# Dark matter, bound states and unitarity

Kallia Petraki

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# Frontiers in particle dark matter searches

(very simplistic summary)

## Past decades

Most research focused on

**$m_{DM}$ ~ 100 GeV ~ $m_{W,Z}$**

(e.g. prototypical WIMP scenario)

## Current frontiers

### Heavy dark matter

**$m_{DM} \gtrsim$ TeV**

Not constrained by colliders.

→ Experimentally probed by existing / upcoming **telescopes** e.g. HESS, IceCube, CTA, Antares

### Light dark matter

**$m_{DM} \lesssim$ few GeV**

Not constrained by older direct detection experiments

→ Development of new generation of **direct detection** experiments

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# Heavy ($m_{DM} \gtrsim$ TeV) dark matter

How does the phenomenology of dark matter look like?
(in popular scenarios, e.g. thermal-relic DM)

**New type of dynamics emerges:**
Long-range interactions

$$\lambda_B \sim \frac{1}{\mu v_{\rm rel}}, \frac{1}{\mu \alpha} \lesssim \frac{1}{m_{\rm mediator}} \sim \text{interaction range}$$

$\mu$: reduced mass ($m_{\rm DM}/2$)

**Does this occur in models we care about?**

- WIMPs with m > few TeV
- WIMPs with m < TeV co-annihilating with coloured/charged particles
- Self-interacting DM

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...type... ...merges:

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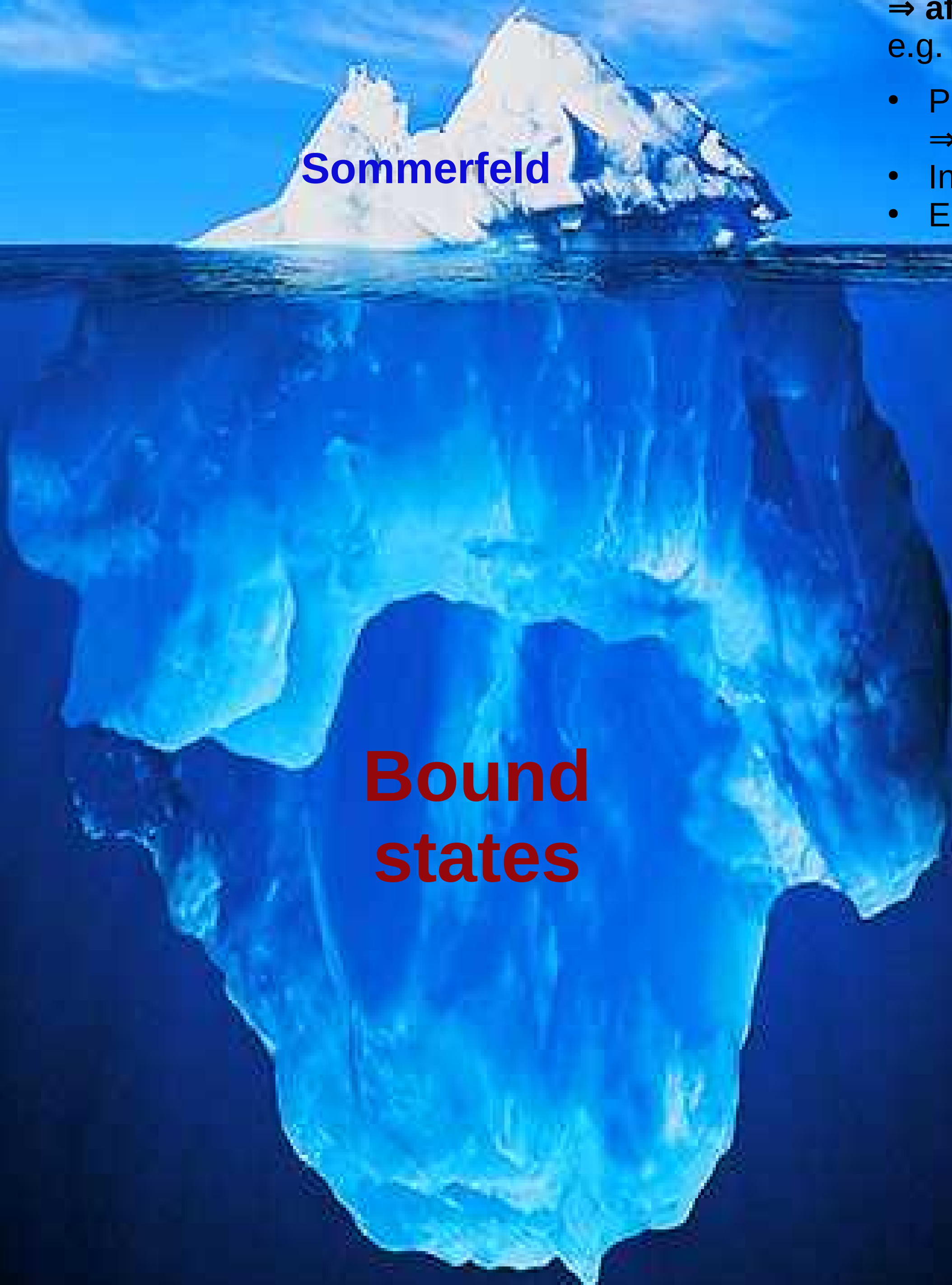
**What changes when the interactions are long-ranged?**

# Implications of non-perturbative effects




**Distortion of scattering-state wavefunctions**
**⇒ affects all cross-sections**
e.g. annihilation, elastic scattering

- Production in early universe, e.g. freeze-out
  ⇒ changes correlation of parameters (mass – couplings)
- Indirect detection signals
- Elastic scattering

**Unstable bound states (positronium-like)**
**⇒ extra annihilation channel**

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- Novel low-energy indirect detection signals
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- Inelastic scattering in direct detection experiments (?)

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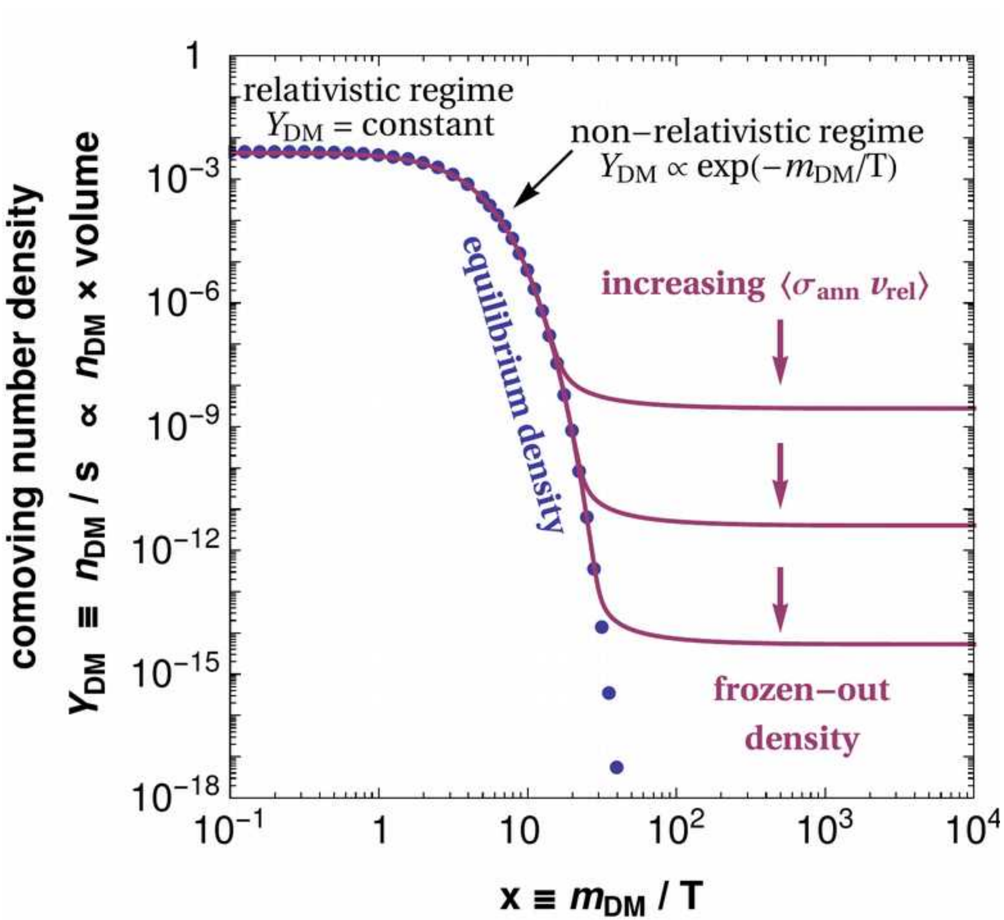
# Dark matter production via **thermal freeze-out**




**time**

***$T > m_{DM}$***

DM kept in chemical & kinetic equilibrium with the plasma, via

***$X + \bar{X} \leftrightarrow f + \bar{f}$***

$n_{DM} \sim T^3$ or $Y_{DM}$ = constant

***$T < m_{DM}$***

$Y_{DM} \propto \exp(-m_{DM}/T)$, while still in equilibrium

***$T < m_{DM} / 25$***

Density too small, annihilations stall

**⇒ Freeze-out!**

$$\Omega \simeq 0.26 \times \left( \frac{1 pb \cdot c}{\sigma_{ann} v_{rel}} \right)$$

**1 pb ~ $\sigma_{Weak}$**

**WIMP miracle!**

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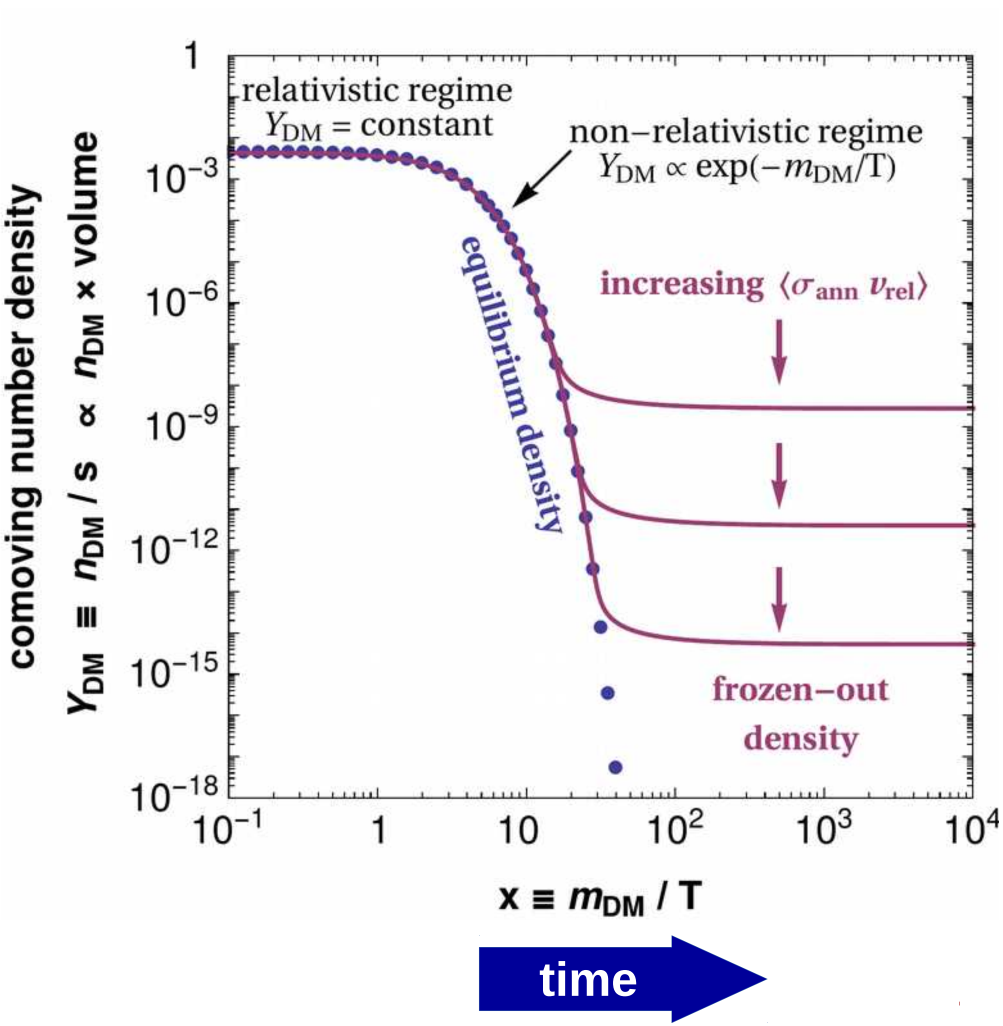


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**WIMP mirac**

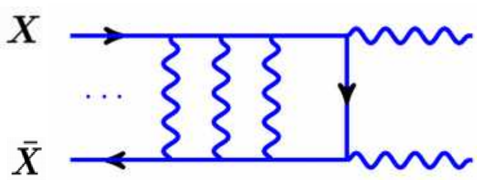
**!**
“Canonical value” assumes contact-type interactions

## Long-range interactions and freeze-out:
## A dark U(1) sector

# Dark U(1) model: Dirac DM $X,\bar{X}$ coupled to $\gamma_D$

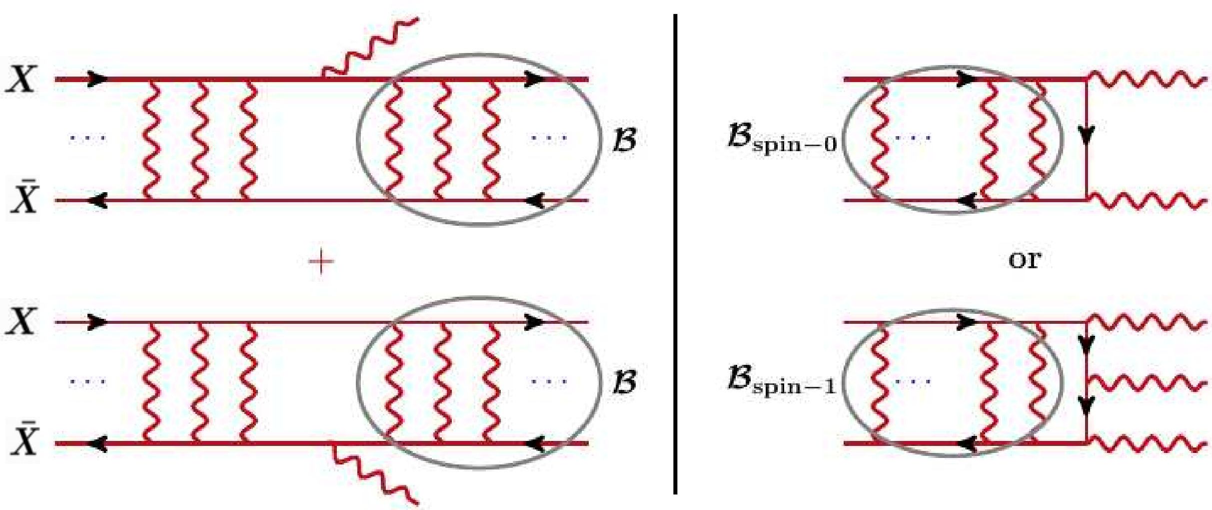
**Direct annihilation**

$X + \bar{X} \to 2\gamma_D$

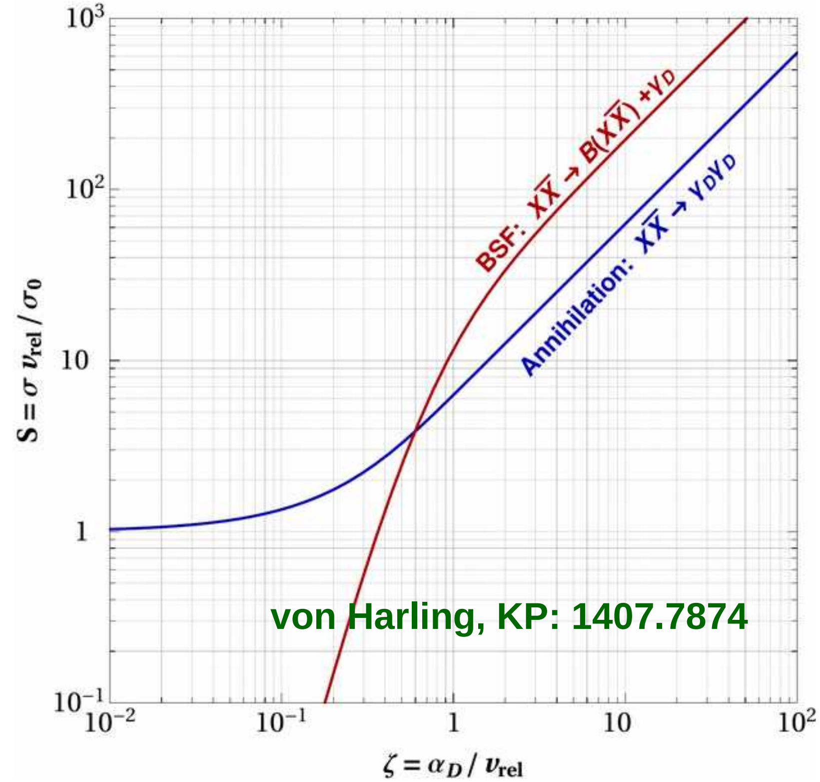



$$\sigma_{\rm ann} v_{\rm rel} = \frac{\pi\alpha_D^2}{m_X^2} \times S_{\rm ann}(\alpha_D/v_{\rm rel})$$

**Bound-state formation and decay**




$$\sigma_{\rm BSF} v_{\rm rel} = \frac{\pi\alpha_D^2}{m_X^2} \times S_{\rm BSF}(\alpha_D/v_{\rm rel})$$



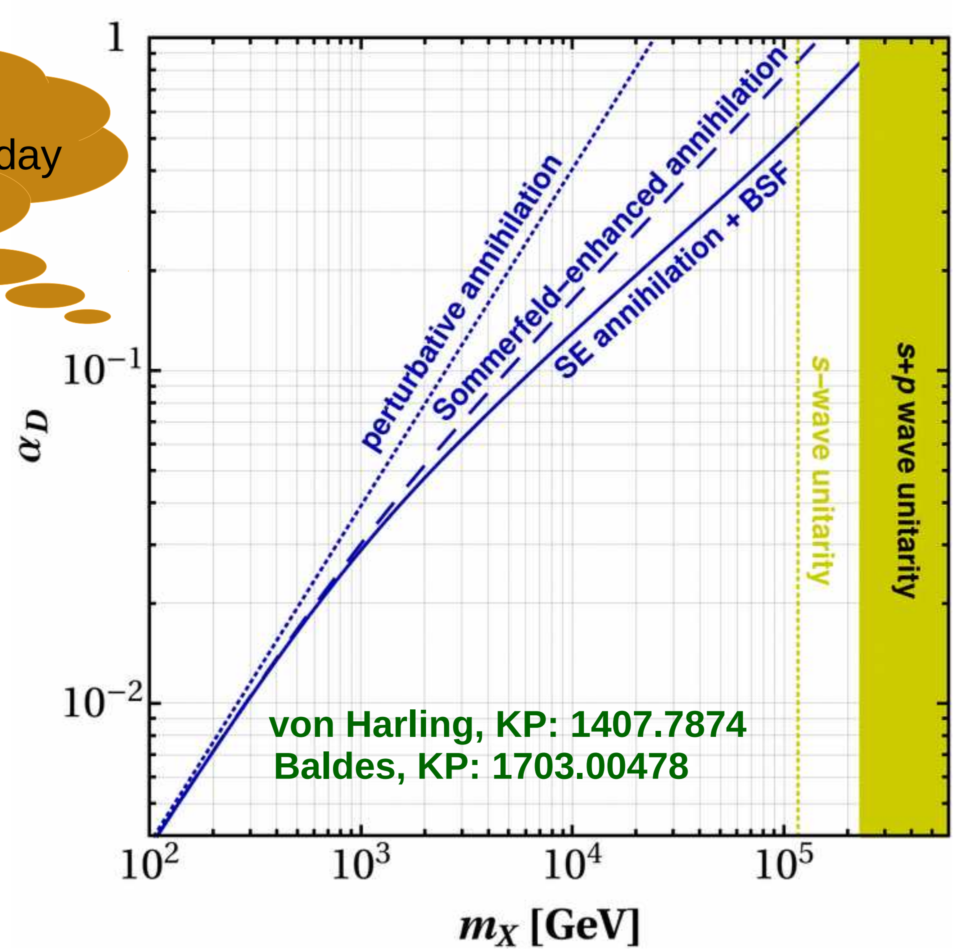

$$S_{\rm ann} \simeq \left(\frac{2\pi\zeta}{1-e^{-2\pi\zeta}}\right) \xrightarrow{\zeta \gtrsim 1} 2\pi\zeta$$

$$S_{\rm BSF} \simeq \left(\frac{2\pi\zeta}{1-e^{-2\pi\zeta}}\right) \frac{2^9\zeta^4 e^{-4\zeta \,{\rm arccot}\zeta}}{3(1+\zeta^2)^2} \xrightarrow{\zeta \gtrsim 1} 3.13 \times 2\pi\zeta$$

# Thermal freeze-out with long-range interactions

**Dark U(1) model: Dirac DM $X,\bar{X}$ coupled to $\gamma_D$**

Important because it determines DM interactions today (direct, indirect detection)



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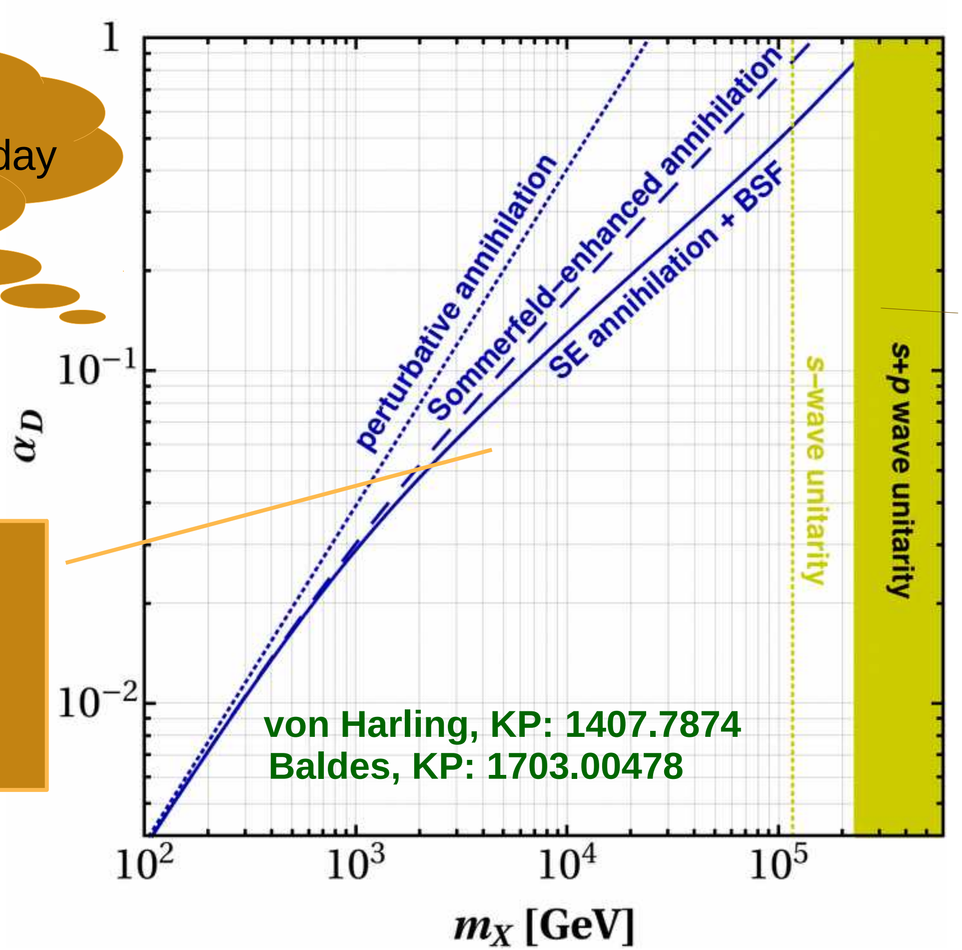



Long-range effects indeed become at $m_{DM} \gtrsim$ few TeV.

Verifies expectation from unitarity arguments!

Dominant annihilation mode: ***s*-wave**.

Dominant BSF mode: ***p*-wave**

Same order!

Higher partial waves Important / dominant in multi-TeV regime.

**DM may be even heavier!**

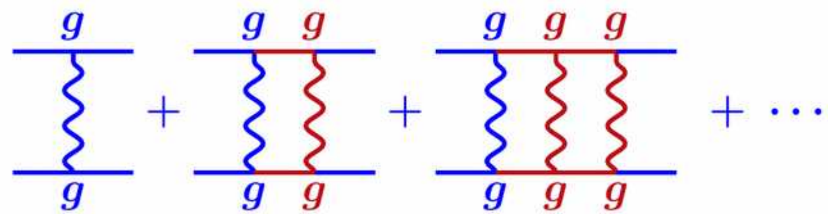
## The origin of non-perturbative effects at perturbative coupling

# What just happened?

## Making sense of the ladder diagrams

Every mediator exchange introduces an
$\alpha = g^2/(4\pi)$ suppression in the amplitude.
How did we get an enhancement and bound states?

Bound-state ladder



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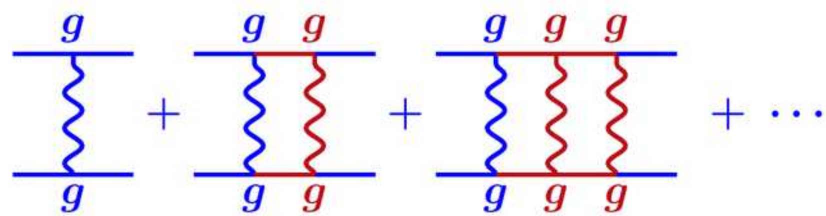


Energy and momentum exchange scale with $\alpha$!

- Momentum transfer: $|\vec{q}| \sim \mu\alpha$.
- Energy transfer: $q^0 \sim |\vec{q}|^2/\mu \sim \mu\alpha^2$.
- Off-shellness of interacting particles: $q^0 \sim |\vec{q}|^2/\mu \sim \mu\alpha^2$.

one boson exchange $\sim \alpha \times \frac{1}{(\mu\alpha)^2} \propto \frac{1}{\alpha}$

each added loop
$$\sim \alpha \times \int dq^0 d^3q \, \frac{1}{q_1 - m_1} \frac{1}{q_2 - m_2} \frac{1}{q_\gamma^2}$$
$$\sim \alpha \times (\mu\alpha^2)(\mu\alpha)^3 \frac{1}{\mu\alpha^2} \frac{1}{\mu\alpha^2} \frac{1}{(\mu\alpha)^2}$$
$$\sim 1$$

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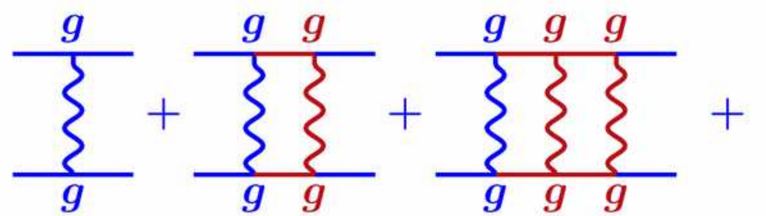


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**1/α scaling responsible for non-perturbative effects**

**(not largeness of coupling)**

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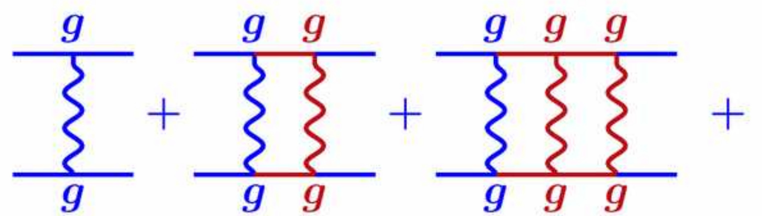


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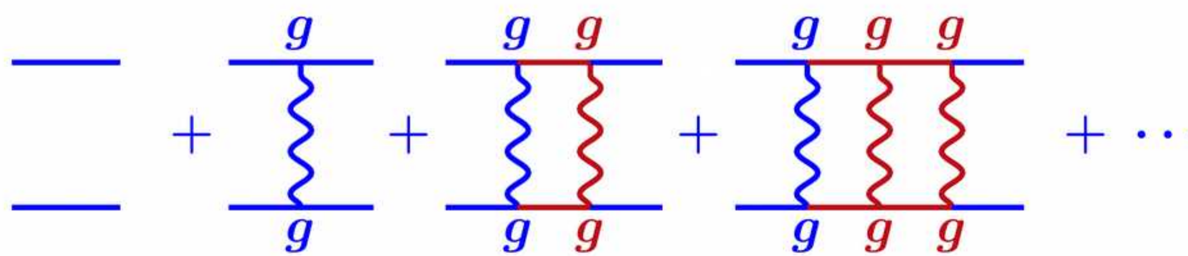
(not largeness of coupling)

All this breaks down if
$m_\gamma > \mu\alpha$
→ *contact-type interaction*

## What just happened?
# Making sense of the ladder diagrams

Every mediator exchange introduces an
$\alpha = g^2/(4\pi)$ suppression in the amplitude.
How did we get an enhancement and bound states?

Scattering state ladder




Energy and momentum exchange scale with both $\alpha$ and $v_{\rm rel}$!

$\mu v_{\rm rel}$ is the *expectation value* of the momentum in CM frame,
the quantum uncertainty scales with $\alpha$.

The Sommerfeld effect appears when
*quantum uncertainty* $\sim$ *expectation value.*

# Unitarity and long-range interactions

# Partial-wave unitarity limit

$$S^\dagger S = 1 \quad \xrightarrow{S=1+iT} \quad -i(T - T^\dagger) = T^\dagger T$$

**Project on a partial wave and insert complete set of states on RHS**

$$\Downarrow$$

$$\sigma^{(\ell)}_{\rm inel} \leqslant \frac{\pi(2\ell+1)}{k^2_{\rm cm}} \quad \xrightarrow{\rm non-rel} \quad \frac{\pi(2\ell+1)}{\mu^2 v^2_{\rm rel}} \quad \xrightarrow{\mu = M_{\rm DM}/2} \quad \frac{4\pi(2\ell+1)}{M^2_{\rm DM} v^2_{\rm rel}}$$

[Griest, Kamionkowski (1990); Hui (2001)]

**Physical meaning:**
**saturation of probability for inelastic scattering**

# Partial-wave unitarity limit

$$\boxed{\sigma_{\rm inel}^{(\ell)} v_{\rm rel} \leqslant \sigma_{\rm uni}^{(\ell)} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M_{\rm DM}^2 v_{\rm rel}}}$$

**Implies upper bound on the mass of thermal-relic DM**

**Griest, Kamionkowski (1990)**

$$\sigma_{\rm ann} v_{\rm rel} \simeq 2.2\times 10^{-26}\ {\rm cm}^3/{\rm s} \leqslant \frac{4\pi}{M_{\rm DM}^2 v_{\rm rel}}$$

$$\langle v_{\rm rel}^2\rangle^{1/2} = (6T/M_{\rm DM})^{1/2} \xrightarrow[M_{\rm DM}/T\,\approx\,25]{\rm freeze-out} 0.49$$

$$\Rightarrow M_{\rm uni} \simeq 117\ {\rm TeV}$$

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**Two assumptions to be questioned**

1. "one does not expect $\sigma v_{\text{rel}} \propto 1/v_{\text{rel}}$ for annihilation channels in a non-relativistic expansion."
2. The $s$-wave yields the dominant contribution to the annihilation cross-section.

# Partial-wave unitarity limit

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**Implies upper bound on t... of thermal-relic DM**

**What are the underlying dynamics of heavy thermal-relic DM?**

**What interactions can approach / attain the unitarity limit?**

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...ns

...questioned

1. “one does not expect $\sigma v_{\rm rel} \propto 1/v_{\rm rel}$ for annihilation channels in a ... ”

**What are the implications for experiments?**

dominant contribution to the annihilation cross-section.

# Partial-wave unitarity limit

$$\boxed{\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M^2_{\rm DM} v_{\rm rel}}}$$

## 1) Velocity dependence of $\sigma_{uni}$

Assuming $\sigma v_{rel}$ = constant, setting it to maximal (inevitably for a fixed $v_{rel}$) and thermal averaging is formally incorrect!

⇒ Unitarity violation at larger $v_{rel}$, non-maximal cross-section at smaller $v_{rel}$.

Sommerfeld-enhanced inelastic processes exhibit exactly this velocity dependence at large couplings / small velocities, e.g. in QED

$$\sigma^{\ell=0}_{\rm ann} v_{\rm rel} \simeq \frac{\pi\alpha_D^2}{M^2_{\rm DM}} \times \frac{2\pi\alpha_D/v_{\rm rel}}{1-\exp(-2\pi\alpha_D/v_{\rm rel})} \xrightarrow{\alpha_D \gg v_{\rm rel}} \frac{2\pi^2\alpha_D^3}{M^2_{\rm DM} v_{\rm rel}}$$

**⇒ Velocity dependence of $\sigma_{uni}$ definitely *not* unphysical!**

Baldes, KP: 1703.00478

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**Parametric**

**1) ~~Velocity~~ dependence of $\sigma_{uni}$**

***What can we learn?***

For a contact-type interaction, mediated by heavy particle with $m_{\rm med} \gtrsim M_{\rm DM}$,

$$\sigma_{\rm ann} v_{\rm rel} \sim \frac{\alpha_D^2 M_{\rm DM}^2}{m_{\rm med}^4} \lesssim \frac{4\pi}{M_{\rm DM}^2 v_{\rm rel}}.$$

Approaching unitarity limit requires large coupling (no surprise)

$$\alpha_D \sim m_{\rm med}^4 / M_{\rm DM}^4 \gtrsim 1.$$

Calculation violates unitarity if

$$m_{\rm med} < \alpha_D^{1/2} M_{\rm DM} \lesssim \alpha_D M_{\rm DM}.$$

Comparison between physical scales
$\Rightarrow$ violation signals new effect at play!

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***What can we learn?***

Including the Sommerfeld enhancement, for a light mediator, e.g. dark QED

$$\sigma_{\rm ann} v_{\rm rel} \simeq \frac{2\pi^2 \alpha_D^3}{M_{\rm DM}^2 v_{\rm rel}} \lesssim \frac{4\pi}{M_{\rm DM}^2 v_{\rm rel}}.$$

Unitarity indicates range of validity

$$\alpha_D \lesssim 0.86$$

Only numerical bound on a dimensionless coupling
$\Rightarrow$ include (resummed) higher order corrections

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**1) Velocity dependence of $\sigma_{uni}$**

Proper thermal average and taking into account delayed chemical decoupling

$$M_{\rm uni} \simeq 117 \text{ TeV} \quad \longrightarrow \quad M_{\rm uni} \simeq 198 \text{ TeV}$$

$s$-wave annihilation

Baldes, KP: 1703.00478

# Partial-wave unitarity limit

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## 2) Higher partial waves

In direct annihilation processes, *s*-wave dominates.

- For contact-type interactions, higher $\ell$ are $v_{\rm rel}^{2\ell}$ suppressed:

$$\sigma_{\rm ann} v_{\rm rel} = \sum_{\ell} \sum_{r=0}^{\infty} c_{\ell r}\, {v_{\rm rel}}^{2\ell+2r}$$

- For long-range interactions:

$$\sigma^{(\ell=0)} v_{\rm rel} \sim \frac{\pi \alpha_D^2}{M_{\rm DM}^2} \times \left(\frac{2\pi\alpha_D/v_{\rm rel}}{1-e^{-2\pi\alpha_D/v_{\rm rel}}}\right) \xrightarrow{\alpha_D \gg v_{\rm rel}} \frac{2\pi^2\alpha_D^3}{M_{\rm DM}^2 v_{\rm rel}}$$

$$\sigma^{(\ell=1)} v_{\rm rel} \sim \frac{\pi \alpha_D^2}{M_{\rm DM}^2} v_{\rm rel}^2 \times \left(\frac{2\pi\alpha_D/v_{\rm rel}}{1-e^{-2\pi\alpha_D/v_{\rm rel}}}\right)\left(1+\frac{\alpha_D^2}{v_{\rm rel}^2}\right) \xrightarrow{\alpha_D \gg v_{\rm rel}} \frac{2\pi^2\alpha_D^5}{M_{\rm DM}^2 v_{\rm rel}}$$

Same $v_{\rm rel}$ scaling (as expected from unitarity!), albeit $v_{\rm rel}^2 \to \alpha_D^2$ suppression.

Baldes, KP: 1703.00478

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However, DM may annihilate via formation and decay of bound states.

dark QED

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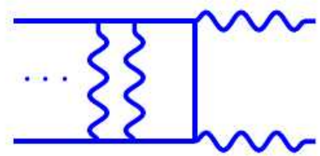

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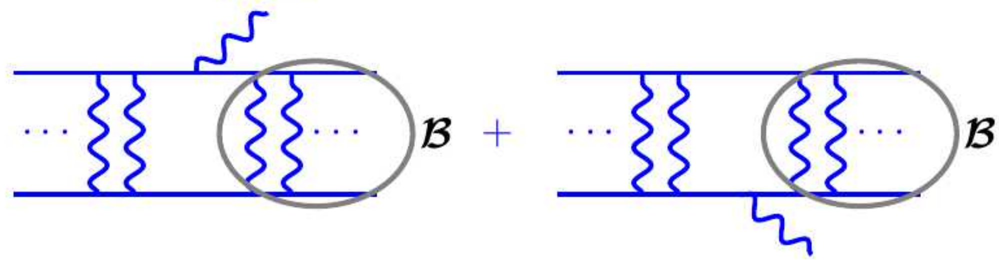


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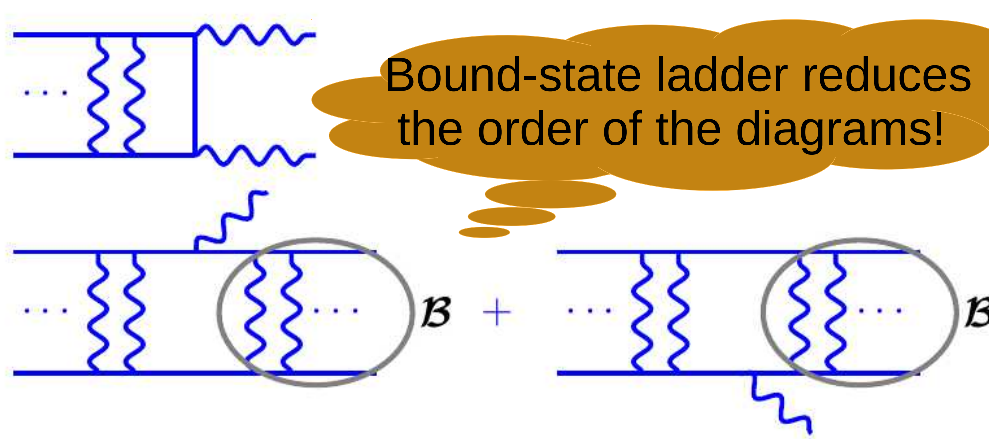
**same order!**

$$\sigma_{\rm BSF}^{(\ell=1)} v_{\rm rel} \xrightarrow{\alpha_D \gg v_{\rm rel}} 3.13 \times \frac{2\pi^2 \alpha_D^3}{M_{\rm DM}^2 v_{\rm rel}}$$




Bound-state ladder reduces the order of the diagrams!

Both *s*-wave and *p*-wave saturate their unitarity limit at $\alpha_D \simeq 0.86$.
$\Rightarrow$ Consider combined bound on the DM mass, $M_{\rm uni}^{s+p} \simeq 276$ TeV.

**Higher partial waves important for DM destruction in early universe**
**$\Rightarrow$ higher $M_{\rm DM}$ AND no general $M_{\rm uni}$ on thermal-relic DM !**

Baldes, KP: 1703.00478

# Partial-wave unitarity limit

$$\boxed{\sigma^{(\ell)}_{\rm inel} v_{\rm rel} \leqslant \sigma^{(\ell)}_{\rm uni} v_{\rm rel} = \frac{4\pi(2\ell+1)}{M^2_{\rm DM} v_{\rm rel}}}$$

**Can be approached or attained only by long-range interactions**

**Baldes, KP: 1703.00478**

Generic conclusion:

In viable thermal-relic DM scenarios, expect long-range behaviour at $m_{DM} \gtrsim$ few TeV!

- **Freeze-out**
  Sommerfeld & BSF alter predicted mass – coupling relation. Important for all experimental probes.

- **Indirect detection**
  Sommerfeld & BSF must be considered in computing signals. Novel lower energy signals produced in BSF.

# Neutralino-squark co-annihilation scenarios

# Squark-neutralino co-annihilation scenarios

- Degenerate spectrum → soft jets → evade LHC constraints
- Large stop-Higgs coupling reproduces measured Higgs mass and brings the lightest stop close in mass with the LSP

⇒ DM density determined by “effective” Boltzmann equation

$$n_{\rm tot} = n_{\rm LSP} + n_{\rm NLSP}$$

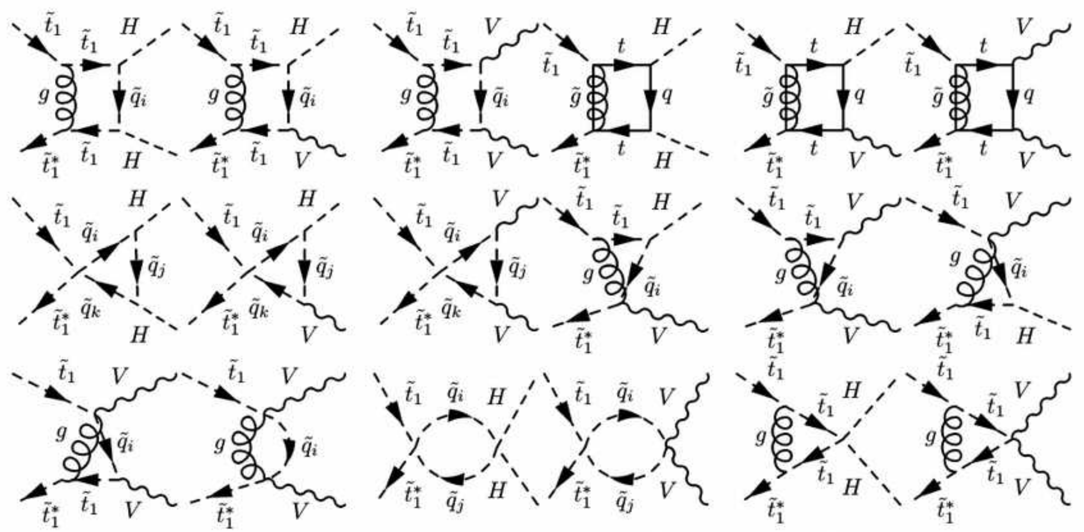
$$\sigma^{\rm eff}_{\rm ann} = [\, n^2_{\rm LSP}\, \sigma^{\rm LSP}_{\rm ann} + n^2_{\rm NLSP}\, \sigma^{\rm NLSP}_{\rm ann} + n_{\rm LSP}\, n_{\rm NLSP}\, \sigma^{\rm LSP-NLSP}_{\rm ann}\, ]/n^2_{\rm tot}$$

**Scenario probed in colliders.**
**Important to compute DM density accurately!**
→ **QCD corrections**

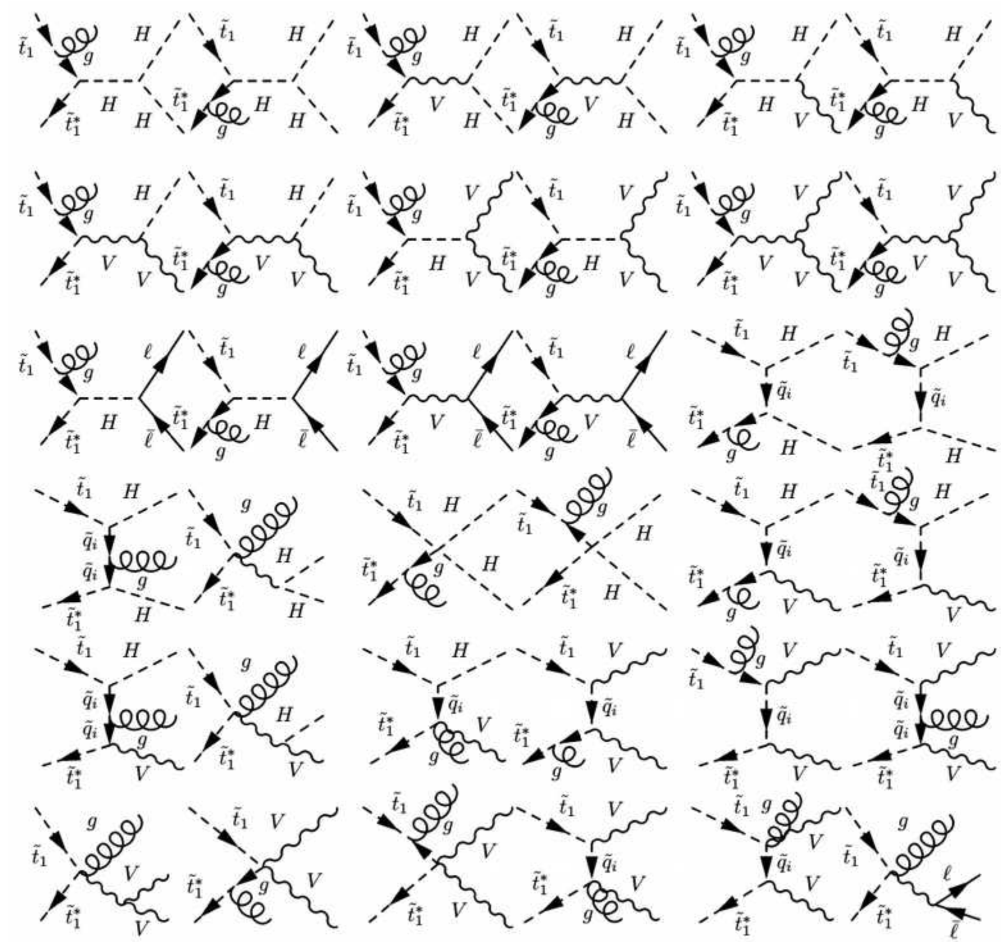
# QCD corrections to stop annihilation

[Klasen+ (since 2014), DM@NLO]

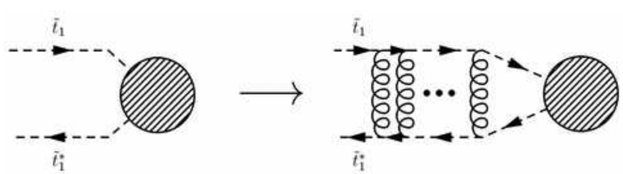
## QCD loop corrections



## Gluon emission



### Sommerfeld effect



broadly, the most important

# QCD corrections to stop annihilation

[Klasen+ (since 2014), DM@NLO]

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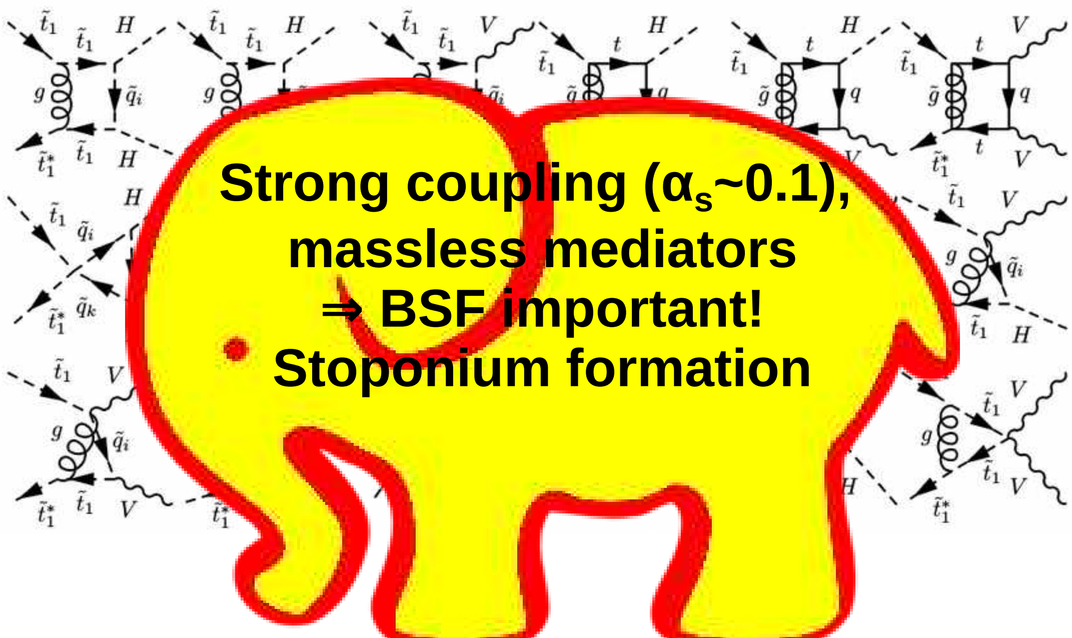
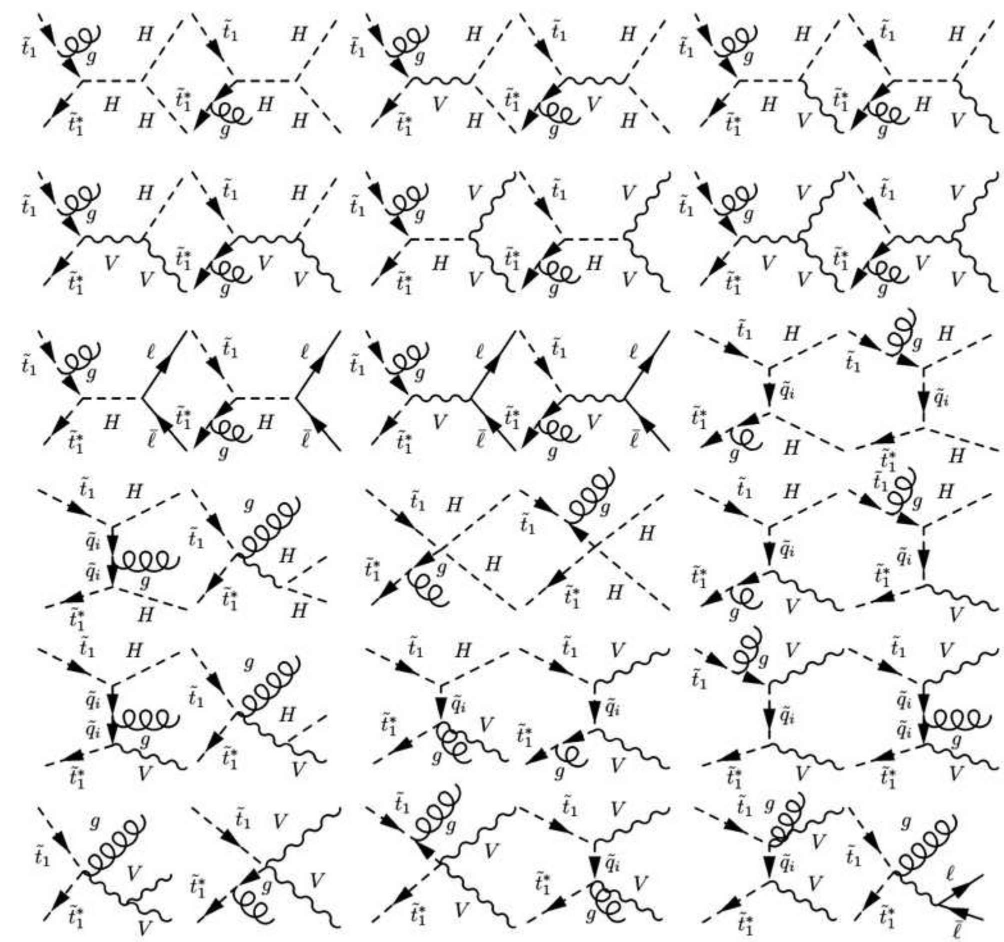


**Strong coupling ($\alpha_s$~0.1), massless mediators**
**⇒ BSF important!**
**Stoponium formation**

## Gluon emission



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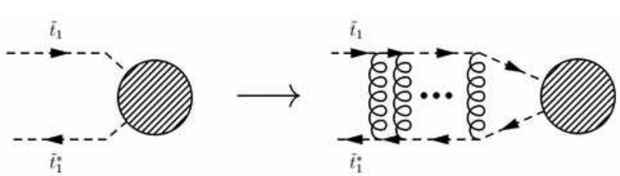


broadly, the most important

# DM coannihilation with scalar colour triplet

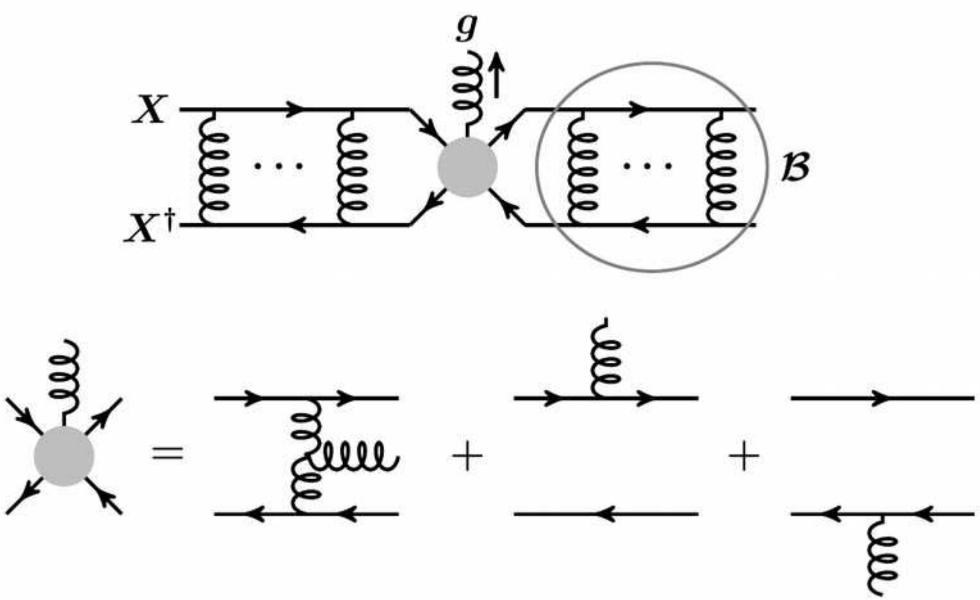
**MSSM-inspired toy model**

$$
\begin{aligned}
\mathcal{L} \supset\ & \frac{1}{2}\overline{\chi^c}\, i\partial\!\!\!/ \chi - \frac{1}{2} m_\chi \overline{\chi^c}\chi \\
+\ & \Big[(\partial_\mu + i g_s G^a_\mu T^a) X\Big]^\dagger \Big[(\partial^\mu + i g_s G^{a,\mu} T^a) X\Big] - m_X^2 |X|^2 \\
+\ & (\chi \leftrightarrow X, X^\dagger) \text{ interactions in chemical equilibrium during freeze-out}
\end{aligned}
$$

# DM coannihilation with scalar colour triplet

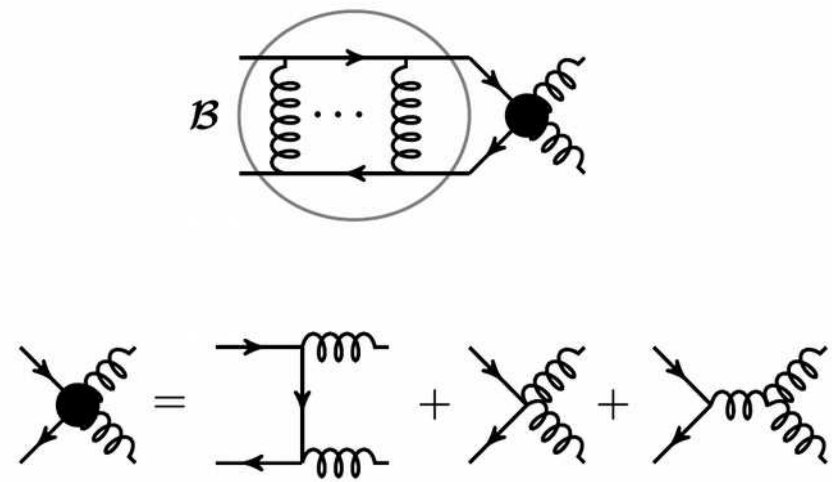
## MSSM-inspired toy model

### Bound-state formation and decay




Allowed transitions

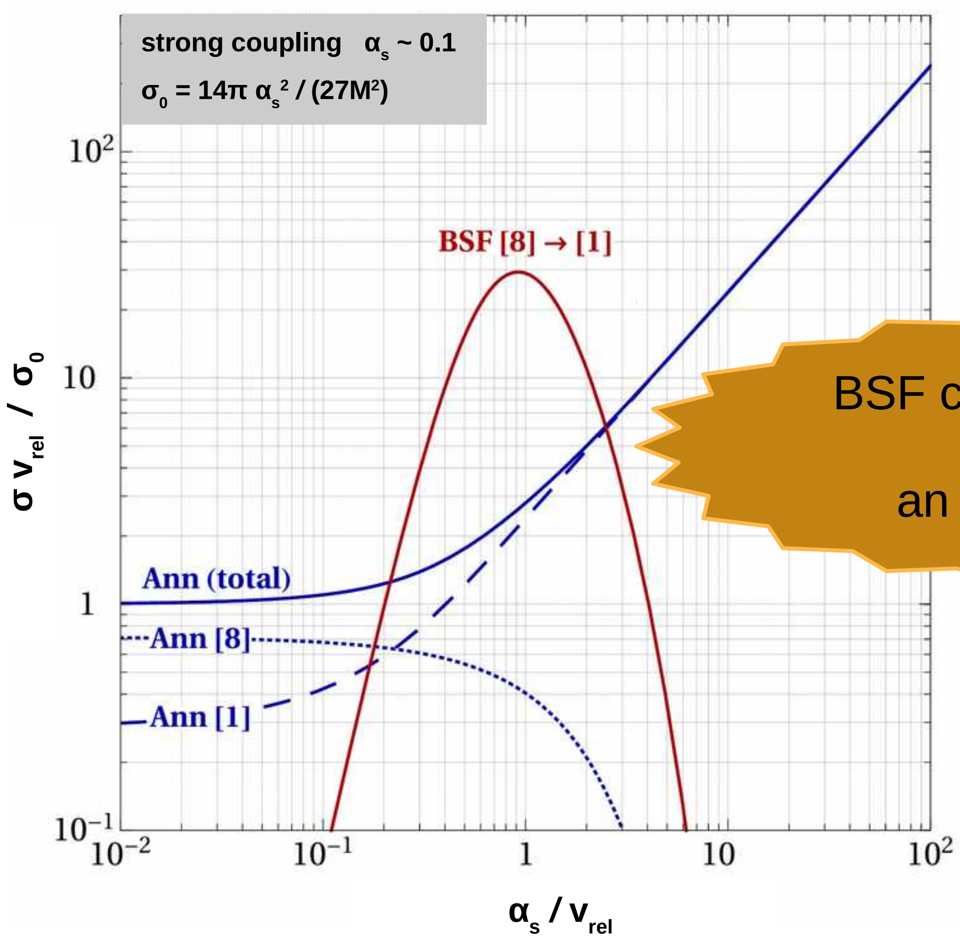
$$(X+X^\dagger)_{[8]} \rightarrow (X+X^\dagger)_{[1]} + g_{[8]}$$
$$(X+X^\dagger)_{[1]} \rightarrow (X+X^\dagger)_{[8]} + g_{[8]}$$
$$(X+X^\dagger)_{[8]} \rightarrow (X+X^\dagger)_{[8]} + g_{[8]}$$



# DM coannihilation with scalar colour triplet

## MSSM-inspired toy model

### Bound-state formation *vs* Annihilation

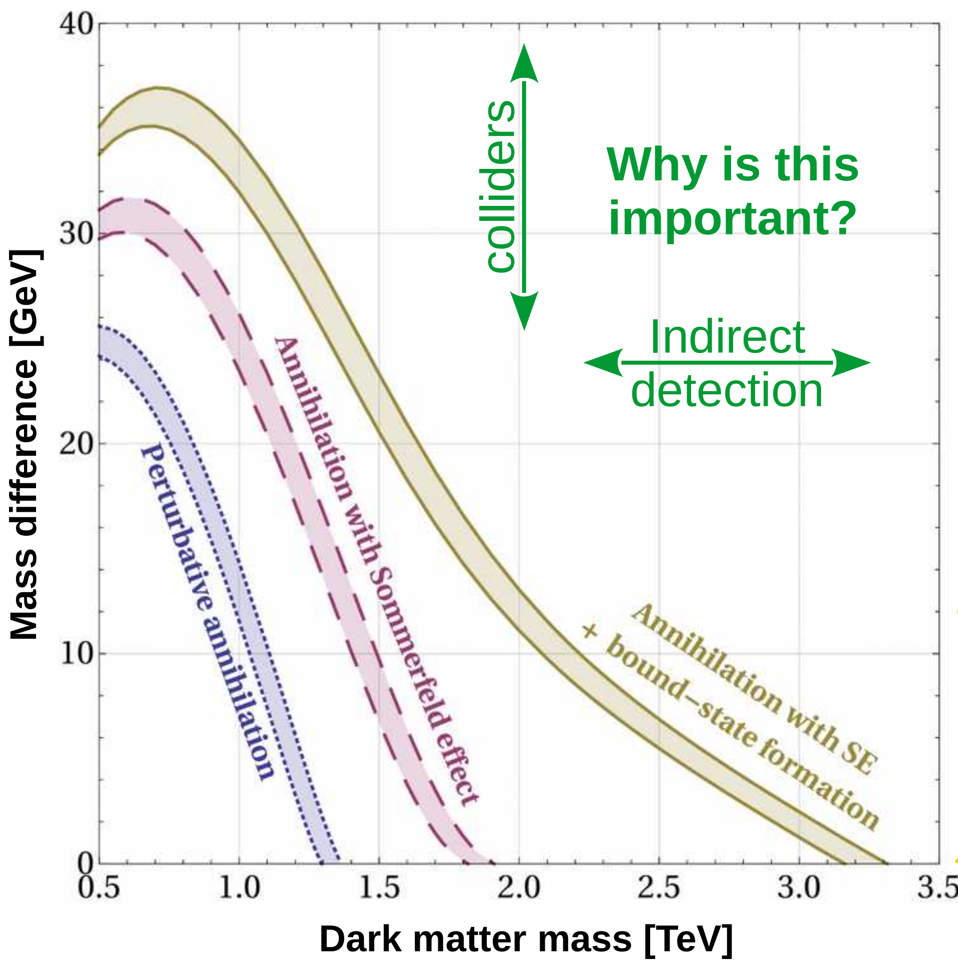



BSF can exceed Annihilation by more than an order of magnitude!

Harz, KP: 1805.01200

# DM coannihilation with scalar colour triplet

## MSSM-inspired toy model




**Why is this important?**

Effect on relic density: much much larger than obs uncertainty in $\Omega_{DM}$

**Not the final picture!**

**Harz, KP: 1805.01200**

## The Higgs as a *light* force mediator

# Squark-neutralino co-annihilation scenarios

- Degenerate spectrum → soft jets → evade LHC constraints
- Large stop-Higgs coupling reproduces measured Higgs mass and brings the lightest stop close in mass with the LSP

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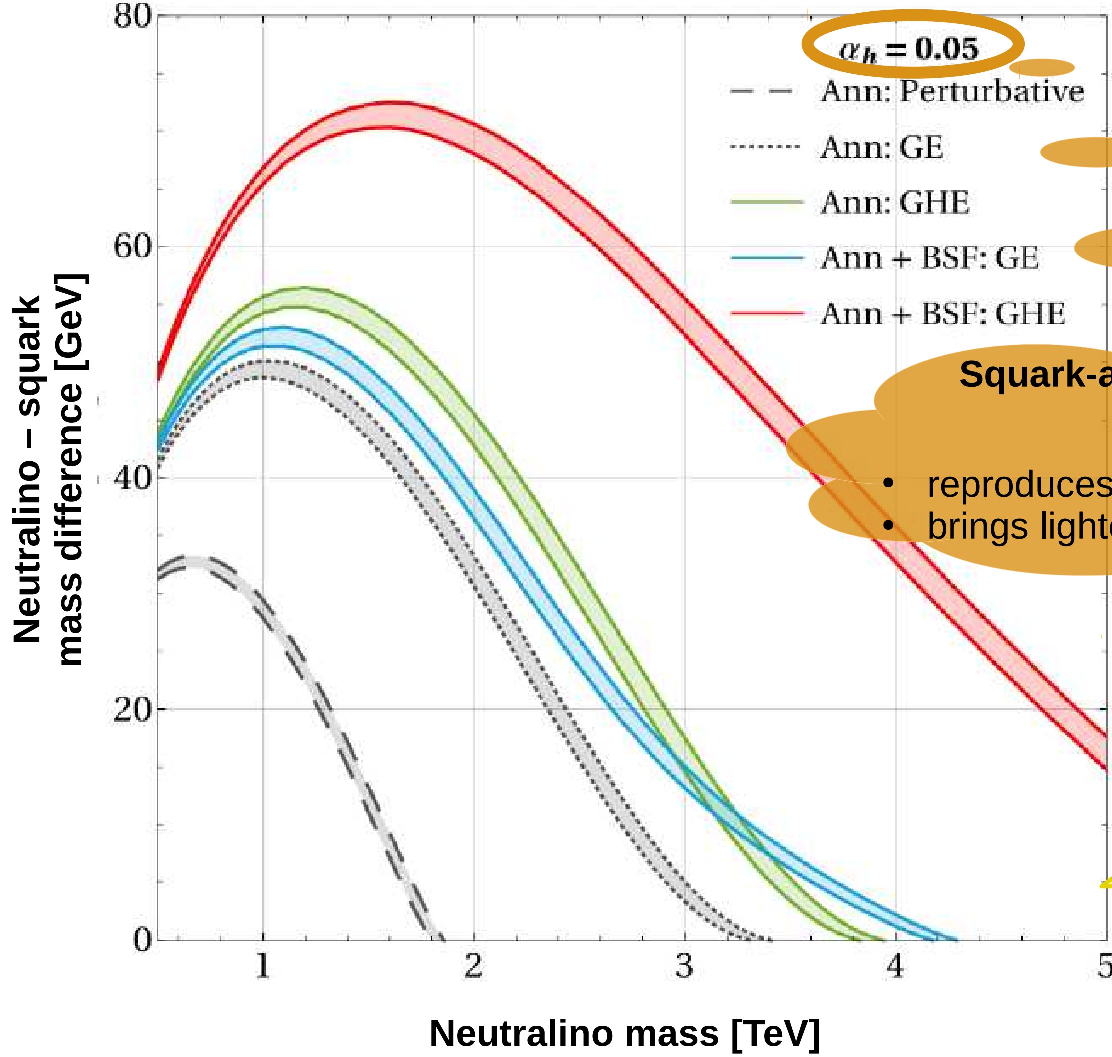
**Scenario probed in colliders.**
**Important to compute DM density accurately!**
→ **QCD corrections**

# DM coannihilation with scalar colour triplet

## MSSM-inspired toy model

## The effect of the Higgs-mediated potential




**Squark-antisquark-Higgs coupling**

Large $\alpha_h$

- reproduces measured Higgs mass
- brings lightest stop close in mass with LSP

**Not the final picture!**

**Harz and KP: 1711.03552, 1901.10030**

# The Higgs as a light mediator

- Sommerfeld enhancement of direct annihilation — Harz, KP: 1711.03552
- Binding of bound states — Harz, KP: 1901.10030

- **Formation of bound states via Higgs (*doublet*) emission ?**

Capture via emission of neutral scalar suppressed,
due to selection rules: quadruple transitions

March-Russel, West 0812.0559
KP, Postma, Wiechers: 1505.00109
An, Wise, Zhang: 1606.02305
KP, Postma, de Vries: 1611.01394

Capture via emission of charged scalar [or its Goldstone mode]
very very rapid: monopole transitions !

Ko,Matsui,Tang: 1910:04311
Oncala, KP: 1911.02605
Oncala, KP: 2101.08666
Oncala, KP: 2101.08667

Sudden change in effective Hamiltonian precipitates transitions.
Akin to atomic transitions precipitated by $\beta$ decay of nucleus.

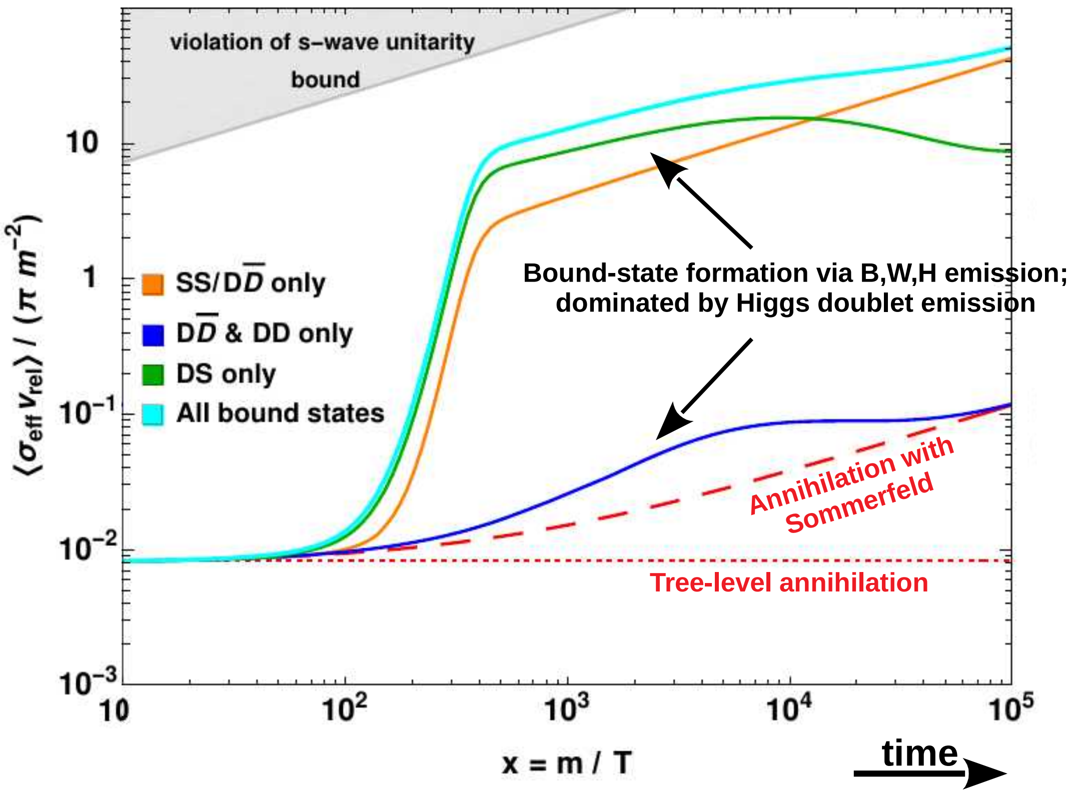
# Renormalisable Higgs-portal WIMP models

***S***inglet-***D***oublet coupled to the Higgs: $\boldsymbol{L \supset -y\,\bar{D}\,H\,S}$

$m_D \simeq m_S \rightarrow D$ and $S$ co-annihilate.

Freeze-out begins before the EWPT if $m_{DM} > 5\text{TeV}$

$\mathbf{M_{DM} = 20\ TeV, \quad \alpha_H = y^2 / (4\pi) = 0.2}$




**Oncala, KP: 2101.08666/7**

# Renormalisable Higgs-portal WIMP models

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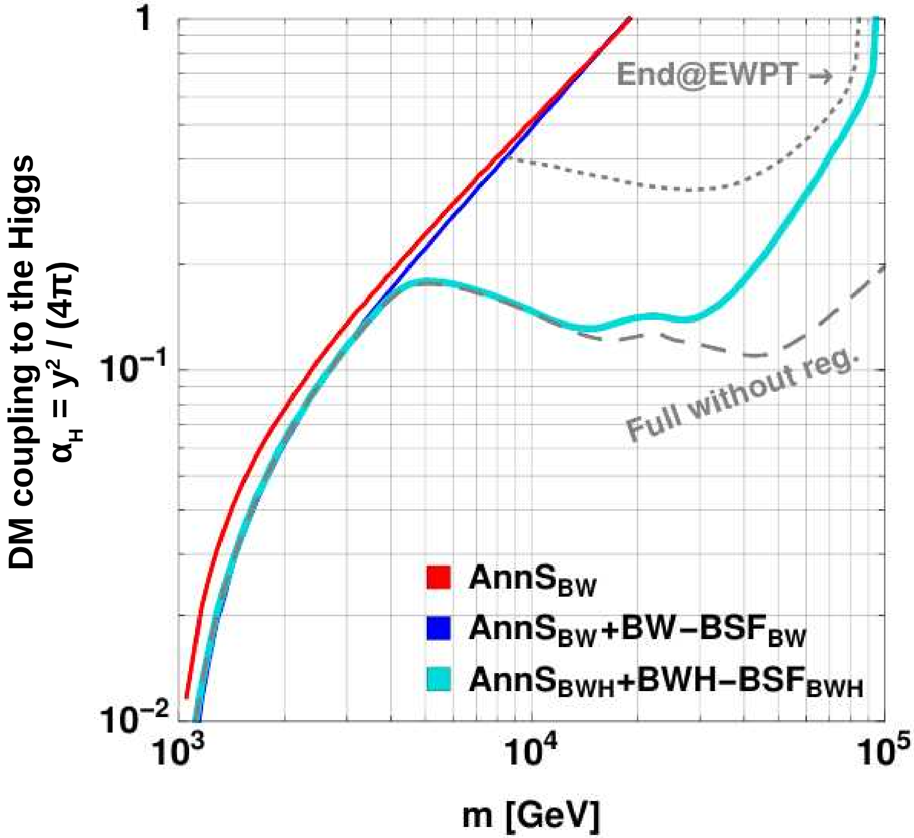



**Huge effect!**

**~ $10^2$ in relic density!**

**Impels reconsideration of Higgs-portal models (incl. neutralino-squark coann scenarios)**

**Oncala, KP: 2101.08666/7**

# Conclusions

- **Bound states impel complete reconsideration of thermal decoupling at / above the TeV scale: *emergence of a new type of inelasticity***

  Unitarity limit can be approached / attained only by long-range interactions
  ⇒ bound states play very important role! **Baldes, KP: 1703.00478**

  There is no unitarity limit on the mass of thermal relic DM!

- **Experimental implications:**
  - DM heavier than anticipated: multi-TeV probes very important

    ⇒ build the 100 TeV collider :)
  - Indirect detection:

    Enhanced rates due to BSF
    Novel signals: low-energy radiation emitted in BSF
    Indirect detection of asymmetric DM
  - Colliders: improved detection prospects due increased mass gap in coannihilation scenarios

- **Effects *not* limited freeze-out scenario:**
  freeze-in, asymmetric DM, self-interacting DM, stable bound states

# Extra slides

# Thermal freeze-out with bound states

## Boltzmann equations

free particles: $$\frac{dn}{dt} + 3Hn = -\langle\sigma^{\rm ann} v_{\rm rel}\rangle\left(n^2 - n^{\rm eq\ 2}\right) - \sum_{\mathcal{B}}\left(\langle\sigma^{\rm BSF}_{\mathcal{B}}\, v_{\rm rel}\rangle\, n^2 - \Gamma^{\rm ion}_{\mathcal{B}}\, n_{\mathcal{B}}\right)$$

bound states: $$\frac{dn_{\mathcal{B}}}{dt} + 3Hn_{\mathcal{B}} = +\left(\langle\sigma^{\rm BSF}_{\mathcal{B}}\, v_{\rm rel}\rangle\, n^2 - \Gamma^{\rm ion}_{\mathcal{B}}\, n_{\mathcal{B}}\right) - \Gamma^{\rm dec}_{\mathcal{B}}\left(n_{\mathcal{B}} - n^{\rm eq}_{\mathcal{B}}\right) - \sum_{\mathcal{B}'\neq\mathcal{B}}\left(\Gamma^{\rm trans}_{\mathcal{B}\to\mathcal{B}'} n_{\mathcal{B}} - \Gamma^{\rm trans}_{\mathcal{B}'\to\mathcal{B}} n_{\mathcal{B}'}\right)$$

| Processes | | Detailed balance |
|---|---|---|
| Bound state formation (BSF)<br>Ionisation (ion) | $X + \bar{X} \to \mathcal{B}(X\bar{X}) + \gamma_D$<br>$\mathcal{B}(X\bar{X}) + \gamma_D \to X + \bar{X}$ | $\langle\sigma^{\rm BSF}_{\mathcal{B}} v_{\rm rel}\rangle (n^{\rm eq})^2 = \Gamma^{\rm ion}_{\mathcal{B}}\, n^{\rm eq}_{\mathcal{B}}$ |
| Decay (dec) | $\mathcal{B}(X\bar{X}) \to 2\gamma_D$ or $3\gamma_D$ | |
| Transitions (trans) | $\mathcal{B}(X\bar{X}) \to \mathcal{B}'(X\bar{X}) + \gamma_D$<br>$\mathcal{B}(X\bar{X}) + \gamma_D \to \mathcal{B}'(X\bar{X})$ | $\Gamma^{\rm trans}_{\mathcal{B}\to\mathcal{B}'}\, n^{\rm eq}_{\mathcal{B}} = \Gamma^{\rm trans}_{\mathcal{B}'\to\mathcal{B}}\, n^{\rm eq}_{\mathcal{B}'}$ |

# Thermal freeze-out with bound states

## Boltzmann equations

free particles: $$\frac{dn}{dt} + 3Hn = -\langle\sigma^{\rm ann} v_{\rm rel}\rangle \left(n^2 - n^{{\rm eq}\ 2}\right) - \sum_{\mathcal{B}} \left(\langle\sigma_{\mathcal{B}}^{\rm BSF}\, v_{\rm rel}\rangle\, n^2 - \Gamma_{\mathcal{B}}^{\rm ion}\, n_{\mathcal{B}}\right)$$

bound states: $$\frac{dn_{\mathcal{B}}}{dt} + 3Hn_{\mathcal{B}} = + \left(\langle\sigma_{\mathcal{B}}^{\rm BSF}\, v_{\rm rel}\rangle\, n^2 - \Gamma_{\mathcal{B}}^{\rm ion}\, n_{\mathcal{B}}\right) - \Gamma_{\mathcal{B}}^{\rm dec}\left(n_{\mathcal{B}} - n_{\mathcal{B}}^{\rm eq}\right) - \sum_{\mathcal{B}' \neq \mathcal{B}} \left(\Gamma_{\mathcal{B}\to\mathcal{B}'}^{\rm trans} n_{\mathcal{B}} - \Gamma_{\mathcal{B}'\to\mathcal{B}}^{\rm trans} n_{\mathcal{B}'}\right)$$

Typically at least one rate is large enough

$$\Gamma_{\mathcal{B}}^{\rm ion} + \Gamma_{\mathcal{B}}^{\rm dec} + \Gamma_{\mathcal{B}}^{\rm trans} \gg H$$

to keep bound states close to equilibrium

$\Rightarrow$ set $dn_{\mathcal{B}}/dt + 3Hn_{\mathcal{B}} \simeq 0$

$\Rightarrow$ get algebraic equations for $n_{\mathcal{B}}$ in terms of $n$, $n_{\mathcal{B}}^{\rm eq}$

$\Rightarrow$ re-employ it in Boltzmann equation for $n$

Ellis, Luo, Olive: 1503.07142

**Complete treatement:**
**Binder, Filimonova, Petraki, White 2112.00042**

# Thermal freeze-out with bound states

## Boltzmann equations and effective cross-section

free particles: $$\frac{dn}{dt} + 3Hn = -\langle \sigma^{\rm ann} v_{\rm rel}\rangle \left(n^2 - n^{\rm eq\ 2}\right) - \sum_{\mathcal{B}} \left(\langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle\, n^2 - \Gamma_{\mathcal{B}}^{\rm ion}\, n_{\mathcal{B}}\right)$$

bound states: $$\frac{dn_{\mathcal{B}}}{dt} + 3Hn_{\mathcal{B}} = + \left(\langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle\, n^2 - \Gamma_{\mathcal{B}}^{\rm ion}\, n_{\mathcal{B}}\right) - \Gamma_{\mathcal{B}}^{\rm dec}\left(n_{\mathcal{B}} - n_{\mathcal{B}}^{\rm eq}\right) - \sum_{\mathcal{B}' \neq \mathcal{B}} \left(\Gamma_{\mathcal{B}\to\mathcal{B}'}^{\rm trans} n_{\mathcal{B}} - \Gamma_{\mathcal{B}'\to\mathcal{B}}^{\rm trans} n_{\mathcal{B}'}\right)$$

$$\frac{dn}{dt} + 3Hn = -\langle \sigma^{\rm eff} v_{\rm rel}\rangle \left(n^2 - n^{\rm eq\ 2}\right)$$

where, neglecting bound-to-bound transitions,

$$\langle \sigma^{\rm eff} v_{\rm rel}\rangle \equiv \langle \sigma^{\rm ann} v_{\rm rel}\rangle + \sum_{\mathcal{B}} \langle \sigma_{\mathcal{B}}^{\rm BSF} v_{\rm rel}\rangle \times \frac{\Gamma_{\mathcal{B}}^{\rm dec}}{\Gamma_{\mathcal{B}}^{\rm dec} + \Gamma_{\mathcal{B}}^{\rm ion}}$$

**Attractor solution is the equilibrium density**

efficiency factors

$$r_{\mathcal{B}} = \sum_{\mathcal{B}'} \Gamma_{\mathcal{B}'}^{\rm dec} \left(\mathbb{\Gamma}^{\rm ion} + \mathbb{\Gamma}^{\rm dec} + \mathbb{\Gamma}^{\rm trans} - \mathbb{T}\right)^{-1}_{\mathcal{B}'\mathcal{B}}$$

**Binder, Filimonova, Petraki, White 2112.00042**

# Thermal freeze-out with bound states

## Effective cross-section

$$\frac{dn}{dt} + 3Hn = -\langle \sigma^{\rm eff} v_{\rm rel} \rangle \left( n^2 - n^{\rm eq\ 2} \right)$$

where, neglecting bound-to-bound transitions,

$$\langle \sigma^{\rm eff} v_{\rm rel} \rangle \equiv \langle \sigma^{\rm ann} v_{\rm rel} \rangle + \sum_{\mathcal{B}} \langle \sigma^{\rm BSF}_{\mathcal{B}} v_{\rm rel} \rangle \times \frac{\Gamma^{\rm dec}_{\mathcal{B}}}{\Gamma^{\rm dec}_{\mathcal{B}} + \Gamma^{\rm ion}_{\mathcal{B}}}$$

At $T \gg$ Binding Energy $\Rightarrow \Gamma^{\rm ion}_{\mathcal{B}} \gg \Gamma^{\rm dec}_{\mathcal{B}}$,

$$\langle \sigma^{\rm BSF}_{\mathcal{B}} v_{\rm rel} \rangle \frac{\Gamma^{\rm dec}_{\mathcal{B}}}{\Gamma^{\rm dec}_{\mathcal{B}} + \Gamma^{\rm ion}_{\mathcal{B}}} \simeq \langle \sigma^{\rm BSF}_{\mathcal{B}} v_{\rm rel} \rangle \frac{\Gamma^{\rm dec}_{\mathcal{B}}}{\Gamma^{\rm ion}_{\mathcal{B}}} = \frac{n^{\rm eq}_{\mathcal{B}}}{(n^{\rm eq})^2} \Gamma^{\rm dec}_{\mathcal{B}}$$

$$\simeq \frac{g_{\mathcal{B}}}{g_X^2} \left( \frac{4\pi}{m_X T} \right)^{3/2} \times e^{|E_{\mathcal{B}}|/T}\ \Gamma^{\rm dec}_{\mathcal{B}}$$

$$\Downarrow$$

Independent of actual BSF cross-section!

$\Gamma^{\rm dec}_{\mathcal{B}} \propto (\sigma^{\rm ann} v_{\rm rel}) \rightarrow$ modest increase over the direct annihilation, but increases exponentially as $T$ drops.

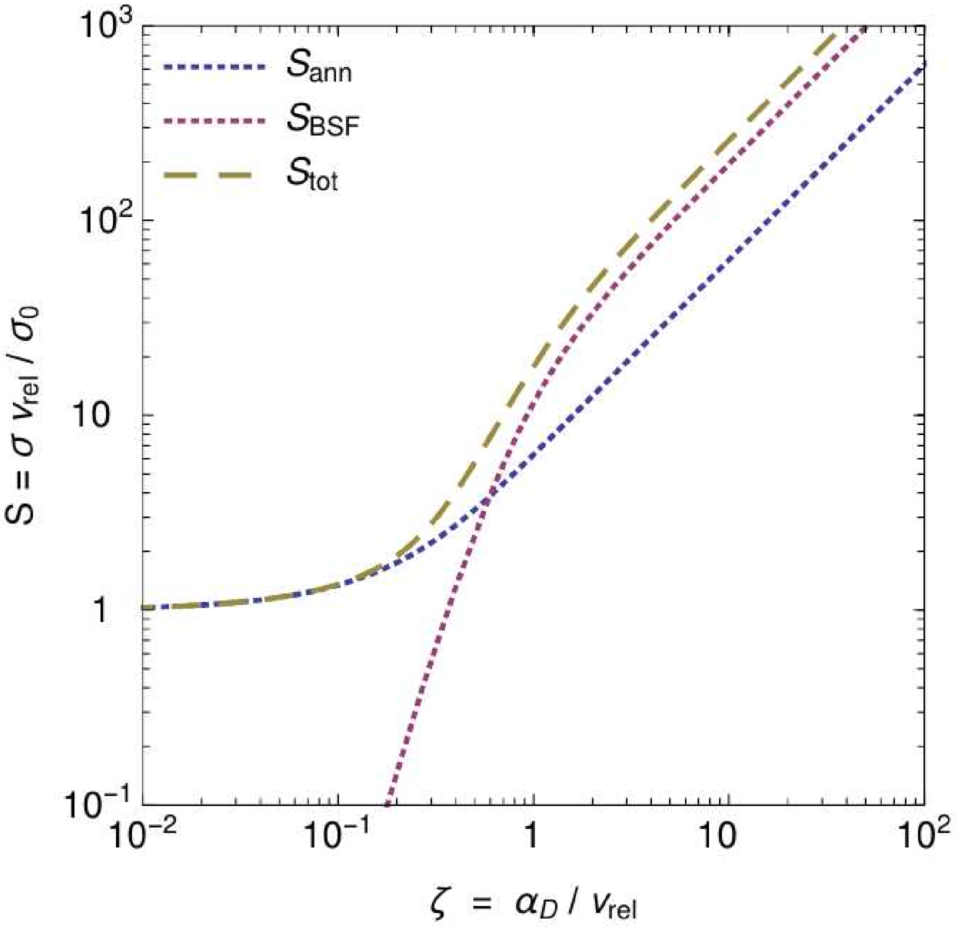
At $T \lesssim$ Binding Energy $\Rightarrow \Gamma^{\rm ion}_{\mathcal{B}} \ll \Gamma^{\rm dec}_{\mathcal{B}}$,

$$\langle \sigma^{\rm BSF}_{\mathcal{B}} v_{\rm rel} \rangle \frac{\Gamma^{\rm dec}_{\mathcal{B}}}{\Gamma^{\rm dec}_{\mathcal{B}} + \Gamma^{\rm ion}_{\mathcal{B}}} \simeq \langle \sigma^{\rm BSF}_{\mathcal{B}} v_{\rm rel} \rangle.$$

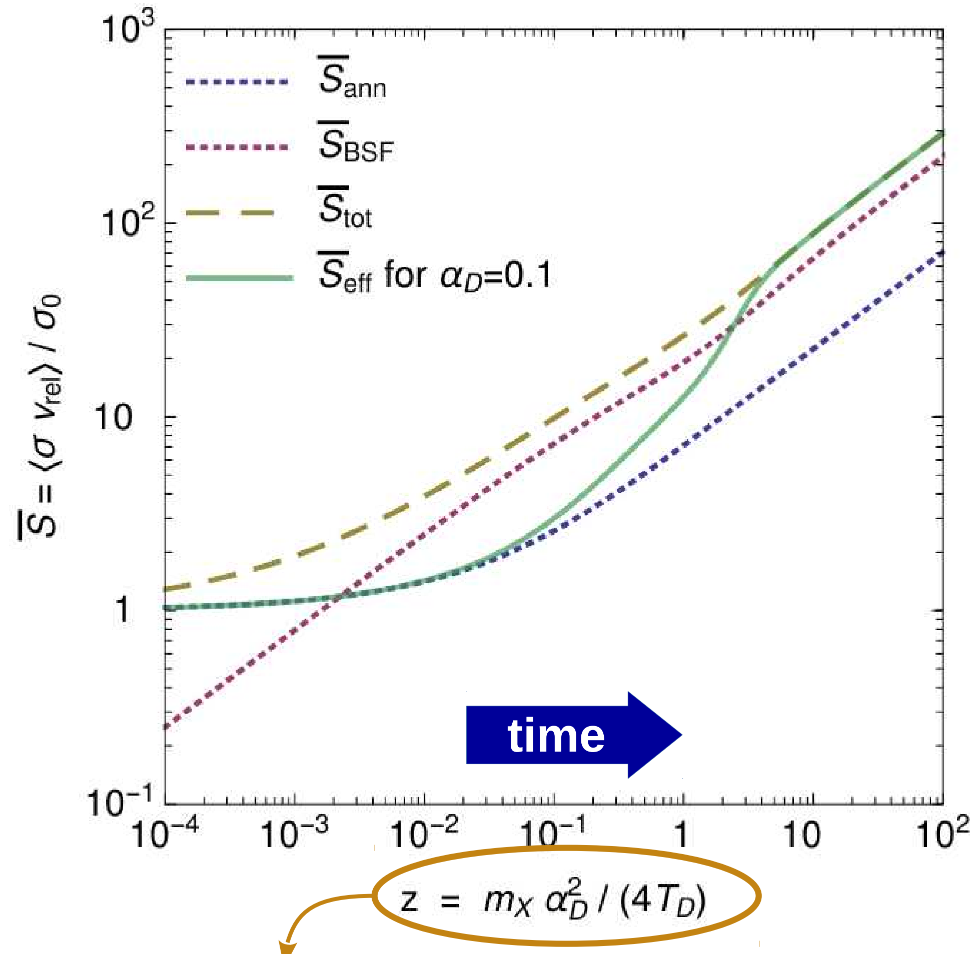
Typically, most of DM destruction due to BSF occurs in this regime.

# Effective cross-section in dark U(1) model

**Cross-sections**




**Thermally averaged cross-sections**




**binding energy / temperature**

# Thermal freeze-out with bound states

## Boltzmann equations and effective cross-section

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bound states: $$\frac{dn_{\mathcal{B}}}{dt} + 3Hn_{\mathcal{B}} = + \left(\langle \sigma^{\rm BSF}_{\mathcal{B}} v_{\rm rel}\rangle n^2 - \Gamma^{\rm ion}_{\mathcal{B}} n_{\mathcal{B}}\right) - \Gamma^{\rm dec}_{\mathcal{B}} \left(n_{\mathcal{B}} - n^{\rm eq}_{\mathcal{B}}\right) - \sum_{\mathcal{B}' \neq \mathcal{B}} \left(\Gamma^{\rm trans}_{\mathcal{B}\to\mathcal{B}'} n_{\mathcal{B}} - \Gamma^{\rm trans}_{\mathcal{B}'\to\mathcal{B}} n_{\mathcal{B}'}\right)$$

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efficiency factors

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**Binder, Filimonova, Petraki, White 2112.00042**

**Bound-to-bound transitions only enhance the total effective cross-section!**

# A corollary

## Saha equilibrium for metastable bound states

$$\boxed{\frac{n_{\mathcal{B}}}{n_{\mathcal{B}}^{\rm eq}} = \left(\frac{n_{\rm free}}{n_{\rm free}^{\rm eq}}\right)^2 - \left[\left(\frac{n_{\rm free}}{n_{\rm free}^{\rm eq}}\right)^2 - 1\right] r_{\mathcal{B}}}$$

**Binder, Filimonova, Petraki, White 2112.00042**

$$r_{\mathcal{B}} = \sum_{\mathcal{B}'} \Gamma^{\rm dec}_{\mathcal{B}'} \left(\mathbb{\Gamma}^{\rm ion} + \mathbb{\Gamma}^{\rm dec} + \mathbb{\Gamma}^{\rm trans} - \mathbb{1}\right)^{-1}_{\mathcal{B}'\mathcal{B}}$$

$r_{\mathcal{B}} = 0$ | $r_{\mathcal{B}} = 1$

$$\frac{n_{\mathcal{B}}}{n_{\mathcal{B}}^{\rm eq}} = \left(\frac{n_{\rm free}}{n_{\rm free}^{\rm eq}}\right)^2$$

Standard Saha equilibrium

$$\frac{n_{\mathcal{B}}}{n_{\mathcal{B}}^{\rm eq}} = 1$$

Particles with decay rate > Hubble