



# MadNIS

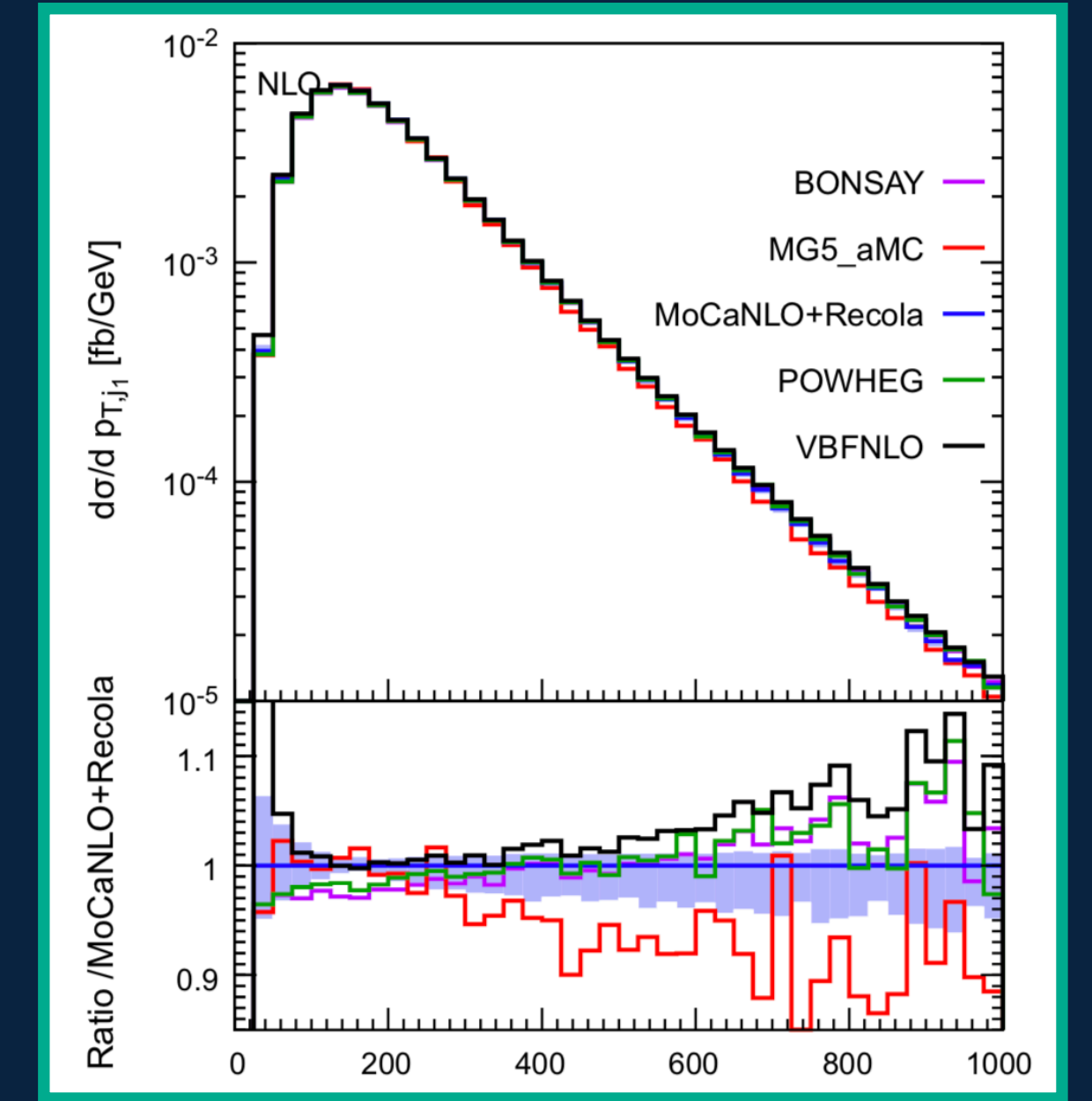
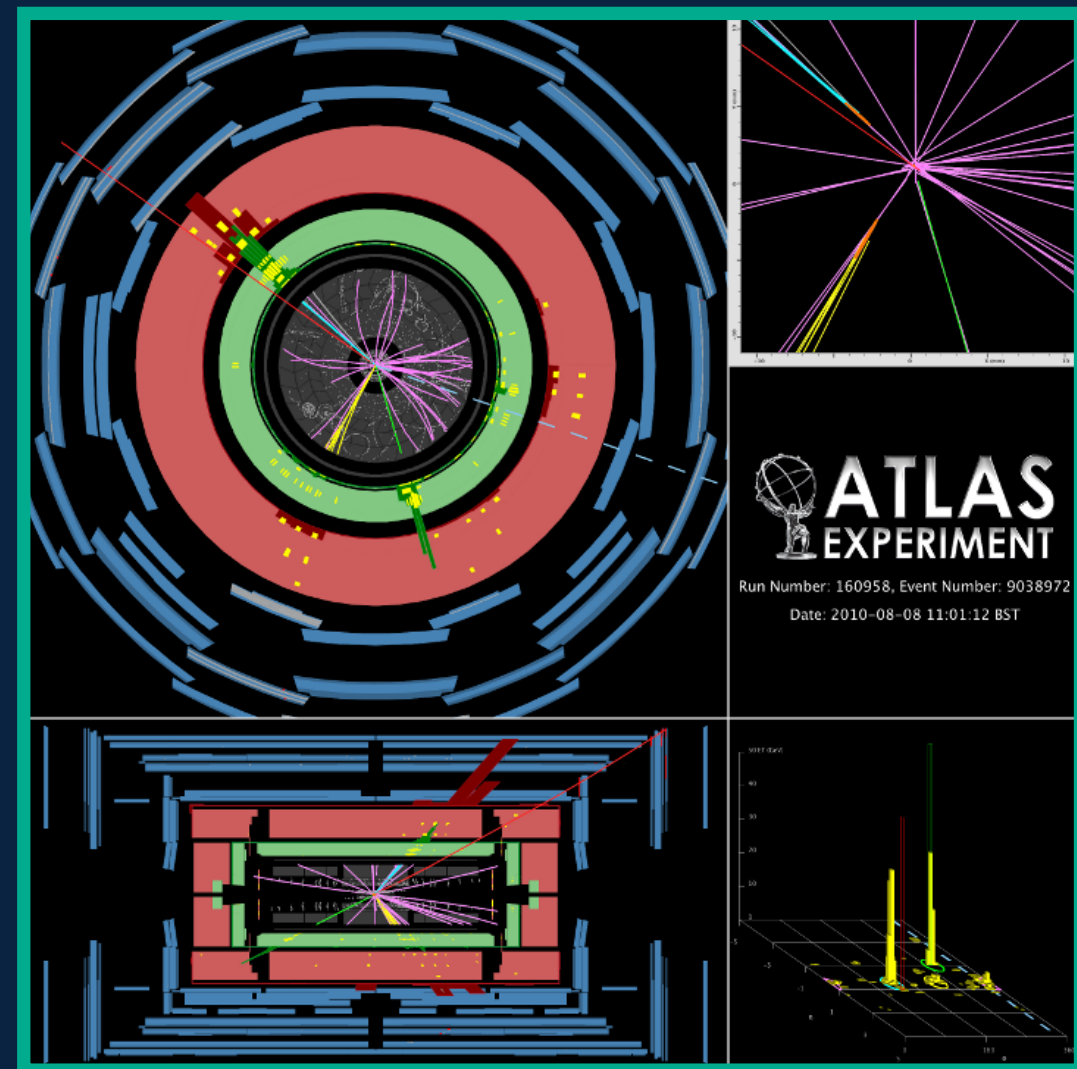
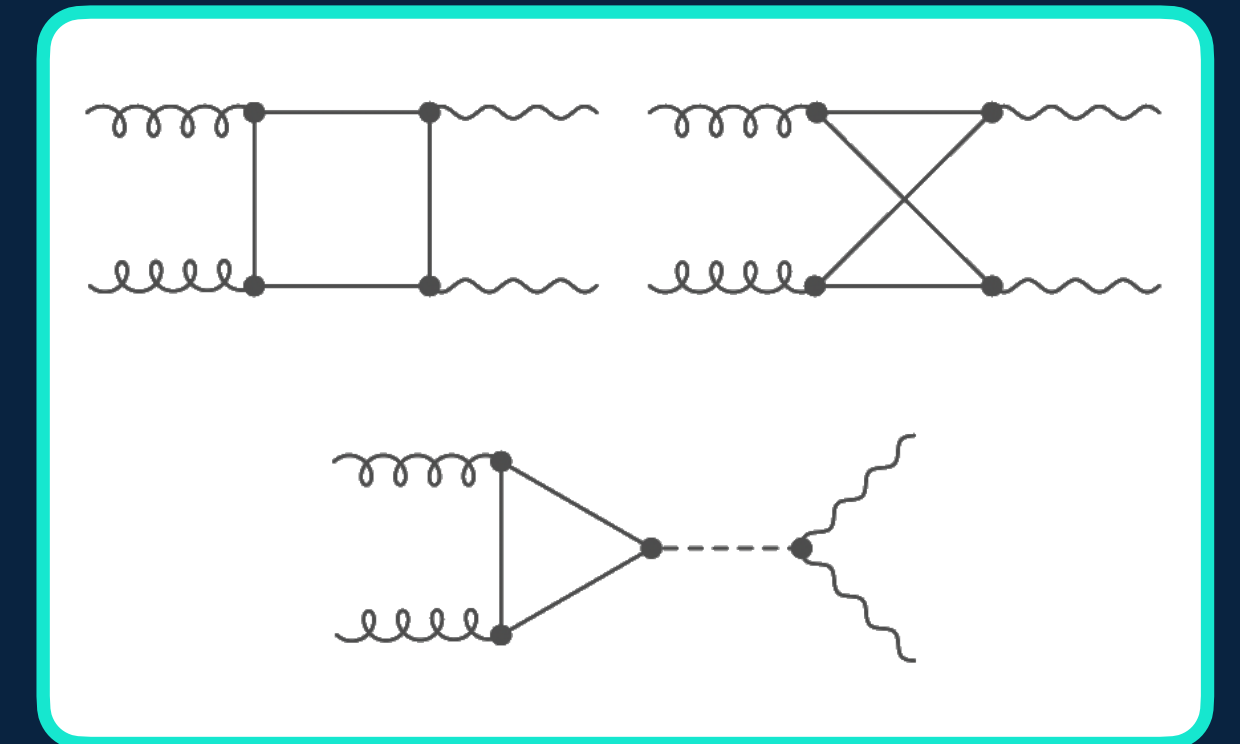
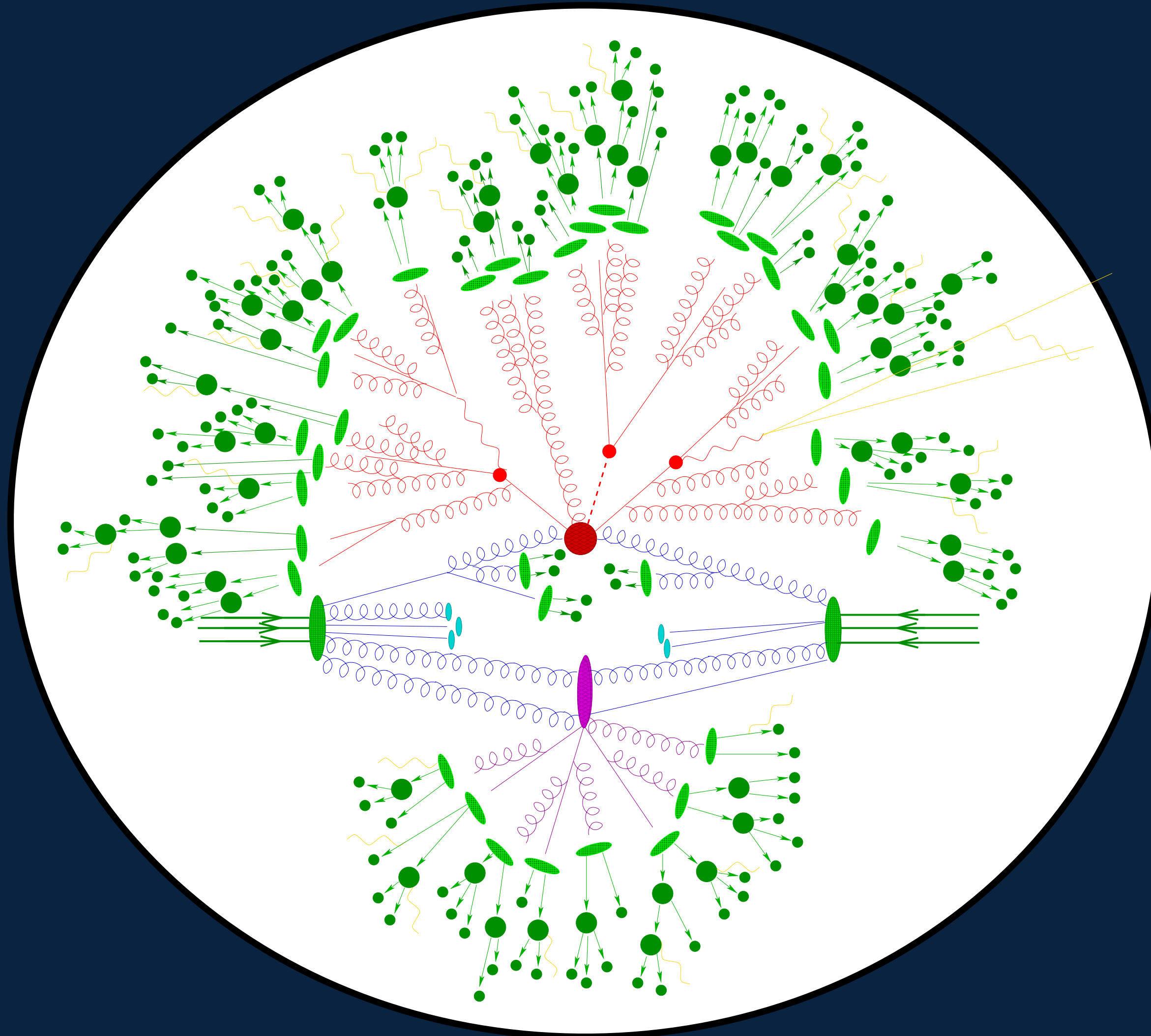
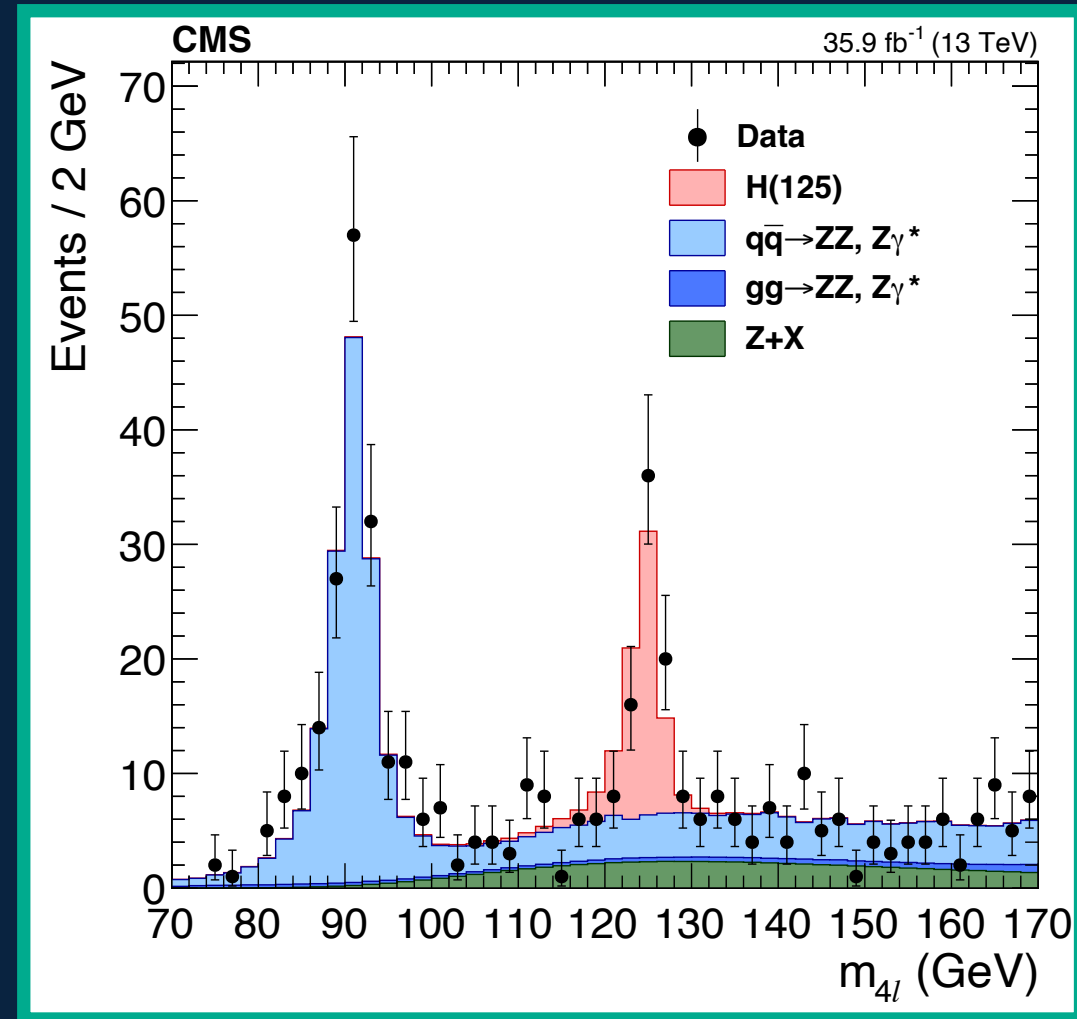
Neural networks for multi-channel  
integration in MadGraph

ML4Jets - Rutgers 2022

Ramon Winterhalder — UC Louvain

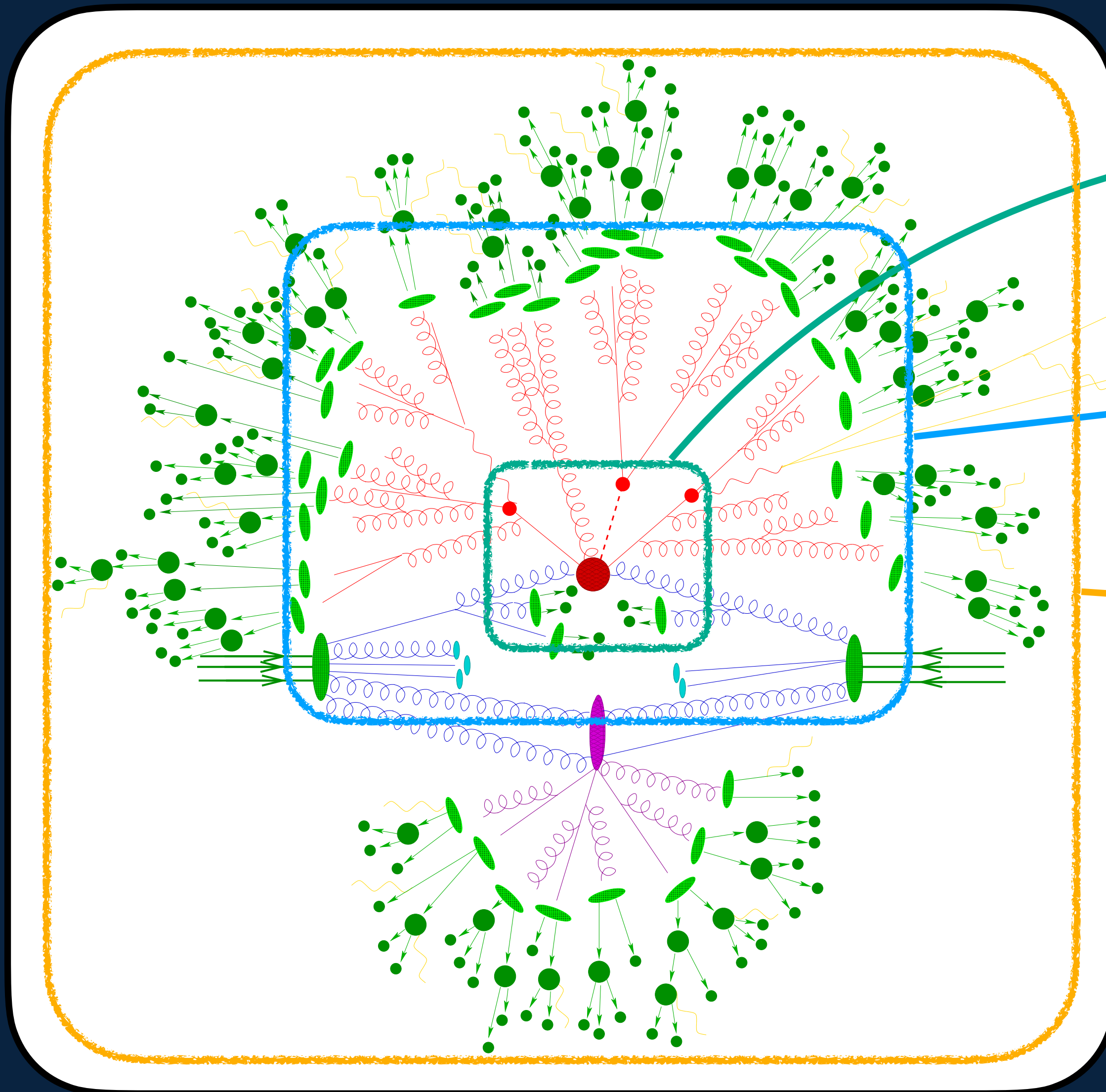
In collaboration with Anja Butter, Theo Heimel, Joshua Isaacson,  
Claudius Krause, Fabio Maltoni, Olivier Mattelaer and Tilman Plehn

# Data analysis in HEP





# Simulations for the LHC



## ◆ Hard Process

- ✿ Depends on the Model (SM, SUSY,...)
- ✿ Perturbative QCD

## ◆ Parton Showering

- ✿ Universal (QCD)

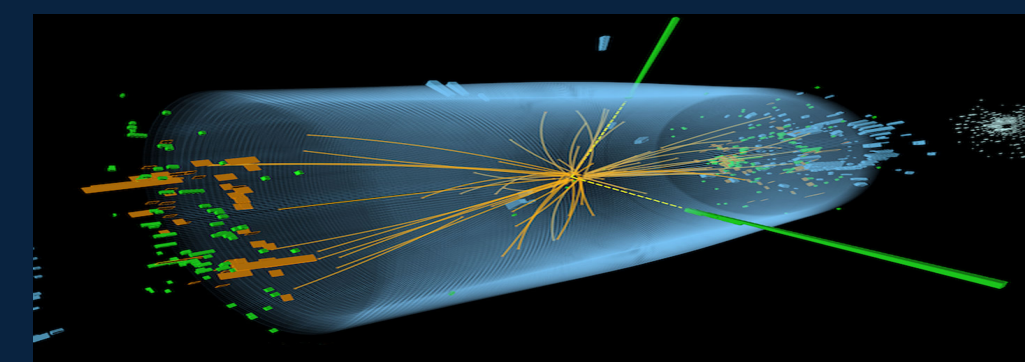
## ◆ Hadronization

- ✿ Model-based, universal

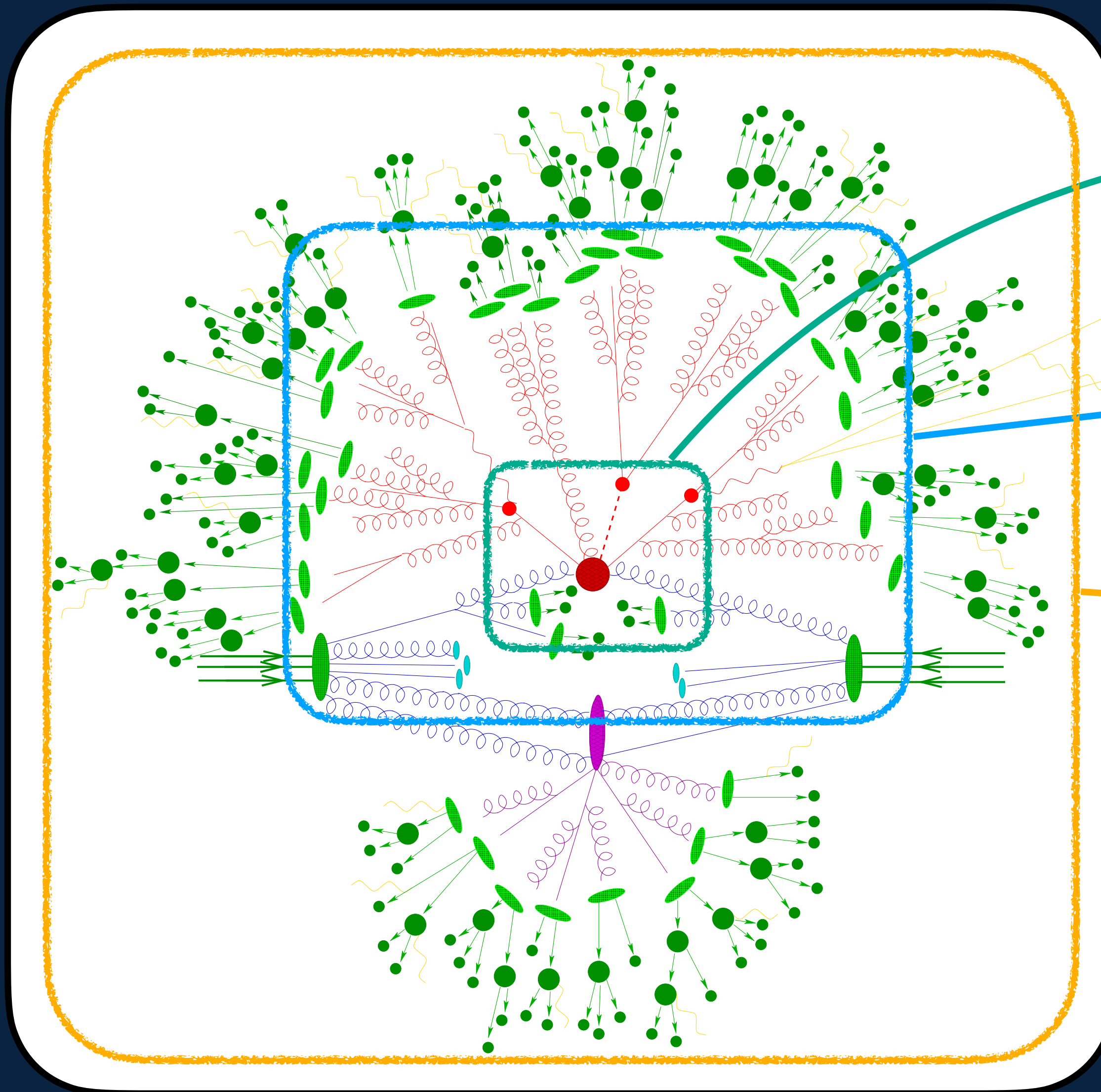
## ◆ Underlying Event

- ✿ Model-based, non-universal

## ◆ Detector Simulation



# Simulations for the LHC



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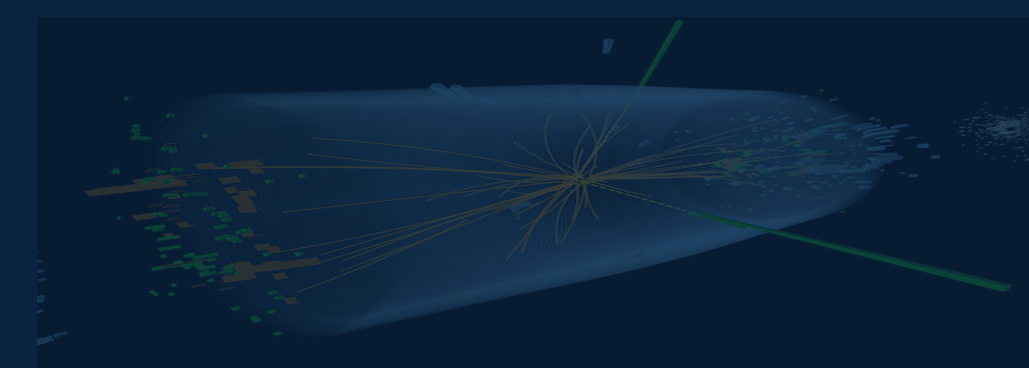
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# Theory predictions in HEP

$$M_{\lambda,c,\dots}(p_a, p_b | p_1, \dots, p_n) : \mathbb{M} \rightarrow \mathbb{C}$$

**Quantum numbers:**  
spin, colour charge etc.

**Kinematics:**  
Momenta in Minkowski  
space, masses, etc.

$$\sigma = \frac{1}{\text{flux}} \sum_{a,b} \int dx_a dx_b f(x_a) f(x_b) \int d\Phi_n \langle |M_{\lambda,c,\dots}(p_a, p_b | p_1, \dots, p_n)|^2 \rangle$$

**Cross section:**  
more generally,  
differential observables\*

**PDFs:**  
convolution over all  
possible initial state  
configurations

**Phase-space  
integral:**  
over final state  
kinematics

**Squared amplitude:**  
summed over final states,  
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# Monte Carlo Integration

Standard Monte Carlo integration

$$I = \int_V d^d x f(x) \simeq \frac{1}{N} \sum_{j=1}^N f(x_j) = \langle f \rangle_x \quad \sigma_I \simeq \sqrt{\frac{\langle f^2 \rangle_x - \langle f \rangle_x^2}{N-1}}$$

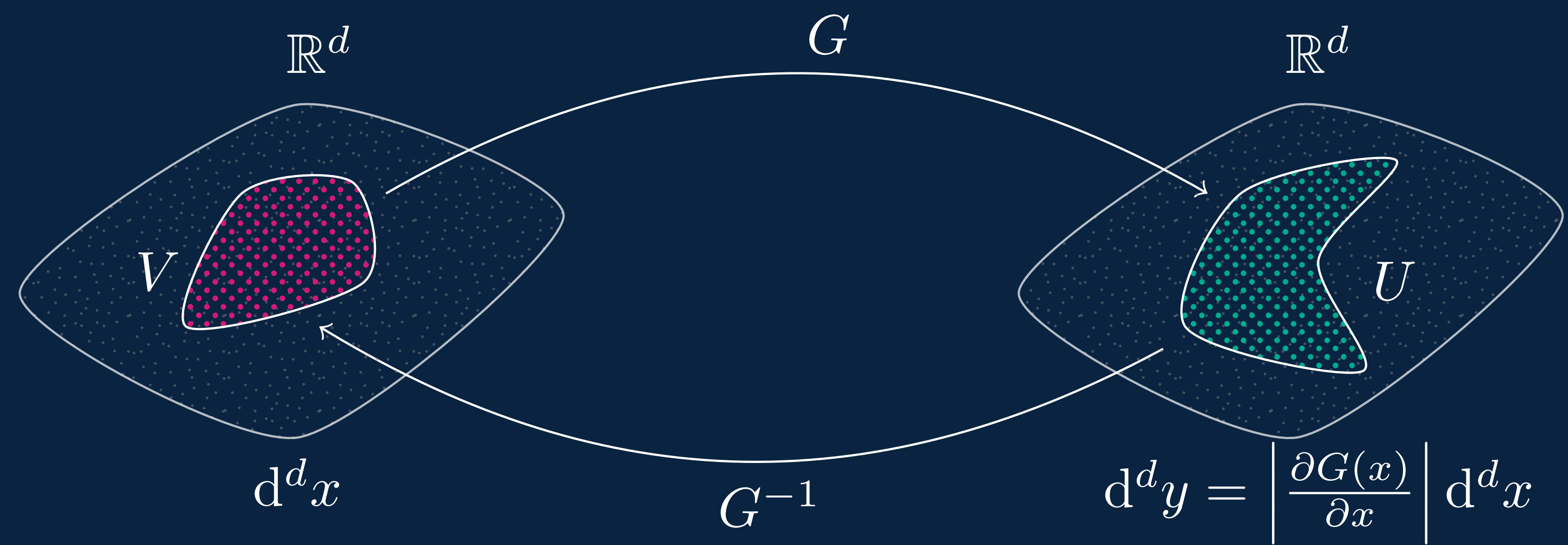
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Mapping

$$y = G(x) \quad g(x) = \left| \frac{\partial G(x)}{\partial x} \right|$$





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## Importance Sampling

$$I = \int_U d^d y \frac{f(x)}{g(x)} \Big|_{x=G^{-1}(y)} \simeq \langle f/g \rangle_y \quad \sigma_I \simeq \sqrt{\frac{\langle (f/g)^2 \rangle_y - \langle f/g \rangle_y^2}{N-1}}$$

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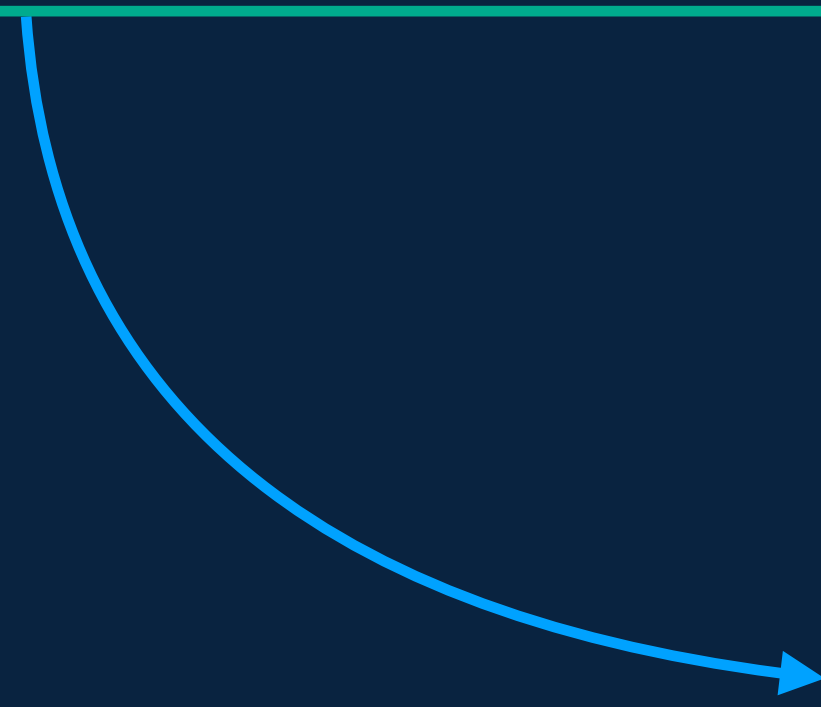
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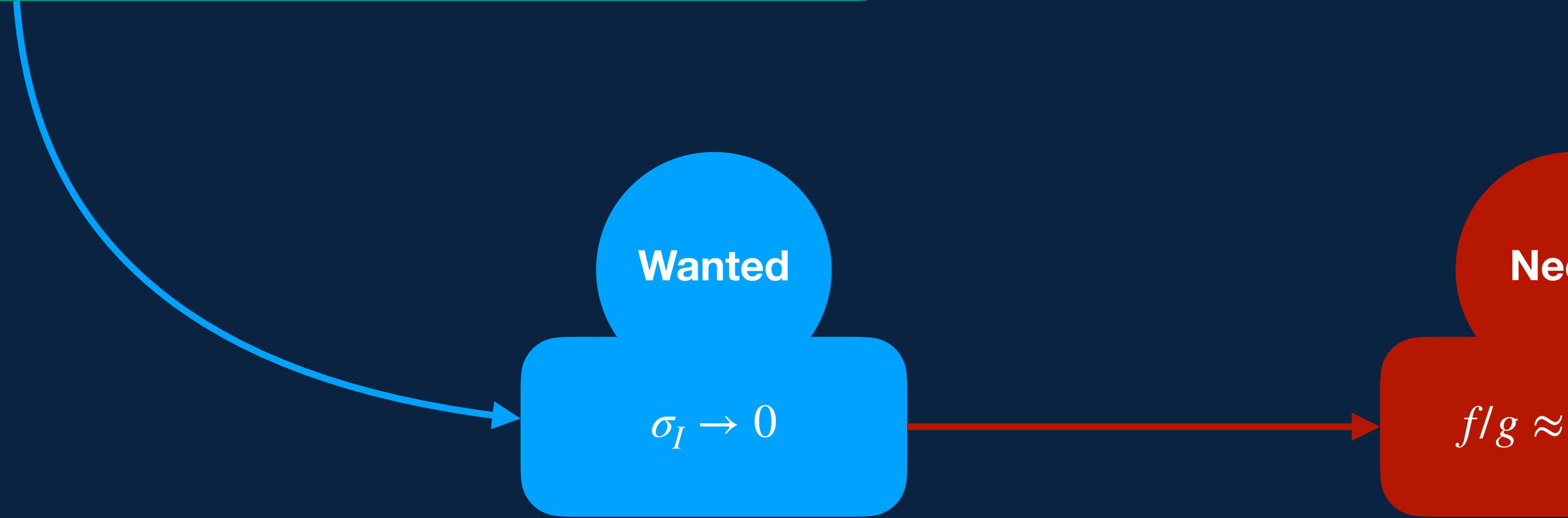
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**Wanted**

$$\sigma_I \rightarrow 0$$

**Needs**

$$f/g \approx \text{const}$$


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## Multi-Channel Integration $\longrightarrow \sum \alpha_i(x) = 1$

$$I = \int_V d^d x f(x) = \sum_i \int_V d^d x \alpha_i(x) f(x) = \sum_i \int_{U_i} d^d y_i \alpha_i(x) \frac{f(x)}{g_i(x)} \Big|_{x \equiv x(y_i)}$$

Mapping

$$y = G(x) \quad g(x) = \left| \frac{\partial G(x)}{\partial x} \right|$$

Channel Mappings

$$y_i = G_i(x) \quad g_i(x) = \left| \frac{\partial G_i(x)}{\partial x} \right|$$



# Neural Multi-Channel Monte Carlo

Multi-Channel Importance Sampling

$$I = \sum_i \int_{U_i} \alpha_i(x) \frac{f(x)}{g_i(x)} dG_i(x)$$

# Neural Multi-Channel Monte Carlo

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## Multi-Channel Importance Sampling

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## Neural Channel Mappings

$$G_i(x) \rightarrow G_i^\theta(x), \quad g_i^\theta(x) = \left| \frac{\partial G_i^\theta(x)}{\partial x} \right|$$

**n-dimensional remapping**

- tractable Jacobian with **normalizing flow**

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→ So far: improvement factors of ~2-4 are achieved [2001.05486, 2001.05478, 2001.10028, 2112.09145]

# Neural Multi-Channel Monte Carlo

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## Multi-Channel Importance Sampling

$$I = \sum_i \int_{U_i} \alpha_i(x) \frac{f(x)}{g_i(x)} dG_i(x)$$

Possible  
Improvements?

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## Multi-Channel Importance Sampling

$$I = \sum_i \int_{U_i} \alpha_i(x) \frac{f(x)}{g_i(x)} dG_i(x)$$

### Neural Channel Weights

$$\alpha_i(x) \rightarrow \alpha_i^\phi(x), \quad \sum_i \alpha_i^\phi(x) = 1$$

**k-dimensional regression**

- with boundary condition

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# Neural Multi-Channel Monte Carlo

18

## Multi-Channel Importance Sampling

$$I = \sum_i \int_{U_i} \alpha_i(x) \frac{f(x)}{g(x|i)} dG(x|i)$$

### Neural Channel Weights

$$\alpha_i(x) \rightarrow \alpha_i^\phi(x), \quad \sum_i \alpha_i^\phi(x) = 1$$

#### k-dimensional regression

- with boundary condition

### Conditional Neural Channel Mappings

$$G_i(x) \rightarrow G^\theta(x|i), \quad g^\theta(x|i) = \left| \frac{\partial G^\theta(x|i)}{\partial x} \right|$$

#### n-dimensional remapping

- tractable Jacobian with **normalizing flow**
- conditioned on channel

→ So far: improvement factors of ~2-4 are achieved [2001.05486, 2001.05478, 2001.10028, 2112.09145]

# Neural Multi-Channel Monte Carlo

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Multi-Channel Importance Sampling

$$I = \sum_i \int_{u_i} \frac{f(x)}{g(x|i)} dG(x|i)$$

**Further Improvements?**

Neural Channel Weights

$$\alpha_i(x) \rightarrow \alpha_i^\phi(x), \quad \sum_i \alpha_i^\phi(x) = 1$$

k-dimensional regression

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Conditional Neural Channel Mappings

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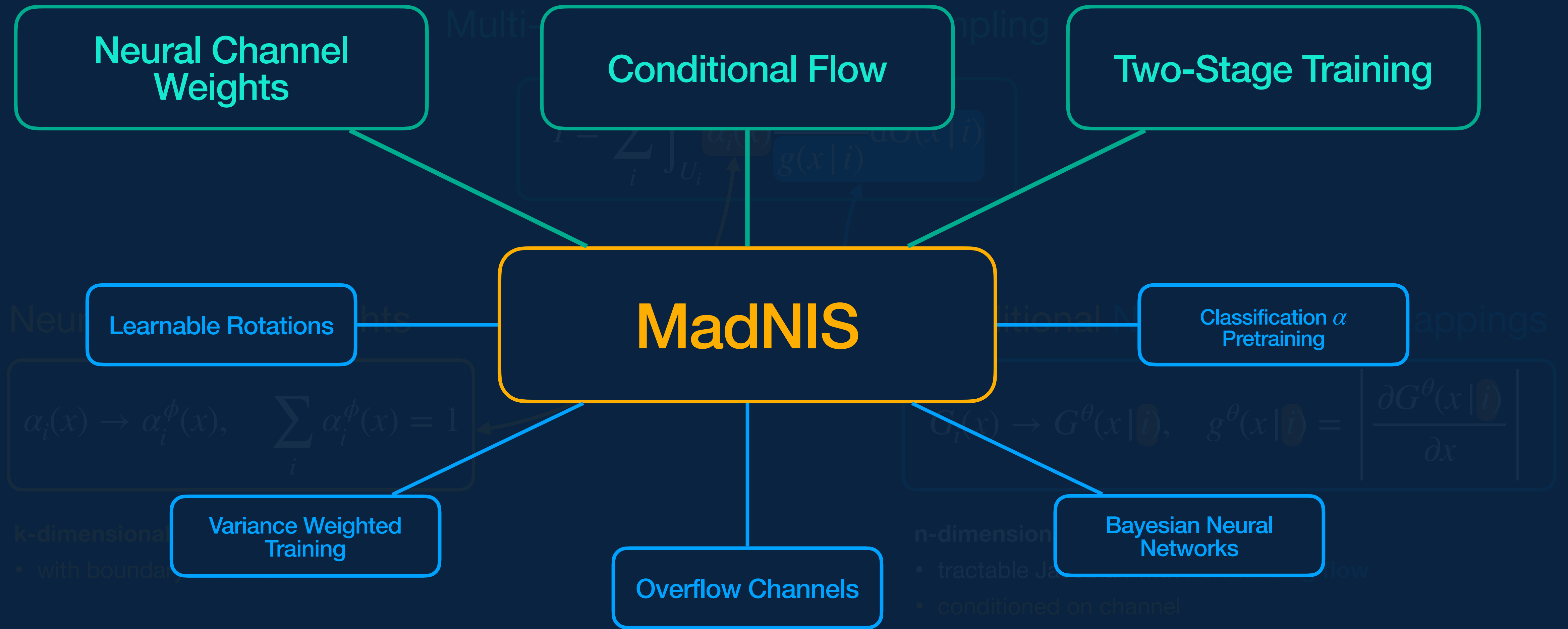
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# Neural Multi-Channel Monte Carlo



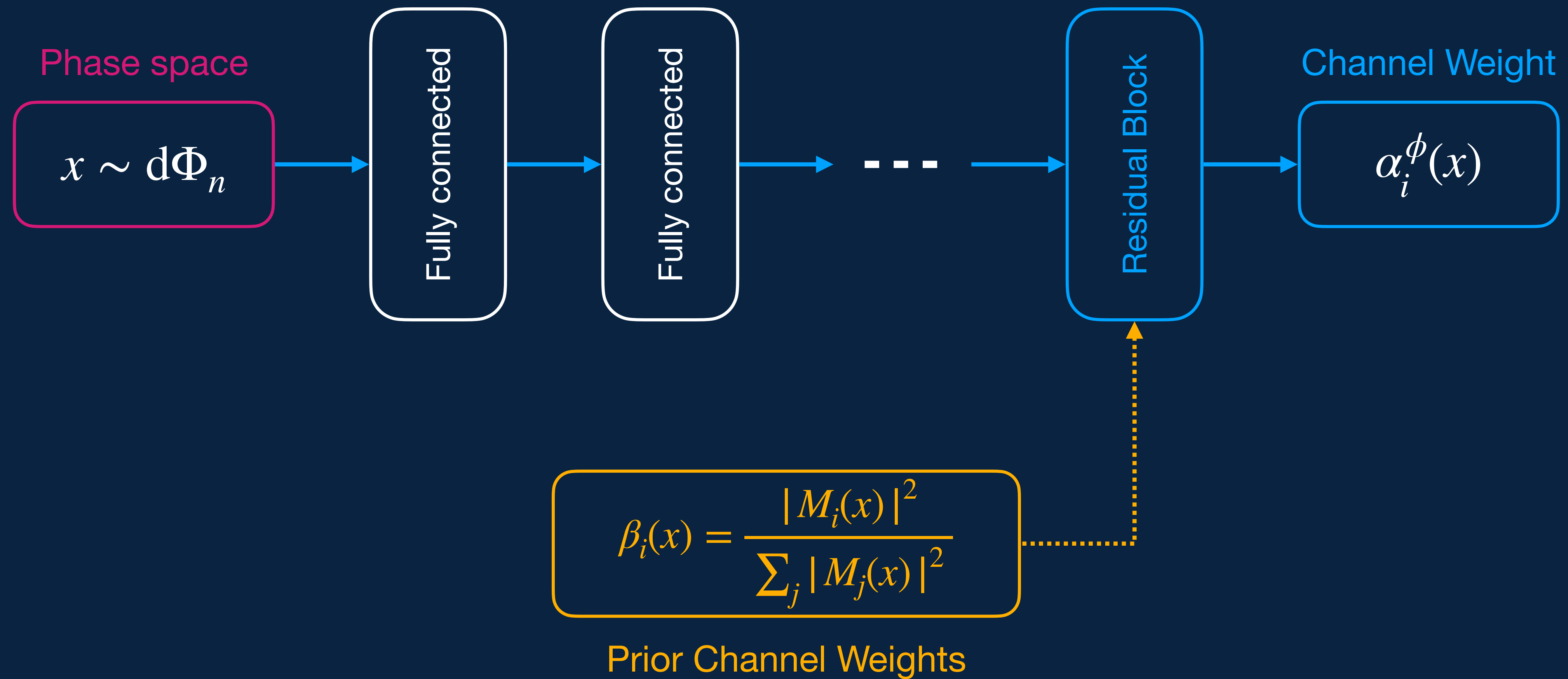
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**MadNIS**

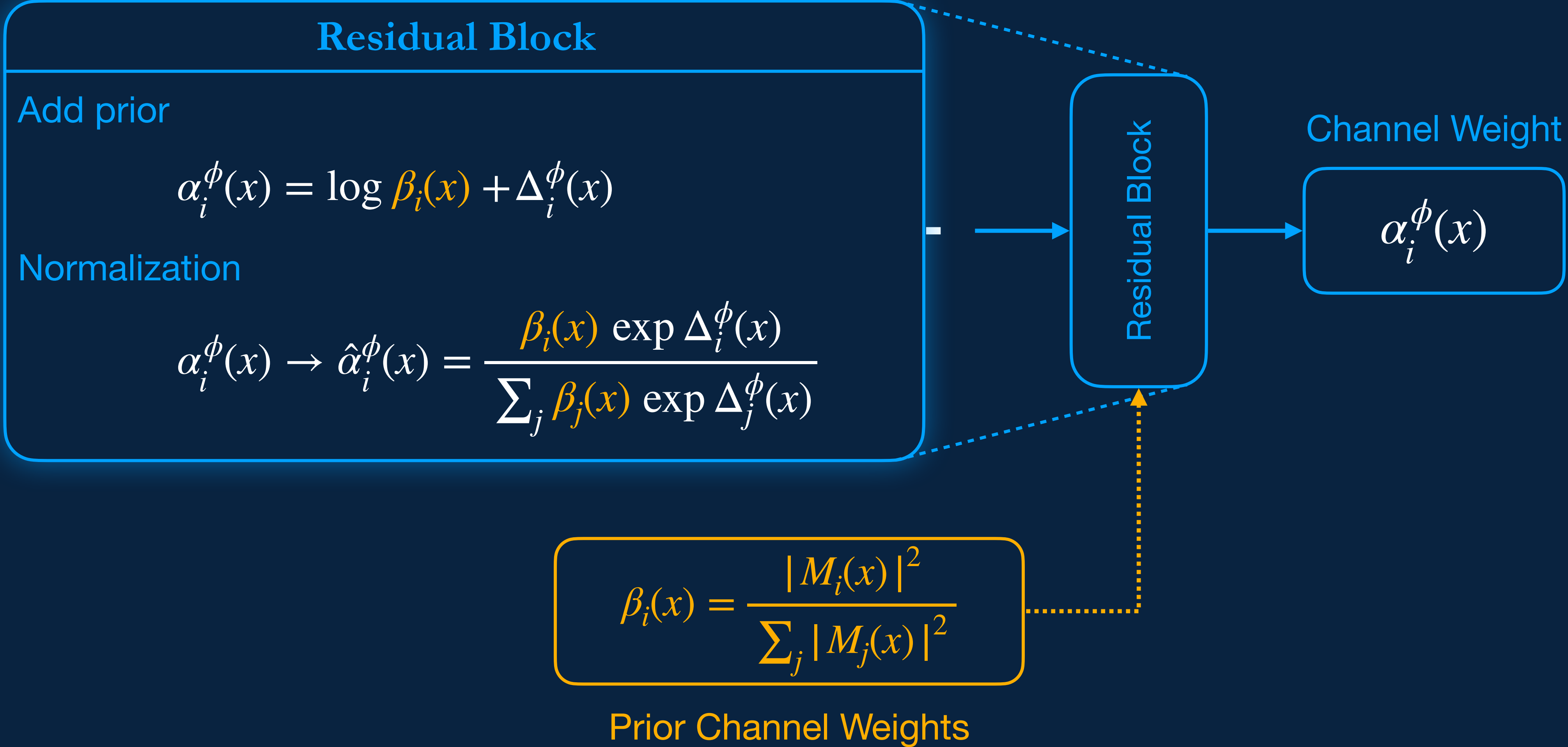
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**Neural Channel Weights**

# Neural Channel Weights



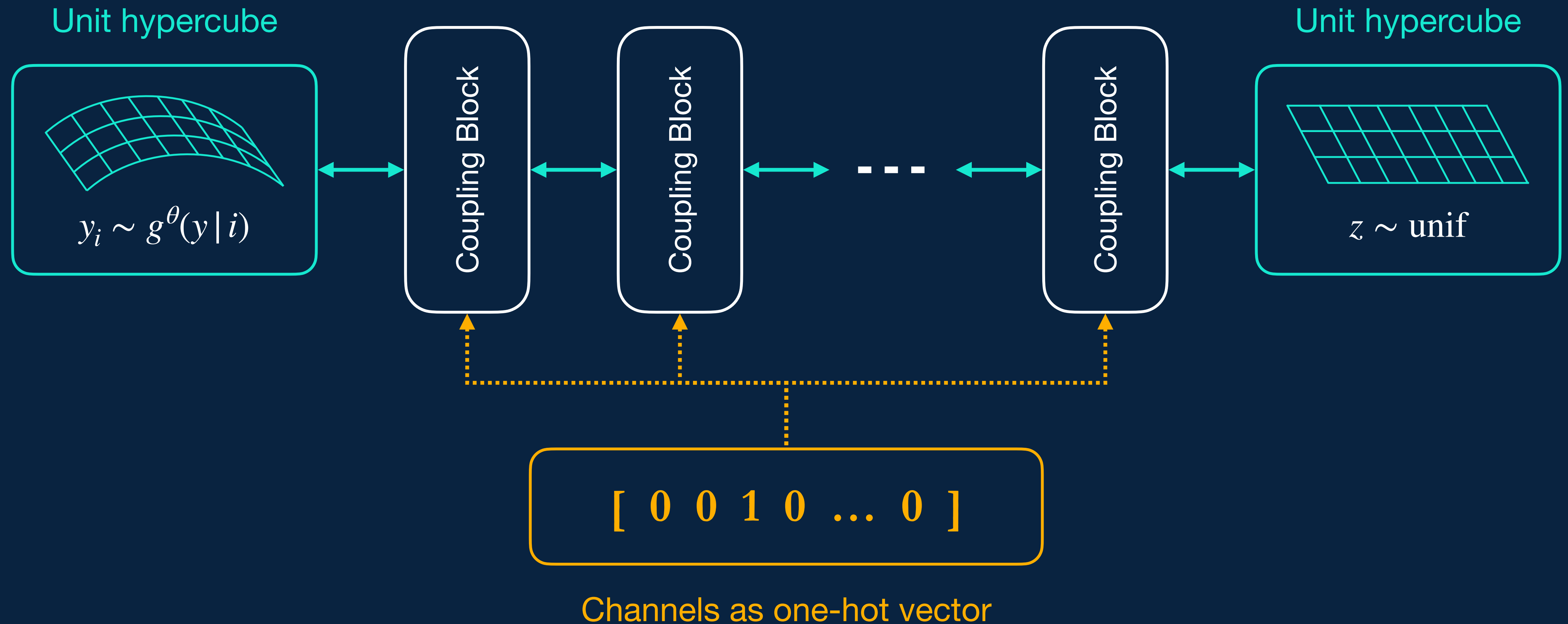
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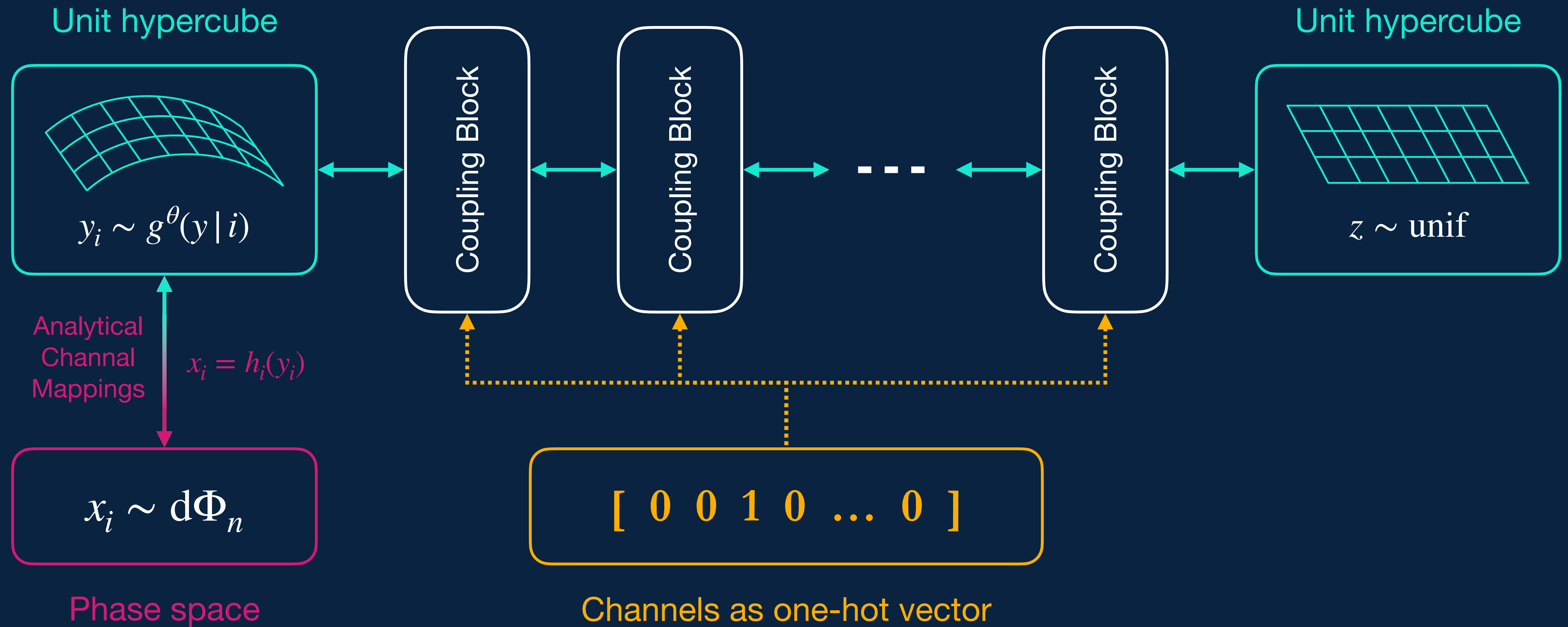


# Conditional Normalizing Flow

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# Conditional Normalizing Flow



## Example

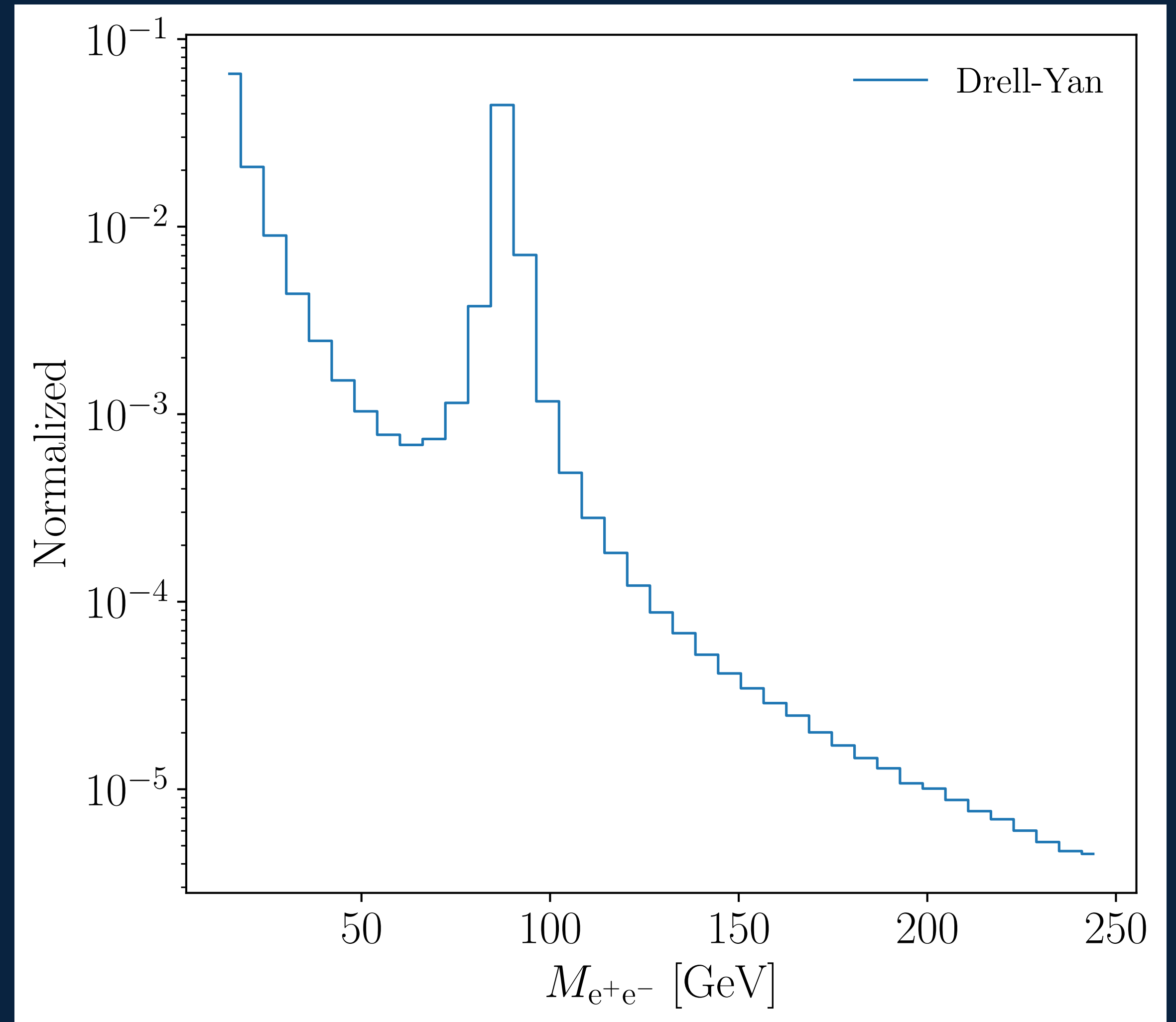
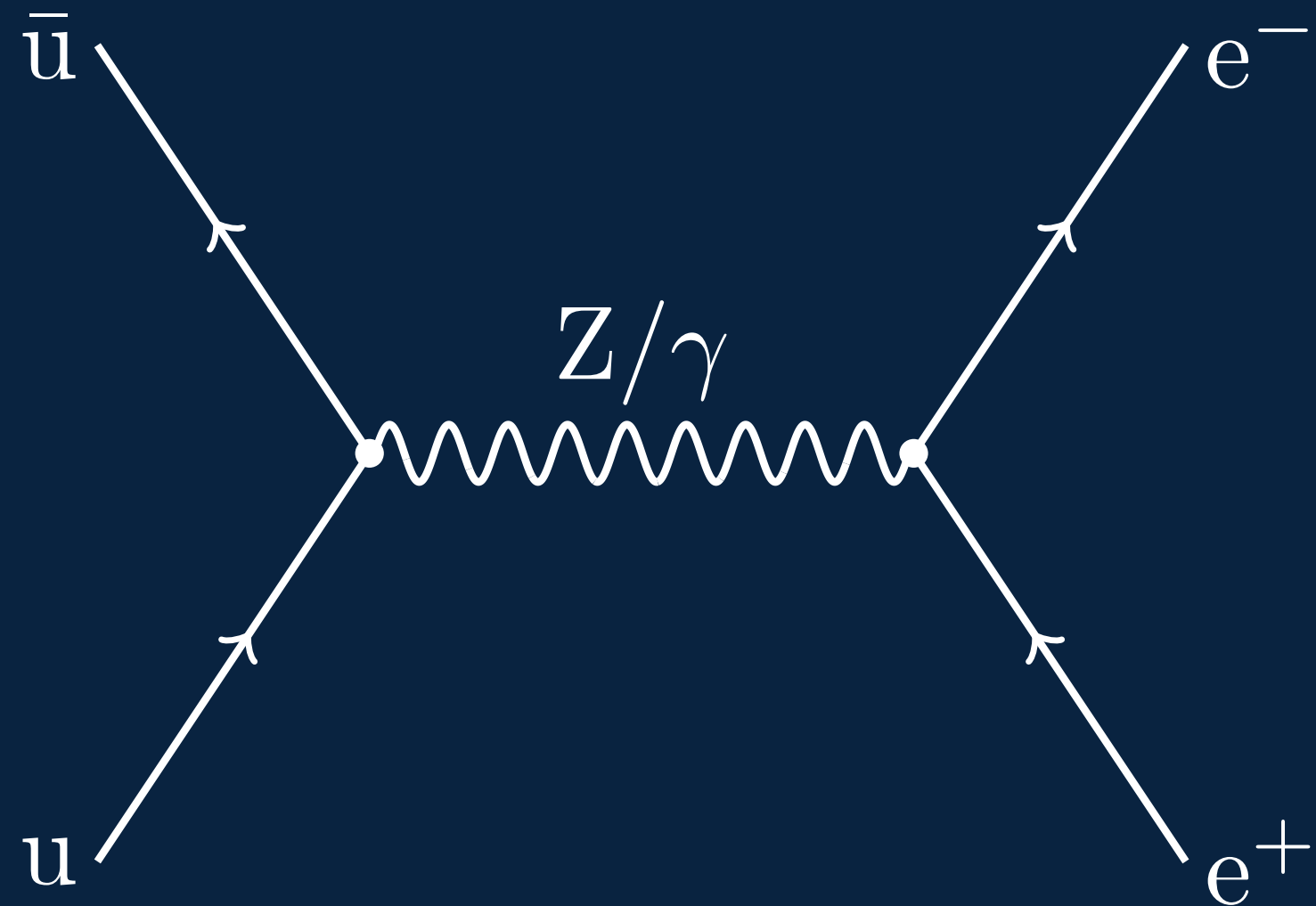
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**Drell-Yan + BSM**

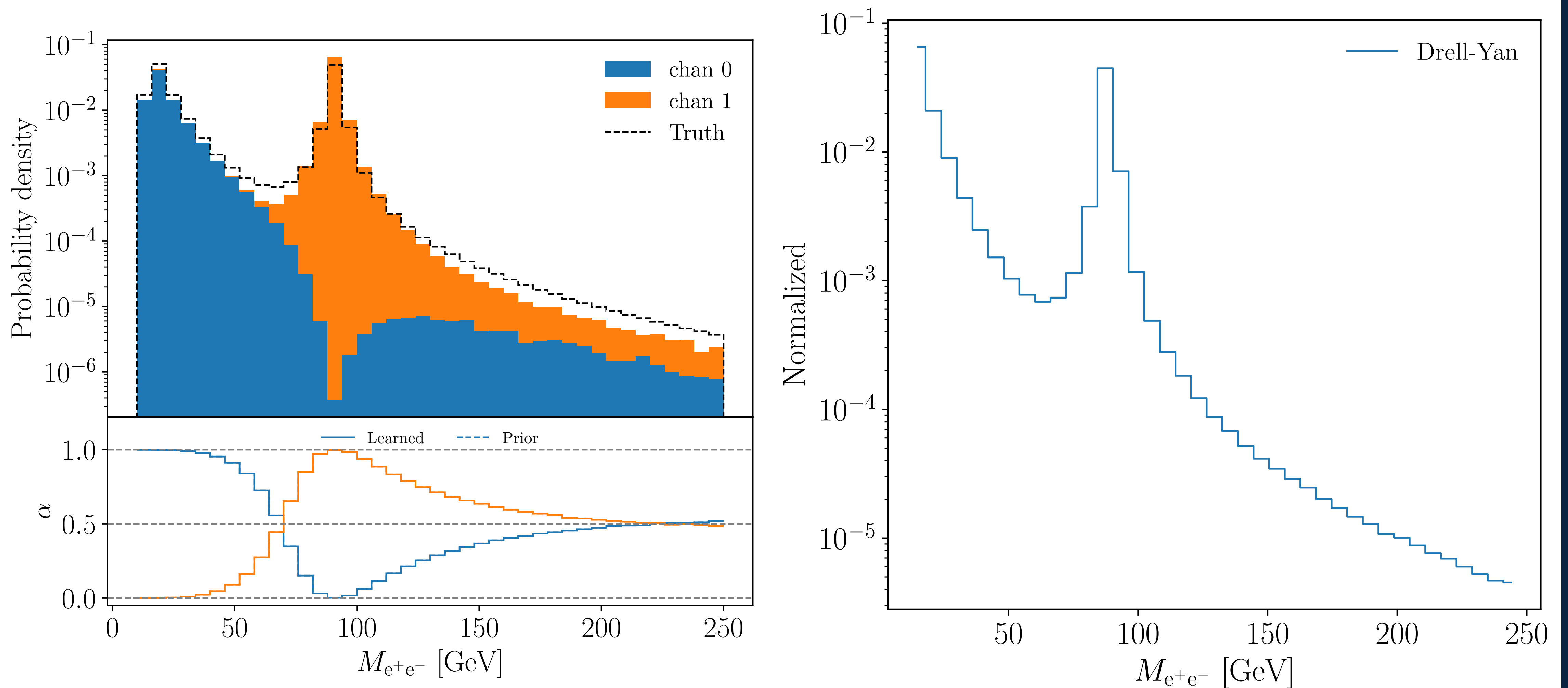
# Example: Drell-Yan

## Implementation

- Custom amplitude in TENSORFLOW2
- Custom PS mappings in TENSORFLOW2
- PDFs from [PDFFLOW \[2009.06635\]](#)

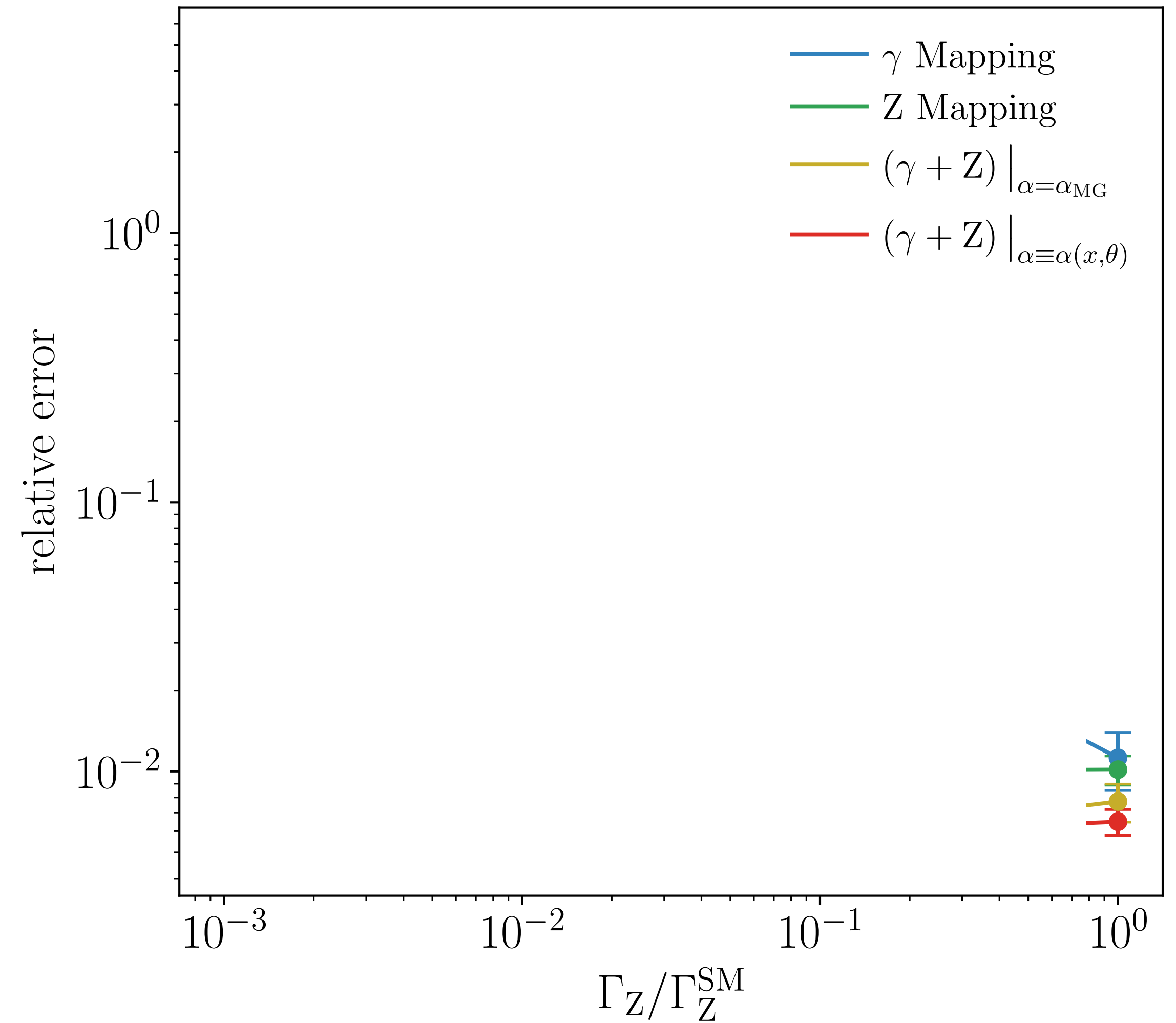
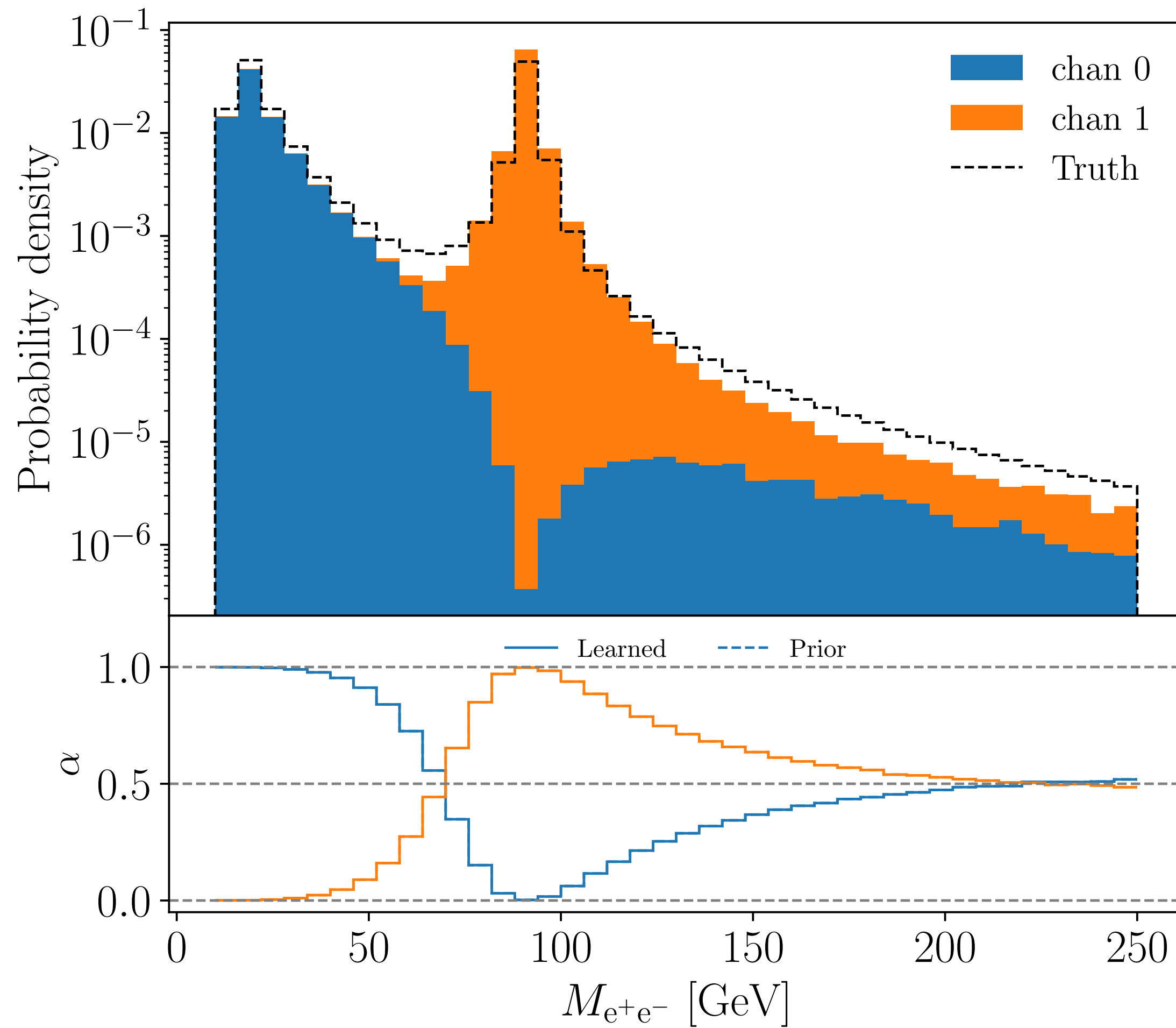


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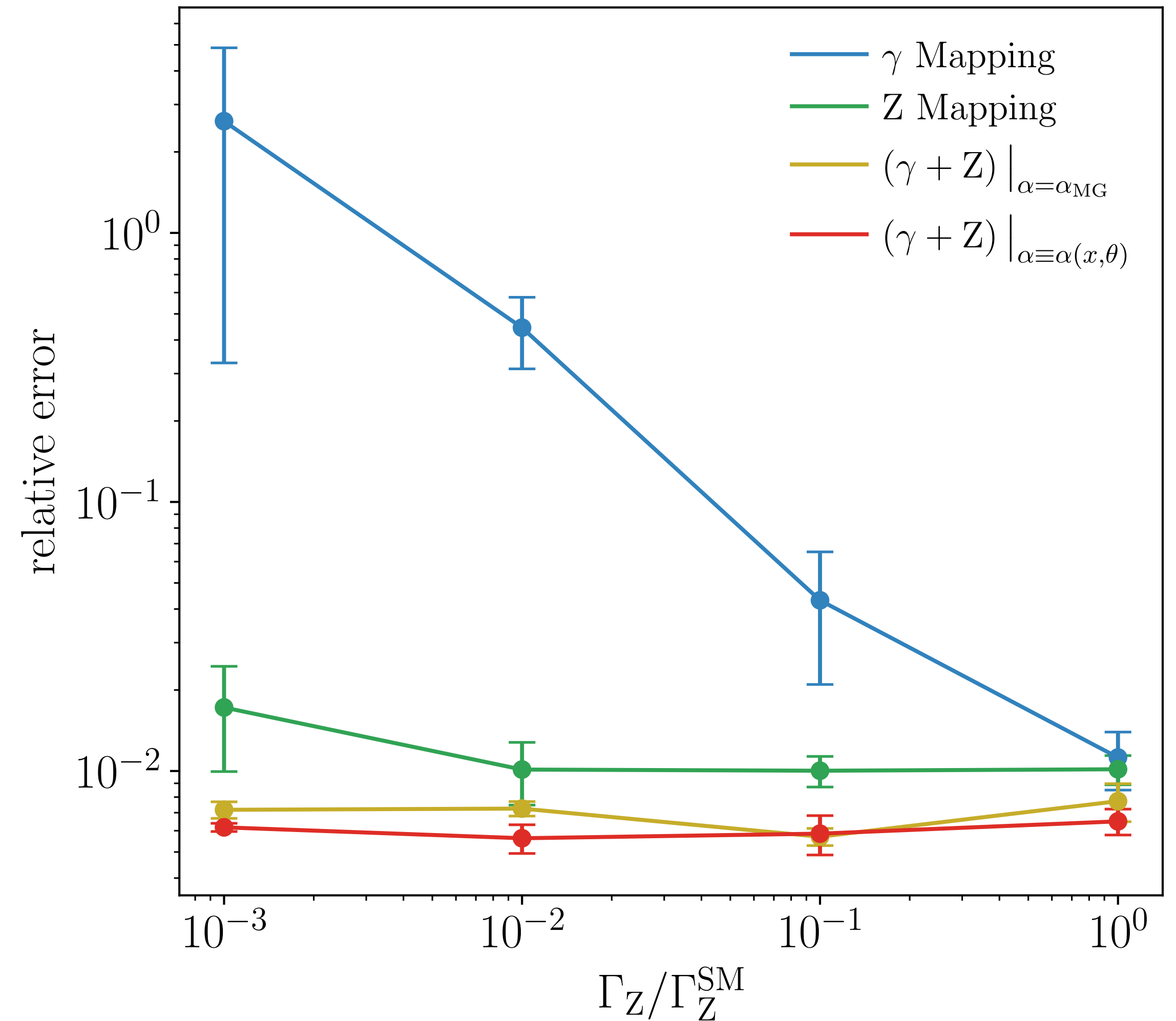
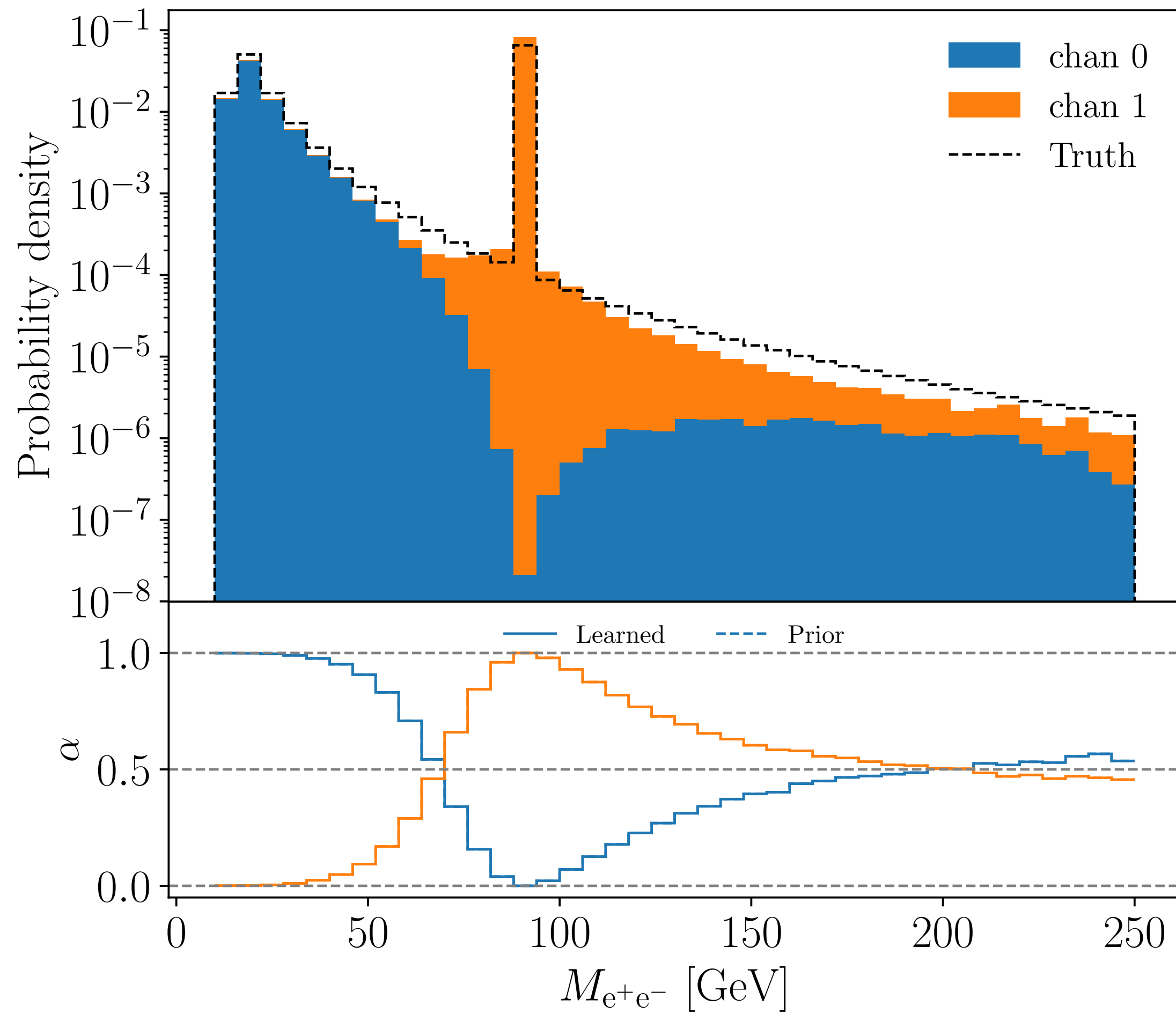




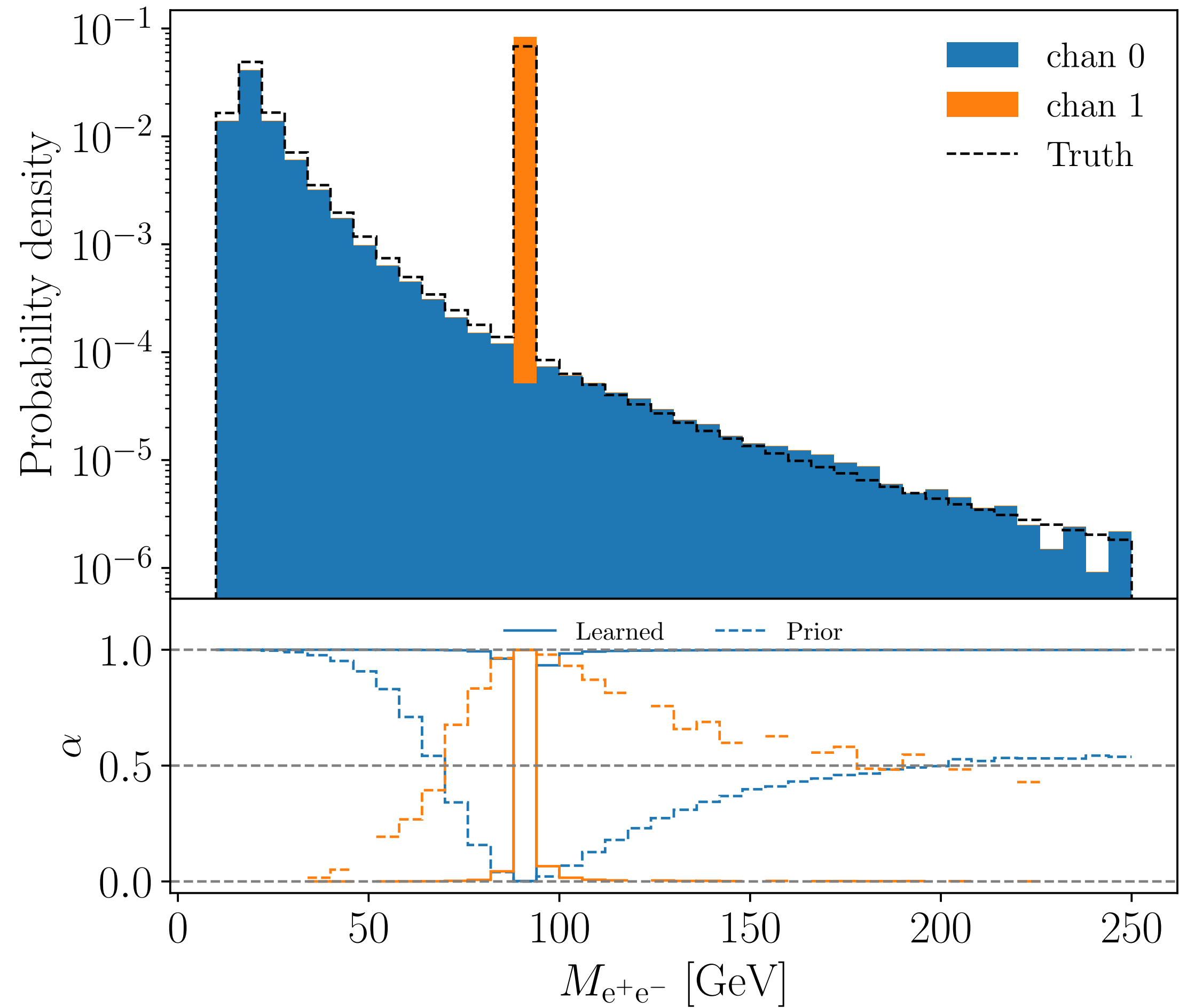
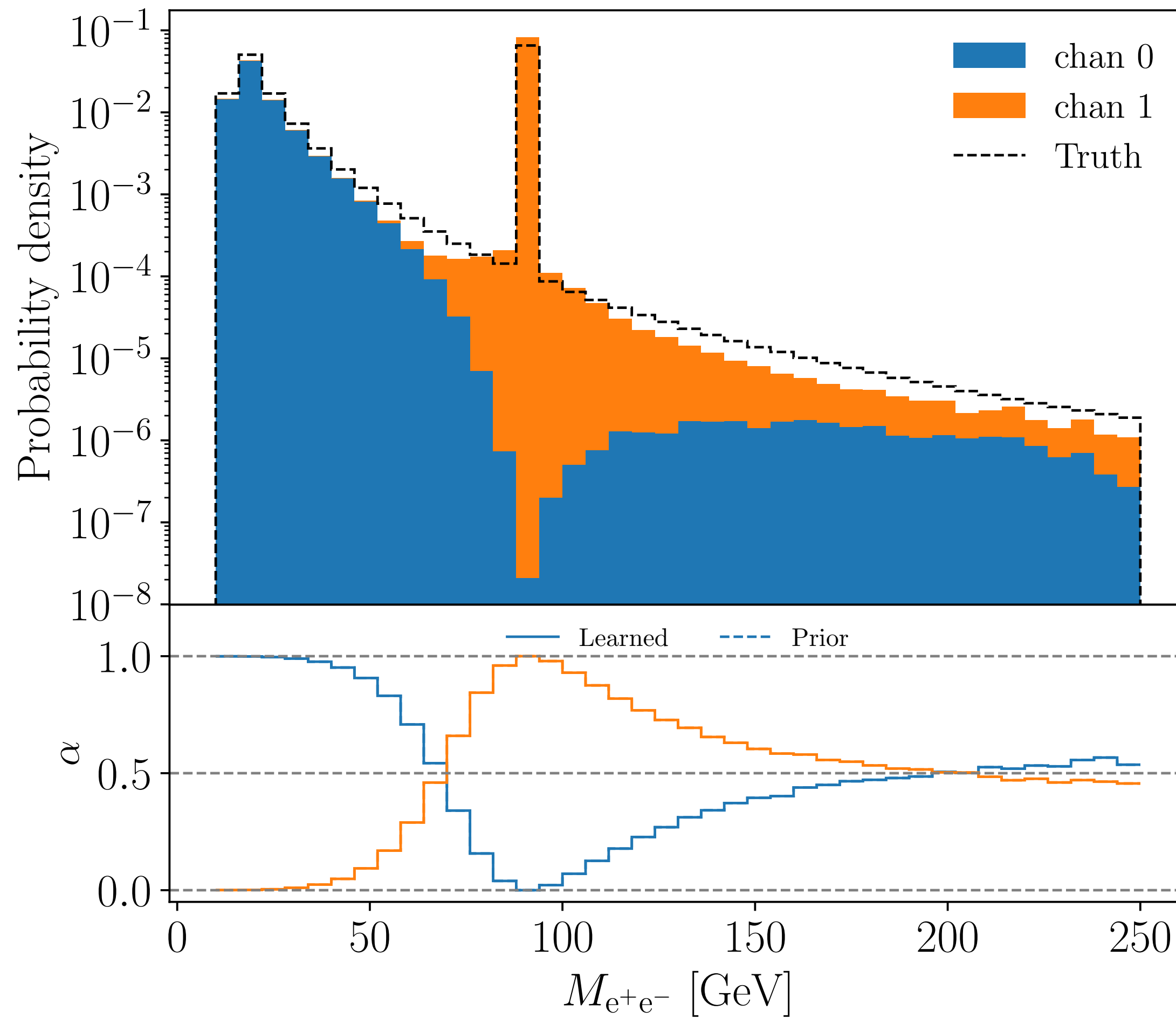
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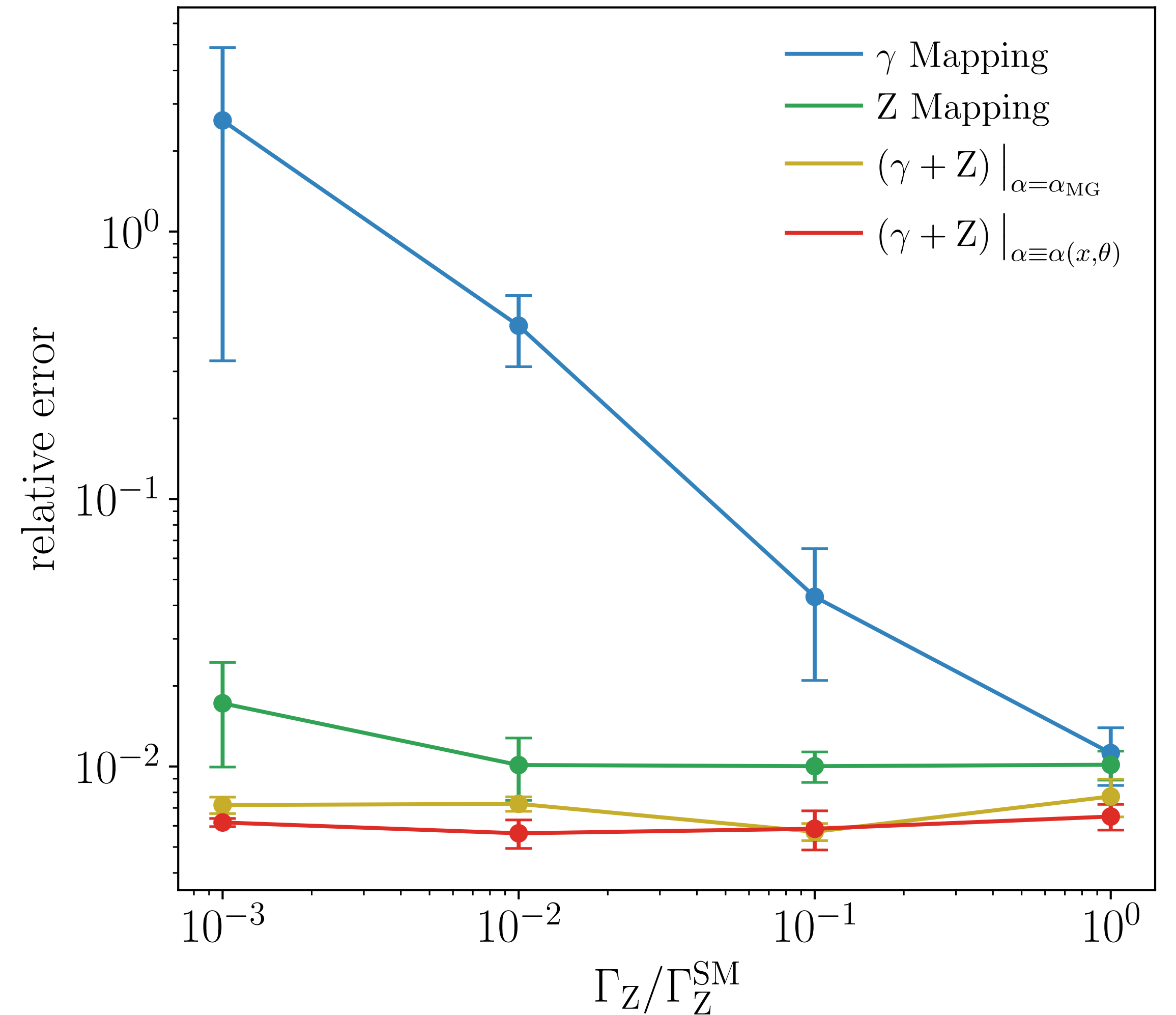
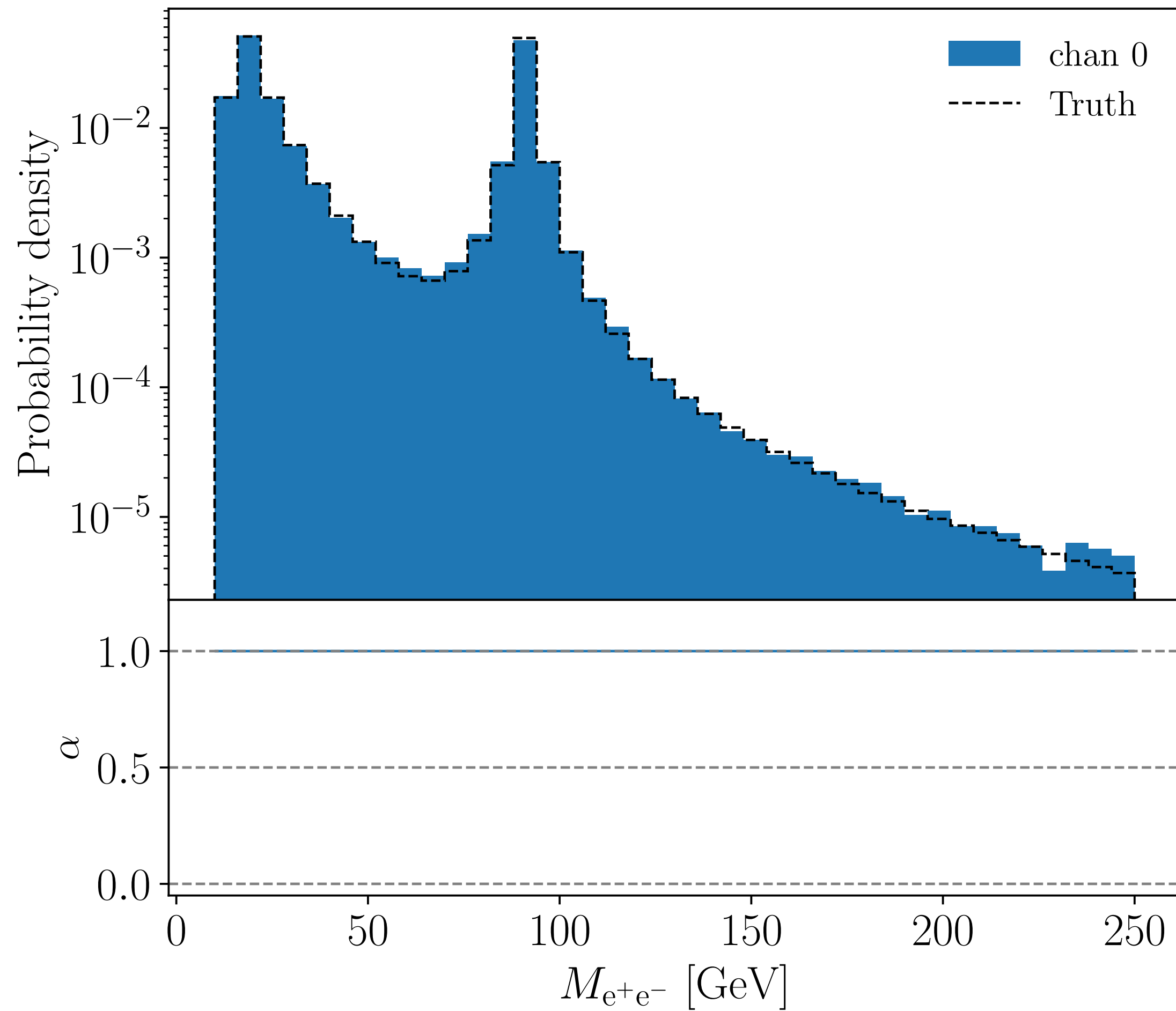
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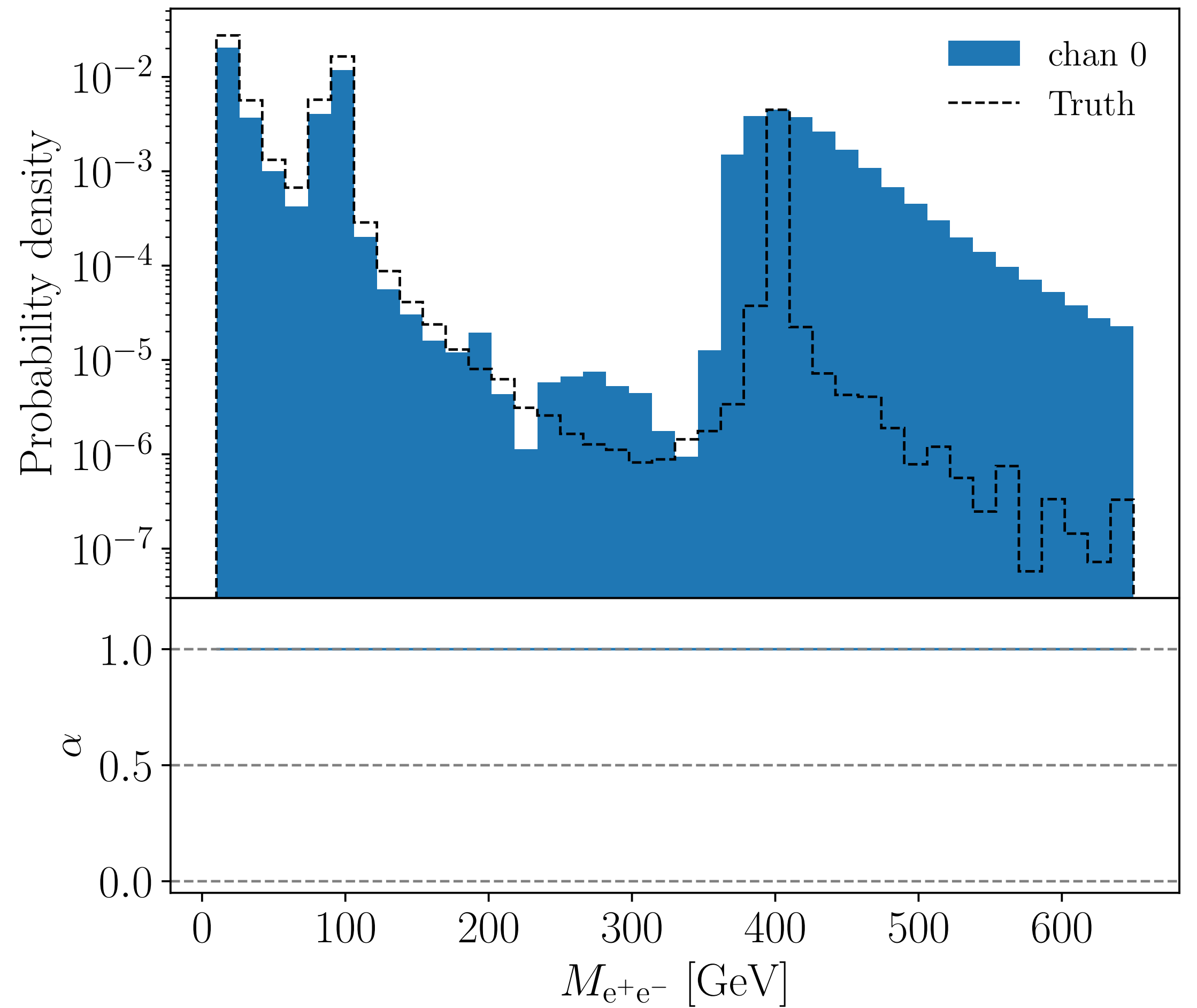
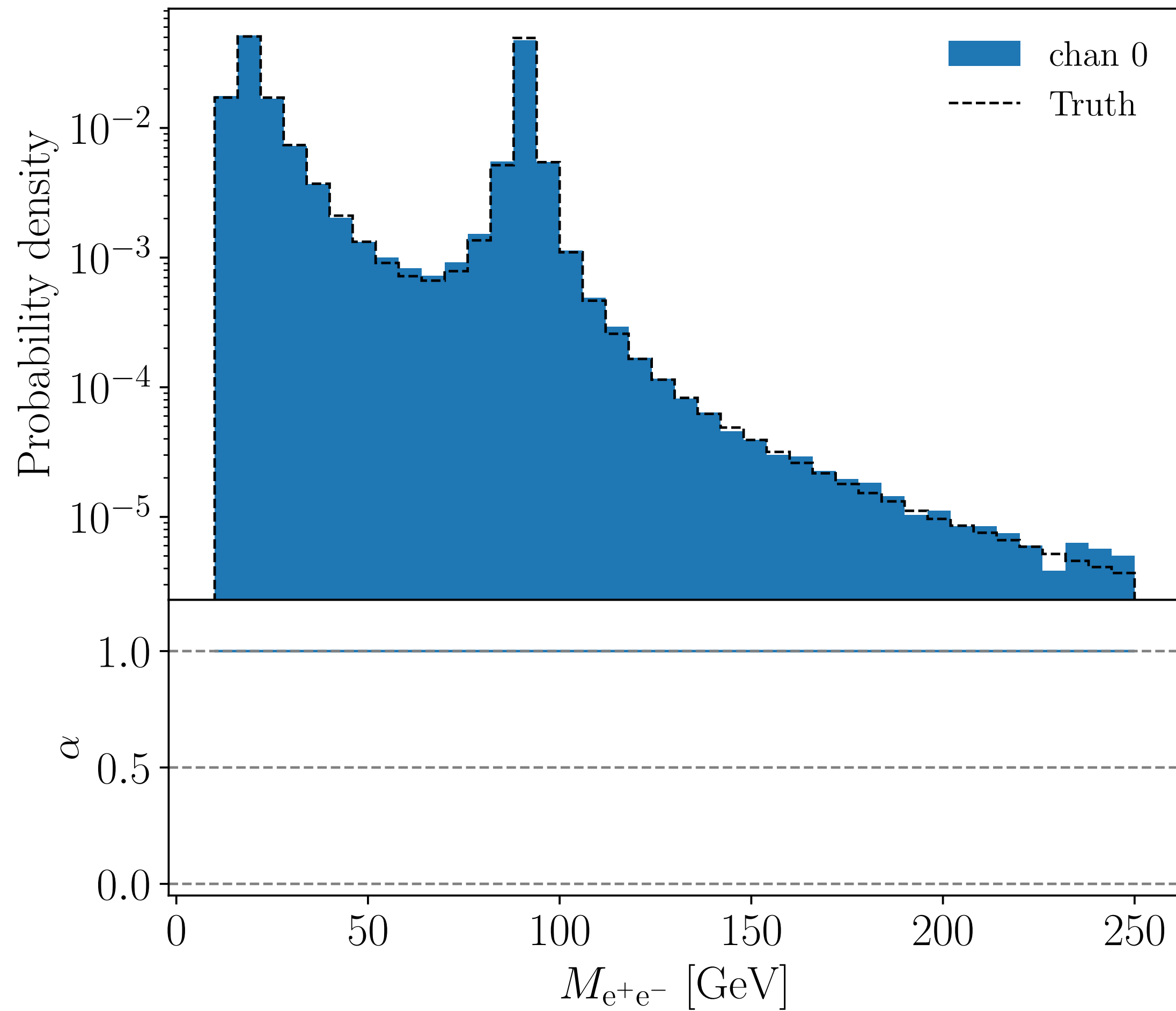
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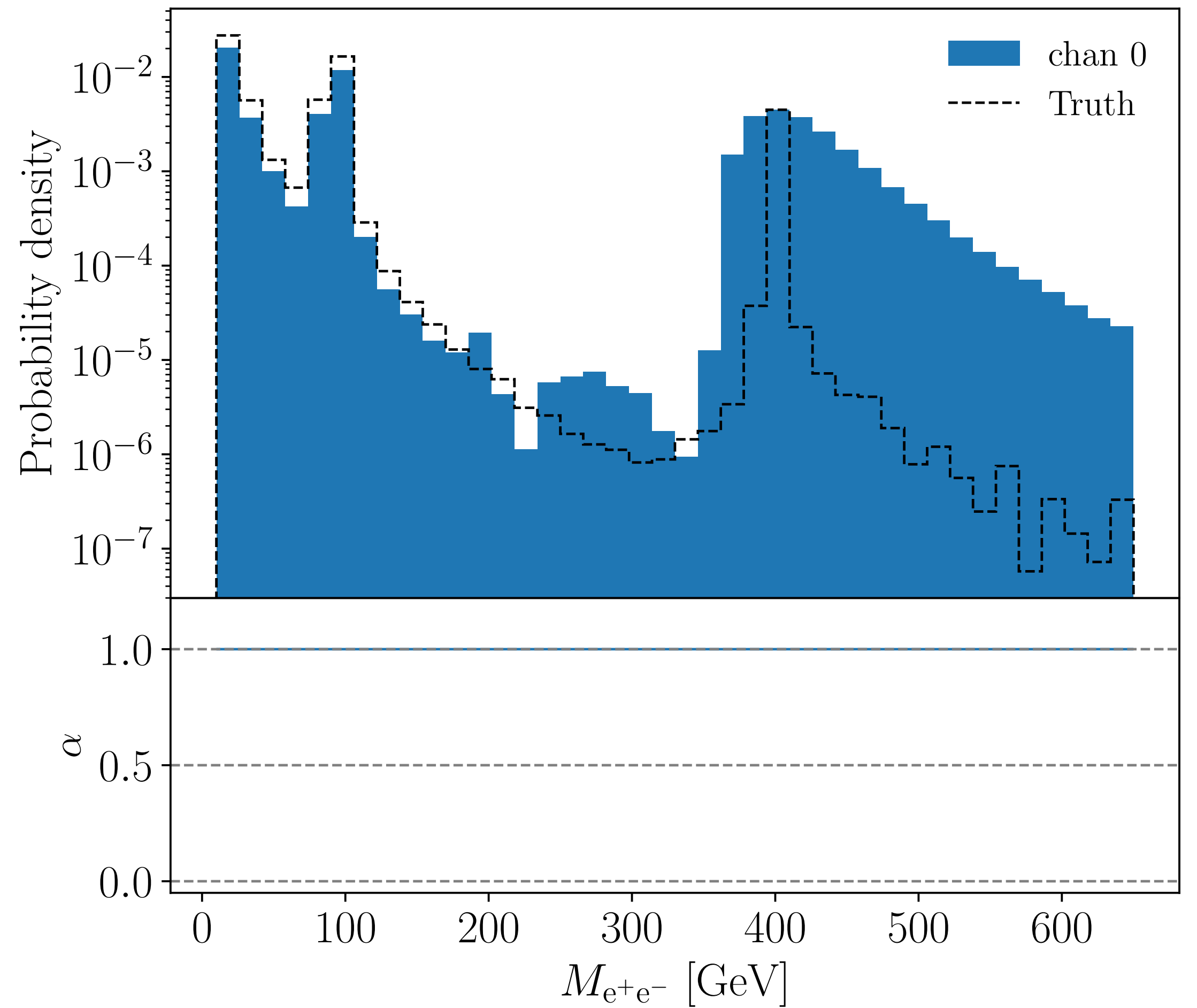
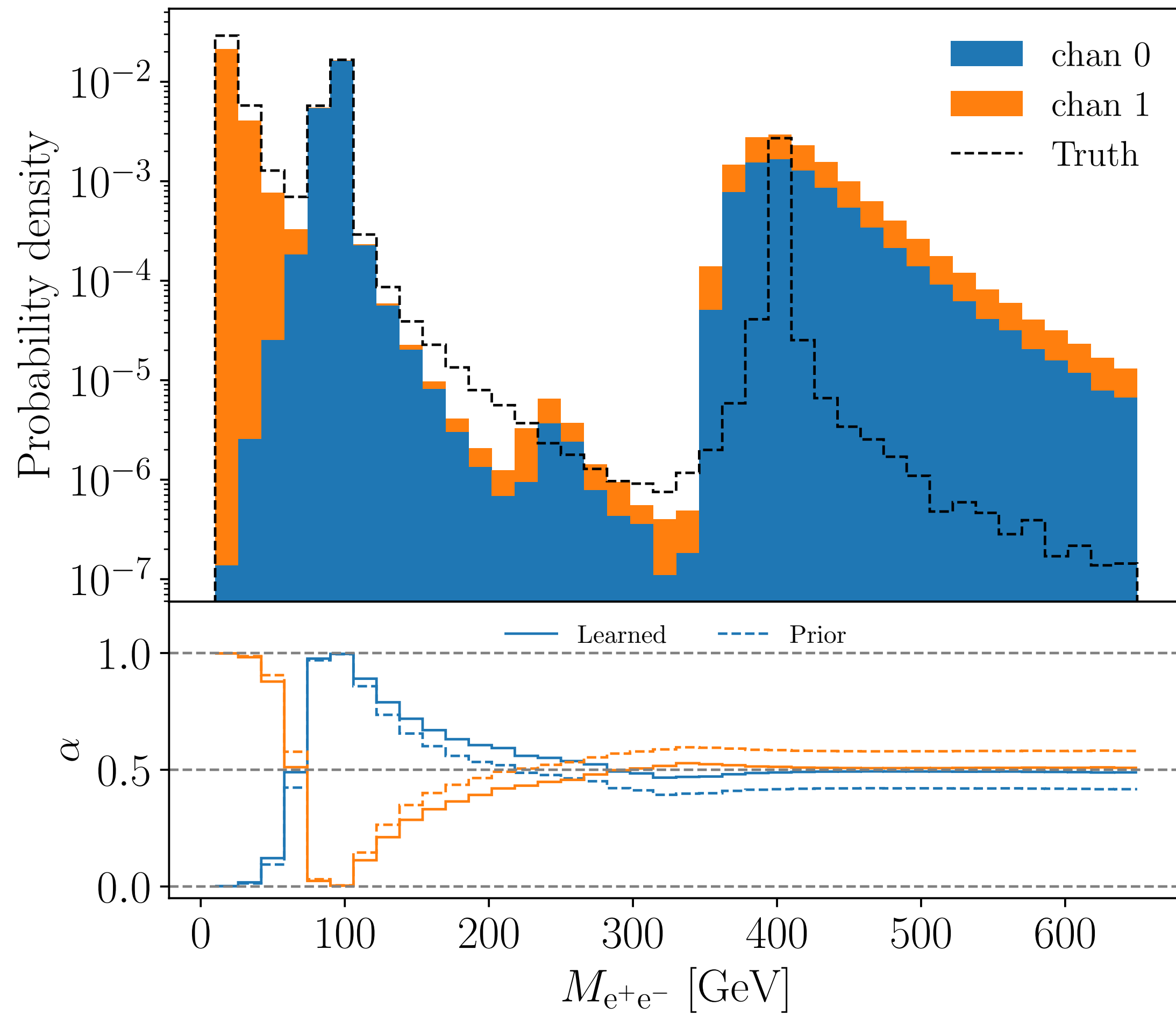
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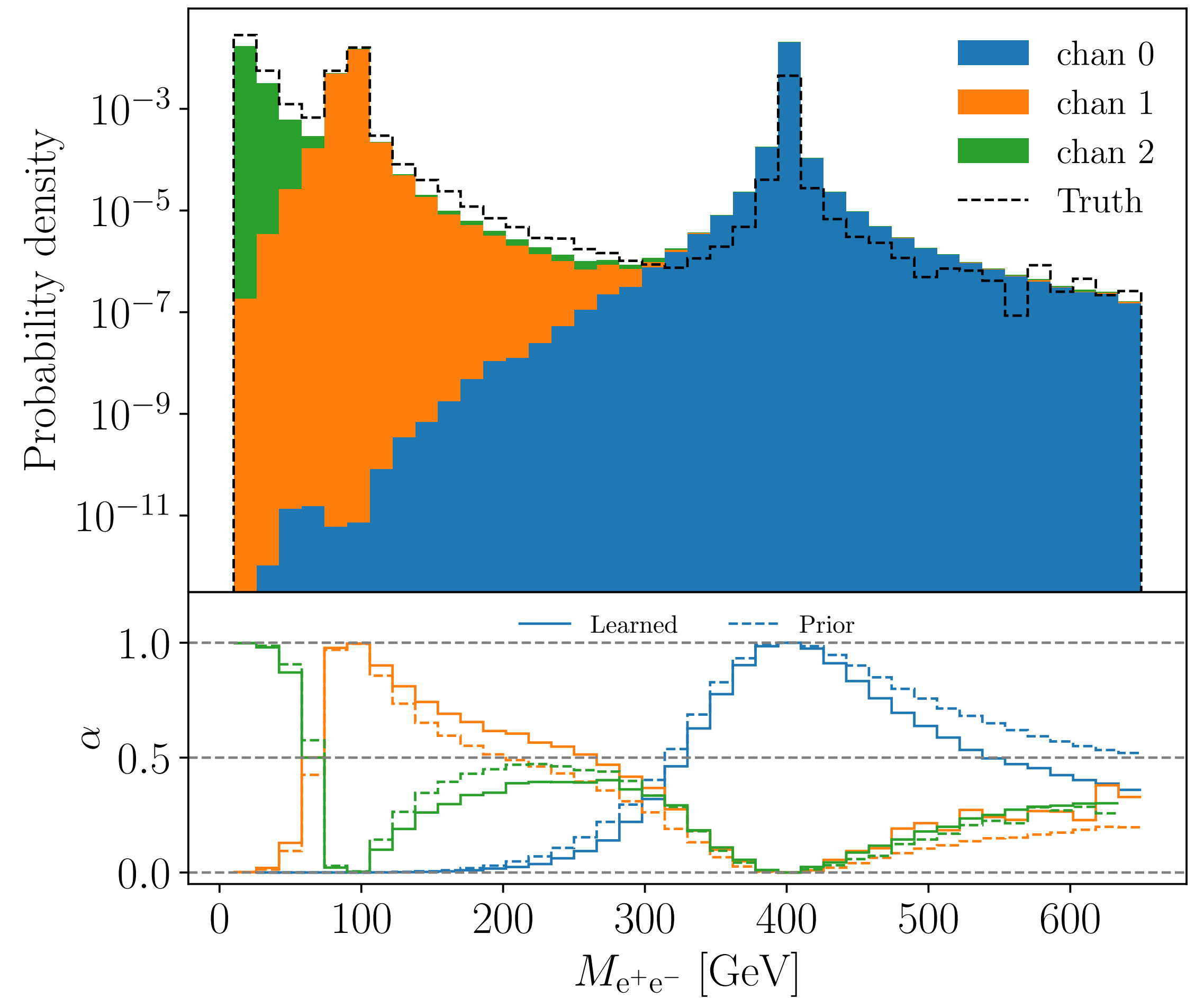
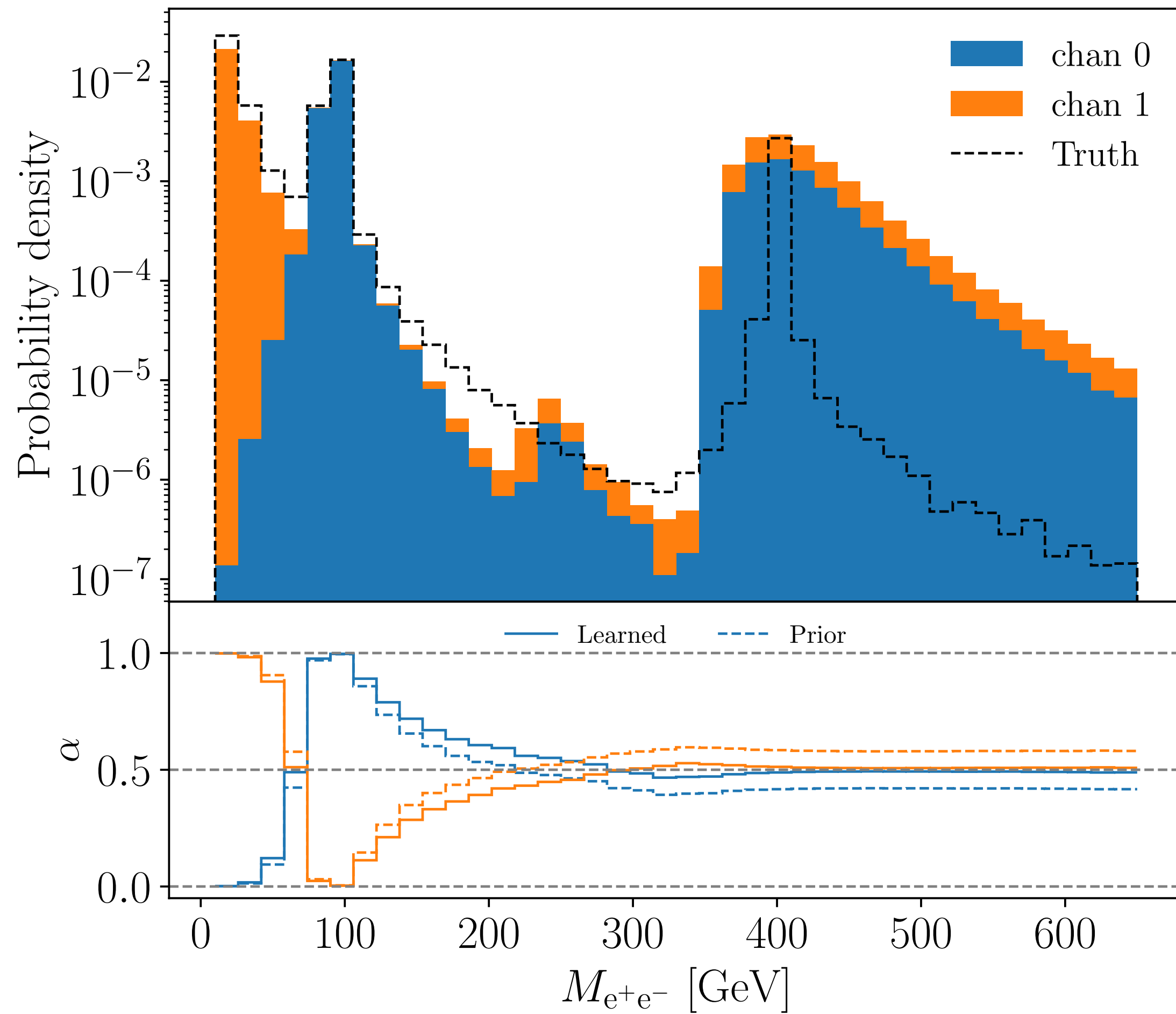


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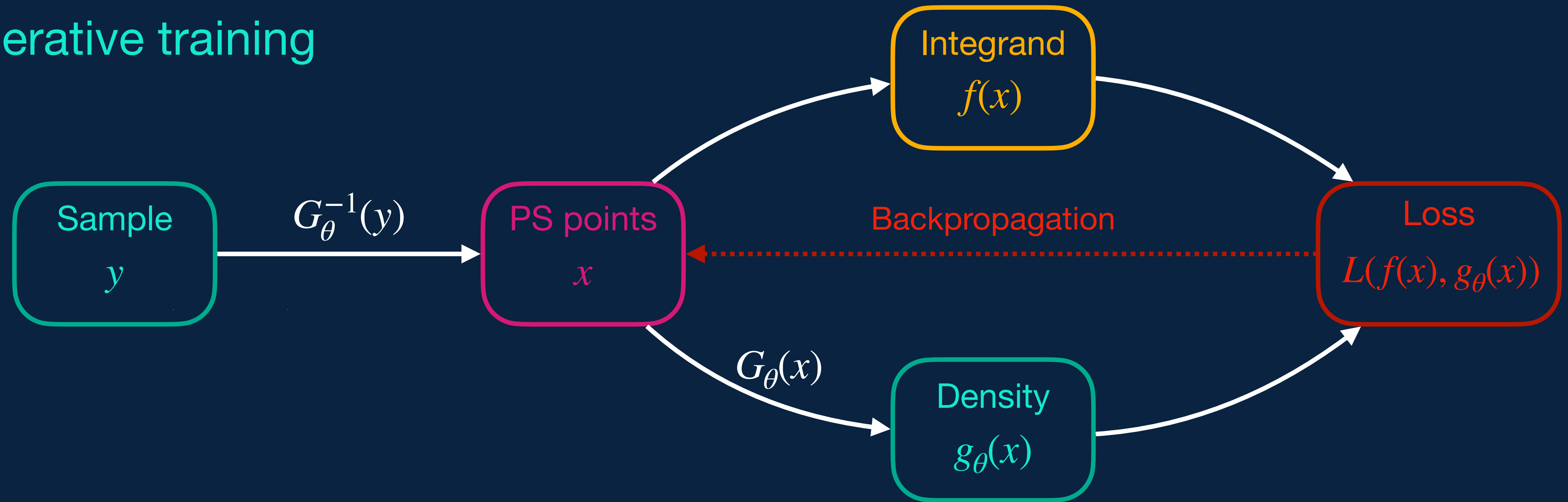


**MadNIS**

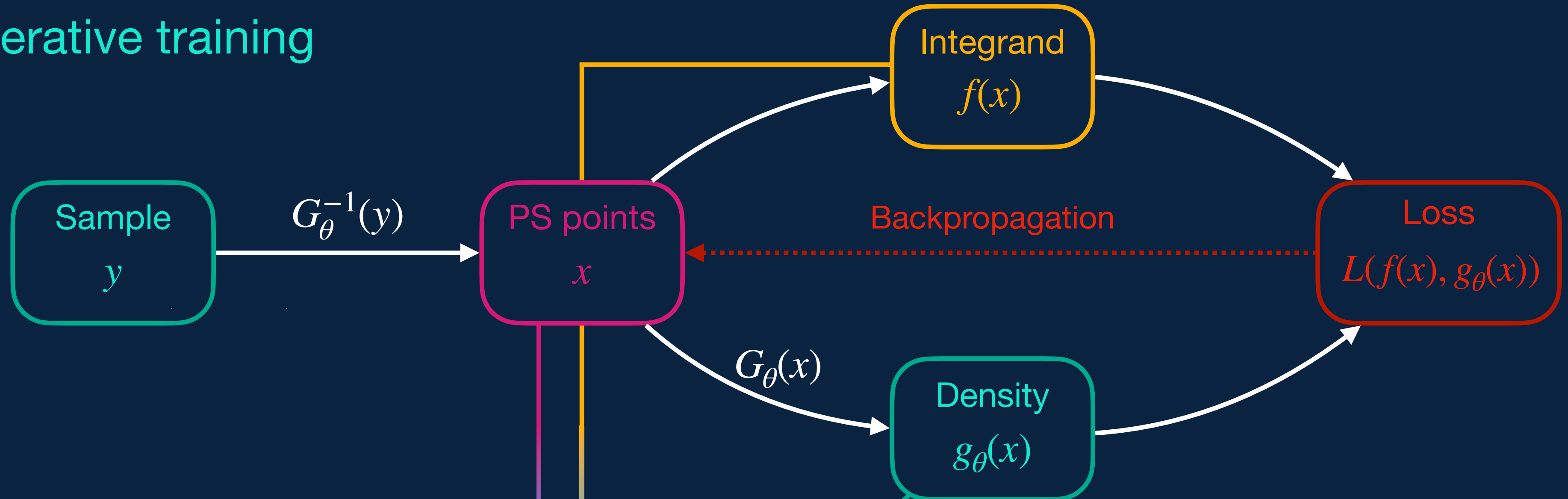
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**Two-Stage Training**

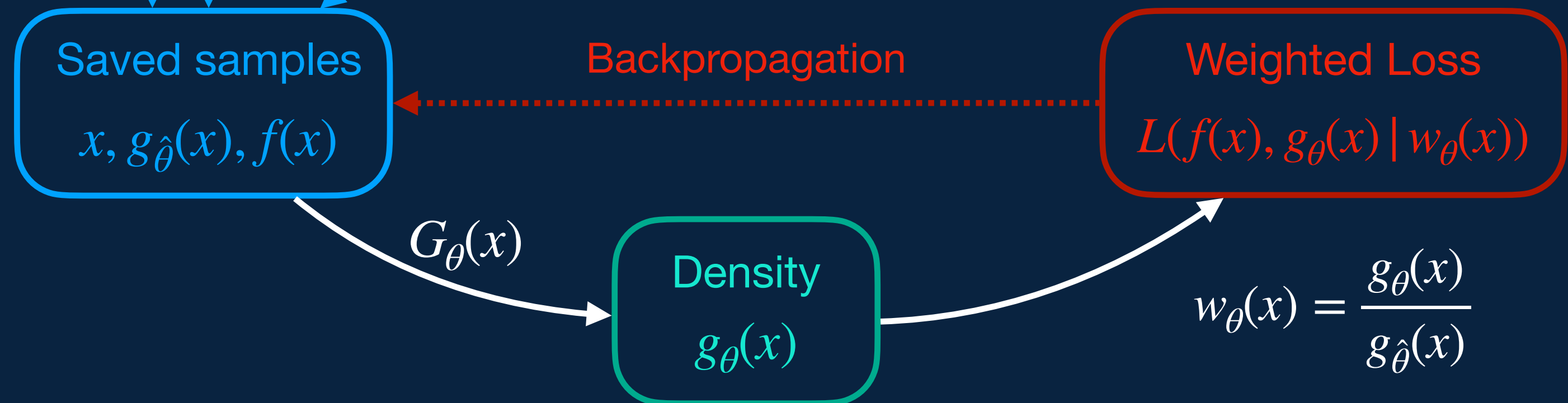
## Generative training



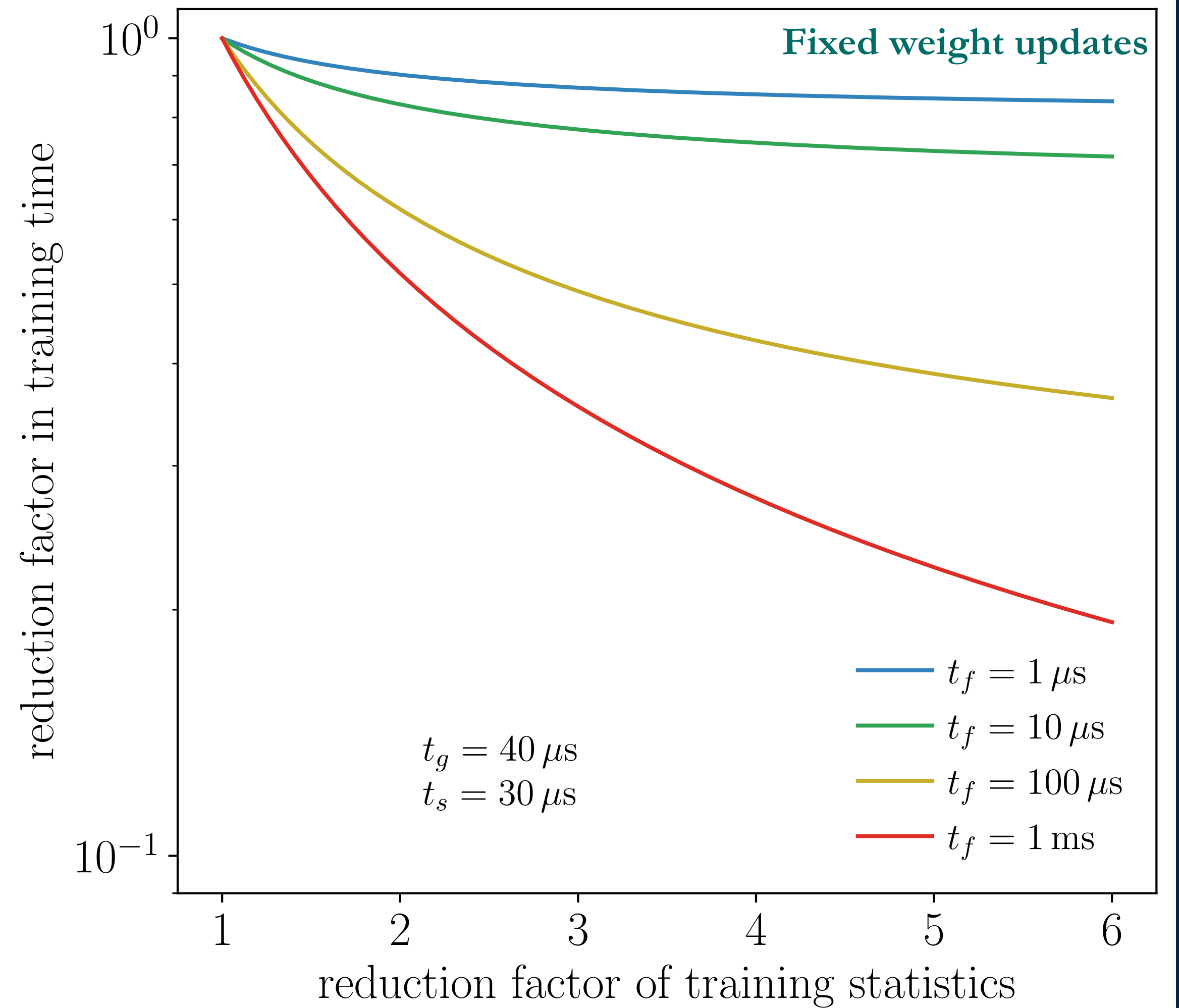
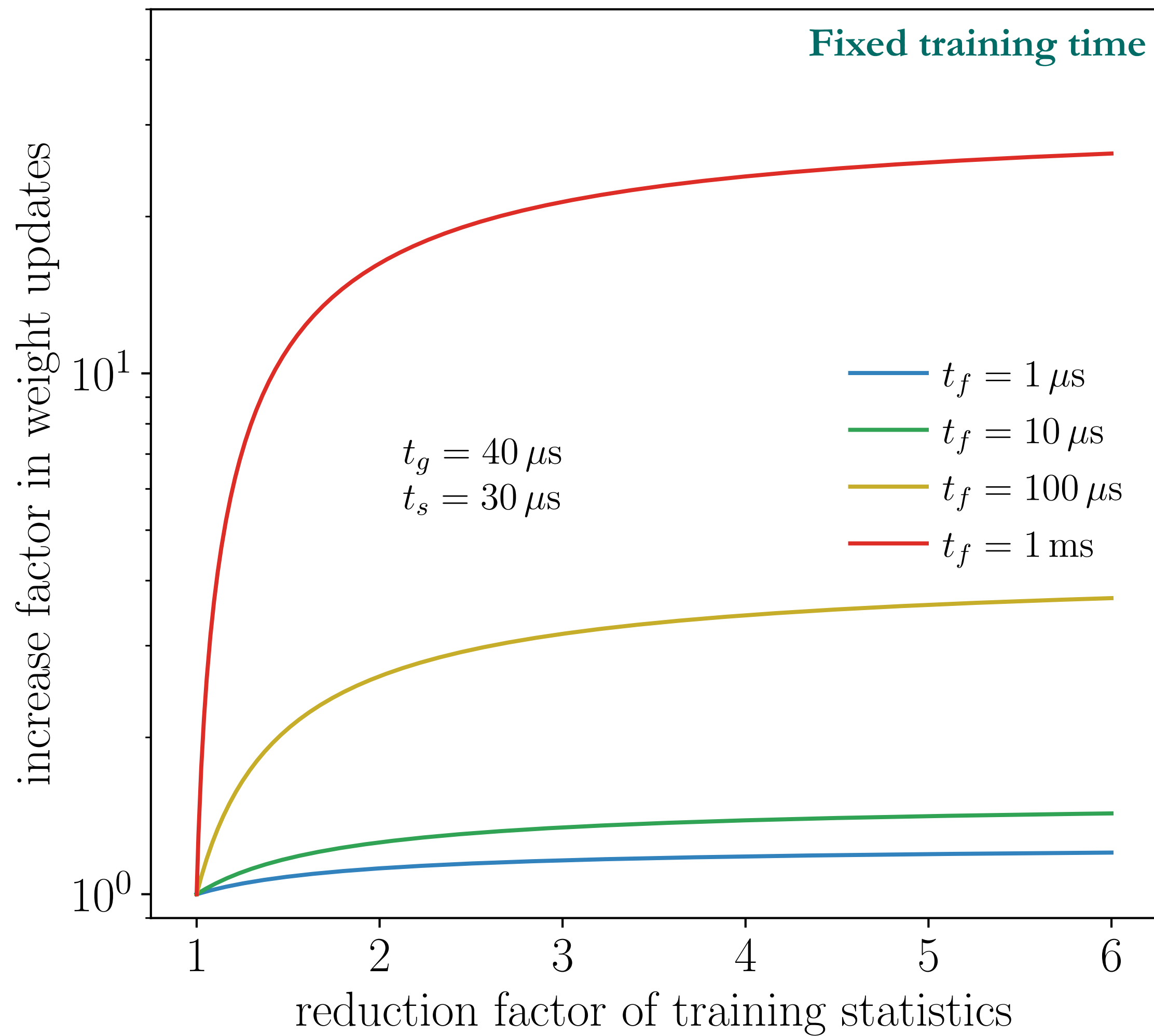
## Generative training



## Sample training



# Two-Stage Training





# Summary and Outlook

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- Channel mappings are **important**
- Multi-channel is **more efficient** and **trained simultaneously** with the flow
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## Outlook

- Implementation of **MadNIS** into **MadGraph**
- Test performance on **real LHC examples**: (eg. multi-leg, NLO, complicated cuts, ...)
- Make matrix elements run on the **GPU** and **differentiable**