

# Computing anomalous dimensions of strongly-coupled CFTs from supergravity

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with Bobev, Duboeuf, Galli, Giambrone, Guarino, Josse, Nicolai, Petrini,  
Robinson, Samtleben, Sterckx, Trigiante, van Muiden

# The importance of Kaluza-Klein spectra

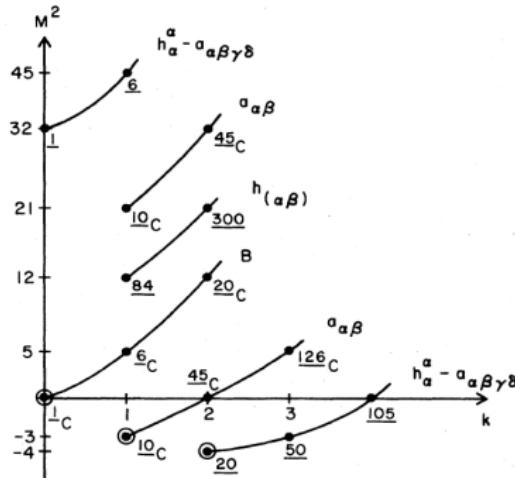
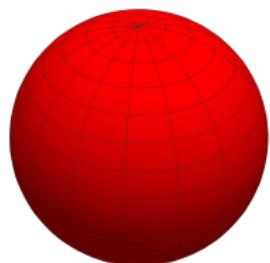


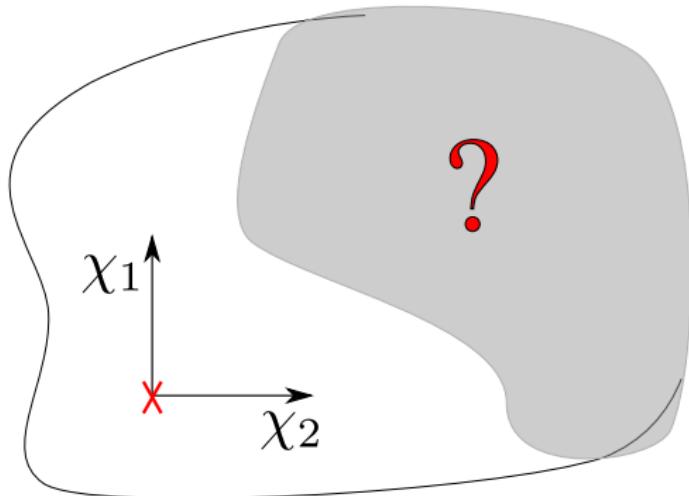
FIG. 2. Mass spectrum of scalars.

- ▶ AdS/CFT: conformal dimensions
- ▶ Stability of non-SUSY vacua?

# AdS/CFT correspondence

Moduli  $\Leftrightarrow$  (exactly) marginal deformations

$$L_{\text{CFT}} \rightarrow L_{\text{CFT}} + \chi_i \mathcal{O}^i$$

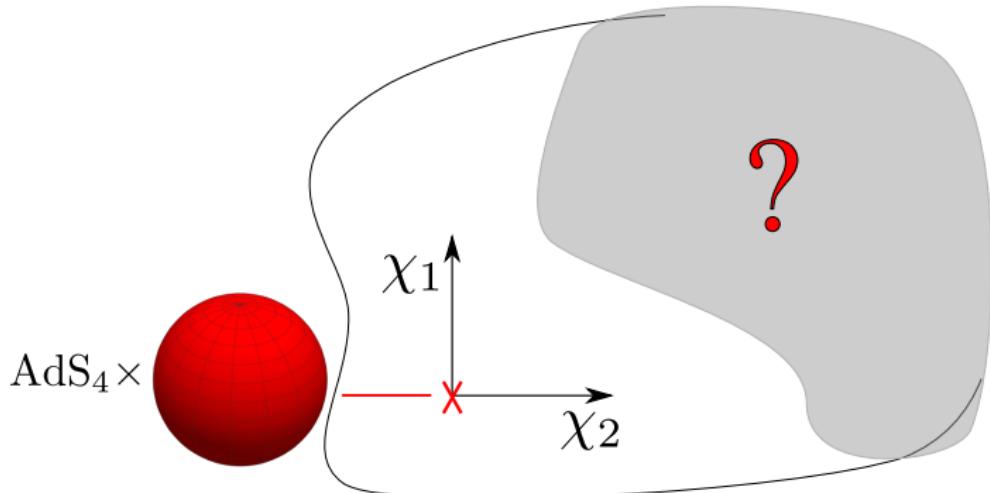


Kaluza-Klein masses  $\Leftrightarrow$  anomalous dimensions

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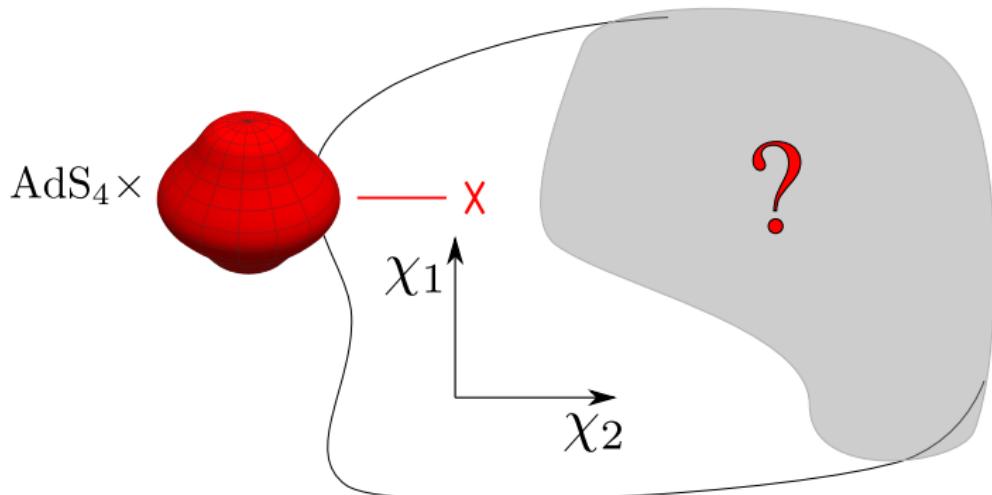


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## Computing Kaluza-Klein spectra is hard

- ▶ Free scalar on  $S^1$ :

$$0 = \partial_x^2 \phi(x, y) + \partial_y^2 \phi(x, y),$$
$$\phi(x, y) = \phi^{(k)}(x) e^{i k y/R}, \quad m^2 = \frac{k^2}{R^2}.$$

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- ▶ SUGRA: (linearised) EoMs mix metric & fluxes  $\Rightarrow$  eigenmodes?

$$\nabla_Q f^{QMN P} + \frac{1}{2} F^{QMNP} \nabla_Q h_R{}^R - \nabla_Q \left( h^{QR} F_R{}^{MNP} \right) - 3 \nabla^Q \left( h^{S[M} F_{QS}{}^{NP]} \right) = -\frac{1}{288} \epsilon^{MNPQ_1\dots Q_8} F_{Q_1\dots Q_4} f_{Q_5\dots Q_8}.$$

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- ▶ Previously, only two cases understood:

- ▶ Spin-2 fields [Bachas, Estes '11] ✓
- ▶  $M_{int} = \frac{G}{H}$  ✓

## Another tool: Consistent truncations

- ▶ Non-linear truncation to subset of KK-modes
  - ▶ Solutions are solutions to higher-dim theory
  - ▶ Compute subset of masses for any vacuum.
  - ▶ Results can be misleading!

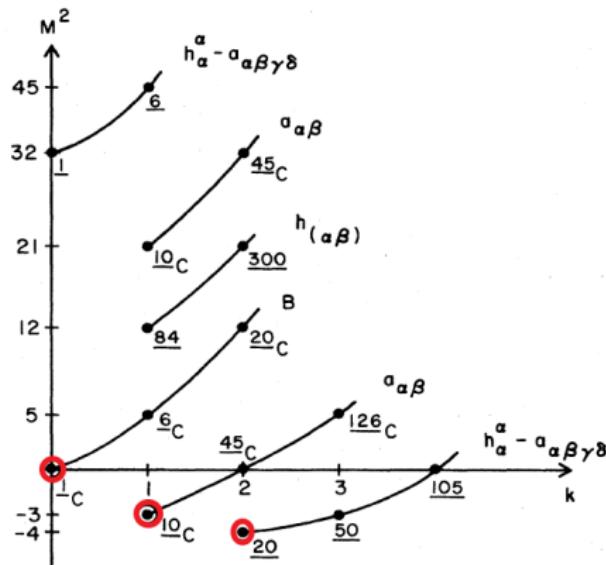
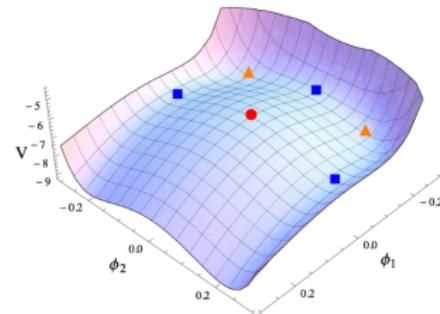


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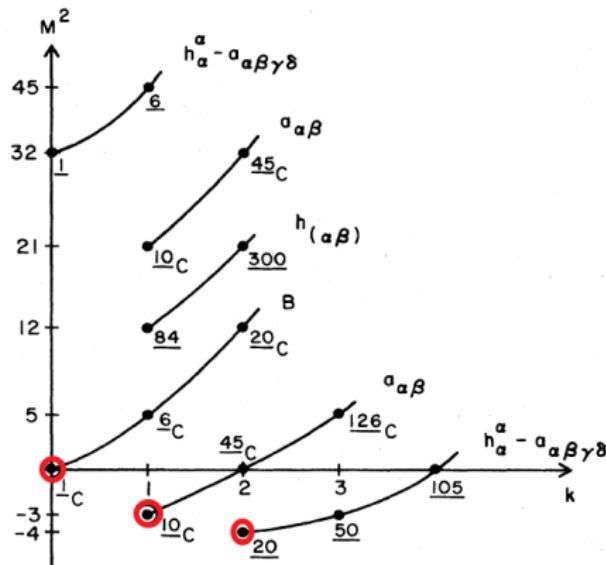
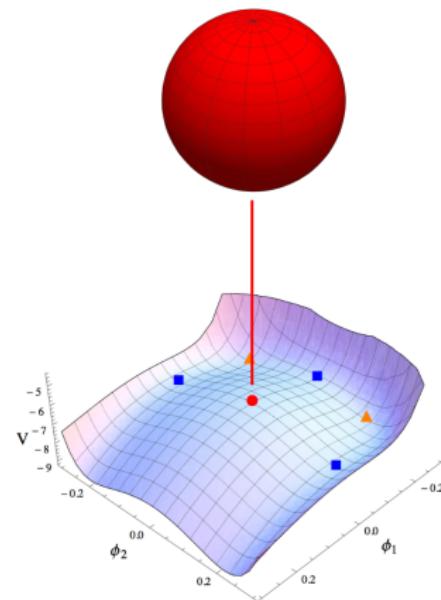


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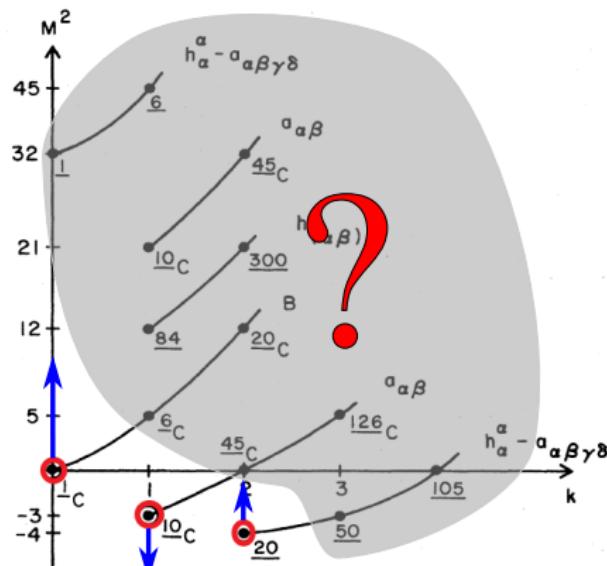
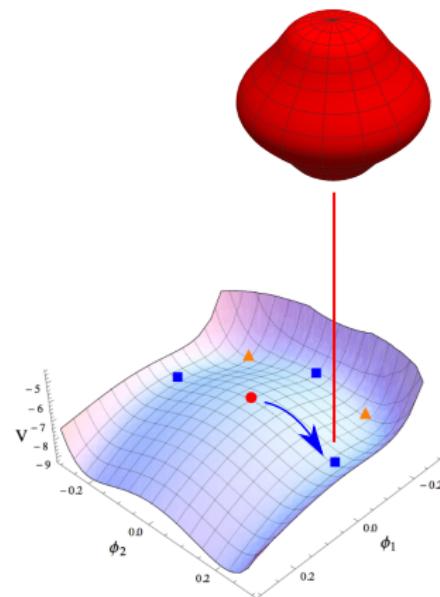


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## Another tool: Consistent truncations

- ▶ No
  - ▶ Sol
  - ▶ Co
  - ▶ Res

[EM, Samtleben '20]

Extend this to full KK spectrum using ExFT!  
Exploit hidden structures

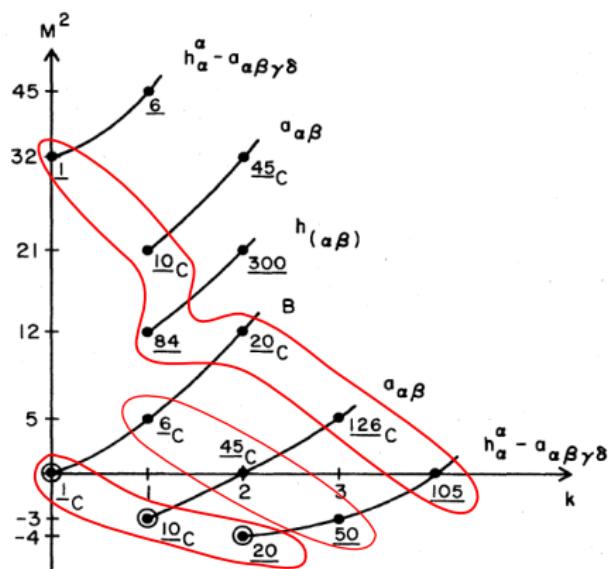
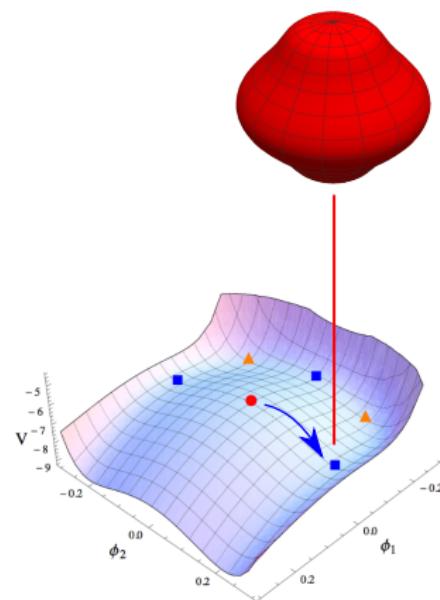
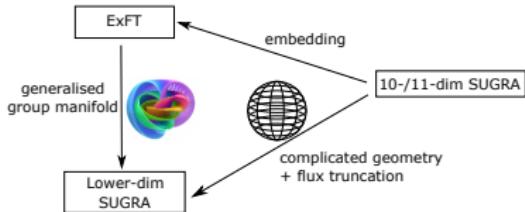


FIG. 2. Mass spectrum of scalars.



# Exceptional Field Theory & consistent truncations



## Kaluza-Klein spectroscopy

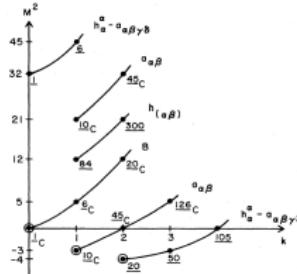
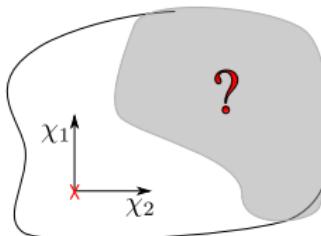
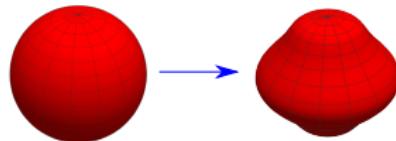


FIG. 2. Mass spectrum of scalars.

## Applications



KK Spectroscopy  
beyond consistent truncations



## Consistent truncation

## Non-linear embedding of lower-dimensional theory into 10-/11-d supergravity

- ▶ All solutions of lower-d SUGRA  $\rightarrow$  solutions of 10-/11-d SUGRA
  - ▶ Non-linearity: highly non-trivial!
  - ▶ Symmetry arguments crucial

NB: No well-controlled AdS vacua of String Theory have scale separation

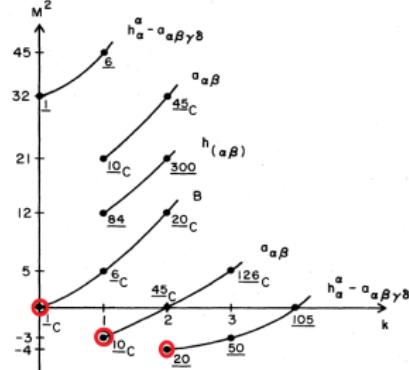
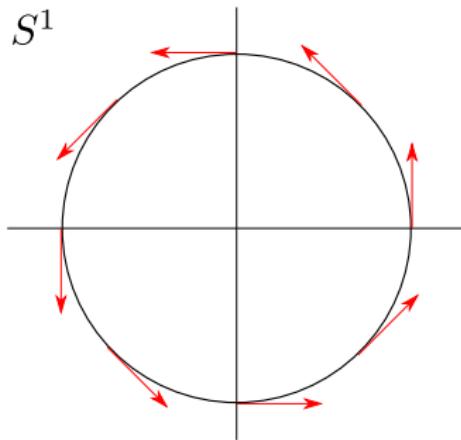


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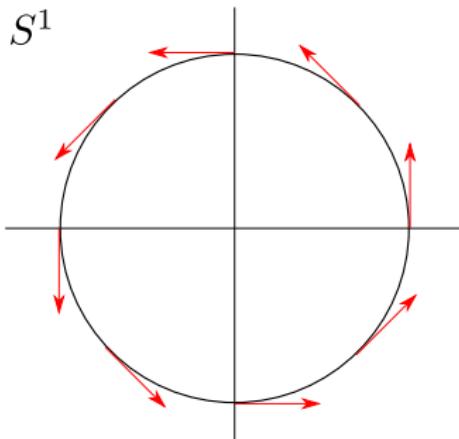
## Consistent truncation on group manifold

Symmetry arguments crucial for consistency, e.g.  
group manifold



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$$U_m{}^\mu \in \mathrm{GL}(D)$$

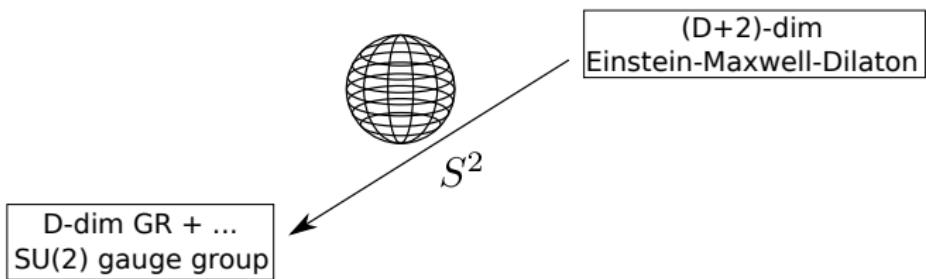
$$\mathcal{L}_{U_m} U_n = f_{mn}{}^\rho U_\rho$$

$$g_{\mu\nu}(x, y) = g_{mn}(x) (U^{-1})_\mu{}^m(y) (U^{-1})_\nu{}^n(y)$$

# Larger symmetry groups from generalising geometry

Symmetry argument for other consistent truncations?

$$S = \int d^{D+2}x \sqrt{|g|} \left( R_g - (\nabla\phi)^2 - e^{\alpha\phi} F^2 \right)$$



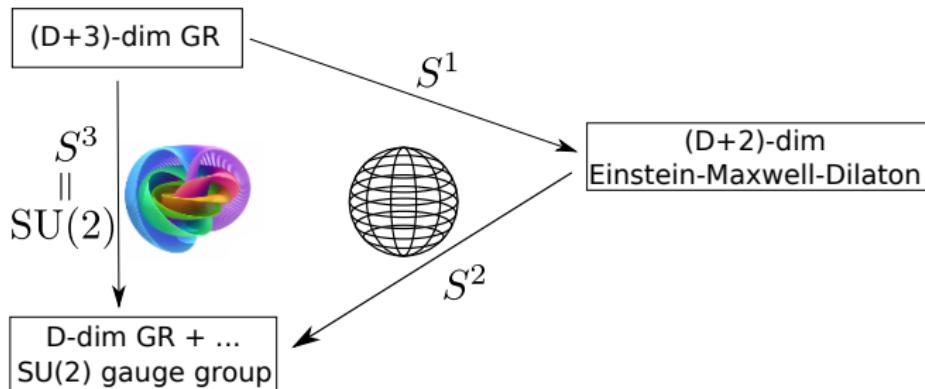
$$\begin{aligned} ds_{D+2}^2 &= Y^{\frac{1}{D}} \left( \Delta^{\frac{1}{D}} ds_D^2 + g^{-2} \Delta^{-\frac{D-1}{D}} T_{ij}^{-1} \mathfrak{D}\mu^i \mathfrak{D}\mu^j \right), \\ e^{\sqrt{\frac{2(D)}{D+1}} \hat{\phi}} &= \Delta^{-1} Y^{\frac{D-1}{D+1}}, \\ F_2 &= \frac{1}{2} \epsilon_{ijk} \left( g^{-1} \Delta^{-2} \mu^i \mathfrak{D}\mu^j \wedge \mathfrak{D}\mu^k - 2g^{-1} \Delta^{-2} \mathfrak{D}\mu^i \wedge \mathfrak{D}T_{jl} T_{km} \mu^l \mu^m - \Delta^{-1} F_{(2)}^{ij} T_{kl} \mu^l \right). \end{aligned}$$

**[Cvetic, Lü, Pope '00]**

# Larger symmetry groups from generalising geometry

Symmetry argument for other consistent truncations?

$$S = \int d^{D+3}x \sqrt{|G|} (R_G)$$



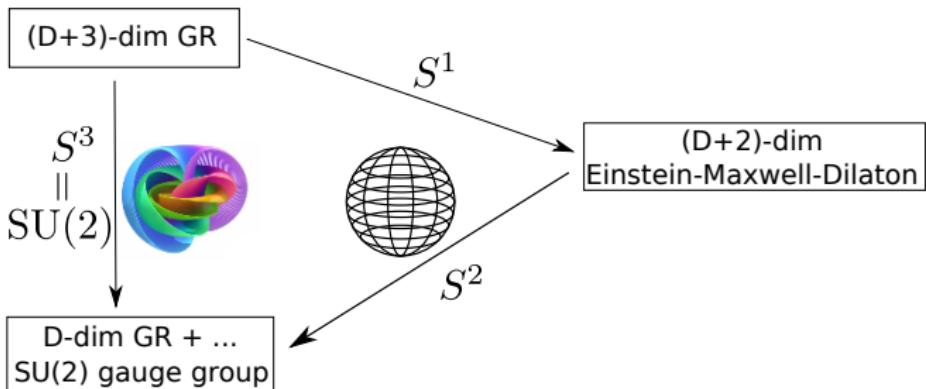
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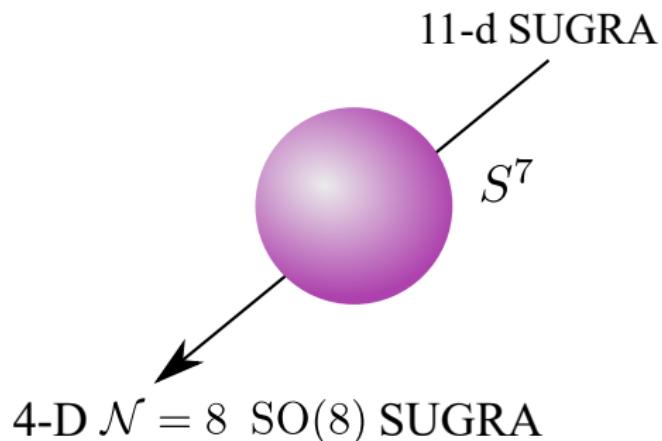
$$\mathcal{L}_{U_m} U_n = f_{mn}{}^p U_p$$

$$G_{\mu\nu}(\textcolor{blue}{x}, \textcolor{red}{y}) = \textcolor{blue}{G}_{mn}(x) (\textcolor{red}{U}^{-1})_\mu{}^m(y) (\textcolor{red}{U}^{-1})_\nu{}^n(y)$$

[Cvetic, Lü, Pope, Gibbons '03]

# Consistent truncations beyond group manifolds

Consistent truncations of 10-d/11-d SUGRA beyond  
group manifolds?



[de Wit, Nicolai '82]

# Exceptional Field Theory

..., [Berman, Perry '10], [Coimbra, Strickland-Constable, Waldram '11],  
[Hohm, Samtleben, '13], ...

Exceptional Field Theory: Unify metric + fluxes of supergravity

11-d SUGRA on  $M_4 \times C_7$ :

$$\{g, C_{(3)}, C_{(6)}, \dots\} = \mathcal{M}_{MN} \in \frac{E_{7(7)}}{\text{SU}(8)}.$$

Diffeo + gauge transf  $\rightarrow$  generalised vector field  $V^M \in \mathbf{56}$  of  $E_{7(7)}$   
Lie derivative  $\rightarrow$  generalised Lie derivative

$$\mathcal{L}_V = V^M \partial_M - (\partial \times_{adj} V) = \text{diffeo + gauge transf},$$

$$\text{with } \partial_M = (\partial_i, \partial^{ij}, \partial^{ijklm}, \dots) = (\partial_i, 0, \dots, 0).$$

# Exceptional Field Theory = reformulation of supergravity

Exceptional Field Theory: Reformulation of 10-/11-d supergravity

$$\{g, C_{(3)}, C_{(6)}, \dots\} = \mathcal{M}_{MN}$$

$$L = R - \frac{1}{48} F_{\mu\nu\lambda\rho} F^{\mu\nu\lambda\rho} + \dots$$

with  $F_{\mu\nu\rho\lambda} = 4\partial_{[\mu}C_{\nu\rho\lambda]}.$

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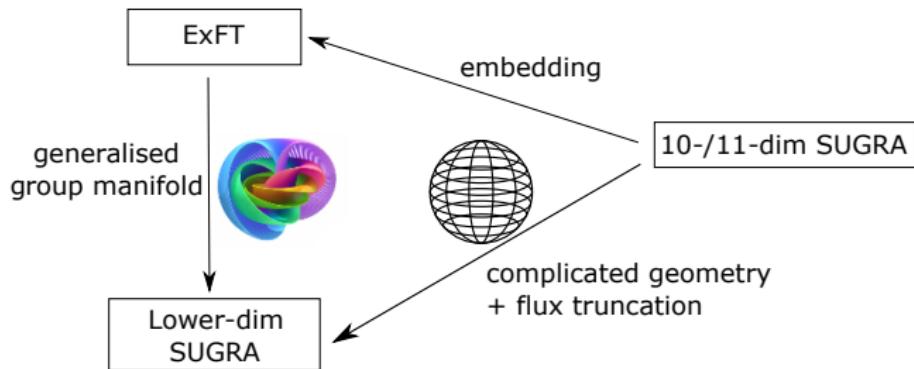
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Generalised Lie derivative  $\Rightarrow$  generalised Ricci scalar

Similar for type II theories & other dimensions

# Exceptional Field Theory and consistent truncations

Consistent truncations to max. gSUGRA captured by  
“generalised group manifolds” in ExFT



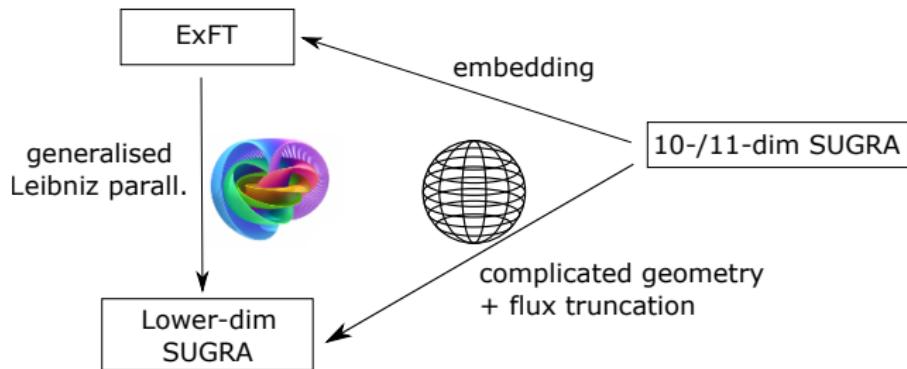
$$U_A{}^M \in E_{7(7)}$$

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C$$

$$\mathcal{M}_{MN}(x, Y) = M_{AB}(x)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

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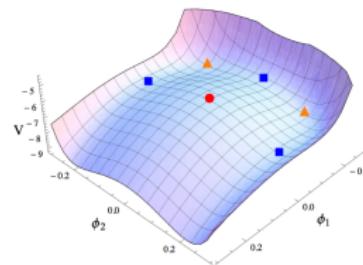
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## Implications for AdS vacua

e.g. deformations of  $\text{AdS}_4 \times S^7$ ,  $\text{AdS}_5 \times S^5$ , ...

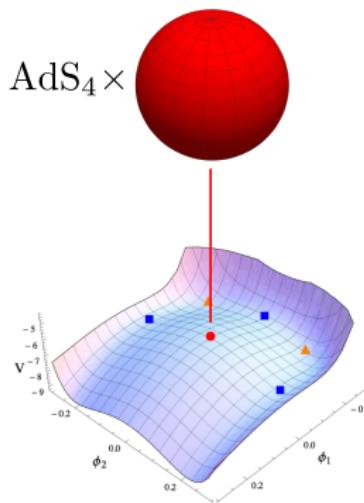
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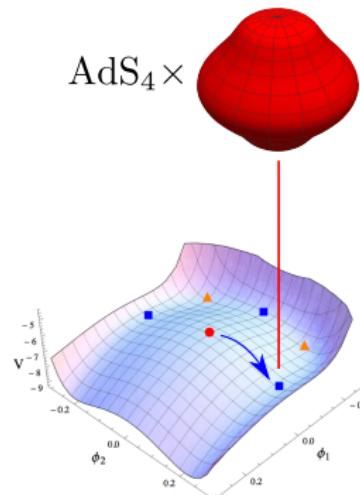
$$\mathcal{M}_{MN}(\textcolor{blue}{x}, Y) = \delta_{AB} (U^{-1})_M{}^A(Y) (U^{-1})_N{}^B(Y)$$



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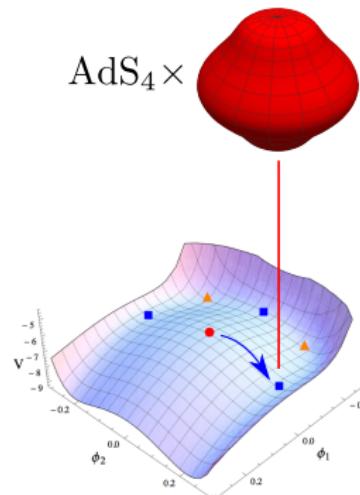


Warped compactifications with few/no remaining (super-)symmetries

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Warped compactifications with few/no remaining (super-)symmetries

“Hidden” group structure!

# Kaluza-Klein spectroscopy

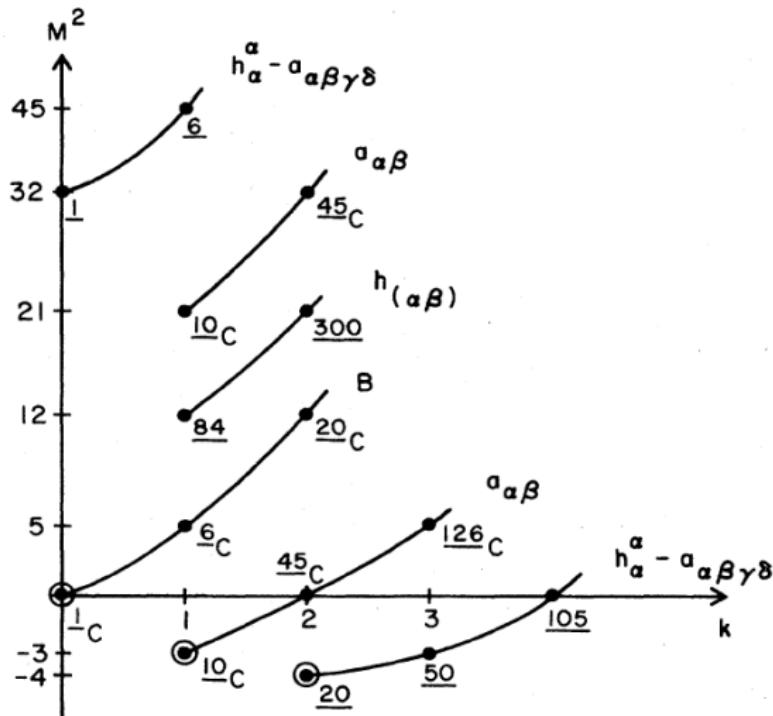


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## KK spectroscopy strategy

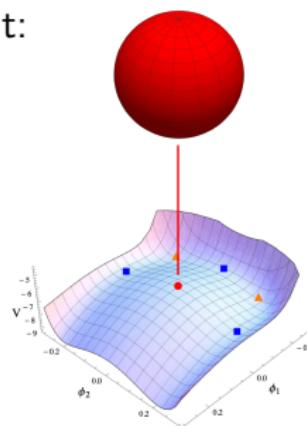
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ExFT KK Ansatz:  $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{harmonics}}_{\text{linear}}$

First at max symmetric point:

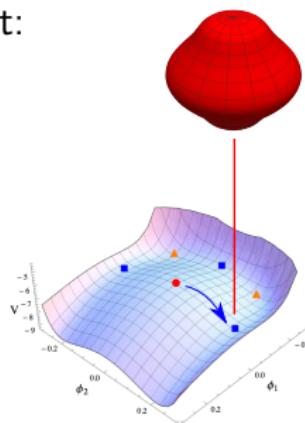


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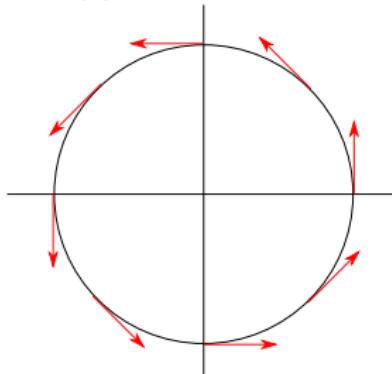
Then at less symmetric point:



Warped compactifications with few/no remaining (super)symmetries!  
Spectrum along RG flow!

## KK spectroscopy

$U_A^M \in E_{7(7)}$  give basis for all fields



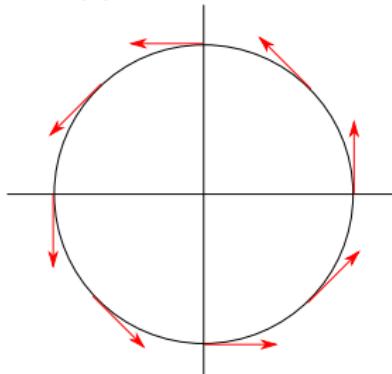
Only need scalar harmonics:  $\mathcal{Y}_\Sigma$

c.f.  $h_{ij}(x, y) = \sum_\ell h^{(\ell)}(x) \mathcal{Y}_{(ij)}^{(\ell)}(y), \quad c_{ijk}(x, y) = \sum_\ell c^{(\ell)}(x) \mathcal{Y}_{[ijk]}^{(\ell)}(y)$

“ $\mathcal{N} = 8$  supermultiplet contains all SUGRA fields”

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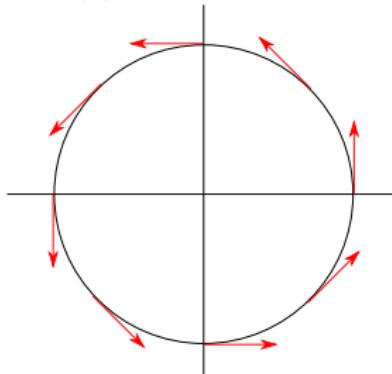


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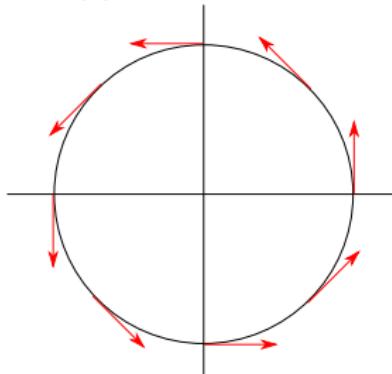
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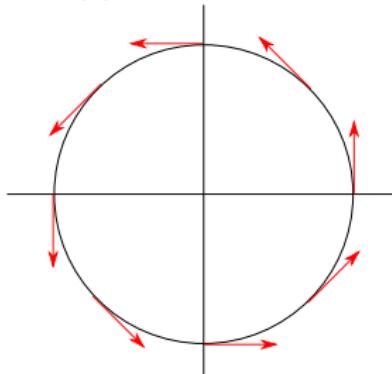


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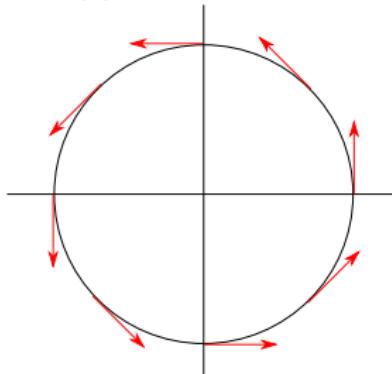
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KK Ansatz = consistent truncation  $\otimes$  scalar harmonics

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Immediate mass diagonalisation for any vacuum!

## Mass matrix

- ▶ Lower-dim info:

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C ,$$

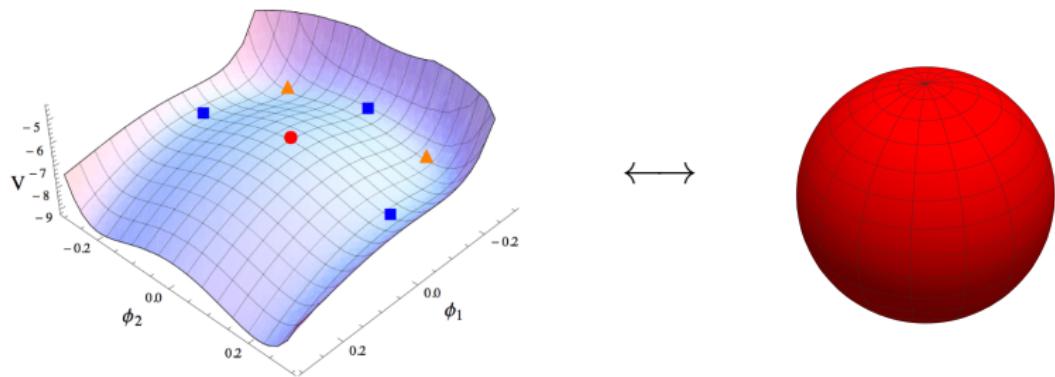
- ▶ Higher-dim info:

$$\mathcal{L}_{U_A} \mathcal{Y}_\Sigma = L_{K_A} \mathcal{Y}_\Sigma = T_{A\Sigma}{}^\Omega \mathcal{Y}_\Omega .$$

**Algebraic** mass matrix:

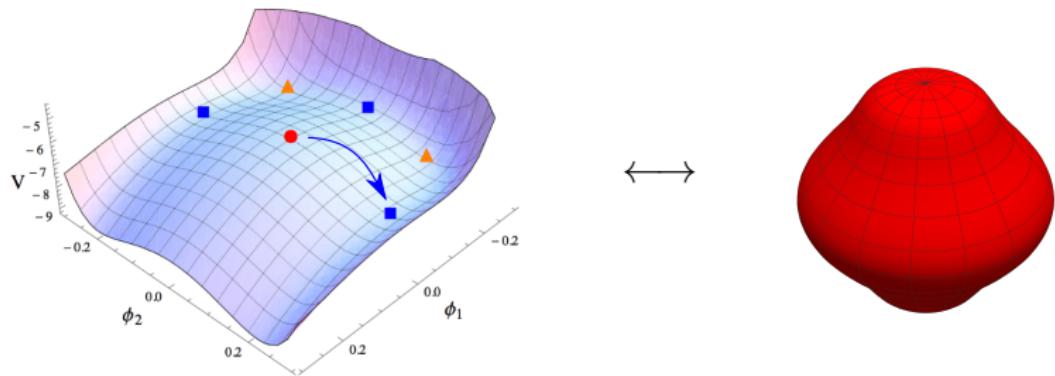
$$M^2 = X^2 + T^2 + XT .$$

# KK spectroscopy at less symmetric point



KK Ansatz:  $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{scalar harmonics}}_{\text{linear}}$

# KK spectroscopy at less symmetric point



KK Ansatz:  $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{scalar harmonics}}_{\text{linear}}$

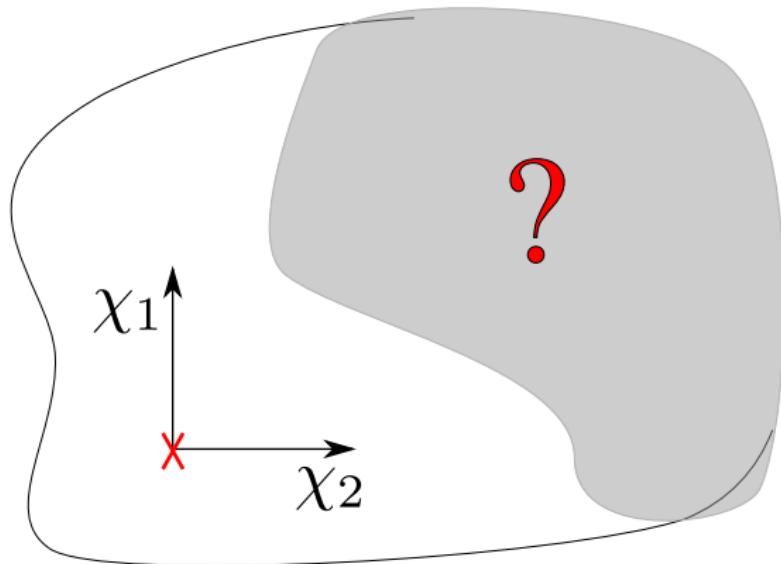
Use same harmonics as for max. symmetric point

Multiplication by  $E_{7(7)}$  matrix,  $M_{AB}(x)$ !

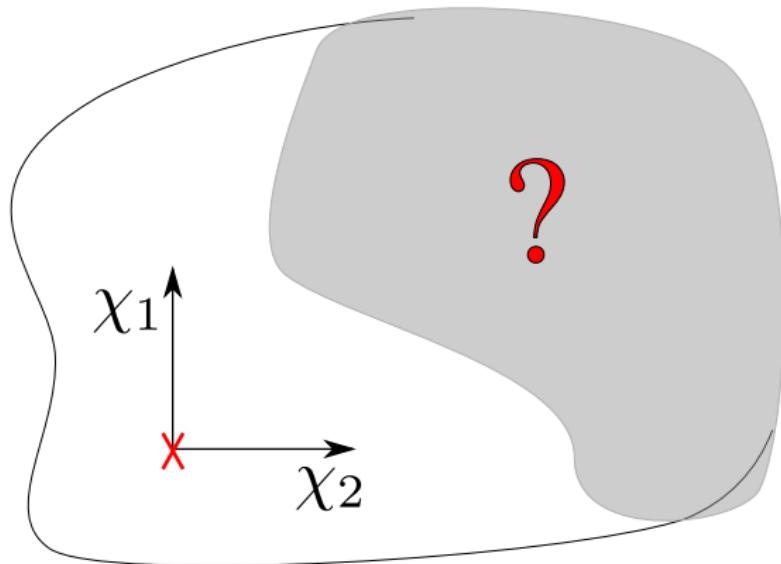
## KK Spectroscopy Summary

- ▶ Only scalar harmonics of maximally symmetric point (round sphere)
- ▶ ExFT KK Ansatz  $\implies$  Differential problem  $\rightarrow$  algebraic problem
- ▶ Compute full spectrum for any vacuum in consistent truncation
- ▶ Spectrum for compactifications with few/no remaining (super-)symmetries

# Applications



# Applications



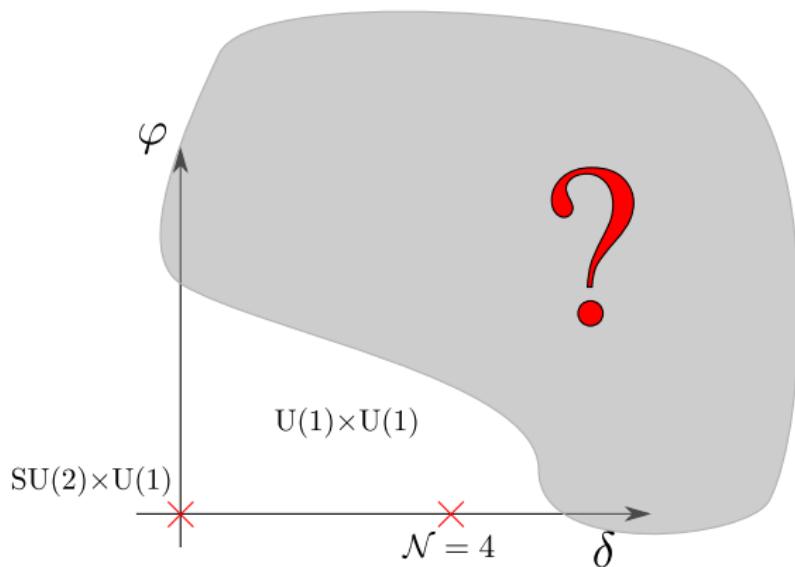
1. Global properties of conformal manifold
2. Non-SUSY AdS

## Ex 1. $\mathcal{N} = 2$ AdS<sub>4</sub> family

$[\mathrm{SO}(6) \times \mathrm{SO}(1, 1)] \ltimes \mathbb{R}^{12}$  supergravity

2 moduli  $(\varphi, \delta) \in \mathbb{R}_{\geq 0}^2$  in 4-d theory  $\Leftrightarrow \mathcal{N} = 2$  conformal manifold

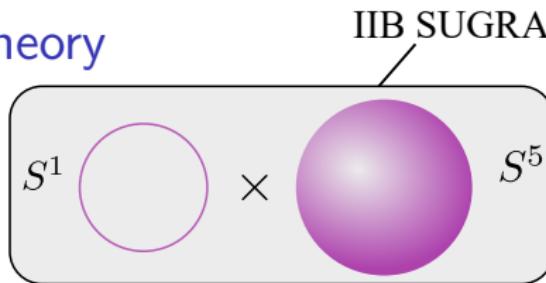
[Guarino, Sterckx, Trigiante '20], [Bobev, Gautason, van Muiden '21]



Expected to be compact e.g. [Perlmutter, Rasteli, Vafa, Valenzuela, '20]

## Ex 1. Uplift to IIB string theory

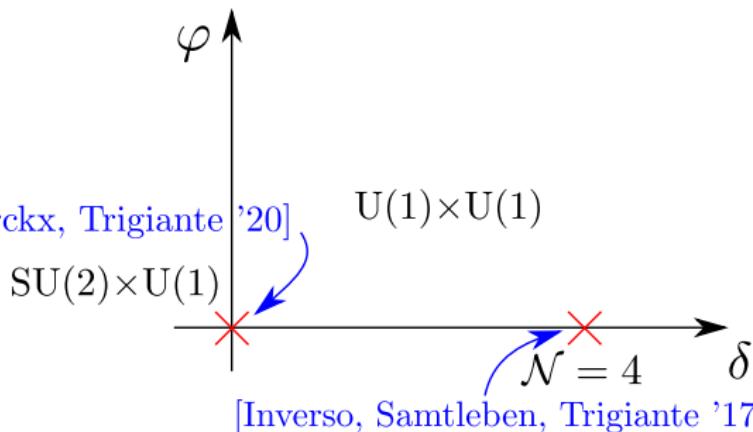
[Inverso, Samtleben, Trigiante '16]



4-D  $[\text{SO}(6) \times \text{SO}(1,1)] \ltimes \mathbb{R}^{12}$  SUGRA

$\text{AdS}_4 \times S^5 \times S^1$  "S-fold" of IIB

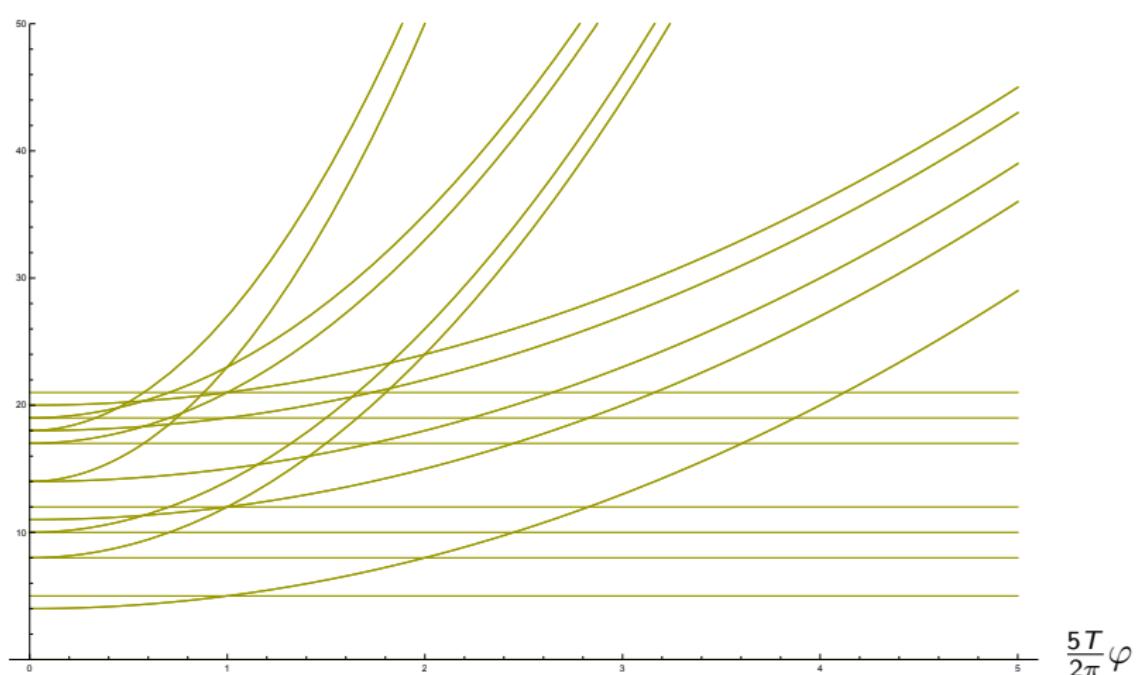
[Guarino, Sterckx, Trigiante '20]



# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along $\varphi$ direction

$$\ell_{S^1} = 0$$

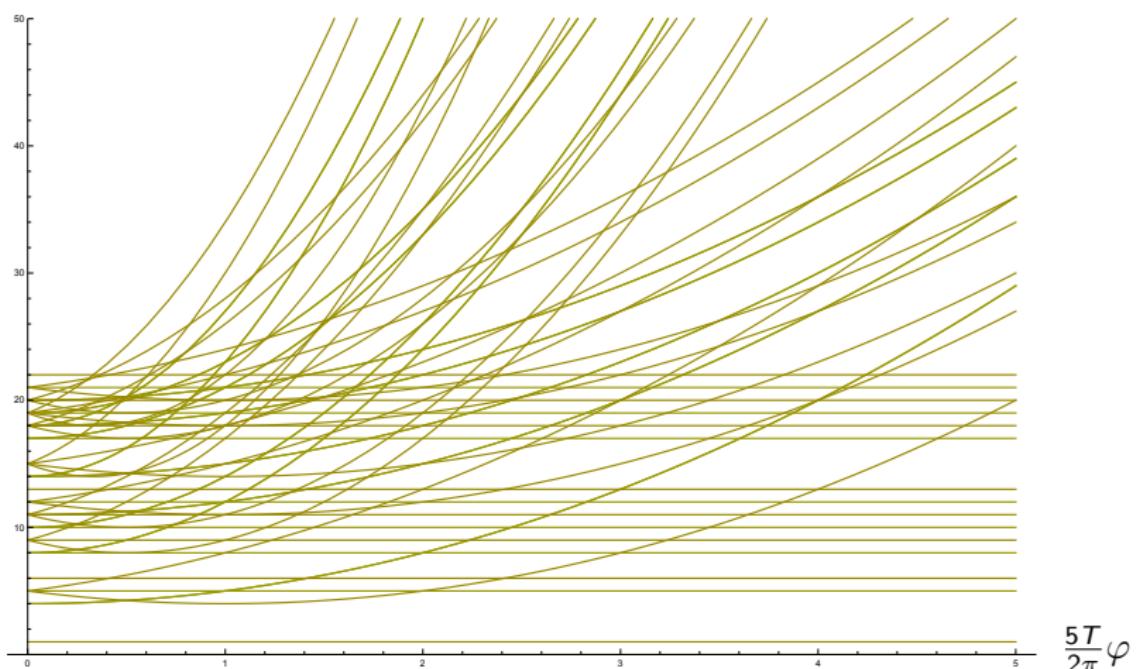
[Giambrone, EM, Samtleben, Trigiante '21]



# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along $\varphi$ direction

$$\ell_{S^1} \leq 1$$

[Giambrone, EM, Samtleben, Trigiante '21]

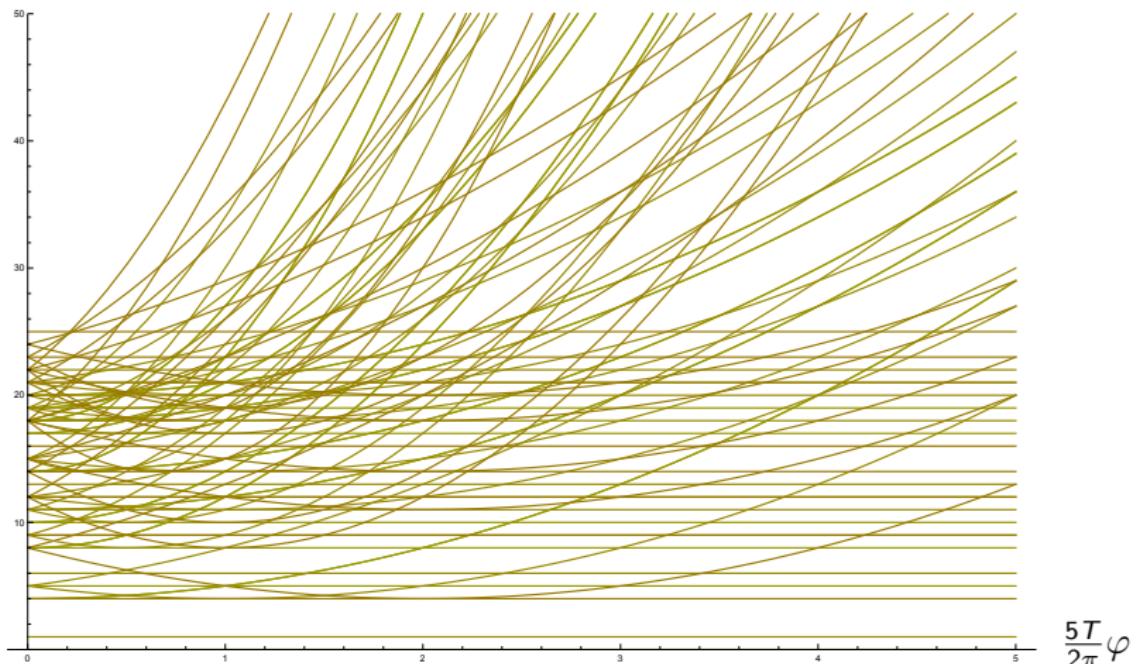


# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along $\varphi$ direction

$$\ell_{S^1} \leq 2$$

[Giambrone, EM, Samtleben, Trigiante '21]

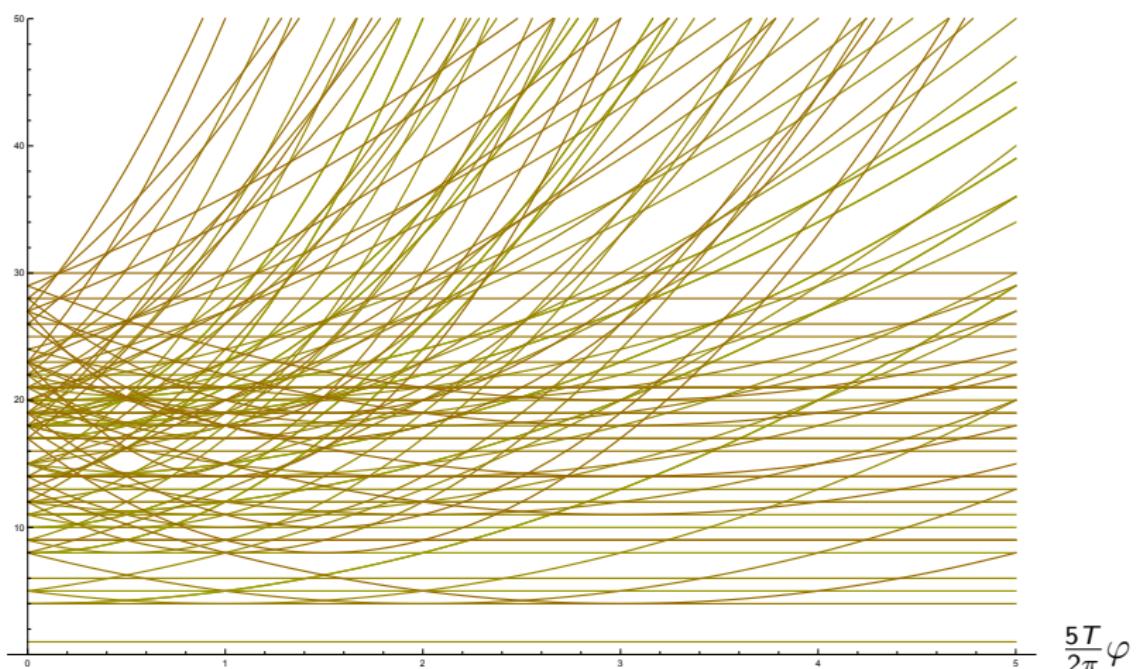
$$m^2 L^2$$



# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along $\varphi$ direction

$$\ell_{S^1} \leq 3$$

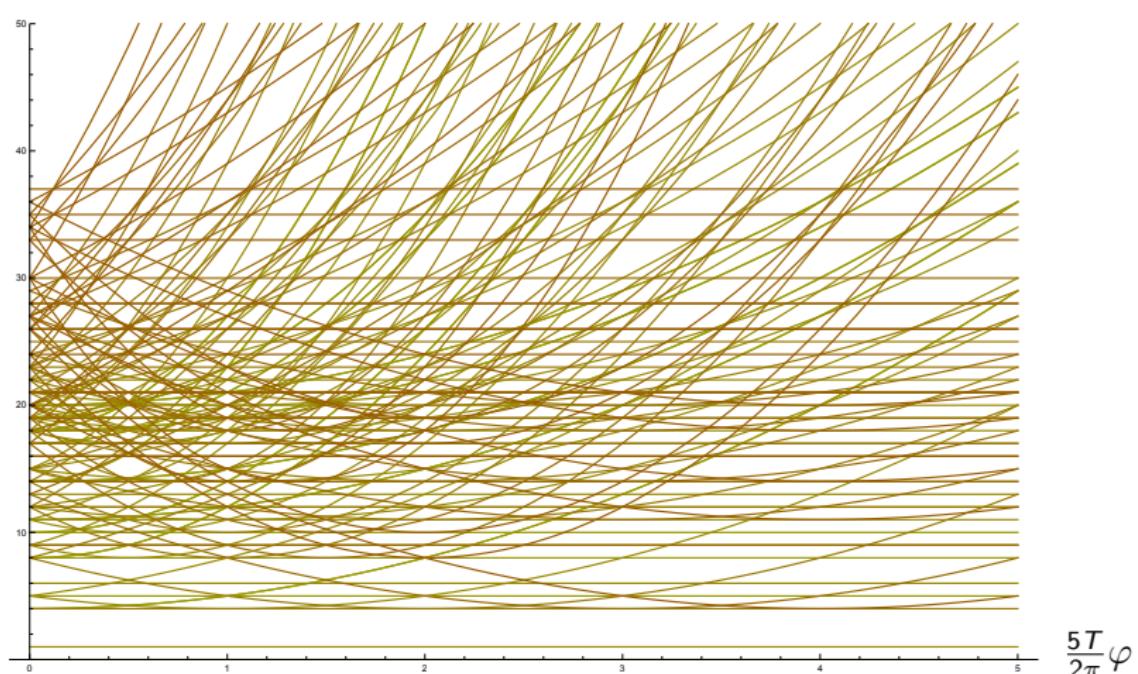
[Giambrone, EM, Samtleben, Trigiante '21]



# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along $\varphi$ direction

$$\ell_{S^1} \leq 4$$

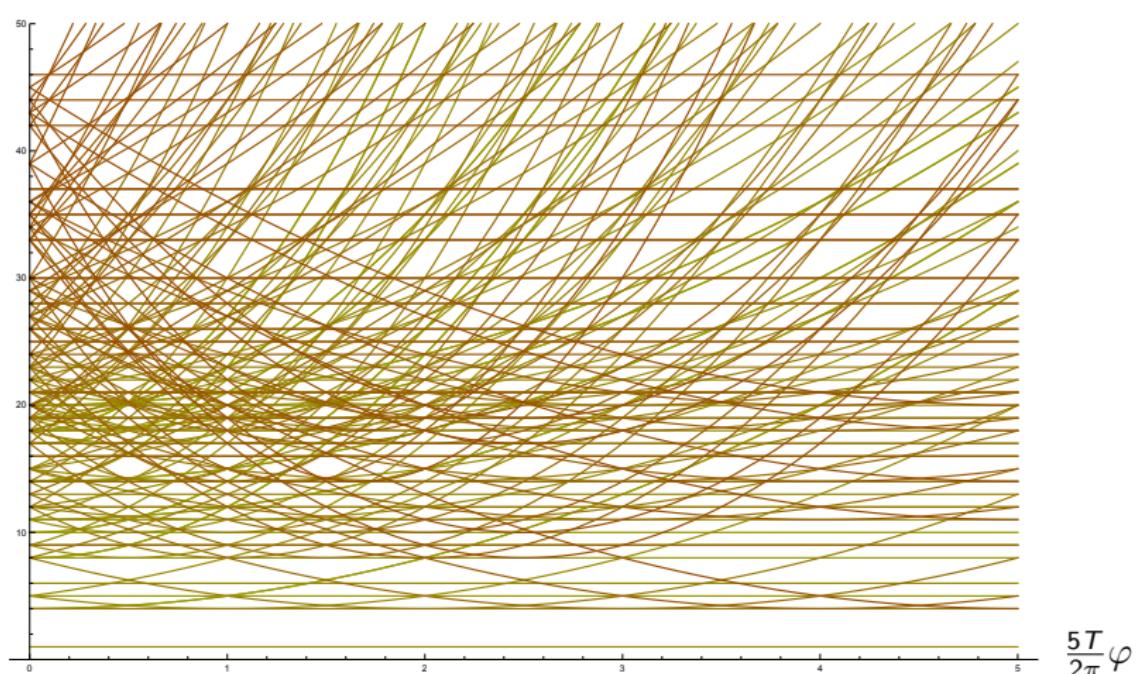
[Giambrone, EM, Samtleben, Trigiante '21]



# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along $\varphi$ direction

$$\ell_{S^1} \leq 5$$

[Giambrone, EM, Samtleben, Trigiante '21]

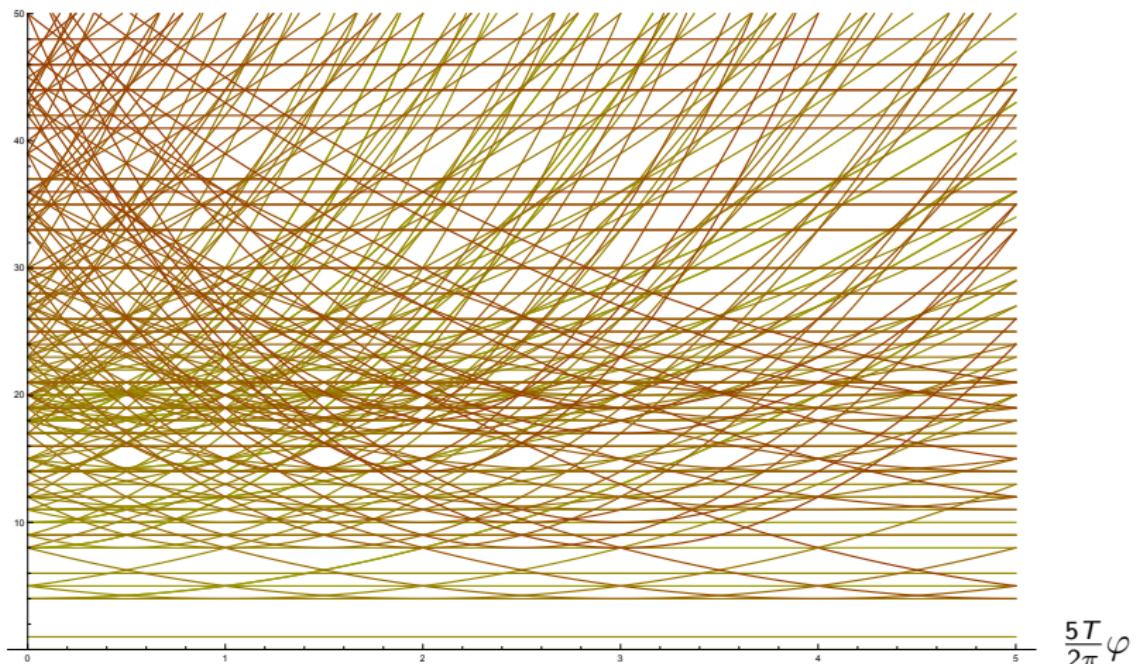


# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along $\varphi$ direction

$$\ell_{S^1} \leq 6$$

[Giambrone, EM, Samtleben, Trigiante '21]

$$m^2 L^2$$

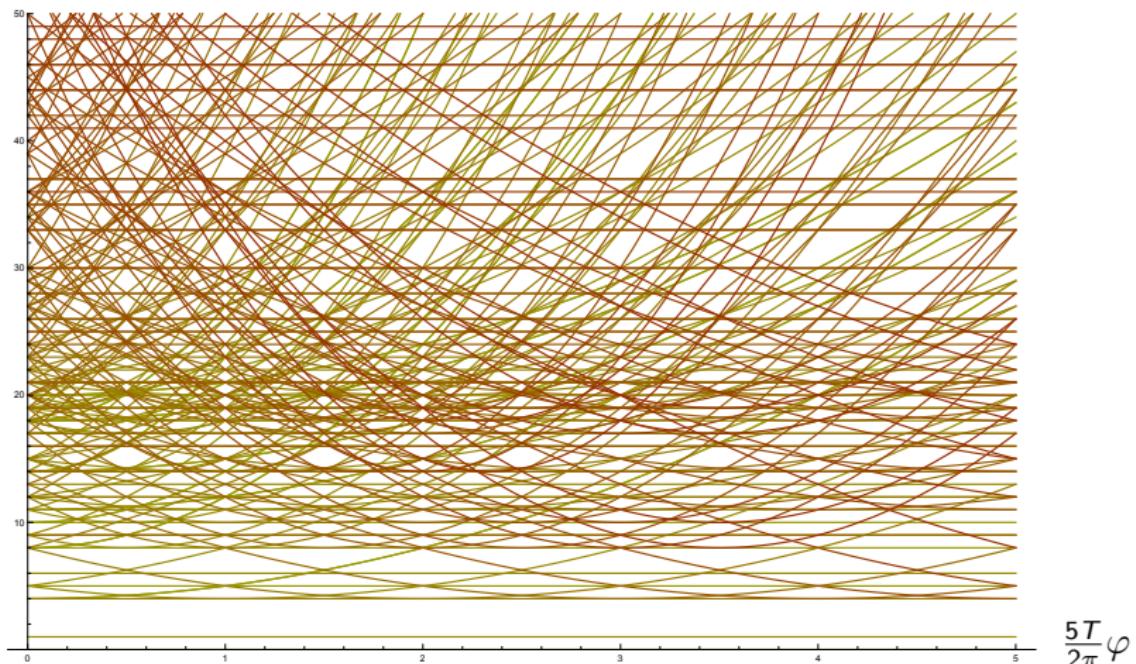


# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along $\varphi$ direction

$$\ell_{S^1} \leq 7$$

[Giambrone, EM, Samtleben, Trigiante '21]

$$m^2 L^2$$

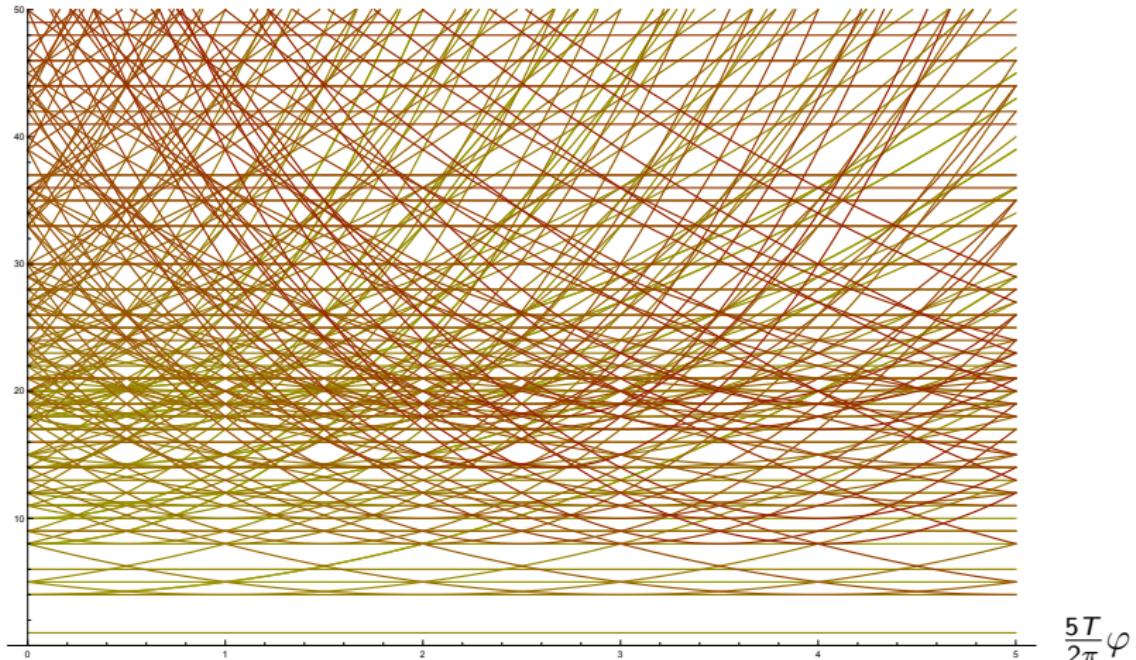


# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along $\varphi$ direction

$$\ell_{S^1} \leq 8$$

[Giambrone, EM, Samtleben, Trigiante '21]

$$m^2 L^2$$

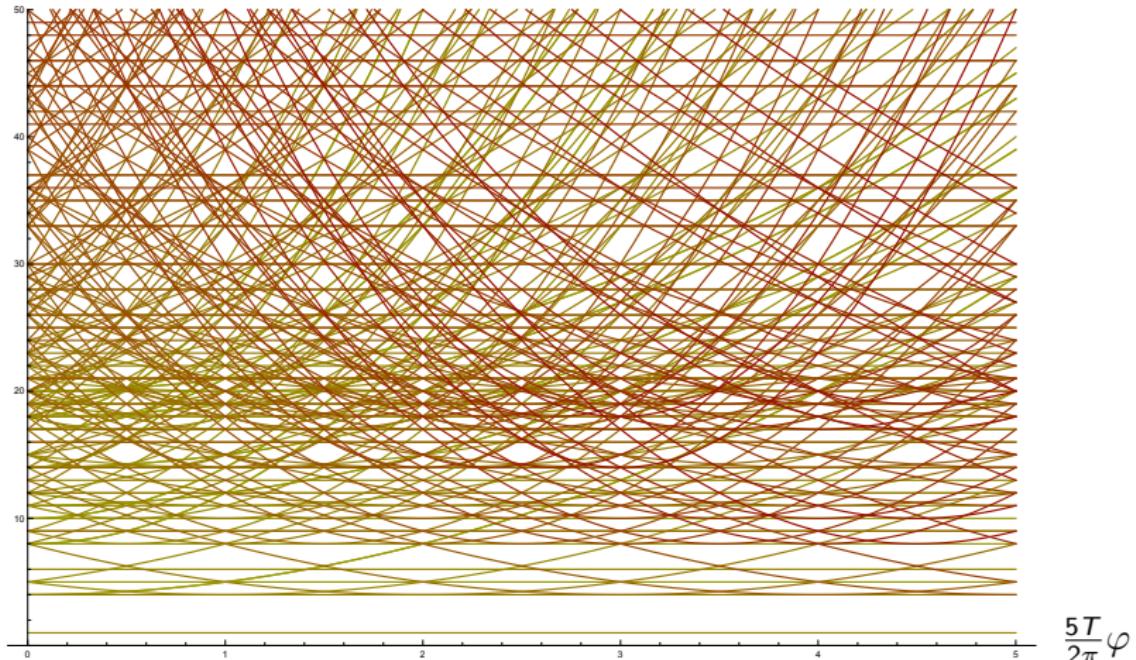


# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold $\text{AdS}_4 \times S^5 \times S^1$ KK spectrum along $\varphi$ direction

$$\ell_{S^1} \leq 9$$

[Giambrone, EM, Samtleben, Trigiante '21]

$$m^2 L^2$$



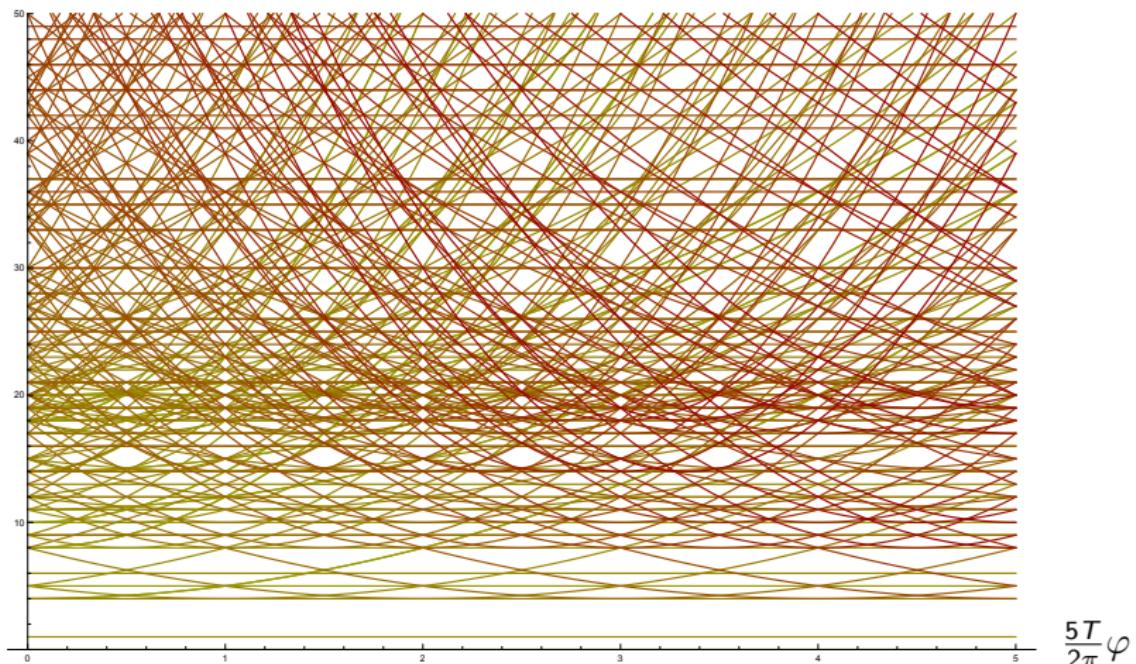
# Ex 1. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\text{AdS}_4 \times S^5 \times S^1$  KK spectrum along  $\varphi$  direction

$$\ell_{S^1} \leq 10$$

[Giambrone, EM, Samtleben, Trigiante '21]

$$m^2 L^2$$



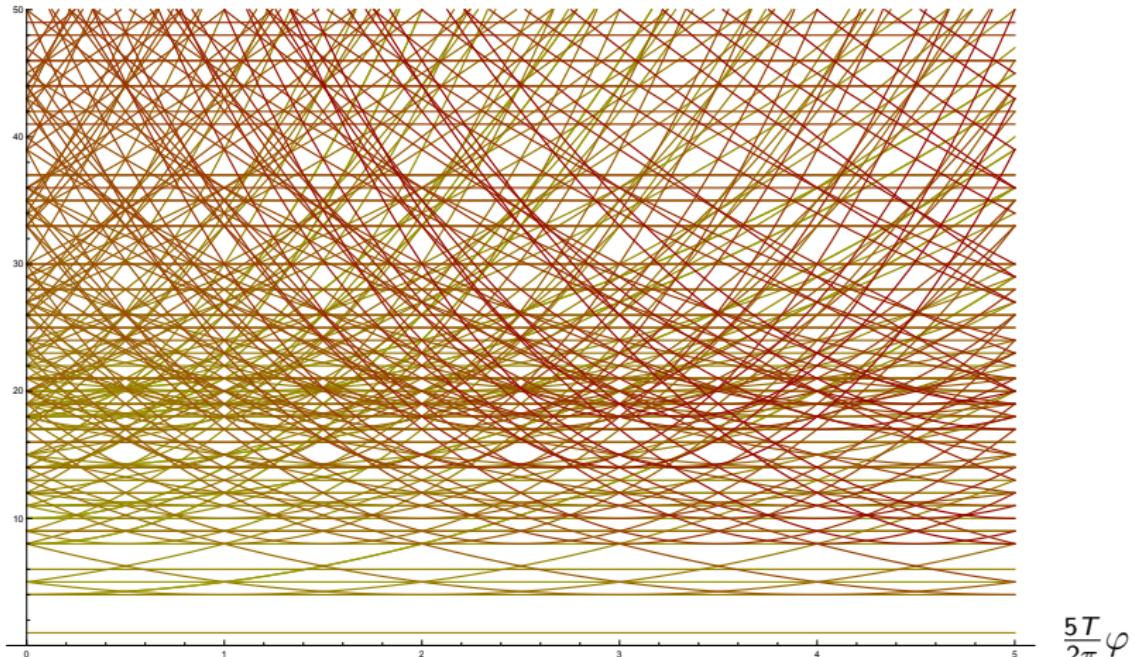
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[Giambrone, EM, Samtleben, Trigiante '21]

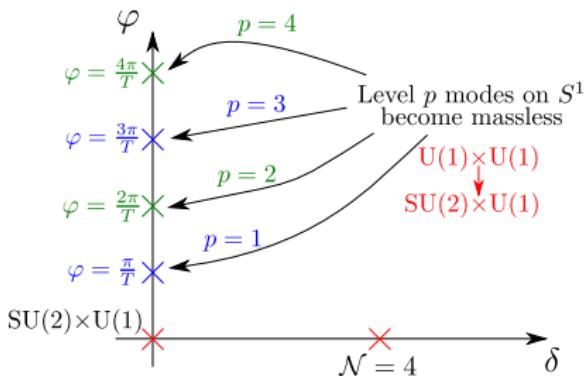
$$m^2 L^2$$



$$\varphi \sim \varphi + \frac{2\pi}{T}, \quad T \text{ radius of } S^1$$

## Ex 1. Space invaders

Higher KK modes become massless when  $\varphi = \frac{p\pi}{T}$ ,  $p \in \mathbb{Z}$   
[Giambrone, EM, Samtleben, Trigiante '21]



Spectrum identical for  $\varphi = \frac{2p\pi}{T}$ ,  $p \in \mathbb{Z}$

Spectrum differs for  $\varphi = \frac{(2p+1)\pi}{T}$ ,  $p \in \mathbb{Z}$

# Ex 1. KK spectrum along $\mathcal{N} = 2$ conformal manifold

[Giambrone, EM, Samtleben, Trigiante '21]

- ▶  $\varphi \in \mathbb{R}^+$  is a 4-d artefact
- ▶  $\varphi \in [0, \frac{2\pi}{T})$  in 10 dimensions
- ▶ KK spectrum as fct of  $\varphi$ :

$$\Delta = \frac{1}{2} + \sqrt{\frac{17}{4} + \frac{1}{2}R^2 - J(J+1) - 2k(k+1) + \ell(\ell+4) + 4\left(\frac{\pi n}{T} - j\varphi\right)^2}.$$

Lorentz spin:  $J$

SU(2) spin:  $k$

U(1)<sub>R</sub> charge:  $R$

U(1)  $\subset$  SU(2) Cartan:  $j$

$S^5$  level:  $\ell$

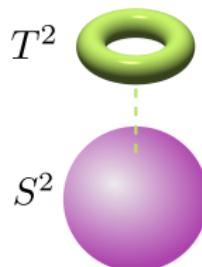
$S^1$  level:  $n$

- ▶ KK spectrum as fct of  $\delta$ : non-compact? [Bobeck, Gautason, van Muiden '21],  
[Cesàro, Larios, Varela '21]

## Ex 1. $\varphi$ as complex structure deformation

[Giambrone, EM, Samtleben, Trigiante '21]

- ▶  $\varphi$ -family:  $\text{AdS}_4 \times S^5 \times S^1$ :  $S^5 \rightarrow S^3 \times S^2$
- ▶  $S^3$  Hopf fibre &  $S^1$ :



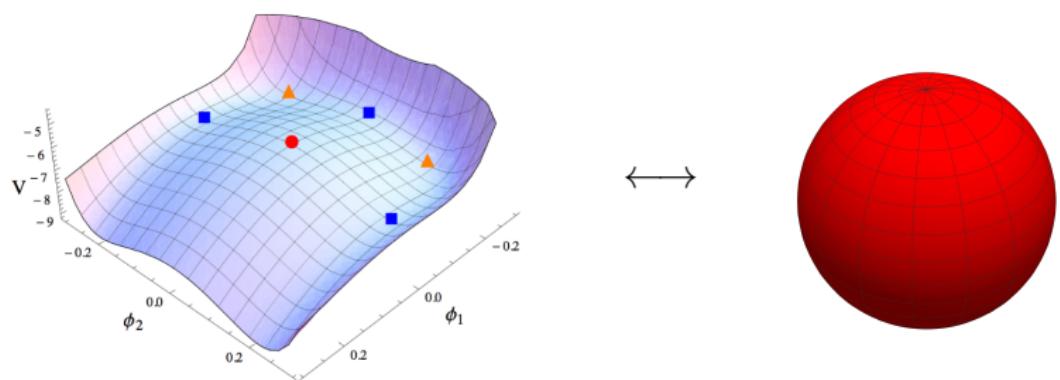
$$\tau = \frac{i}{4\pi} - \frac{\varphi T}{2\pi}$$

$$\varphi \rightarrow \varphi + \frac{2\pi}{T} \implies \tau \rightarrow \tau - 1$$

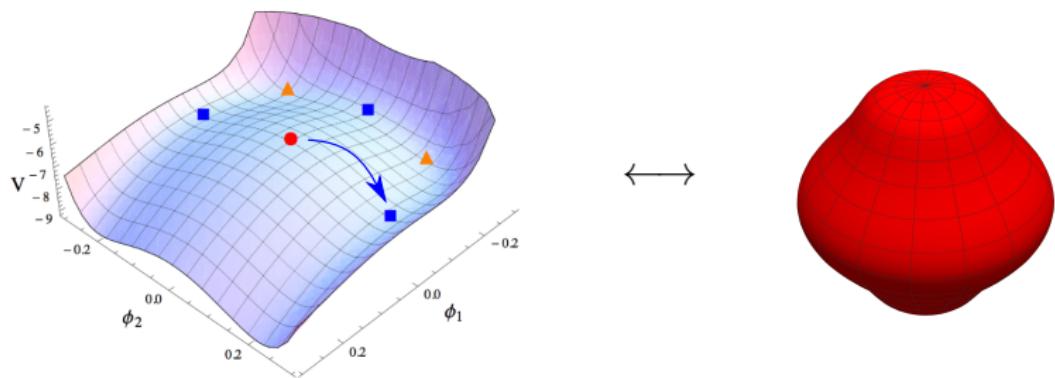
- ▶  $\varphi$  deformation: locally  $\rightarrow$  coordinate transformation

Similar in other S-fold vacua [Cesàro, Larios, Varela '22]

## Application to non-SUSY vacua



## Application to non-SUSY vacua

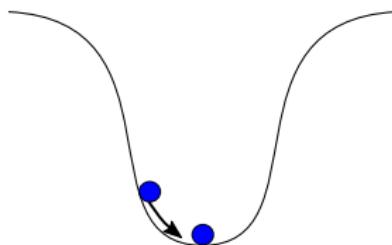


Can compute spectrum for non-SUSY vacua!

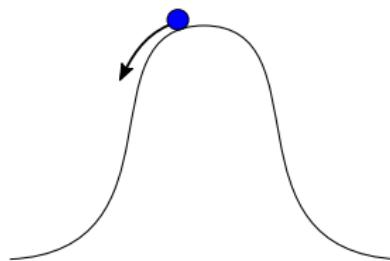
# Stability of non-SUSY AdS vacua

Non-SUSY vacua typically suffer from instabilities

Stable

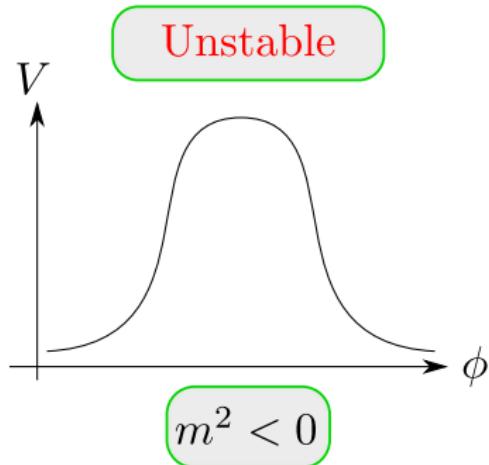
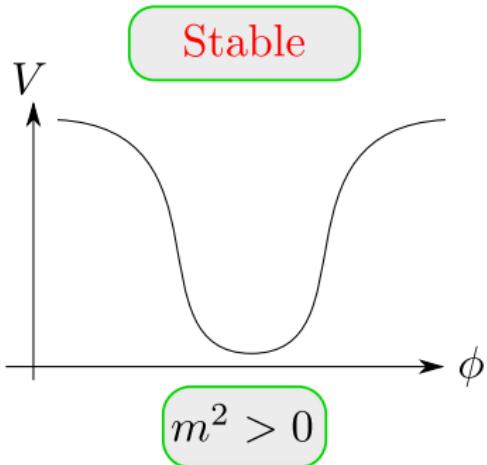


Unstable



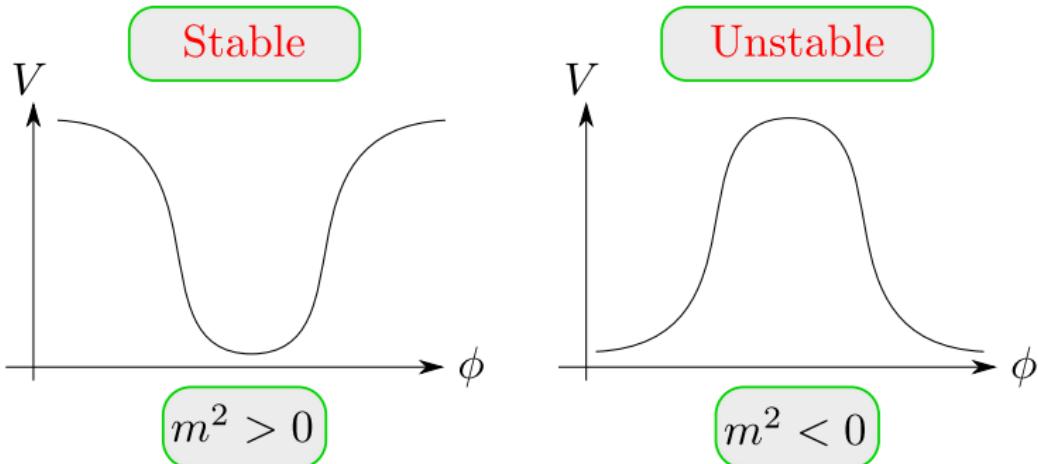
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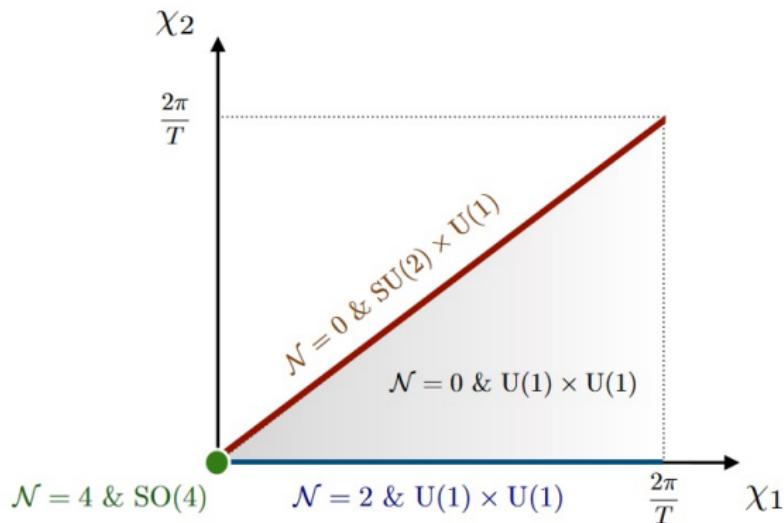
Non-SUSY vacua typically suffer from instabilities



Caveat: anti-de Sitter spacetime  $m^2 < -m_{BF}^2$  for instability

## Ex 2. Non-SUSY flat deformations

2 other flat directions  $\chi_1, \chi_2$  of 4-D supergravity [Guarino, Sterckx '21]



Non-supersymmetric conformal manifold?

## Ex 2. Non-SUSY exactly marginal deformations

Non-SUSY exactly marginal deformations not expected to exist

### Evidence for a miracle

[Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

- ▶ Perturbative stability
- ▶ Non-perturbative stability
- ▶  $\frac{1}{N}$  corrections

$\chi_1, \chi_2$  deformations are locally coordinate transformations!

## Ex 3. Warning: Kaluza-Klein instability

Is zero-mode stability enough?

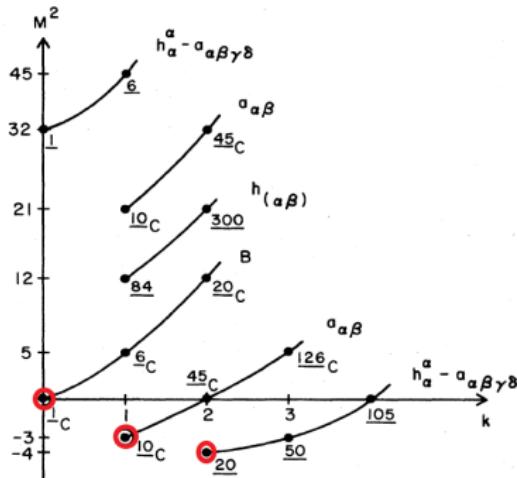
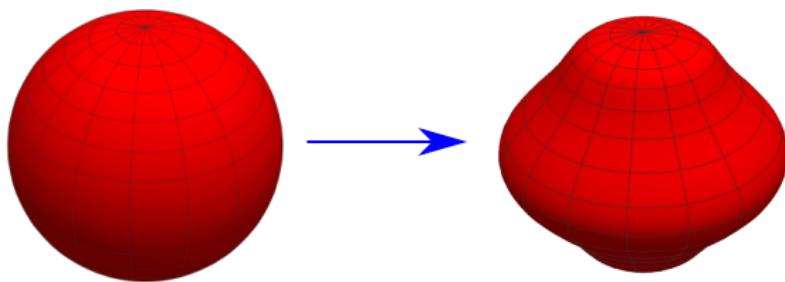


FIG. 2. Mass spectrum of scalars.

Higher KK modes can still be tachyonic!

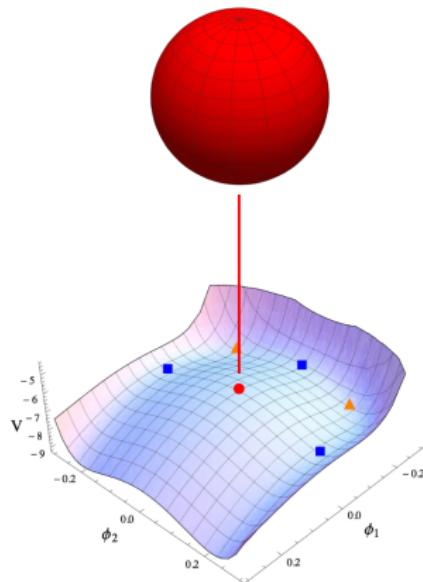
[EM, Nicolai, Samtleben '20]

## KK Spectrometry beyond consistent truncations



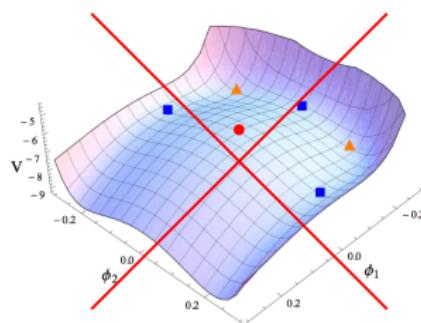
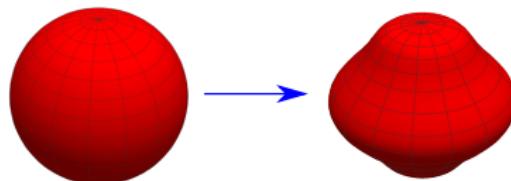
# KK spectrum beyond consistent truncations

Deformations not triggered by  $\mathcal{N} = 8$  scalars?



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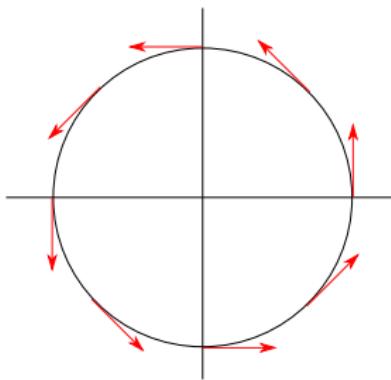
e.g. generic single-trace RG flows of  $\mathcal{N} = 4$  SYM

## Generalised parallelisability

[Du Boef, EM, Samtleben '22]

$U_A{}^M \in E_{7(7)}$  give basis for all fields

$$\text{but, } \mathcal{L}_{U_A} U_B = X_{AB}{}^C(Y) U_C .$$



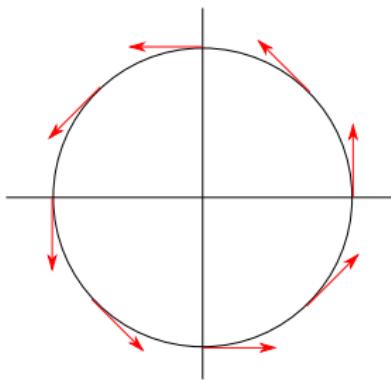
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Only need scalar harmonics:  $\mathcal{Y}_\Sigma$

$$\mathcal{M}_{MN}(x, Y) = (\delta_{AB} + j_{AB}{}^\Sigma(x) \mathcal{Y}_\Sigma)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

$$j_{AB}{}^\Sigma \in \mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$$

## Applications

Compute KK spectrum of generic single-trace deformations, outside  $\mathcal{N} = 8$  SUGRA

### Examples

- ▶  $\mathcal{N} = 1$  and  $\mathcal{N} = 0$   $\text{AdS}_4 \times$  Squashed  $S^7$ :  $\frac{\text{USp}(4)}{\text{SU}(2)}$ , not symmetric space  
→ Full spectrum for first time [Du Boef, EM, Samtleben '22]

$$L[J] \otimes [p, q, r] \otimes \{s\} : \quad \Delta = 1 + \frac{5}{3}s + \frac{1}{3}\sqrt{(3J + 2s^2)^2 + 5\mathcal{C}(p, q, r)}.$$

- ▶  $\beta$ -deformation of  $\text{AdS}_5 \times S^5$   
→ Anomalous dimensions along conformal manifold  
[Galli, Josse, EM, Petrini, *work in progress*]

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# Conclusions

ExFT: Compute full KK spectrum for warped compactifications with few/no remaining (super-)symmetries

- ▶ New holographic tests (comparison with SUSY index) & predictions [Bobev, EM, Robinson, Samtleben, van Muiden '21]
- ▶ Danger of trusting lower-dimensional supergravity!
- ▶ Higher KK modes crucial for physics
  - ▶ Compactness of conformal manifold
  - ▶ Perturbatively stable non-SUSY AdS, also in mIIA [Guarino, EM, Samtleben '21]
  - ▶ Higher KK modes can trigger instabilities [EM, Nicolai, Samtleben '20]

# Conclusions

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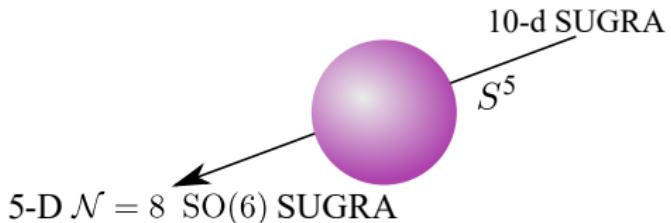
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Outlook:

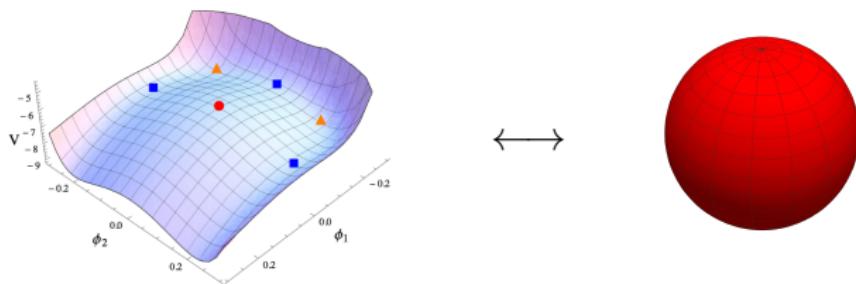
- ▶ Vacua of less SUSY truncations?
- ▶ Correlation functions?

Thank you!

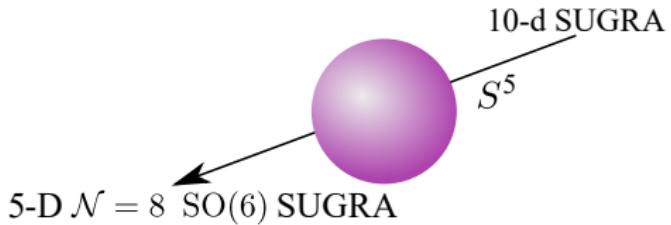
## Ex 0. Holographic dual of LS SCFT



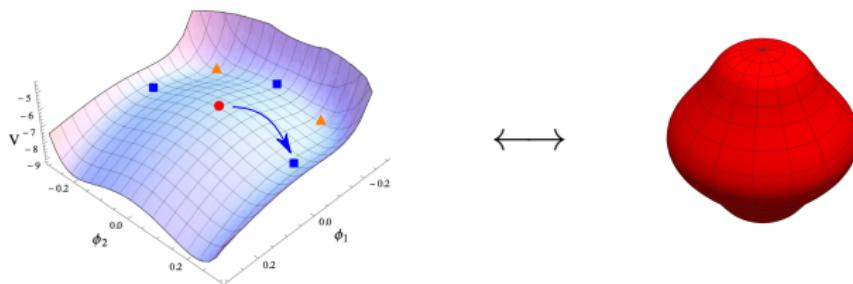
$\mathcal{N} = 2$   $SU(2)_F \times U(1)_R$   $AdS_5$  vacuum [Khavaev, Pilch, Warner '00]



## Ex 0. Holographic dual of LS SCFT



$\mathcal{N} = 2$   $SU(2)_F \times U(1)_R$   $AdS_5$  vacuum [Khavaev, Pilch, Warner '00]



## Ex 0. Checks & Predictions for LS SCFT

[Bobev, EM, Robinson, Samtleben, van Muiden '21]

Full spectrum of single-trace primary operators:

$$\Delta = 1 + \sqrt{7 - 3|j_1 + j_2| + \frac{3}{4}(r^2 - 2(p+2y)^2 + 2\ell(\ell+4) - 4k(k+1))}$$

Lorentz spin:  $j_1, j_2$

SU(2)<sub>F</sub> spin:  $k$

U(1)<sub>R</sub> charge:  $r$

$S^5$  level:  $\ell$

U(1)<sub>P</sub> × U(1)<sub>Y</sub> charges:  $p, y$

Unprotected operators with finite  $\Delta$  at strong coupling!

Semi-short multiplets match superconformal index

## Ex 2. KK Spectroscopy

[Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

KK spectroscopy → full KK spectrum

Perturbatively stable!

$$\Delta = \frac{3}{2} + a + \frac{1}{2} \sqrt{9 + 2\ell(\ell+4) + 4\ell_1(\ell_1+1) + 4\ell_2(\ell_2+1) + 2 \left( \frac{2n\pi}{T} + j_1\chi_1 + j_2\chi_2 \right)^2}$$

Position within  $\mathcal{N} = 4$  multiplet:  $a$

SO(4) spin:  $\ell_1, \ell_2$

Charges under  $U(1) \times U(1)$  Cartan:  $j_1, j_2$

$S^5$  level:  $\ell$

$S^1$  level:  $n$

## Ex 2. Non-perturbative stability?

[Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

- ▶ Probe-brane analysis:  $T > Q$   
Branes more stable than in SUSY case!
- ▶ No Ooguri-Vafa instability [Ooguri, Vafa '16]
- ▶  $S^1$  and  $S^5$  protected against “bubble of nothing” [Witten '82]
- ▶ D3-brane bubble of nothing [Bomans, Cassani, Dibitetto, Petri '21] ??

## Ex 2. $\frac{1}{N}$ corrections

[Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]

Flat directions lifted by  $\frac{1}{N}$  corrections?

Protection by diffeomorphism symmetry

- ▶  $\chi_1, \chi_2 \rightarrow$  coordinate transformations (locally)
- ▶  $\chi_1, \chi_2$  do not appear in diffeo-invariant quantities

Also applies to  $\mathcal{N} = 1$  exactly marginal deformations

[Bobev, Gautason, van Muiden '21]

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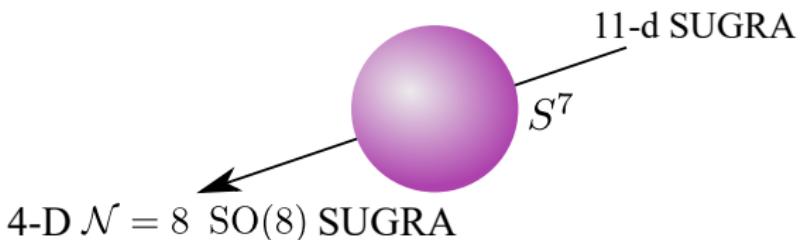
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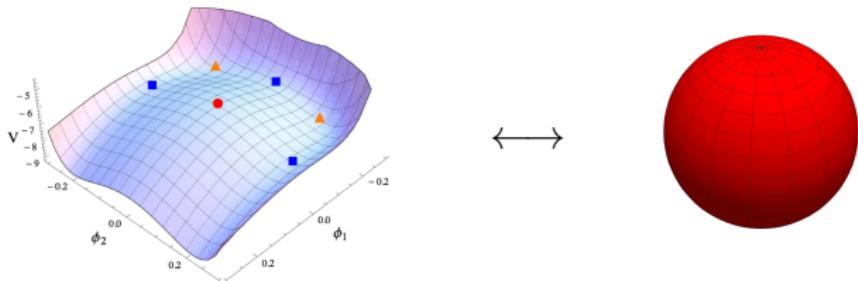
Corrections from D5-instantons?

$\text{vol}_{S^5 \times S^1}$  independent of  $\chi_1, \chi_2$

## Ex 3. Tachyonic KK modes

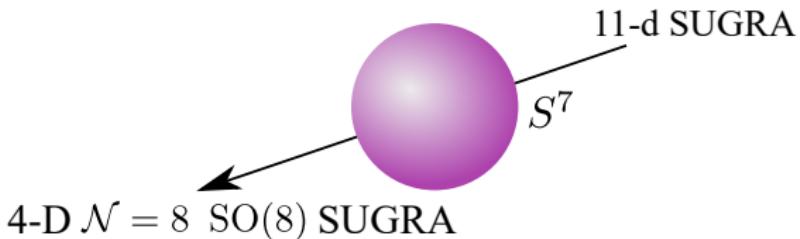


- ▶ Only one non-SUSY vacuum that is stable in 4-d! [Fischbacher, Pilch, Warner '10], [Comsa, Firsching, Fischbacher '19]
- ▶ Non-SUSY  $\text{SO}(3) \times \text{SO}(3)$   $\text{AdS}_4$  vacuum [Warner '83]

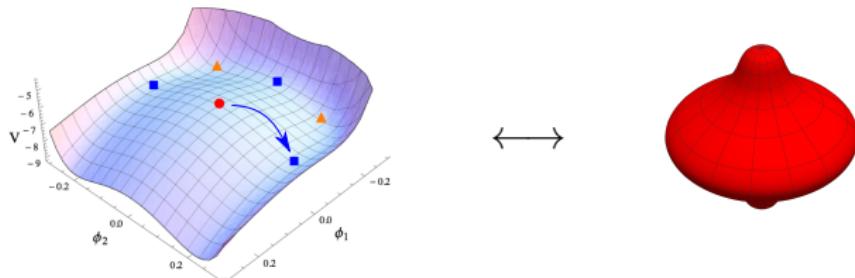


- ▶ Instability?

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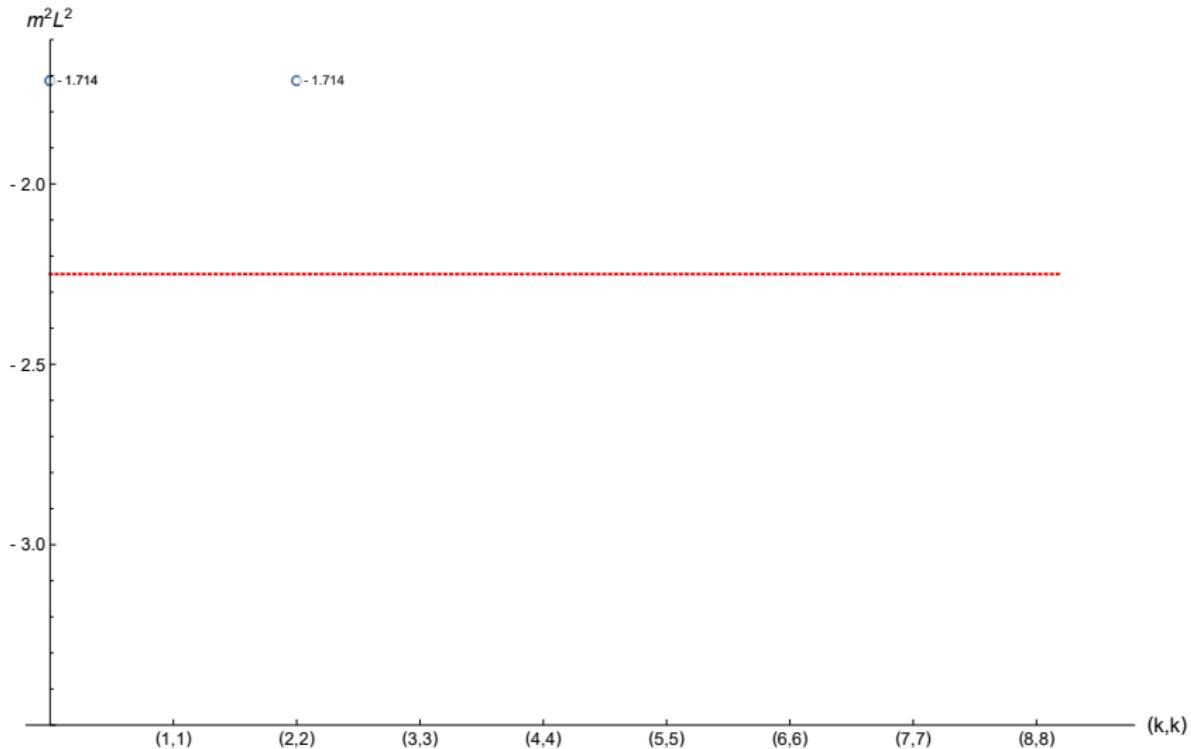


- ▶ Instability?

## Ex 3. Tachyonic KK modes

Modes  $\ell = 0$ :  $\mathcal{N} = 8$  supergravity multiplet

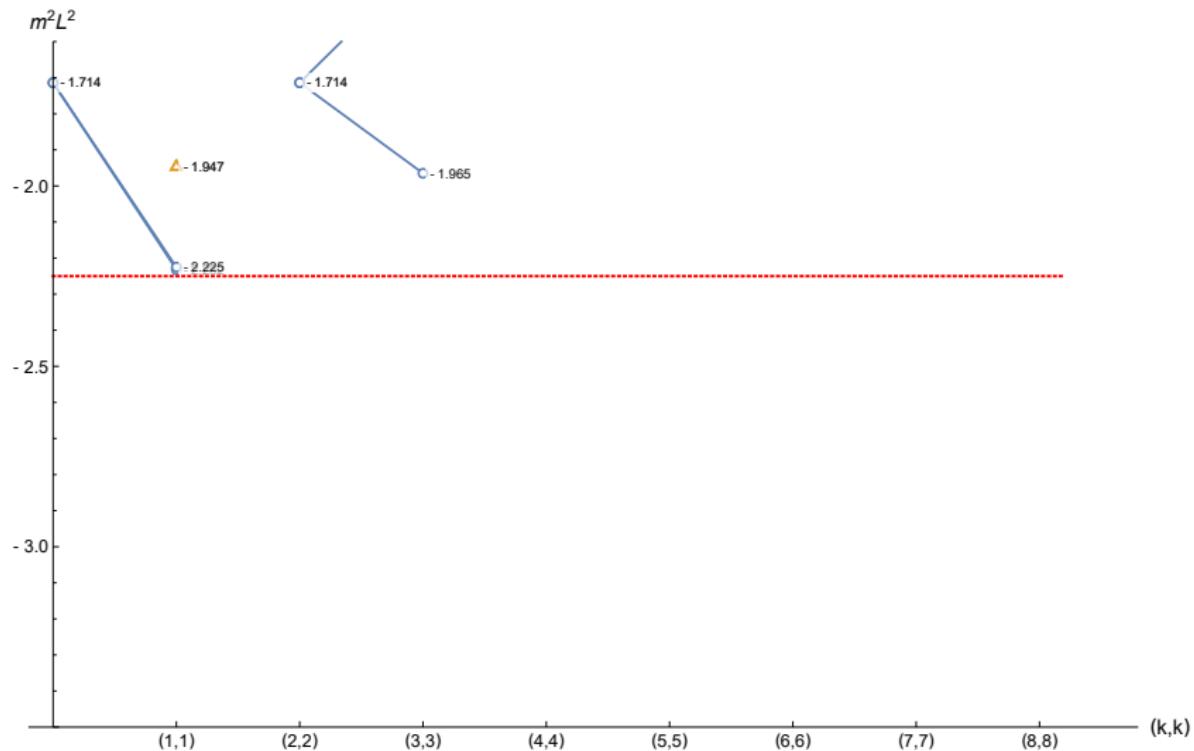
[Fischbacher, Pilch, Warner '10]



## Ex 3. Tachyonic KK modes

Modes  $\ell \leq 1$ : still stable!

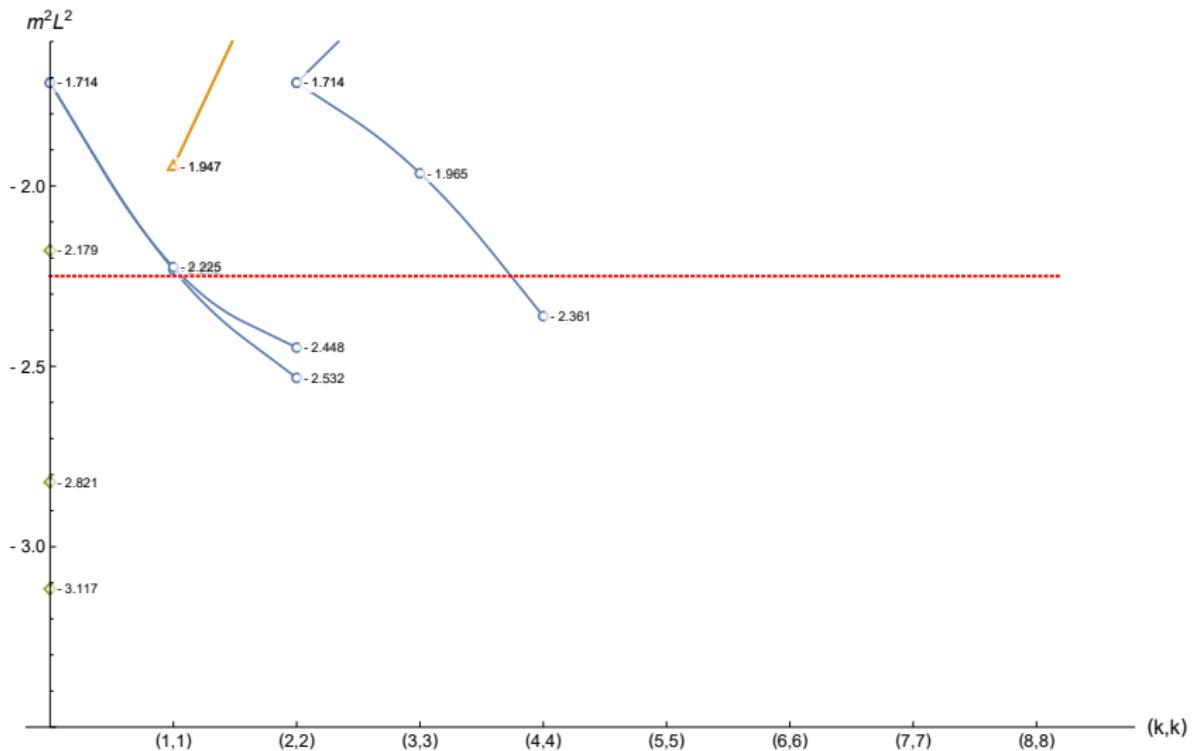
[EM, Nicolai, Samtleben '20]



## Ex 3. Tachyonic KK modes

Modes  $\ell \leq 2$ : **tachyons!**

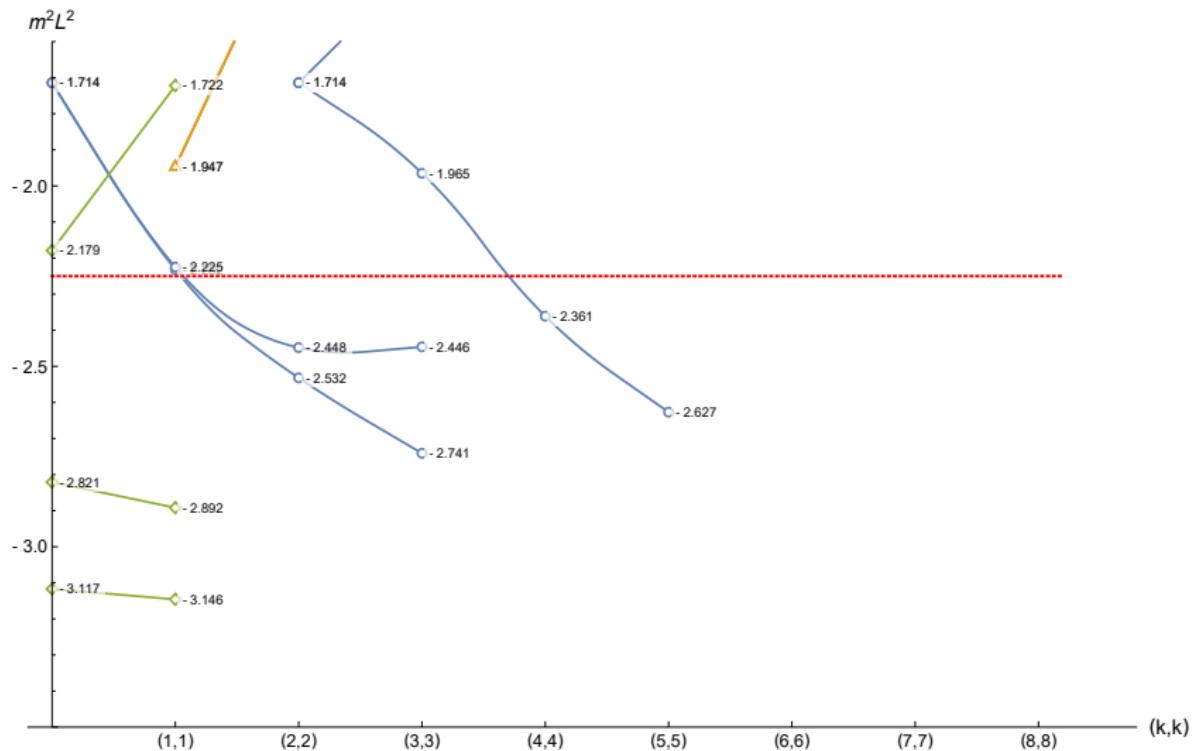
[EM, Nicolai, Samtleben '20]



## Ex 3. Tachyonic KK modes

Modes  $\ell \leq 3$

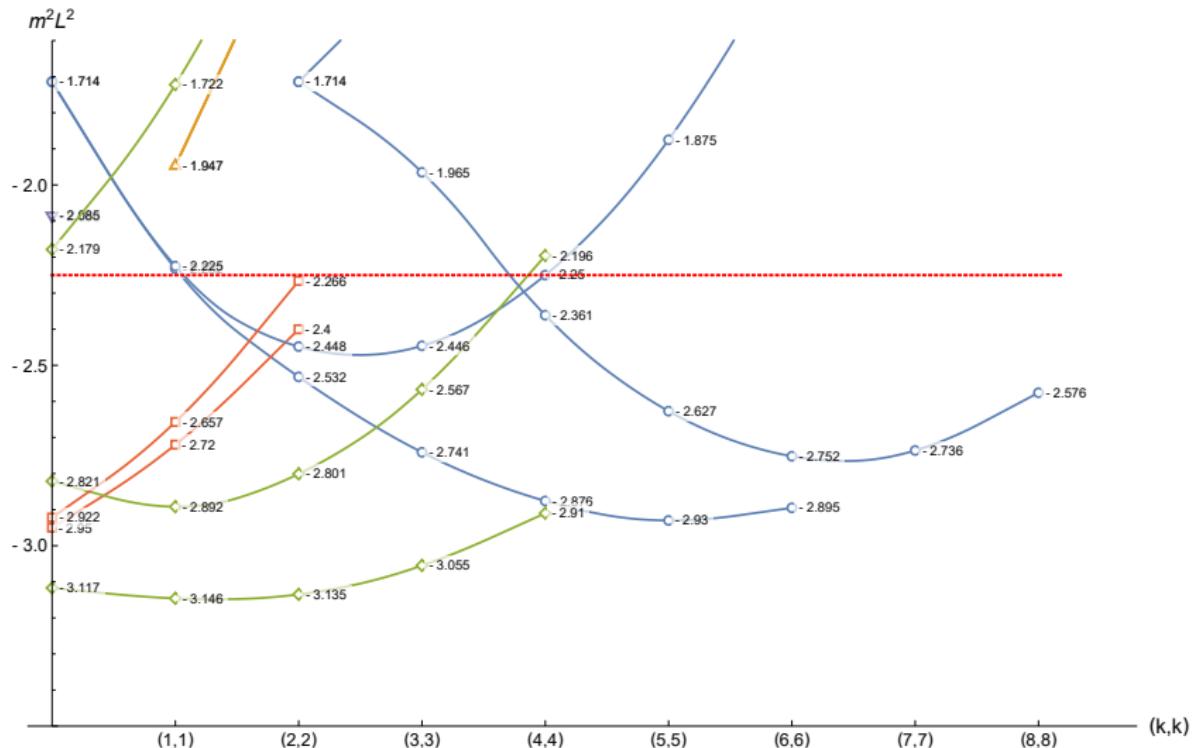
[EM, Nicolai, Samtleben '20]



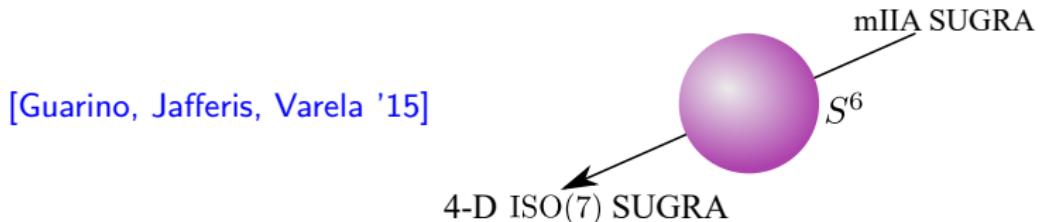
# Ex 3. Tachyonic KK modes

Modes  $\ell \leq 6$

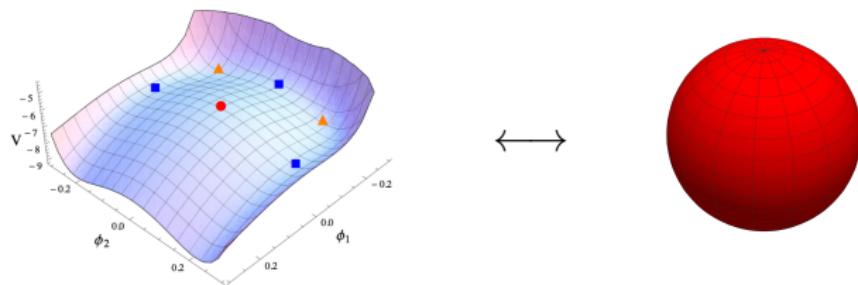
[EM, Nicolai, Samtleben '20]



## Ex 4. Perturbatively stable non-SUSY $\text{AdS}_4$ vacua

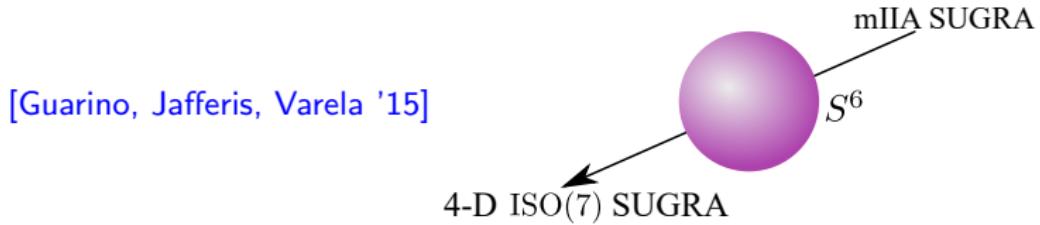


- ▶ 7 stable non-SUSY  $\text{AdS}_4$  vacua

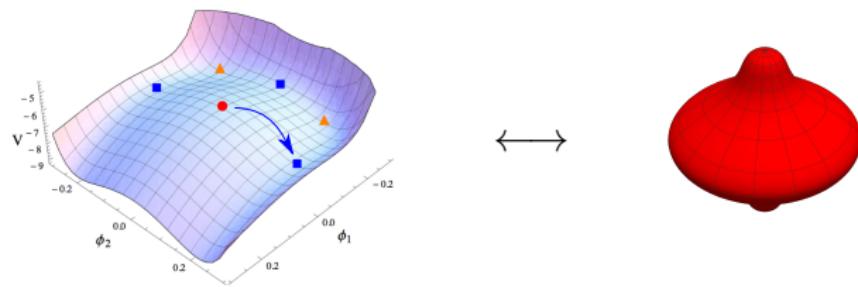


- ▶  $G_2$  invariant + 6 less symmetric non-SUSY  $\text{AdS}_4$ , stable in 4-D

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- ▶  $G_2$  invariant + 6 less symmetric non-SUSY  $\text{AdS}_4$ , stable in 4-D

## Ex 4. Stability of $G_2$ vacuum in mIIA

- Analytic spectrum:

$$L^2 \mathbb{M}_{(\text{scalar})}^2 = (\ell + 2)(\ell + 3) - \frac{3}{2} \mathcal{C}_{G_2} \geq 0.$$

$\ell$ :  $S^6$  KK level

$\mathcal{C}_{G_2}$ :  $G_2$  Casimir

$G_2$  vacuum is perturbatively stable in mIIA SUGRA  
[Guarino, EM, Samtleben '21]

- No signs of Ooguri-Vafa instability [Guarino, Tarrio, Varela '20]
- Protected against “bubble of nothing”
- May suffer from different non-perturbative instabilities [Bomans, Cassani, Dibitetto, Petri '21]

## Ex 4. Stability of six other $AdS_4$ vacua in mIIA

Evidence for perturbative stability in mIIA SUGRA  
[Guarino, EM, Samtleben '21]

- ▶ Numerical evaluation up to level  $\ell = 4$ :
  - ▶ no tachyons
  - ▶ lowest-lying masses increase monotonically with level
- ▶ No signs of Ooguri-Vafa instability [Guarino, Tarrio, Varela '20]
- ▶ Protected against “bubble of nothing”
- ▶ Other non-perturbative instabilities?