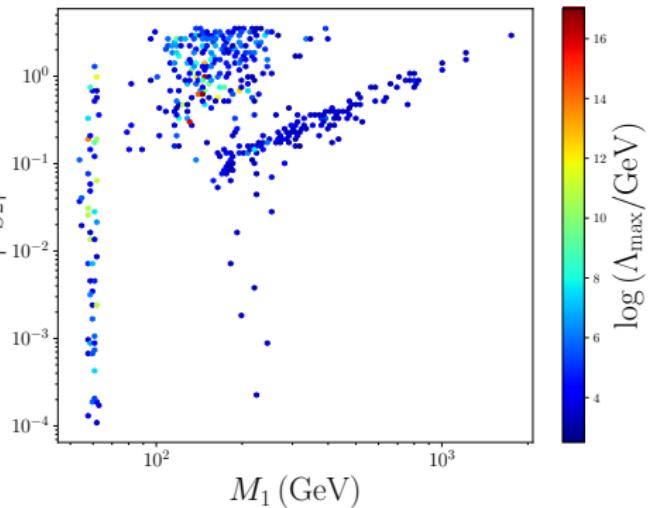


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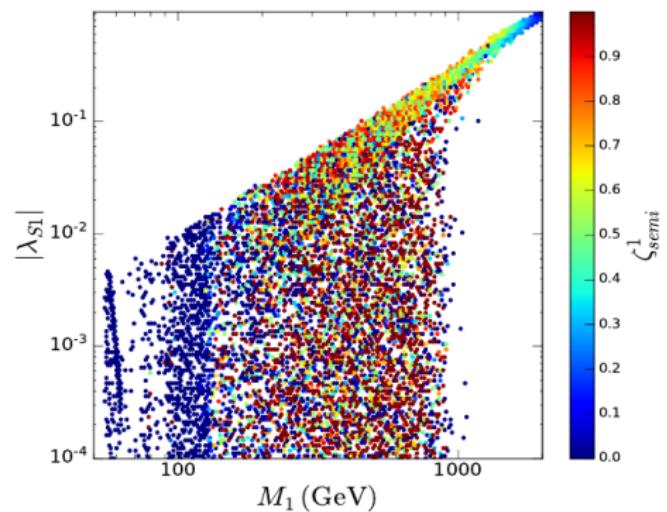


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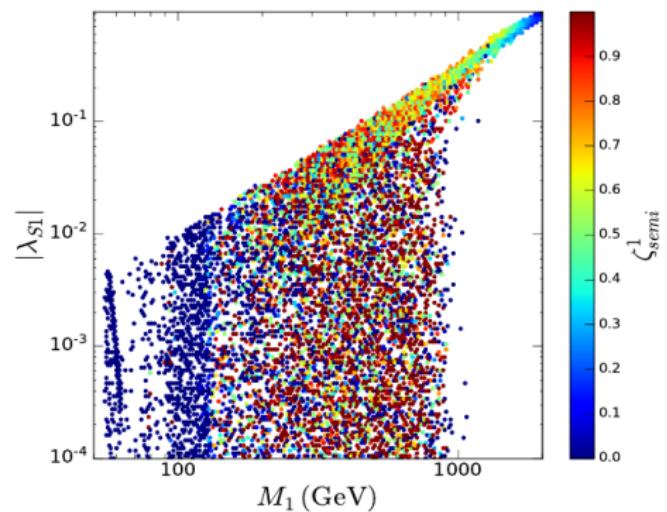
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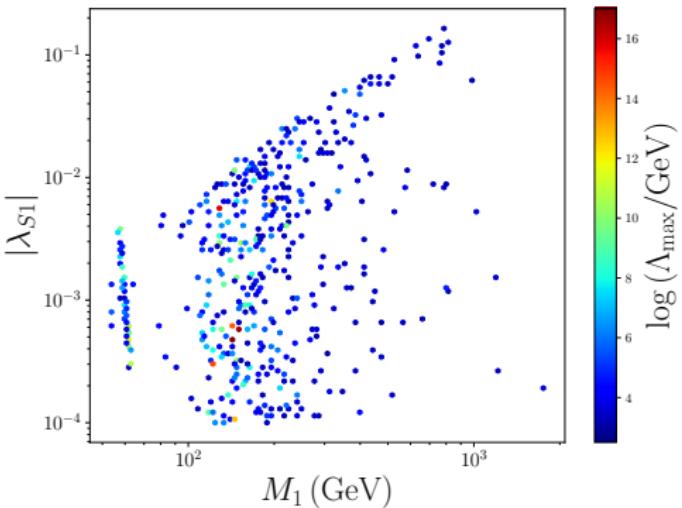


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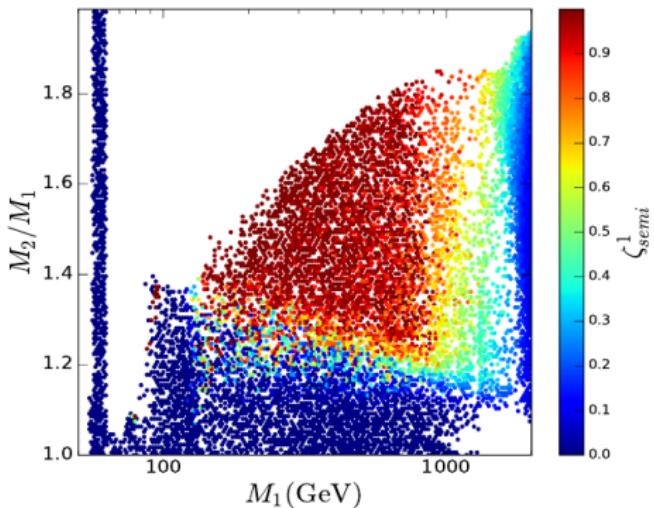


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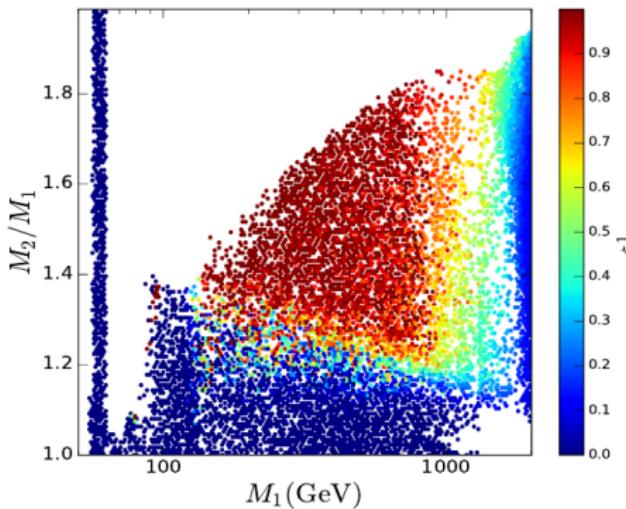
Combination of constraints

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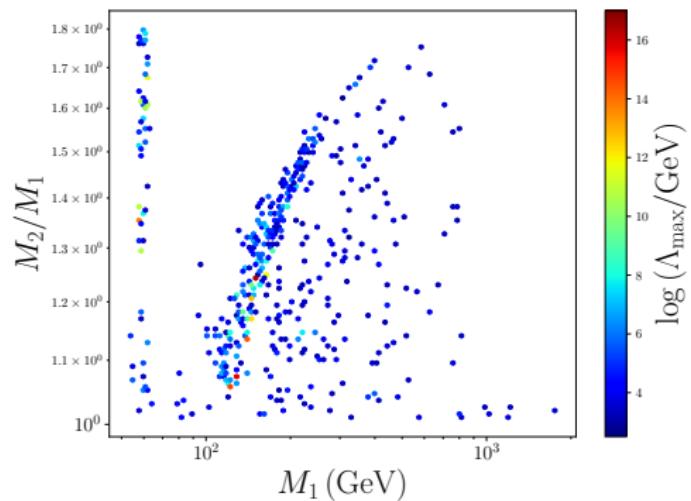


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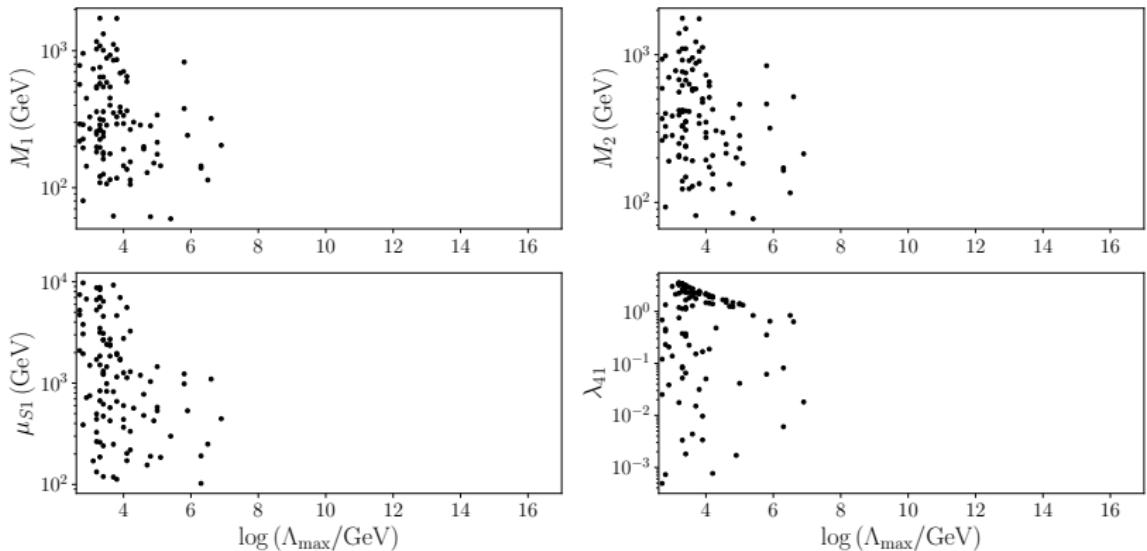


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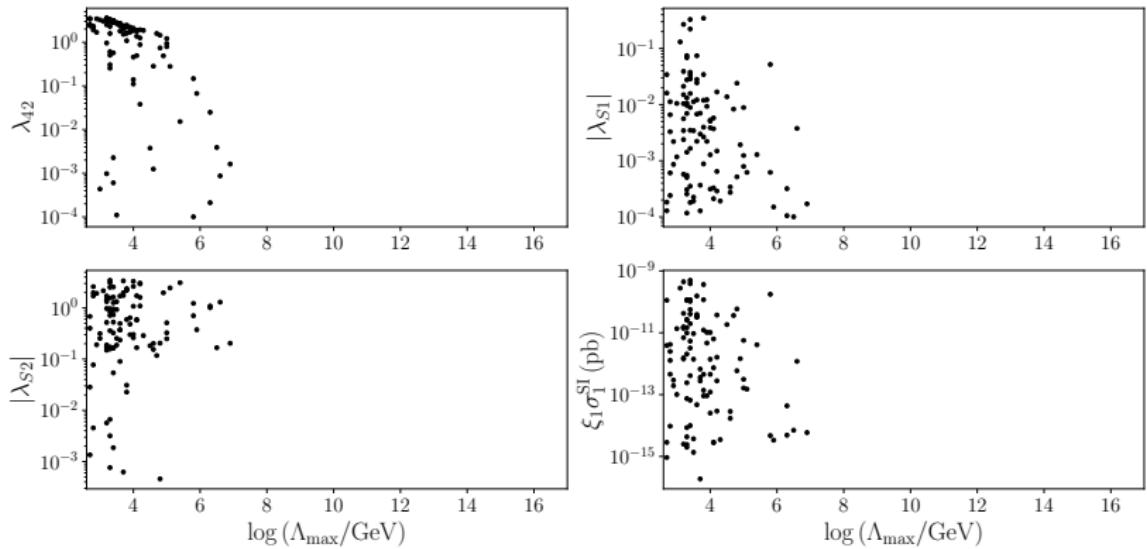


Combination of constraints

For values of $\lambda_{412}, \lambda_{3i}, \mu_{S2} \neq 0$



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Remarks

- The model remains viable at high energy scales, even for all the parameters taking non-zero values.
- The most restrictive condition is the scalar potential stability.
- The vacuum stability is not guaranteed for all the points in the parameter space. We found that the Higgs self-coupling can take negative values since its β -function at 2-loops level depends on λ_{Si} .
- The case $\lambda_{S1} < 0$ **and** $\lambda_{S2} < 0$ is totally excluded (does not fulfills positivity).
- The values of M_i , λ_{4i} , λ_{3i} and λ_{S1} must be restricted at $\Lambda \sim m_t$ in order to guarantee the viability of the model at high energy scales.

Thanks.