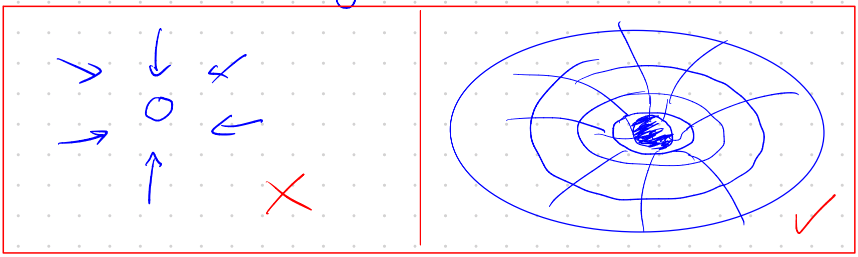
Equivalence Principle

Einstein's insight on gravity



gravity is the metric tensor describing the curvature of spacetime

Big Question:

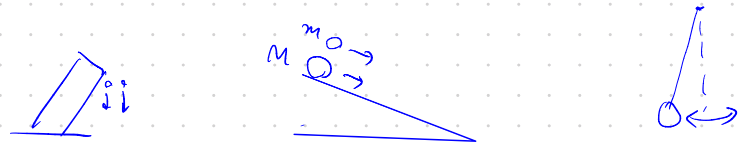
How Equivalence Principle (EP) leads us to the mathematical strategy of describing gravity as the geometry of a curved manifold

WEP

EEP

SEP

Weak Equivalence Principle (WEP)



"gravity is universal"

Newton's second Law

$$F_g = ma$$

$$\Downarrow$$

$$m_g(-\vec{\nabla}\Phi) = m_i a$$

gravitational response (m_g) — inertial mass (m_i)

$$\Downarrow$$

$$a = \left(\frac{m_g}{m_i}\right)(-\vec{\nabla}\Phi)$$

$\frac{m_g}{m_i}$: the "charge" of response (kinematically)

($m_g \propto m_i$)

if $m_g = C m_i$,

$$= C(-\vec{\nabla}\Phi)$$

set $C=1$

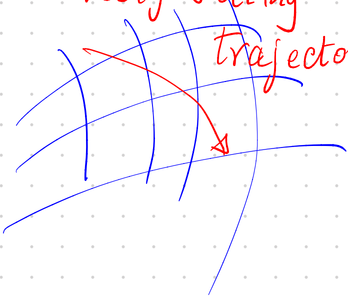
Newtonian mechanics $\Rightarrow$ WEP : $\boxed{m_i = m_g}$

$$\vec{a} = -\vec{\nabla}\Phi$$

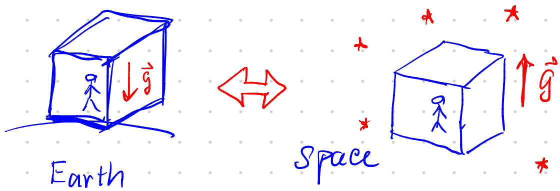
The response of matter to gravitation is universal. Every object falls at the same rate in gravity, indep. of the composition of the object.

freely-falling trajectory = inertial trajectory (unaccelerated)

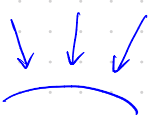
↳ means "subjects only to gravity"



WEP



Caution: $g = g(y)$ (gravity) ⇒ Only small enough region



WEP: there is no way (no experiment) to disentangle the effects of a grav. f. from those of being in a uniformly accelerated frame, simply by observing the behavior of free-falling particles.
in small enough region of spacetime

EEP
(Einstein's Equivalence Principle)

In small enough region of spacetime, the laws of physics reduce to those of SR; it is impossible to detect the existence of a grav. f. by means of local experim't

Remark 1 "gravity" couples to all forms of energy and momentum

$$m_g = m_i$$

?

$$m_g = m_p + m_e - B.E.$$

SR ($E = mc^2$)

ถ้าใช้ WEP จะงง

Remark 2 Local Lorentz frame

SR: We can define inertial frame (no external force) ⇒ acceleration can be well-defined

EEP: no such things of grav. neutral
(gravity is inescapable)

inertial frame = ? ⇒ acceleration due to gravity cannot be well-defined

"unaccelerated" ⟺ "freely-falling"

* gravity is not a "force"
↳ the cause of acceleration

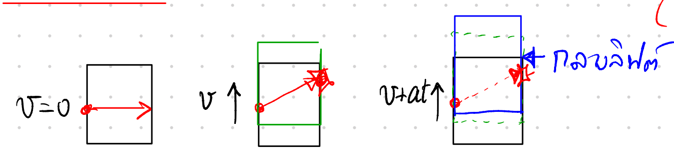
<u>Note</u>

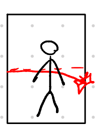
SR: Galilian's Principle of Relativity (only mechanical laws) ⇒ Einstein's Principle of Relativity (all laws of physics: mechanics + EM + nuclear + ⋯)

GR: WEP (Newton's laws: mech. + grav.) ⇒ EEP (all laws of physics mech. + EM + ⋯ BUT not gravitation)

⇒ SEP (mech + EM + ⋯ + self-gravity)

<u>Elevator</u>

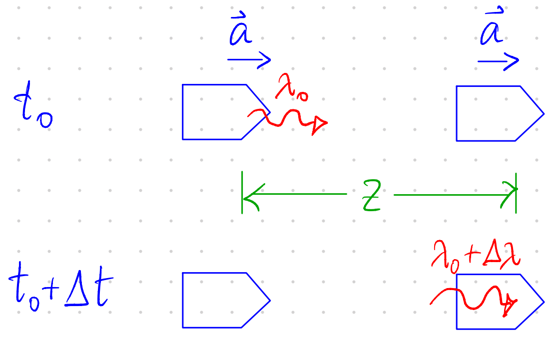




light bending

EEP

↳ gravitational redshift



$(\Delta v << c)$

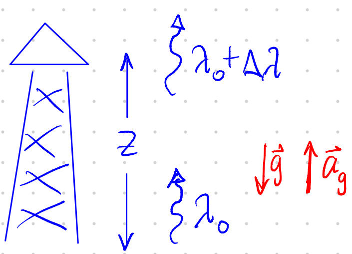
$$\Delta v = a\Delta t = a\frac{z}{c}$$

$$\Delta t = \frac{z}{c}$$

from SR, rel. Doppler effect

$$\lambda_{obs} = \lambda_0\sqrt{\frac{1+\beta}{1-\beta}} \approx \lambda_0 + \lambda_0\beta \quad ; \beta = \frac{\Delta v}{c}$$

$$\frac{\Delta\lambda}{\lambda_0} \approx \frac{\Delta v}{c} = \frac{az}{c^2}$$



EEP,

$$\frac{\Delta\lambda}{\lambda_0} = \frac{a_g z}{c^2}$$

(grav. redshift)

$$\vec{g} = -\vec{\nabla}\phi$$

$$\vec{a}_g = \vec{\nabla}\phi$$

$$\frac{\Delta\lambda}{\lambda_0} = \frac{1}{c}\int \vec{\nabla}\phi \, dt$$

$$= \frac{1}{c}\int \frac{\partial \phi}{\partial z} \frac{dt}{dz} \, dz$$

$\frac{dt}{dz} \to \frac{1}{c}$

$$= \frac{1}{c^2}\int \partial_z \phi \, dz$$

$$\frac{\lambda_1 - \lambda_0}{\lambda_0} = \frac{1}{c^2}\Delta\phi \implies \lambda_1 = \lambda_0\left(1 + \frac{\Delta\phi}{c^2}\right)$$

$$\frac{\lambda_1}{\lambda_0} = \frac{\Delta t_1}{\Delta t_0} = (1 + \Delta\phi) \qquad (c=1)$$

$$\Delta t_1 = (1 + \Delta\Phi)\Delta t_0$$

the clock on the tower appears to run more quickly!!

$$\frac{\Delta t(y)}{\Delta t_\infty} = 1 - \Delta\Phi \qquad \text{(grav. time dilation)}$$

↖ top of the tower at $y=\infty$

$$ds^2 = -(1 - 2\Delta\Phi)dt^2 + dx^2 + dy^2 + dz^2$$

curved spacetime ↗   ↑ t in inertial frame