

PELICAN: deep jets with permutation and Lorentz equivariance

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THE UNIVERSITY OF
CHICAGO

ML4Jets 2022

Jet-images — deep learning edition

[Luke de Oliveira](#), [Michael Kagan](#), [Lester Mackey](#), [Benjamin Nachman](#) & [Ariel Schwartzman](#)

Journal of High Energy Physics Regular Article - Theoretical Physics | [Open Access](#) | [Published: 25 January 2017](#)

1558 Accesses | 180 Citations

[A preprint version](#) of this article is available on arXiv.

ABSTRACT

Building on the notion of a jet image, we use deep learning techniques based on convolutional neural networks. Modern deep learning algorithms are motivated by feature driven approaches where these features are learned by the network to improve performance. This interplay between physics and machine learning algorithms is generally used to discover new particles and structures in jets.

Deep learning in color: towards automated quark/gluon jet discrimination

[Patrick T. Komiske](#), [Eric M. Metodiev](#) & [John Thaler](#)

Journal of High Energy Physics Regular Article - Theoretical Physics | [Open Access](#) | [Published: 18 October 2018](#)

1764 Accesses | 162 Citations

[A preprint version](#) of this article is available on arXiv.

ABSTRACT

Artificial intelligence offers the potential to revolutionize collider physics. To establish its utility, we apply convolutional neural networks designed by physicists. Our approach treats a jet image, with intensity given by the number of particles, by adding color to the images, where the color is the momentum in charged particle directions. We find that charged particle counts. Overall, we find that the neural networks are surprisingly good at discriminating observables. This suggests that deep learning may be useful to improve jet discrimination in imperfect simulations.

Regular Article - Theoretical Physics | [Open Access](#) | [Published: 18 October 2018](#)

Pulling out all the tops with computer vision and deep learning

[Sebastian Macaluso](#)

Journal of High Energy Physics Regular Article - Experimental Physics | [Open Access](#) | [Published: 02 May 2017](#)

888 Accesses | 77 Citations

[A preprint version](#) of this article is available on arXiv.

ABSTRACT

We apply computer vision techniques (CNN) — to build a hierarchical top tagger (of Kasieczka et al) has been shown to have performance to state-of-the-art. We introduce a number of novel image preprocessing, top tagging, and BDTs based on high-level features. We find a wide range of tagging strategies that outperform QCD background rejection. We find that (non-merged) top jets can be identified straightforwardly extended to other topologies here may be useful to

Regular Article - Experimental Physics | [Open Access](#) | [Published: 02 May 2017](#)

Deep-learning top taggers or the end of QCD?

[Gregor Kasieczka](#), [Tilman Plehn](#), [Michael Russell](#) & [Torben Schell](#)

Journal of High Energy Physics Regular Article - Experimental Physics | [Open Access](#) | [Published: 02 May 2017](#)

830 Accesses | 137 Citations

[A preprint version](#) of this article is available on arXiv.

ABSTRACT

Machine learning based top tagging from the LHC. Top tagging performance is compared to the DeepTop approach and a novel network architecture to improve top tagging performance. Model production channels are used to study multivariate QCD-based top tagging performance, establishing a baseline for multivariate hypothesis-

Neural Message Passing for Jet Physics

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[Submitted on 27 Jul 2017 (v1), last revised 23 Apr 2018 (this version, v3)]

Deep-learned Top Tagging with a Lorentz Layer

Anja Butter, Gregor Kasieczka, Tilman Plehn, Michael Russell

Regular Article - Theoretical Physics | [Open Access](#) | [Published: 15 January 2019](#)

Energy flow networks: deep sets for particle jets

[Patrick T. Komiske](#), [Eric M. Metodiev](#) & [Jesse Thaler](#)

Journal of High Energy

1334 Accesses | 101

[A preprint version](#)

ABSTRACT

A key question for machine learning is how to learn from collisions of particles, we build up or “point clouds”. Additionally, we introduce Energy Flow Networks. We also develop Particle Flow Networks. The inclusion of additional features, such as a per-particle energy flow, improves the overall event-level classification performance compared to existing event representations. We demonstrate the power of these networks on the collider task of dijet classification.

performance compared to existing methods. We also show how the learned event representation can be directly visualized, providing insight into the inner workings of the model. These architectures lend themselves to efficiently processing and analyzing events for a wide variety of tasks at the Large Hadron Collider. Implementations and examples of our architectures are available online in our [EnergyFlow](#) package.

Open Access

Acc

Equivariant energy flow networks for jet tagging

Matthew J. Dolan and Ayodele Ore

Phys. Rev. D **103**, 074022 – Published 27 April 2021

[Submitted on 8 Jun 2020]

Lorentz Group Equivariant Neural Network for Particle Physics

[Alexander Bogatskiy](#), [Brandon Anderson](#), [Jan T. Offermann](#), [Marwah Roussi](#), [David W. Miller](#), [Risi Kondor](#)

We present a neural network architecture that is fully equivariant with respect to transformations under the Lorentz group, a fundamental symmetry of space and time in physics. The architecture is based on the theory of the finite-dimensional representations of the Lorentz group and the equivariant nonlinearity involves the tensor product. For classification tasks in particle physics, we demonstrate that such an equivariant architecture leads to drastically simpler models that have relatively few learnable parameters and are much more physically interpretable than leading approaches that use CNNs and point cloud approaches. The competitive performance of the network is demonstrated on a public classification dataset [27] for tagging top quark decays given energy-momenta of jet constituents produced in proton-proton collisions.

[Submitted on 27 Jul 2017 (v1), last revised 23 Apr 2018 (this version, v3)]

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Journal of High Energy Physics

1334 Accesses | 101 Citations

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ABSTRACT

A key question for machine learning in particle physics is how to learn from collisions of particles, we build up representations for “point clouds”. Additionally, we introduce Energy Flow Networks (EFNs) and Particle Flow Networks (PFNs). We also develop Particle Flow Networks (PFNs) with the inclusion of additional information. Our models feature a per-particle representation and an overall event-level representation. We demonstrate the power of these models on the collider task of dijet tagging. Our performance compared to existing methods. We also show how the learned event representation can be directly visualized, providing insight into the inner working model. These architectures lend themselves to efficiently processing and analyzing a wide variety of tasks at the Large Hadron Collider. Implementations and example architectures are available online in our [EnergyFlow](#) package.

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[Submitted on 5 Jul 2021]

Particle Convolution for High Energy Physics

[Chase Shimmin](#)

We introduce the Particle Convolution layer, an equivariant neural network layer for particle physics. The particle convolution layer is implemented for various tasks, q/g tagging and more. Moreover, we show that our model significantly outperforms existing methods. We speculate that by generalizing convolutional symmetry to the current state-of-the-art, we can achieve better performance.

[Submitted on 20 Jan 2022 (v1), last revised 29 May 2022 (this version, v5)]

An Efficient Lorentz Equivariant Graph Neural Network for Jet Tagging

[Shiqi Gong](#), [Qi Meng](#), [Jue Zhang](#), [Huilin Qu](#), [Congqiao Li](#), [Sitian Qian](#), [Weitao Du](#), [Zhi-Ming Ma](#), [Tao Yan](#), [Liu](#)

Deep learning networks for particle physics can be an important tool in many applications. In many applications, spacetime symmetry is incorporated in the design is complex. High-order tensors

Particle Transformer for Jet Tagging

[Huilin Qu](#), [Congqiao Li](#), [Sitian Qian](#) *Proceedings of the 39th International Conference on Machine Learning*, PMLR 162:18281-18292, 2022.

Abstract

Jet tagging is a critical yet challenging classification task in particle physics. While deep learning has

Symmetries in jet data

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- Euclidean symmetries in jet images (CNN's)

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- Permutations of particles (particle clouds, Deep Sets, GNN, MPNN)

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- Euclidean symmetries in jet images (CNN's)
- Permutations of particles (particle clouds, Deep Sets, GNN, MPNN)
- Rotational, boost, Lorentz symmetries

What constraints does full
Lorentz+Permutation symmetry impose?

Invariants of the Lorentz group

6. Second example: unimodular group $SL(n)$

We shall take up the question of *invariants of the unimodular group $SL(n)$* at once for any number of covariant or Latin vectors x, y, \dots and any number of contravariant or Greek vectors ξ, η, \dots .

THEOREM (2.6.A).

$$[xy \dots z], \quad (\xi x), \quad [\xi \eta \dots \zeta]$$

is a complete table of typical basic invariants for the unimodular group.

[H. Weyl, The Classical Groups, 1939]

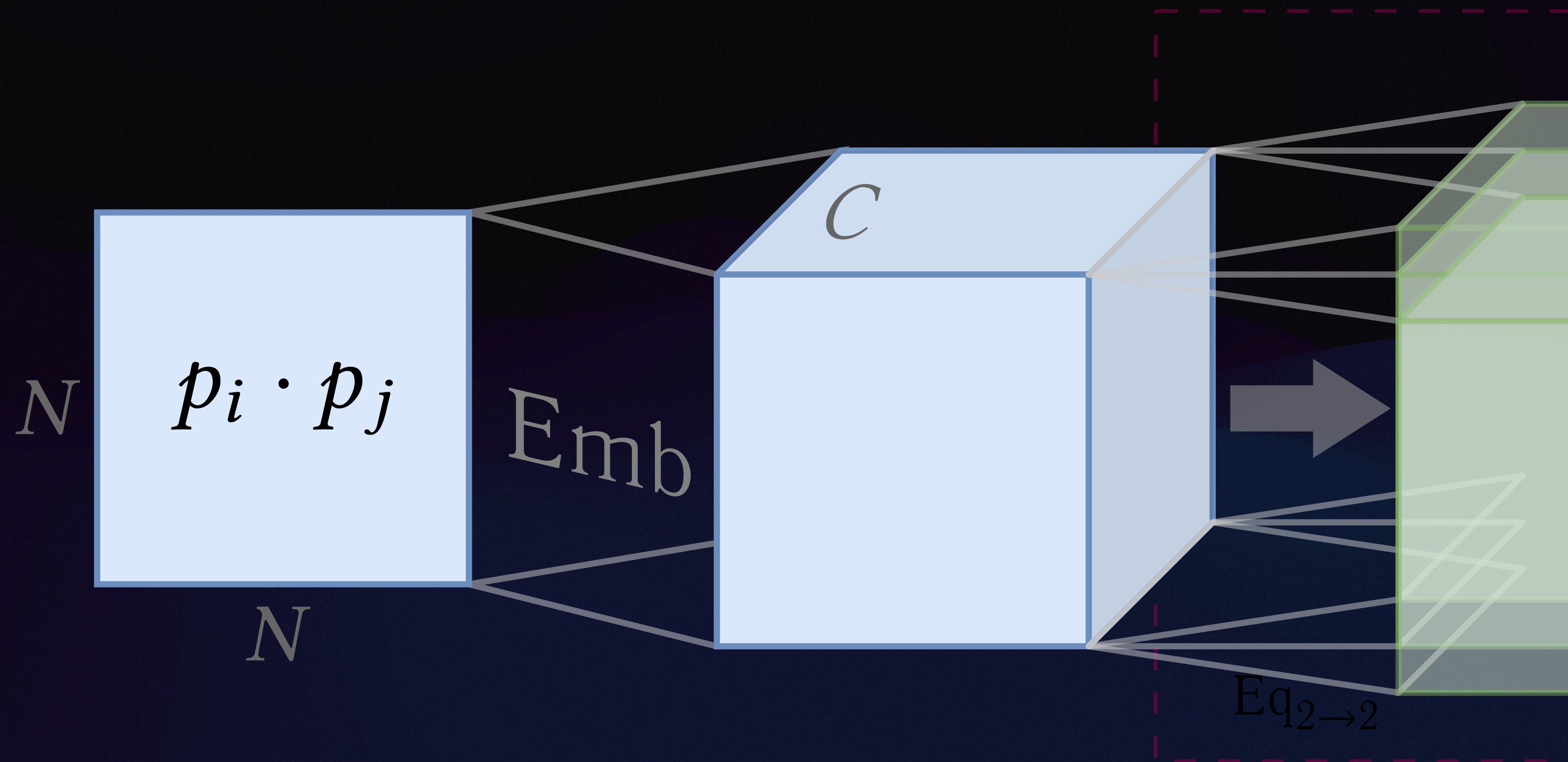
Invariants of the Lorentz group

All Lorentz-invariant symmetric functions of a set of 4-vectors p_i^μ can be expressed as functions of only the dot products $d_{ij} = \eta_{\mu\nu} p_i^\mu p_j^\nu$

$$I(p_1, \dots, p_N) = I\left(\{p_i \cdot p_j\}_{i,j}\right)$$

$$p_i \cdot p_j = p_i^0 p_j^0 - \vec{p}_i \cdot \vec{p}_j$$

Input to the network: matrix of dot products $d_{ij} = p_i \cdot p_j$



Permutation equivariant architecture
with only edge data?

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Permutation Equivariant, Lorentz
Invariant/Covariant Aggregator Net

Equivariant Aggregators



Equivariant Aggregators

- Basic approach — Deep Sets $\rho \left(\sum_i \phi(x_i) \right)$

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- Common approach — GNN/MPNN: $v'_i = f(v_i, \sum_j m_{ij}(v_i, v_j))$
- For pure “edge data”, look for the most general permutation-equivariant mapping

$$T_{ij} \mapsto T'_{ij}$$

$$F \left(\pi \circ T_{i_1 i_2 \dots i_r} \right) = \pi \circ F \left(T_{i_1 i_2 \dots i_r} \right), \quad \pi \in S_N.$$

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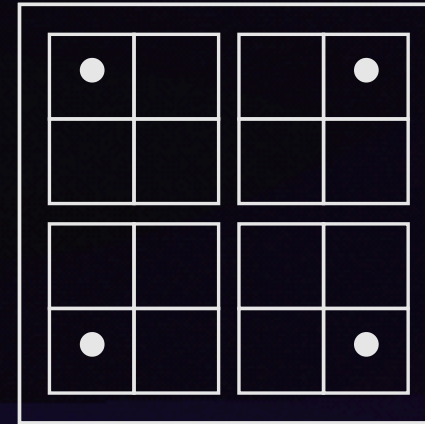
- The space of such linear mappings between $N \times N$ matrices is 15-dimensional

Equivariant Aggregators

Let's illustrate equivariant mappings $T_{ij} \mapsto T'_{ij}$ as rank 4 binary tensors ($N = 2$)

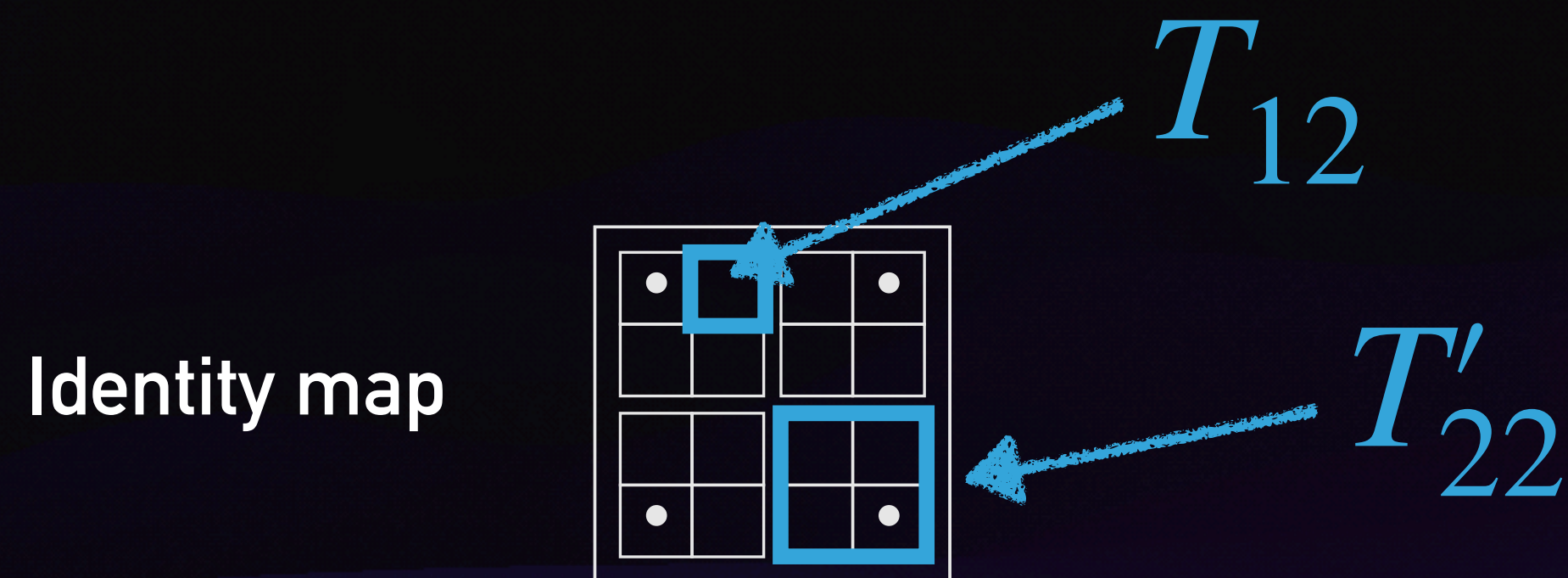
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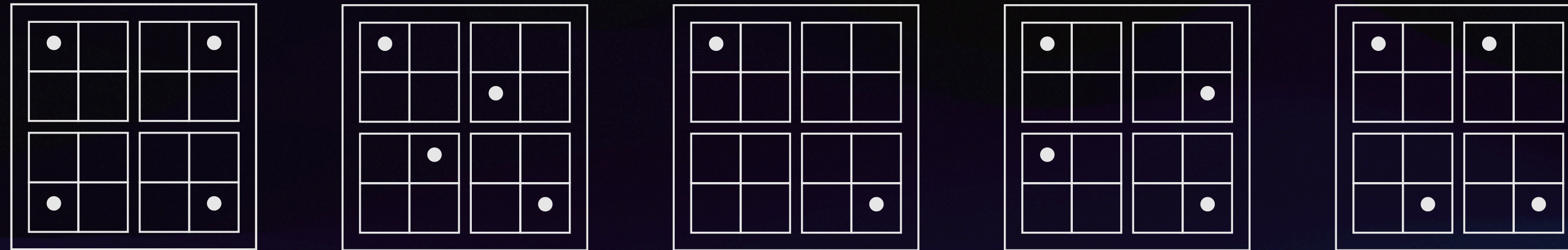
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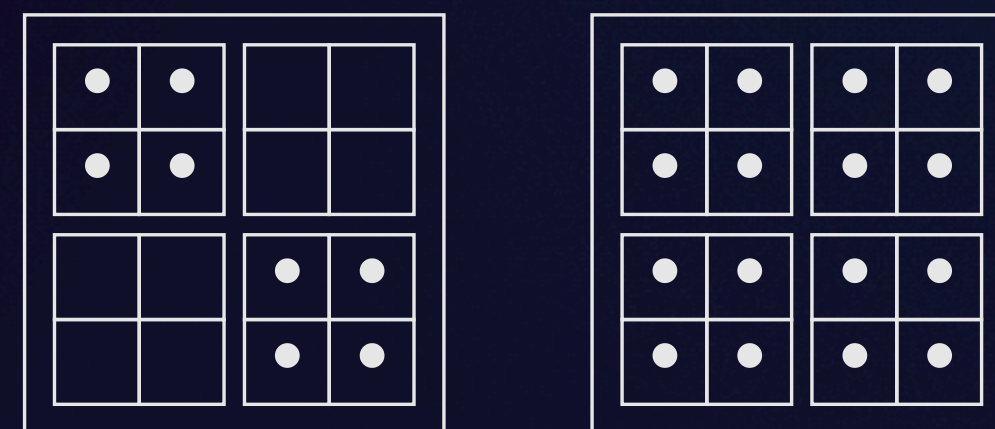
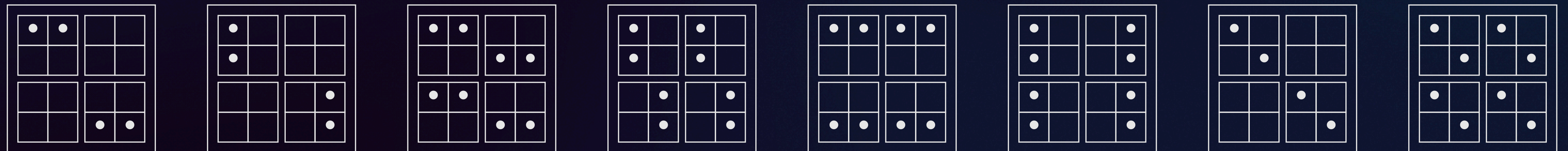
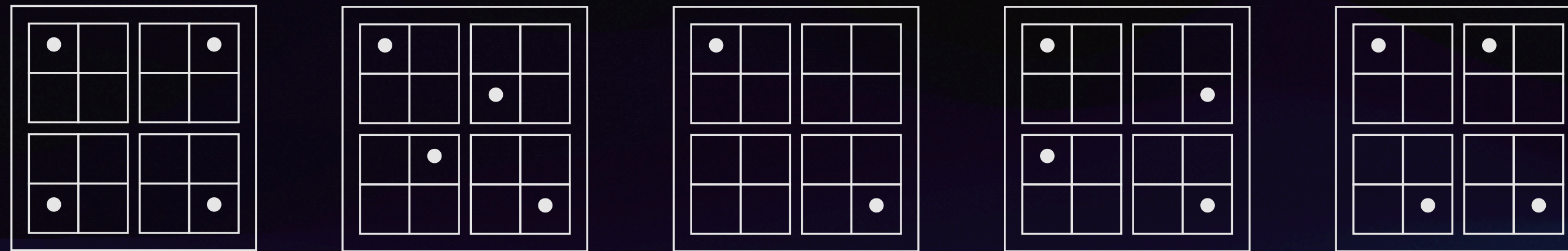
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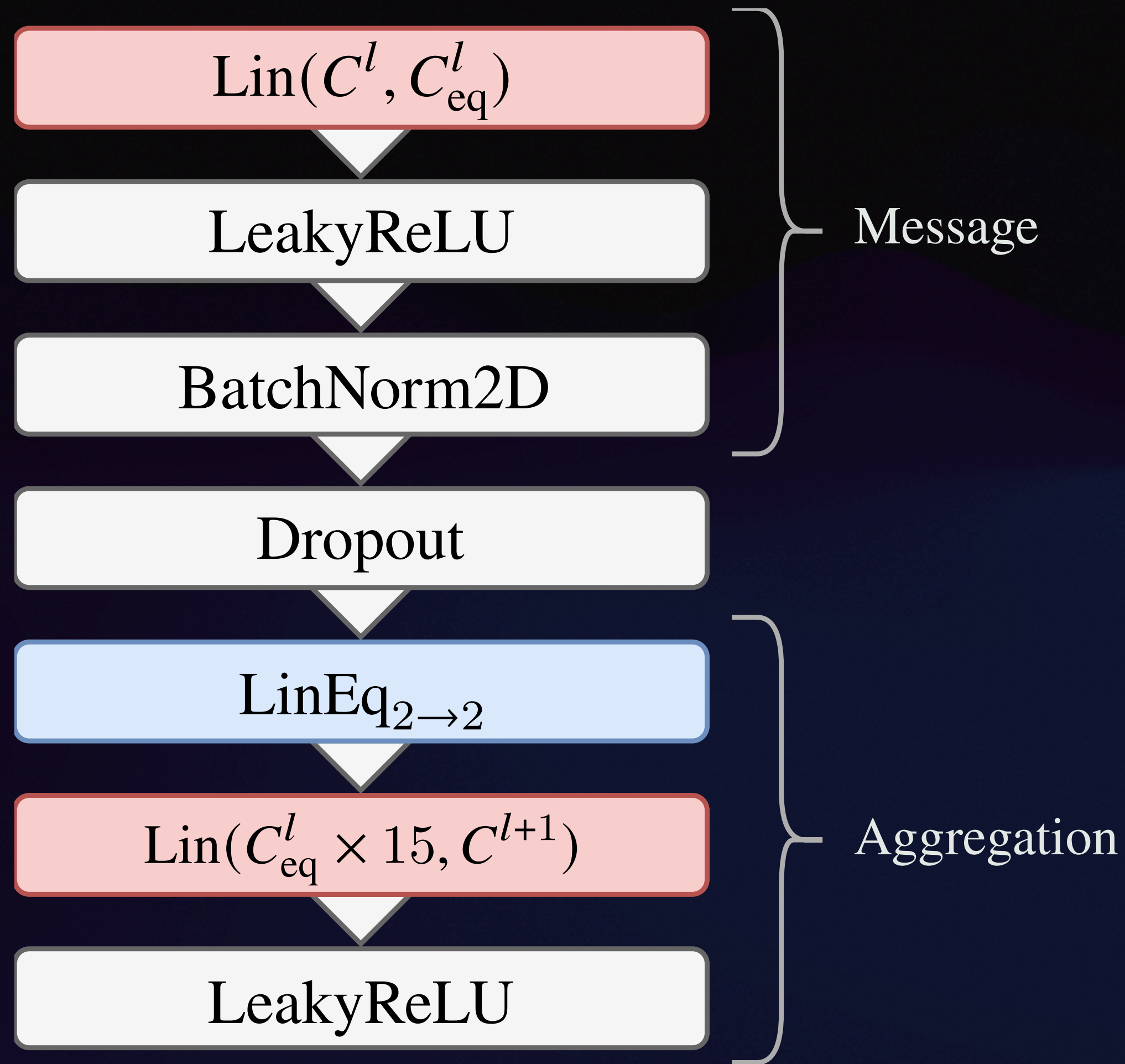


Equivariant Aggregators

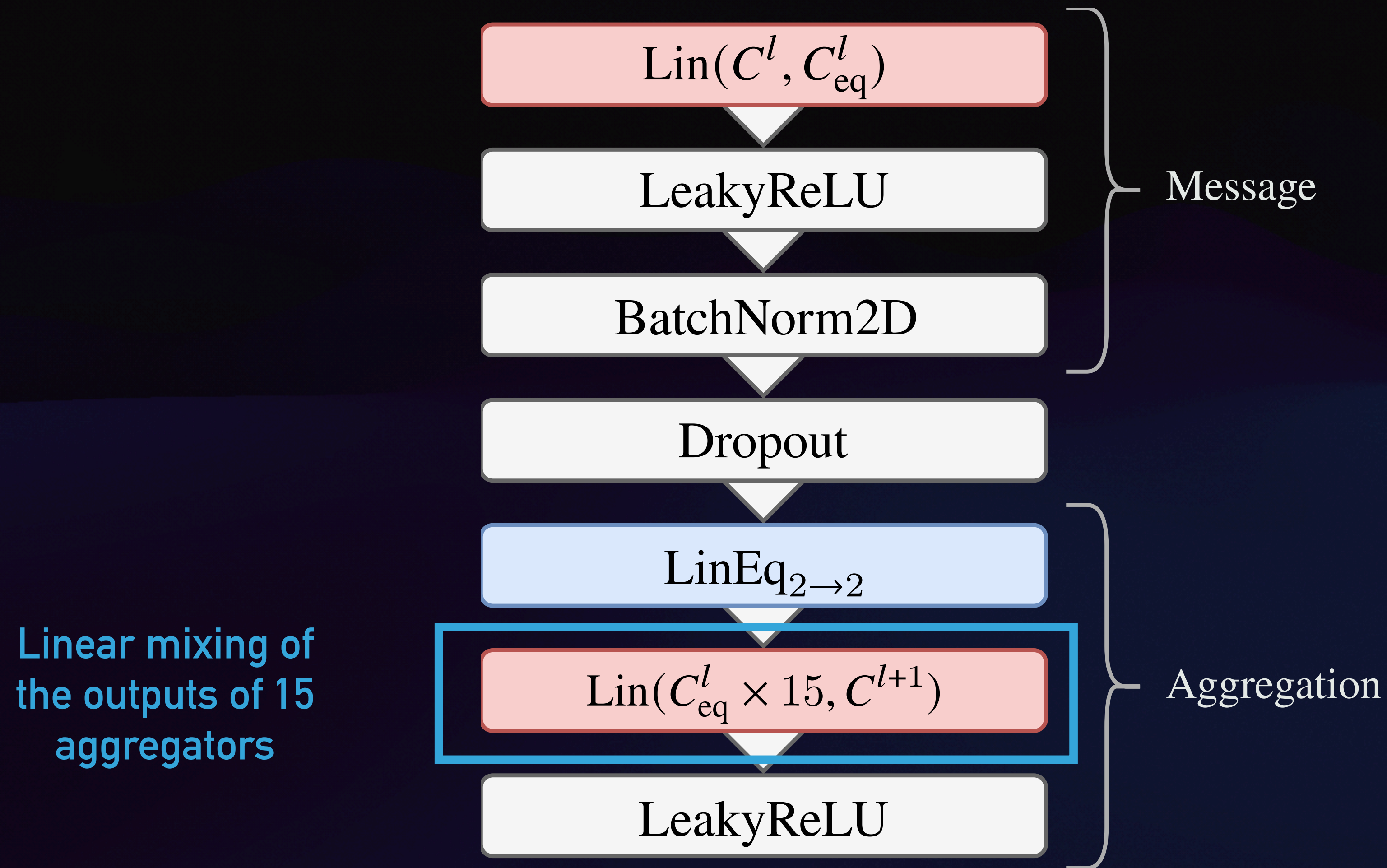
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PELICAN equivariant block



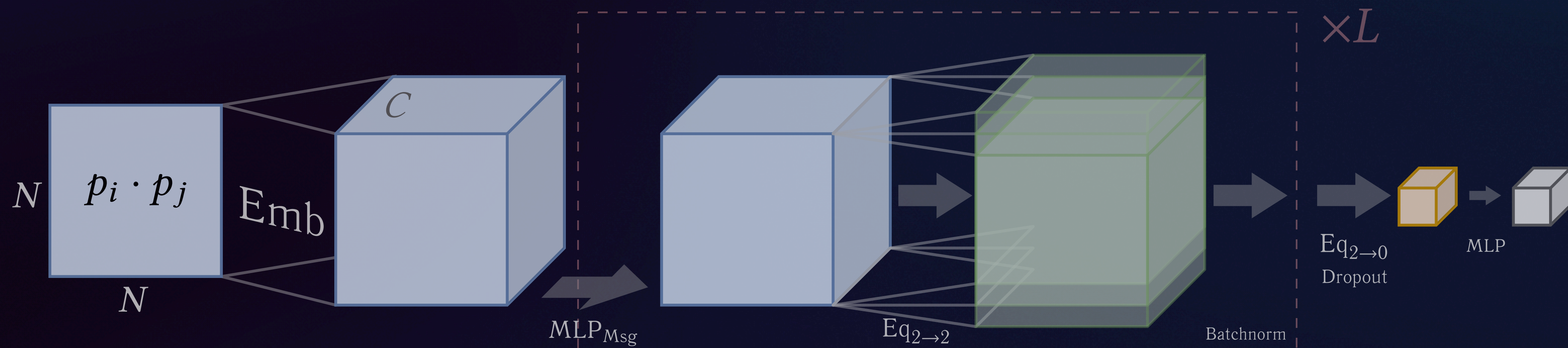
PELICAN equivariant block



PELICAN Classifier

Classifier

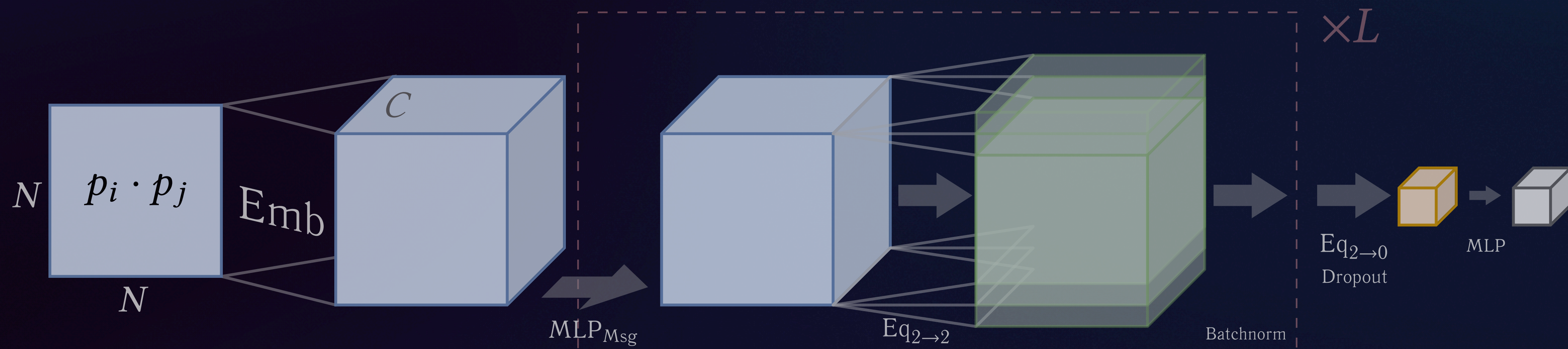
$$p_i \rightarrow d_{ij} \rightarrow \text{Embed} \rightarrow [\text{Eq}_{2 \rightarrow 2}]^L \rightarrow \text{Eq}_{2 \rightarrow 0} \rightarrow \text{MLP}$$



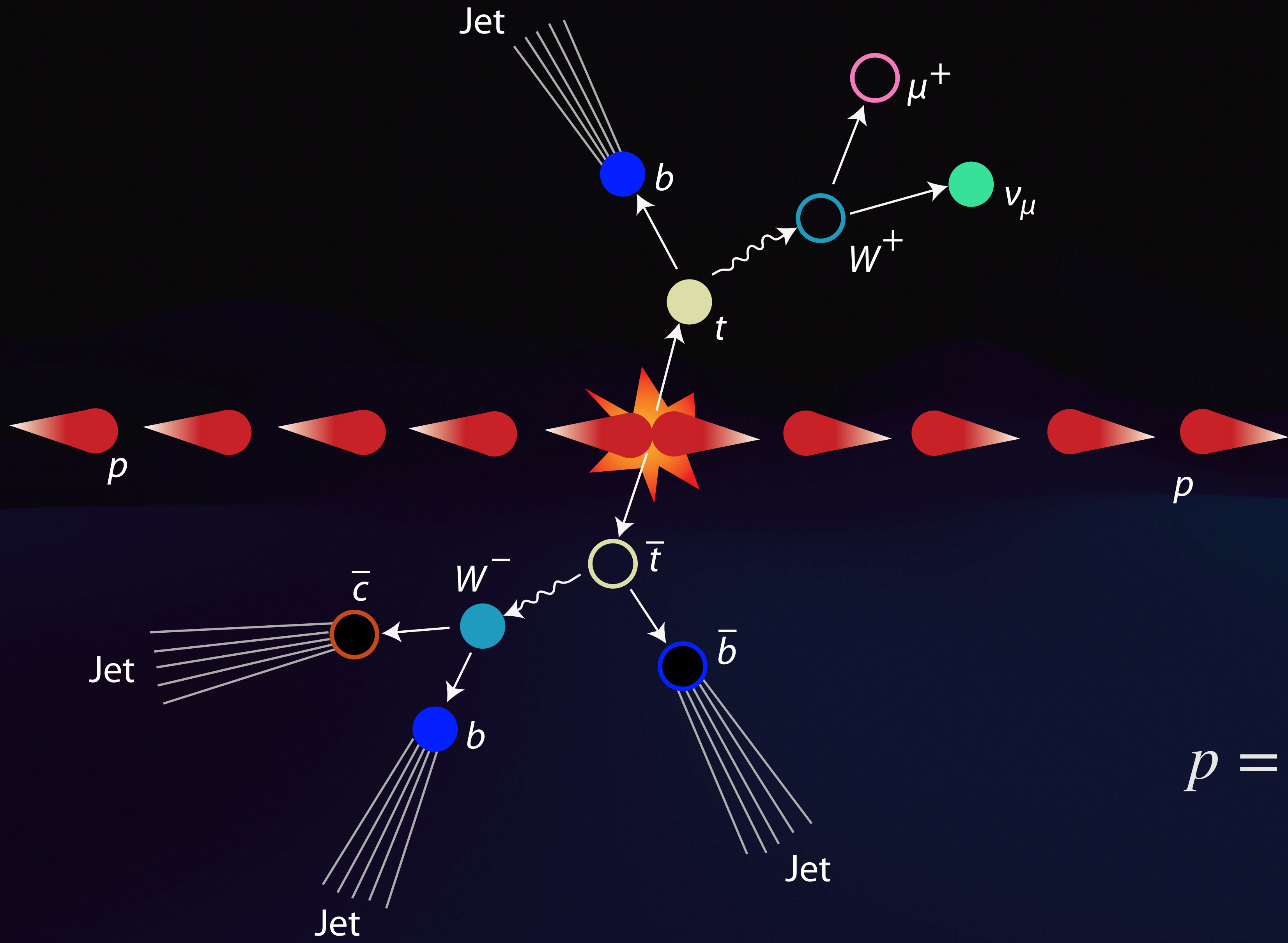
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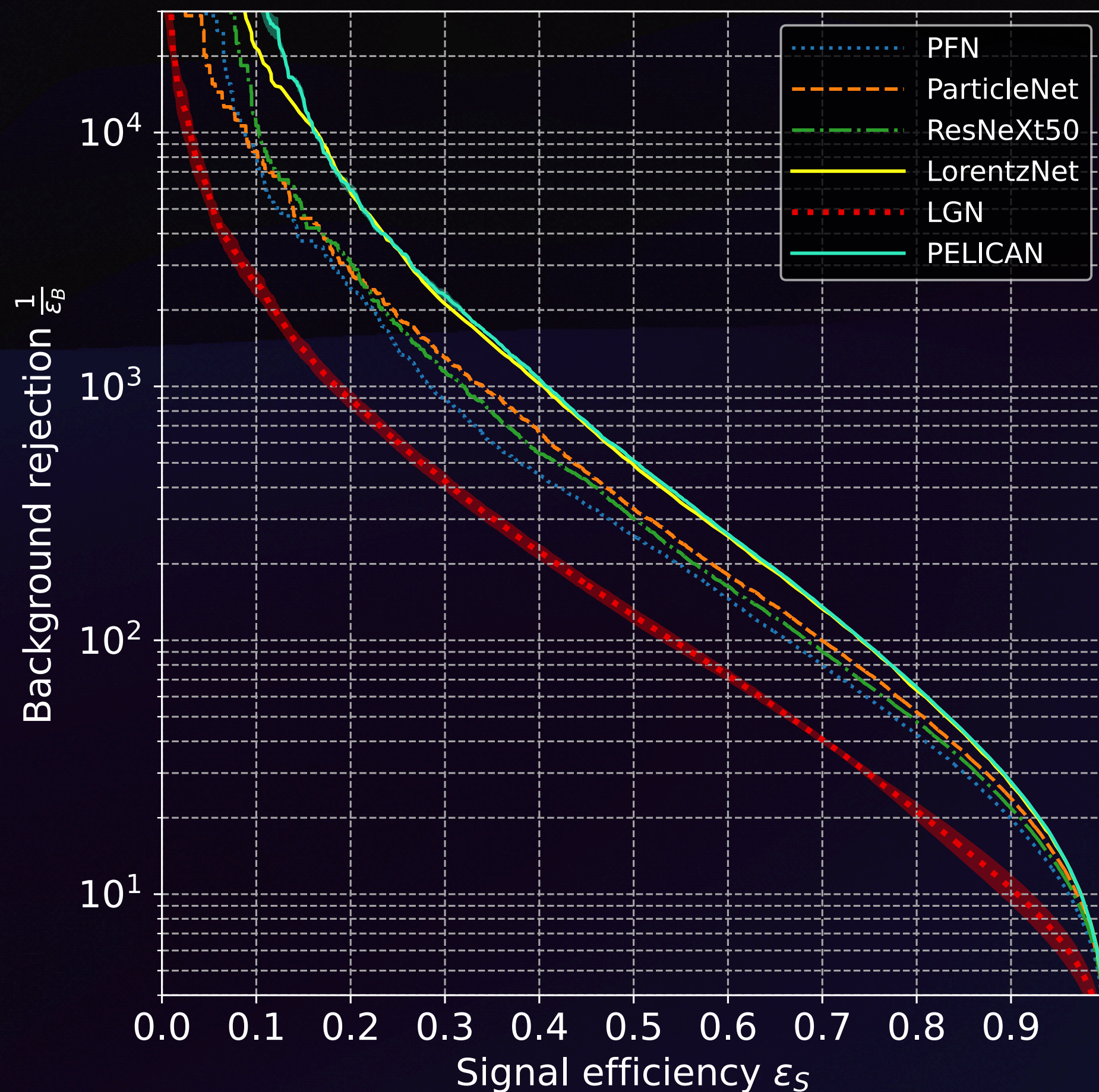
Top-tagging with PELICAN



$$p = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

Top-tagging performance

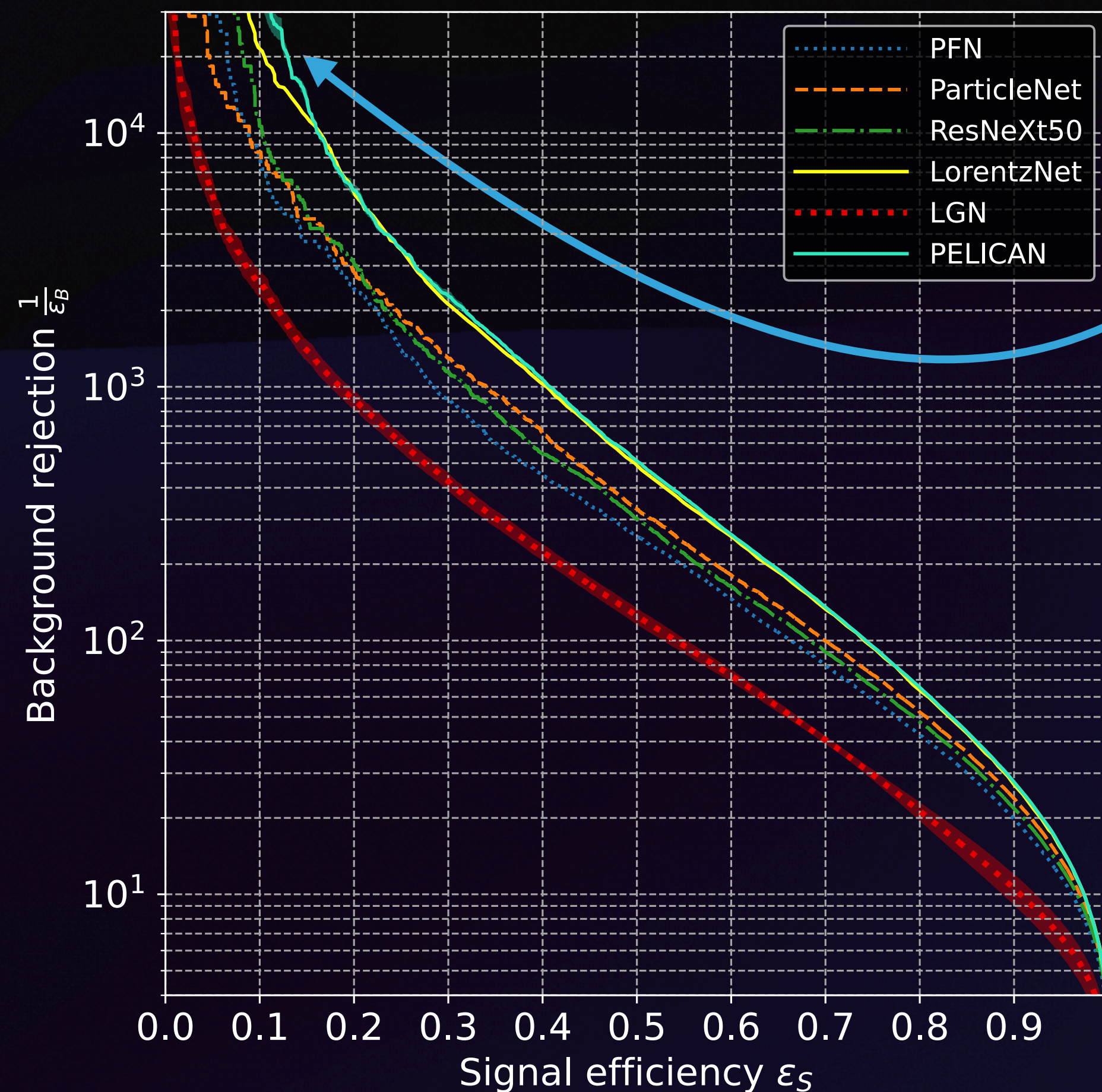
- State of the art top-tagger with 8x fewer params of the previous best tagger
- Exact invariance massively improves sample efficiency



| Architecture | Accuracy | AUC | $1/\epsilon_B$ | # Params |
|-----------------|------------------|------------------|----------------------------------|------------|
| LGN | 0.929(1) | 0.964(14) | 424 ± 82 | 4.5k |
| PFN | 0.932 | 0.982 | 891 ± 18 | 82k |
| ResNeXt | 0.936 | 0.984 | 1122 ± 47 | 1.46M |
| ParticleNet | 0.938 | 0.985 | 1298 ± 46 | 498k |
| LorentzNet | 0.942 | 0.9868 | 2195 ± 173 | 220k |
| Our work | 0.9425(1) | 0.9869(1) | 2289 ± 204 | 46k |

Top-tagging performance

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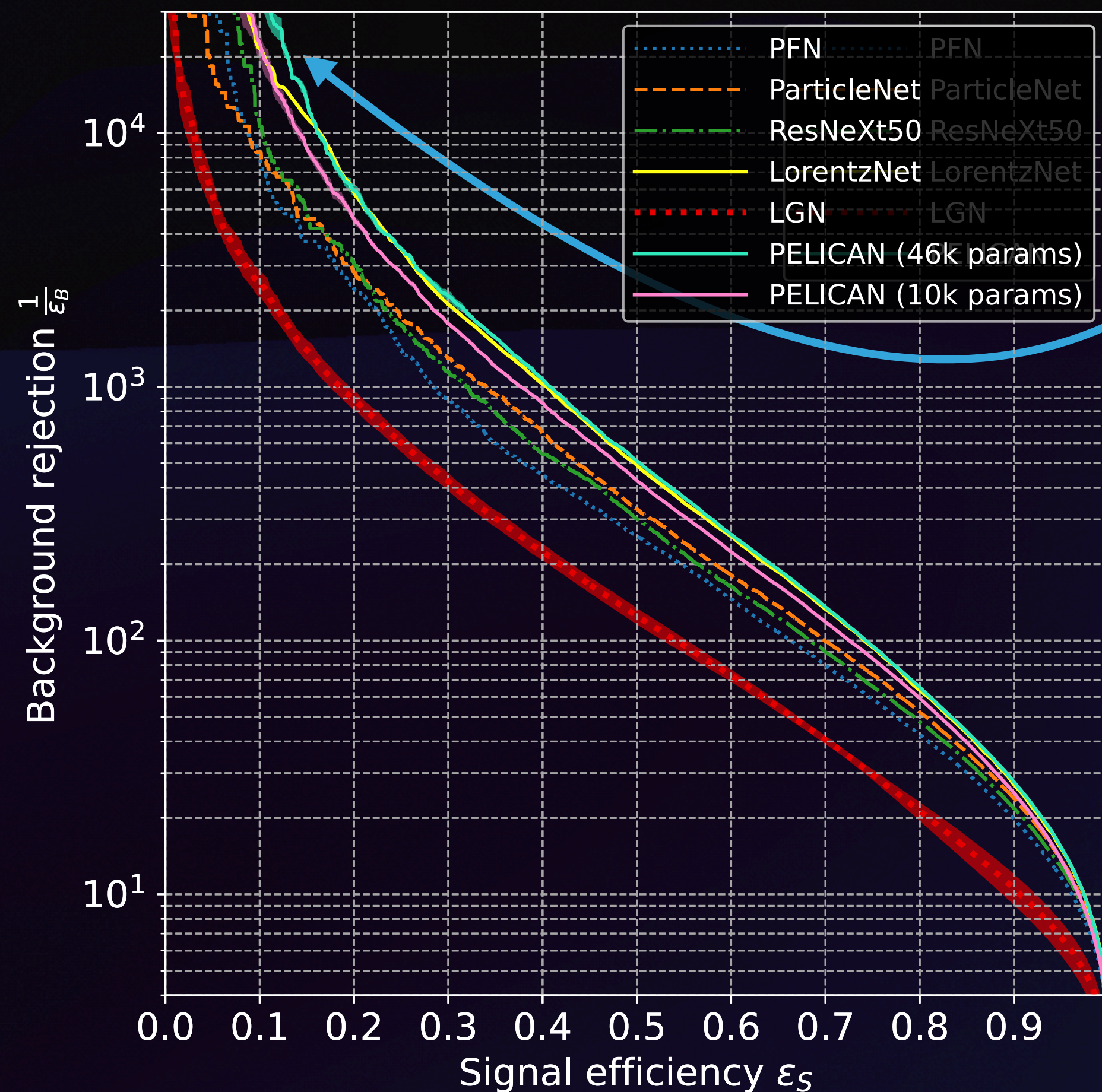


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Regression with PELICAN

Lorentz equivariance

All Lorentz-equivariant mappings $F^\mu(p_1, \dots, p_N)$ have the form

$$F(p_1, \dots, p_N) = \sum_i c_i p_i,$$

where $c_i(p_1, \dots, p_N)$ are Lorentz-invariant, i.e. only functions of d_{ij} .

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What about permutation invariance?

Lorentz-equivariant

$$F^\mu(p_1, \dots, p_N)$$

Lorentz-equivariant

$$F^\mu(p_1, \dots, p_N)$$

Permutation-equivariant

$$c_i(\{d_{jk}\})$$

Lorentz-equivariant

$$F^\mu(p_1, \dots, p_N)$$



Permutation-equivariant

$$c_i \left(\{d_{jk}\} \right)$$

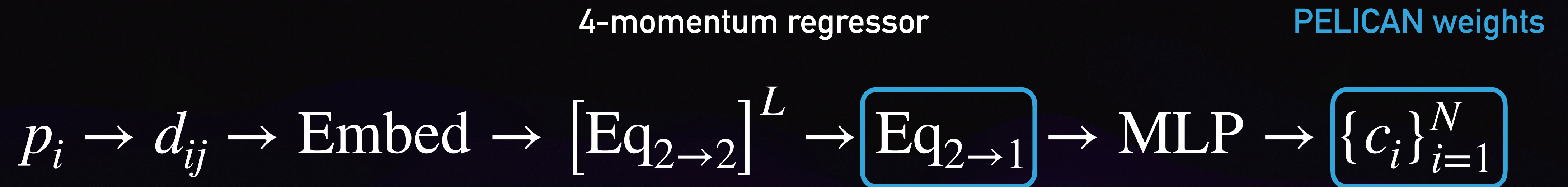
$$F^\mu(p_1, \dots, p_N) = \sum_i c_i p_i^\mu$$

PELICAN 4-momentum Regressor

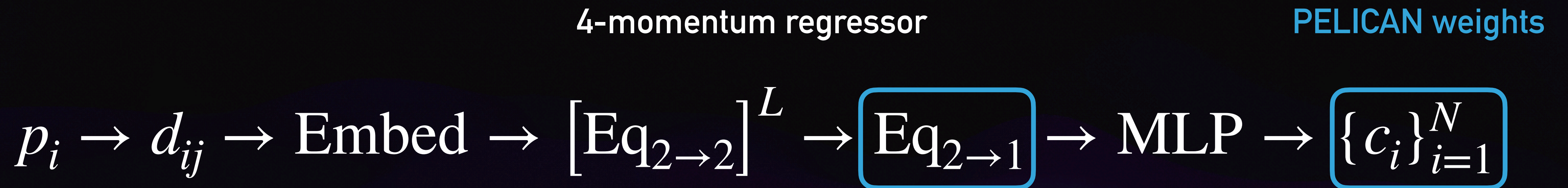
4-momentum regressor

$$p_i \rightarrow d_{ij} \rightarrow \text{Embed} \rightarrow [\text{Eq}_{2 \rightarrow 2}]^L \rightarrow \boxed{\text{Eq}_{2 \rightarrow 1}} \rightarrow \text{MLP} \rightarrow \{c_i\}_{i=1}^N$$

PELICAN 4-momentum Regressor



PELICAN 4-momentum Regressor

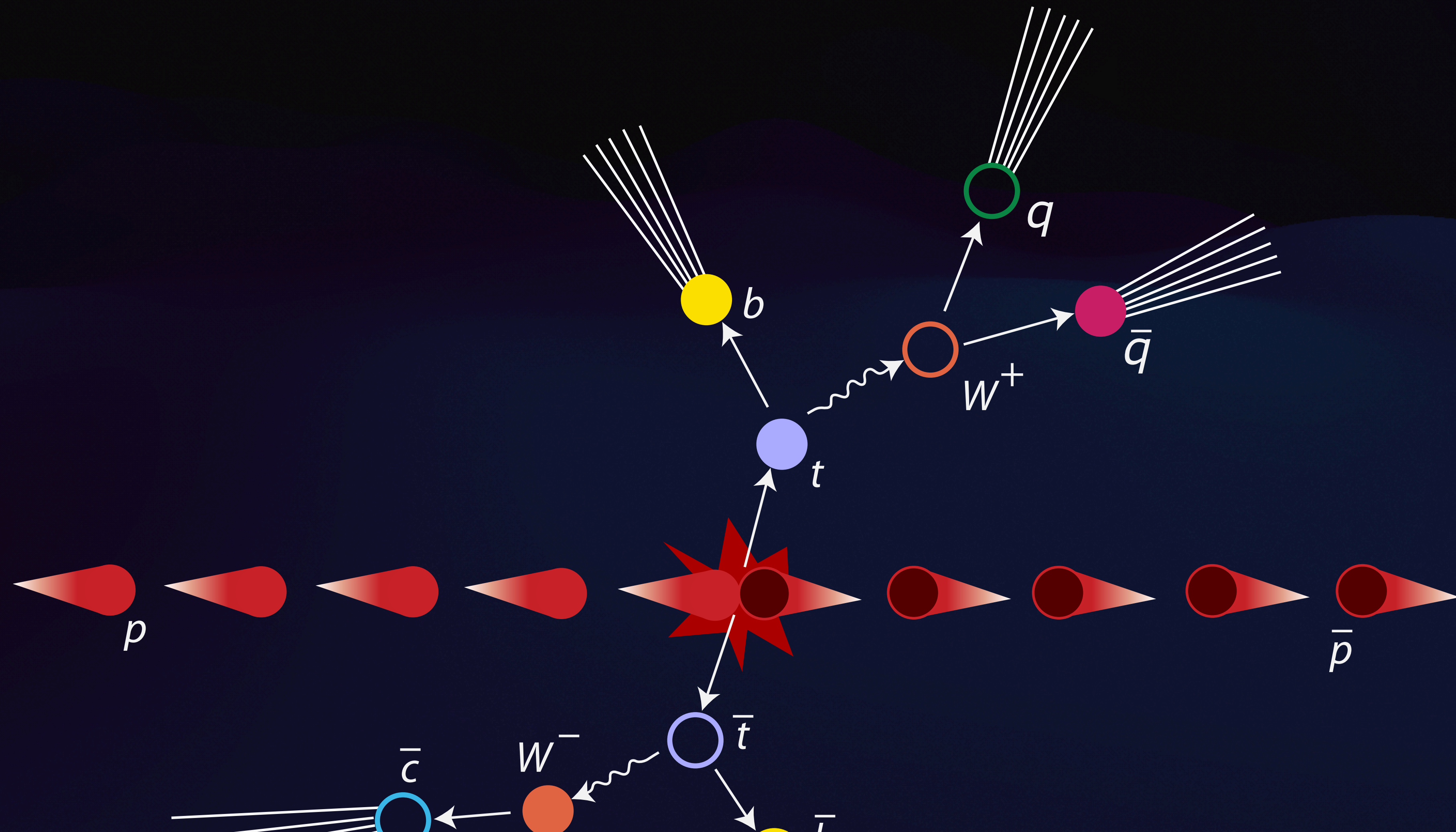


The 4-momentum is reconstructed as $p_{\text{predict}} = \sum_{i=1}^N c_i p_i$

The loss function is a linear combination of $|m_{\text{predict}} - m_{\text{target}}|$ and $|\vec{p}_{\text{predict}} - \vec{p}_{\text{target}}|$

Hadronic W reconstruction inside of a top jet

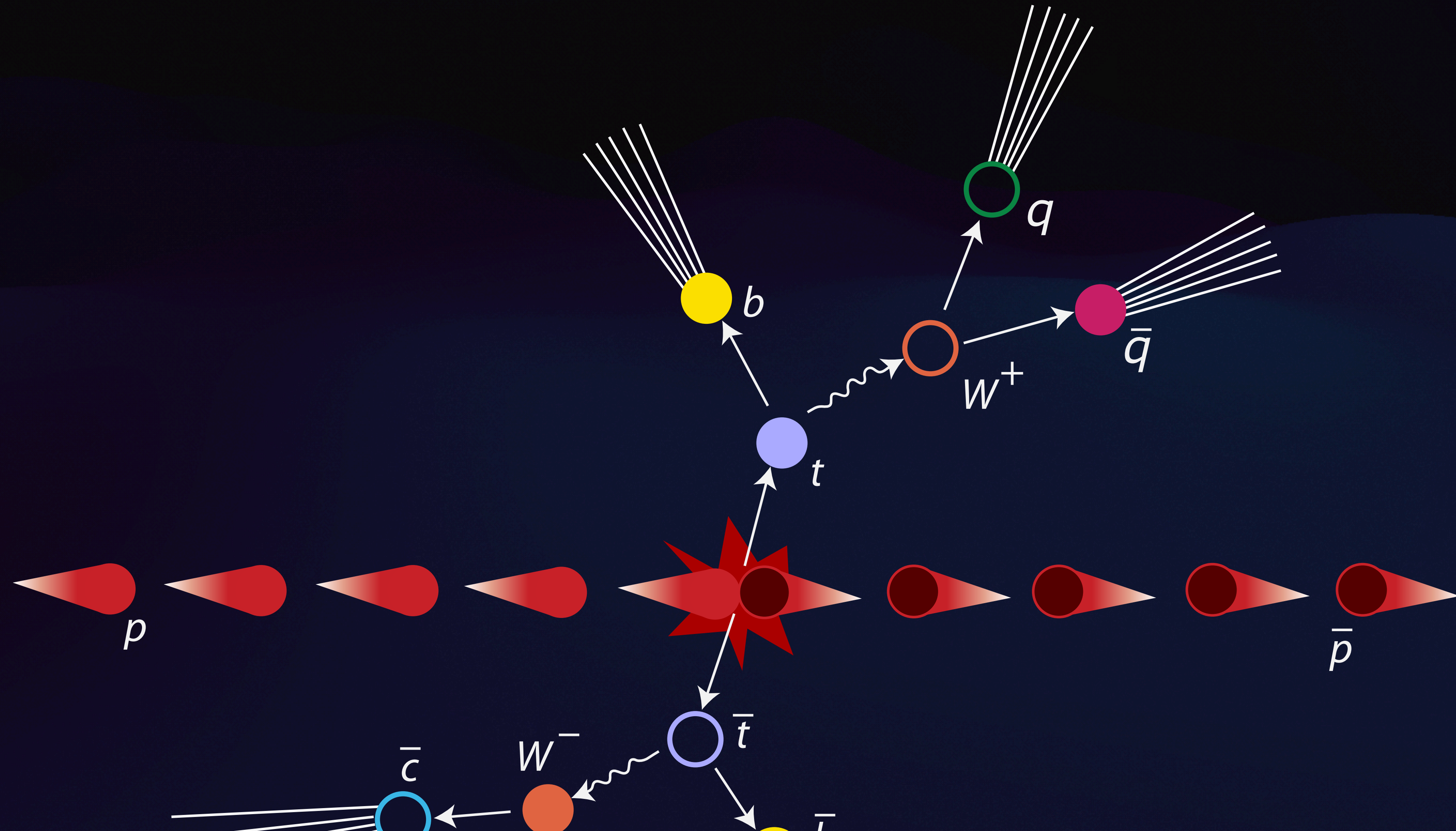
Dataset similar to the top-tagging one:



Hadronic W reconstruction inside of a top jet

Dataset similar to the top-tagging one:

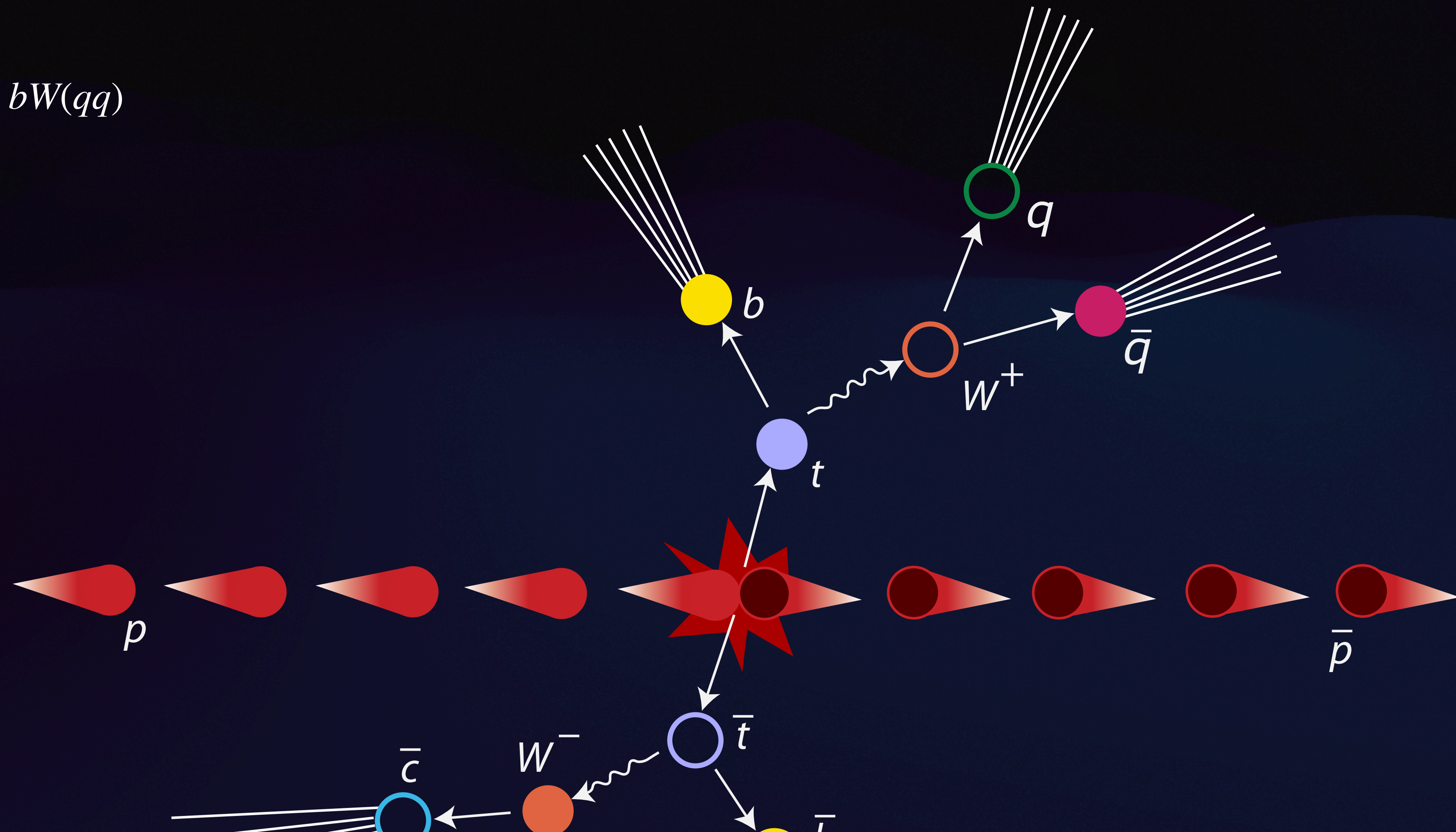
1. Only top jets



Hadronic W reconstruction inside of a top jet

Dataset similar to the top-tagging one:

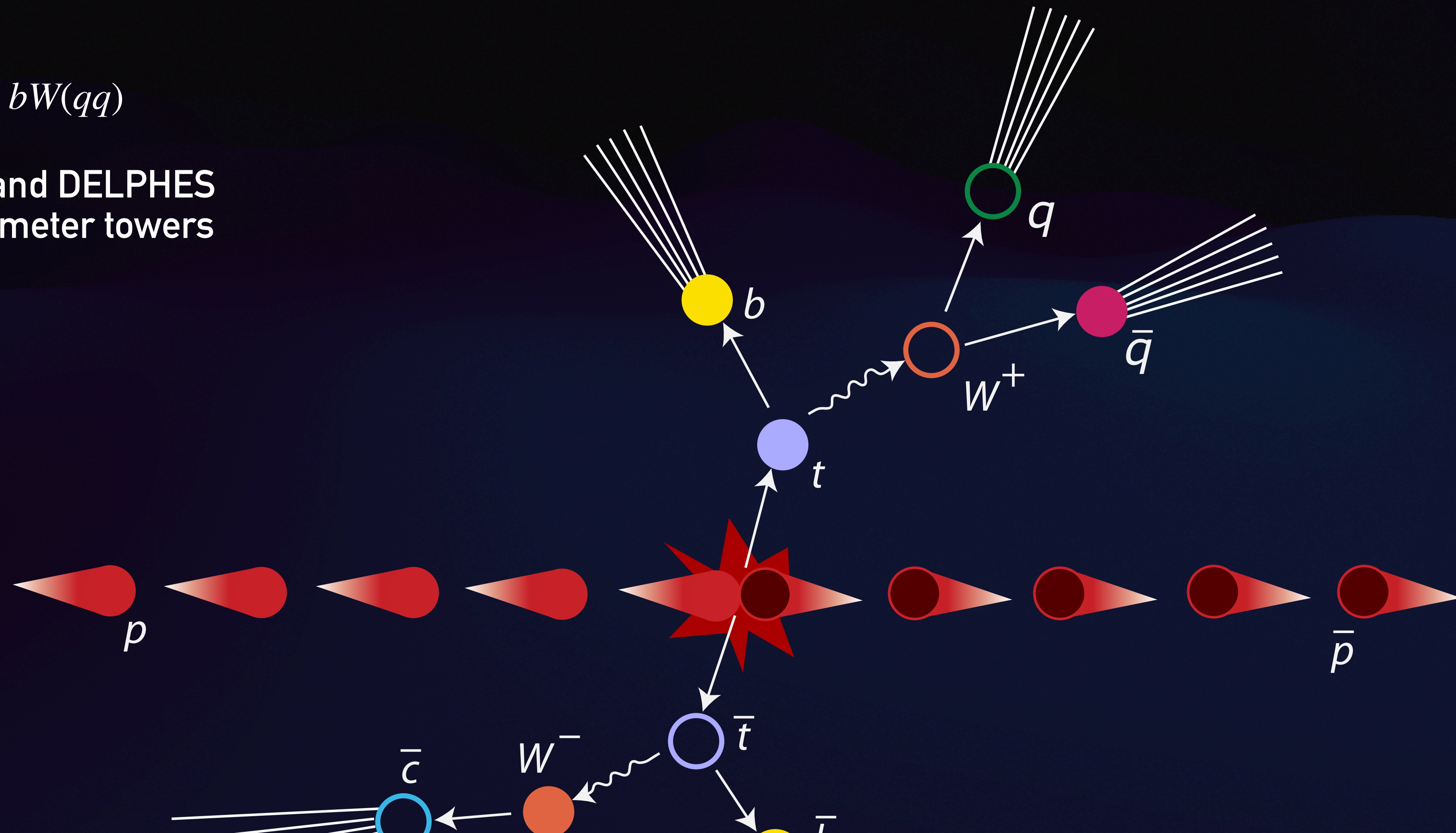
1. Only top jets
2. Only hadronic decays $t \rightarrow bW(qq)$



Hadronic W reconstruction inside of a top jet

Dataset similar to the top-tagging one:

1. Only top jets
2. Only hadronic decays $t \rightarrow bW(qq)$
3. Two versions: truth level and DELPHES reconstructed from calorimeter towers



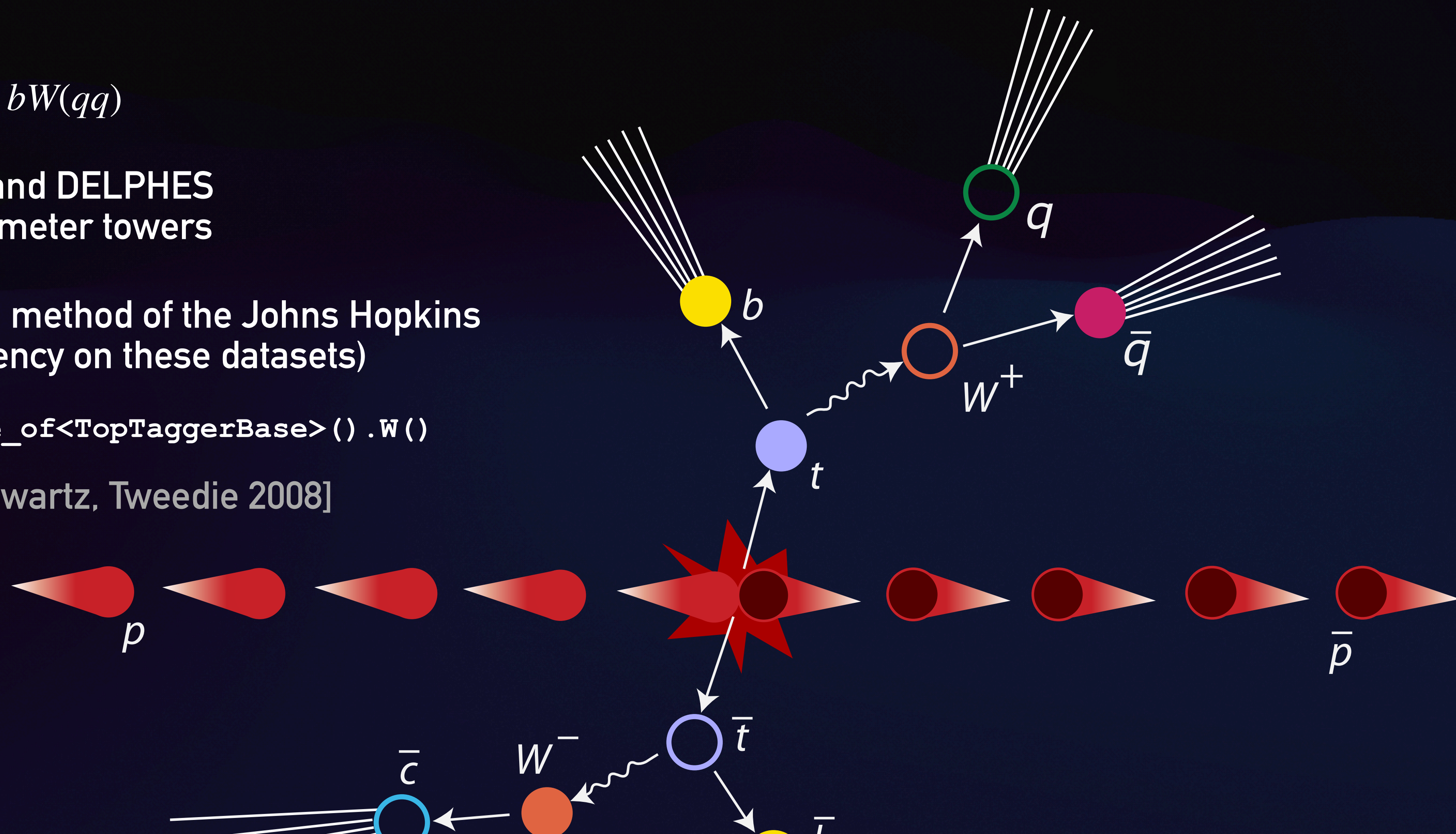
Hadronic W reconstruction inside of a top jet

Dataset similar to the top-tagging one:

1. Only top jets
2. Only hadronic decays $t \rightarrow bW(qq)$
3. Two versions: truth level and DELPHES reconstructed from calorimeter towers
4. Non-ML baseline: built-in method of the Johns Hopkins top-tagger (37%/31% efficiency on these datasets)

```
top_candidate.structure_of<TopTaggerBase>().W()
```

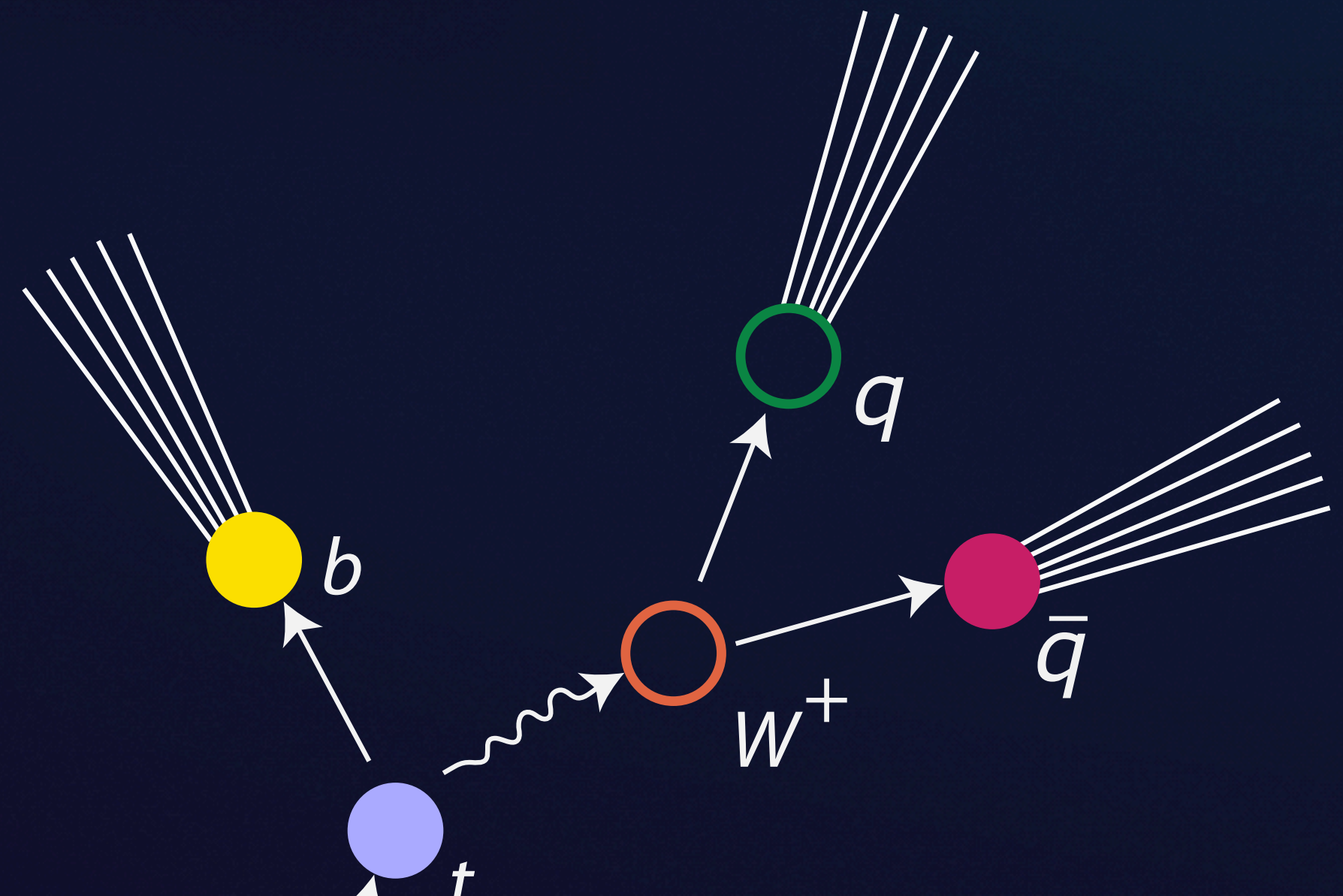
[Kaplan, Rehermann, Schwartz, Tweedie 2008]



Hadronic W reconstruction inside of a top jet

JH = Johns Hopkins Top Tagger
[Kaplan, Rehermann, Schwartz, Tweedie 2008]

PELICAN | JH = PELICAN on JH-tagged events

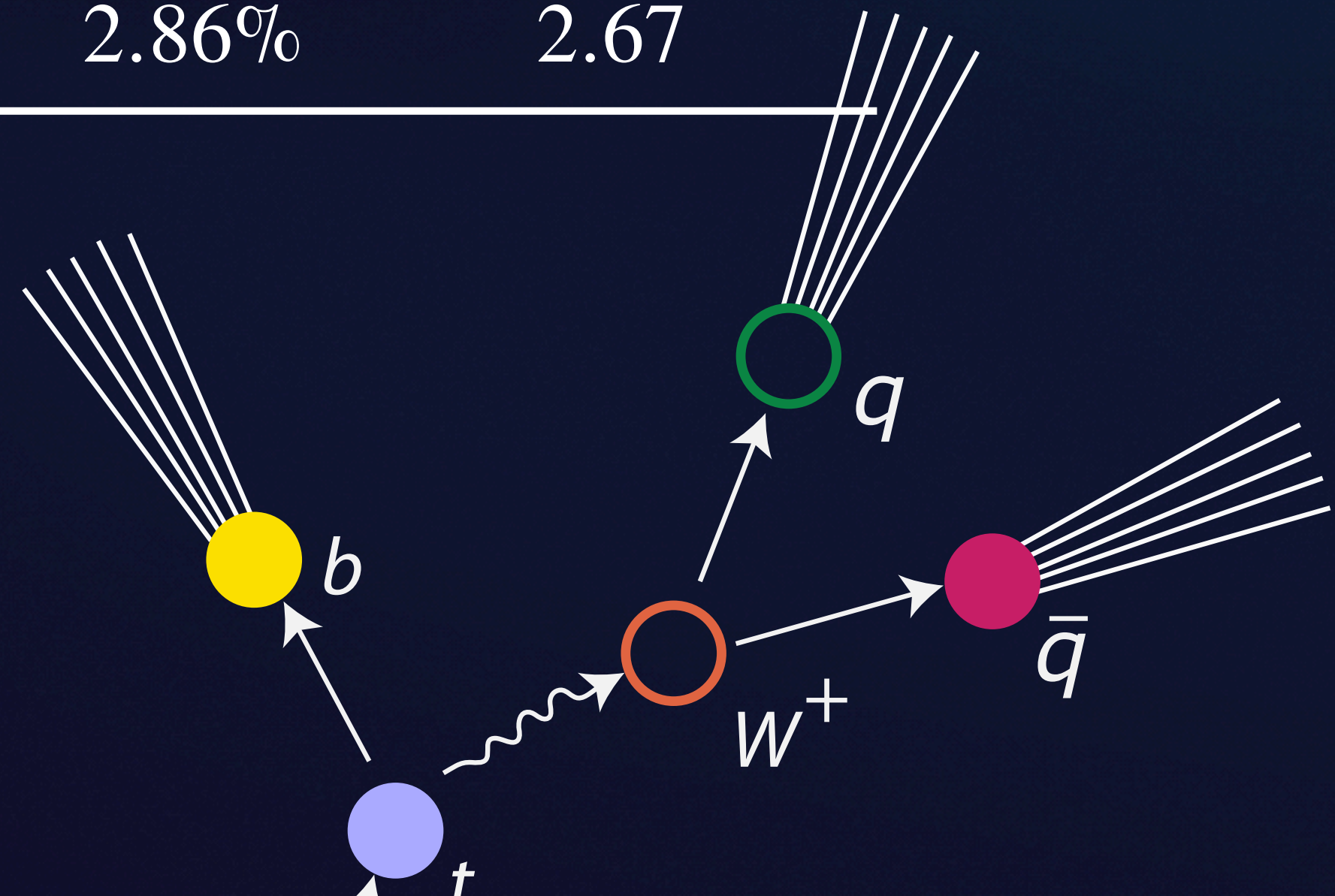


Hadronic W reconstruction inside of a top jet

| | Method | σ_{p_T} (%) | σ_m (%) | σ_ψ (centirad) |
|--------------------|------------|--------------------|----------------|--------------------------|
| Without DELPHES | JH | 0.65% | 1.27% | 0.156 |
| | PELICAN | 0.82% | 1.20% | 0.384 |
| | PELICAN JH | 0.27% | 0.59% | 0.088 |
| With DELPHES | JH | 10.3 % | 8.3 % | 8.73 |
| | PELICAN | 5.51% | 3.22% | 4.16 |
| | PELICAN JH | 3.81% | 2.86% | 2.67 |

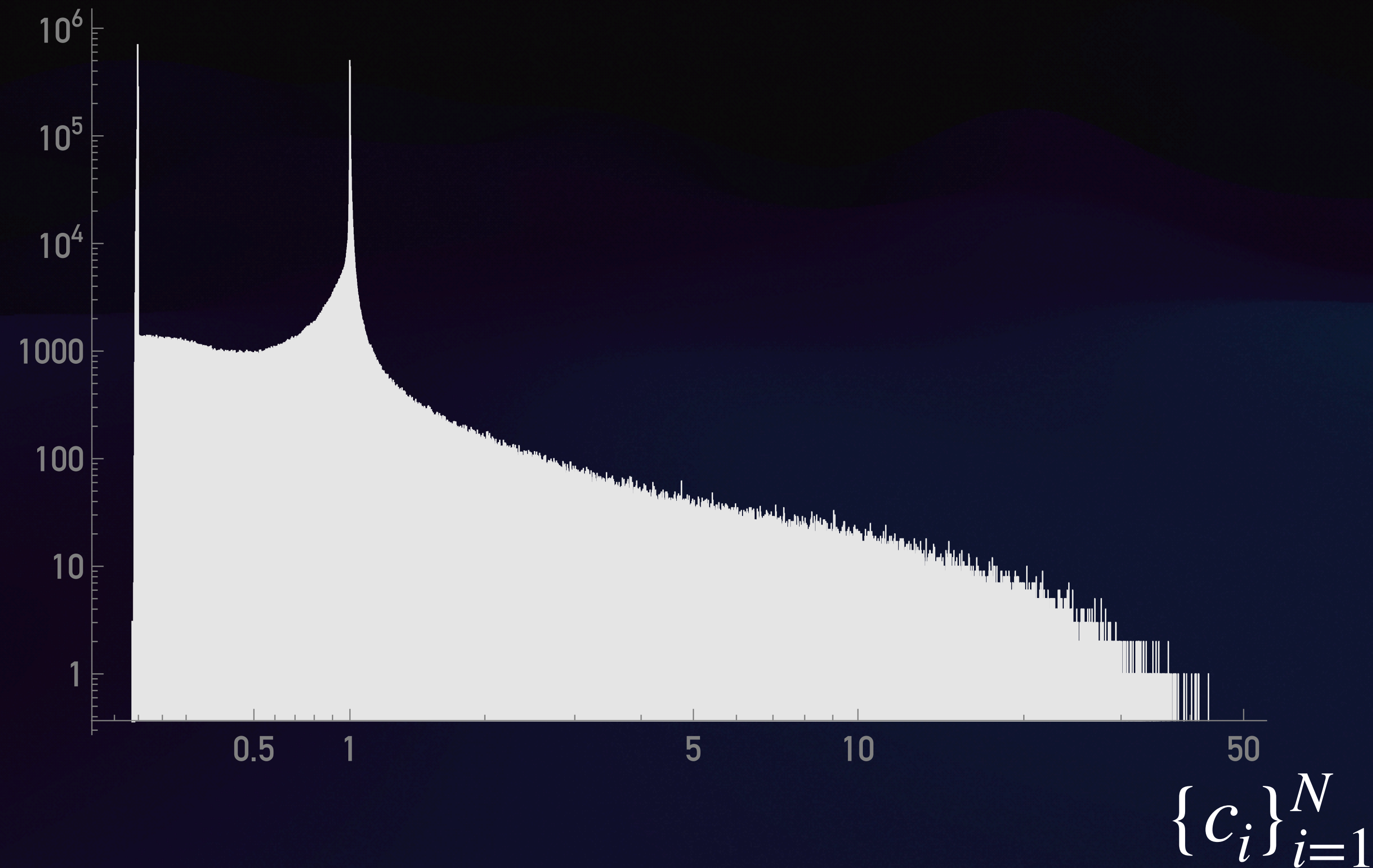
JH = Johns Hopkins Top Tagger
[Kaplan, Rehermann, Schwartz, Tweedie 2008]

PELICAN | JH = PELICAN on JH-tagged events

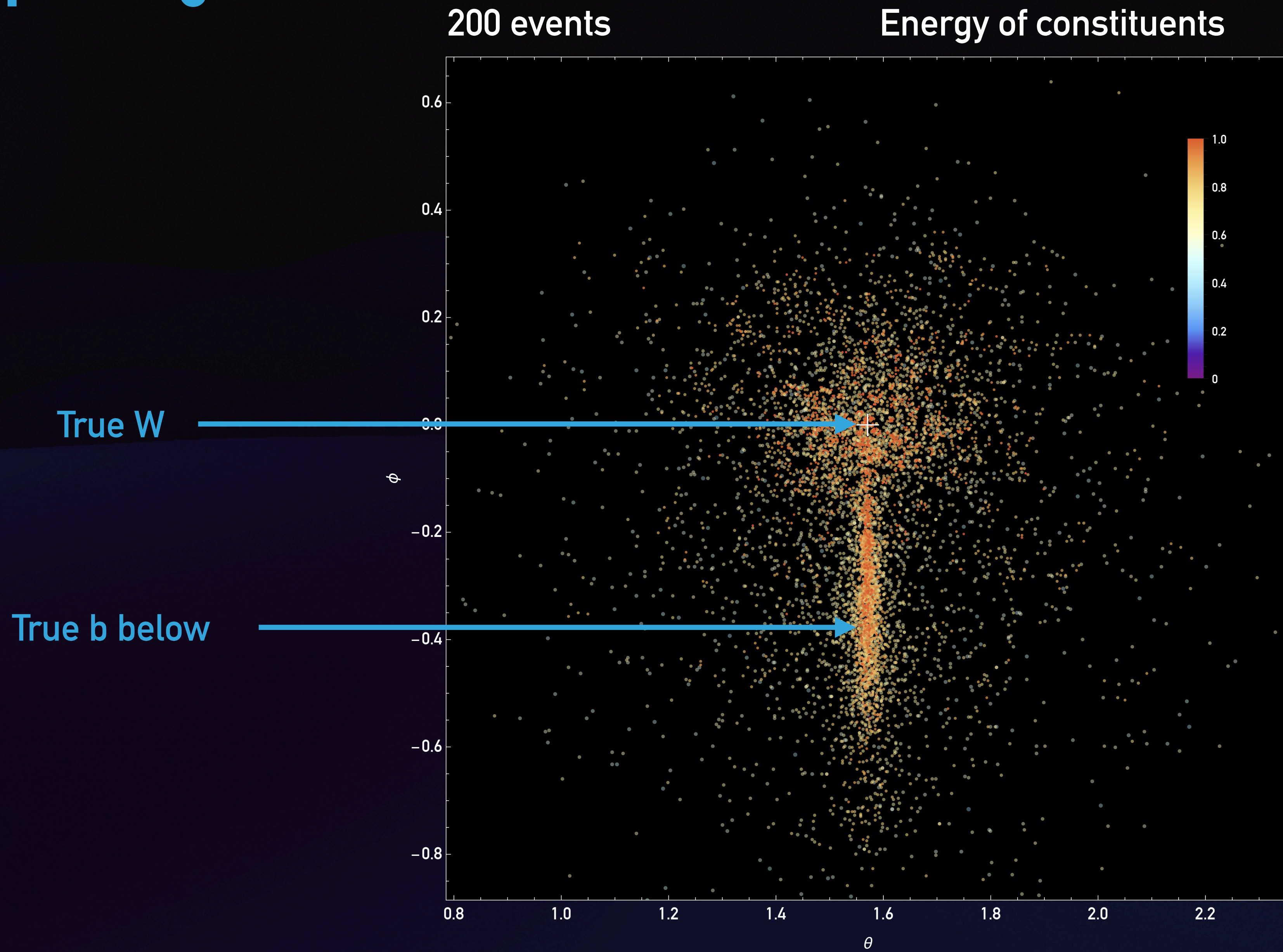


Explaining PELICAN

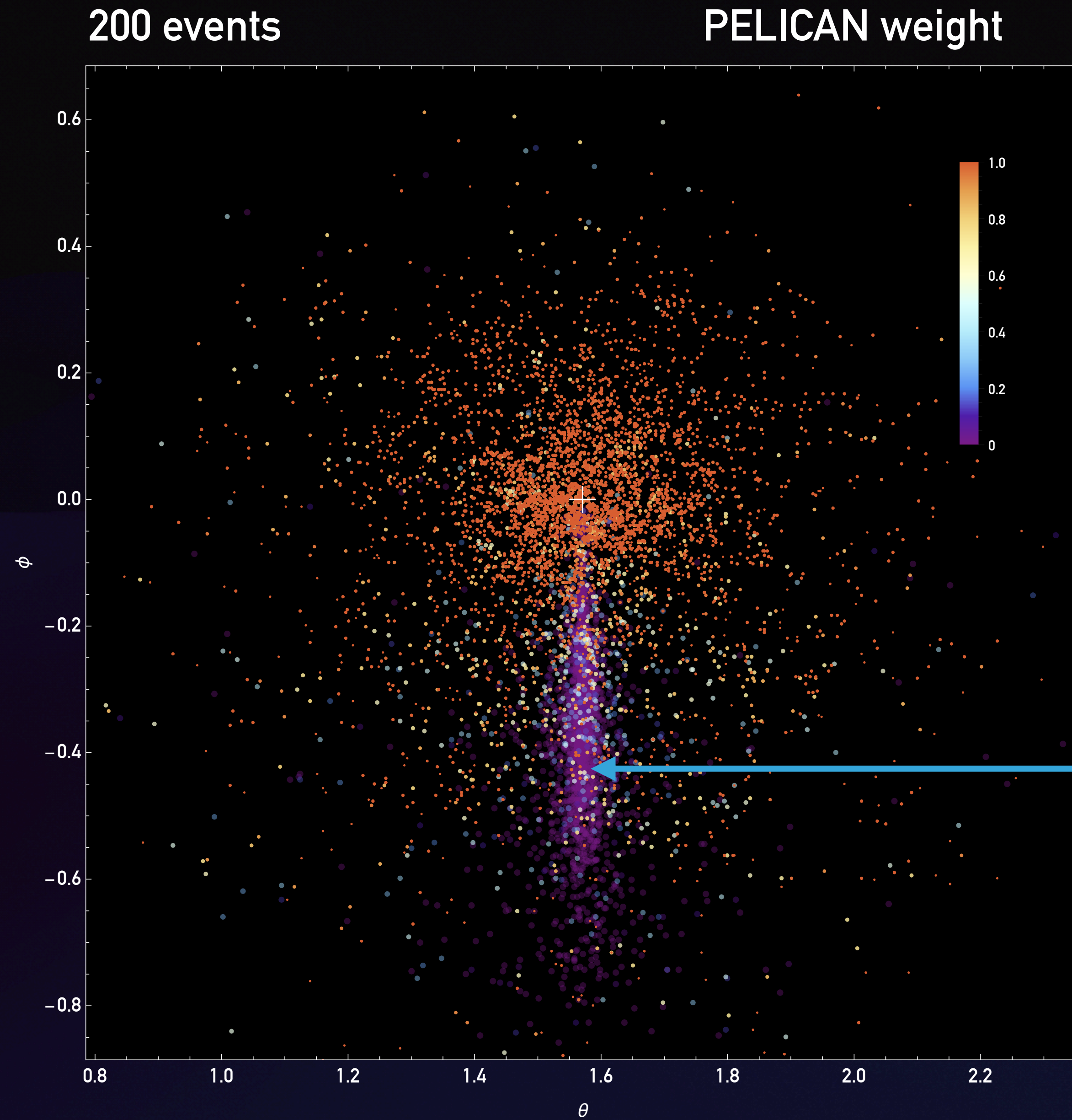
Distribution of output PELICAN weights on truth-level dataset
(~100k events)



Explaining PELICAN



Explaining PELICAN

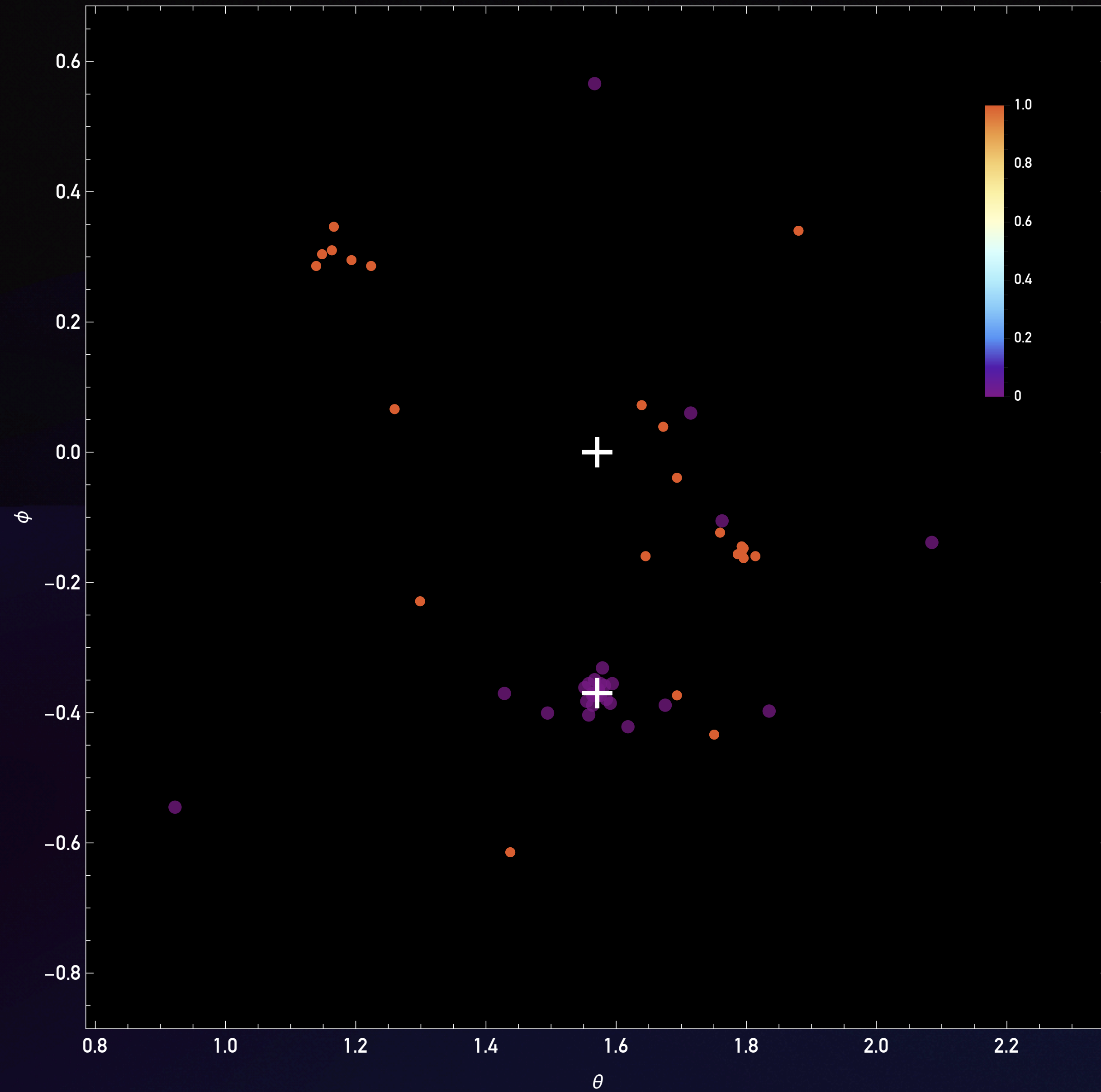


PELICAN learns to separate out the b-quark cluster

Truth level

Single event

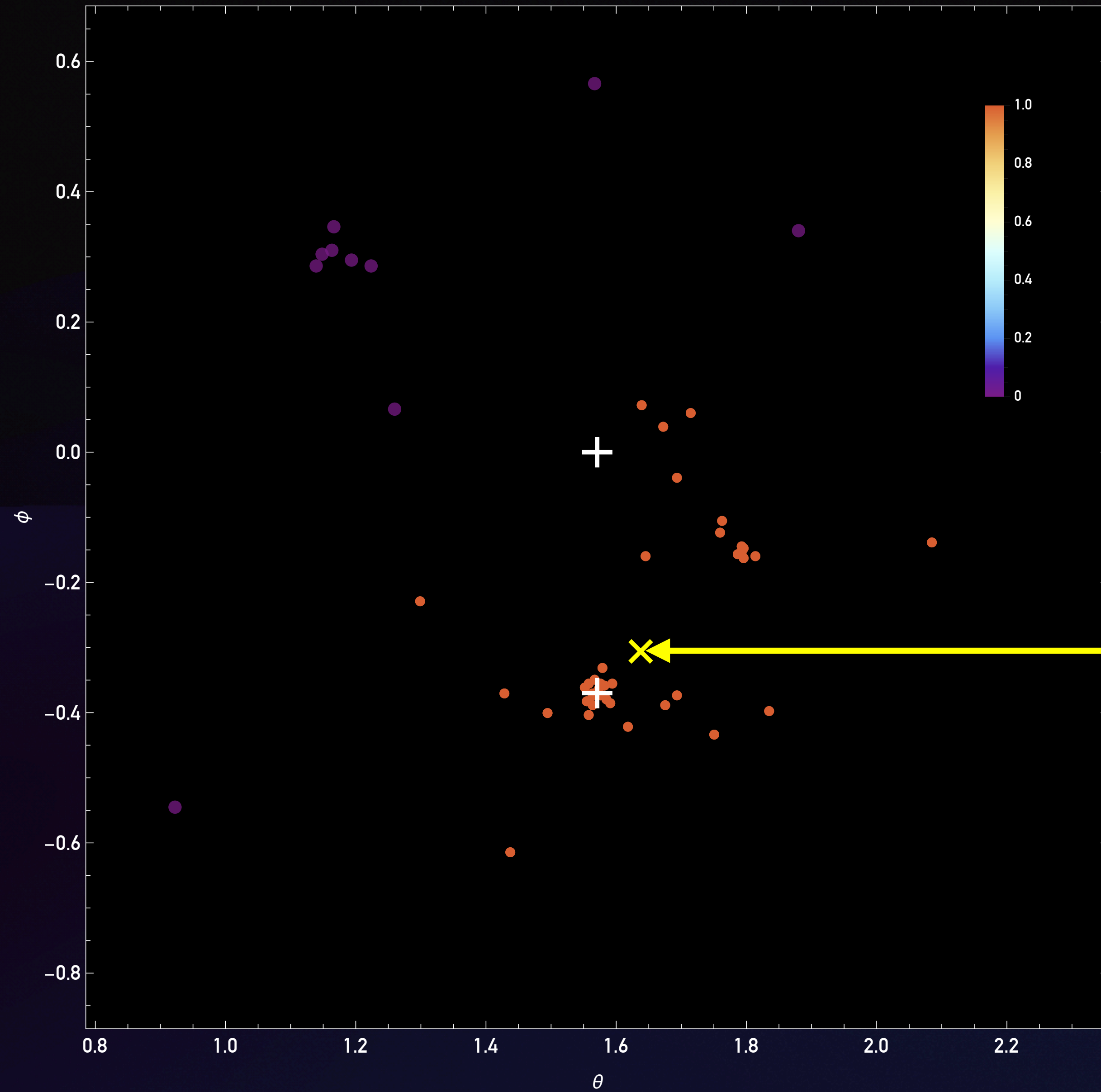
True W products in orange



Truth level

Single event

JH-identified W products in orange

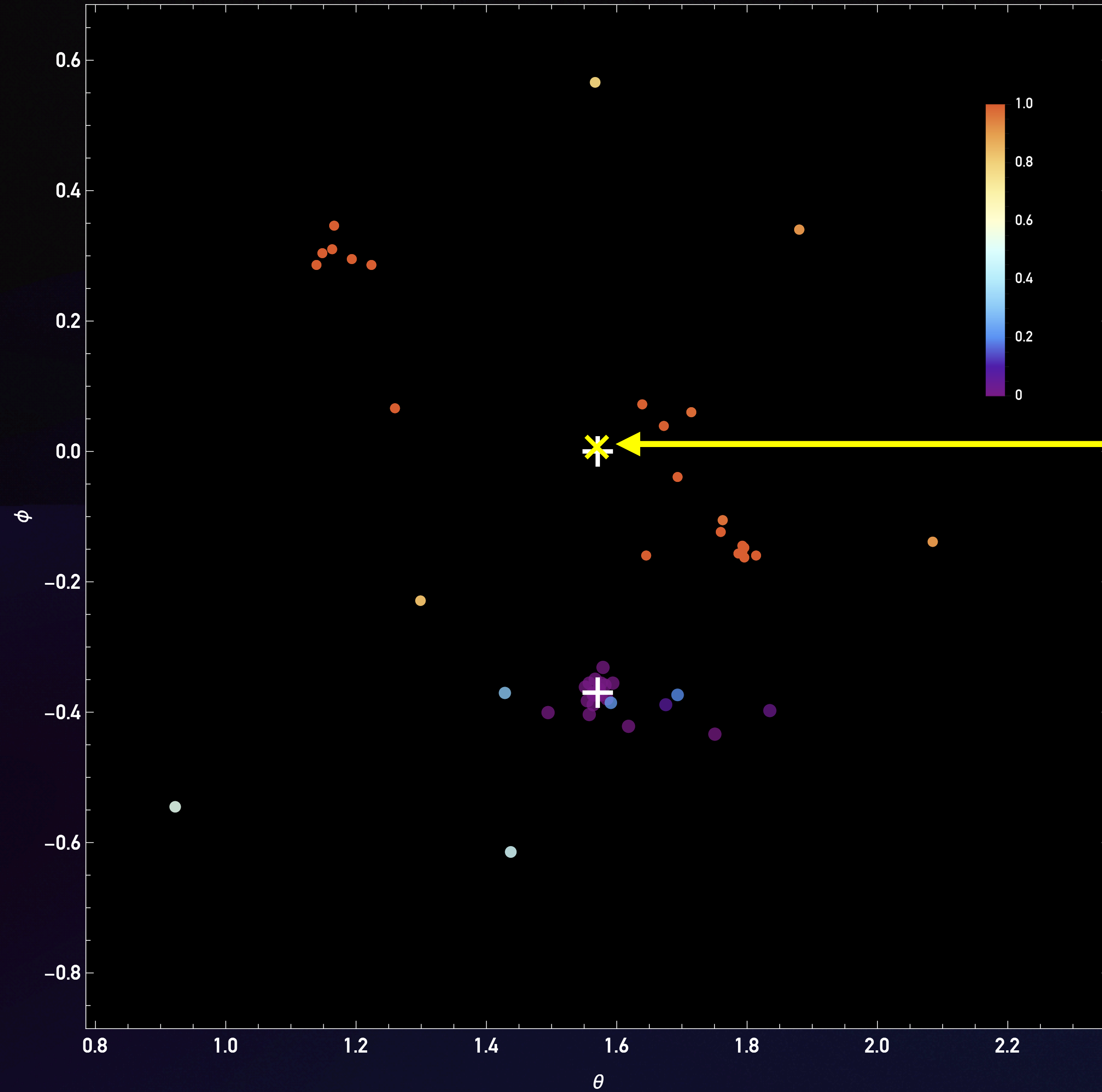


JH-reconstructed W

Truth level

Single event

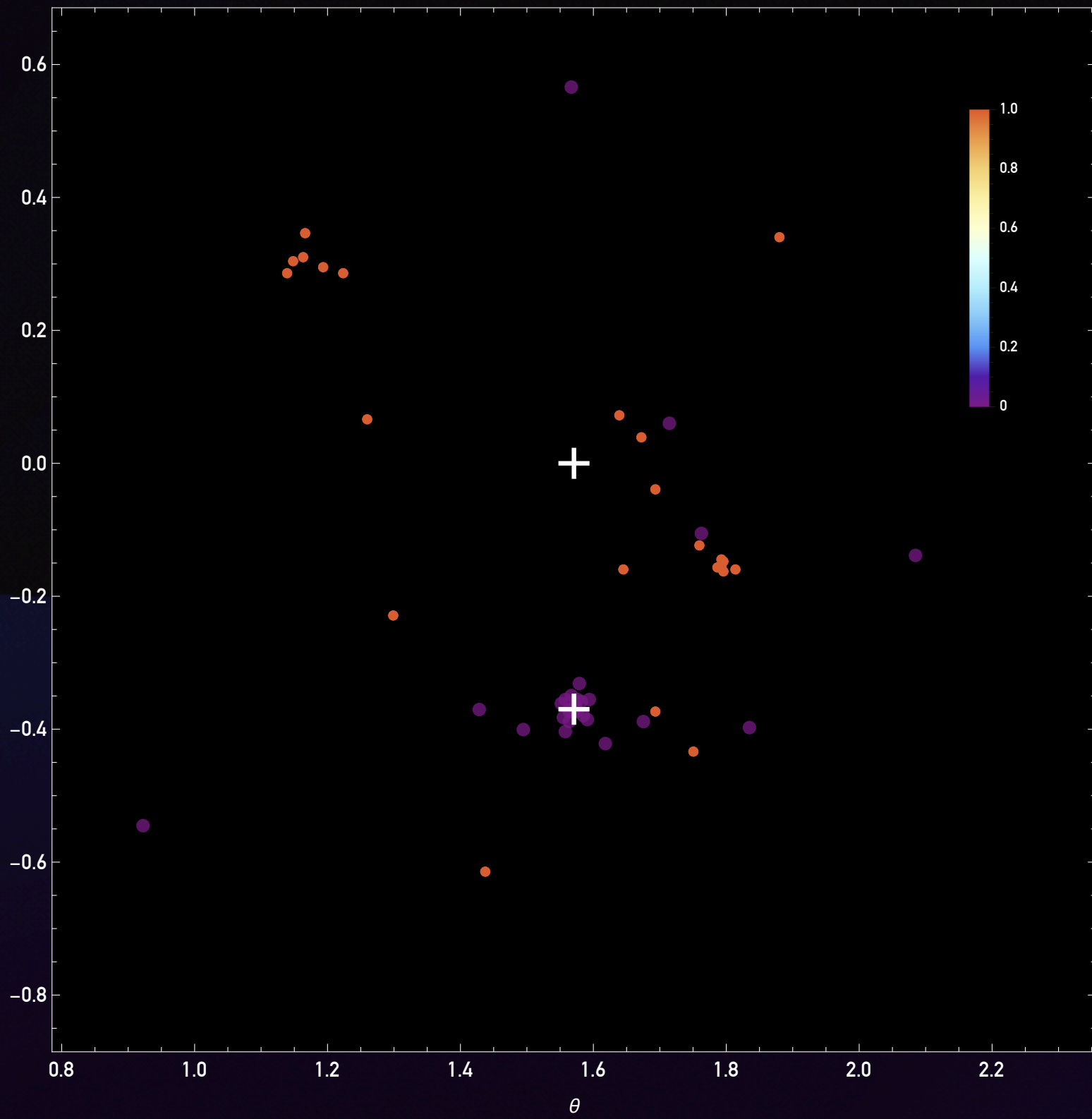
PELICAN weight



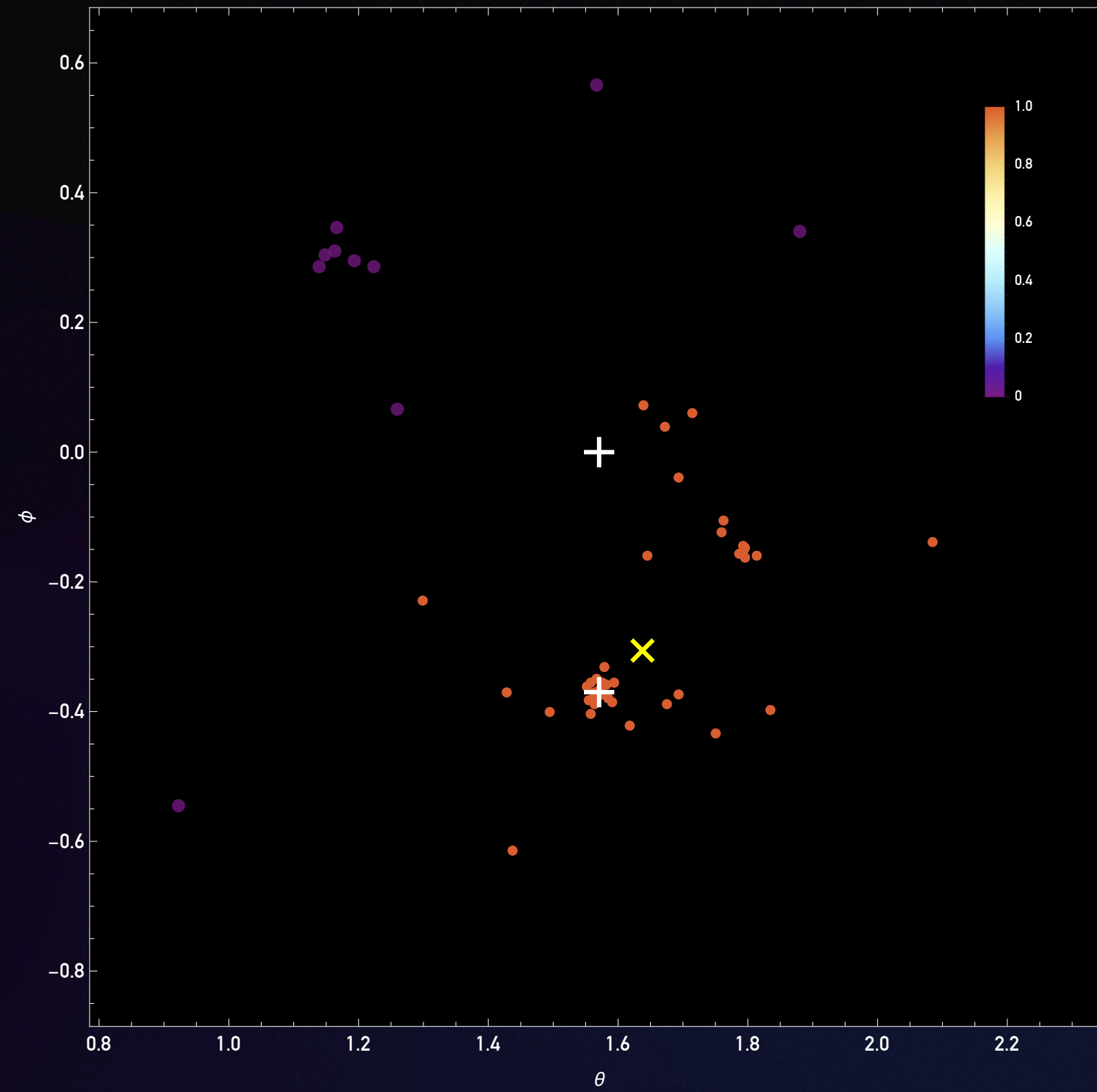
PELICAN-reconstructed W

Truth level

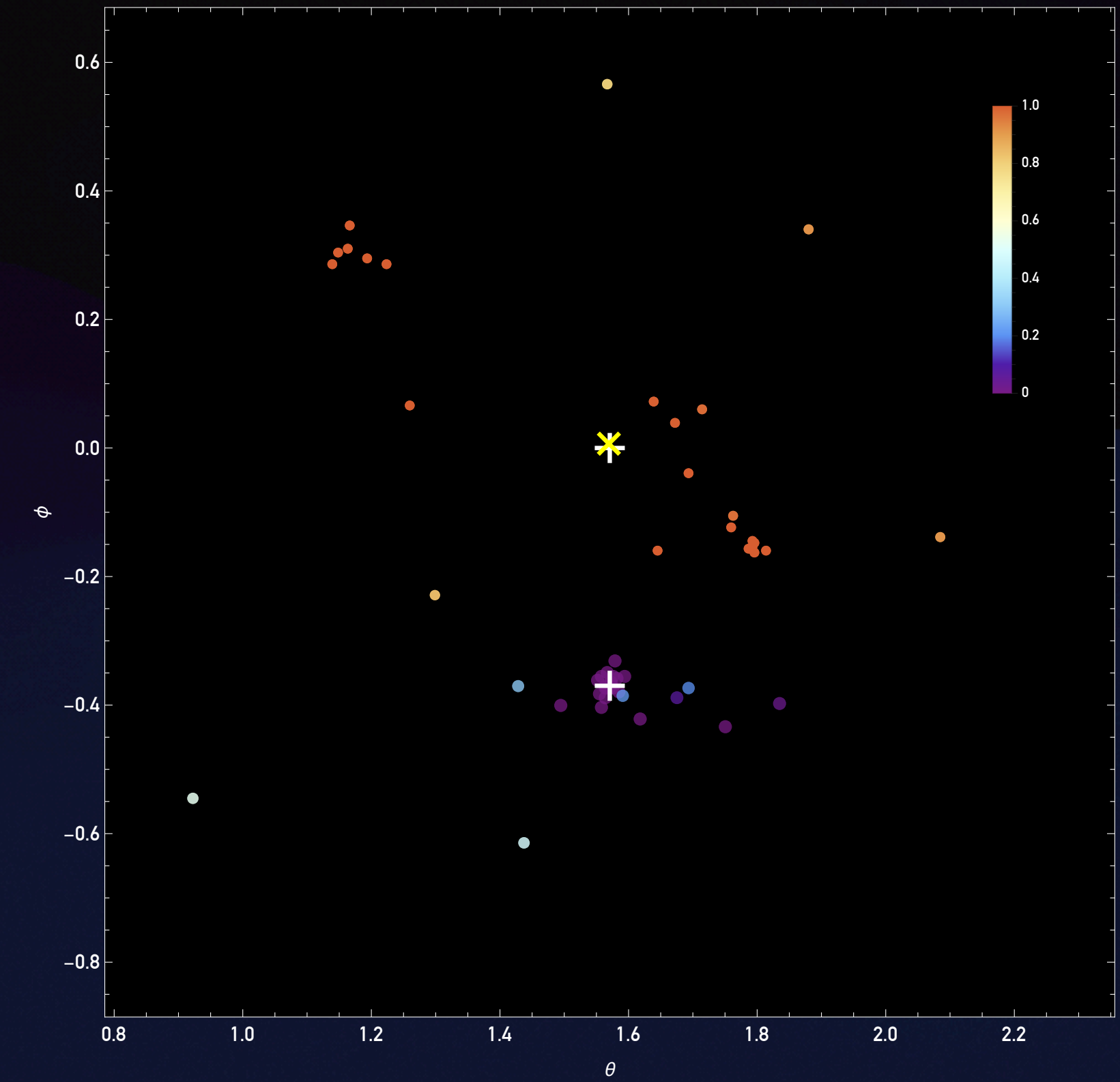
Truth



JH

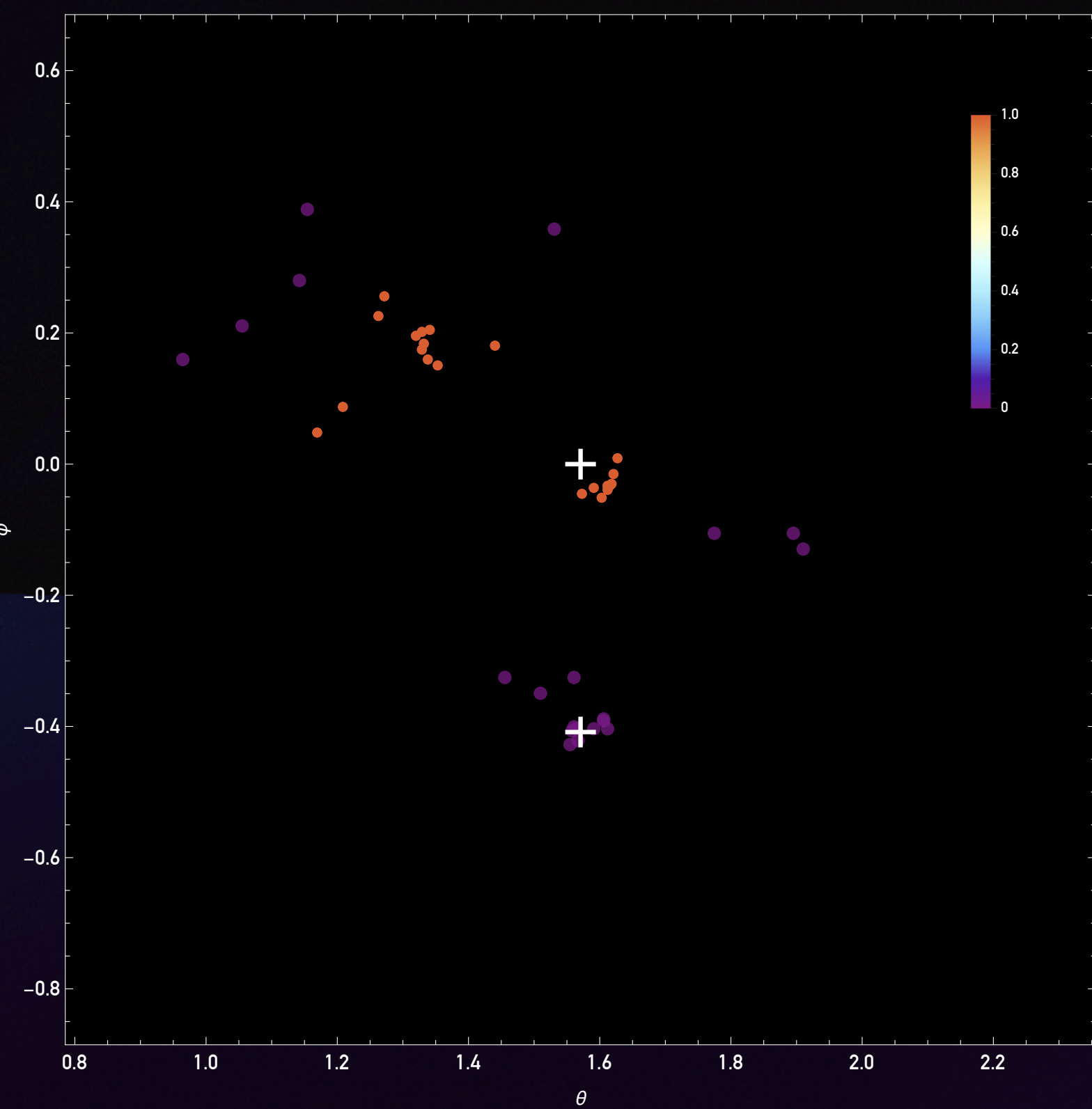


PELICAN weight

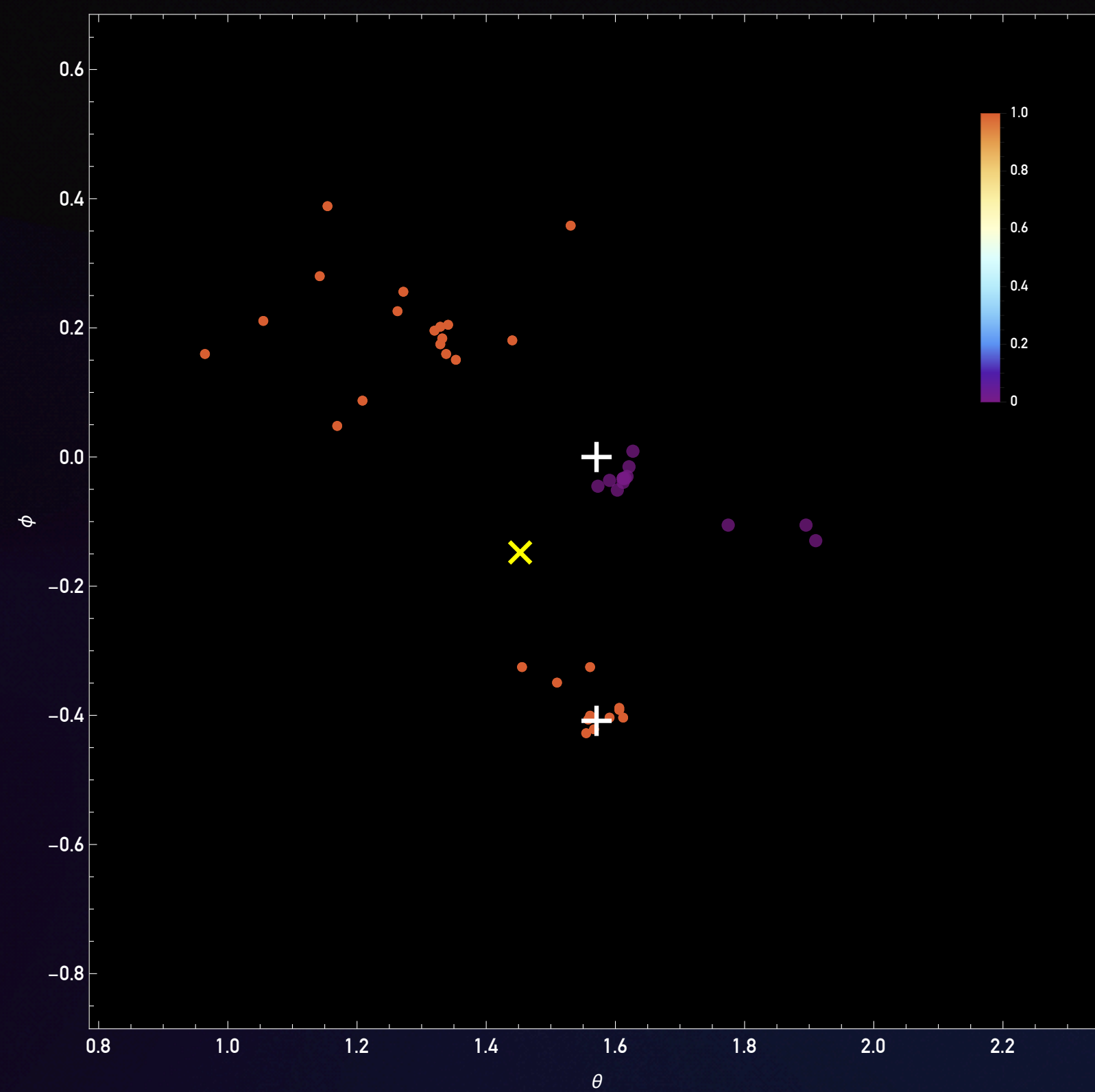


Truth level

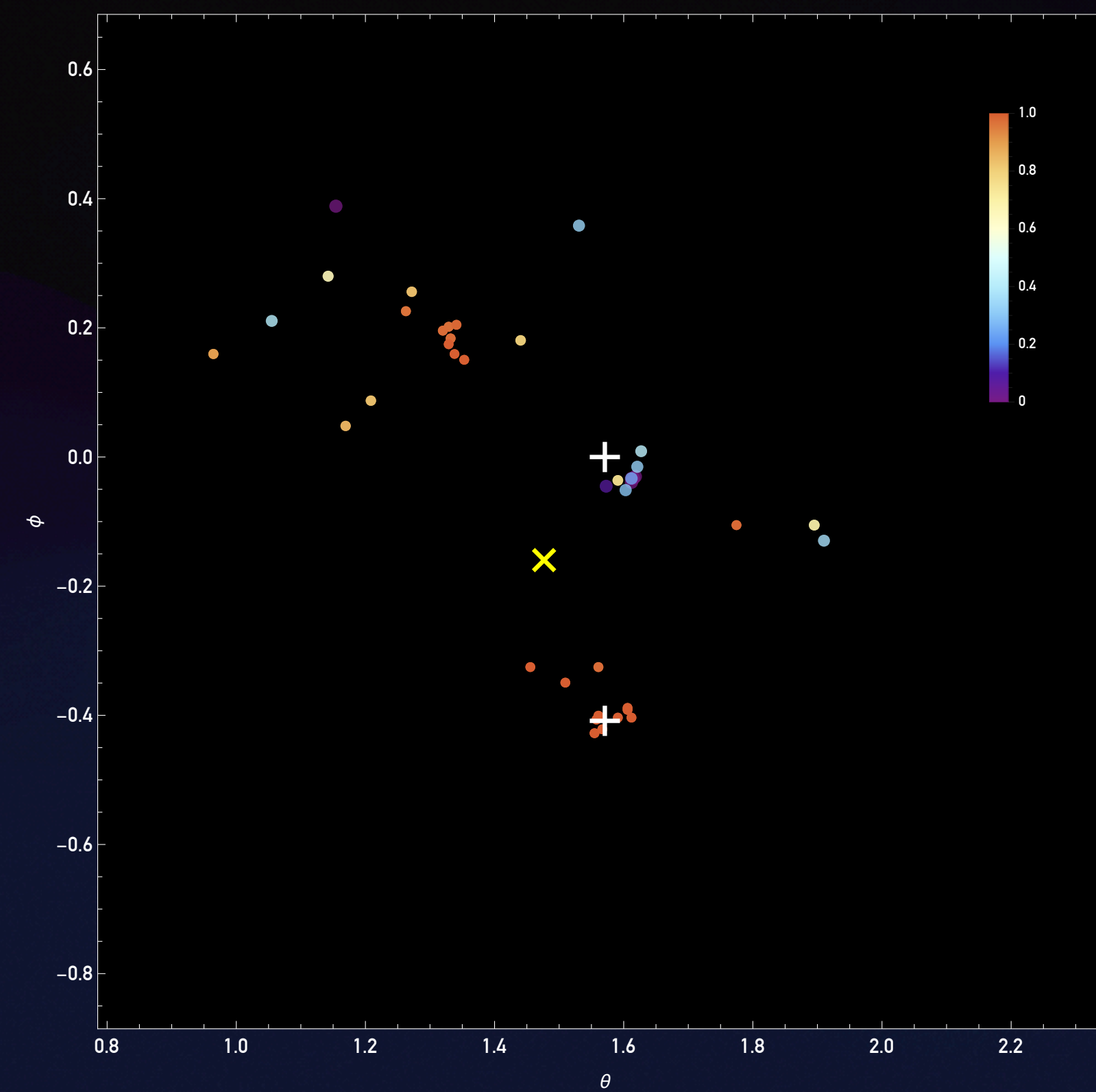
Truth



JH

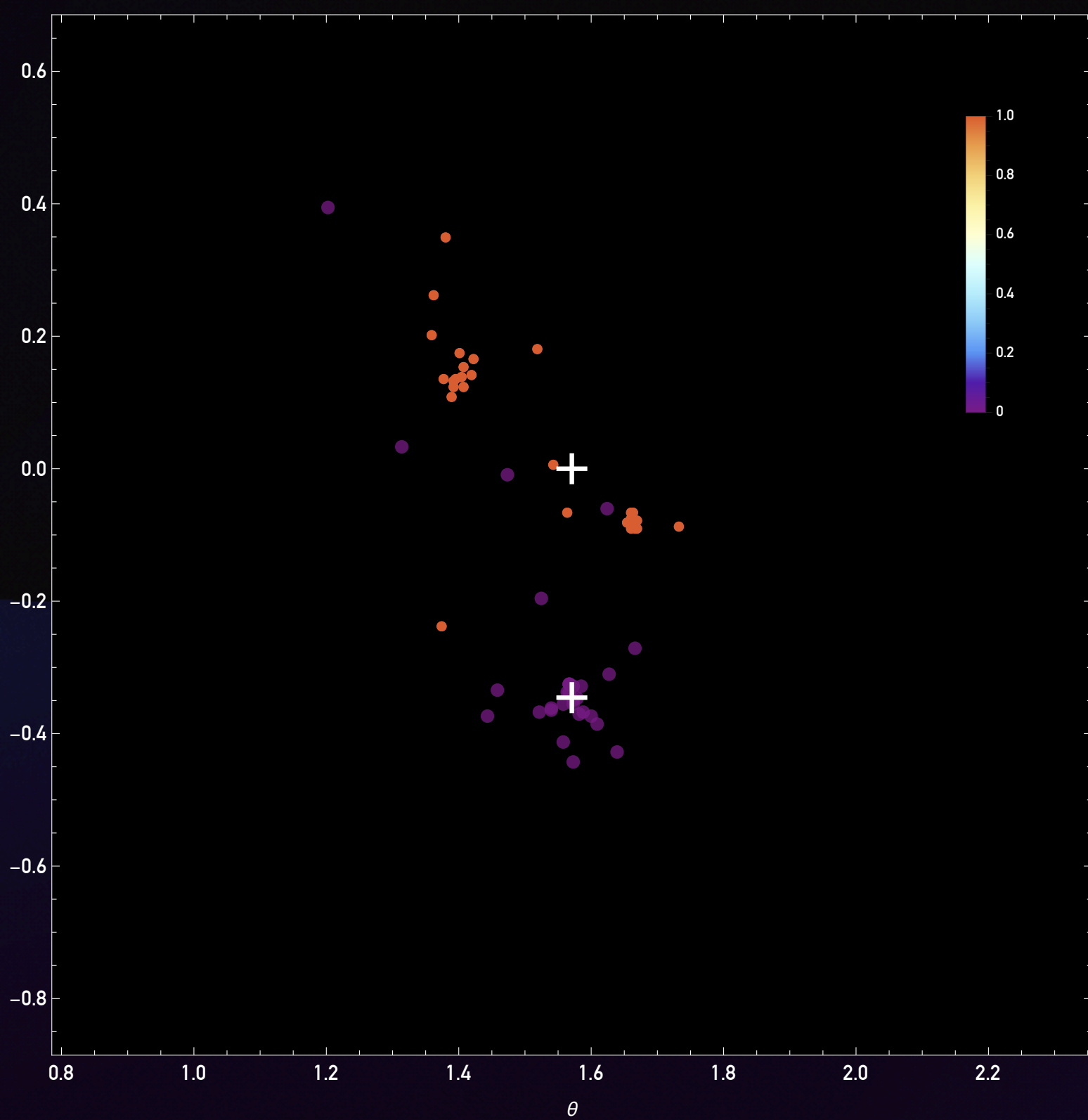


PELICAN weight

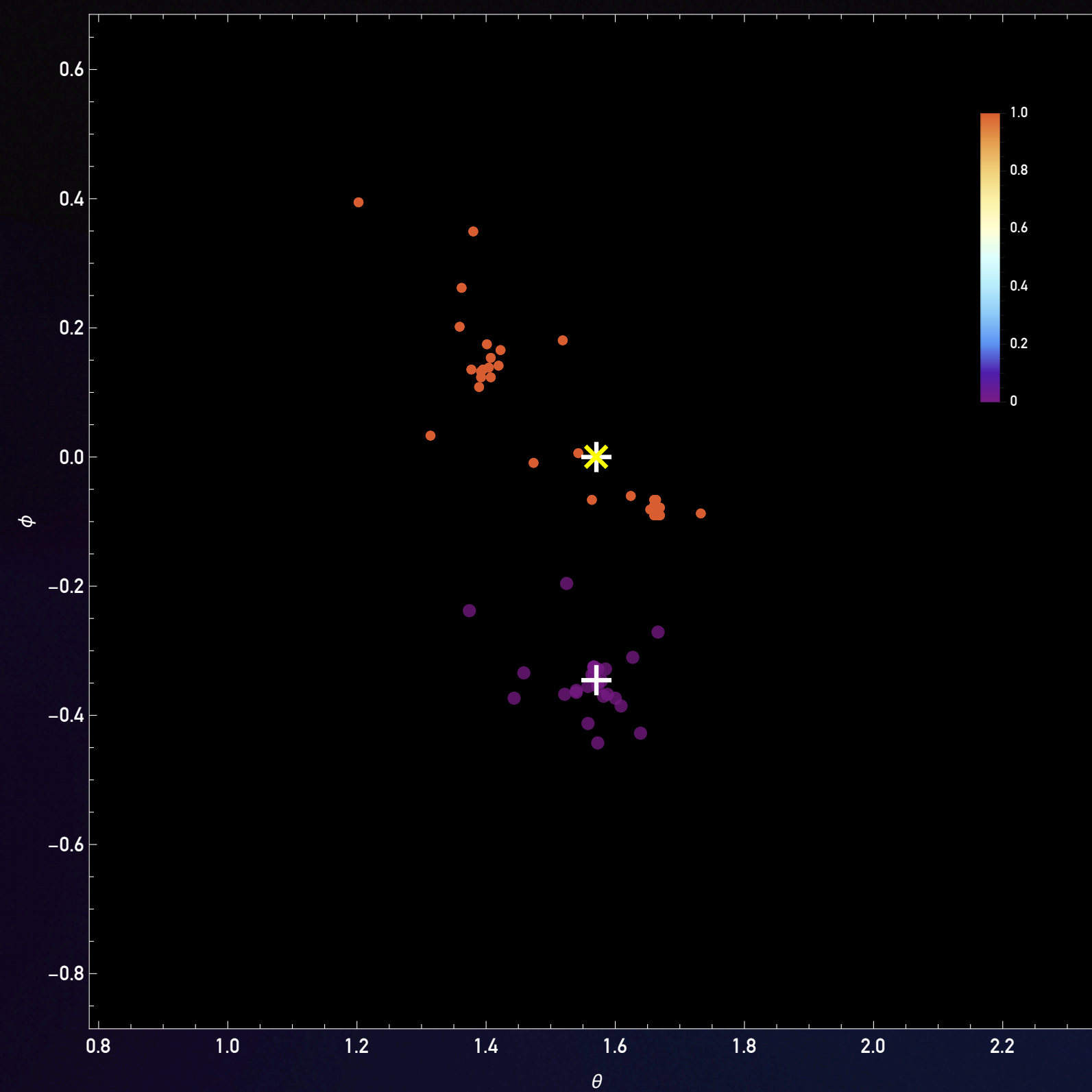


Truth level

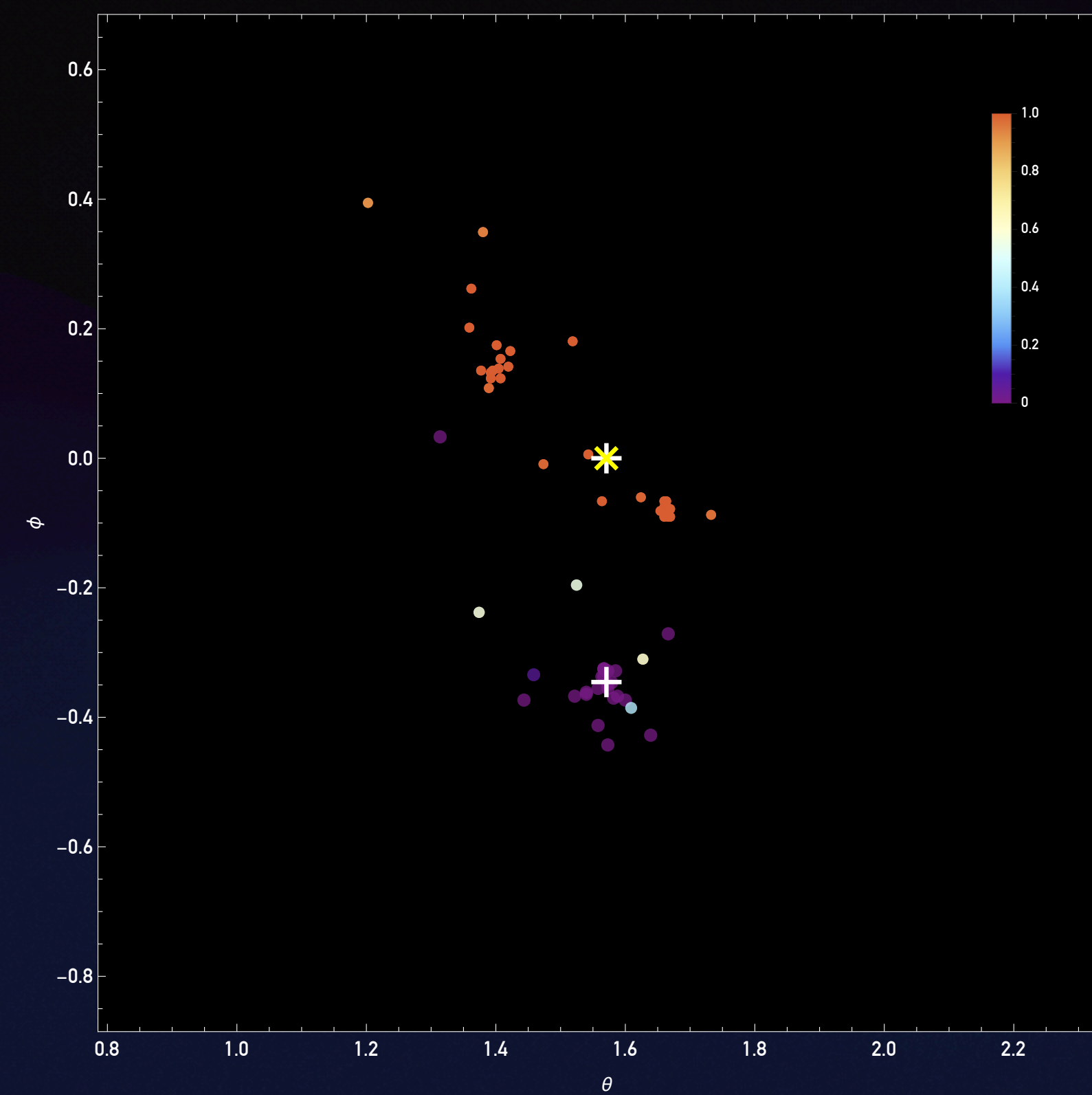
Truth



JH

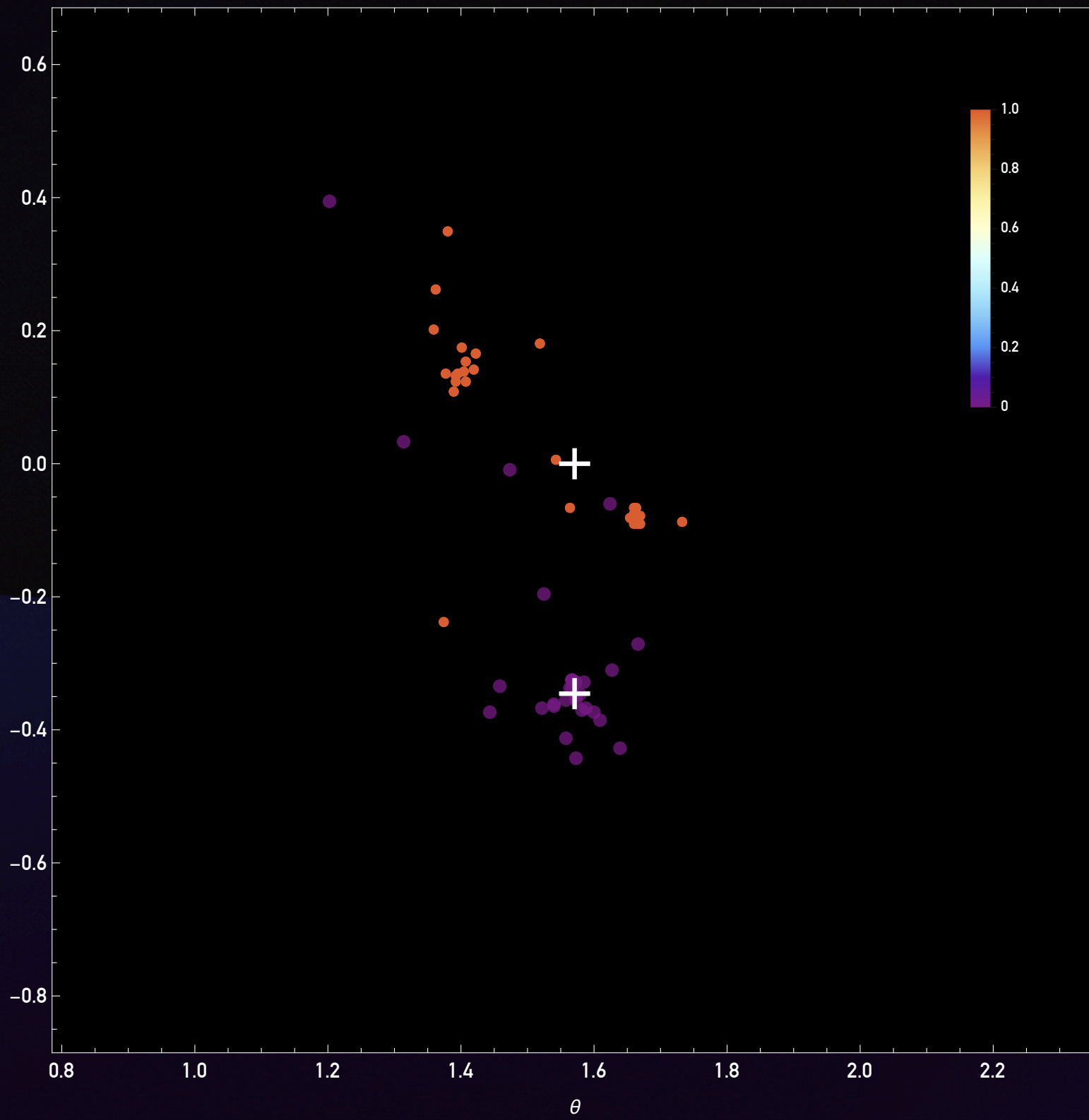


PELICAN weight

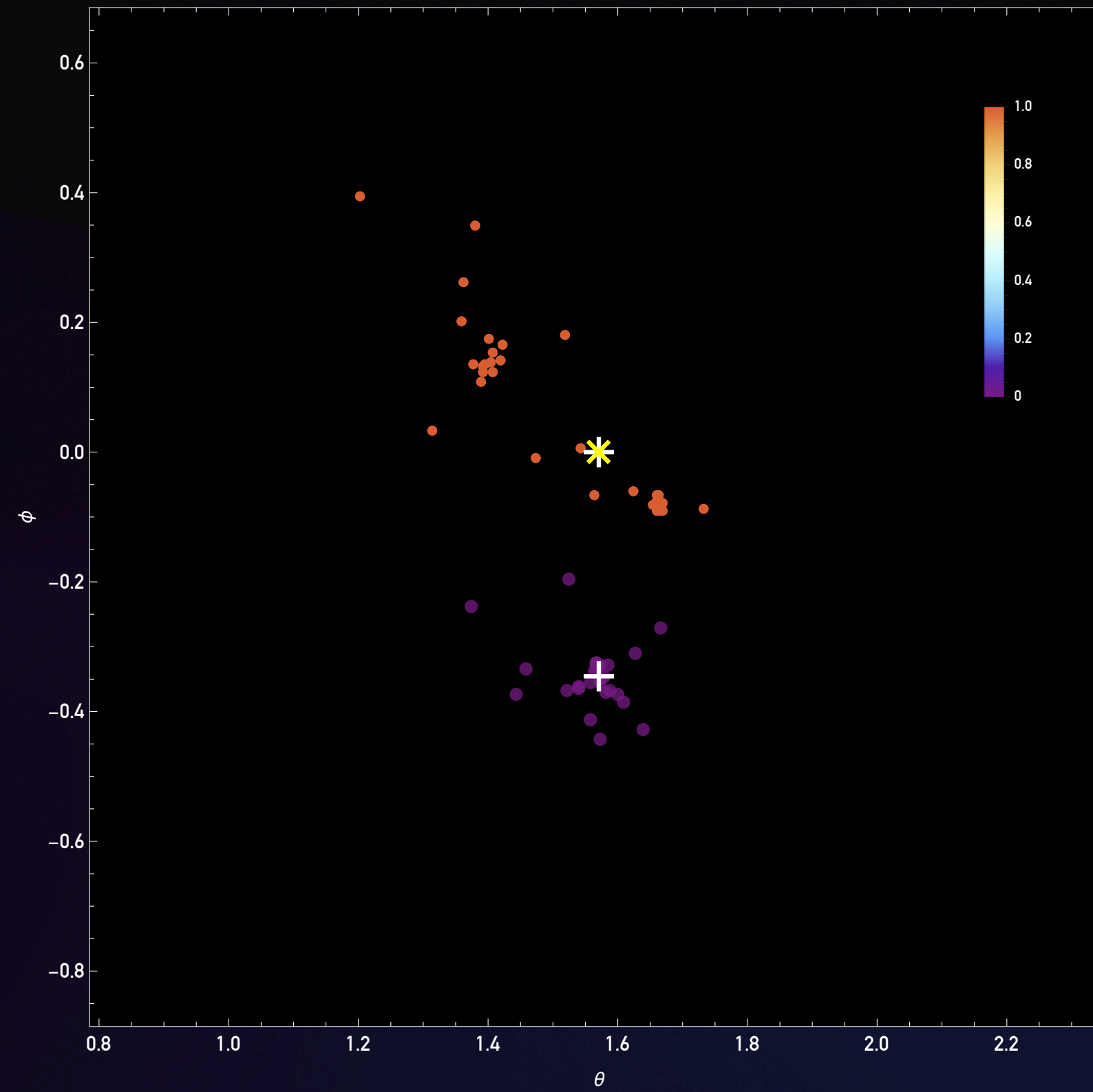


Truth level

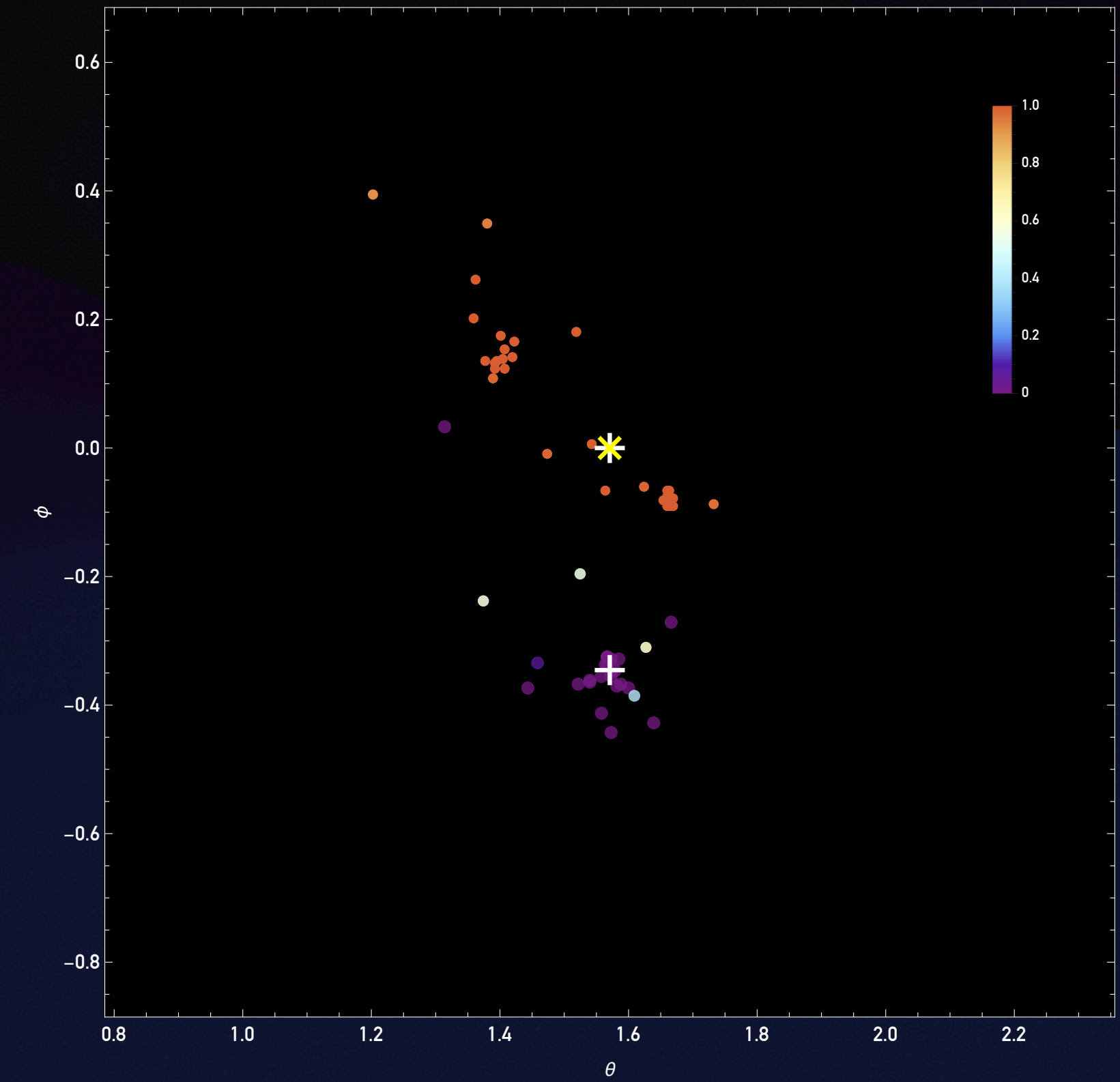
Truth



JH



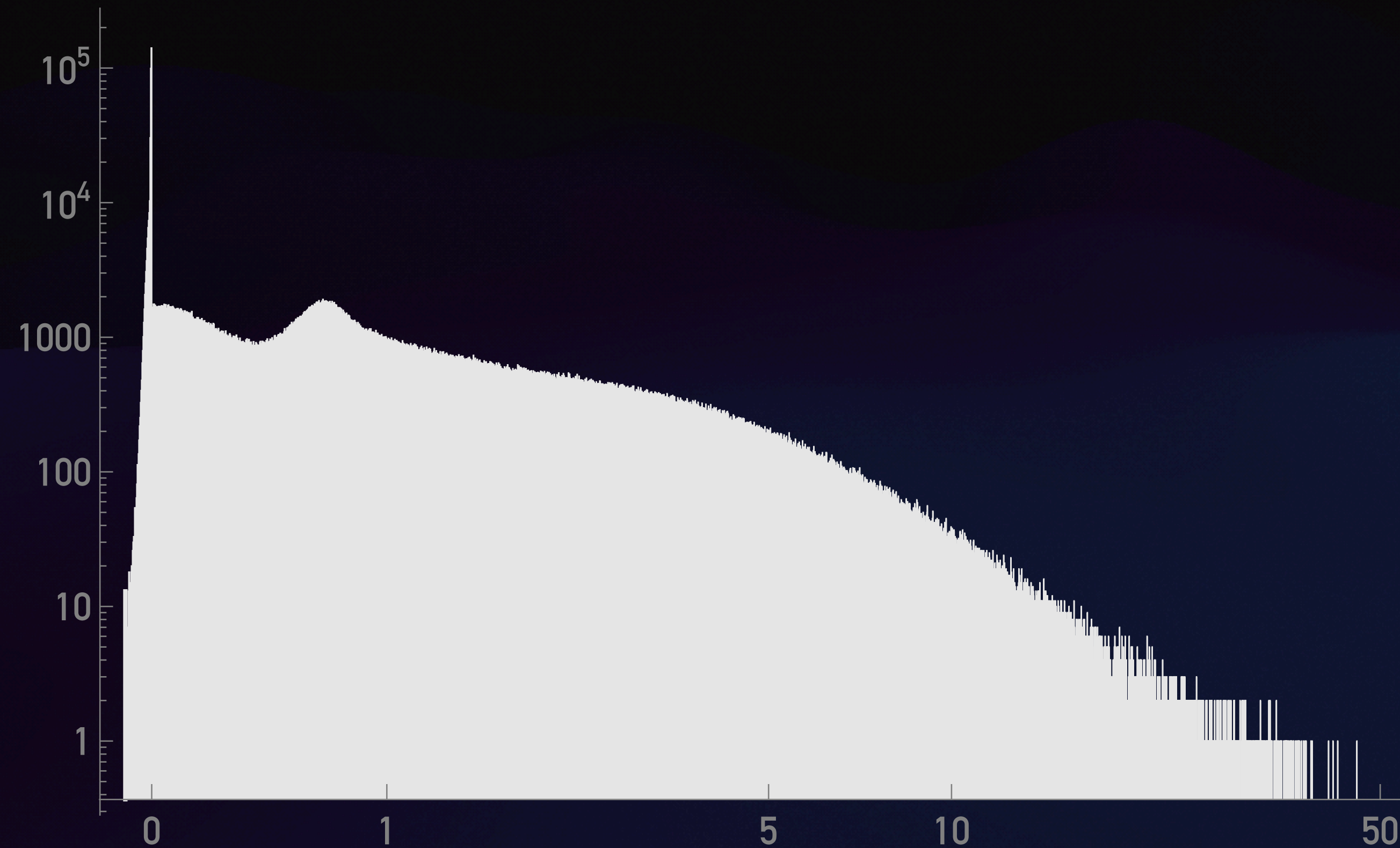
PELICAN weight



And these are only events successfully tagged by JH (~37%)!

DELPHES dataset

Distribution of output PELICAN weights on DELPHES dataset
(~100k events)

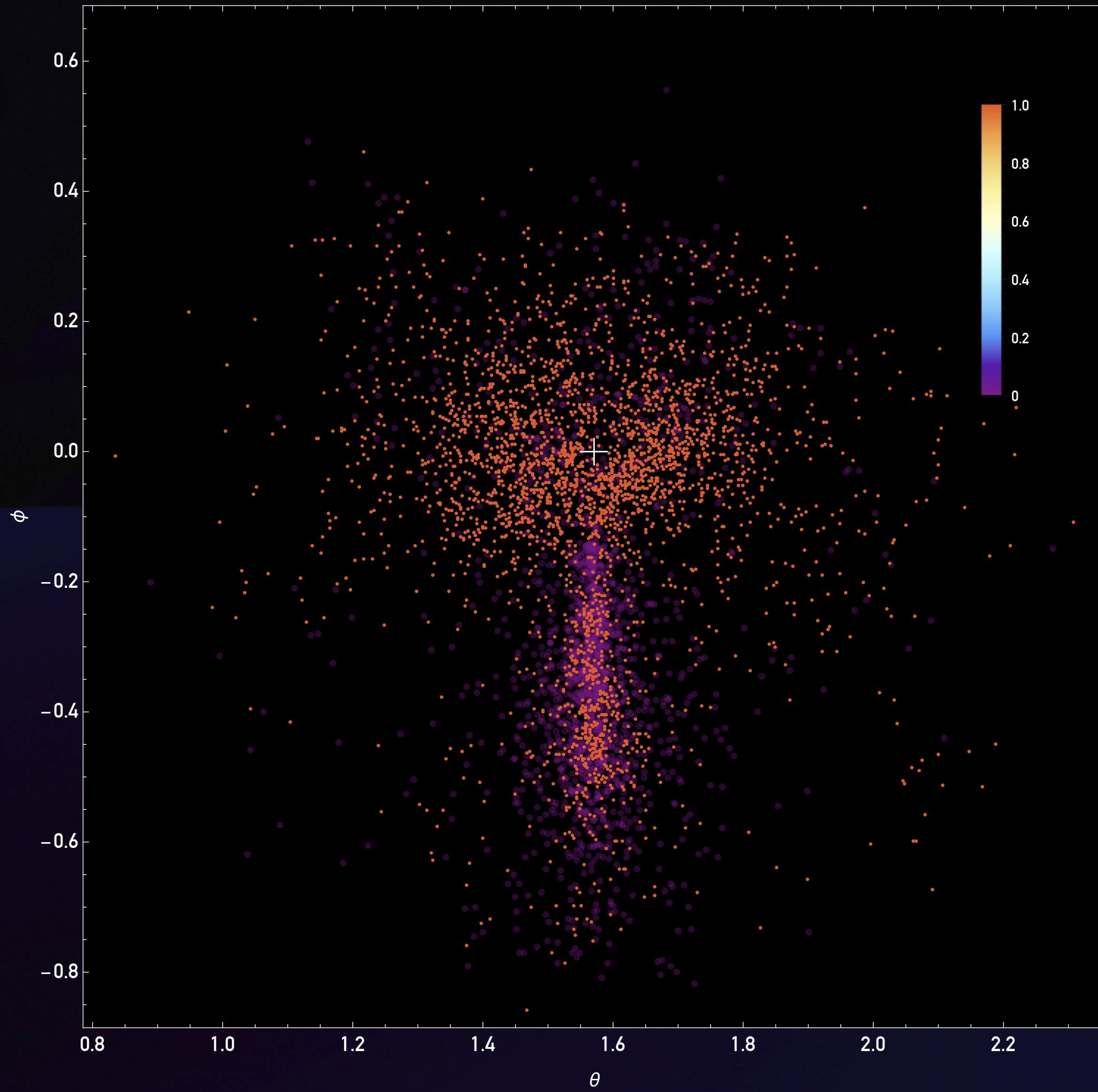


$$\{c_i\}_{i=1}^N$$

DELPHES

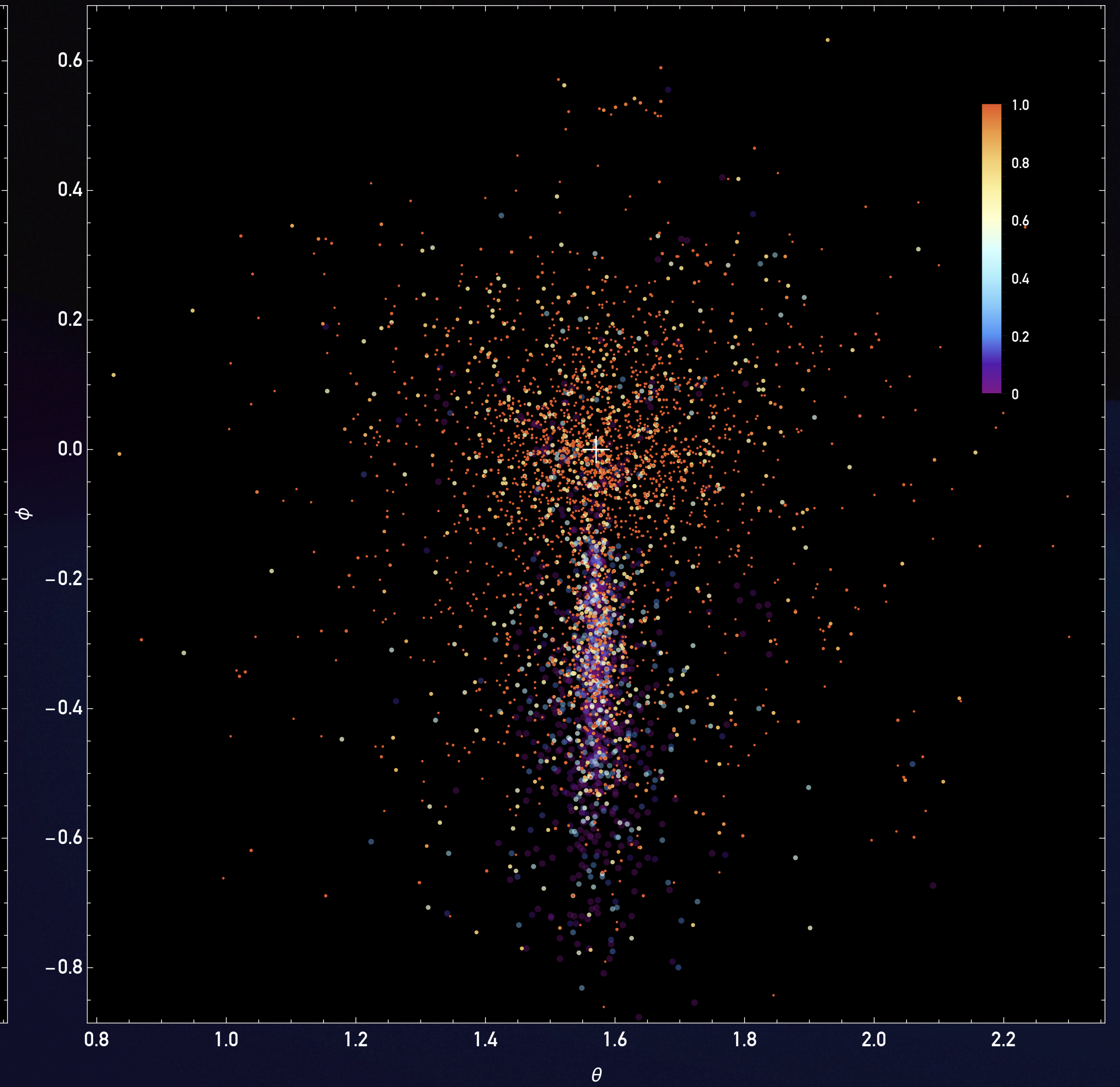
200 JH-tagged events

JH



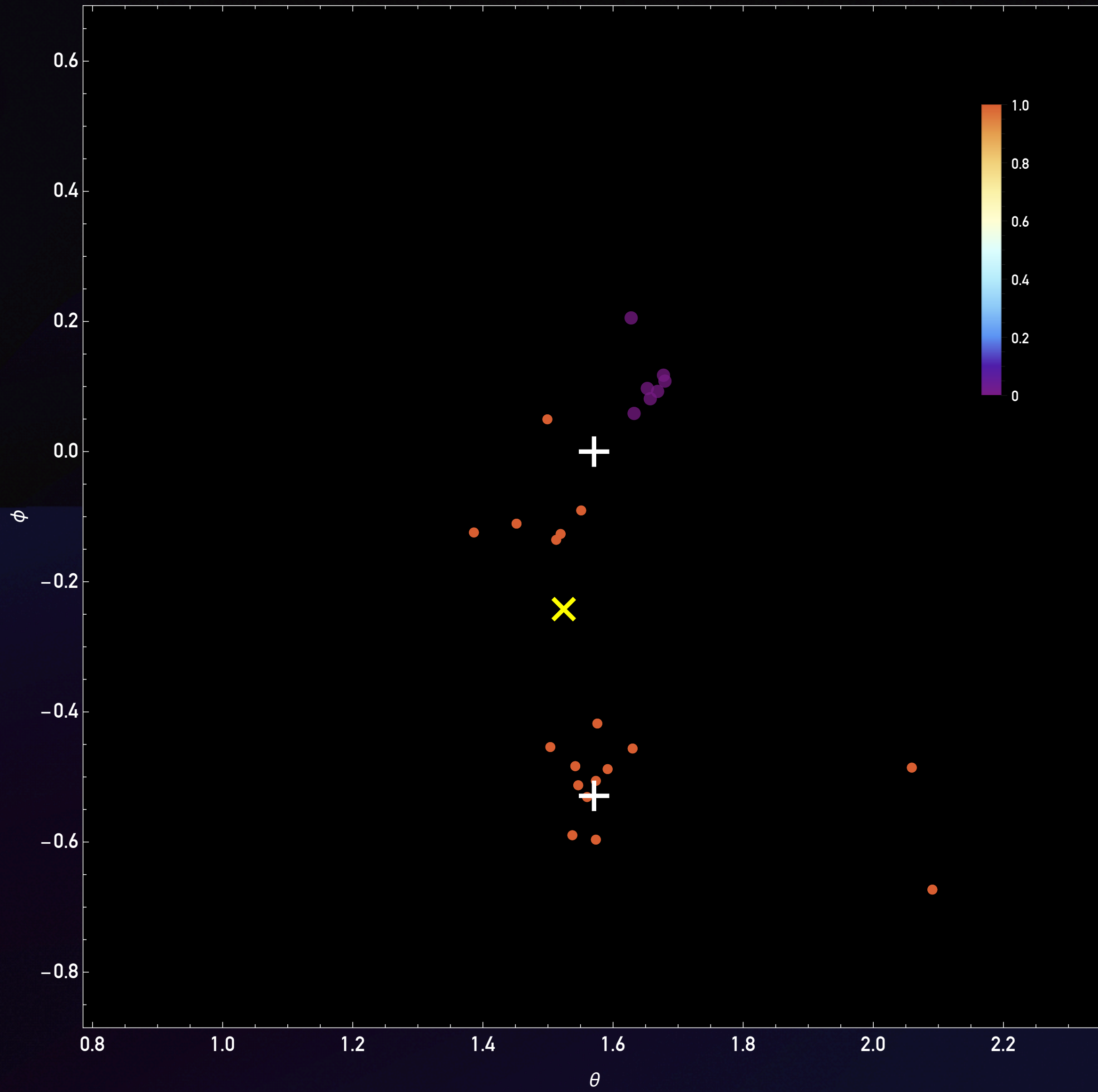
200 random events

PELICAN

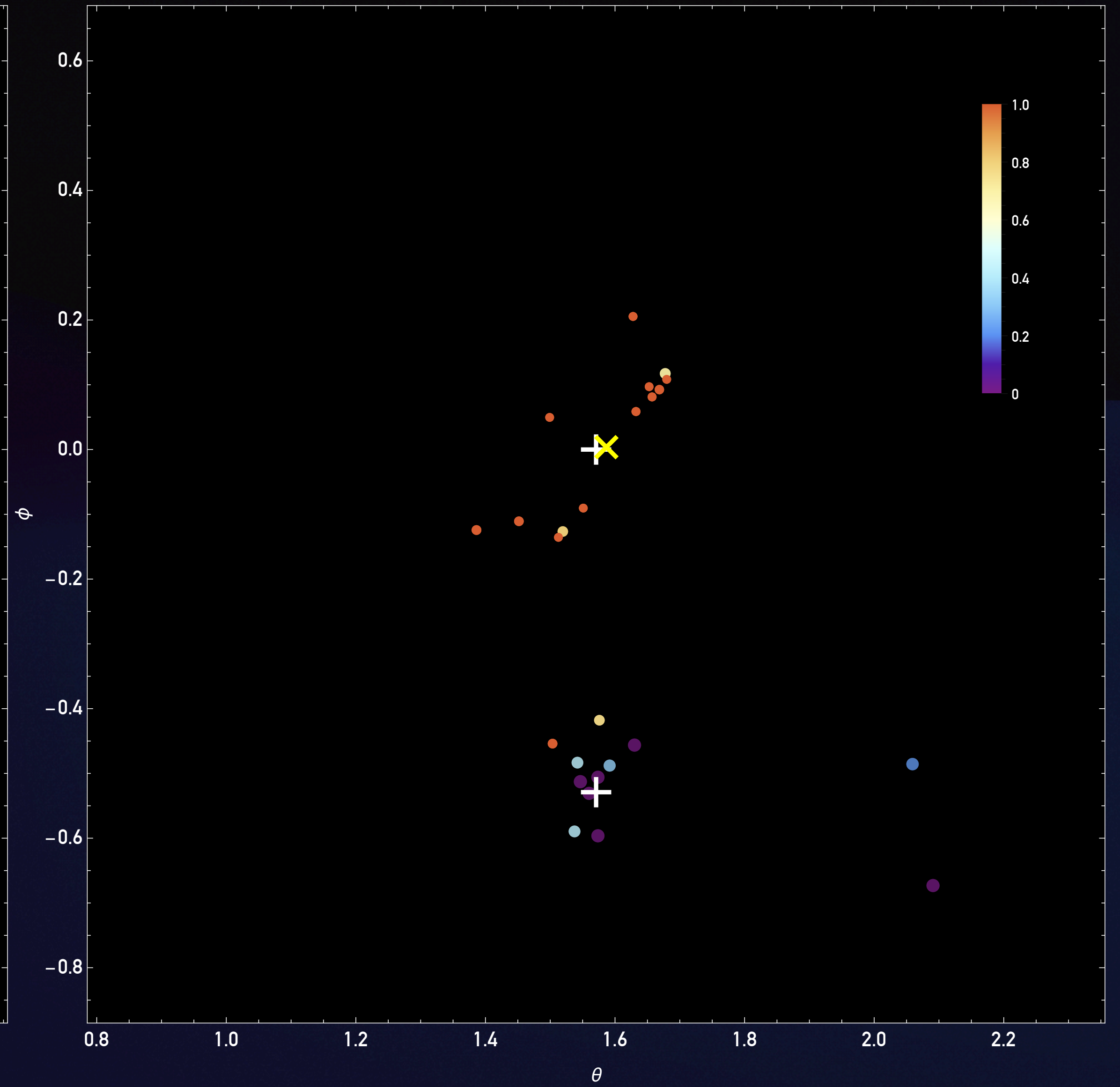


DELPHES

JH

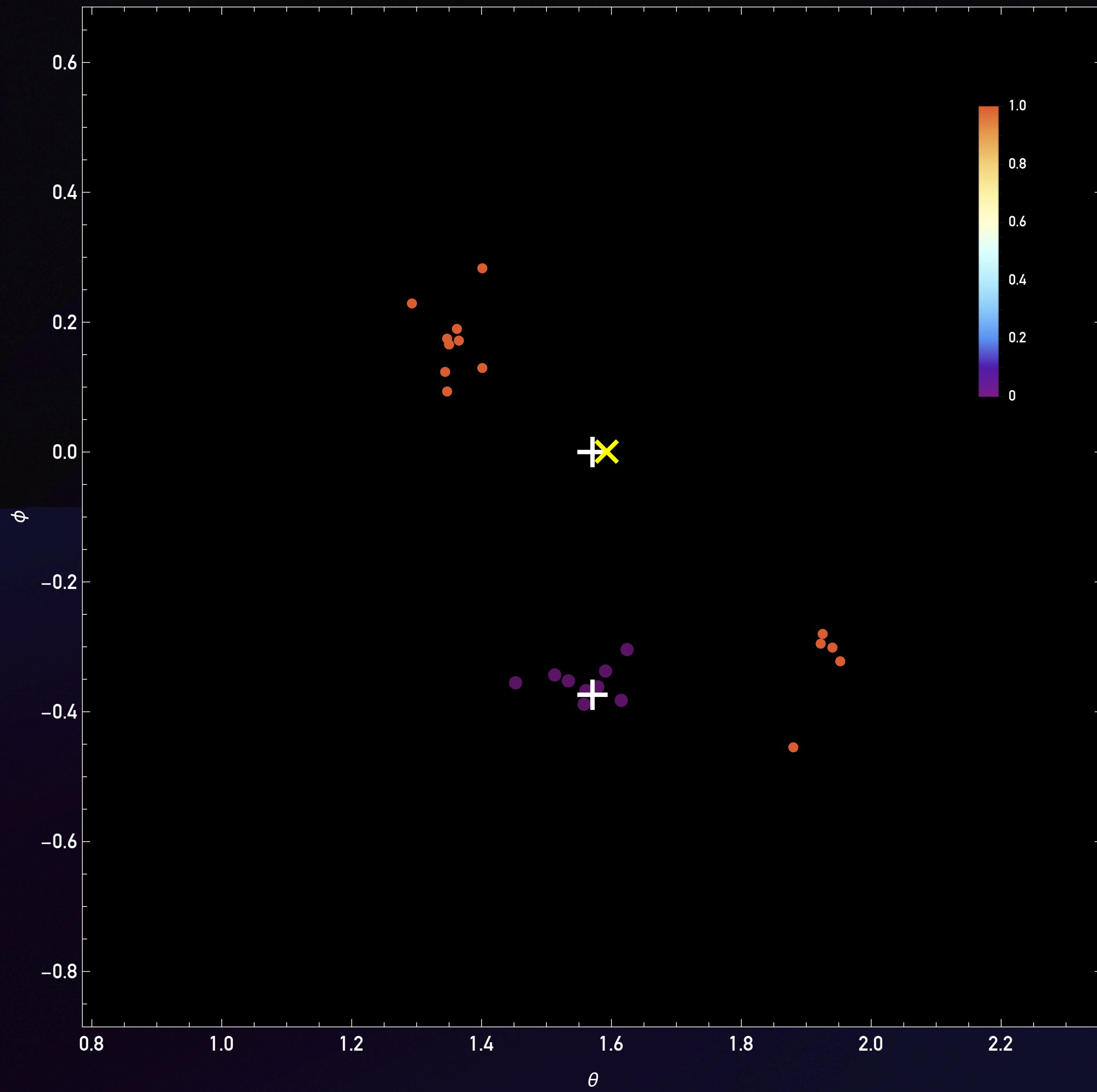


PELICAN

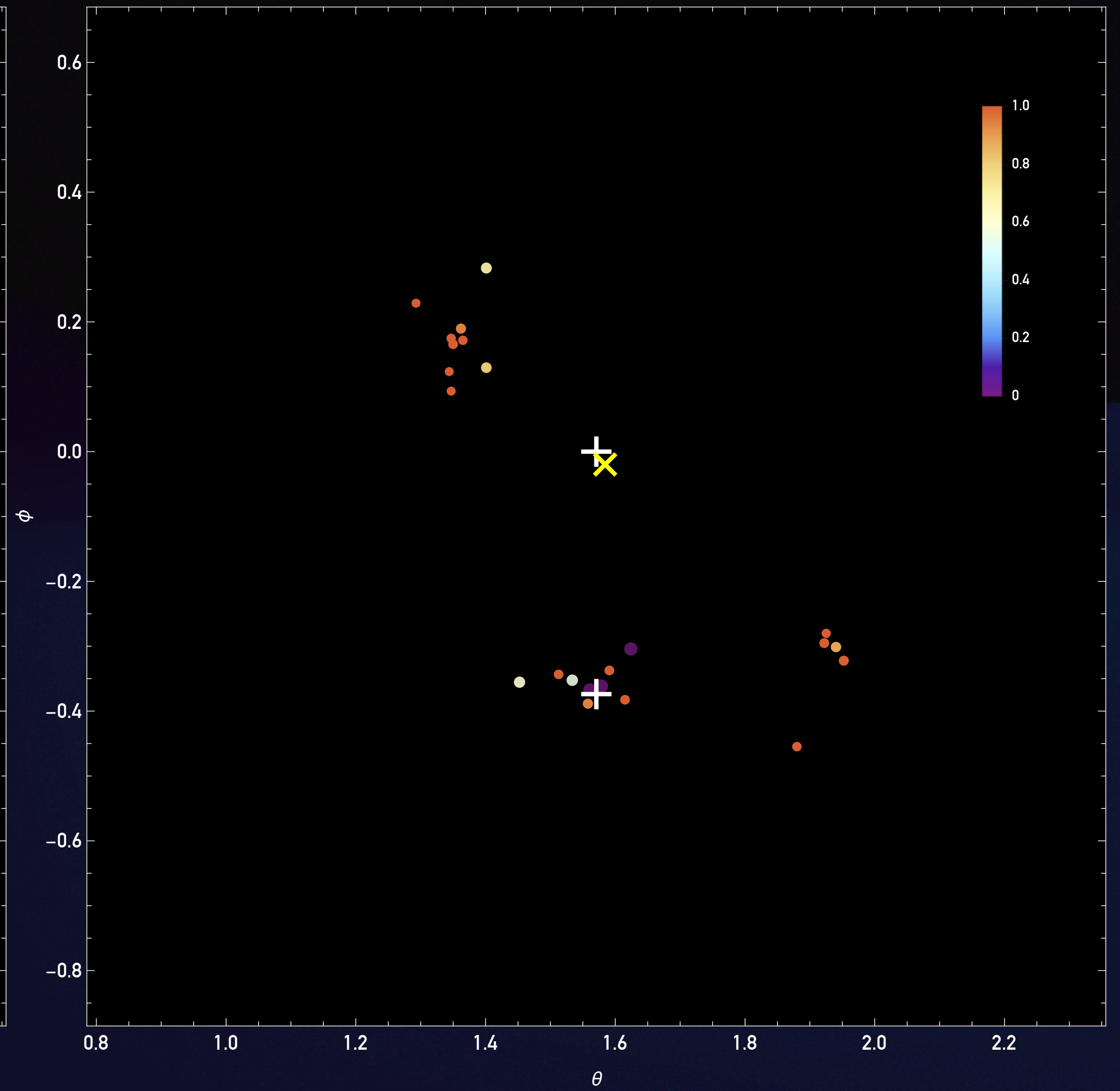


DELPHES

JH

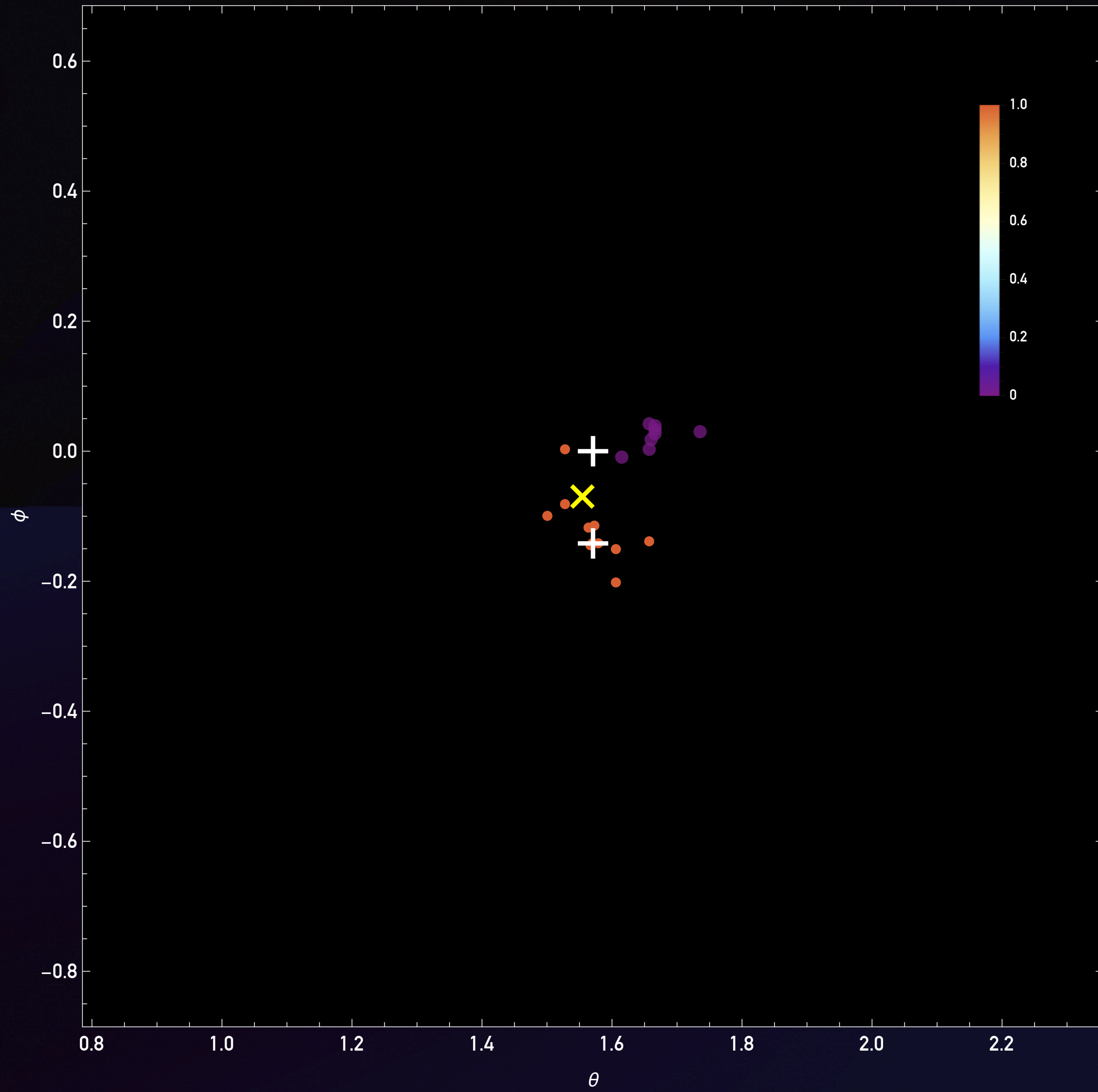


PELICAN

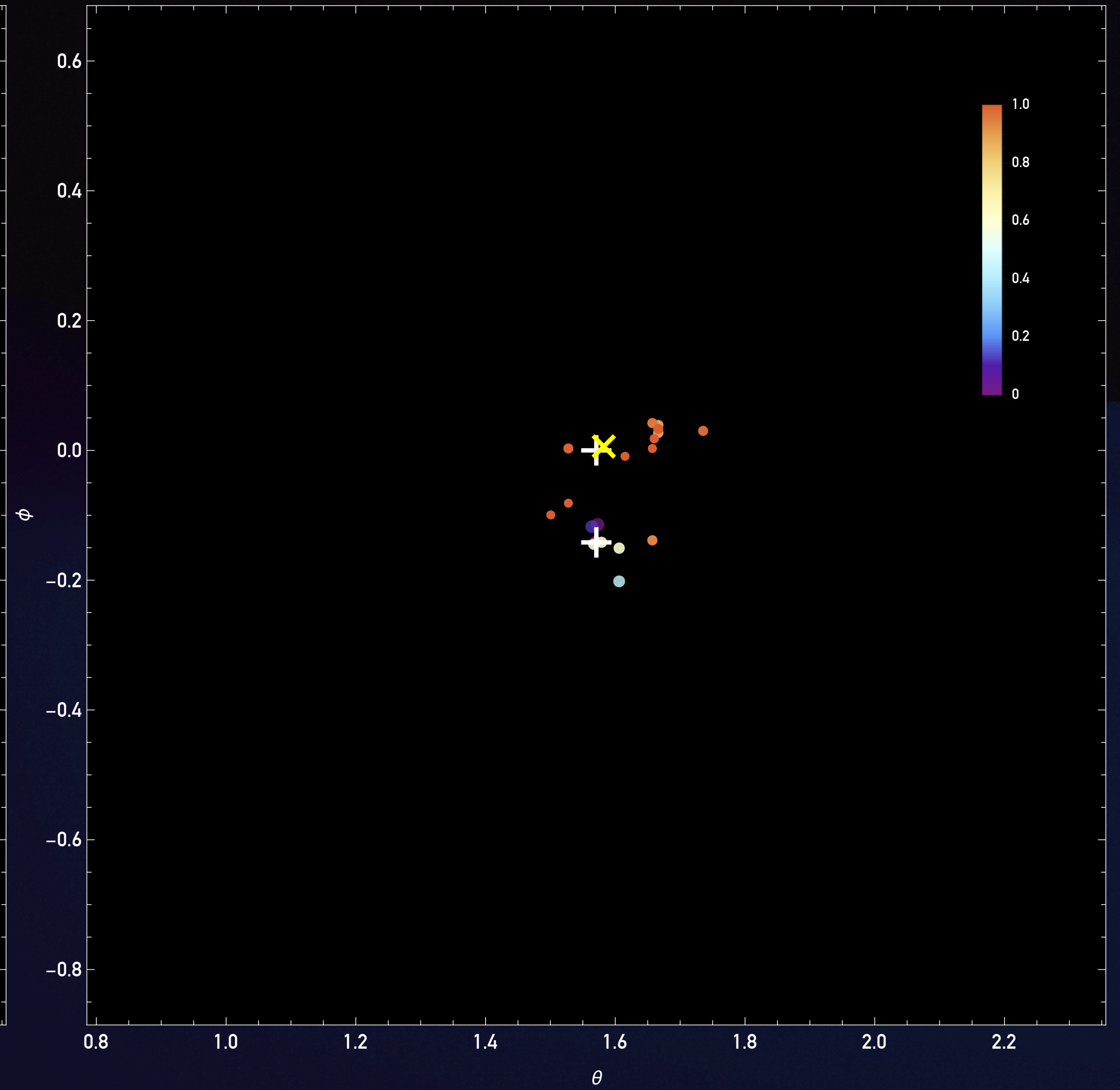


DELPHES

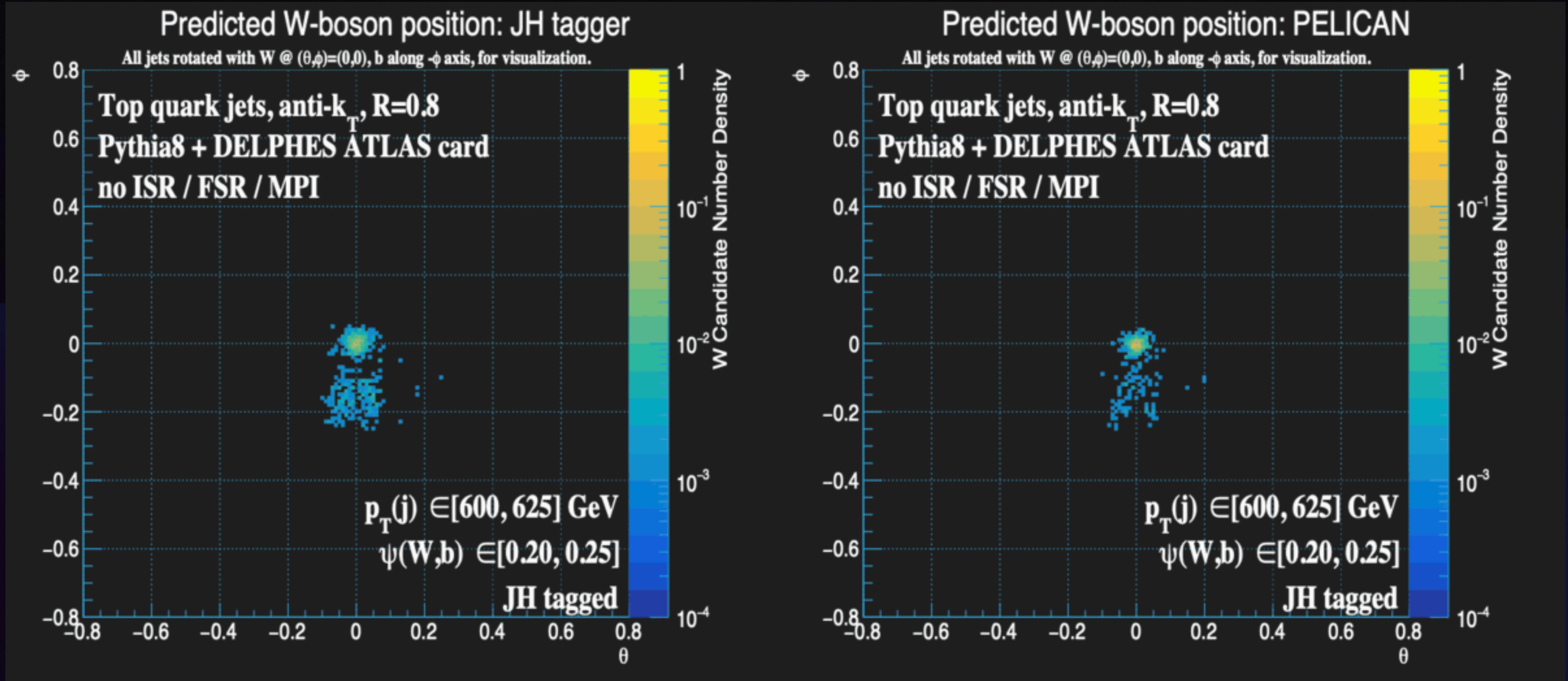
JH



PELICAN

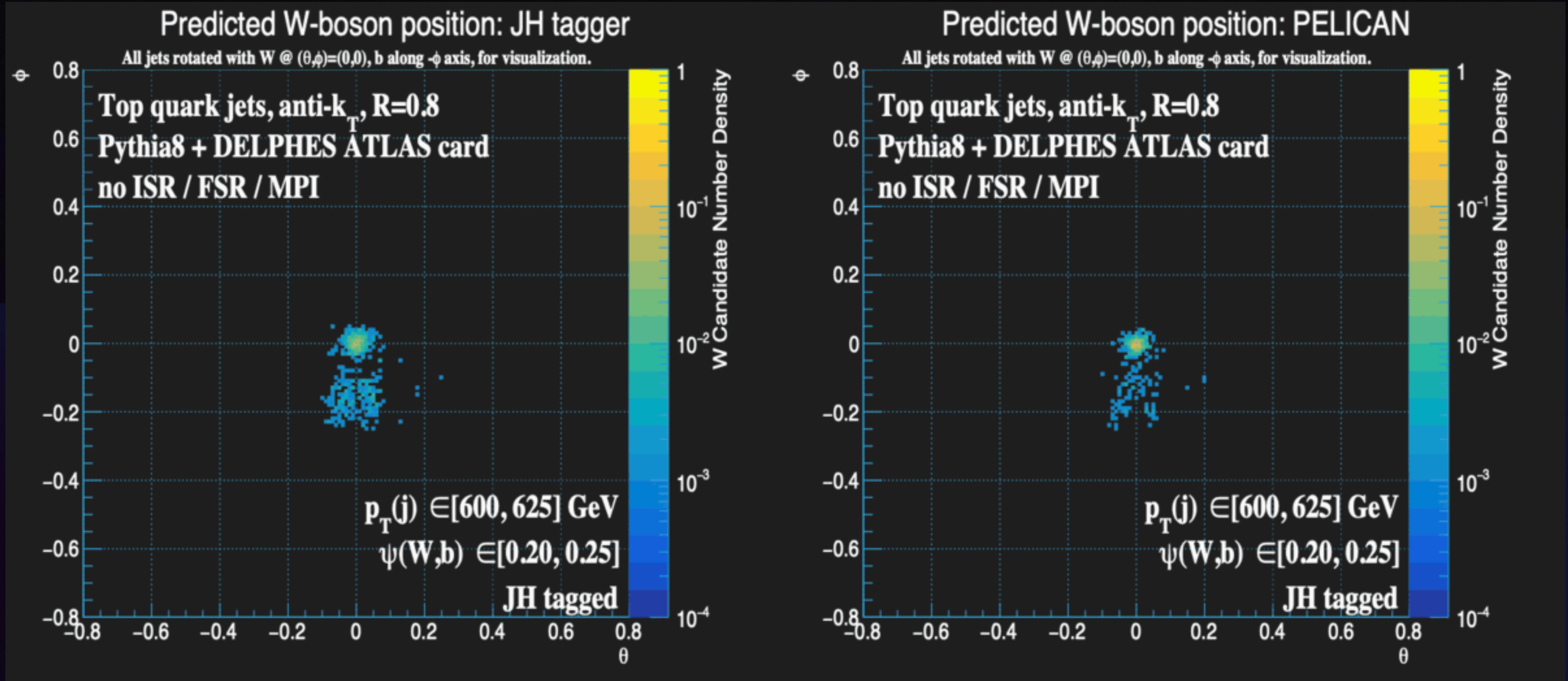


DELPHES



PELICAN is consistently better at identifying the W clusters

DELPHES



PELICAN is consistently better at identifying the W clusters

Other applications

Other applications

- Looking inside jets: parent reconstruction

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- Integrated pipeline: tagging \rightarrow regression

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- IRC safety

Other applications

- Looking inside jets: parent reconstruction
- Integrated pipeline: tagging \rightarrow regression
- Mass measurements
- Polarization tagging (helicity angle distributions)
- IRC safety
- Jet combinatorics?

Watch out for the paper on arXiv today!

