$$D_s^+ \to K^+ K^- K^+$$

Radiation topologies analysis

David Alejandro Barón Ospina

Motivation

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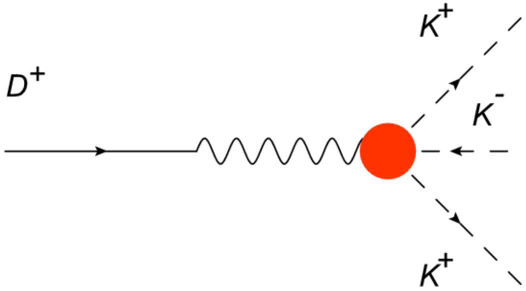

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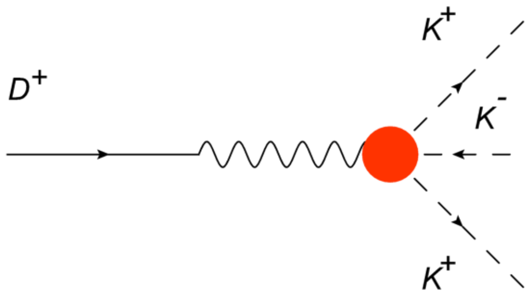

The anhalitaion topology is dominant over the radiation one due to its quark content.

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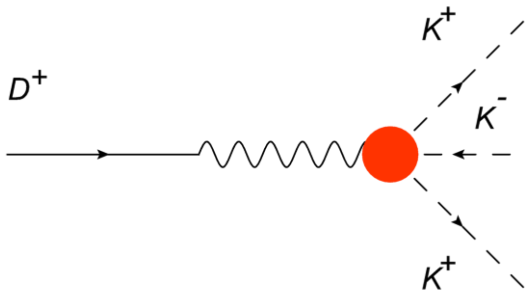
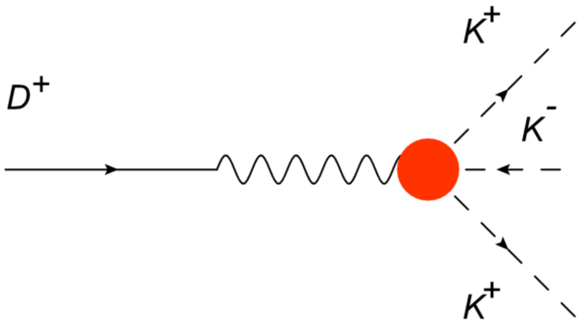
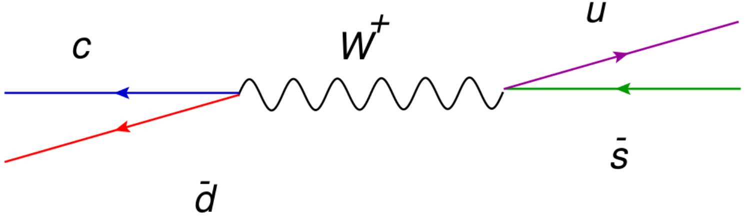


The anhalitaion topology is dominant over the radiation one due to its quark content.

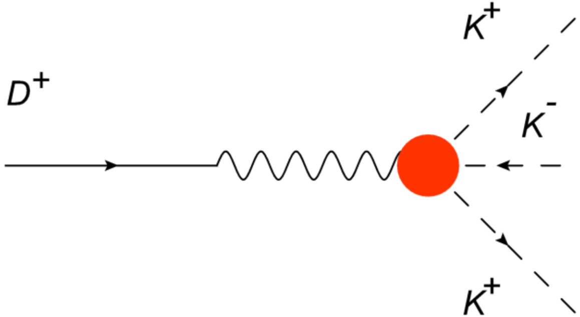
The weak vertex is a constant, and it can be easily factorized from the strong part of the amplitude.



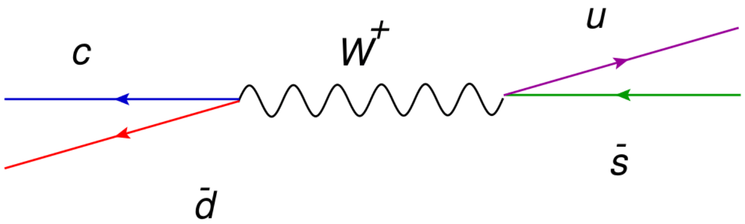
D^+
K^+
K^-
K^+



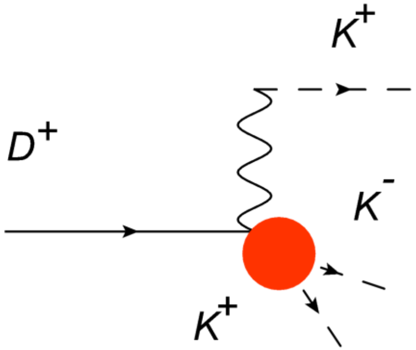
c
W^+
u
$\bar{s}$
$\bar{d}$



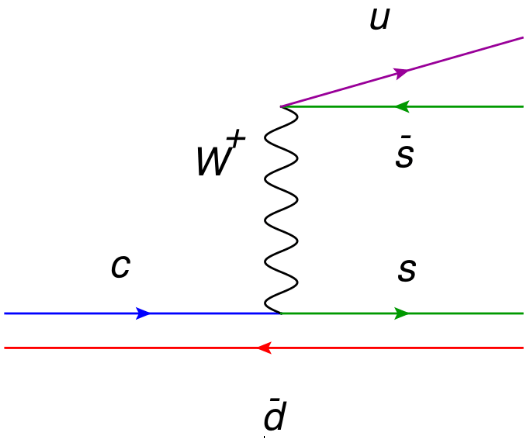
D^+
K^+
K^-
K^+



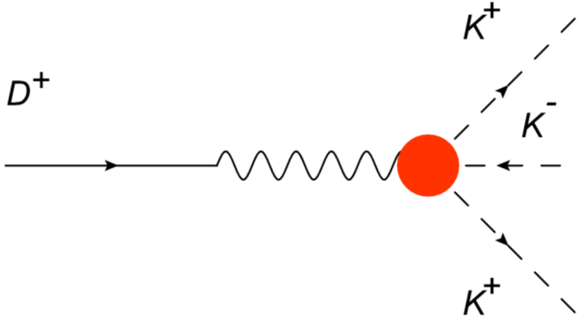
c
W^+
u
$\bar{s}$
$\bar{d}$

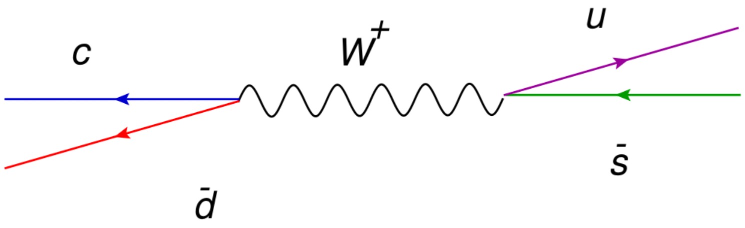


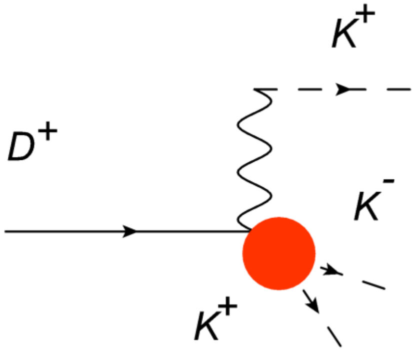
K^+
D^+
K^-
K^+

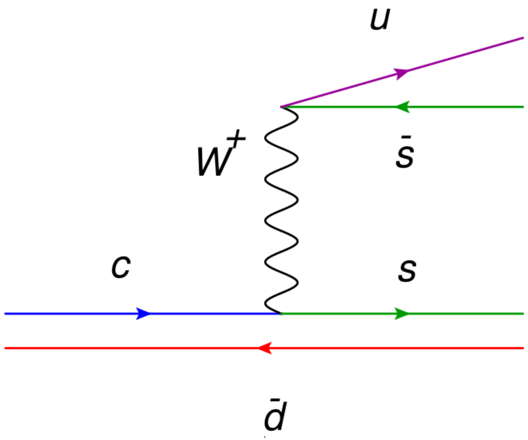


u
W^+
$\bar{s}$
c
s
$\bar{d}$

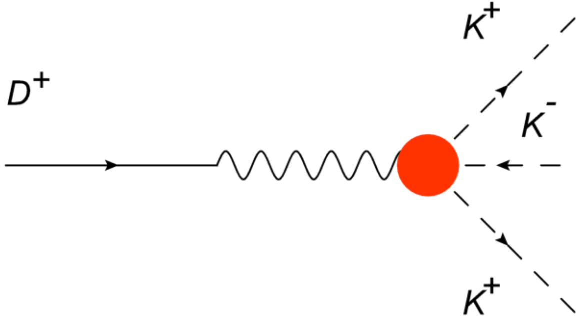


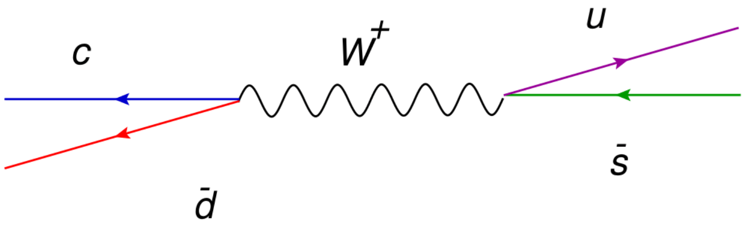


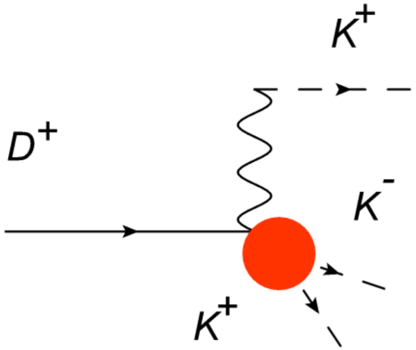


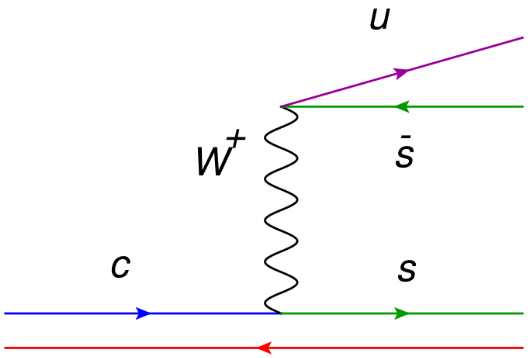


$$K^+ \leftrightarrow u\bar{s}$$



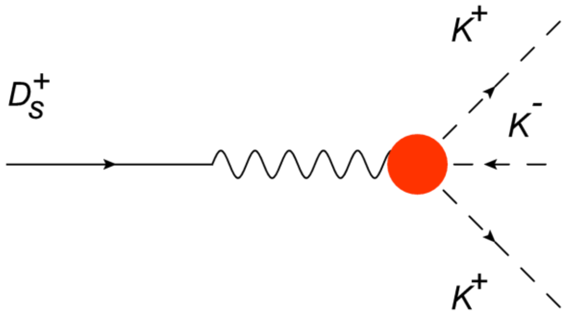


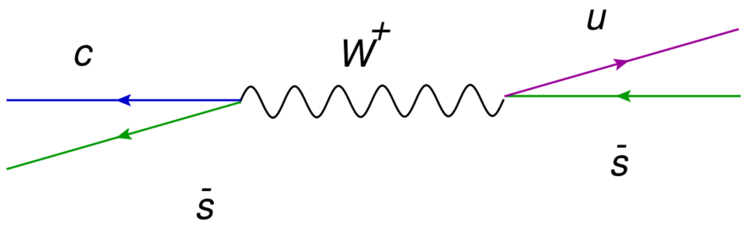


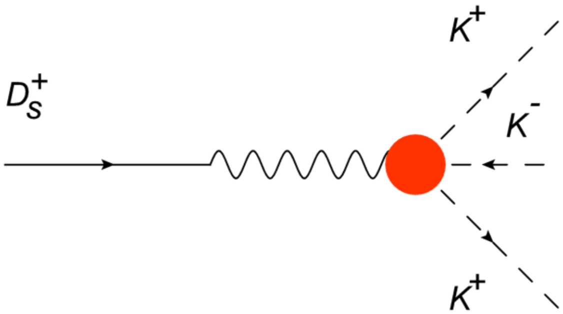


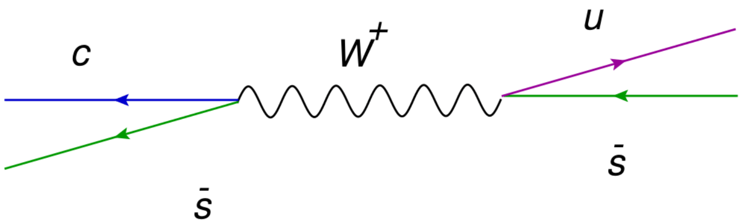
$\boxed{\bar{d}}$ → It needs more interactions at the quark level

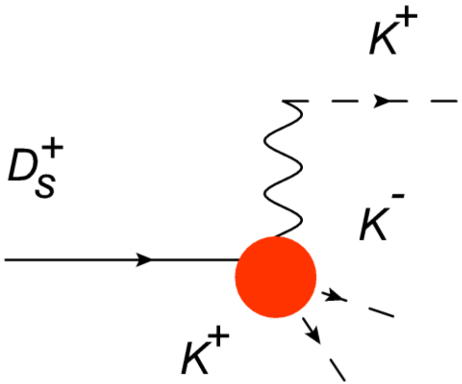
$$K^+ \leftrightarrow u\bar{s}$$

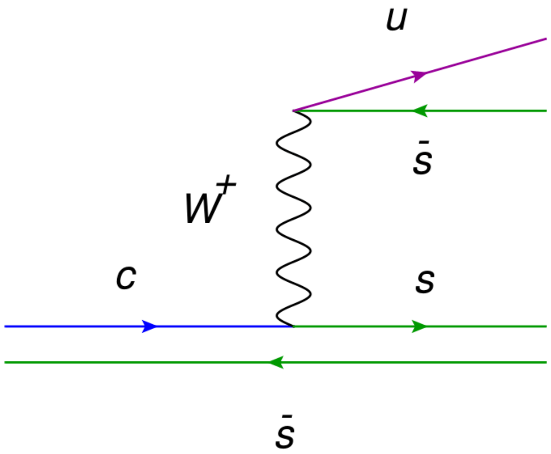

D_s^+
K^+
K^-
K^+

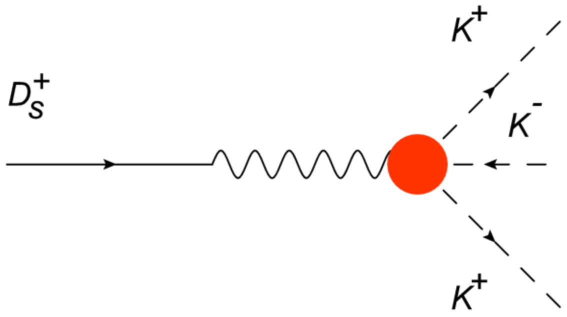

c
W^+
u
$\bar{s}$
$\bar{s}$

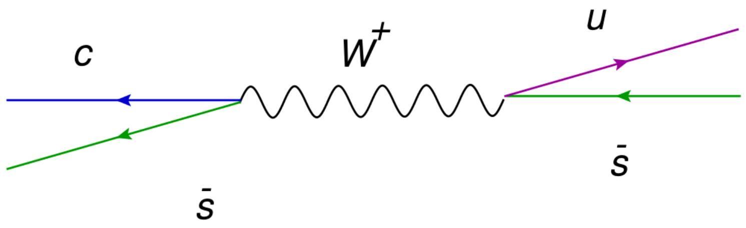

D_S^+
K^+
K^-
K^+

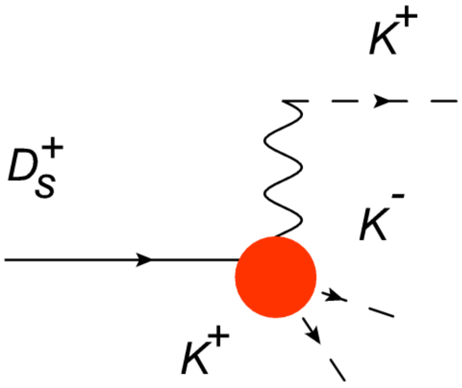

c
W^+
u
$\bar{s}$
$\bar{s}$

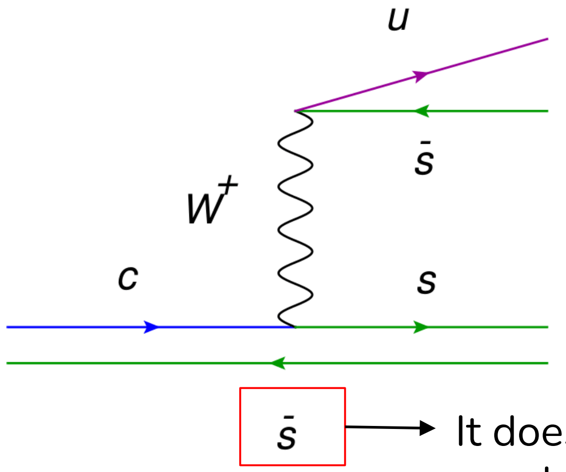

K^+
D_S^+
K^-
K^+


u
$\bar{s}$
W^+
c
s
$\bar{s}$





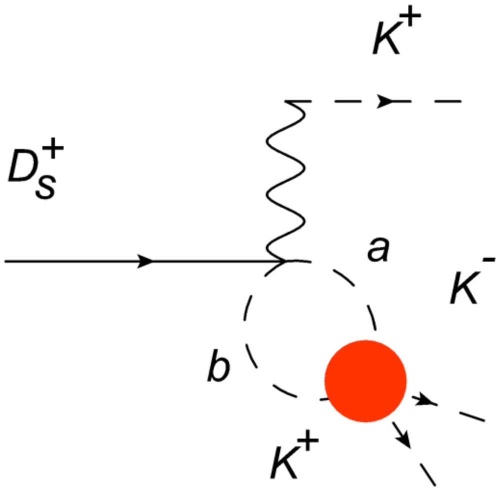




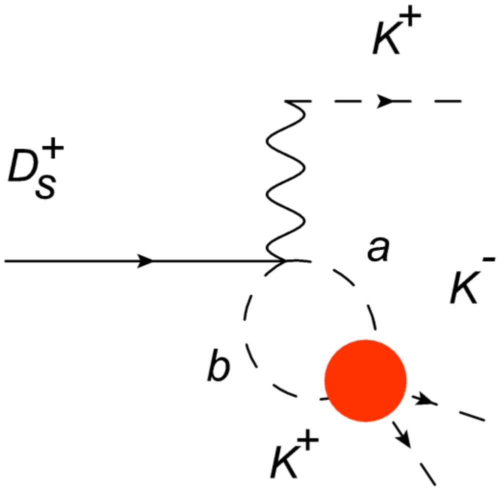
It does not have d quark any longer!

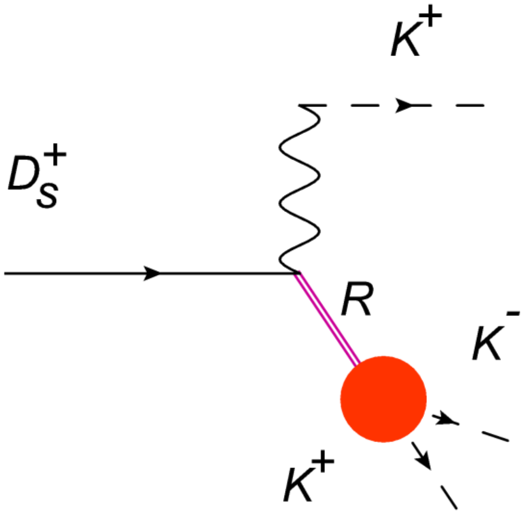
Resonant and non resonant contributions

Resonant and non resonant contributions



Resonant and non resonant contributions





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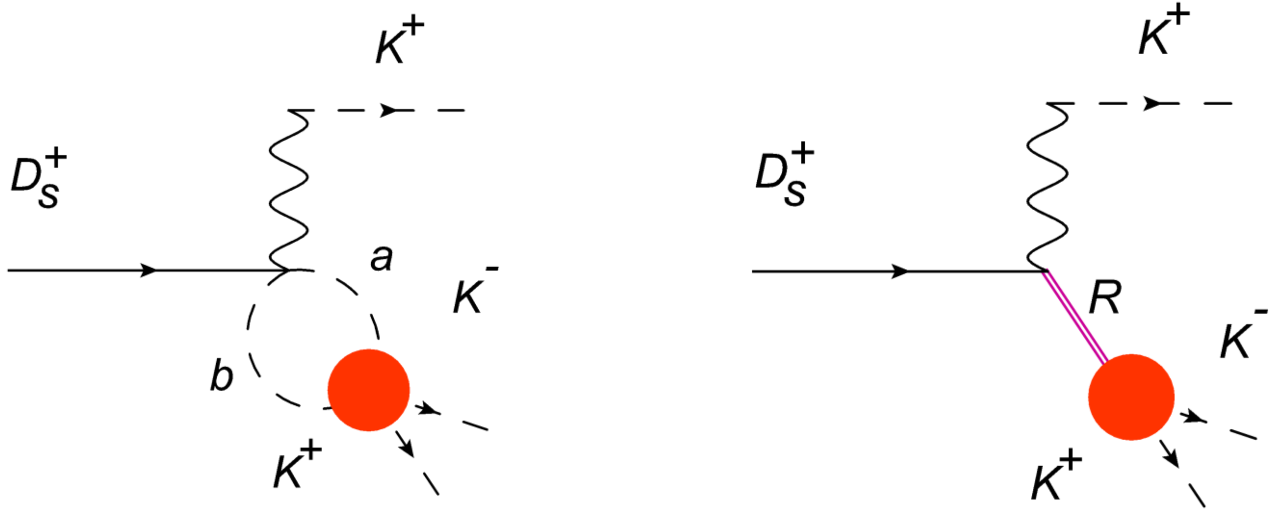

$$R : \phi,\, f_0$$

Non resonant weak vertex

Non resonant weak vertex

$$\bar{c}\gamma^{\mu}\left(1-\gamma^{5}\right)s \rightarrow \frac{i}{2}\alpha Tr\left(\gamma^{\mu}\left(1-\gamma^{5}\right)H_{i}u_{is}^{+}\right)$$

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It has kaons and pions fields

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$$\left\langle 0\left|\bar{c}\gamma^{\mu}\gamma^{5}s\right|D_s^{+}\right\rangle = i f_D P^{\mu}_{D_s^{+}}$$

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$$\left\langle W^{+}\phi^{e}\phi^{f}\left|\right|D_s^{+}\right\rangle = -\frac{g}{2\sqrt{2}}V_{cs}^{*}\frac{f_D}{16f^{2}}P_D^{\mu}\left(\lambda_e\lambda_f+\lambda_f\lambda_e\right)_{ss}\varepsilon_{\mu}^{*}\left(P_W\right)$$

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Resonant weak vertexes

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$$\left\langle W^+ f_0 \right|\left| D_s^+ \right\rangle = V_{cs}^* g \left(S_1 P_D^\mu + S_2 P_R^\mu \right) \varepsilon_\mu^* \left(P_W \right)$$

Resonant weak vertexes

$$\langle W^{+} f_0 || D_s^{+} \rangle = V_{cs}^{*} g \left(S_1 P_D^{\mu} + S_2 P_R^{\mu} \right) \varepsilon_{\mu}^{*} \left(P_W \right)$$

$$\langle W^{+} \phi || D_s^{+} \rangle = V_{cs}^{*} g \sin\theta \Bigg(V_1 \delta^{\alpha v} P_{\phi}^{\mu} + V_2 \delta^{\alpha v} P_{\phi}^{\mu} + V_3 P_D^{\mu} P_D^{\alpha} P_{\phi}^{v}$$

$$+ V_4 P_D^{\mu} P_{\phi}^{\alpha} P_{\phi}^{v} - (\mu \leftrightarrow v) \Bigg) \left(\frac{i}{m_{\phi}} \left(P_{\phi\mu} \varepsilon_v \left(P_{\phi} \right) - P_{\phi v} \varepsilon_{\mu} \left(P_{\phi} \right) \right) \right)^{*} \varepsilon_{\alpha}^{*} \left(P_w \right)$$

Resonant weak vertexes

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$$+ \boxed{V_4} P_D^\mu P_\phi^\alpha P_\phi^v - (\mu \leftrightarrow v) \Big) \left(\frac{i}{m_\phi} \left(P_{\phi\mu} \varepsilon_v \left(P_\phi \right) - P_{\phi v} \varepsilon_\mu \left(P_\phi \right) \right) \right)^* \varepsilon_\alpha^* \left(P_w \right)$$

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$$\left. + \boxed{V_4} P_D^\mu P_\phi^\alpha P_\phi^v - (\mu \leftrightarrow v) \right) \boxed{\left(\frac{i}{m_\phi} \left(P_{\phi\mu} \varepsilon_v \left(P_\phi \right) - P_{\phi v} \varepsilon_\mu \left(P_\phi \right) \right) \right)^*} \boxed{\varepsilon_\alpha^* \left(P_w \right)}$$

Constant weak vertex approximation

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$$\langle W^+ \phi || D_s^+ \rangle = V_{cs}^* g \sin\theta \left(V \delta^{\alpha v} P_D^\mu - (\mu \leftrightarrow v) \right) \left(\frac{i}{m_\phi} (P_{\phi\mu} \varepsilon_v (P_\phi) - P_{\phi v} \varepsilon_\mu (P_\phi)) \right)^* \varepsilon_\alpha^* (P_w)$$

Final states interactions

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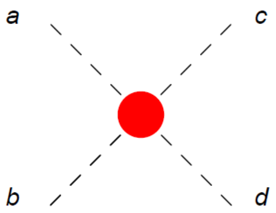
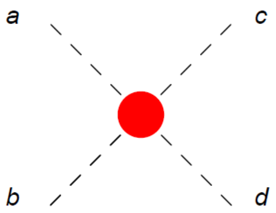


$$= i\kappa_{ab|cd}$$

Final states interactions



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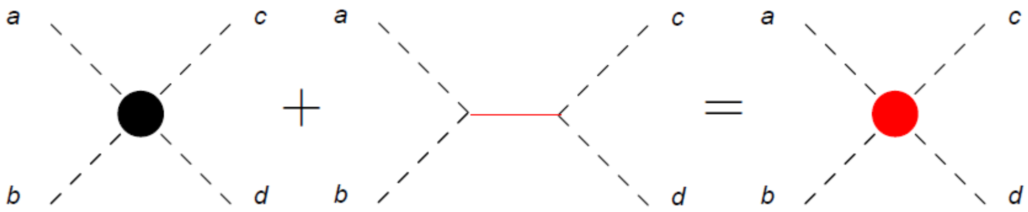
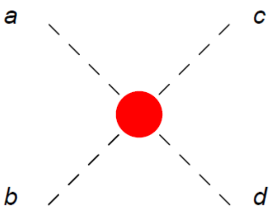
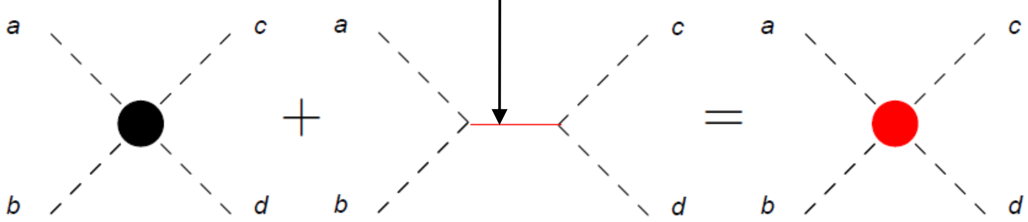


Final states interactions

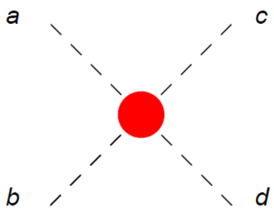
Resonance with quantum numbers J, I



$= i\kappa_{ab|cd}$



Final states interactions



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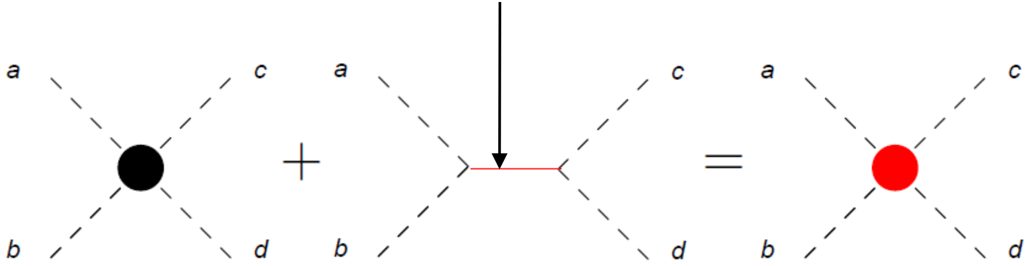
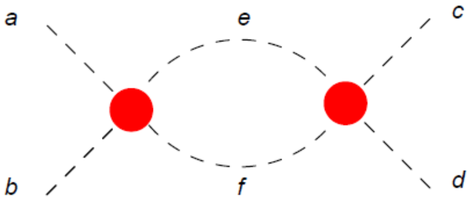


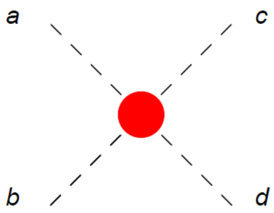


$$= \sum_{ef} i\kappa_{ab|ef}(-\Omega_{ef})\kappa_{ef|cd}$$

$$= \sum_{ef} i\kappa_{ab|ef}M_{ef|cd}$$

Final states interactions

Resonance with quantum numbers J, I



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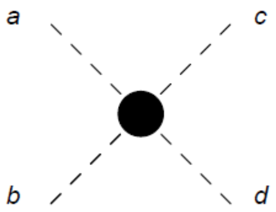
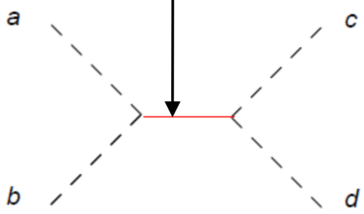
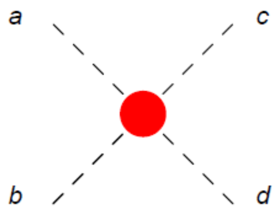
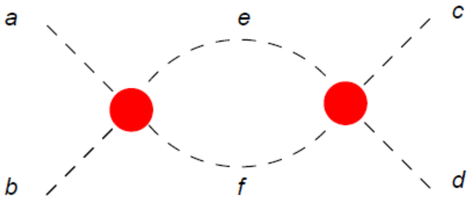


$$+$$



$$=$$





Loop function

$$= \sum_{ef} i\kappa_{ab|ef}(-\Omega_{ef})\kappa_{ef|cd}$$

$$= \sum_{ef} i\kappa_{ab|ef}M_{ef|cd}$$

$$\sum_{ef} i\kappa_{ab|ef} \left(\delta_{ef|cd} + M_{ef|cd} + M^2_{ef|cd} + \ldots\ldots\right) = \sum_{ef} i\kappa_{ab|ef} \sum_{n=0}^{\infty} M^n_{ef|cd}$$

$$\sum_{ef} i\kappa_{ab|ef} \left(\delta_{ef|cd} + M_{ef|cd} + M^2_{ef|cd} + \ldots\ldots\right) = \sum_{ef} i\kappa_{ab|ef} \sum_{n=0}^{\infty} M^n_{ef|cd}$$

$$\langle ab||cd\rangle = \sum_{ef} i\kappa_{ab|ef} \left(\frac{1}{1-M}\right)_{ef|cd}$$

Example: ϕ

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$$\langle K^-(P_1)\,K^+(P_2)\,[K^+(P_3)]\,|V_8^{KK}|\,D_s^+(P_D)\rangle = \frac{i}{2}\left(m_{13}^2 - m_{23}^2\right)\Gamma_{(0)KK}^{(1,0)}$$

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$\Gamma_{(0)ab}^{[J,I]}$ Tree-level amplitude $D_s^+ \to K^+(P_3)a(P_2)b(P_1)$ of the pair $a(P_2)b(P_1)$ component with quantum numbers J, I

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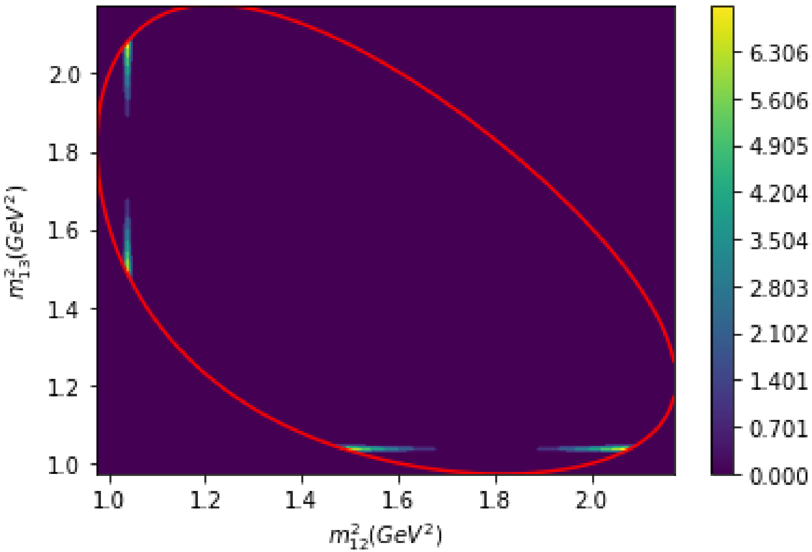
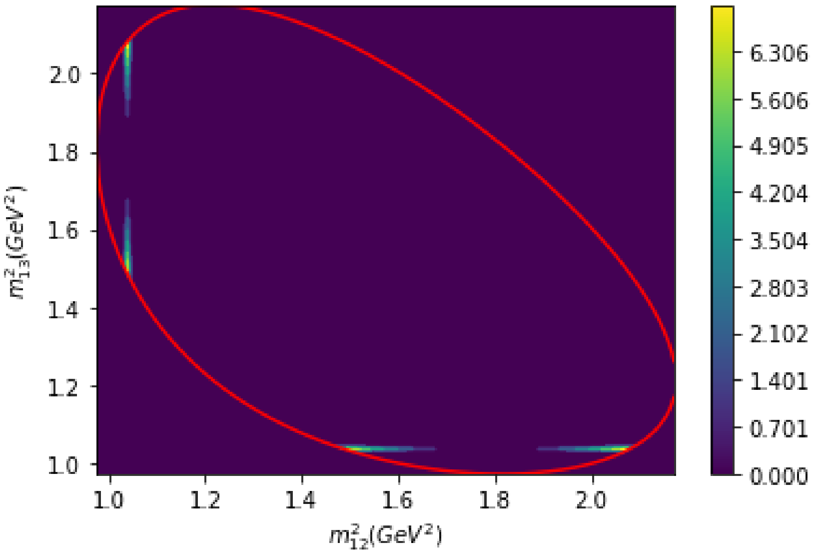
$\Gamma_{(0)ab}^{[J,I]}$ Tree-level amplitude $D_s^+ \to K^+(P_3)a(P_2)b(P_1)$ of the pair $a(P_2)b(P_1)$ component with quantum numbers J, I

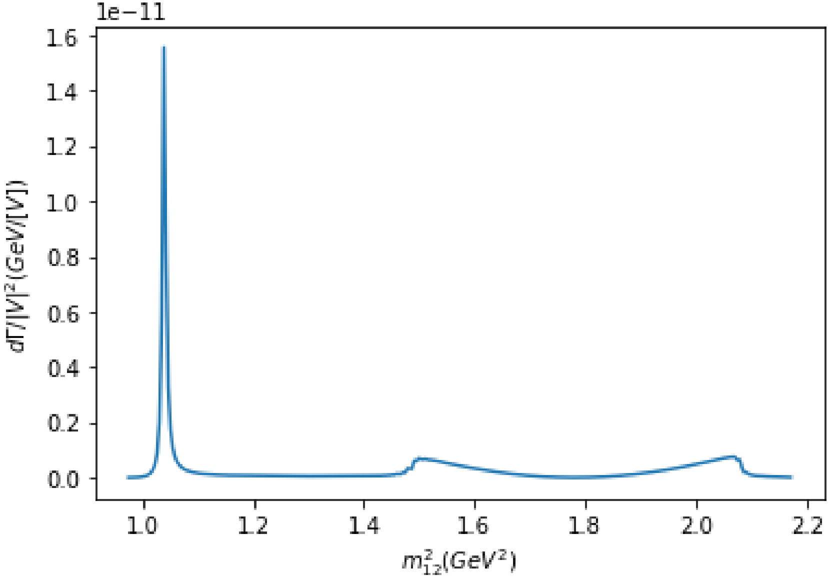
$$\langle K^- K^+ K^+ || D_s^+\rangle = \frac{\frac{i}{2}\left(m_{13}^2 - m_{23}^2\right)\Gamma_{(0)KK}^{(1,0)}}{1-M} + \left(m_{12}^2 \leftrightarrow m_{13}^2\right)$$



2.0
1.8
1.6
1.4
1.2
1.0
$m_{13}^2(GeV^2)$
1.0
1.2
1.4
1.6
1.8
2.0
$m_{12}^2(GeV^2)$
6.306
5.606
4.905
4.204
3.504
2.803
2.102
1.401
0.701
0.000



2.0
1.8
1.6
1.4
1.2
1.0
1.0 1.2 1.4 1.6 1.8 2.0
$m_{12}^2(GeV^2)$
$m_{13}^2(GeV^2)$
6.306
5.606
4.905
4.204
3.504
2.803
2.102
1.401
0.701
0.000



1e−11
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$d\Gamma/|V|^2(GeV/[V])$
1.0 1.2 1.4 1.6 1.8 2.0 2.2
$m_{12}^2(GeV^2)$

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