Thermalization in nuclear collisions

and emergence of the most perfect liquid in nature

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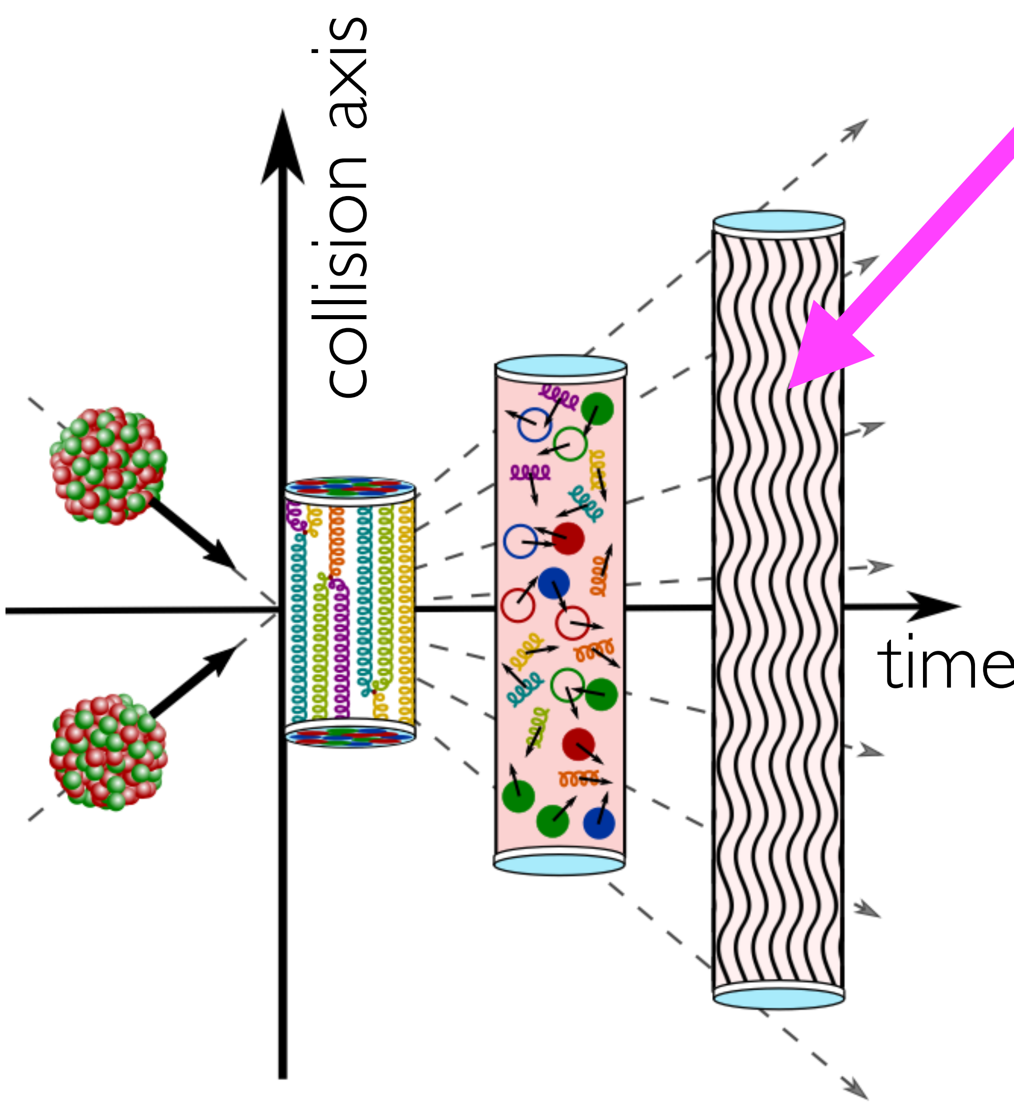
2110.07621 and **2112.12794** with Serantes, Spaliński, Svensson, Withers

for an overview, see, e.g., **2005.12299** with Berges, Mazeliauskas, Venugopalan

Context (2011-2013)

Motivation

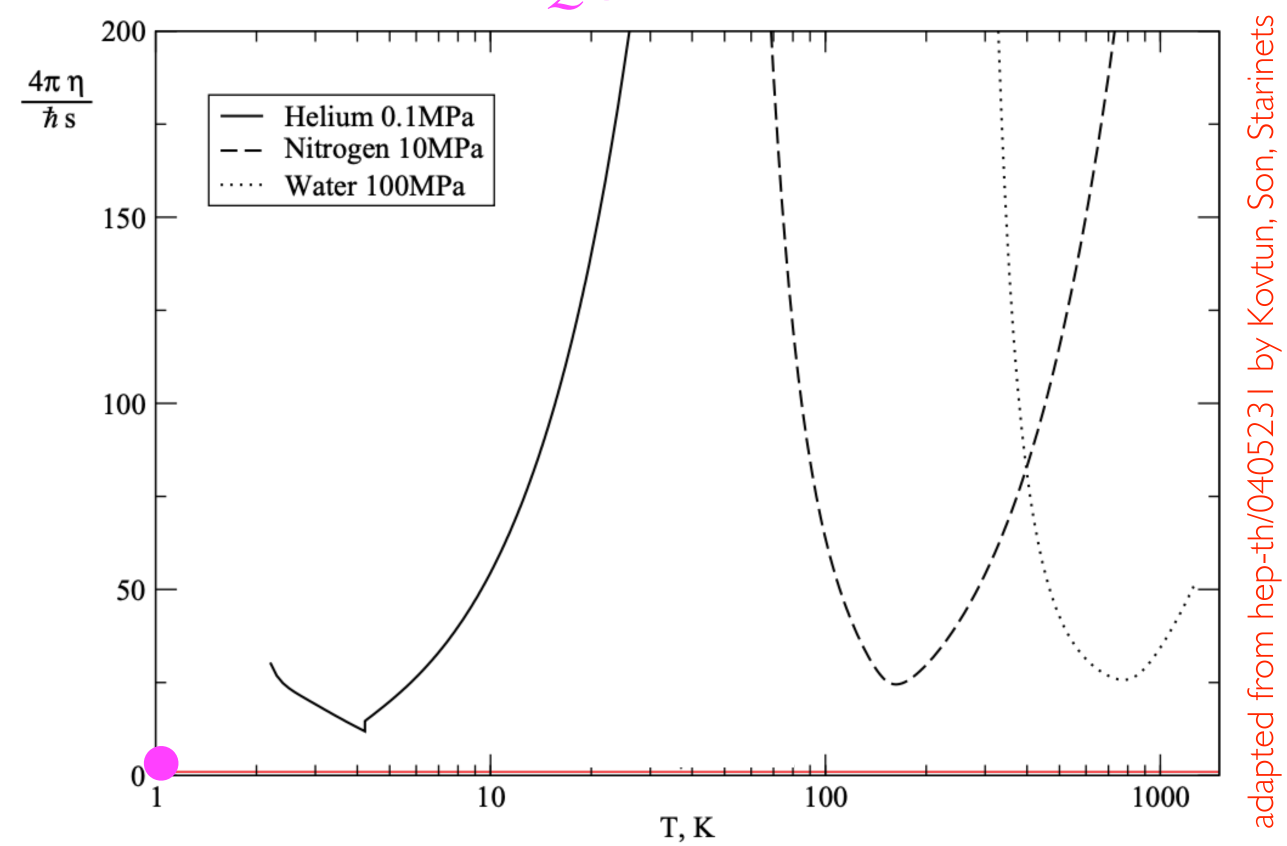
heavy-ion collisions
at RHIC and LHC



2005.12299
with Berges, Mazeliauskas & Venugopalan

the quark-gluon plasma is the most
perfect liquid known in the universe:

$$\left(\frac{\eta}{s}\right)_{QGP} = \mathcal{O}\left(\frac{1}{4\pi}\right)$$



this talk:

what does theory tells up in 2022 about its emergence in experiment?

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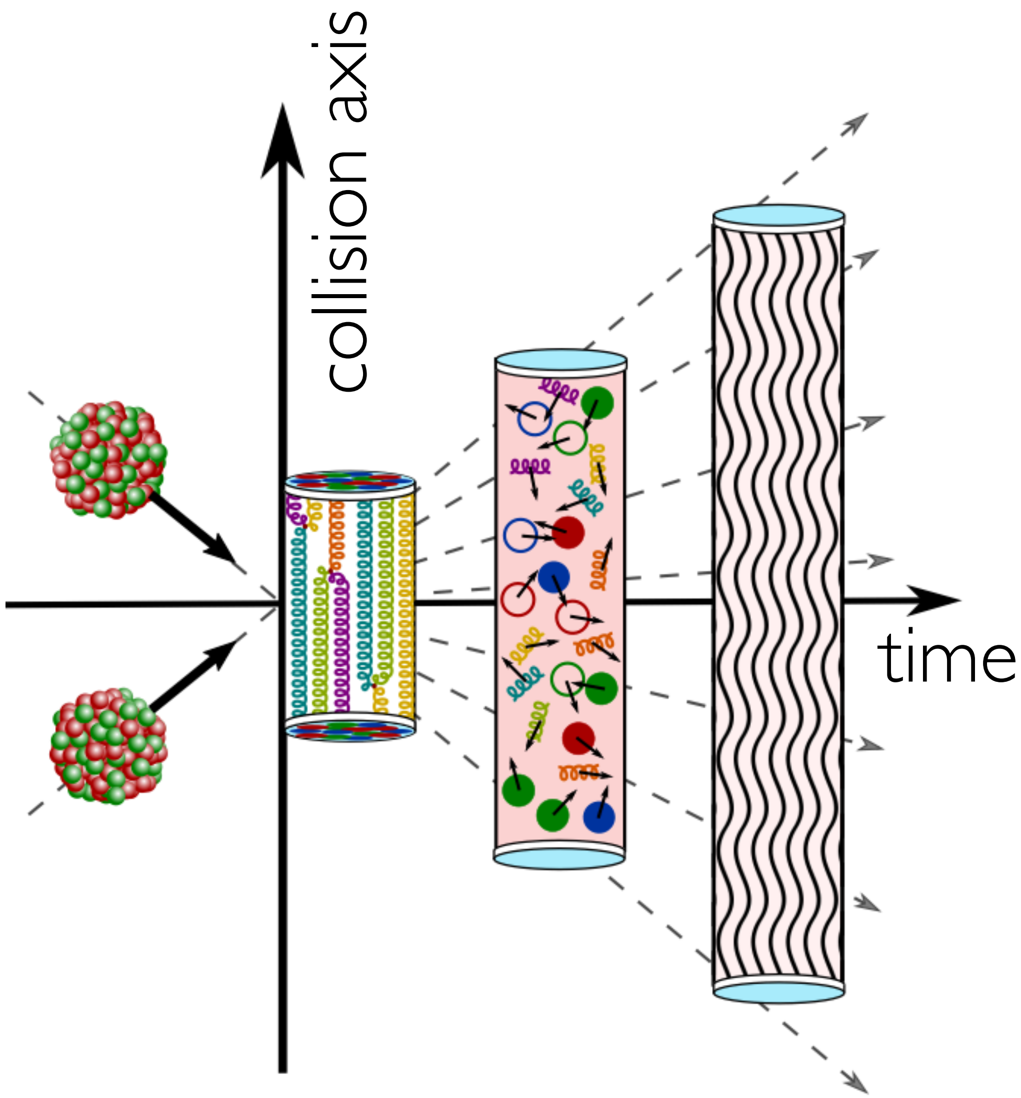

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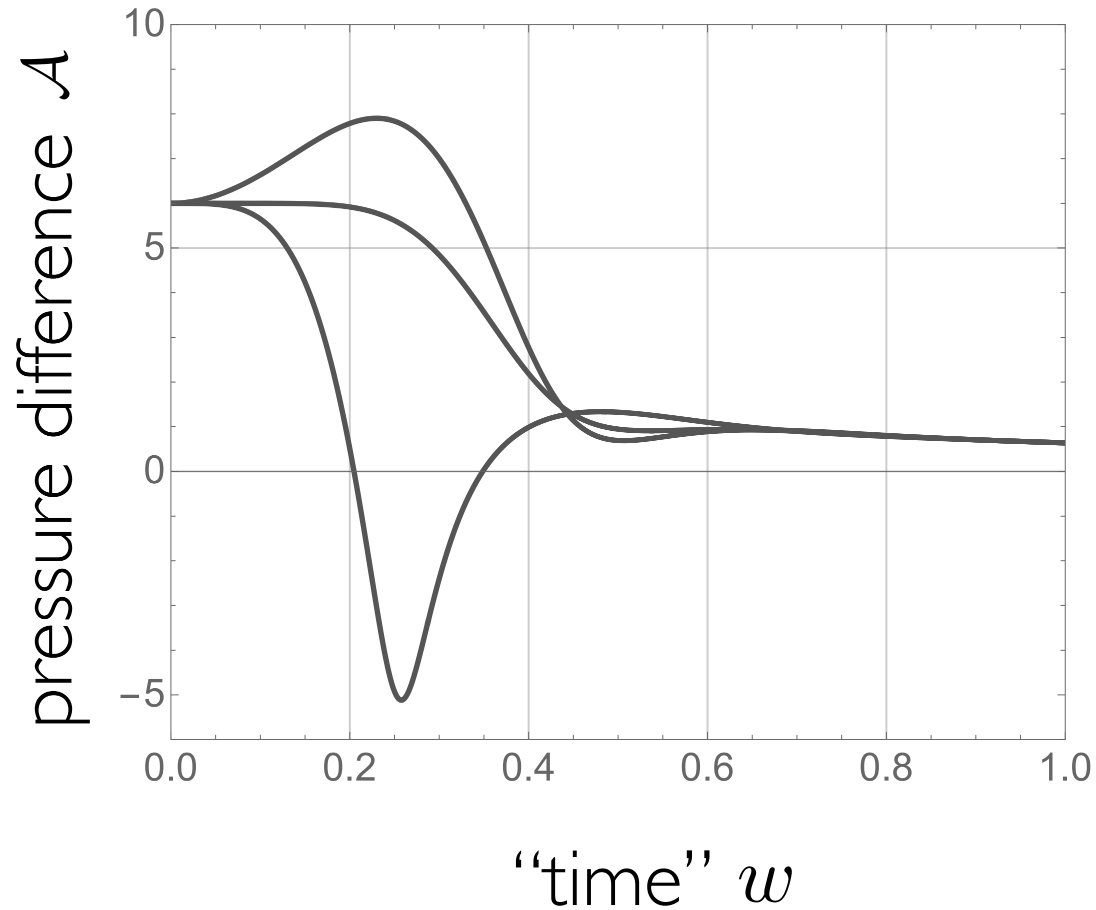
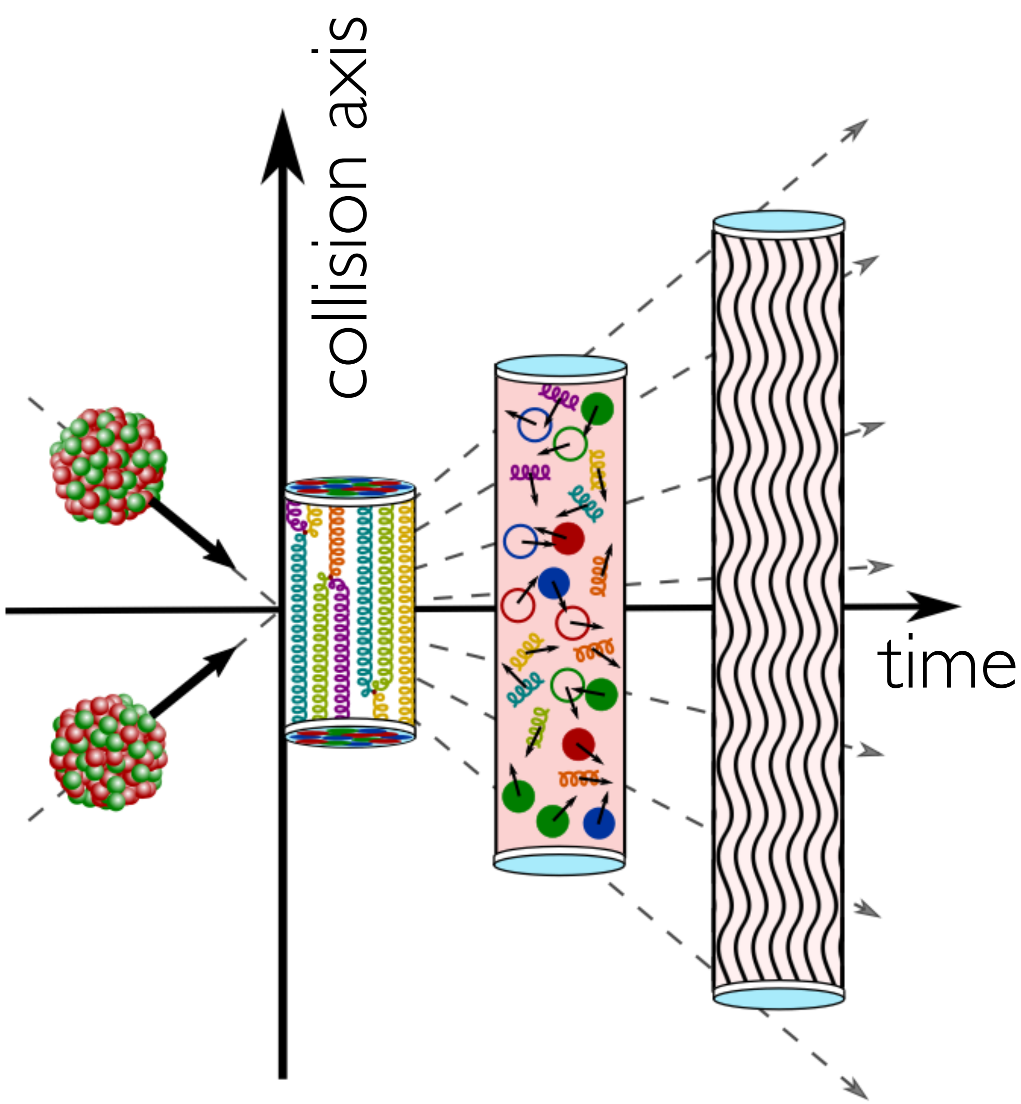
behaviour in
of theoretical models
(here: holography)



1103.3452 with Janik & Witaszczyk

Motivation

heavy-ion collisions
at RHIC and LHC



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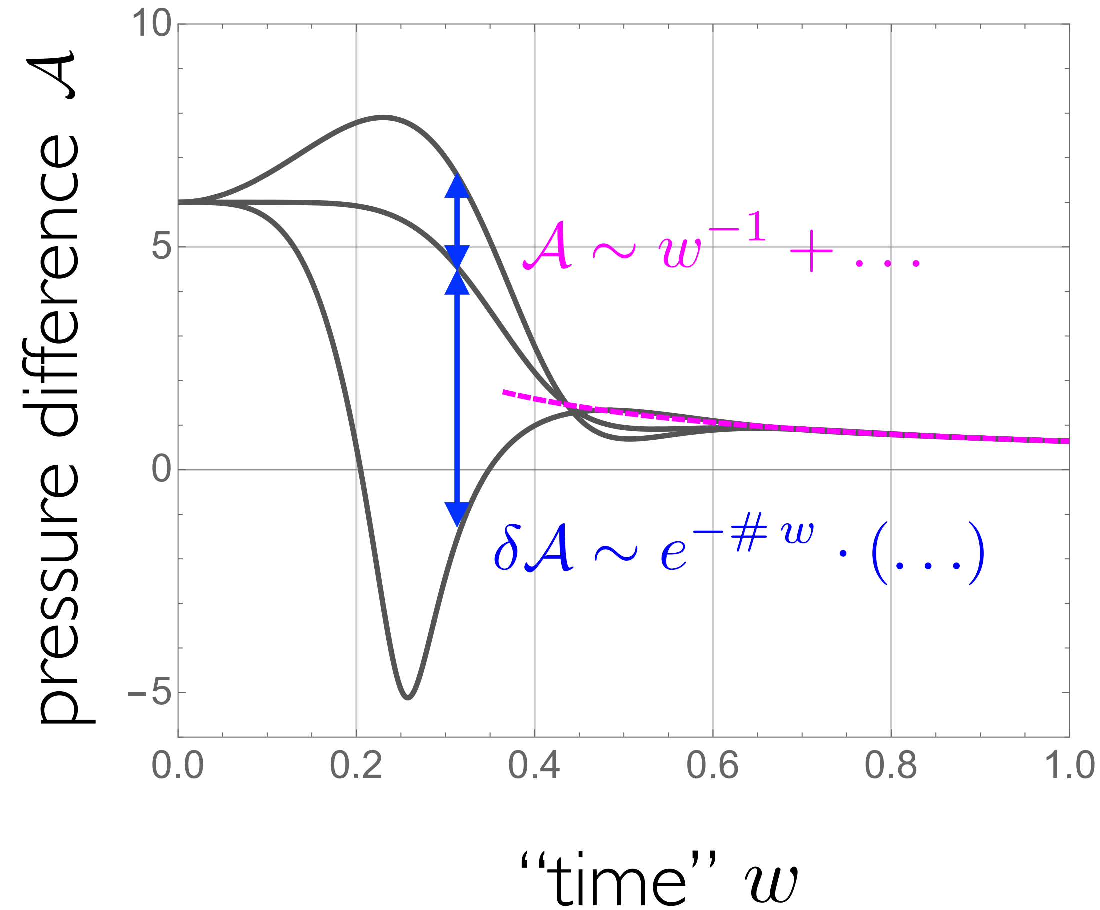

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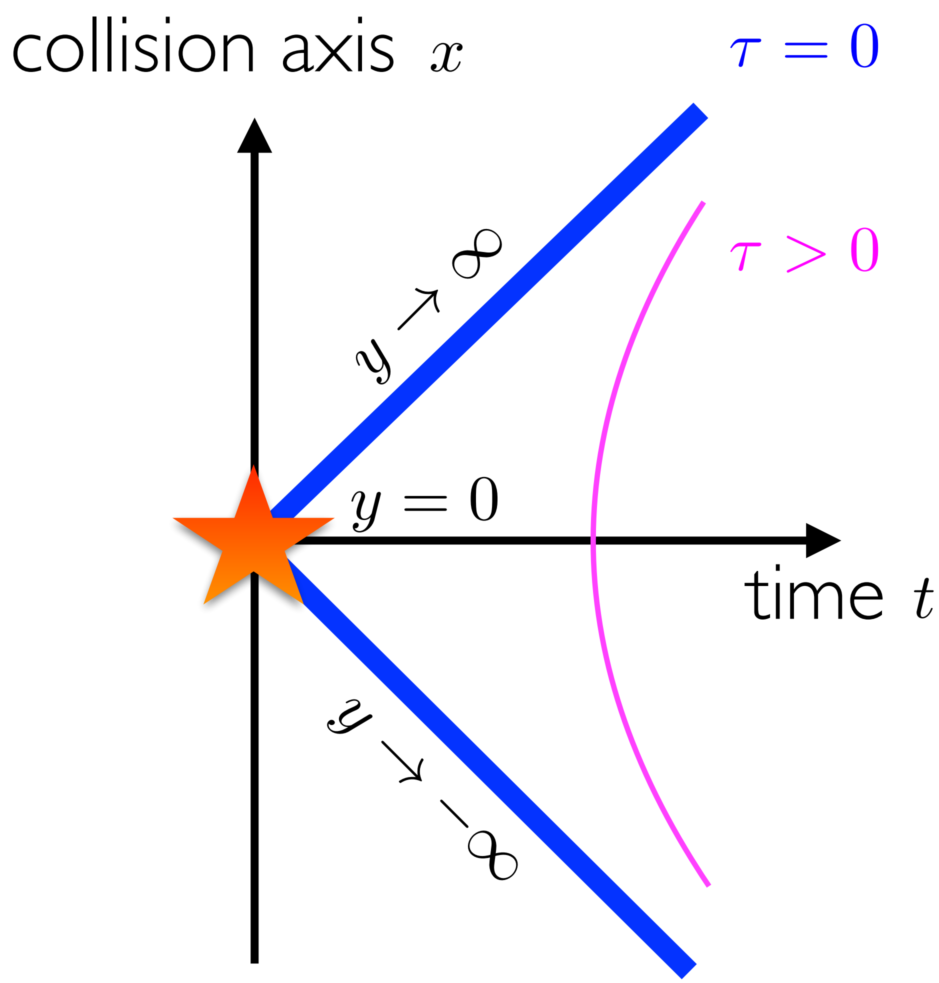
Bjorken flow: basics

Bjorken 1982



+ two transverse coordinates

Bjorken's simplification: physics is the same in all longitudinally boosted reference frame; this is Lorentzian analogue of rotational invariance.

analogue of the radius: $\tau = \sqrt{t^2 - x^2}$ analogue of the angle: $y = \mathrm{arccosh}(t/x)$

Relativistic hydrodynamics: basics

e.g. **1205.5040** by Kovtun; **1707.02282** with Spaliński & Florkowski; **1712.05815** by Romatschke[2]

In relativistic systems like nuclear matter at RHIC and LHC energy and momentum are encoded in the energy-momentum tensor $T^{\mu\nu}$

General energy-momentum tensor has 10 components subject to 4 conservation equations $\nabla_\mu T^{\mu\nu} = 0$

Relativistic hydrodynamics is built on an assumption that if we wait long enough the number of degrees of freedom (independent components of $T^{\mu\nu}$) reduces to these specifying a local equilibrium state

$$T^{\mu\nu} = \mathcal{E}(T) u^\mu u^\nu + \mathcal{P}(T) \left(g^{\mu\nu} + u^\mu u^\nu\right) + \ldots \quad \text{with} \quad u_\alpha \, u^\alpha = -1$$

This stress tensor defines perfect fluid hydrodynamics: $\nabla_\mu (s(T) u^\mu) = 0$

Relativistic hydrodynamics: dissipation

e.g. **1205.5040** by Kovtun; **1707.02282** with Spaliński & Florkowski; **1712.05815** by Romatschke[2]

Realistic fluids dissipate and in hydrodynamics this is encapsulated by some of the corrections to the perfect fluid description

$$T^{\mu\nu} = \mathcal{E}(T)u^{\mu}u^{\nu} + \mathcal{P}(T)\left(g^{\mu\nu} + u^{\mu}u^{\nu}\right) + \pi^{\mu\nu}$$

To the leading order in derivatives, the dissipative terms are

this talks considers conformal fluids

$$\pi^{\mu\nu} = \underbrace{-\eta(T)\,\nabla^{\langle\mu}u^{\nu\rangle}}_{\text{shear term}} \underbrace{\cancel{- \zeta(T)(g^{\mu\nu} + u^{\mu}u^{\nu})\nabla_{\alpha}u^{\alpha}}}_{\text{bulk term}} + \mathcal{O}(\nabla^2)$$

@ conformality:

2 order: 5 terms

0712.2451 by Baier et al.

3 order: ~20 terms

1507.02461 by Grozdanov & Kaplis

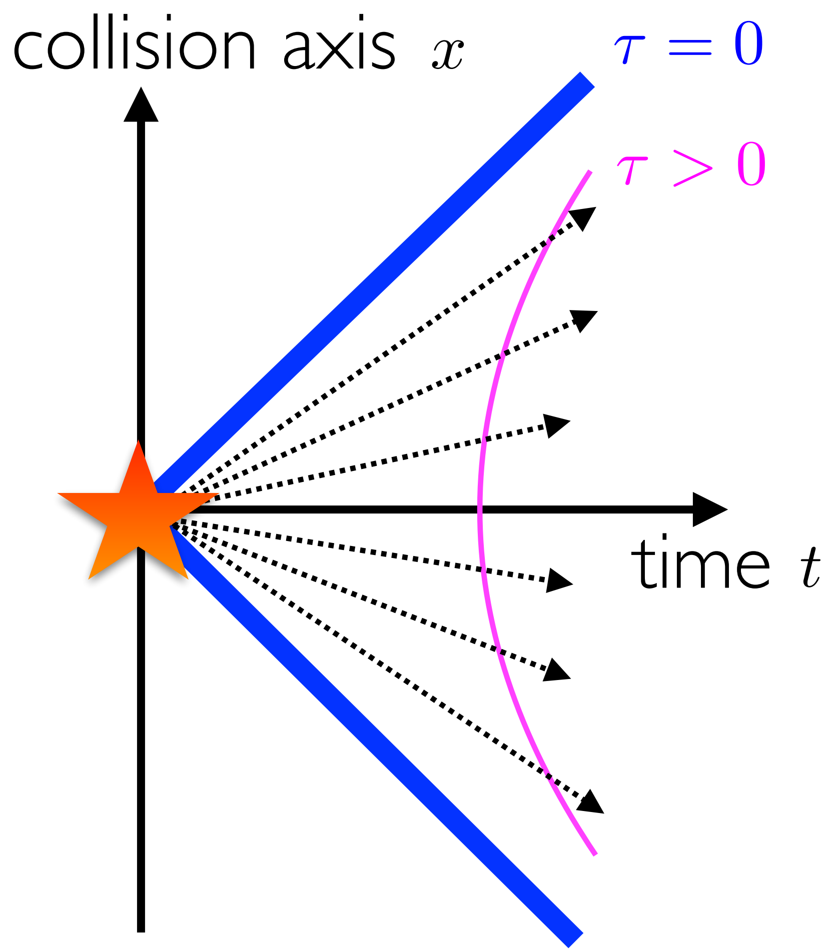
Such corrections $\rightarrow$ allow for dissipation

Such corrections $\rightarrow$ influence properties of solutions of hydrodynamics

Such corrections $\rightarrow$ therefore of interest to pheno at RHIC/LHC

Bjorken flow and relativistic hydrodynamics

e.g. 1707.02282 with Spaliński & Florkowski



Bjorken flow is a comoving flow in Minkowski:

$$u^\mu \partial_\mu = \partial_\tau \quad \text{and} \quad ds^2 = -d\tau^2 + \tau^2 dy^2 + d\mathbf{x}_\perp^2$$

$$\downarrow$$

$$\nabla^\mu u^\nu \sim \frac{1}{\tau} \quad \text{etc}$$

It is an intrinsically nonlinear phenomenon

For conformal fluids $\mathcal{E} = 3\,\mathcal{P} \sim T^4$ and $\eta \sim T^3$

$$\downarrow$$

$$\mathcal{A} \equiv \frac{\pi^\perp_\perp - \pi^y_y}{\mathcal{E}/3} = 8\frac{\eta}{s}\underbrace{\boxed{\frac{1}{\tau\, T(\tau)}}}_{\equiv w^{-1}} + \mathcal{O}(\nabla^2)$$

Motivation

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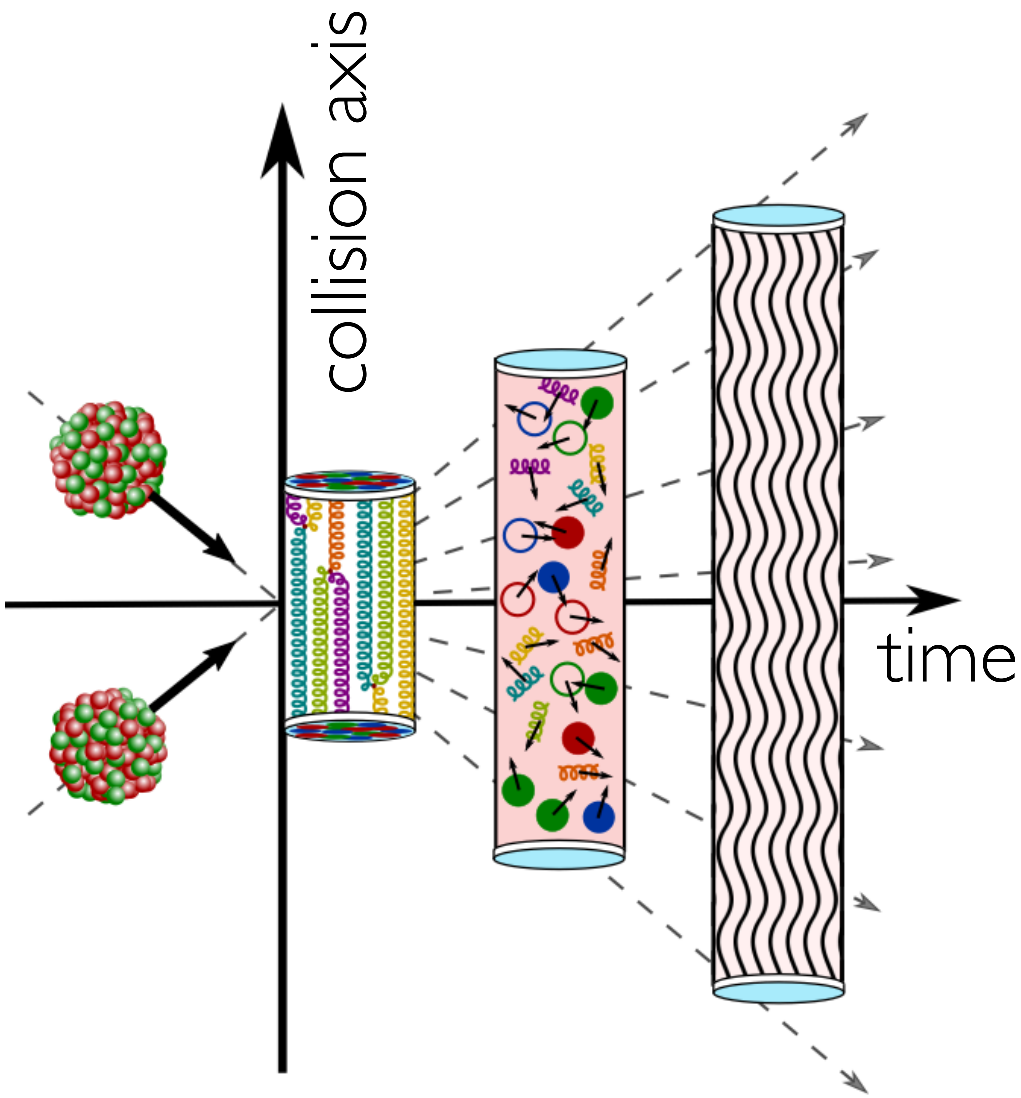

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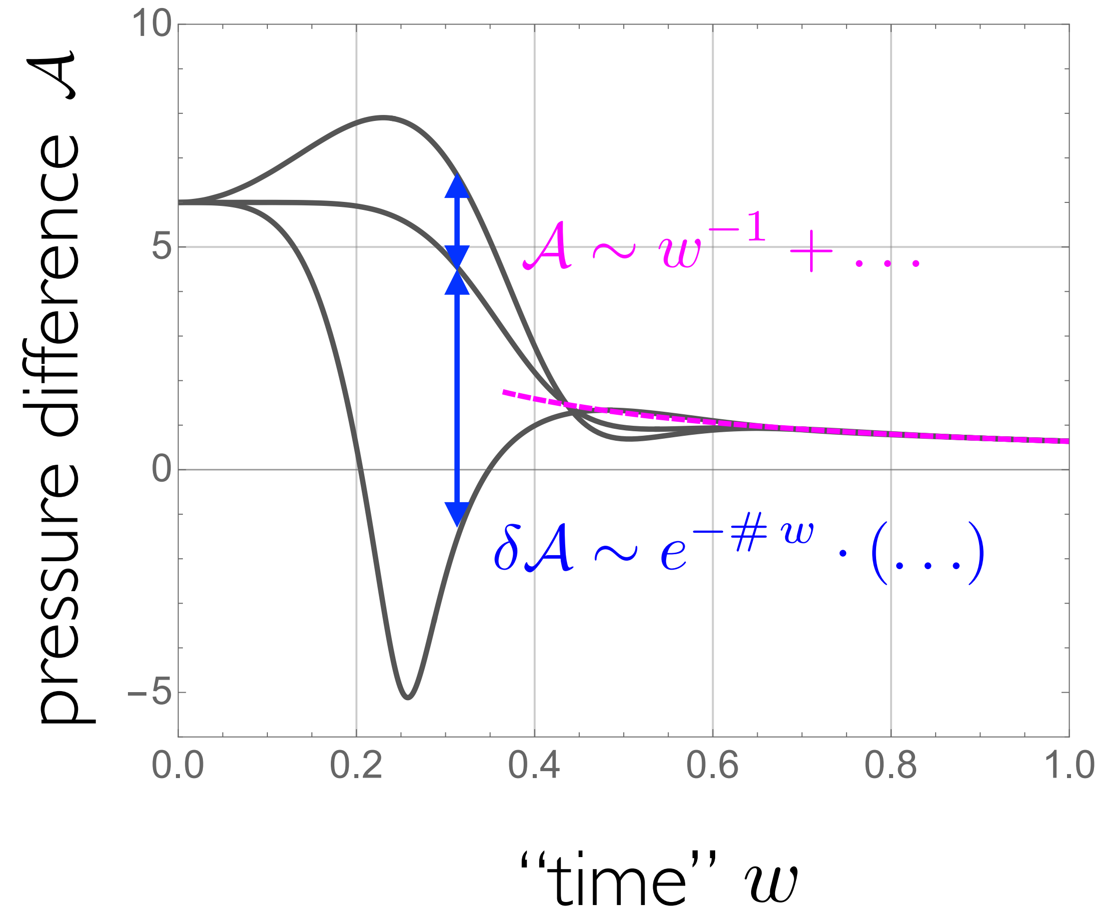

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Bjorken flow as a tool

Asymptotics* of hydrodynamic gradient expansion

$$\pi^{\mu\nu} = -\eta(T)\,\nabla^{\langle\mu}u^{\nu\rangle} + \boxed{\mathcal{O}(\nabla^2)} \longrightarrow \mathcal{O}(\nabla^{n\to\infty})$$

is a question with a foundational, as well as a phenomenological component

The key simplifying feature that triggered progress on this problem is the fact that velocity in Bjorken flow is entirely fixed by the symmetry: $u^\mu\partial_\mu = \partial_\tau$

This allows to define a version of on-shell gradient expansion of the form

$$\mathcal{A} \equiv \frac{\pi^{\perp}_{\ \perp} - \pi^{y}_{\ y}}{\mathcal{E}/3} = 8\frac{\eta}{s}\underbrace{\frac{1}{\tau\,T(\tau)}}_{\equiv w^{-1}} + \mathcal{O}(\nabla^2) = \sum_{n=1}^{\infty} a_n w^{-n} + \ldots$$

$\sim$ Knudsen number: $\left(\frac{\ell_{\mathrm{micro}}}{\ell_{\mathrm{macro}}}\right)^n$

which is soluble among a whole class of models giving rise to hydrodynamics

The main Bjorken flow result

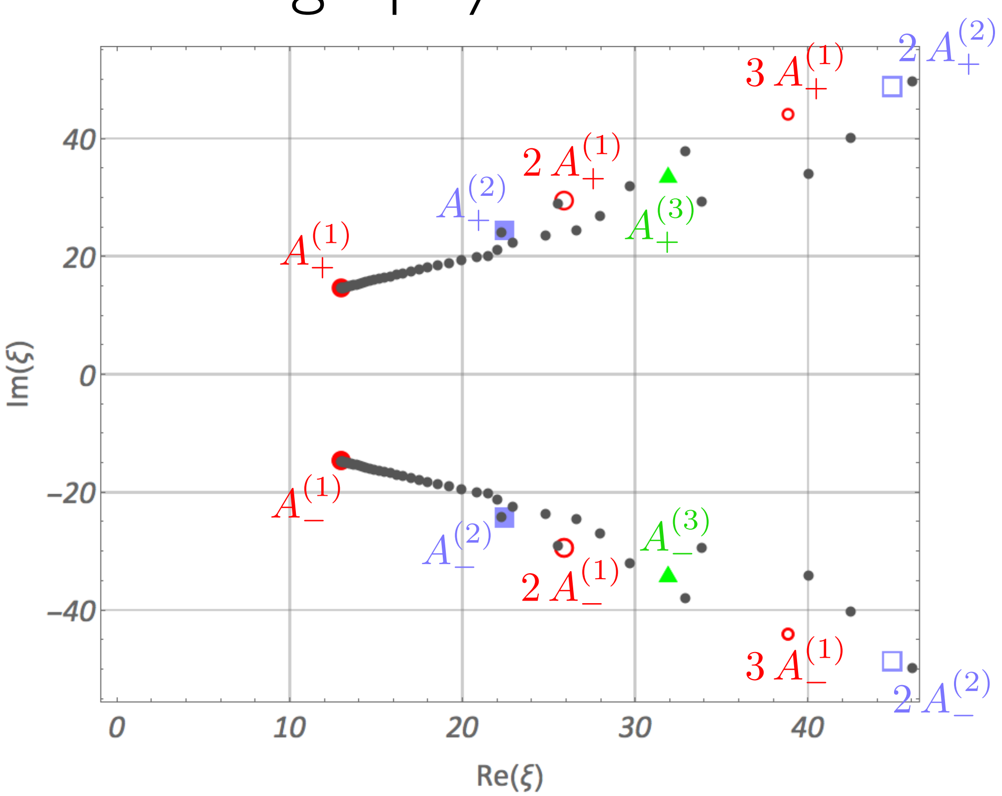
In all but one example of a microscopic model*, one gets that $a_n \sim n!$

Therefore hydrodynamic gradient expansion can diverge factorially on-shell

A standard tool in asymptotic series is a sequence

$$\mathcal{A}(w) \approx \sum_{n=1}^{\infty} \frac{a_n}{w^n} \xrightarrow[\text{Borel trafo.}]{} B\mathcal{A}(\xi) = \sum_{n=1}^{\infty} \frac{a_n}{n!}\,\xi^n \approx \frac{b_0 + \ldots + b_{100}\,\xi^{100}}{c_0 + \ldots + c_{100}\,\xi^{100}}$$

which in holography reveals 1302.0697 with Janik and Witaszczyk



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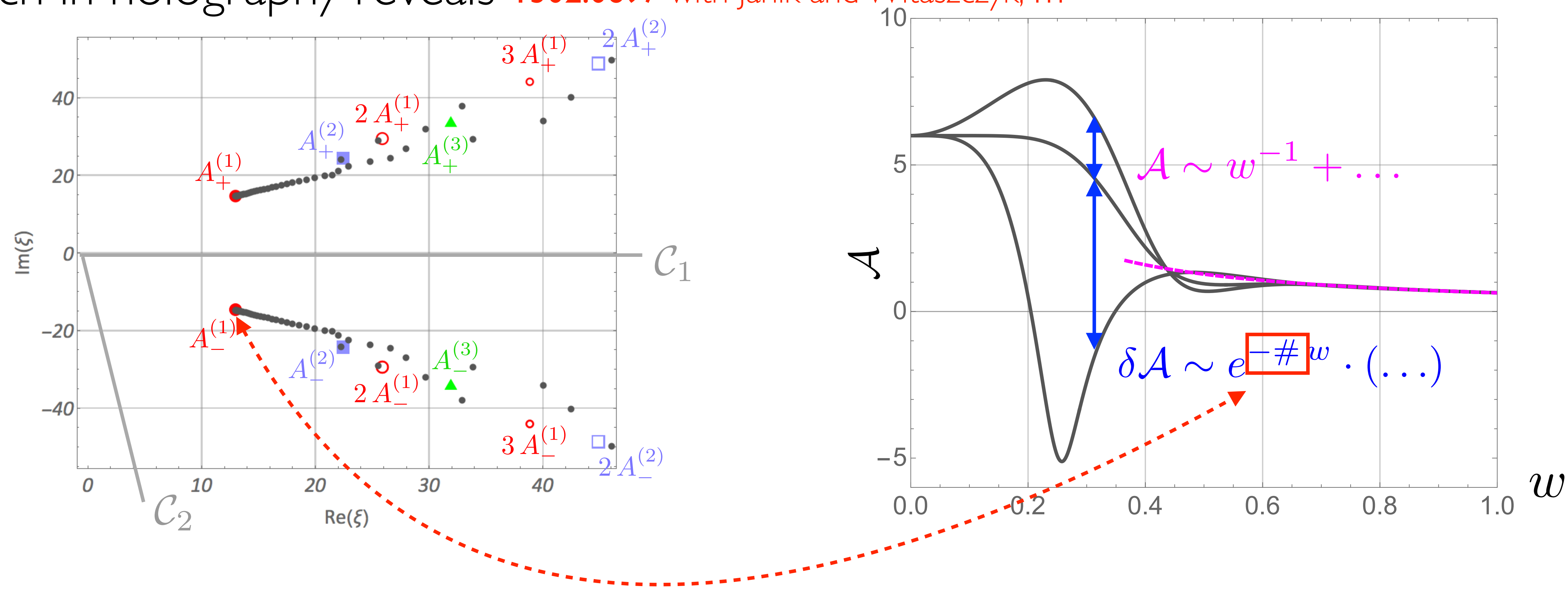

ambiguity in the inverse Borel transform $\left(\int_{\mathcal{C}_1} d\xi - \int_{\mathcal{C}_2} d\xi\right) \left[e^{-w\,\xi}\, w\, B\mathcal{A}(\xi)\right]$

reveals that the 1/w expansion diverges due to transient QNM

Thermalization and perturbative expansions

thermalization	QM with $V = -\frac{1}{2}x^2\left(1-\sqrt{g}\,x\right)^2$
expectation value of $T_{\mu\nu}$	ground state energy
gradient expansion in $\frac{1}{w}$	perturbative series in g
transient effects $e^{-\#w}(\ldots)$	instanton $e^{-1/(3g)}(\ldots)$

This parallel has allowed to borrow tools developed for perturbative expansions in QM and QFT to study thermalization in nuclear collisions

In particular, it led to a paradigm shift from a (n implicitly assumed) geometrically convergent hydrodynamic gradient expansion to a factorially divergent series governed by the notion of optimal truncations

However, the key underlying assumption (the boost invariance) is an idealization; **does this parallel survives in more realistic setups?**

Yes, it does (2021-)

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First results beyond boost invariance

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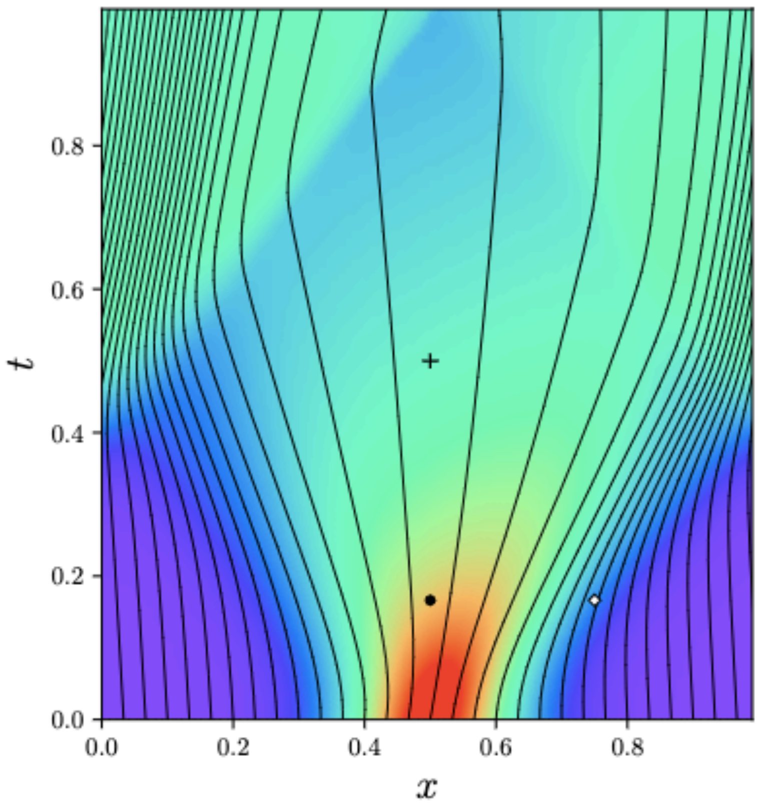
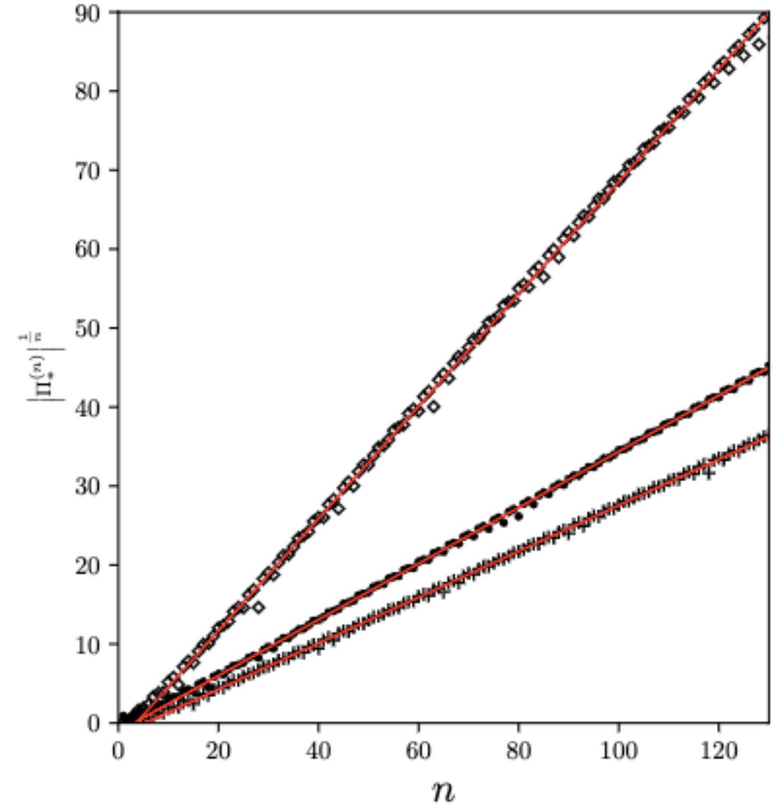
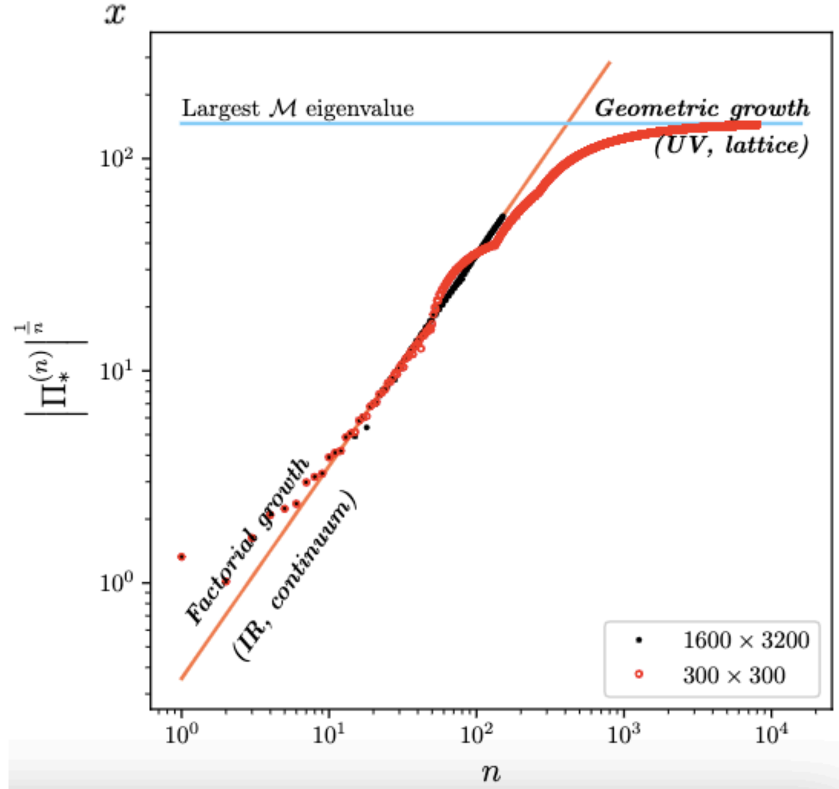






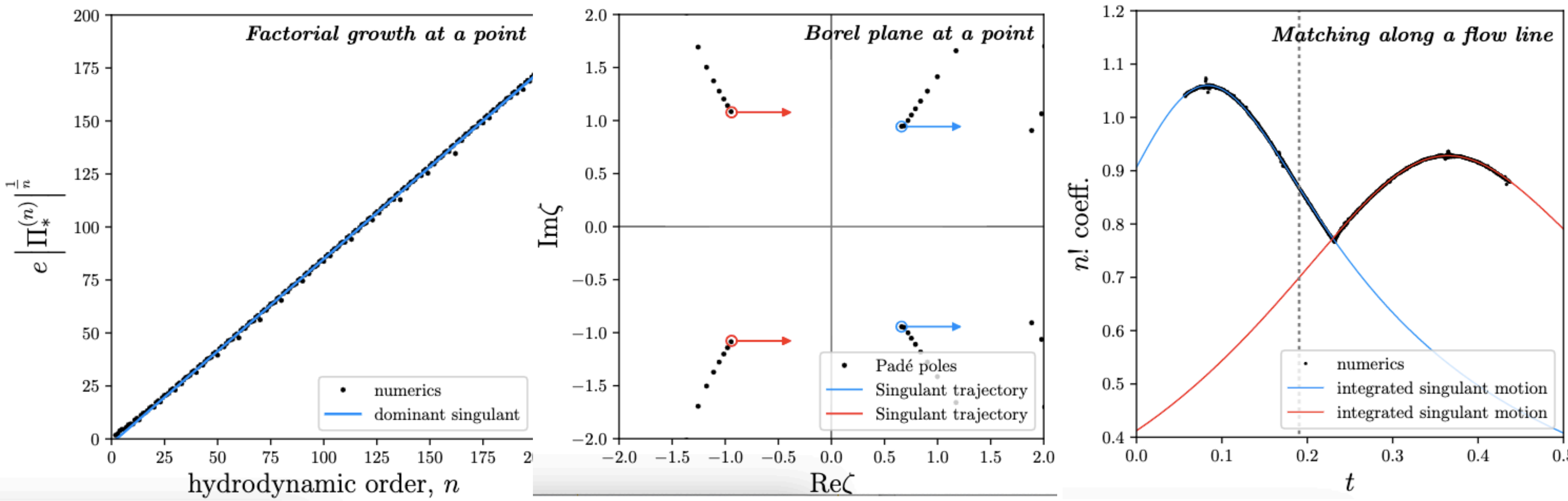
Introducing singulants

2112.12794 with Serantes, Spaliński, Svensson, Withers

Instead of hunting numerically the behaviour $\Pi^{(n)} \sim \Gamma(n)$, we can postulate it

$$\Pi^{(n)} \sim \frac{\Gamma(n)}{\chi(t,x)^n}$$

and study χ as a standalone object — the singulant. This makes sense, in particular we were able to derive and check its equations of motion:



The meaning of singulants

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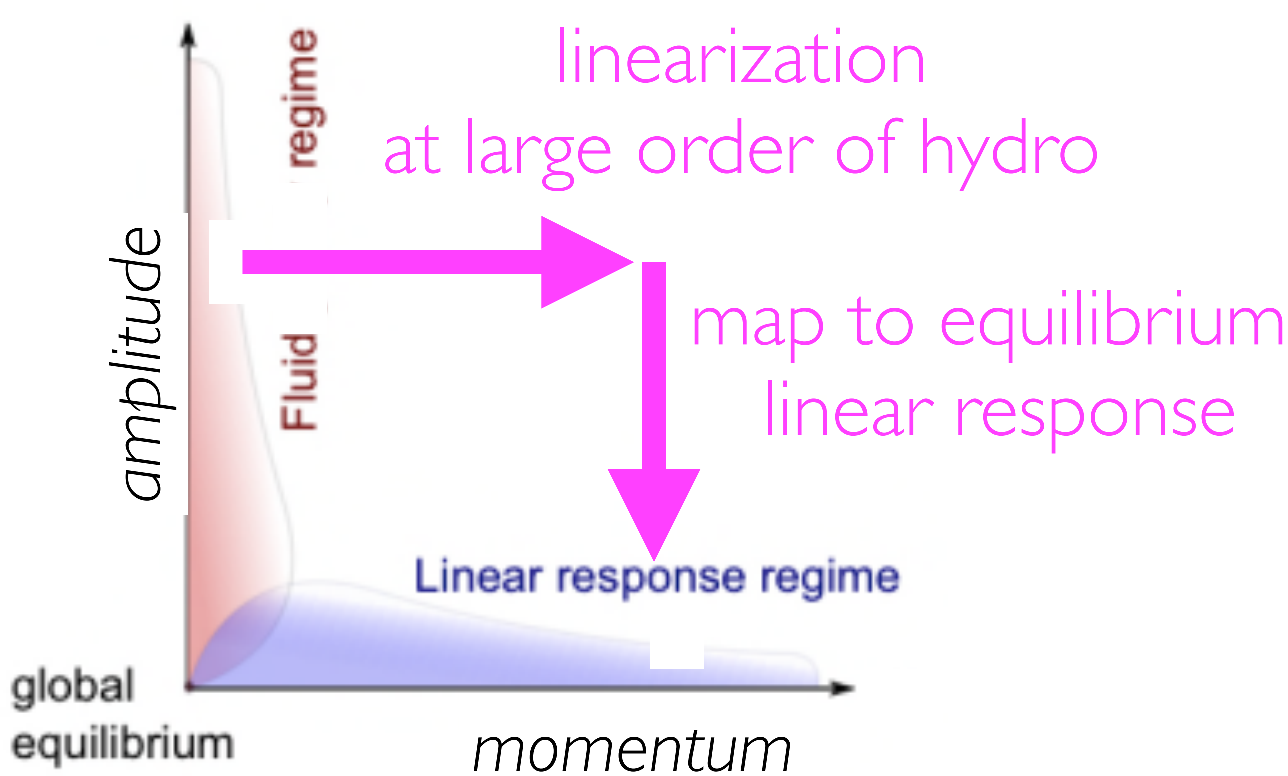
Often upon taking a large number of something, the description simplifies in terms of new emergent degrees of freedom (e.g. the 't Hooft limit)

Here sums of a large number of different grandient terms reorganize themselves in terms of a simple singulant description

In fact, the factorial ansatz leads to a linearization and reveals a new duality between singulants and the linear response theory:



adapted from 1006.3675 by Hubeny & Rangamani

Outlook

Summary

thermalization	QM with $V = -\frac{1}{2}x^2\left(1-\sqrt{g}\,x\right)^2$
expectation value of $T_{\mu\nu}$	ground state energy
gradient expansion in $\frac{1}{w}$	perturbative series in g
transient effects $e^{-\#w}(\ldots)$	instanton $e^{-1/(3g)}(\ldots)$
new: singulants	

Thank you