UNIVERSAL MANIFESTATIONS & MODELLING OF *BOSE-EINSTEIN CONDENSATION*:

From Atomic & Condensed Matter to Cosmological Scales

Joint Quantum Centre WWW.JQC.ORG.UK

Newcastle University

Xmas Theoretical Physics Workshop @Athens 2022

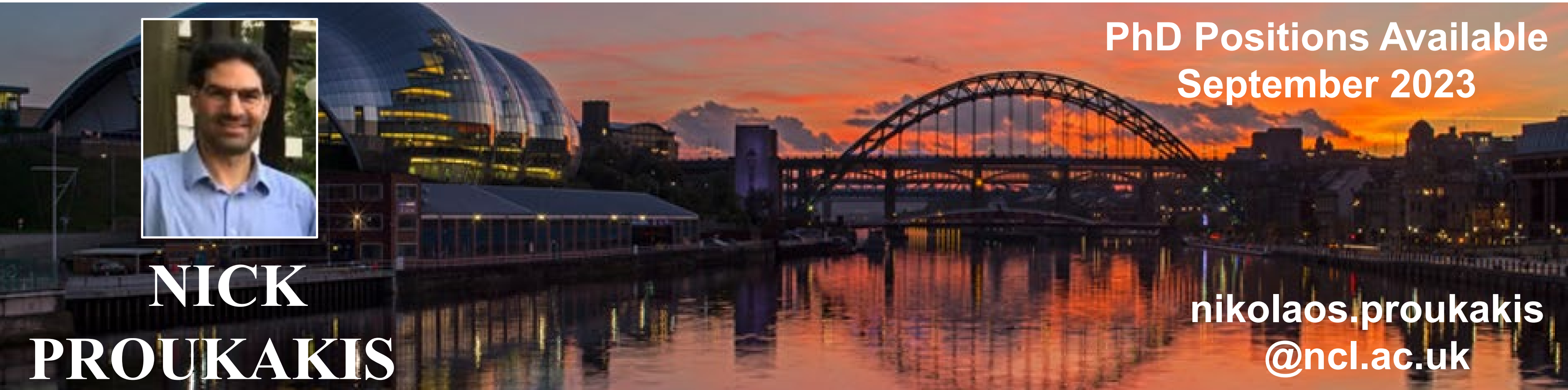
EPSRC Engineering and Physical Sciences Research Council

QUANTERA

The Leverhulme Trust

MARIE CURIE ACTIONS

PRACE PARTNERSHIP FOR ADVANCED COMPUTING IN EUROPE



NICK PROUKAKIS

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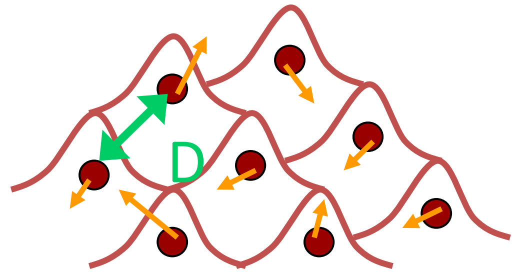
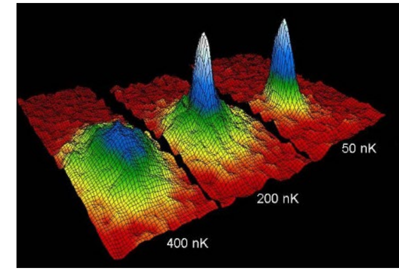
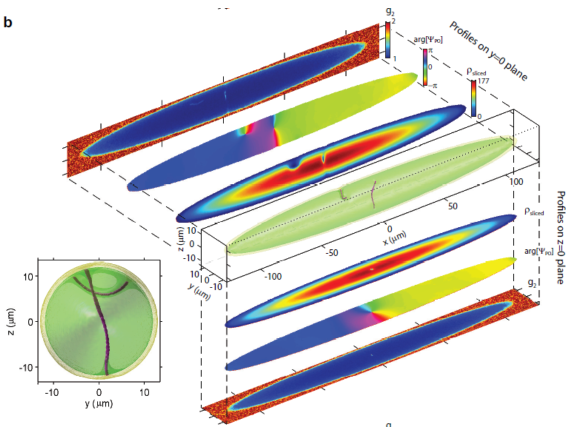
PhD Positions Available September 2023

OUTLINE

Intro to Universality
of
Bose-Einstein Condensation

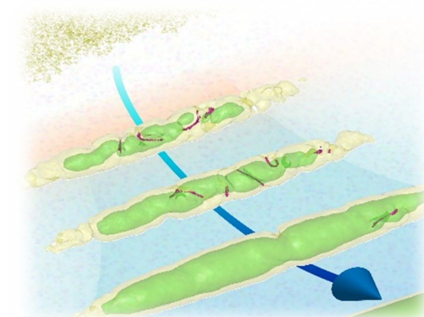


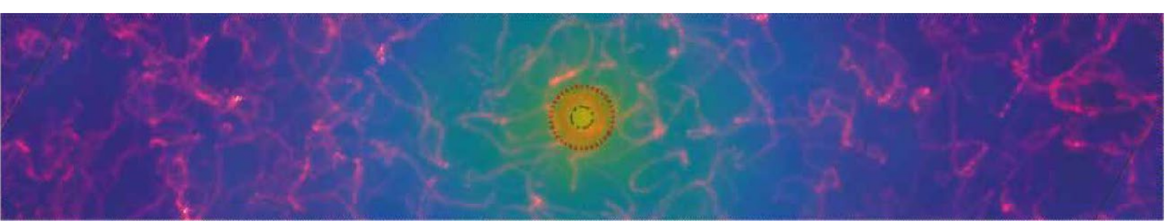




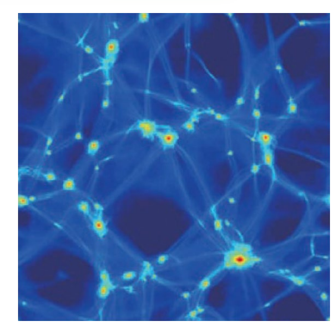
Modelling
&
Characterization
of a Condensate

Universality of
Condensate Formation
in Laboratory Systems





Features
of a
Cosmological
Condensate

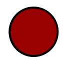


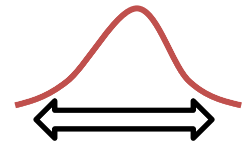
Conclusions

BOSE-EINSTEIN CONDENSATION (BEC)

A Fundamental Principle of Quantum Mechanics:

(Point) Particles can behave like (Extended) Waves





$$\lambda \propto 1/\sqrt{T}$$

How does this affect the behaviour of many particles?

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A Fundamental Principle of Quantum Mechanics:

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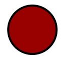
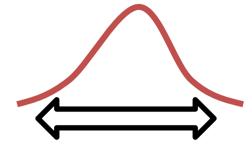




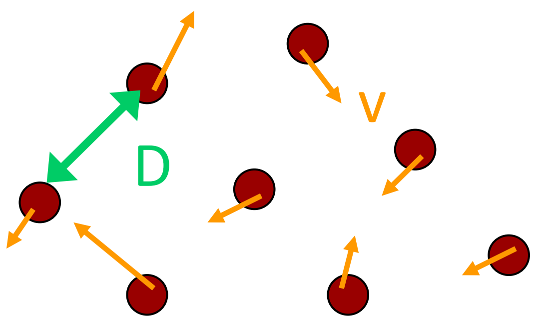
$$\lambda \propto 1/\sqrt{T}$$

How does this affect the behaviour of many particles?

'Classical' Regime

$$\boxed{T > T_C}$$

(Distinguishable Particles)



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A Fundamental Principle of Quantum Mechanics:

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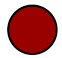
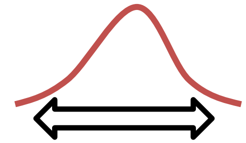




$$\lambda \propto 1/\sqrt{T}$$

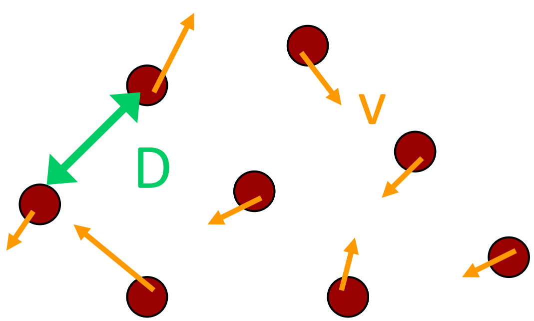
How does this affect the behaviour of many particles?

'Classical' Regime $\boxed{T > T_C}$

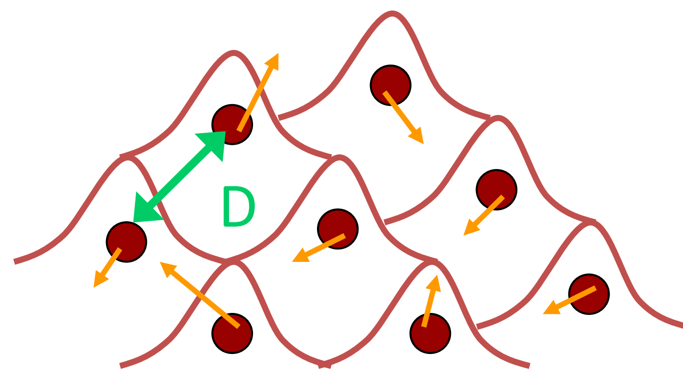
(Distinguishable Particles)

'Quantum' Regime $\boxed{T < T_C}$

(Indistinguishable Particles) → **Fermions** / ***Bosons***



$$\boxed{n\lambda^3 \sim 1}$$



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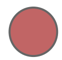

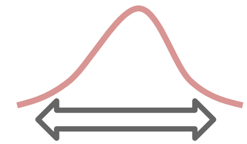

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How does this affect the behaviour of many particles?

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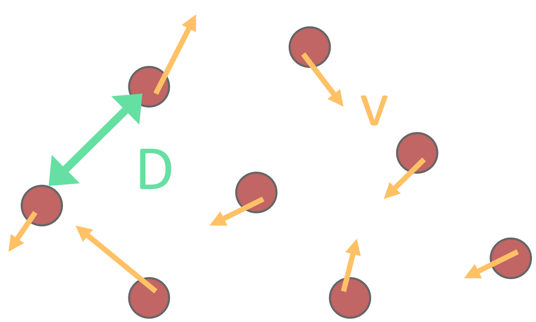
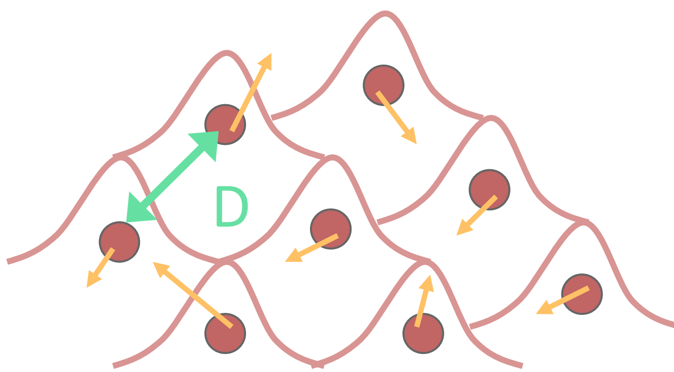


$$\boxed{n\lambda^3 \sim 1}$$

'Quantum' Regime $\boxed{T < T_C}$

(Indistinguishable Particles) → Fermions / ***Bosons***



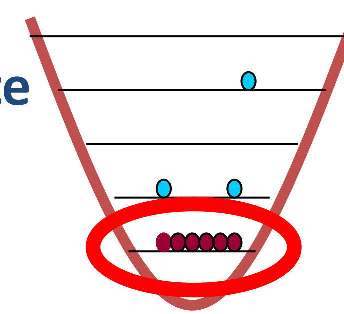
Bose & Einstein (1924-5):

Bosonic **Particles Favour Co-existence in Same Quantum State**

(*'Condensation' in Momentum State*)

→ Emergence of a *Macroscopic* Quantum State

(e.g. at Critical Temperature)



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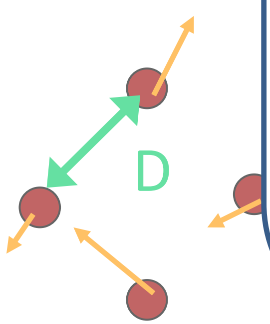
How ... es?

'Classical' Regime

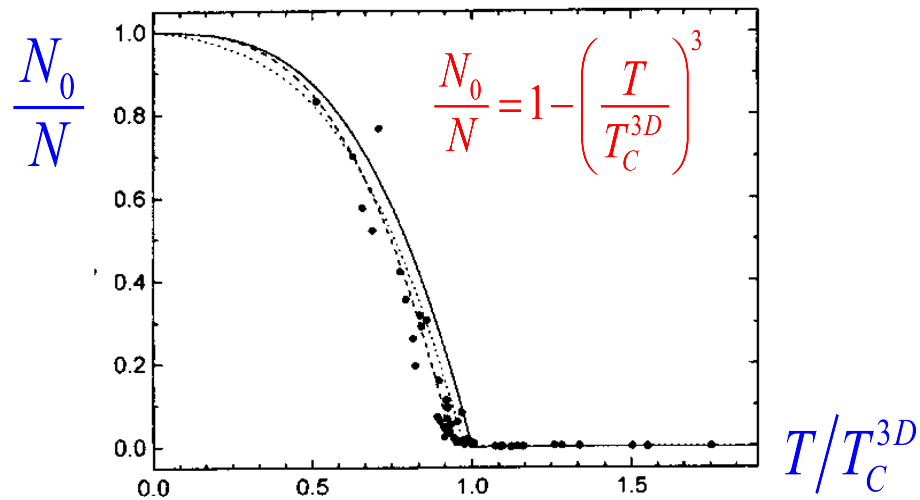
(Distinguishable ... les)

Fermions

Bosons



% OF CONDENSATE ATOMS



Transition Happens Suddenly !

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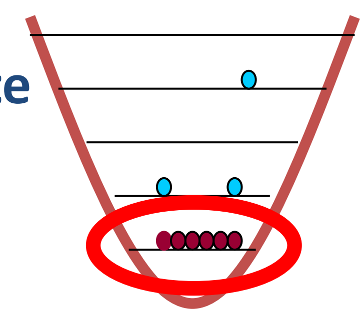
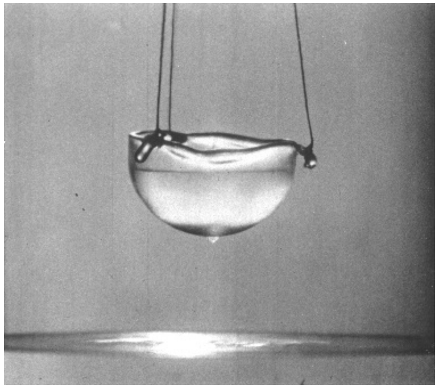
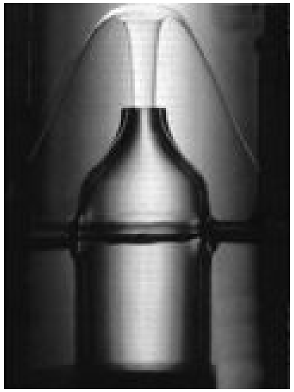


BEC AT PLAY IN VASTLY DIFFERENT PHYSICAL SYSTEMS

ELEMENTARY BOSON	PHYSICAL SYSTEM	DENSITY	TEMP (K)

‘Traditional’ Systems

ELEMENTARY BOSON	PHYSICAL SYSTEM	DENSITY	TEMP (K)
^{4}He Atom	$\boxed{\text{Liquid Helium}}$	10^{22} cm^{-3}	2

Superfluidity interpreted as Bose-Einstein Condensation of Bosonic Atoms
(late 1930’s !)

In General, one often also deals with

--Composite particles (of many bosons/fermions)

--Quasiparticles (Effective/Dressed particles)

→ Condensation of Multi-particle ‘Entities’ (Atoms) incl. Fermionic Systems

Proukakis & Burnett in *Quantum Gases: Finite Temperature & Non-Equilibrium Dynamics*
[Proukakis, Gardiner, Davis & Szymanska (Eds), Imperial College Press (2013)]

BEC AT PLAY IN VASTLY DIFFERENT PHYSICAL SYSTEMS

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ELEMENTARY BOSON	PHYSICAL SYSTEM	DENSITY	TEMP (K)
^{4}He Atom	Liquid Helium	10^{22} cm^{-3}	2
^{3}He Atom Pairs	Liquid Helium	10^{22} cm^{-3}	2×10^{-3}
Cooper Pair	Superconductor	10^{23} cm^{-3}	~ 10
Cooper Pair	Exotic / High T_C Superconductor	10^{21} cm^{-3}	1 – 160 (+)

→ All Different Manifestations of BEC + Intrinsic System Properties

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BEC AT PLAY IN VASTLY DIFFERENT PHYSICAL SYSTEMS

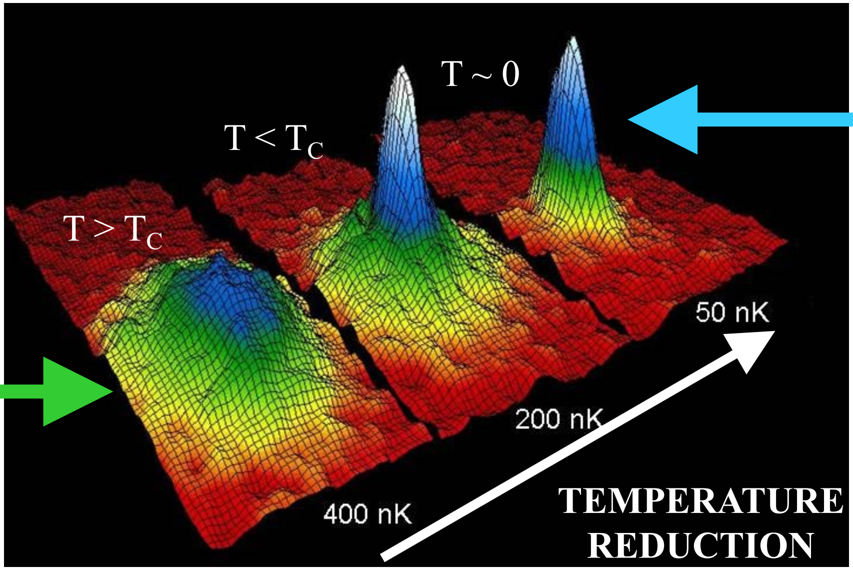
ELEMENTARY BOSON	PHYSICAL SYSTEM	DENSITY	TEMP (K)
Bosonic Ultracold Atom (BEC)	Trapped Atomic Gas	$10^{13} – 10^{15}$ cm^{-3}	$10^{-7} – 5\times10^{-5}$
Fermionic Ultracold Atom Pair (BCS)	Trapped Atomic Gas	$10^{12} – 10^{13}$ cm^{-3}	10^{-7}

Ultracold Atoms

(Dilute, Weakly-Interacting, Trapped)

(H, **Li, Na, K, Rb, Cs,** He*, Yb, Ca, Sr, Cr, Er, …)

CLASSICAL (THERMAL) GAS



BOSE-EINSTEIN CONDENSATE

Bose-Einstein Condensation
Physics Nobel 2001
Cornell, Ketterle, Wieman

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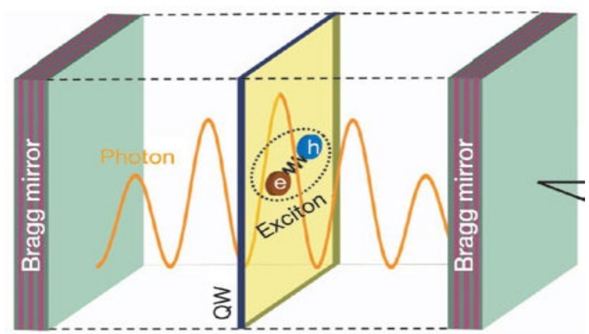
BEC AT PLAY IN VASTLY DIFFERENT PHYSICAL SYSTEMS

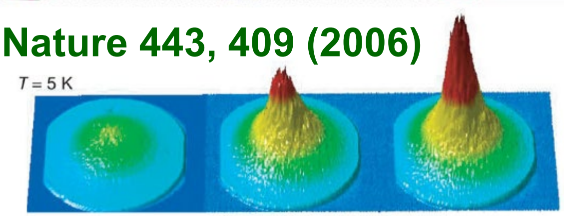
ELEMENTARY BOSON	PHYSICAL SYSTEM	DENSITY	TEMP (K)

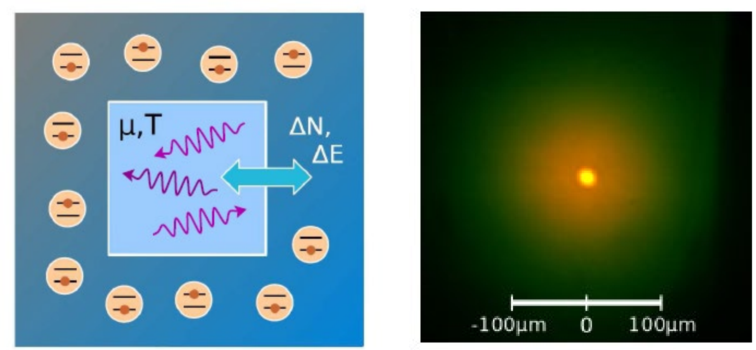
Optical & Magnetic Systems

Exciton–Polaritons	Semiconductor	10^9 cm^{-2}	~ few 10 K
Magnon	Magnetic Insulator	10^{18}-10^{19} cm^{-3}	Room Temp
Photon	Light	10^{11} cm^{-2}	Room Temp



Nature 443, 409 (2006)





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Cooper Pair	Superconductor	10^{23} cm^{-3}	~ 10
Cooper Pair	Exotic / High T_C Superconductor	10^{21} cm^{-3}	1 – 160 (+)
Nucleon Pair (nn / pp)	Neutron Star	10^{39} cm^{-3}	~ few 10^{8}

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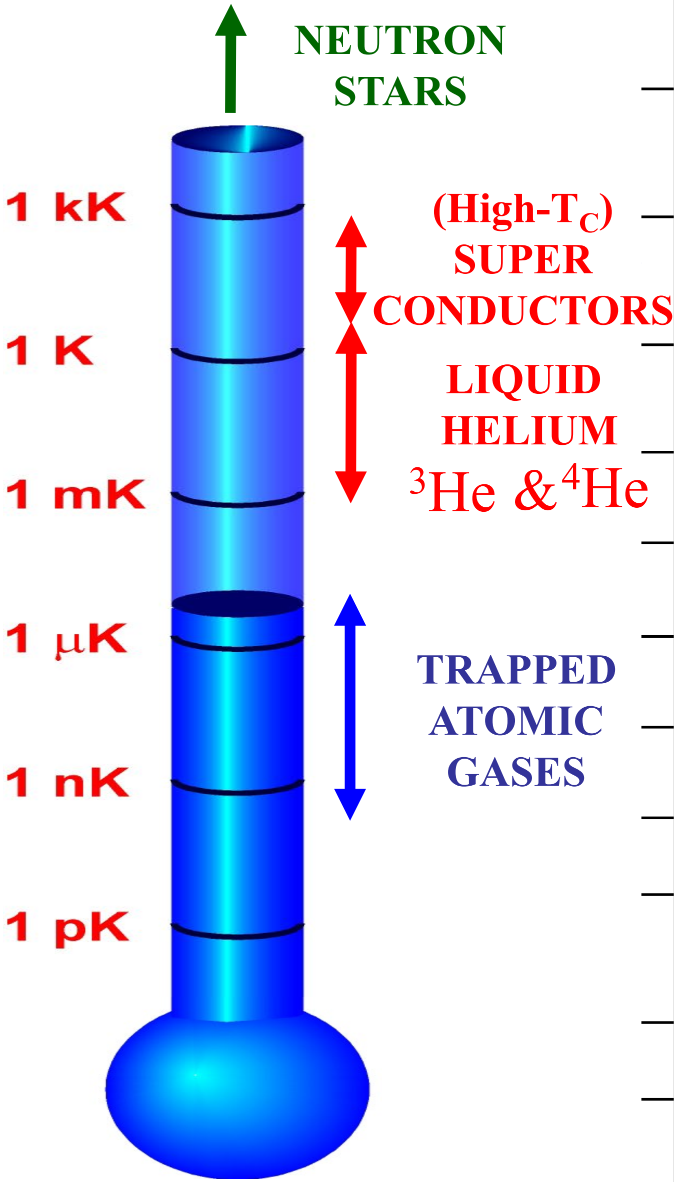

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Semiconductor		
Magnetic Insulator		
Light		
Liquid Helium		
Liquid Helium		
Superconductor		
Exotic / High T$_C$ Superconductor		
Neutron Star	10^{39} cm^{-3}	~ few 10^8

BEC Condition:

$$n\,[\lambda\,(T, m, v)]^d \sim 1$$

"Control Parameters"

* Density n
* Temperature T
* (Boson) Mass m
* Typical Velocity v

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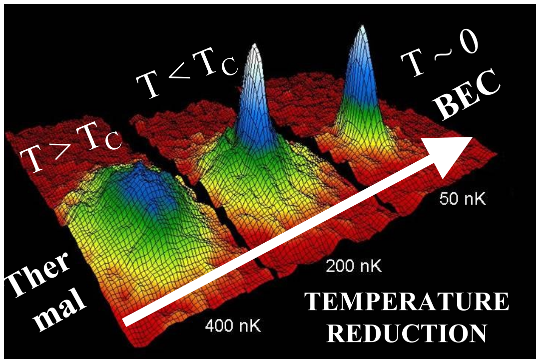
LABORATORY BECs (Weakly-Interacting)

Ultracold Atomic BECs

(3D / 2D / 1D)

[“Equilibrium” State]



Science 269, 198 (1995)

(Lifetime set by 3-Body Losses)

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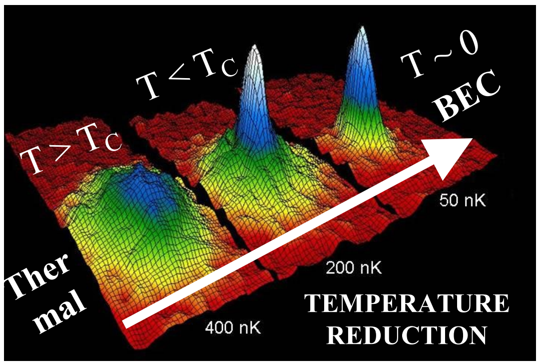

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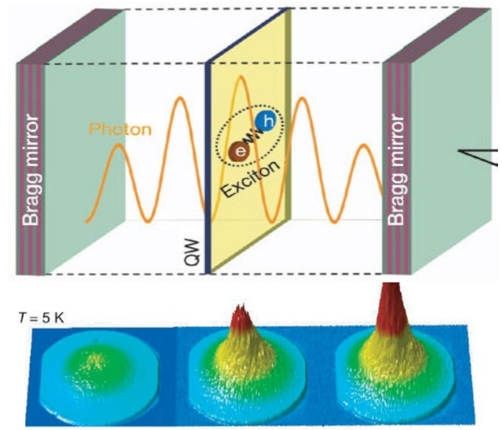
Exciton-Polariton BECs

(2D / 1D)

[(Quasi-)Equilibrium]



Nature 443, 409 (2006)

(Strong Light-Matter Coupling)
Driven—Dissipative System

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Ultracold Atomic BECs

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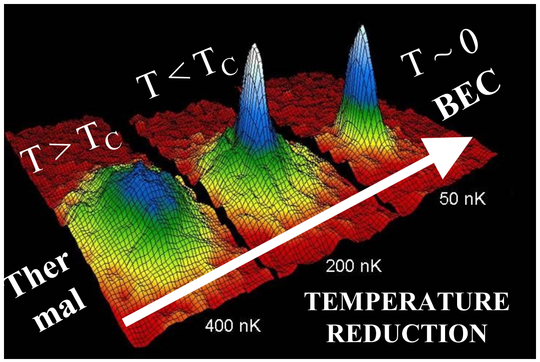

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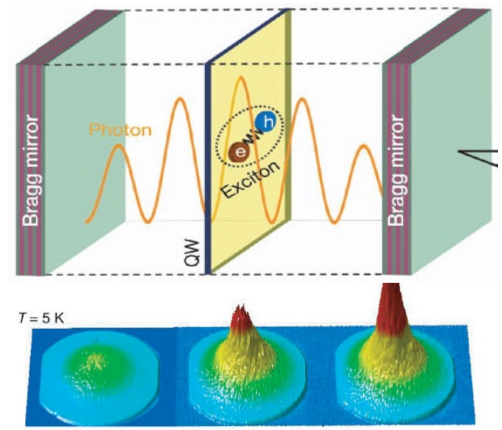

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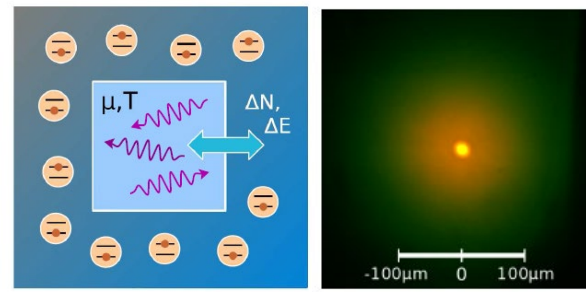
(Strong Light-Matter Coupling)

Driven—Dissipative System

Photon BECs

(2D)

[Quasi-Equilibrium]



Nature 468, 545 (2010)

Photon Gas Thermalisation

with Dye Solution

(Absorption ←→ Re-emission)

LABORATORY BECs (Weakly-Interacting)

Ultracold Atomic BECs

(3D / 2D / 1D)

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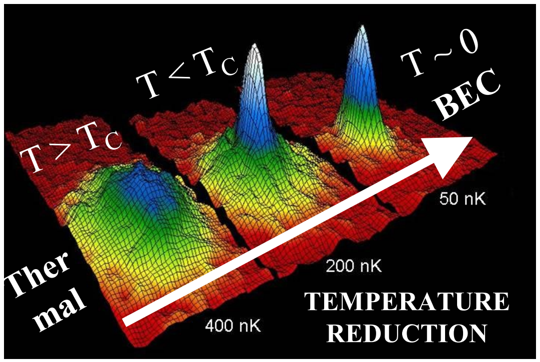

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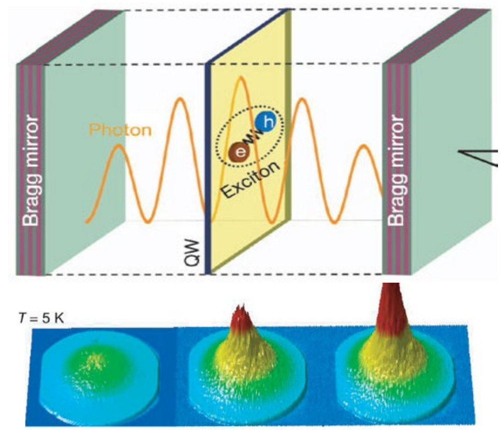

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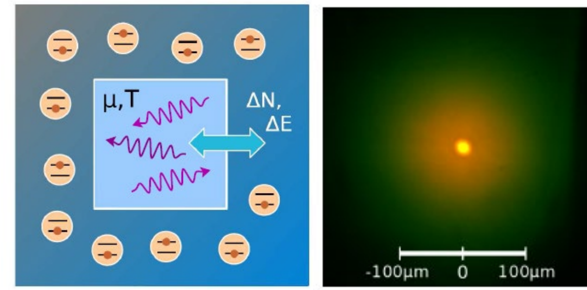


Nature 468, 545 (2010)

Photon Gas Thermalisation
with Dye Solution
(Absorption ←→ Re-emission)

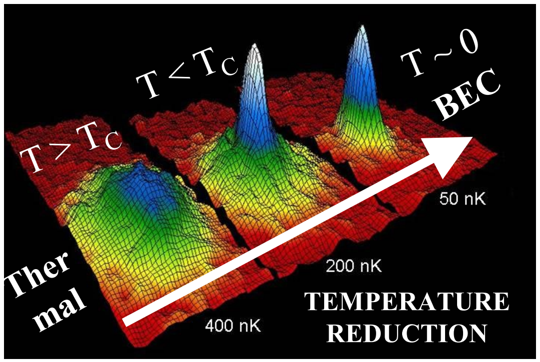
"Real-Time" Control Parameters:

Ultracold Atomic BECs	Exciton-Polariton BECs	Photon BECs
Temperature	Pumping	Temperature
Density	Density	Density
Interaction Strength / Type	Potential Energy Profile	Reservoir Size
Trap Profile	Photon Fraction	
Dispersion		

LABORATORY BECs (Weakly-Interacting)

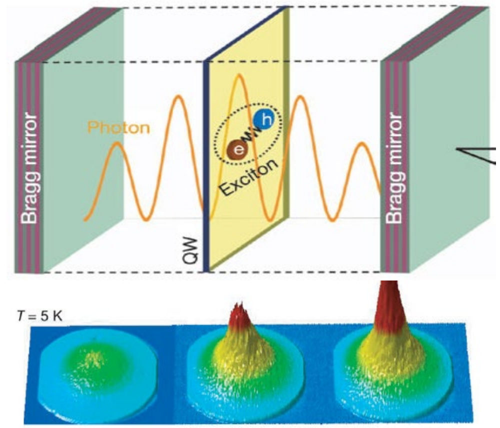
Ultracold Atomic BECs
(3D / 2D / 1D)

["Equilibrium" State]



Exciton-Polariton BECs
(2D / 1D)

[(Quasi-)Equilibrium]



Photon BECs
(2D)

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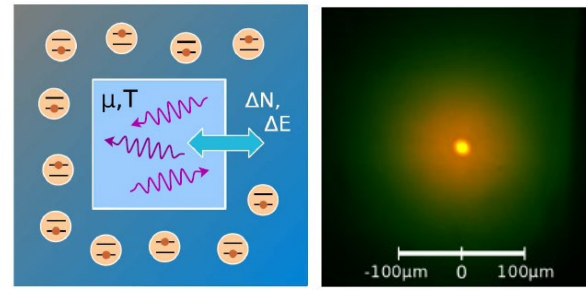

All Above Systems can be Described by a Macroscopic Wavefunction / Field obeying an (appropriate) Nonlinear Schroedinger (***Gross-Pitaevskii***) Equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{TRAP}(r) + g|\Psi|^2 \qquad\qquad\qquad\right)\Psi$$

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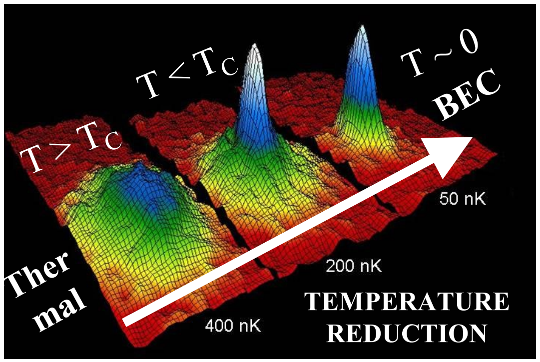

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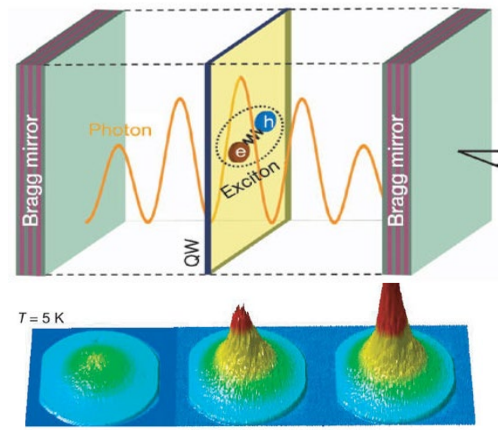
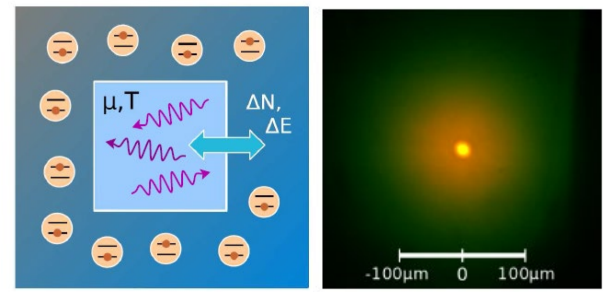


Photon BECs
(2D)

[Quasi-Equilibrium]



All Above Systems can be Described by a Macroscopic Wavefunction / Field obeying an (appropriate) Nonlinear Schroedinger (***Gross-Pitaevskii***) Equation

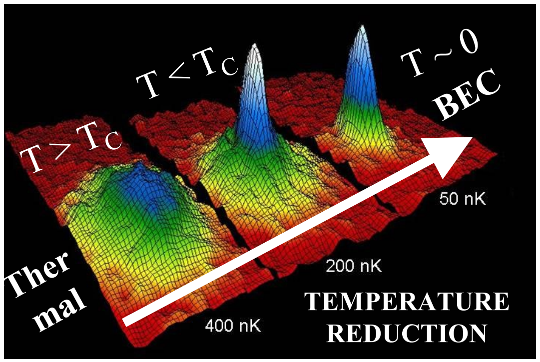
$$i\hbar\frac{\partial\Psi}{\partial t}=\left(-\frac{\hbar^2}{2m}\nabla^2+V_{TRAP}(r)+g|\Psi|^2+[Pumping\ -Loss]\right)\Psi+\left(\begin{array}{c}\text{Coupling to}\\ \text{Non-BEC Modes}\\ \text{including noise}\end{array}\right)$$

→ Systems exhibit diverse features of quantum fluids/liquids, nonlinear optics, etc …

CONDENSATES CHARACTERISED IN THIS TALK

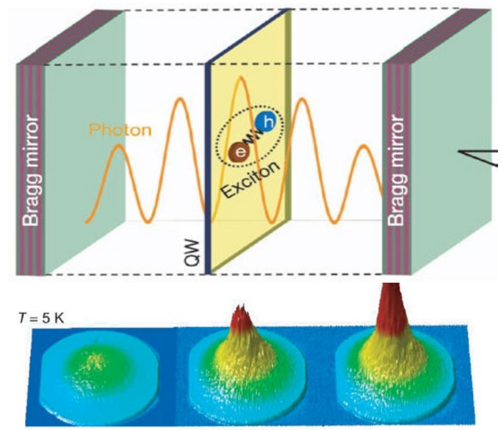
Ultracold Atomic BECs
(3D / 2D / 1D)

[“Equilibrium” State]



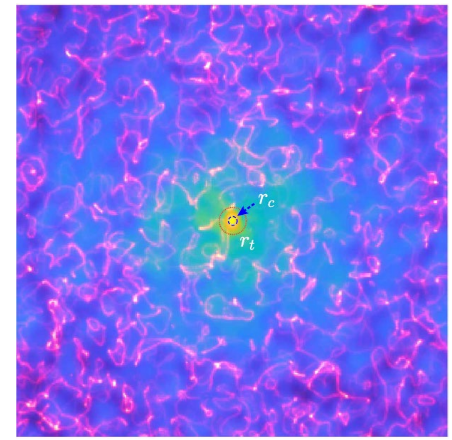
Exciton-Polariton BECs
(2D / 1D)

[(Quasi-)Equilibrium]



Fuzzy Dark Matter
(Galactic-Size Condensation)

[*Hypothesized !*]



Classical Field obeys appropriate (Nonlinear) Schroedinger Equation

$$i\hbar \frac{\partial \Psi}{\partial t} =$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{TRAP}(r) + g|\Psi|^2\right)\Psi$$

$$-iR\Psi$$

Coupling to Non-Condensate Bath

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(r) + g|\Psi|^2\right)\Psi$$

$$+i\frac{1}{2}\left(\frac{P}{1+|\Psi|^2/n_S} - \gamma\right)\Psi$$

Pumping & Dissipation Baths

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + mV_G(r,t) + g|\Psi|^2\right)\Psi$$

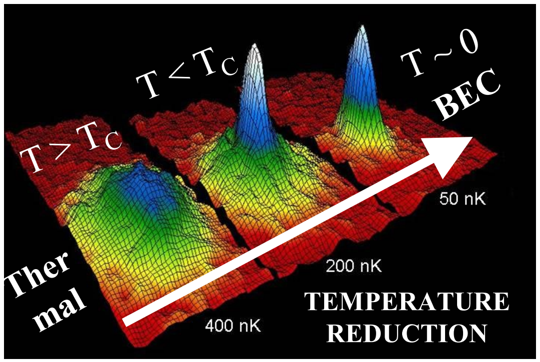
Coupling to Gravitational Field (Poisson Equation)

$$\nabla^2 V_G(r,t) = 4\pi G\left(|\Psi(\mathrm{r,t})|^2 - \langle|\Psi|^2\rangle\right)$$

CONDENSATES CHARACTERISED IN THIS TALK

Ultracold Atomic BECs
(3D / 2D / 1D)

["Equilibrium" State]



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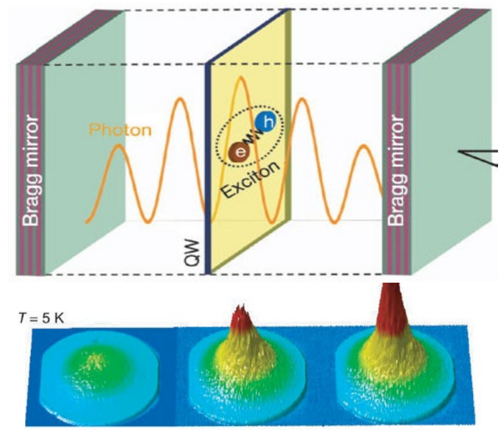

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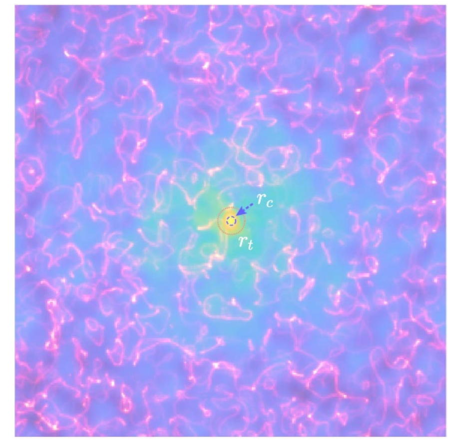

In Practice we often also add Stochastic Noise Terms
(related to bath couplings)

$$i\hbar\frac{\partial\Psi}{\partial t} =$$

(1-iγ)

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{TRAP}(r) + g|\Psi|^2\right)\Psi$$

$$\cancel{-iR\Psi}$$

$$+dW_\gamma$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(r) + g|\Psi|^2\right)\Psi$$

$$+i\frac{1}{2}\left(\frac{P}{1+|\Psi|^2/n_S} - \gamma\right)\Psi$$

$$+dW_{P,\gamma}$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + mV_G(r,t) + g|\Psi|^2\right)\Psi$$

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(Poisson Equation)

$$\nabla^2 V_G(r,t) = 4\pi G\left(|\Psi(\mathrm{r,t})|^2 - \langle|\Psi|^2\rangle\right)$$

CONDENSATES CHARACTERISED IN THIS TALK

Ultracold Atomic BECs

(3D / 2D / 1D)

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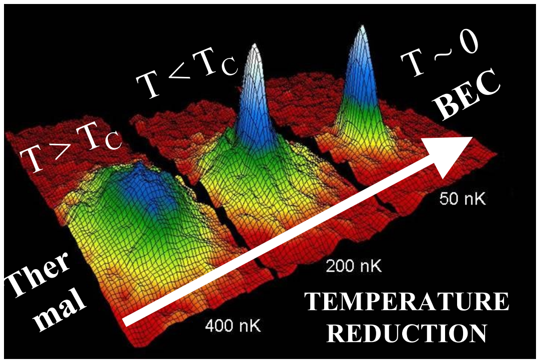

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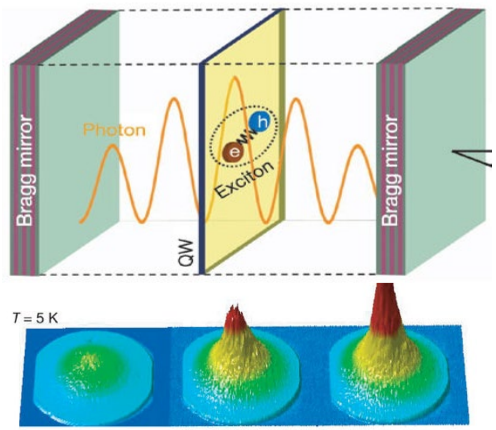

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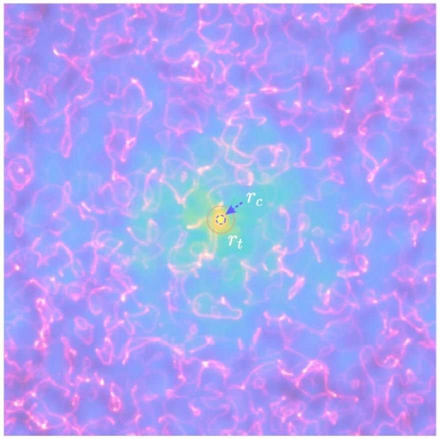

QUESTION #1:

In the Laboratory Condensates
(which can be controlled / monitored)

How Does Coherence Grow
from an Initially Incoherent State?

Comms.Phys. (Nature) 1, 24 (2018)
PRR 2, 033183 (2020)
PRR 3, 013097 (2021)
PRR 3, 013212 (2021)

PRL 121, 095302 (2018)
PRL 125, 095301 (2020)
EPL 133, 17002 (2021)

CONDENSATES CHARACTERISED IN THIS TALK

Ultracold Atomic BECs
(3D / 2D / 1D)
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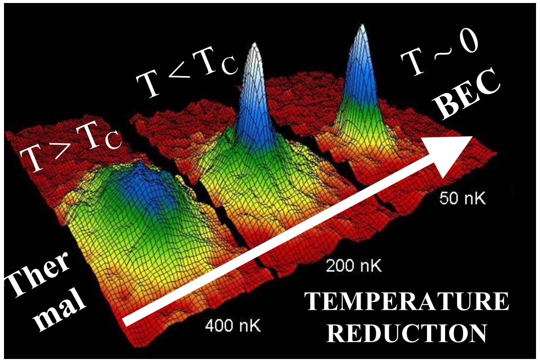

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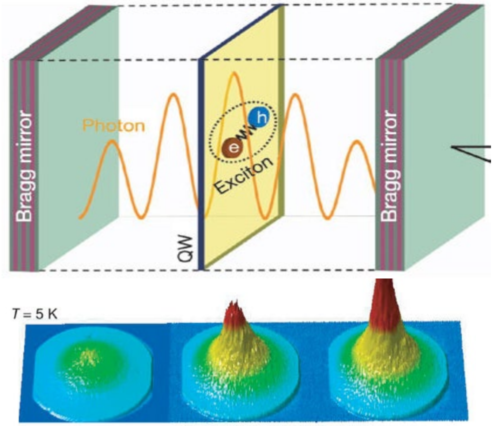

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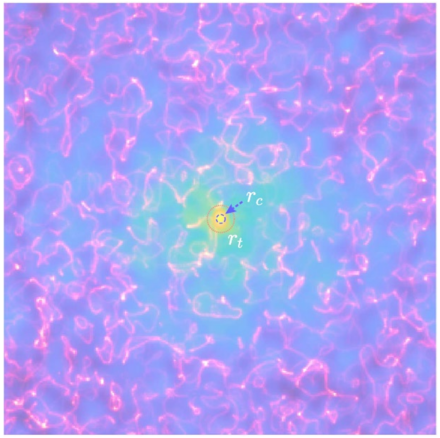

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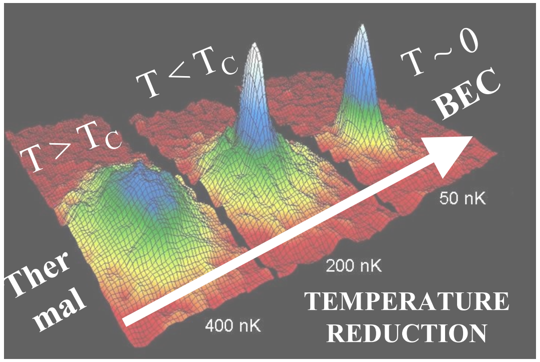

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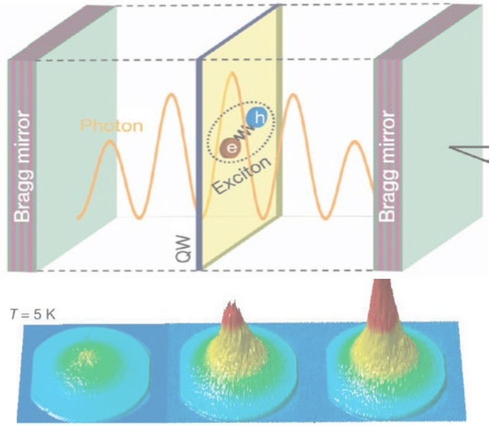

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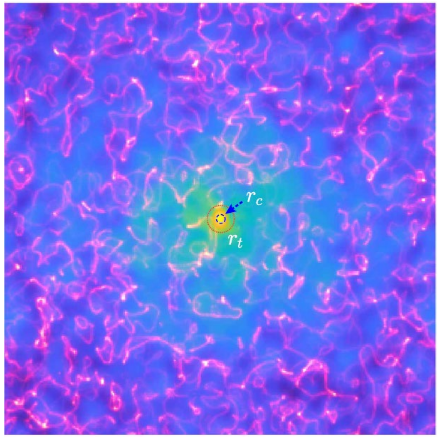

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QUESTION #2:

What does Condensation
have to do with
Dark Matter Distribution
in the Universe ?

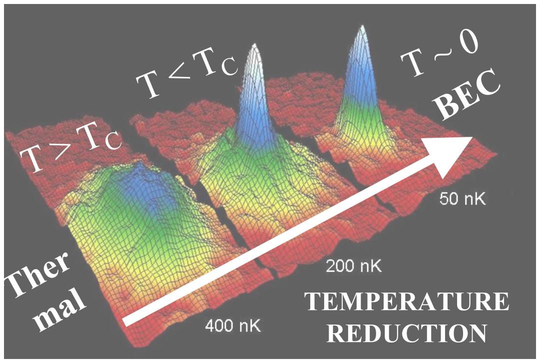
IK Liu, NP Proukakis, G Rigopoulos
arXiv preprint arXiv:2211.02565

CONDENSATES CHARACTERISED IN THIS TALK

Ultracold Atomic BECs

(3D / 2D / 1D)

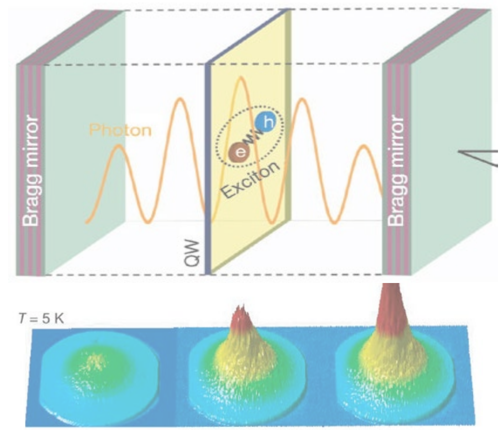
["Equilibrium" State]



Exciton-Polariton BECs

(2D / 1D)

[(Quasi-)Equilibrium]



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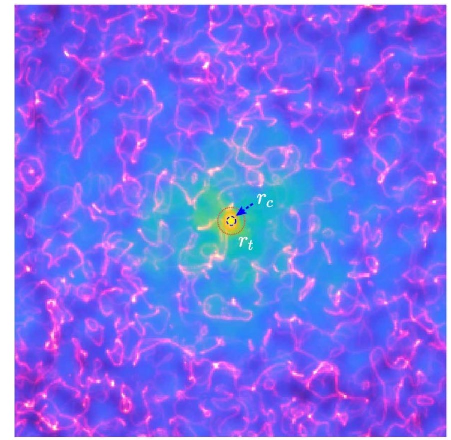
[*Hypothesized !*]



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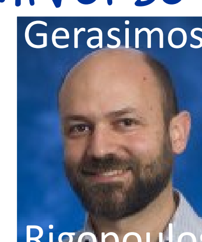
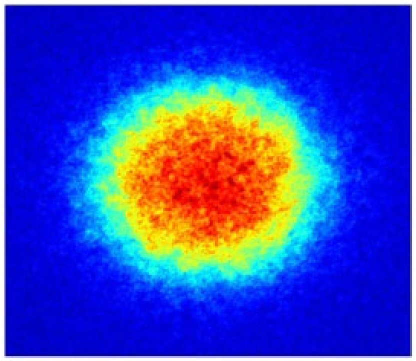


CHARACTERIZING A CONDENSATE STATE

(Dimensionless)
Phase-Space Density

$$\boxed{n\,\lambda^{d} \sim 1\ ?}$$



(A Necessary Condition)

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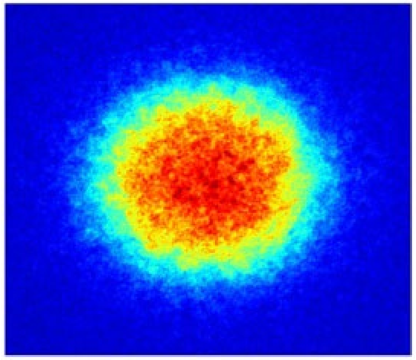


(A Necessary Condition)

Off-Diagonal Long-Range Order (ODLRO)

r ??? r'

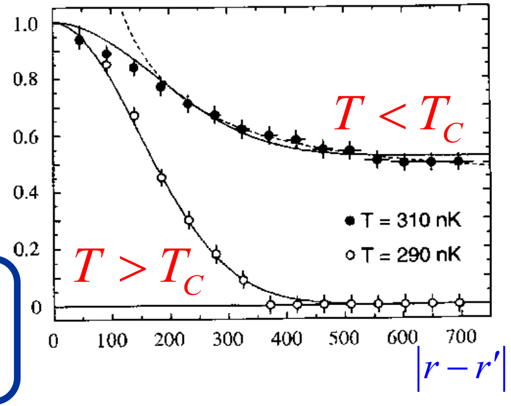
$$\rho(r,r') = \langle \Phi^*(\mathrm{r})\Phi(\mathrm{r}')\rangle$$

→ Constant

as $|r - r'| \to \infty$

Normalizing:

$$\boxed{g^{(1)}(r,r') = \frac{\rho(r,r')}{\sqrt{n(r)}\sqrt{n(r')}}}$$



(Definition: Relevant in 3D & `Thermodynamic Limit')

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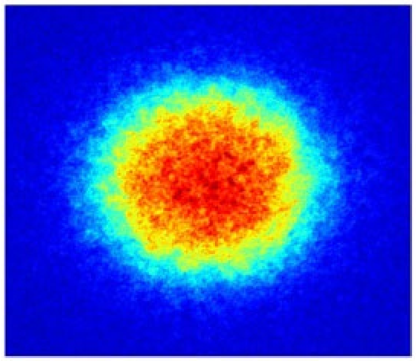

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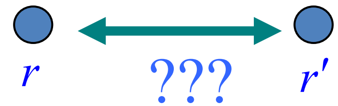

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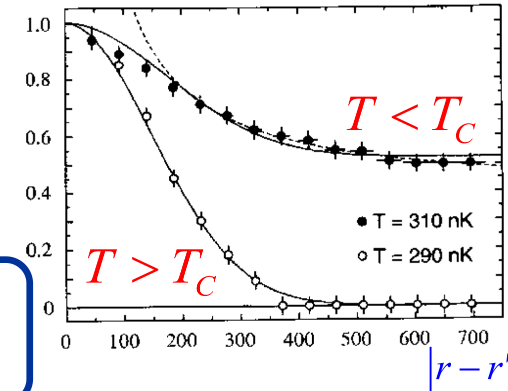

(Definition: Relevant in 3D & `Thermodynamic Limit')

Note:

In 2D, there is no ODLRO

… but… Correlation Function decays slower *(algebraically)*

$$g^{(1)}(0,r)|_{T<T_c} = \left\{ \begin{array}{ll} \text{constant} & (3\mathrm{D}) \\ r^{-\alpha(T)} & (2\mathrm{D}) \end{array} \right\}$$

than corresponding *(exponential)* decay on incoherent side

CHARACTERIZING A CONDENSATE STATE

(Dimensionless)
Phase-Space Density

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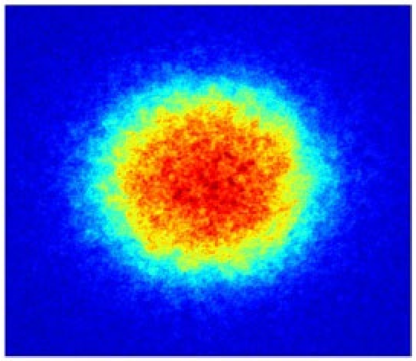

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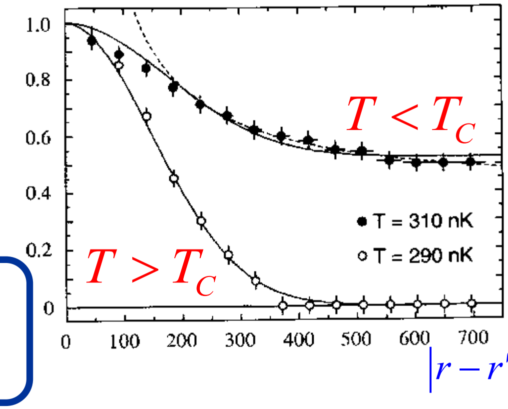

(Definition: Relevant in 3D & `Thermodynamic Limit')

Penrose-Onsager Condensate Mode

(Mode with Largest Eigenvalue)

$$\int dr'\rho(r,r')\,\psi_n(r') = N_n\psi_n(r)$$

If $N_0 \sim N_{Total}$ $\rightarrow$ System is Condensed $[\psi_0(r)]$

Typically consider (Single) Mode with $N_0 \gg N_i$
as (Penrose-Onsager) Condensate Mode

CHARACTERIZING A CONDENSATE STATE

(Dimensionless) Phase-Space Density

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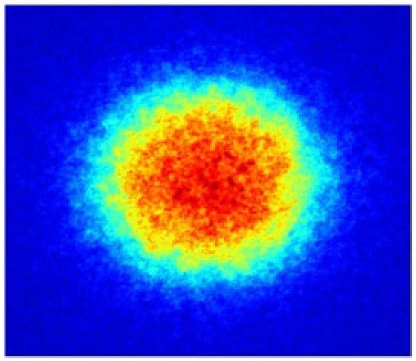

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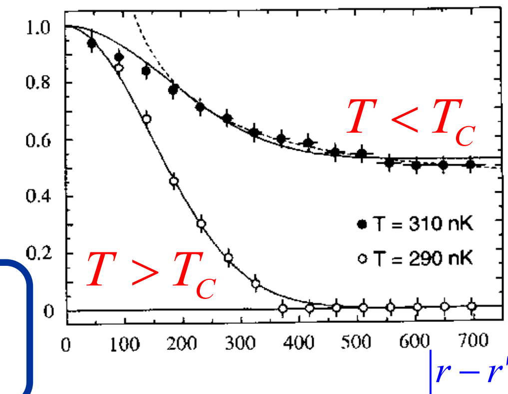


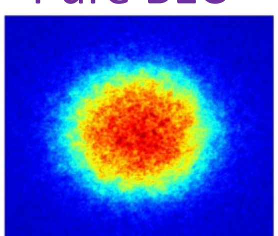
(Definition: Relevant in 3D & `Thermodynamic Limit')

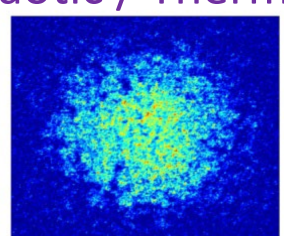
Density-Density Correlation Function

$$\boxed{g^{(2)}(r) = \frac{\langle|\Phi(\mathrm{r})|^4\rangle}{\langle|\Phi(\mathrm{r})|^2\rangle^2}}$$

= 1 = 2

Pure BEC Chaotic / Thermal





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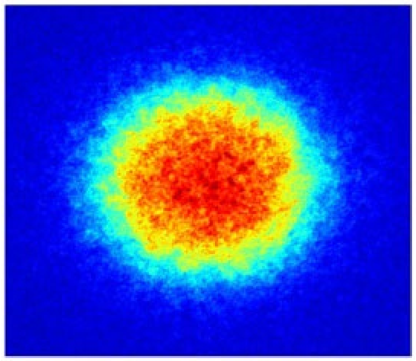

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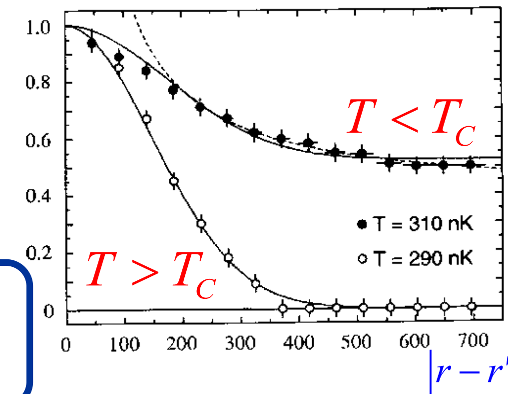

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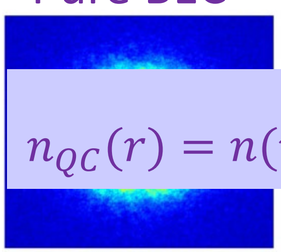
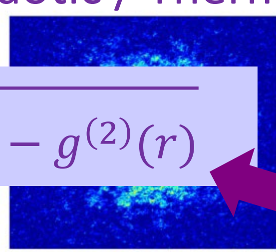
= 1 Pure BEC

= 2 Chaotic / Thermal





$$n_{QC}(r) = n(r)\sqrt{2 - g^{(2)}(r)}$$

Penrose-Onsager Condensate Mode

(Mode with Largest Eigenvalue)

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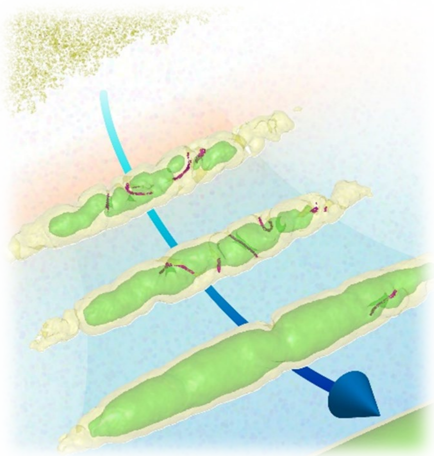
If $N_0 \sim N_{Total}$ → System is Condensed $[\psi_0(r)]$

Typically consider (Single) Mode with $N_0 \gg N_i$ as (Penrose-Onsager) Condensate Mode

If Many Competing Modes with Large / Similar N_i Then System said to have a `*Quasi-Condensate*'

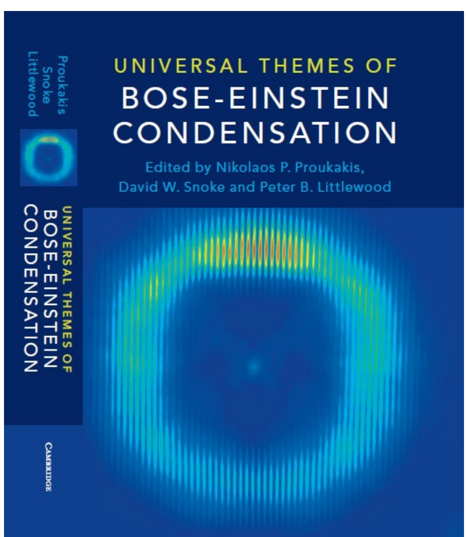
CONDENSATE FORMATION DYNAMICS



How Does Macroscopic Coherence Form
from an Incoherent Initial State?

An Old Problem
Studied Across Diverse Physical Systems

connecting
AMO, Condensed-Matter, Quantum Fluids
with Statistical Physics (& Early Cosmological) Studies

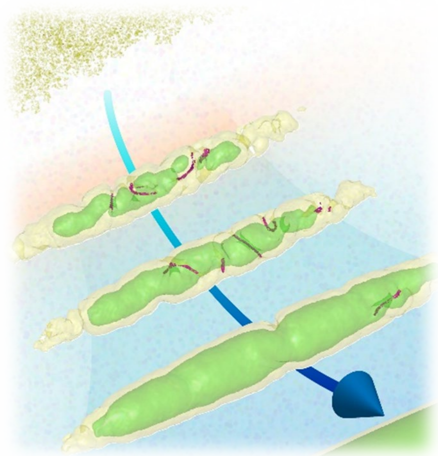


See e.g. review:
Davis, Wright, Gasenzer, Gardiner & **Proukakis**
"Formation of Bose-Einstein Condensate"
[arXiv:1601.06197]
in

Universal Themes of Bose-Einstein Condensation
(Cambridge University Press, 2017)

Edited by
NP Proukakis, DW Snoke & PB Littlewood

CONDENSATE FORMATION DYNAMICS



How Does Macroscopic Coherence Form from an Incoherent Initial State?

An Old Problem
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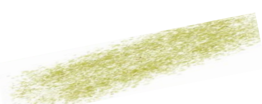
Phase Transition *Schematic*

Equilibrium Perspective

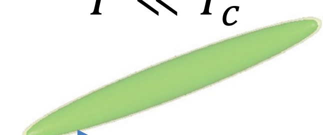
Thermal Equil.

$T \gg T_c$



"Coherent" Equil.

$T \ll T_c$



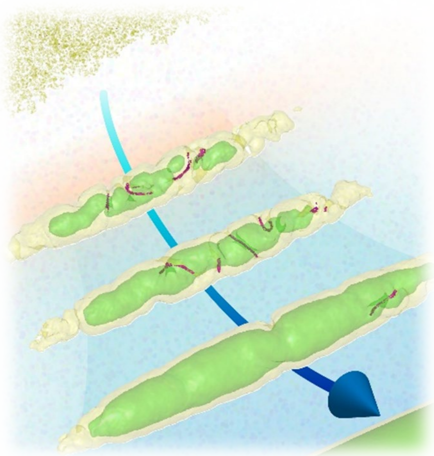
Critical Point

$T = T_c$

Control Parameter

$(T_c - T)$

CONDENSATE FORMATION DYNAMICS



How Does Macroscopic Coherence Form from an Incoherent Initial State?

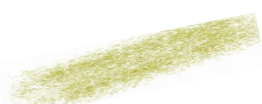
An Old Problem
Studied Across Diverse Physical Systems

Phase Transition *Schematic*

Equilibrium Perspective

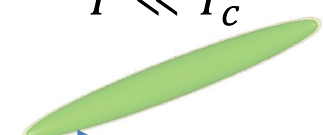
Thermal Equil.

$T \gg T_c$



"Coherent" Equil.

$T \ll T_c$



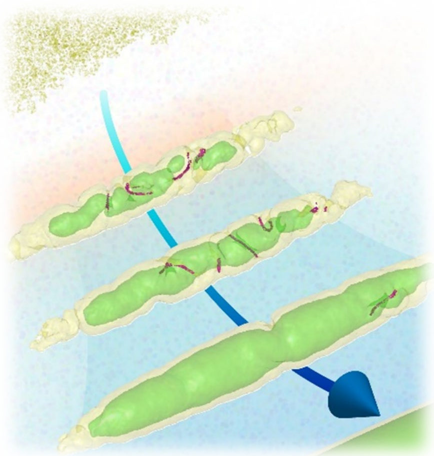
Critical Point (Region)

$T = T_c$

Control Parameter

$(T_c - T)$

CONDENSATE FORMATION DYNAMICS



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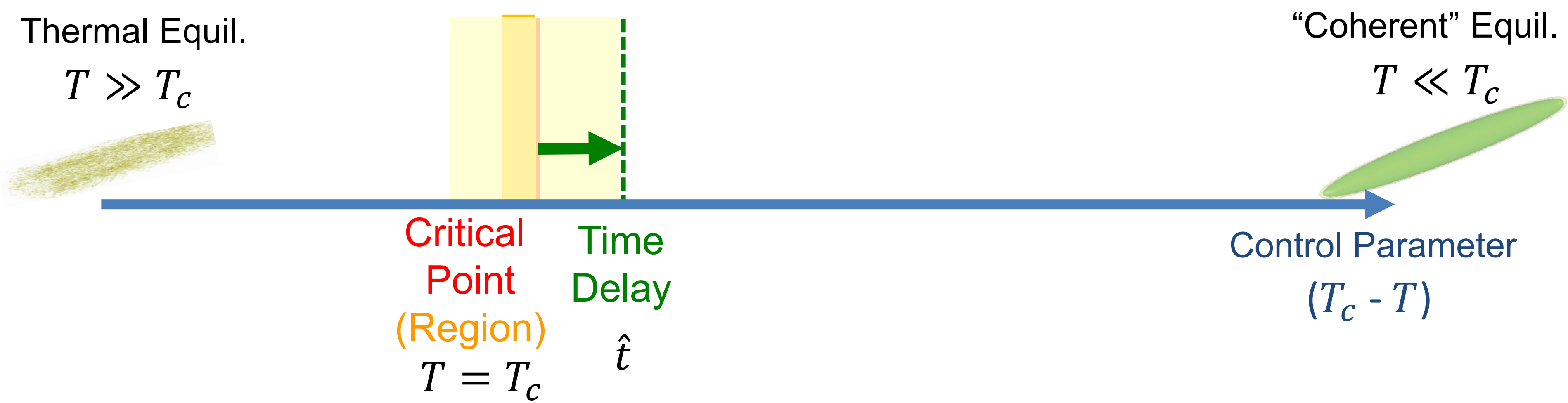
Phase Transition *Schematic*

Dynamical Perspective



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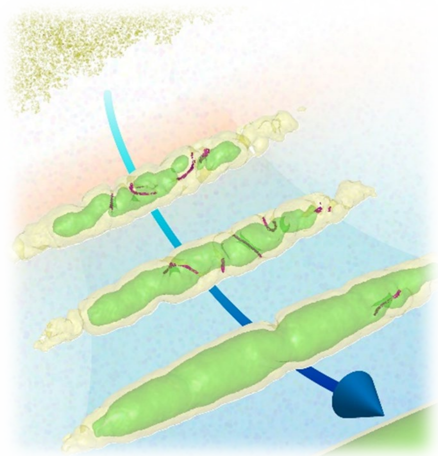

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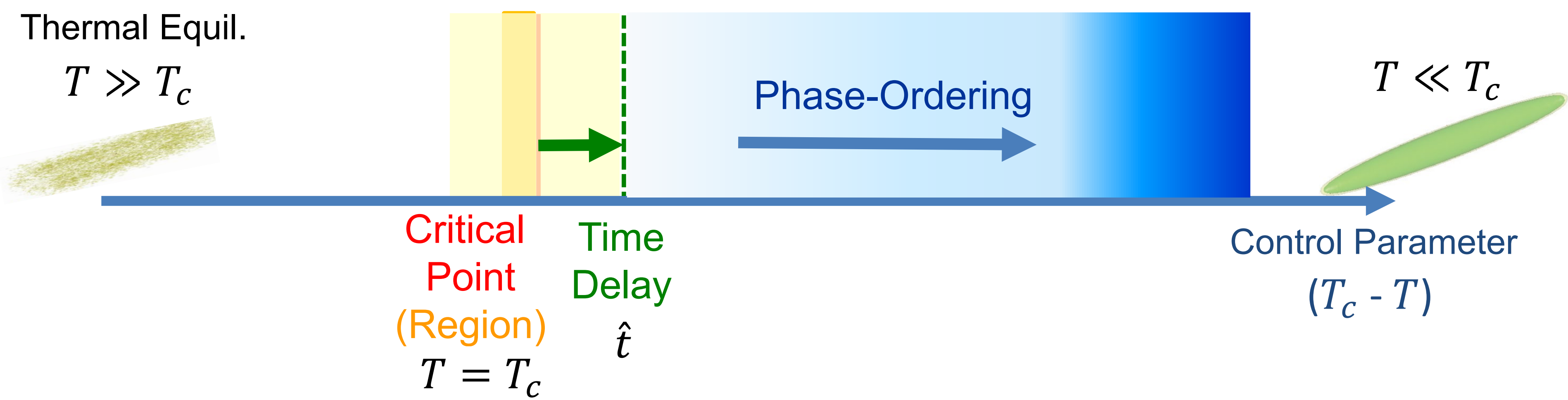
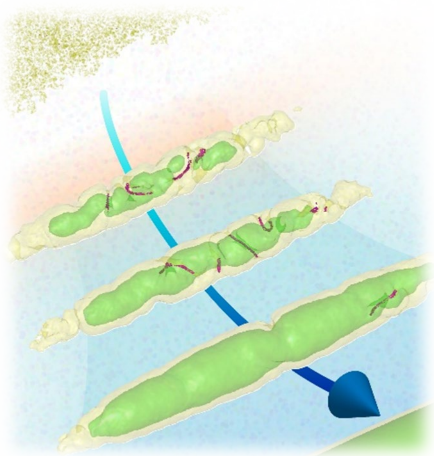


Characterized by Universal Physics / Critical Exponents ν, z

CONDENSATE FORMATION DYNAMICS



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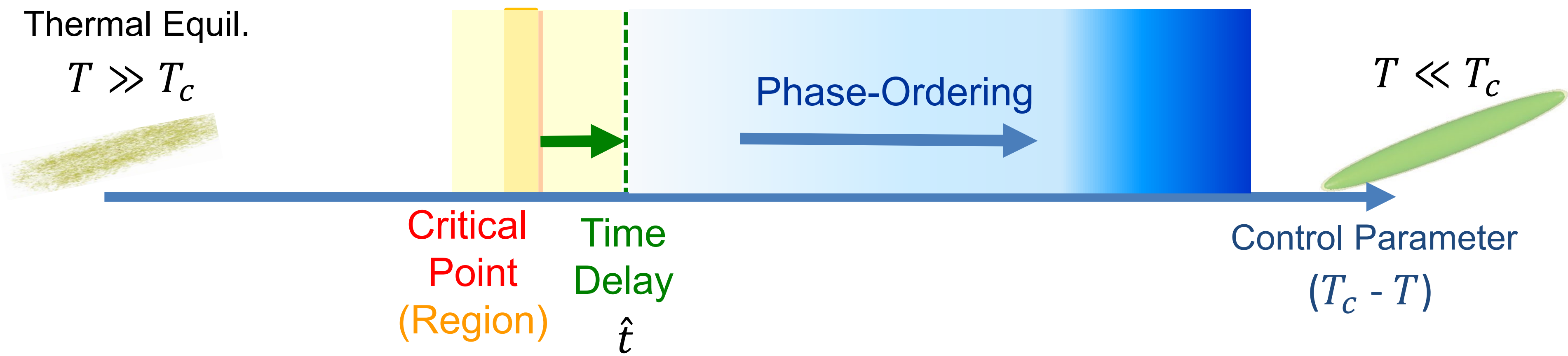


Examples Discussed Here:

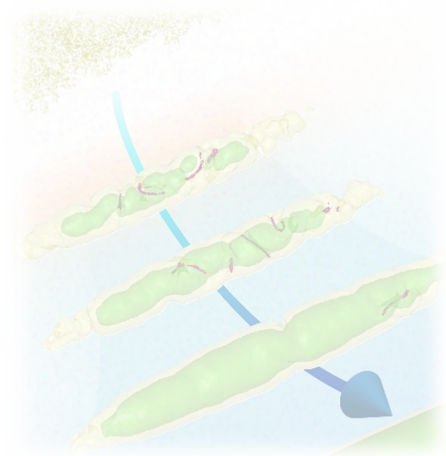
Kibble-Zurek Scaling (3D)

$$f(\mathbf{k},t) = \hat{t}F\left(\frac{t}{\hat{t}}, \hat{\xi}\mathbf{k}\right)$$

Phase-Ordering Scaling (2D)

$$\frac{g^{(1)}(r/L(t),t)}{g^{(1)}_{SS}(r/L(t),t)} \sim F\left(\frac{r}{L(t)}\right)$$

CONDENSATE FORMATION DYNAMICS



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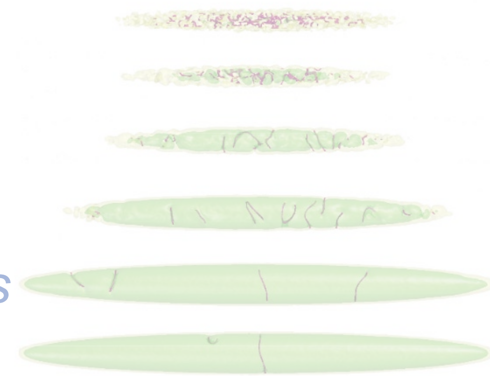

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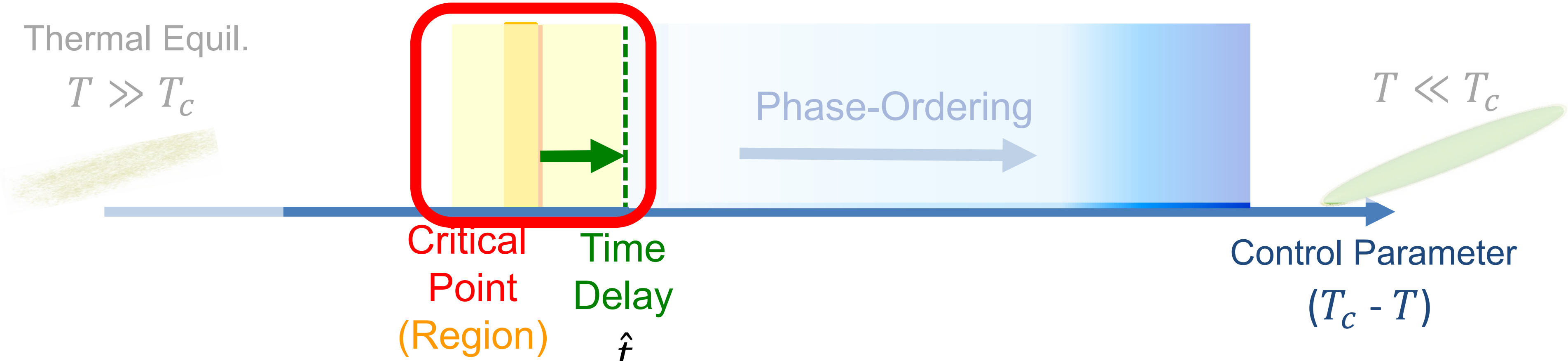

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CONDENSATE FORMATION DYNAMICS

How does System Choose Phase through a Symmetry-Breaking Mechanism ?

COSMOLOGY

Tom Kibble

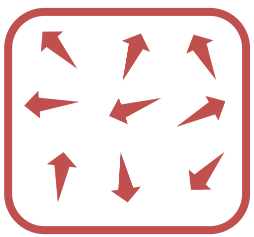
Topology of cosmic domains and strings

T W B Kibble

Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, UK

Journal of Physics A: Mathematical and General 9, 1387 (1976)

"…initial formation of "protodomains" [e.g. strings] as the Universe cools …"
(assuming a 'Hot Big Bang' Model)



Thus we can anticipate the formation of an initial domain structure with the expectation value of ϕ, the order parameter, varying from region to region in a more or less random way. Of course for energetic reasons a constant or slowly varying $\langle\phi\rangle$ is preferred and so much of this initially chaotic variation will quickly die away. The interesting question is whether any residue remains—in particular whether normal regions can be 'trapped' like flux tubes in a superconductor.

CONDENSED MATTER

Wojciech Zurek

Cosmological experiments in superfluid helium?

W. H. Zurek

Theoretical Astrophysics, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

Nature 317, 505 (1985)

discuss the analogy between cosmological strings and vortex lines in the superfluid, and suggest a cryogenic experiment which tests key elements of the cosmological scenario for string formation.

How does System Choose Phase through a Symmetry-Breaking Mechanism ?

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General 9, 1387 (1976)

→ Kibble-Zurek Mechanism

Features Observed in Many Physical Systems:

Superfluid He3, He4, Superconducting Josephson Junctions, Liquid Crystals, Ions, Cold Atoms …

Review: del Campo & Zurek, Int J Mod Phys A 29, 1430018 (2014)

)

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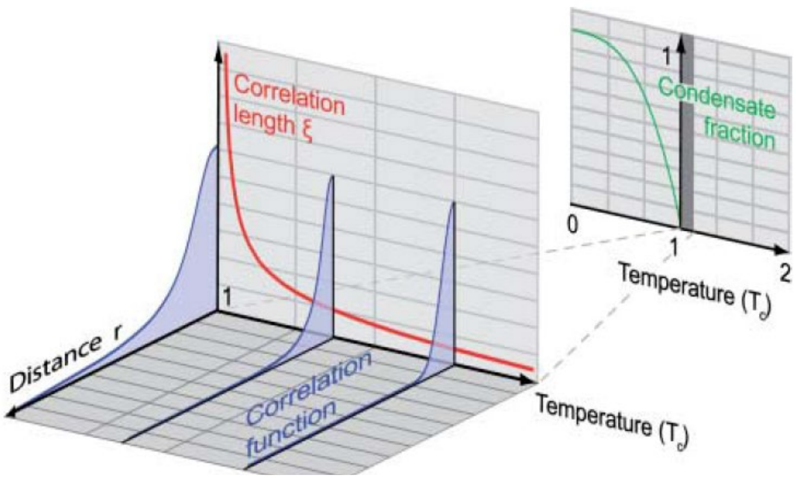
PHASE TRANSITION PHYSICS

In Phase Transition Region, Characteristic Quantities Diverge in Specific Ways

Correlation Length



$$\langle \Psi^\dagger(r)\Psi(0)\rangle \propto \frac{1}{r}\exp(-r/\xi)$$

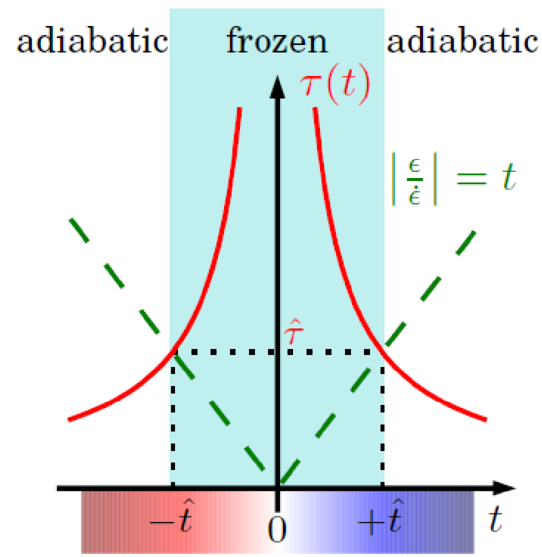
Correlation Length

$$\xi = \xi_0 \left| \frac{T - T_C}{T_C} \right|^{-\nu}$$

ν : *Static* Critical Exponent

ξ_0 : Depends on microphysics

Relaxation Time



Relaxation Time

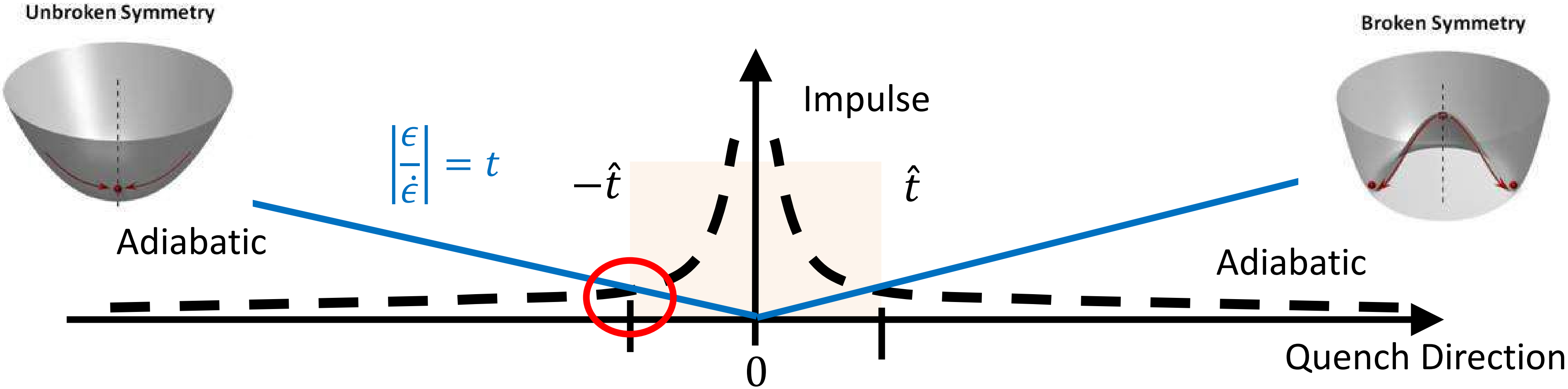
$$\tau = \tau_0 \left| \frac{T - T_C}{T_C} \right|^{-\nu z}$$

z : *Dynamical* Critical Exponent

τ_0 : Depends on microphysics

KIBBLE-ZUREK MECHANISM

Consider a Driven Phase Transition from the Incoherent Region



$\epsilon(t) = t/\tau_Q$

As System enters Critical Region (from incoherent side)
it undergoes "critical slowing down"
(due to diverging relaxation time)

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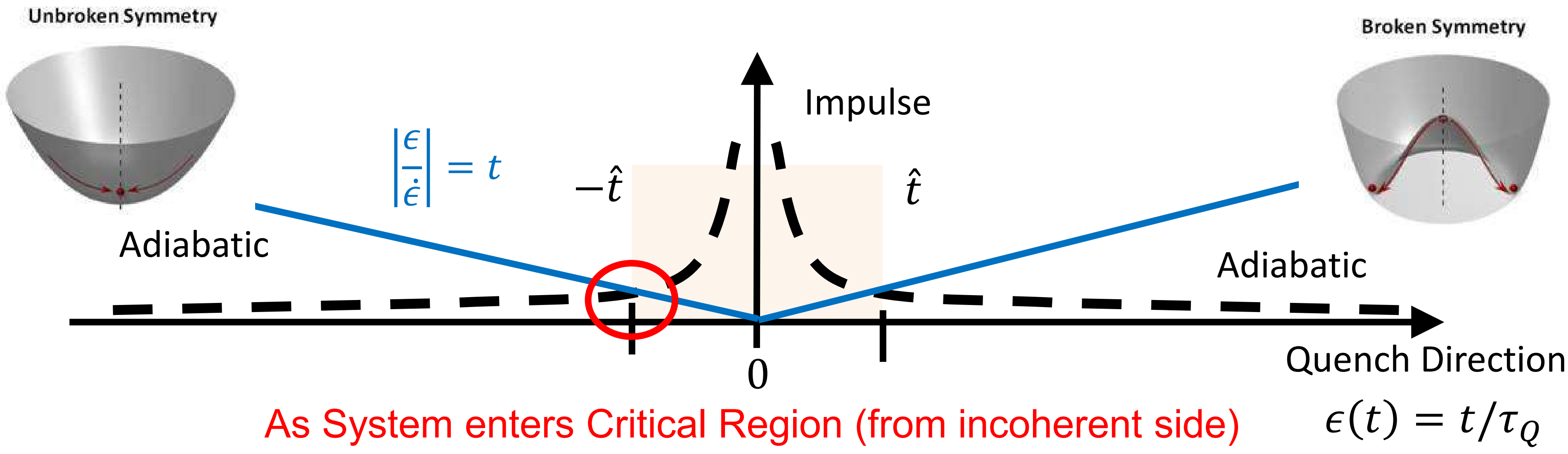

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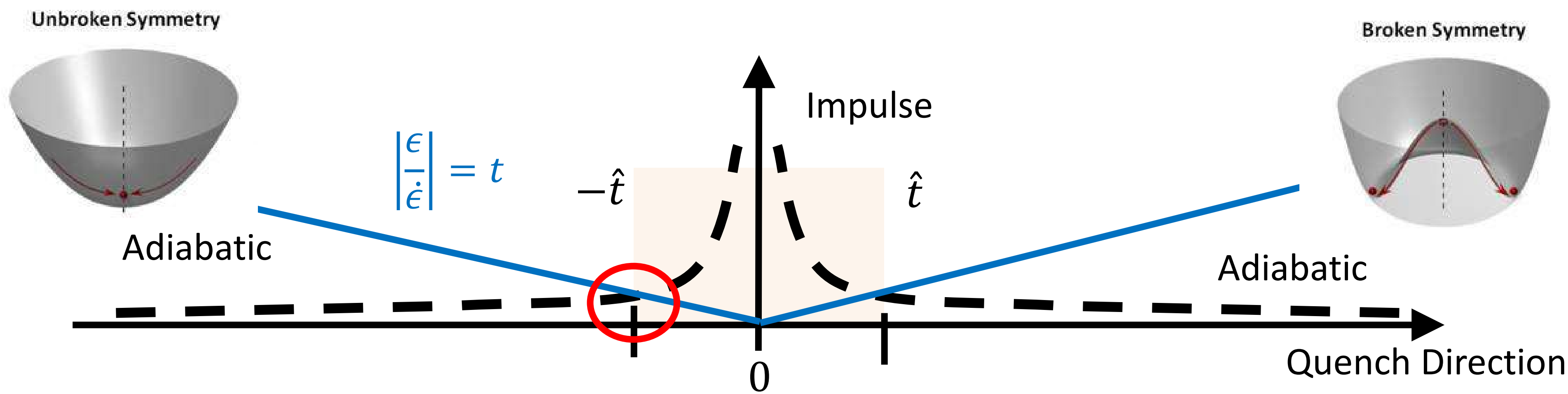

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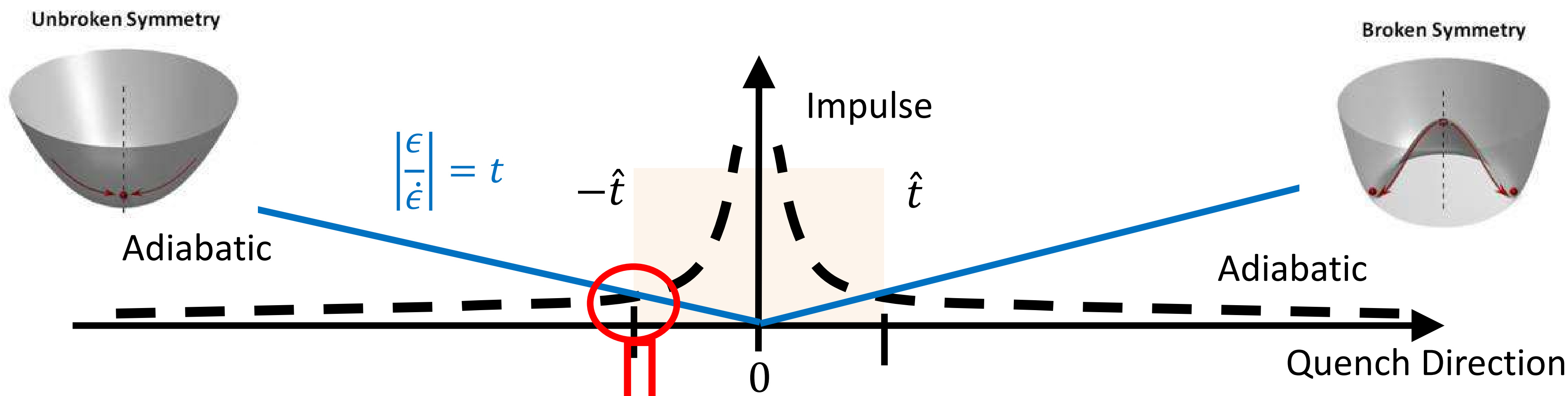

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$$\boxed{\tau(-\hat{t}) = |-\hat{t}\,|}$$

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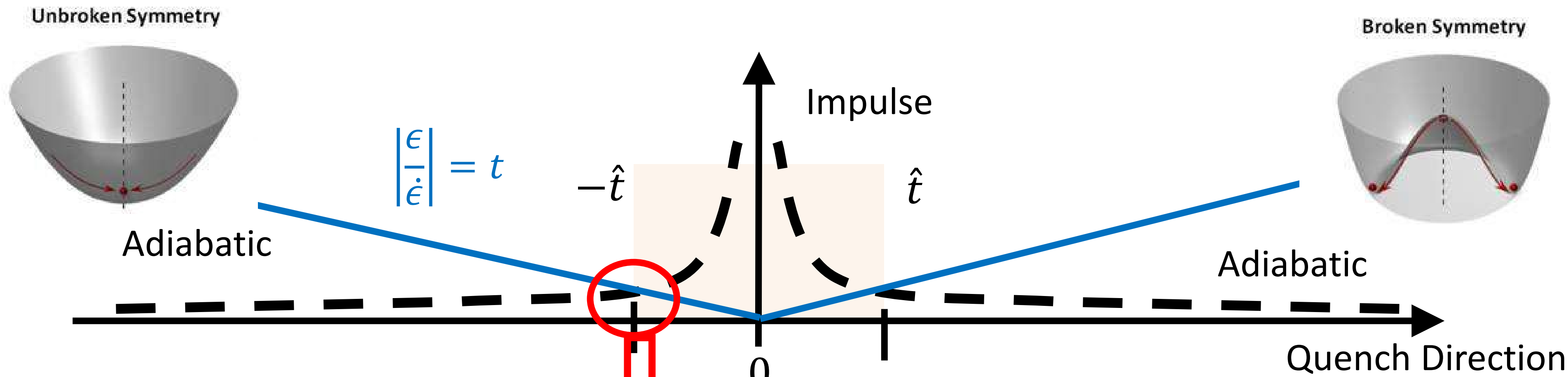

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$\tau_0|\varepsilon(\hat{t})|^{-\nu z}$

$\left|\frac{\varepsilon(-\hat{t})}{\dot{\varepsilon}(-\hat{t})}\right|$

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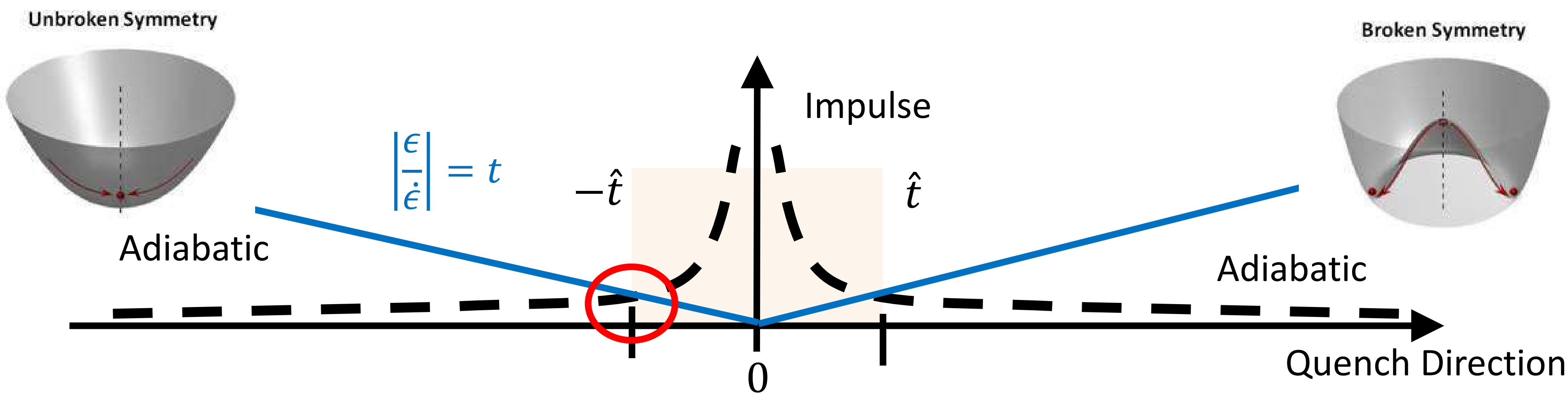

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Obtain Characteristic Scaling Laws in terms of Quench Time (τ_Q) and Critical Exponents

'Freeze-out' Time
$\hat{t} \sim \tau_Q^{z\nu/(1+z\nu)}$

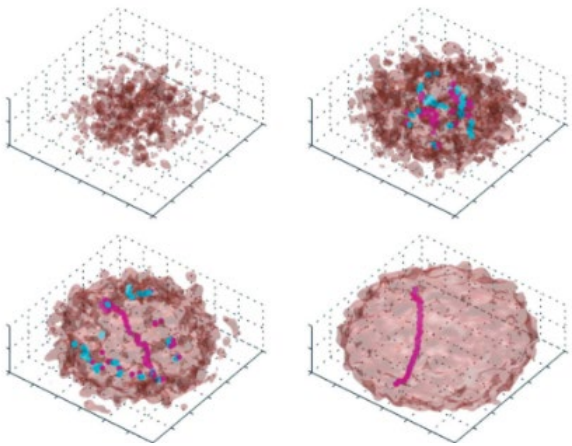
'Freeze-out' Length
$\hat{\xi} \sim (\tau_Q/\tau_0)^{\nu/(1+z\nu)}$

$\Longleftrightarrow$

Number of Emerging Defects
$N \sim (\tau_Q)^{-\alpha}$

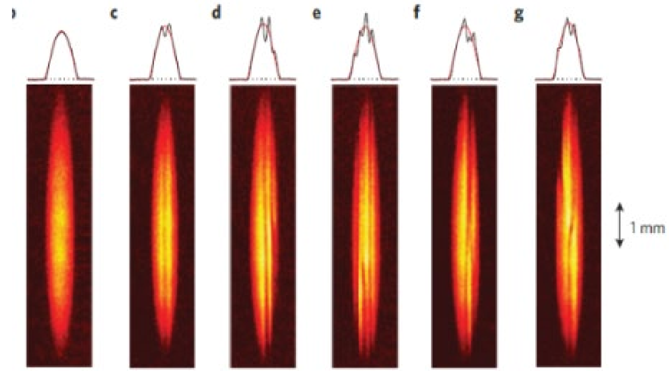
CONTROLLED QUENCH EXPERIMENTS (COLD ATOMS)

3D HARMONIC



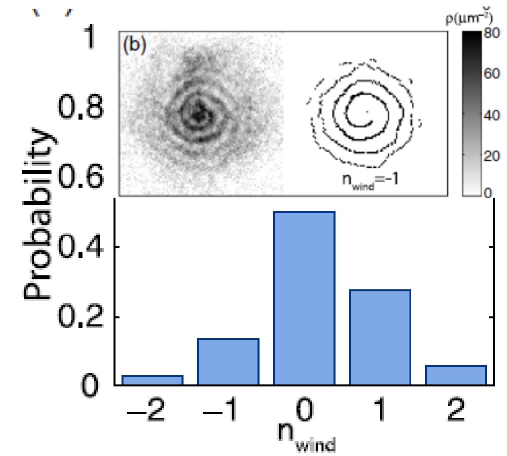
Nature 455, 948 (2008)

ELONGATED 3D



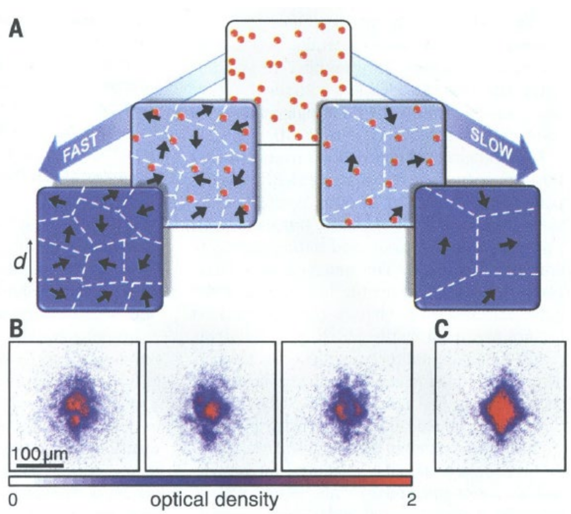
Nat Phys 9, 656 (2013)

RING TRAP



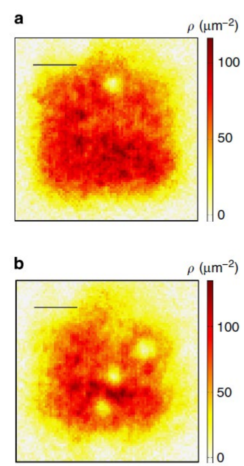
PRL 113, 135302 (2014)

3D BOX TRAP



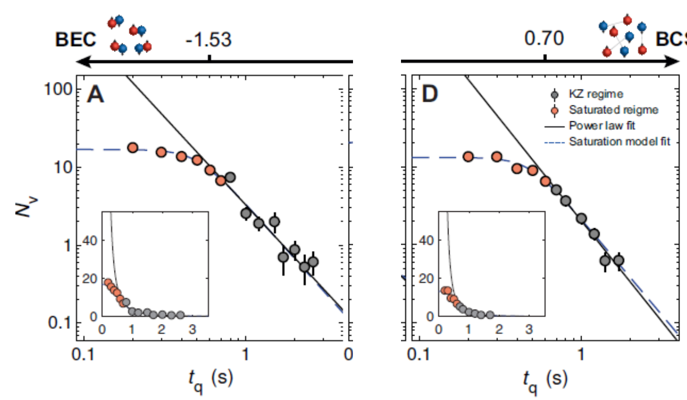
Science 347, 167 (2015)

2D BOX TRAP



Nat. Comms 6, 6162 (2015)

FERMIONIC SUPERFLUID



Nat. Phys. 15, 1227 (2019)

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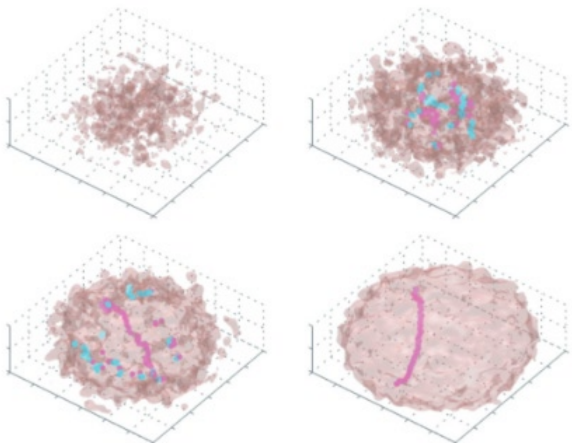

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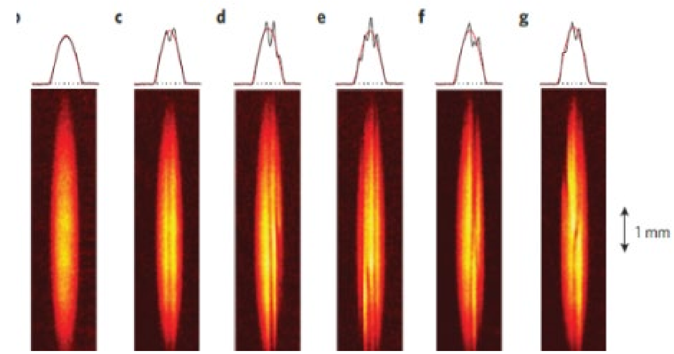


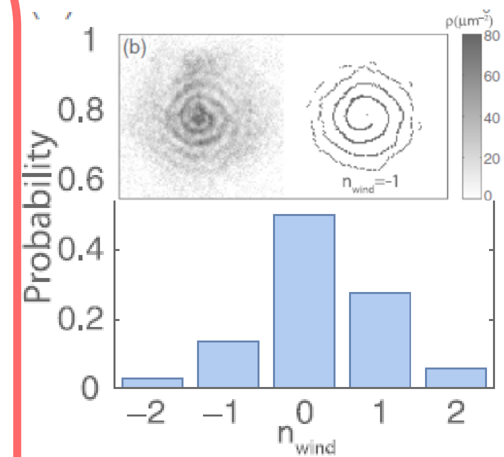
Model Trento Experiments:

Nat Phys 9, 656 (2013)
PRA 94,023628 (2016)
arXiv:2201.08569 (2022)

to shed more light onto early-time / microscopic & universal properties not easily accessible experimentally

Liu *et al,*
Comms.Phys. (Nature) 1, 24 (2018)
PRR 2, 033183 (2020)

RING TRAP



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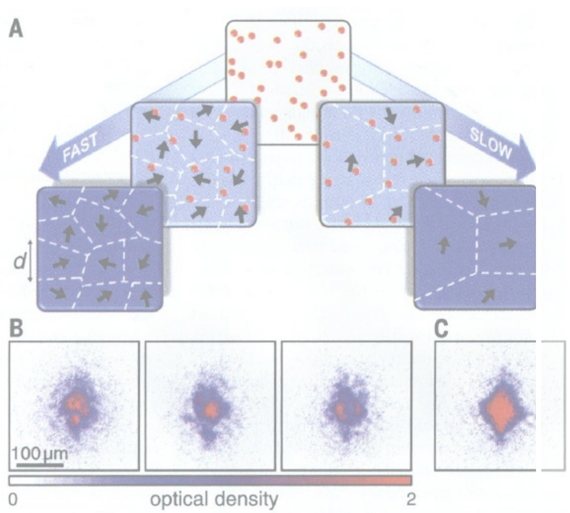

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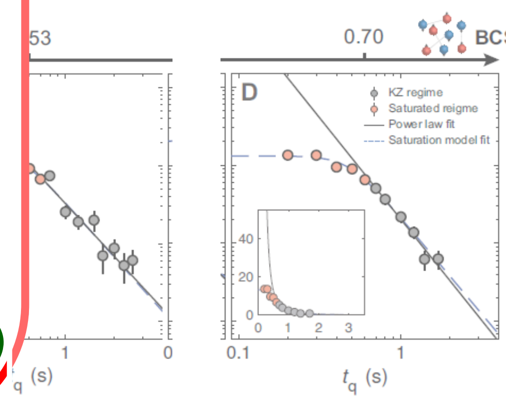
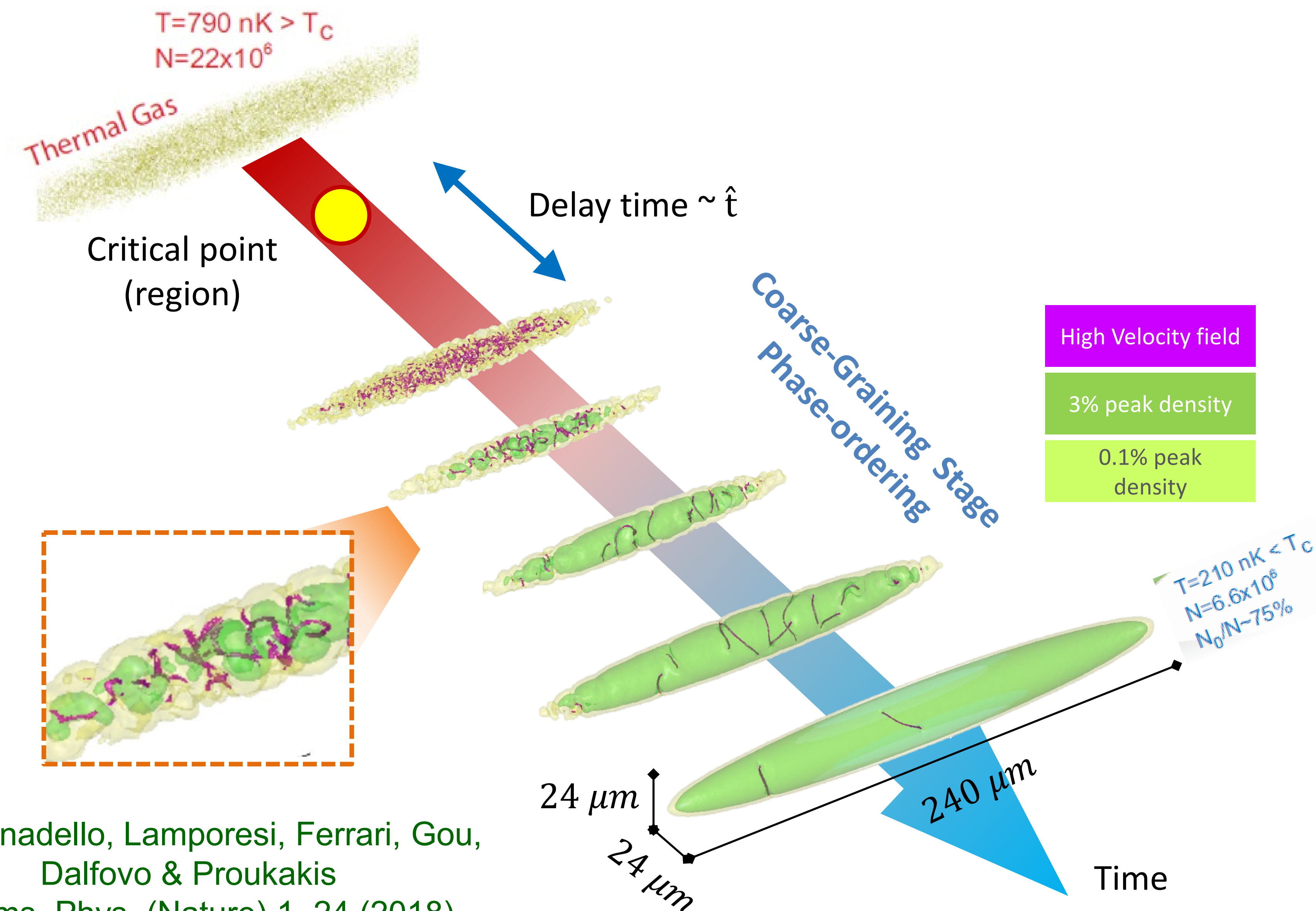


Nat. Comms 6, 6162 (2015)

Nat. Phys. 15, 1227 (2019)

DYNAMICAL VISUALIZATION

Emerging Features of Quenched Growth



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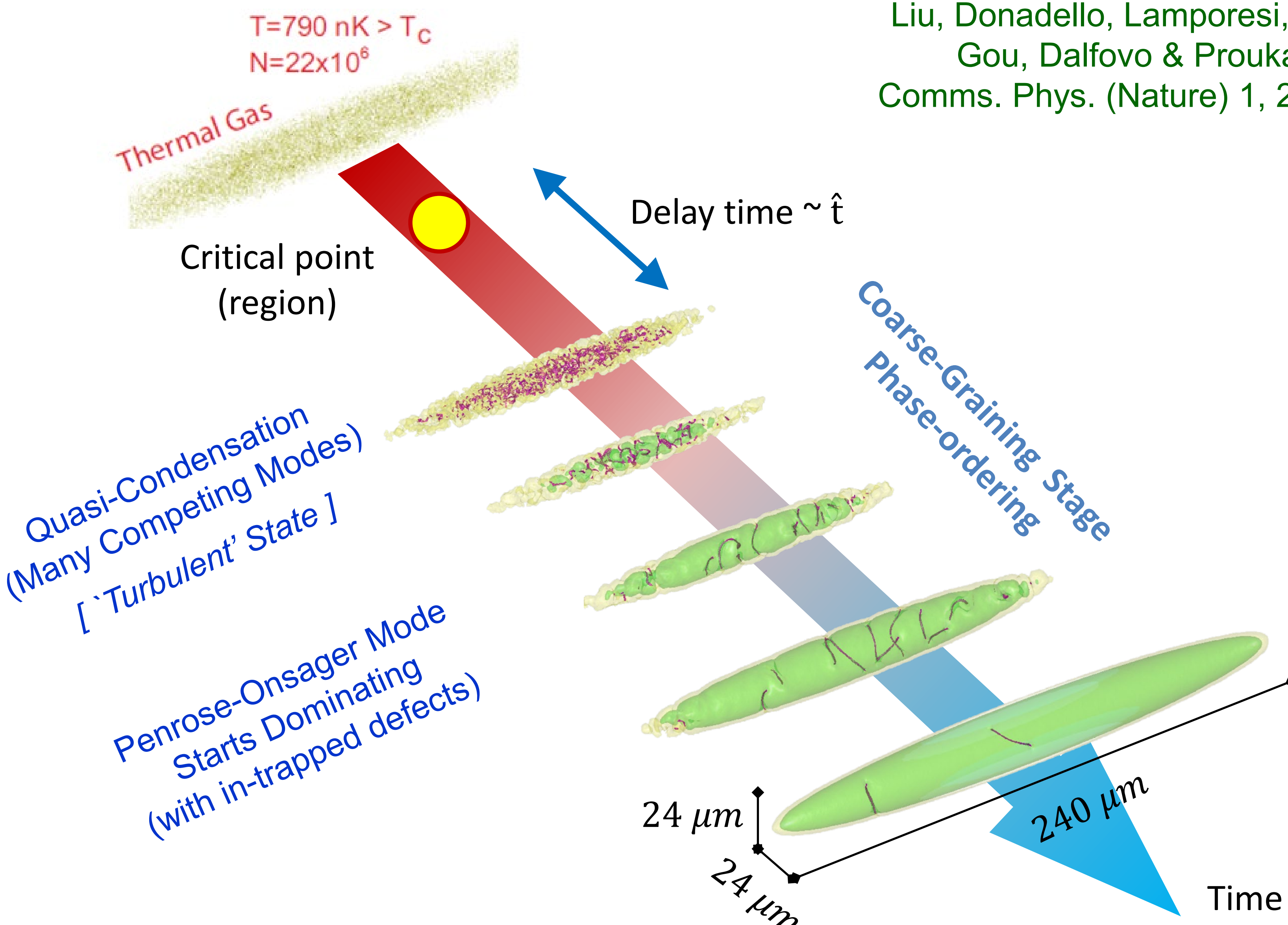

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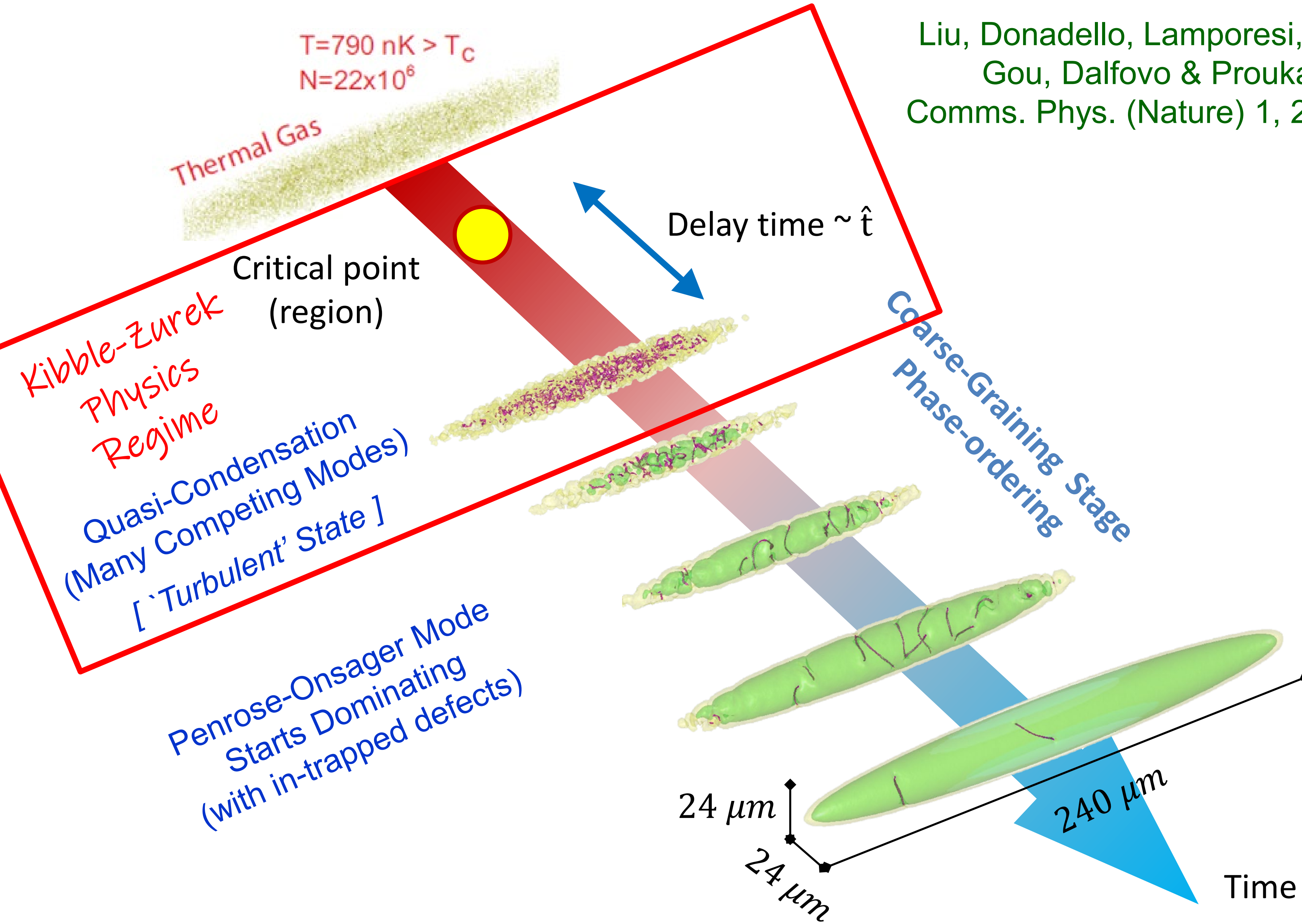
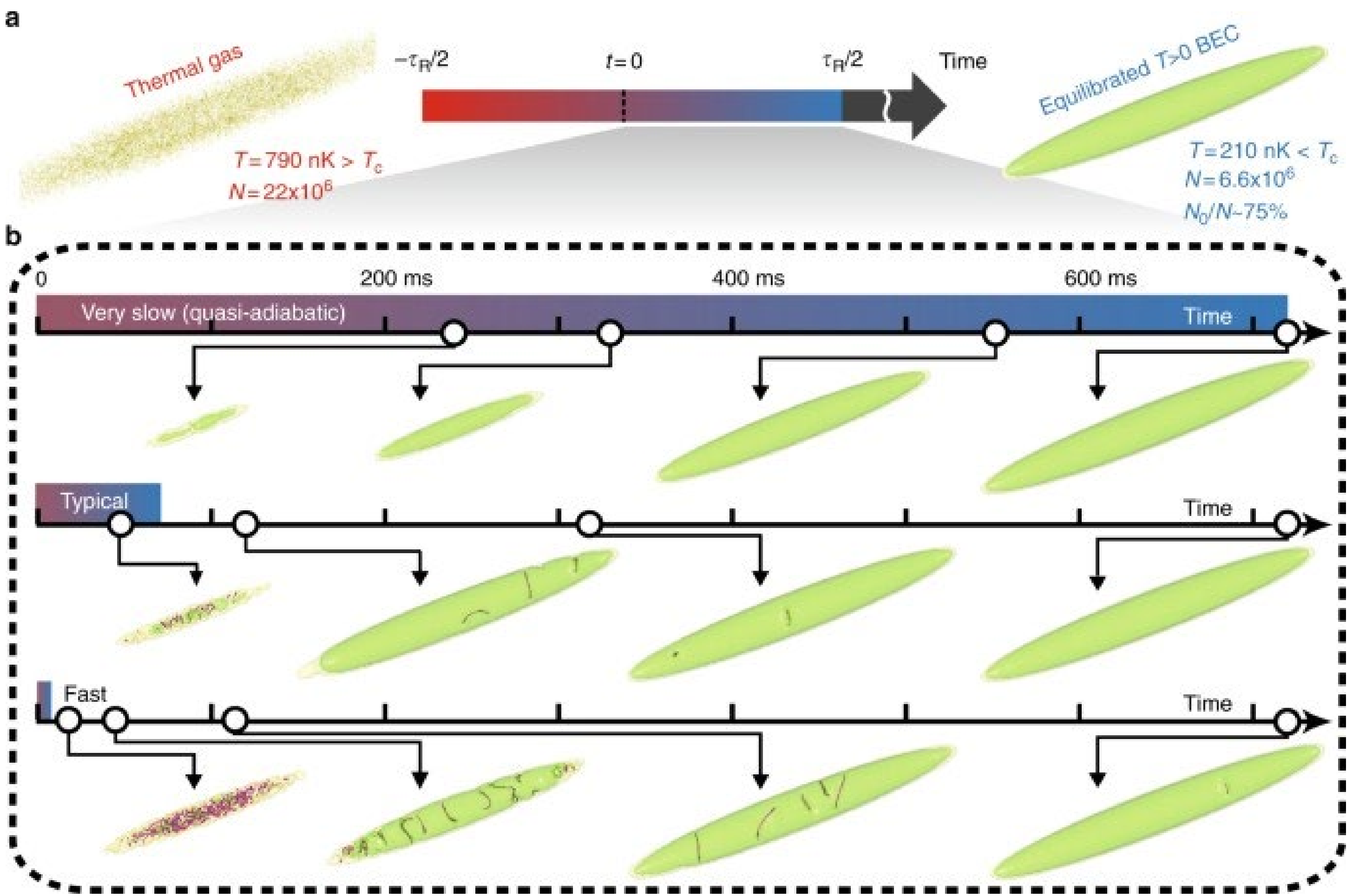


EFFECT OF VARIABLE QUENCH RATE



Liu, Donadello, Lamporesi, Ferrari, Gou, Dalfovo & Proukakis, Comms. Phys. (Nature) 1, 24 (2018)

KIBBLE-ZUREK ANALYSIS: Early Time Dynamics

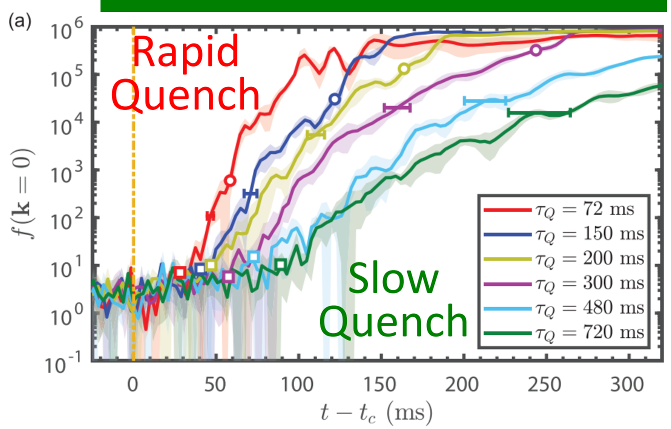
Study Dynamics under Different Quench Ramp Durations

(same Initial & Final conditions)

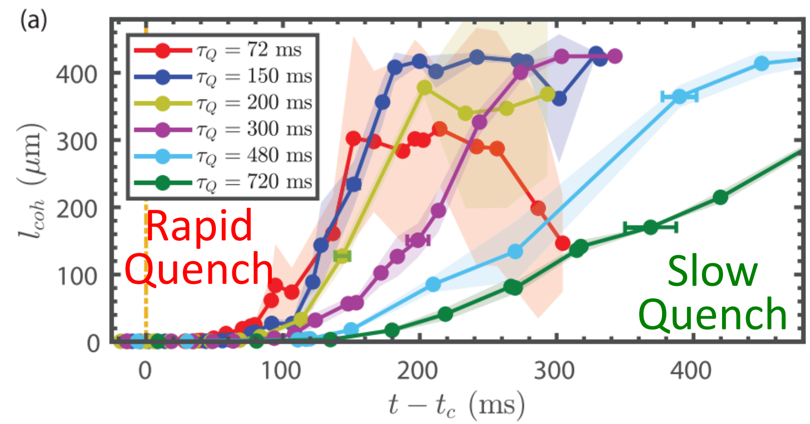
Spectral Function $f(\mathbf{k},t)$ ⟵ Fourier Transform Pair ⟶ $\langle \Psi_c^*(\mathbf{r},t)\Psi_c(\mathbf{r}',t)\rangle$ **Correlation Function**

RAW (Unscaled)

Zero-Momentum Mode Occupation



Coherence Length (Condensate Mode)



Rapid Quenches → Faster Growth of Measure of Coherence

Can we scale out dependence on cooling rate ?

Kibble-Zurek Hypothesis:
All Physical Variables in Critical Region
are universal when scaled as

Time	$t / \hat{t}$
Distance	$r / \hat{\xi}$
Wavevector	$k\, \hat{\xi}$

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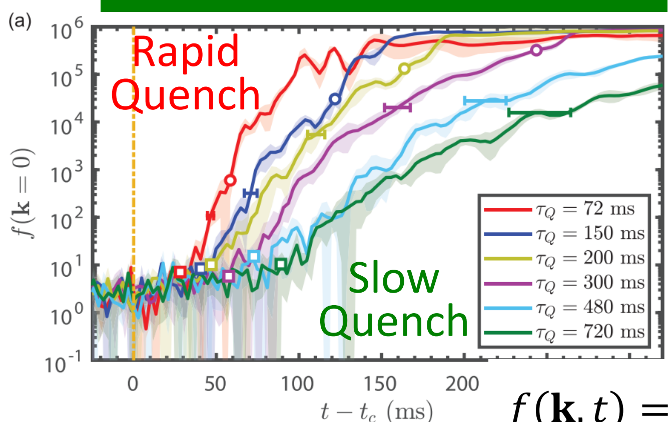
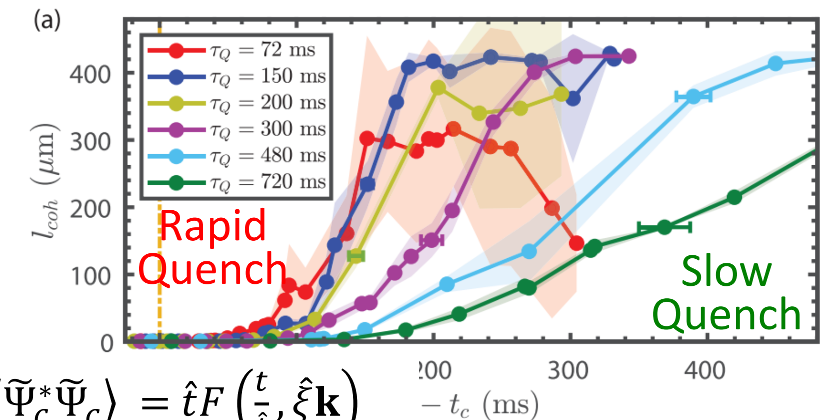


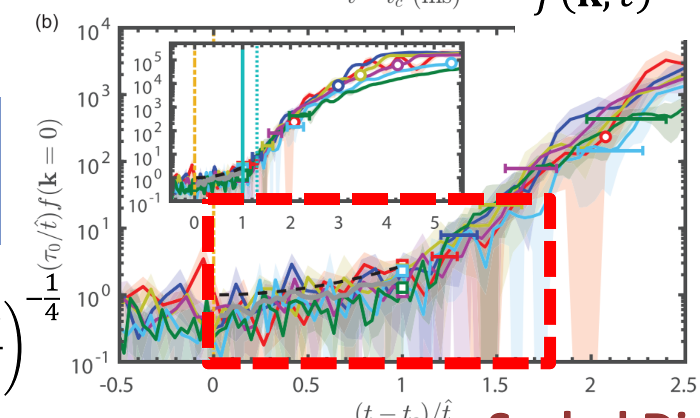


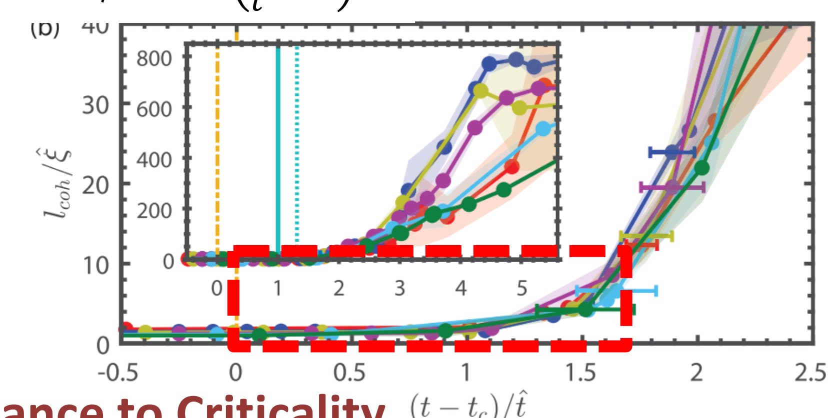
$$f(\mathbf{k},t) = \langle \widetilde{\Psi}_c^* \widetilde{\Psi}_c \rangle = \hat{t} F\left(\frac{t}{\hat{t}}, \hat{\xi}\mathbf{k}\right)$$

$$\hat{t} = \sqrt{\frac{\hbar}{\gamma \mu_f}\tau_Q}$$

SCALED (Kibble-Zurek)

$$\hat{\xi} = \hat{k}^{-1} = \tau_Q^{\frac{1}{4}}\left(\frac{4M^2\mu_f}{\gamma\hbar^3}\right)^{-\frac{1}{4}}$$





Scaled Distance to Criticality

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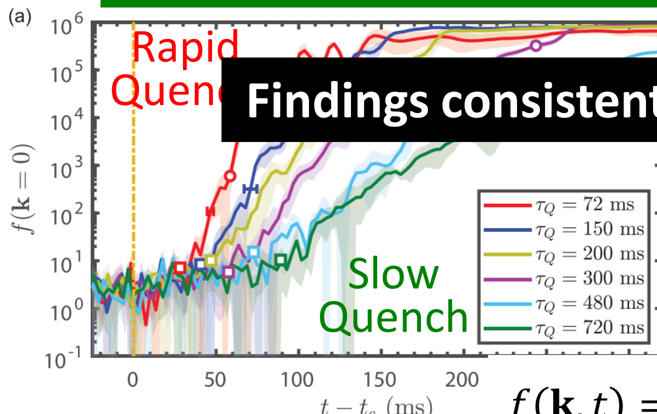
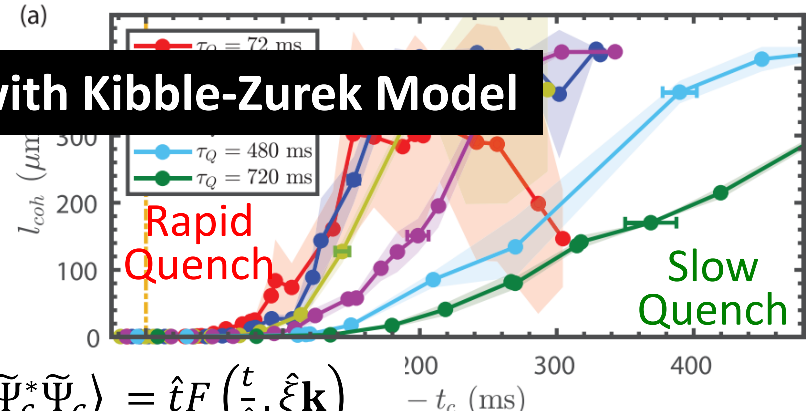




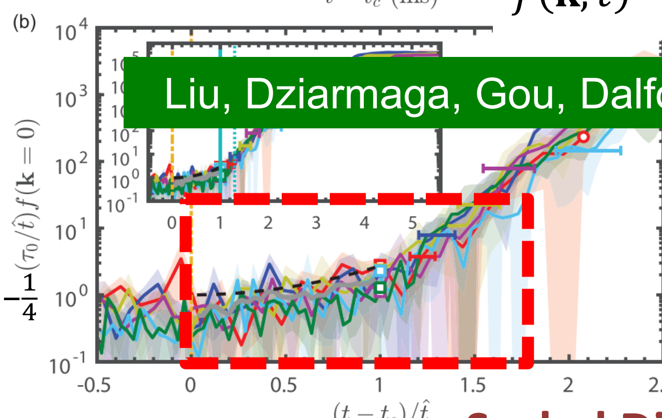
Findings consistent with Kibble-Zurek Model

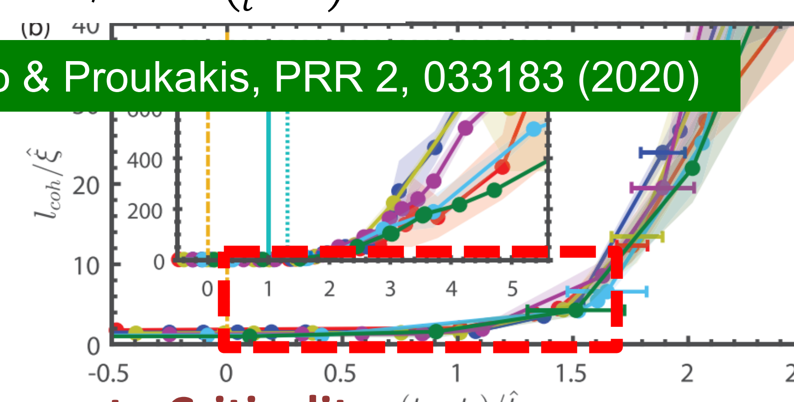
$$f(\mathbf{k},t) = \langle \widetilde{\Psi}_c^* \widetilde{\Psi}_c \rangle = \hat{t} F\left(\frac{t}{\hat{t}}, \hat{\xi}\mathbf{k}\right)$$

$$\hat{t} = \sqrt{\frac{\hbar}{\gamma\mu_f}\tau_Q}$$

SCALED (Kibble-Zurek)

$$\hat{\xi} = \hat{k}^{-1} = \tau_Q^{\frac{1}{4}}\left(\frac{4M^2\mu_f}{\gamma\hbar^3}\right)^{-\frac{1}{4}}$$

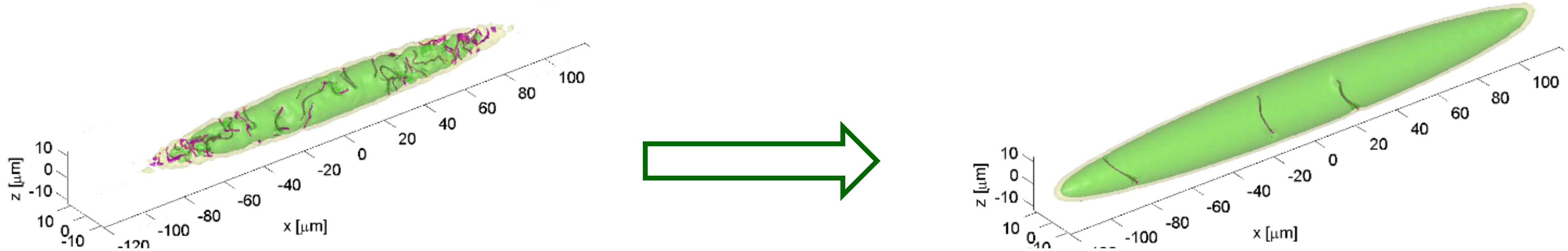




Liu, Dziarmaga, Gou, Dalfovo & Proukakis, PRR 2, 033183 (2020)

Scaled Distance to Criticality

KIBBLE-ZUREK ANALYSIS: Late Time Dynamics



After Crossing Phase Transition

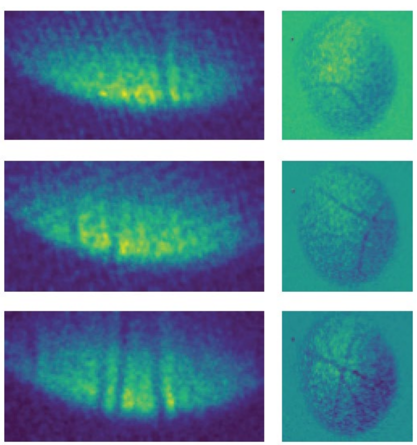
Defect Number Continuously Decay, within a Growing Condensate Density

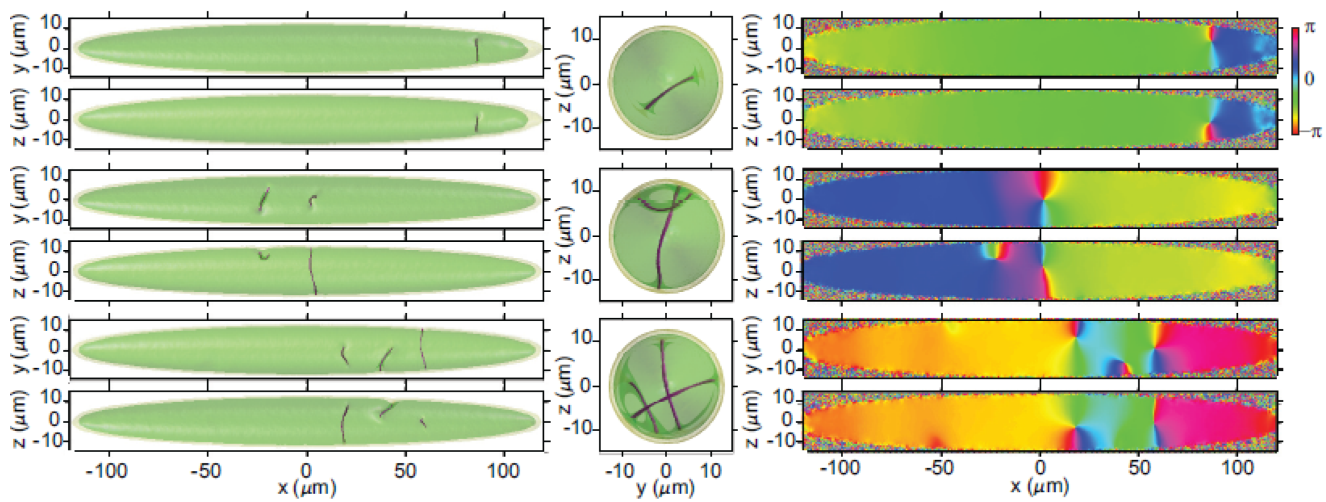
Late-Time Images after Density Saturation Reveal In-Trapped Defects

Consistent with Experimental Picture

EXPERIMENT

THEORY





Harmonic / Anisotropic Nature of System

Makes it Hard to Quantitatively Characterize Predicted Phase-Ordering Scalings

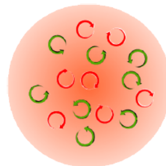
Liu, Donadello, Lamporesi, Ferrari, Gou, Dalfovo & Proukakis, Comms. Phys. (Nature) 1, 24 (2018)

2D HOMOGENEOUS PHASE TRANSITION PHYSICS

In 2d *Equilibrium* would Expect a Berezinskii-Kosterlitz-Thouless (BKT) Phase Transition

Below Threshold

Normal state (disorder)

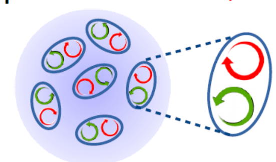


Free Vortices

$$g^{(1)}(r) = e^{-\frac{r}{\xi}}$$

Above Threshold

Superfluid state (order)



Bound Vortex-Antivortex pairs

$$g^{(1)}(r) = r^{-\alpha}$$

with $\alpha \leq \frac{1}{4}$

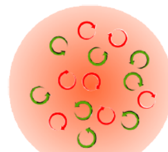
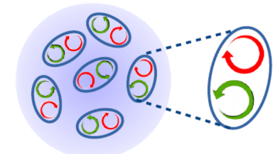
Much Easier to Characterize in a 2D Box (Homogeneous)

Question:

Are Phase Transition, or Dynamical Crossing / Relaxation Affected by Driving & Dissipation (for exciton-polariton systems) ?

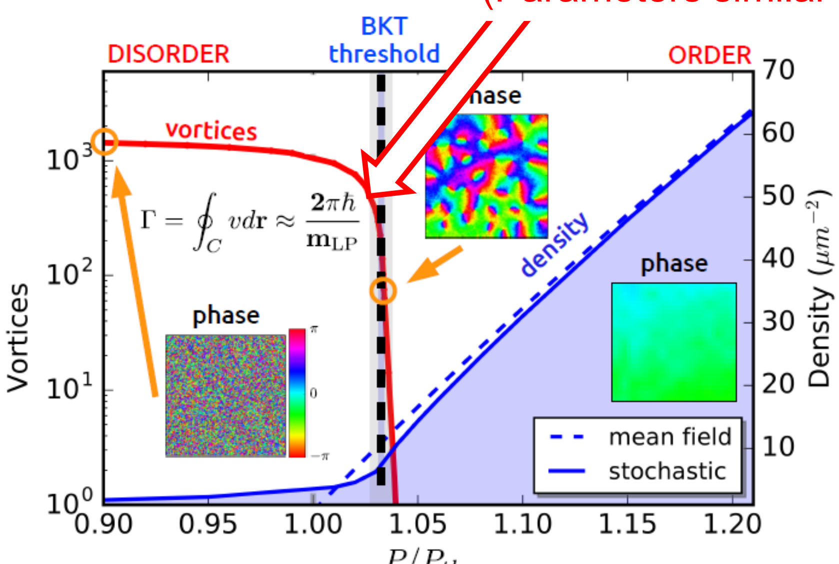
2D PHASE TRANSITION PHYSICS: Driven-Dissipative Case

In 2d *Equilibrium* would Expect a Berezinskii-Kosterlitz-Thouless (BKT) Phase Transition

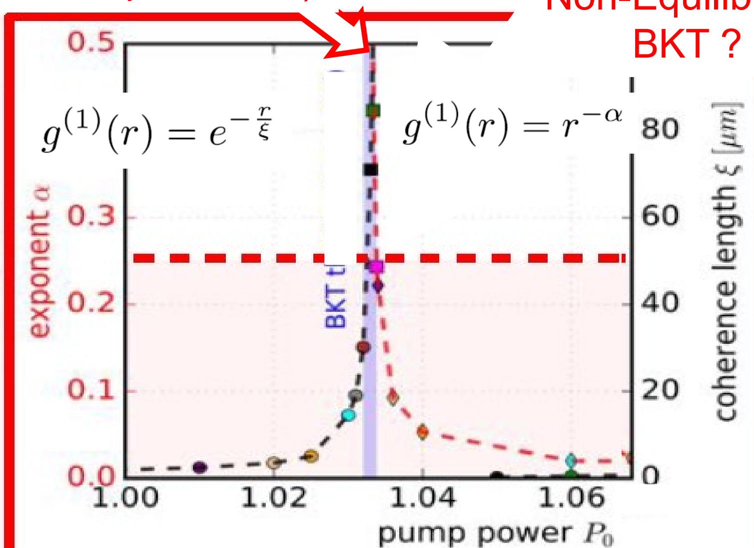
Below Threshold	Above Threshold
Normal state (disorder)	Superfluid state (order)
	
Free Vortices	*Bound Vortex-Antivortex pairs*
$g^{(1)}(r) = e^{-\frac{r}{\xi}}$	$g^{(1)}(r) = r^{-\alpha}$ with $\alpha \leq \frac{1}{4}$

At Threshold, we Find Sharp Vortex Number Decrease, and Coherence Length Increase

(Parameters similar to Yamamoto's Experiments)



Non-Equilibrium BKT ?



Comaron, Carusotto, Szymanska and Proukakis, EPL 133, 17002 (2021)

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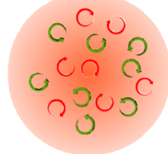

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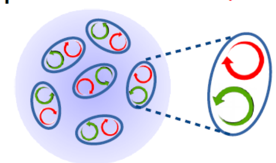
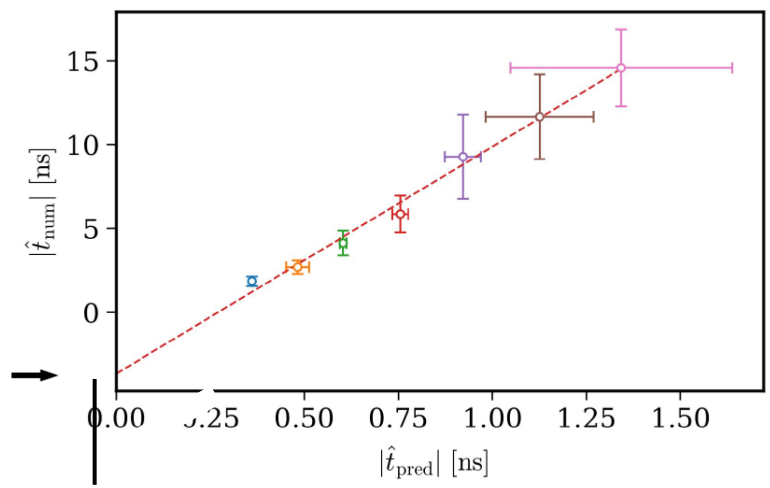


Bound Vortex-Antivortex pairs

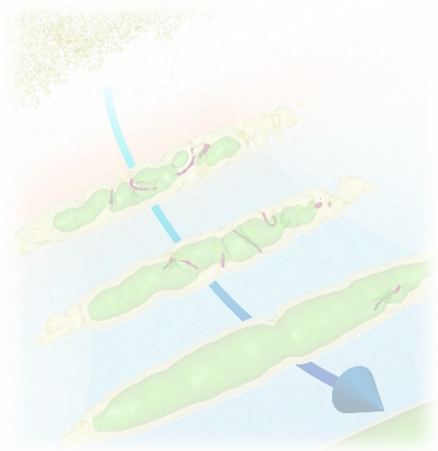
$$g^{(1)}(r) = r^{-\alpha} \quad \text{with } \alpha \leq \frac{1}{4}$$

Perform Linear Quenches
Across the Phase Transition

We have Directly Confirmed
The Kibble-Zurek Prediction
in such Driven-Dissipative System
through
Direct Numerical Comparison
of $\hat{t}_{num}$ vs. $\hat{t}_{pred}$
yielding *Linear* Relation

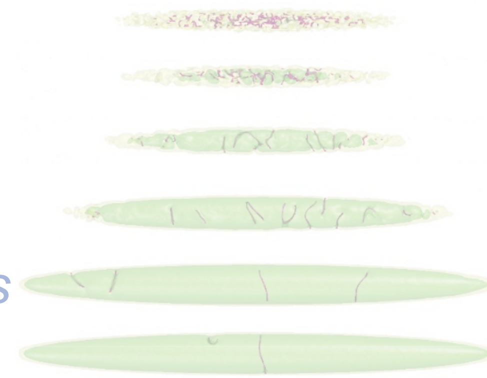


Zamora, Dagvadorj, Comaron, Carusotto, Proukakis and Szymanska, PRL 125, 095301 (2020)



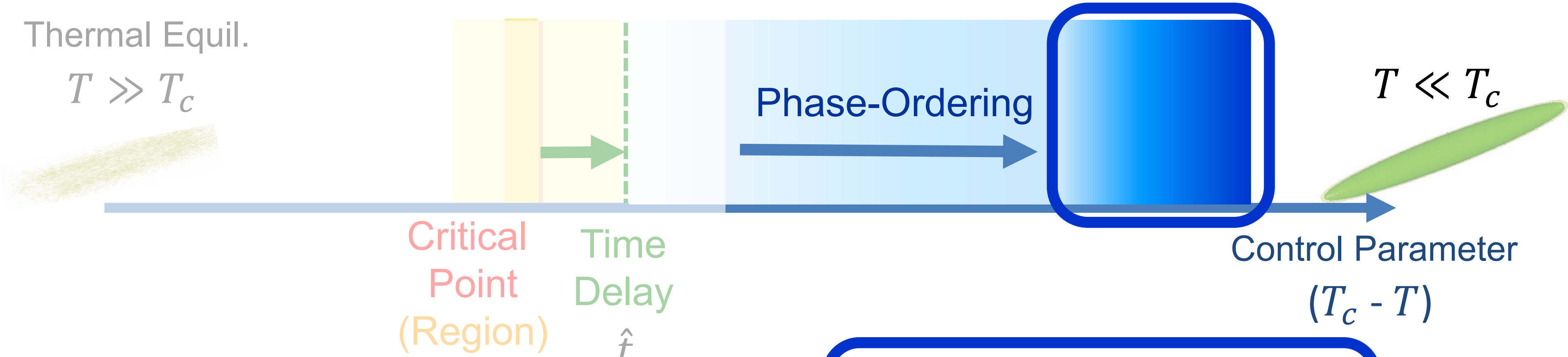
How Does Macroscopic Coherence Form
from an Incoherent Initial State?

An Old Problem
Studied Across Diverse Physical Systems



Phase Transition *Schematic*

Dynamical Perspective



Examples Discussed Here:

Kibble-Zurek Scaling (3D)

$$f(\mathbf{k},t) = \hat{t}F\left(\frac{t}{\hat{t}}, \hat{\xi}\mathbf{k}\right)$$

Phase-Ordering Scaling (2D)

$$\frac{g^{(1)}(r/L(t),t)}{g^{(1)}_{SS}(r/L(t),t)} \sim F\left(\frac{r}{L(t)}\right)$$

Easier to Characterize Following Instantaneous Quench Across Phase Transition

If L(t) >> (Microscopic System Lengthscales) ⇒ (Long-Stage System Morphology is UNIVERSAL (i.e. independent of microscopic details))

Bray, Adv. Phys. 43, 357 (1994)

This leads to *Self-Similar Evolution* characterised in terms of $\left(r/L(t)\right)$

PHASE-ORDERING KINETICS

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1st order (Phase) Correlation Function

should collapse onto a *single* function at sufficiently late-time window

$$g^{(1)}(r,t) \sim g^{(1)}_{SS}(r,t) \bullet F\left(\frac{r}{L(t)}\right) \quad \Rightarrow \frac{g^{(1)}(r/L(t),t)}{g^{(1)}_{SS}(r/L(t),t)} \sim F\left(\frac{r}{L(t)}\right)$$

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Verified in 2D Across Many Systems, including
2D XY Model, Ultracold Atoms, Closed/Open Systems, Exciton-Polariton Condensates

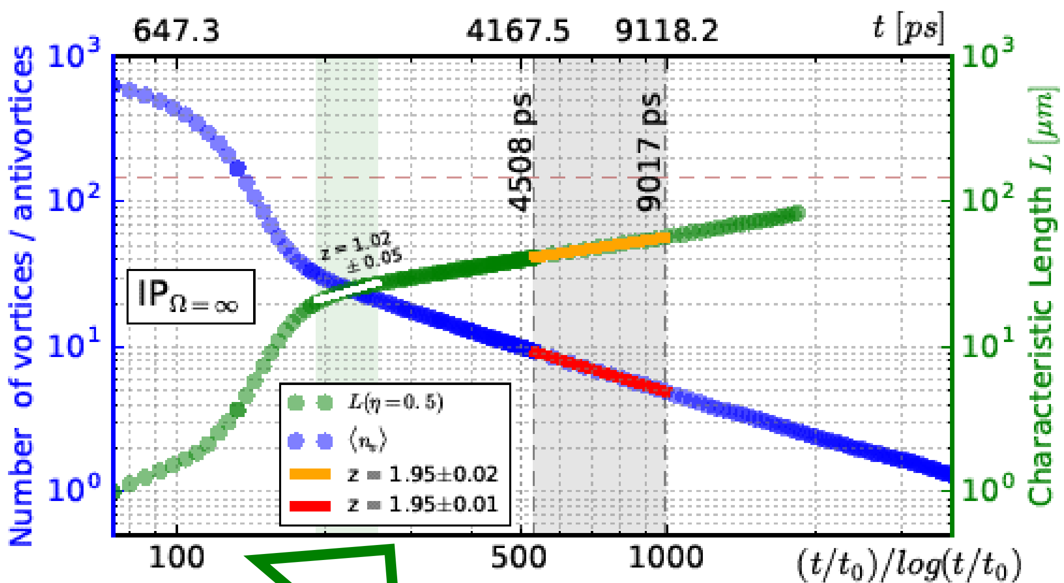
Jelic & Cugliandolo, J. Stat. Mech. P02032 (2011)

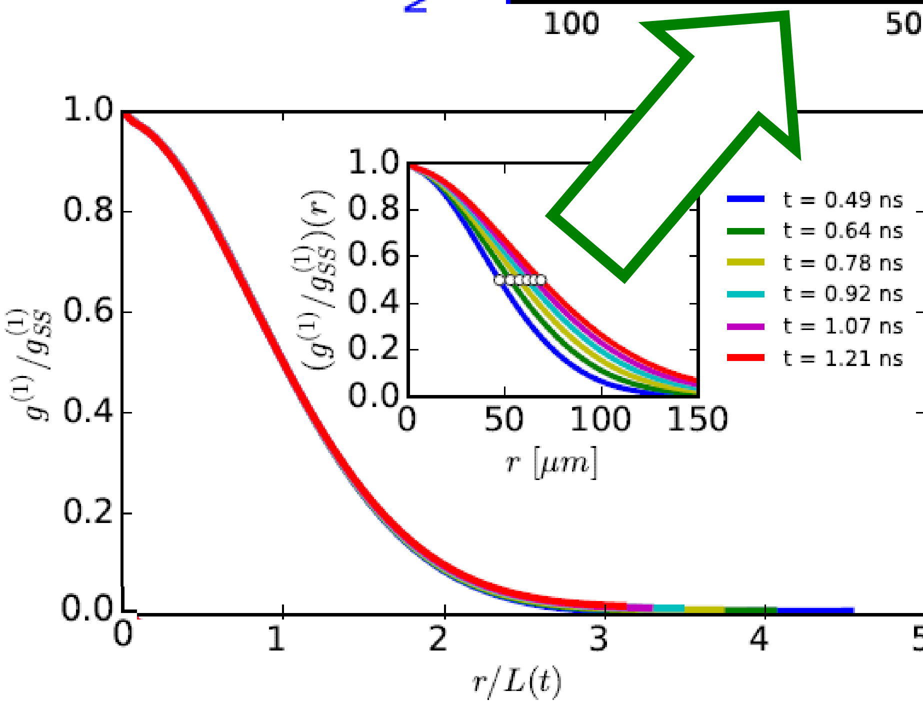
Comaron, Larcher, Dalfovo & Proukakis, PRA 100, 033618 (2019)

Groszek, Comaron, Proukakis & Billam, PRR 3, 013212 (2021)

Comaron, Dagvadorj, Zamora, Carusotto, Proukakis & Szymanska, PRL 121, 095302 (2018)

PHASE-ORDERING KINETICS: EXCITON-POLARITON CASE





Observe Clear Evidence of Logarithmic Corrections

$$L(t) \sim \left(\frac{t}{\log(t/t_0)} \right)^{1/z} \quad \text{with} \quad z \approx 2$$

as in 2D XY Model

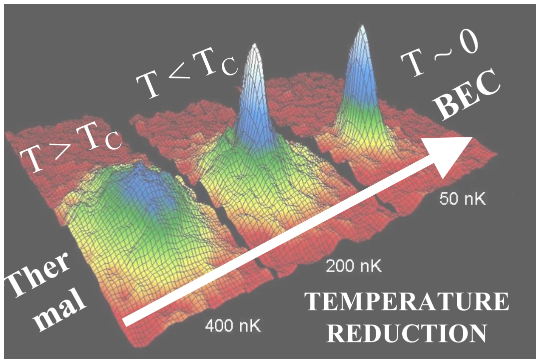
Jelic & Cugliandolo, J. Stat. Mech. P02032 (2011)

confirming BKT nature of phase transition when crossed dynamically

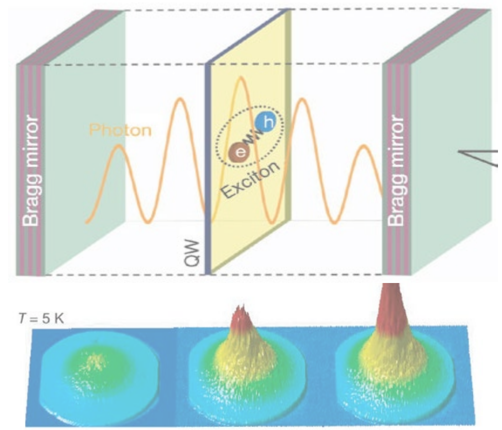
Comaron, Dagvadorj, Zamora, Carusotto, Proukakis & Szymanska, PRL 121, 095302 (2018)

CONDENSATES CHARACTERISED IN THIS TALK

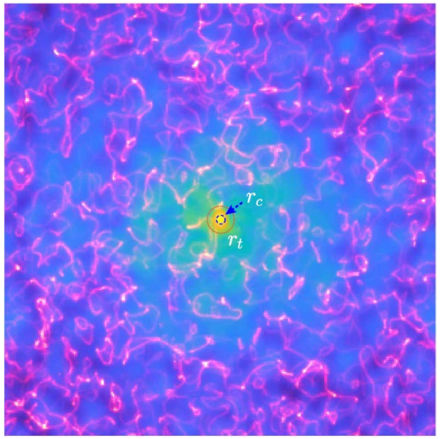
Ultracold Atomic BECs
(3D / 2D / 1D)
["Equilibrium" State]



Exciton-Polariton BECs
(2D / 1D)
[(Quasi-)Equilibrium]



Fuzzy Dark Matter
(Galactic-Size Condensation)
[*Hypothesized !*]



QUESTION #1:

In the Laboratory Condensates
(which can be controlled / monitored)

How Does Coherence Grow from an Initially Incoherent State?

Comms.Phys. (Nature) 1, 24 (2018)
PRR 2, 033183 (2020)
PRR 3, 013097 (2021)
PRR 3, 013212 (2021)

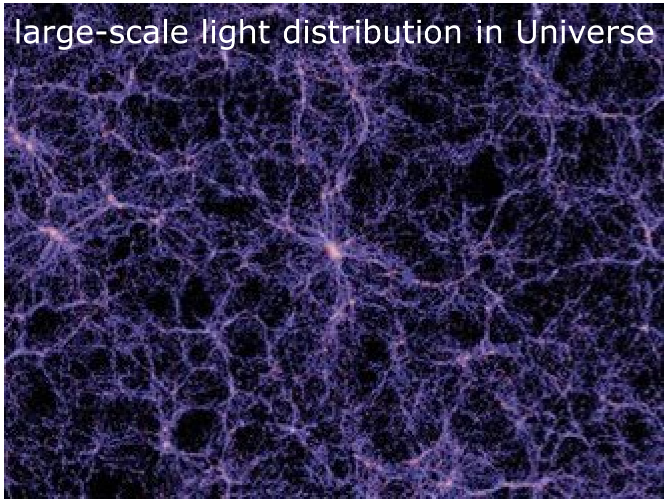
PRL 121, 095302 (2018)
PRL 125, 095301 (2020)
EPL 133, 17002 (2021)

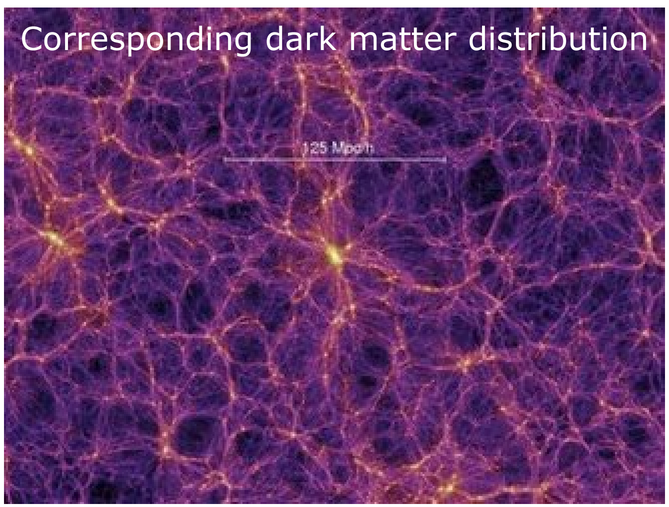
QUESTION #2:

What does Condensation have to do with Dark Matter Distribution in the Universe ?

IK Liu, NP Proukakis, G Rigopoulos
arXiv preprint arXiv:2211.02565

Millenium Simulations



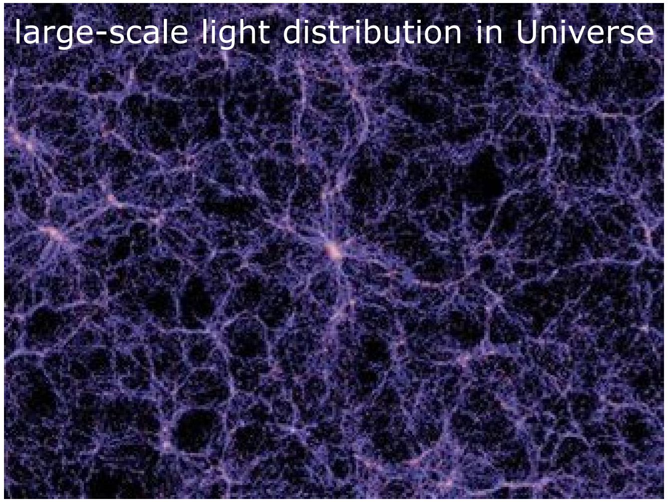


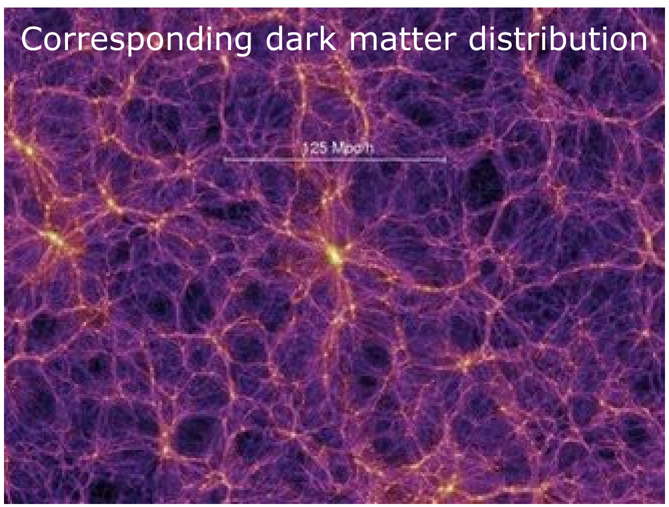
More than 10 billion particles
Cubic region (2 billion light-years)

https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/

MILLENIUM SIMULATION

Millenium Simulations





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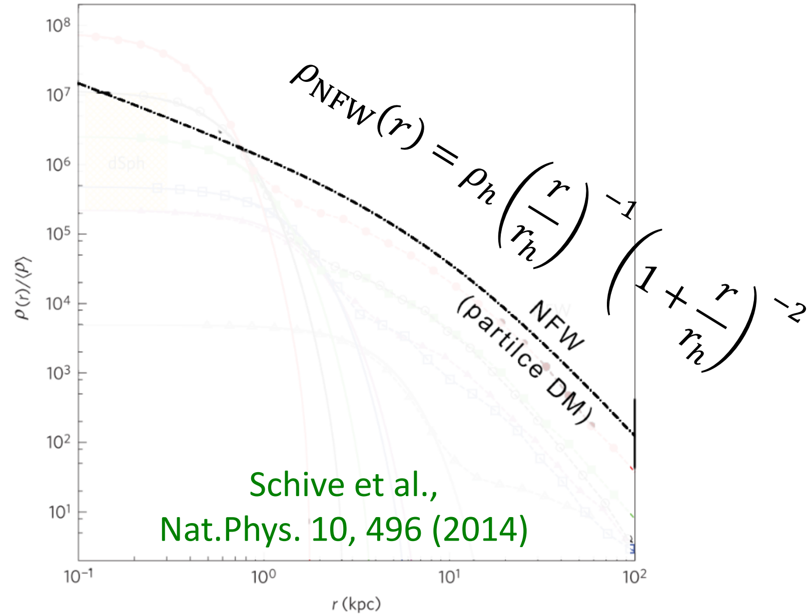
CDM Model (N-Body Simulations)

Excellent Large-Scale Description

NFW: Navarro, Frenk & White, ApJ, 462, 563 (1996)



... but ...

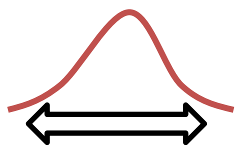
some `small-scale' issues identified

e.g. `Cusp-core problem'

$\rho_{\mathrm{NFW}}(r) \sim r^{-1}$

as r → 0

Fuzzy Dark Matter Model



$$\lambda = \frac{h}{p}$$

Ultralight Axions $m \sim 10^{-22}\ eV/c^2$

Typical galactic Halo Velocities

$$\lambda \sim O(1)\ kpc$$

Galactic-Scale Condensation !

Hu et al., PRL 85, 1158 (2000)

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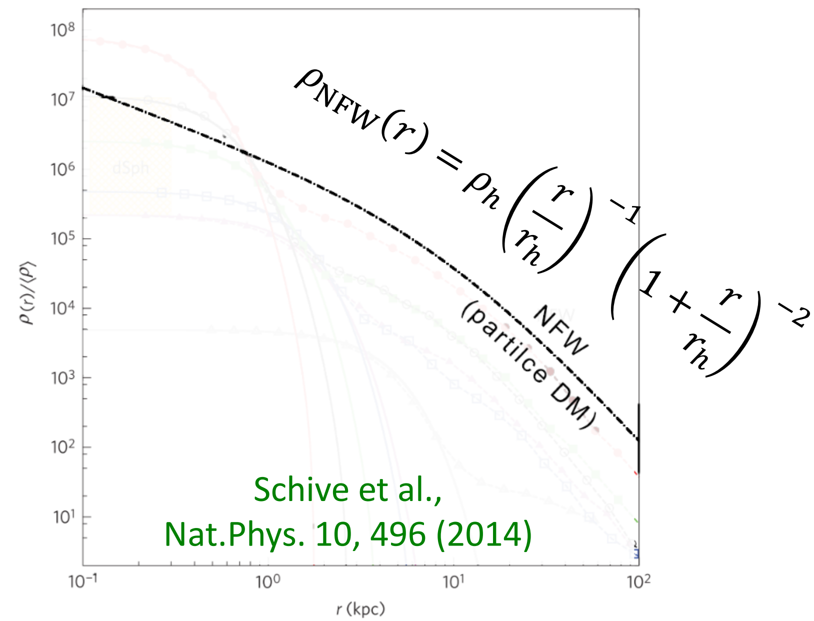

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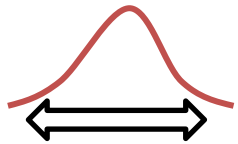

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Schrödinger-Poisson equations

$$i\hbar \frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + \mathrm{m}\Phi(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$

$$\nabla^2\Phi(\mathbf{r},t) = 4\pi G[\rho(\mathbf{r},t) - \bar{\rho}]$$

$$\rho(\mathbf{r},t) = |\Psi(\mathbf{r},t)|^2$$

Recent Reviews:

Marsh, Phys. Rep. 643, 1 (2016)

Hui, Astr. & Astroph. Review 59, 247 (2021)

Ferreira Astr. & Astroph. Review 29, 7 (2021)

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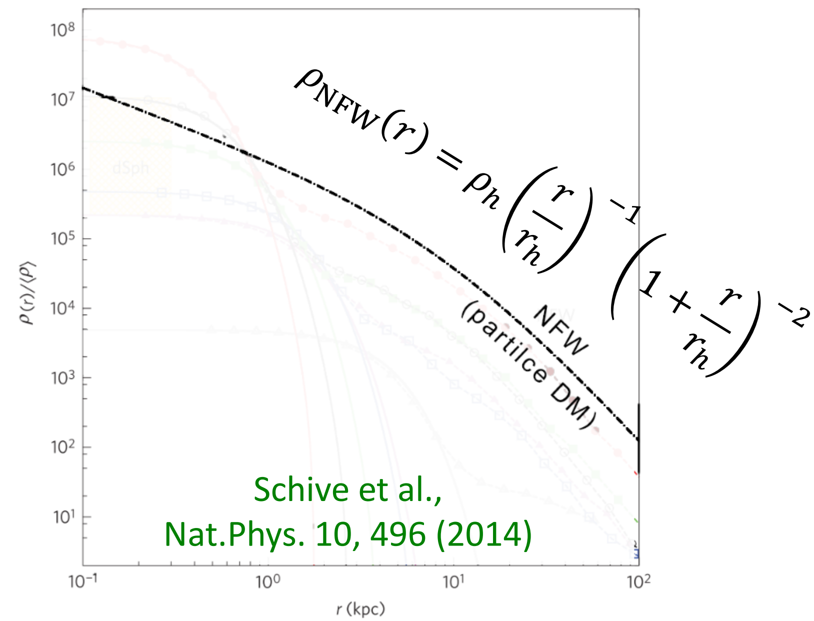

Schive et al.,
Nat.Phys. 10, 496 (2014)

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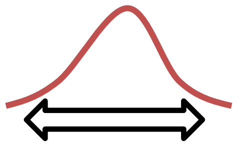

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$$\rho(\mathbf{r},t) = |\Psi(\mathbf{r},t)|^2$$

→ Can also add self-interactions *a la* Gross-Pitaevskii Equation

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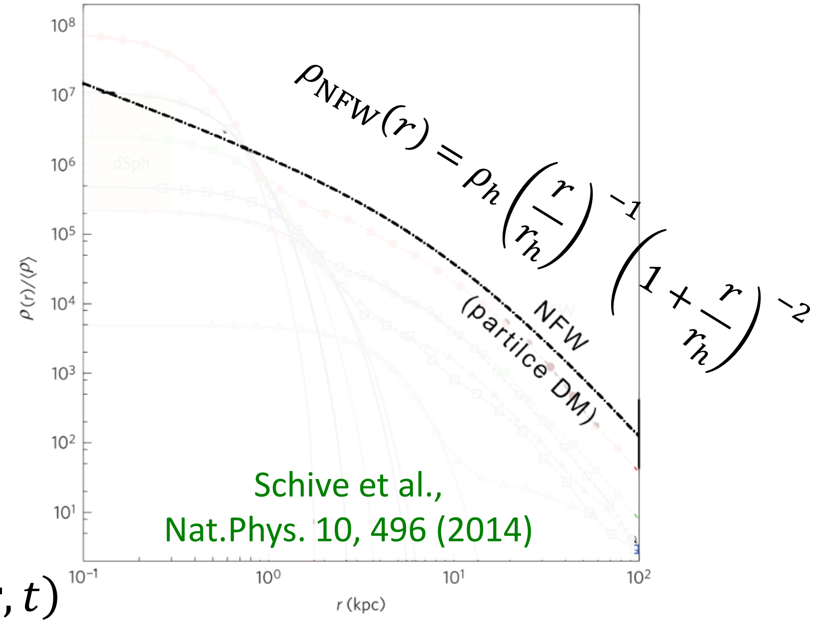

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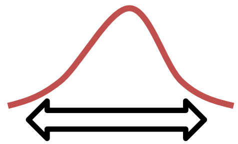
COLD DARK MATTER vs. FUZZY DARK MATTER

Fuzzy Dark Matter Model



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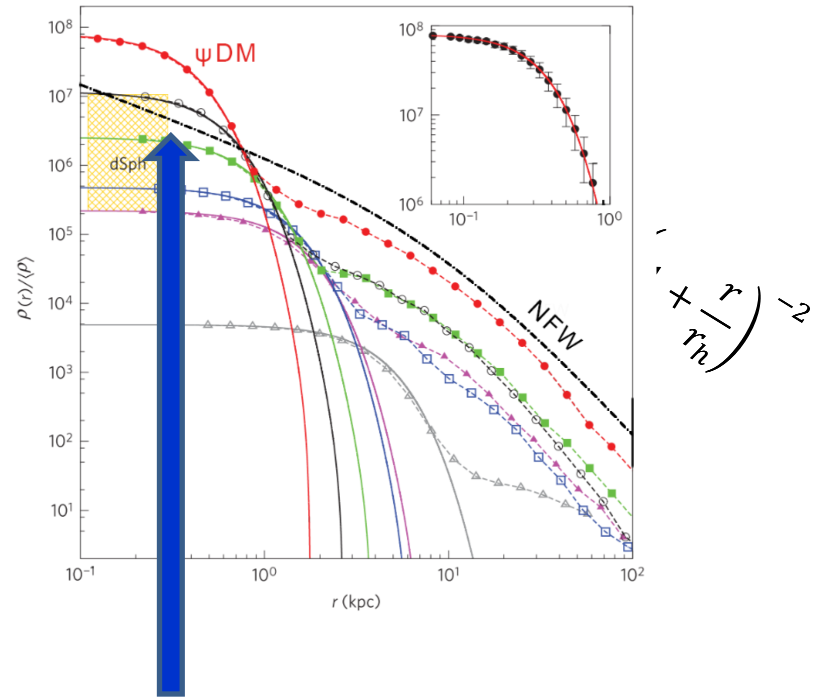

FDM / ψDM appears to solve short-scale density divergence

Schrödinger-Poisson equations

$$i\hbar \frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + \mathrm{m}\Phi(\mathbf{r},t)\right]\Psi(\mathbf{r},t)$$

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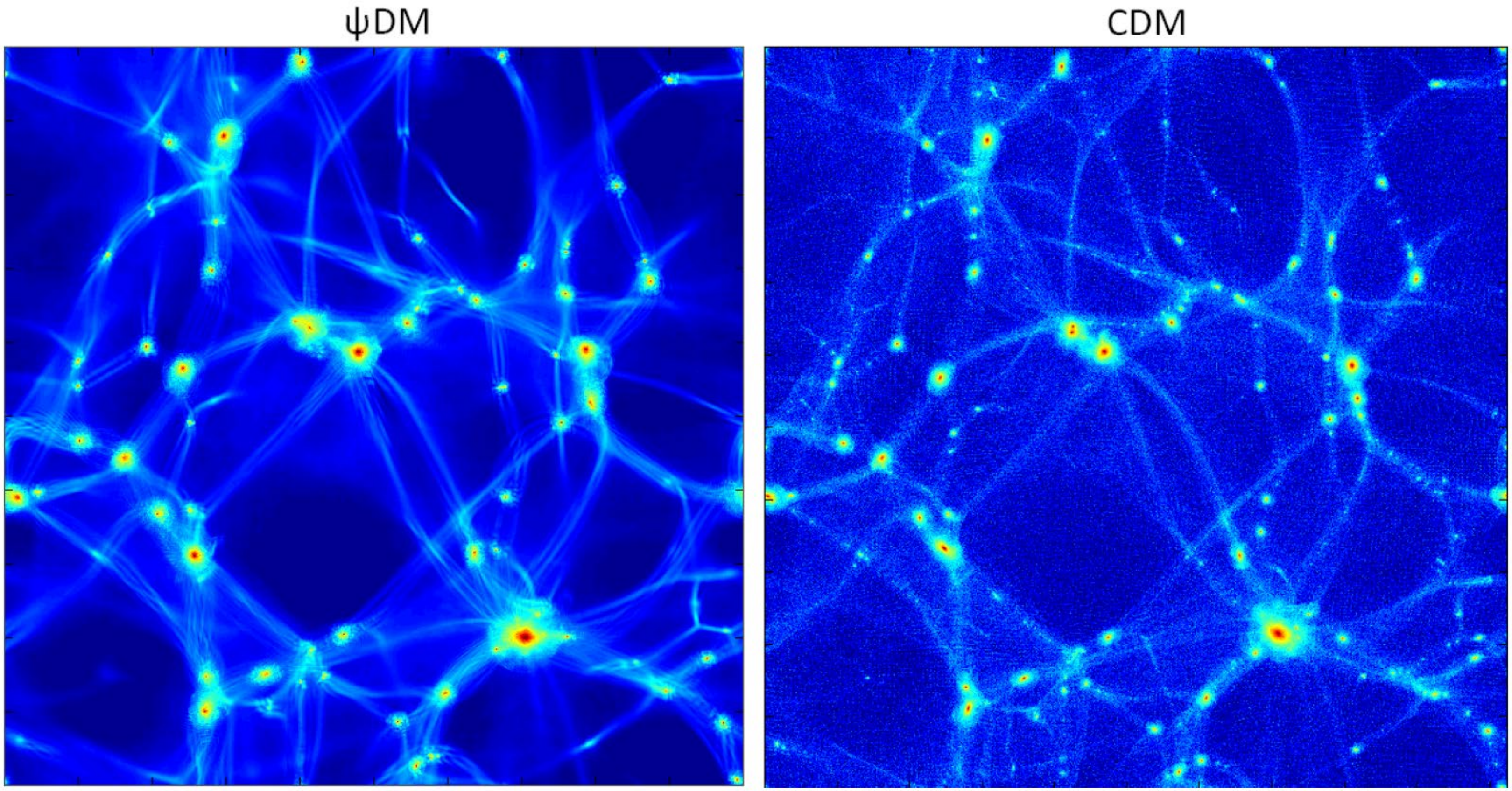
COLD DARK MATTER vs. FUZZY DARK MATTER

Fuzzy Dark Matter Model **CDM Model (N-Body Simulations)**



Schive et al., Nat.Phys. 10, 496 (2014)

Fuzzy Dark Matter Effectively Reproduces CDM Large-Scale Predictions!

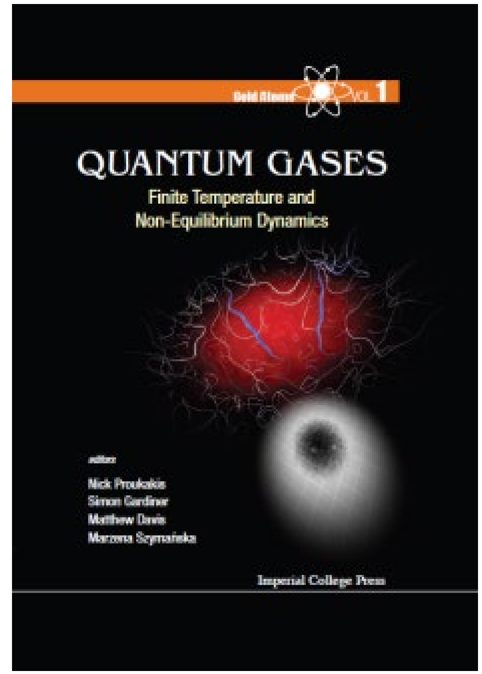
... and it offers an immediate `cure' of the CDM short-scale `anomaly' ...

COLD DARK MATTER vs. FUZZY DARK MATTER

Fuzzy Dark Matter Model | **CDM Model (N-Body Simulations)**

ψDM | CDM

Analogous to
well-known discussions
(in cold quantum matter context)
about relation between
Kinetic / Boltzmann Equations
and
Classical Field Description



Schive et al., Nat.Phys. 10, 496 (2014)

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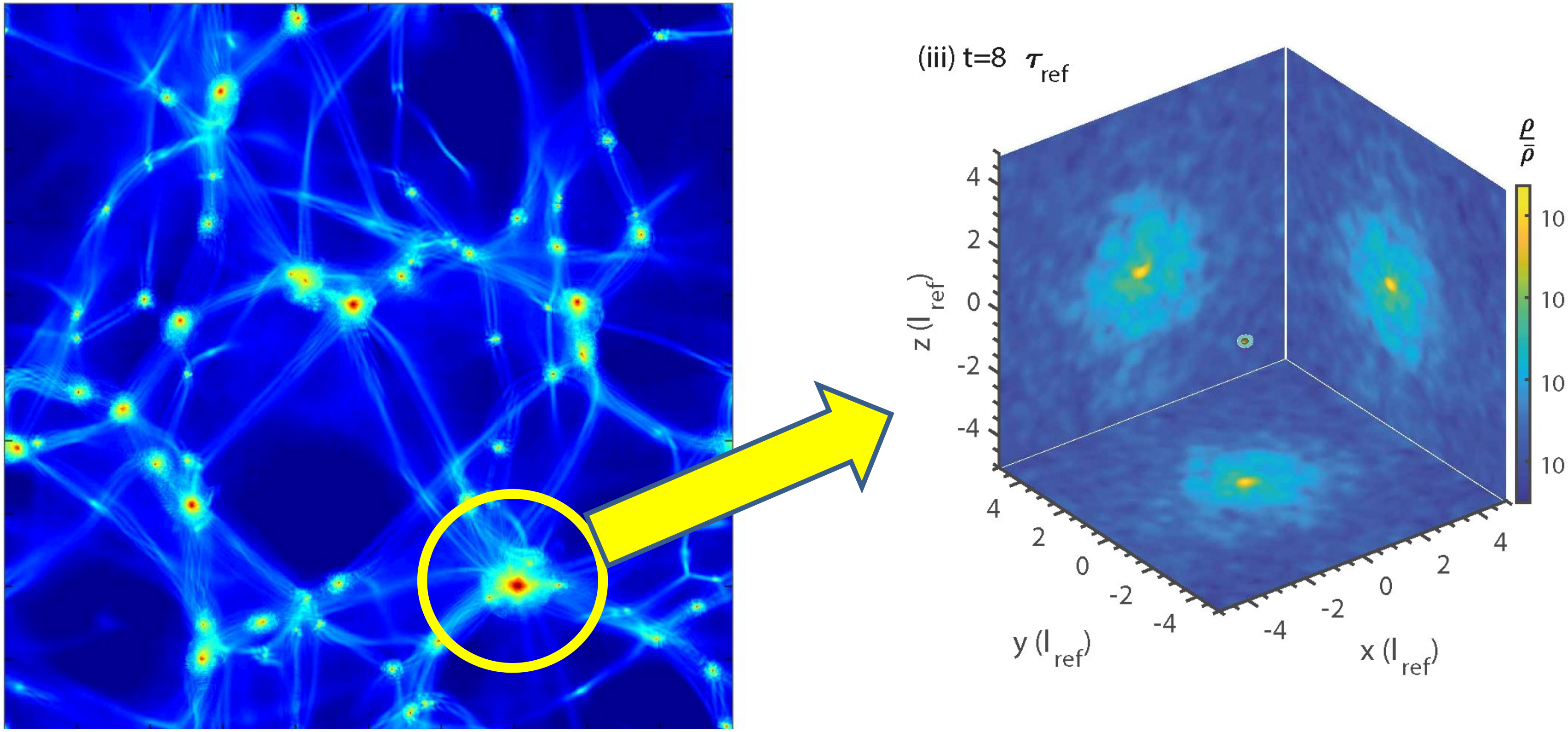
Fuzzy Dark Matter Model

Focus on Isolated Virialized Core + Halo (Idealized Scenario)

ψDM



Schive et al., Nat.Phys. 10, 496 (2014)

& Analyze its Coherence Properties

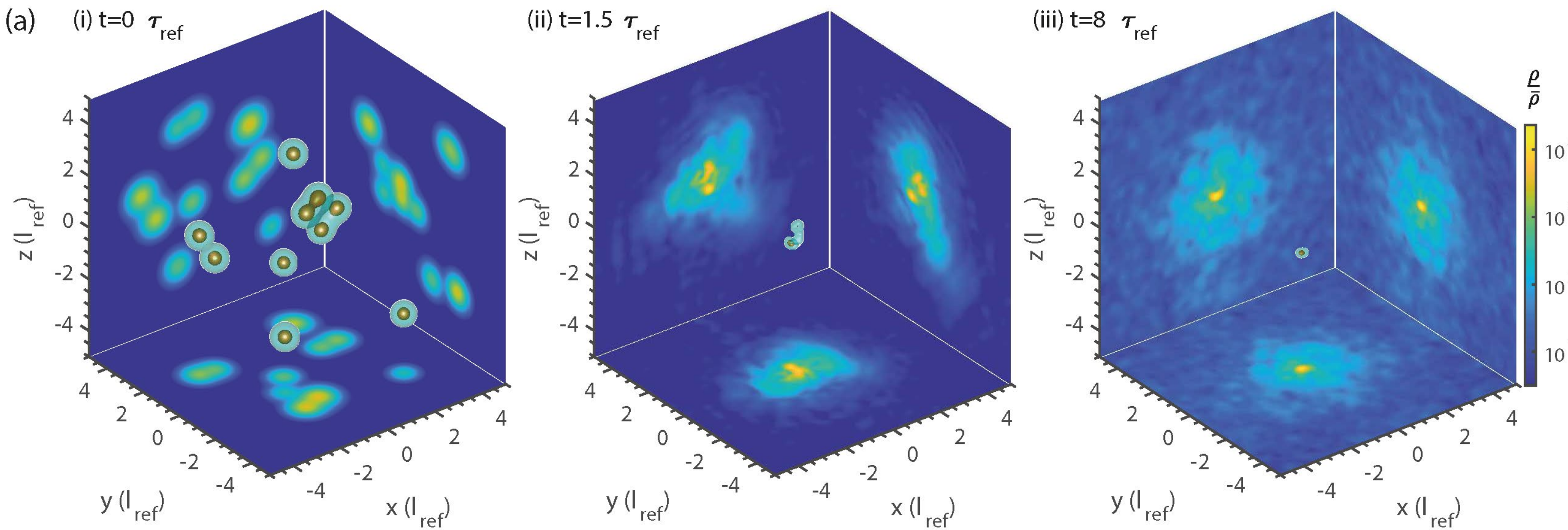
Liu, Proukakis, Rigopoulos, arXiv:2211.02565

GENERATION OF ISOLATED VIRIALIZED FDM CORE + HALO

Mocz et al., MNRAS 471, 4559 (2017)
Chan et al., arXiv:2110.11882

Soliton merger simulation

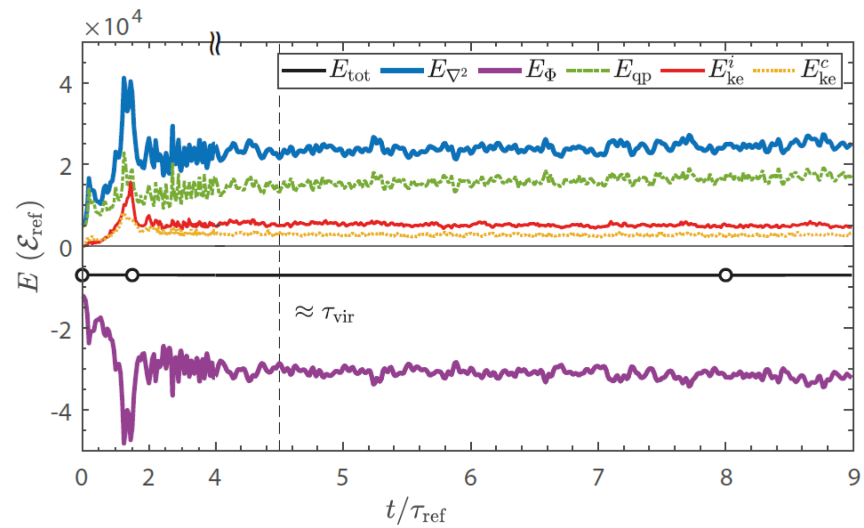
Primary sample $(M = 100 M_{\rm ref})$



$$E_{\rm ref} = \hbar\sqrt{G\rho_{\rm ref}} \qquad \tau_{\rm ref} = (G\rho_{\rm ref})^{-1/2} \qquad l_{\rm ref} = \left(\frac{\hbar^2}{m^2 G\rho_{\rm ref}}\right)^{1/4}$$

$$\rho_{\rm ref} = 10^3 M_\odot {\rm kpc}^{-3}$$
$$m_{\rm ref} = 2.5 \times 10^{-23}\ {\rm eV}$$

$$M_{\rm ref} \approx 1.26 \times 10^6 M_\odot$$
$$\tau_{\rm ref} \approx 14.91\ {\rm Gyr}$$
$$l_{\rm ref} \approx 10.81\ {\rm kpc}$$

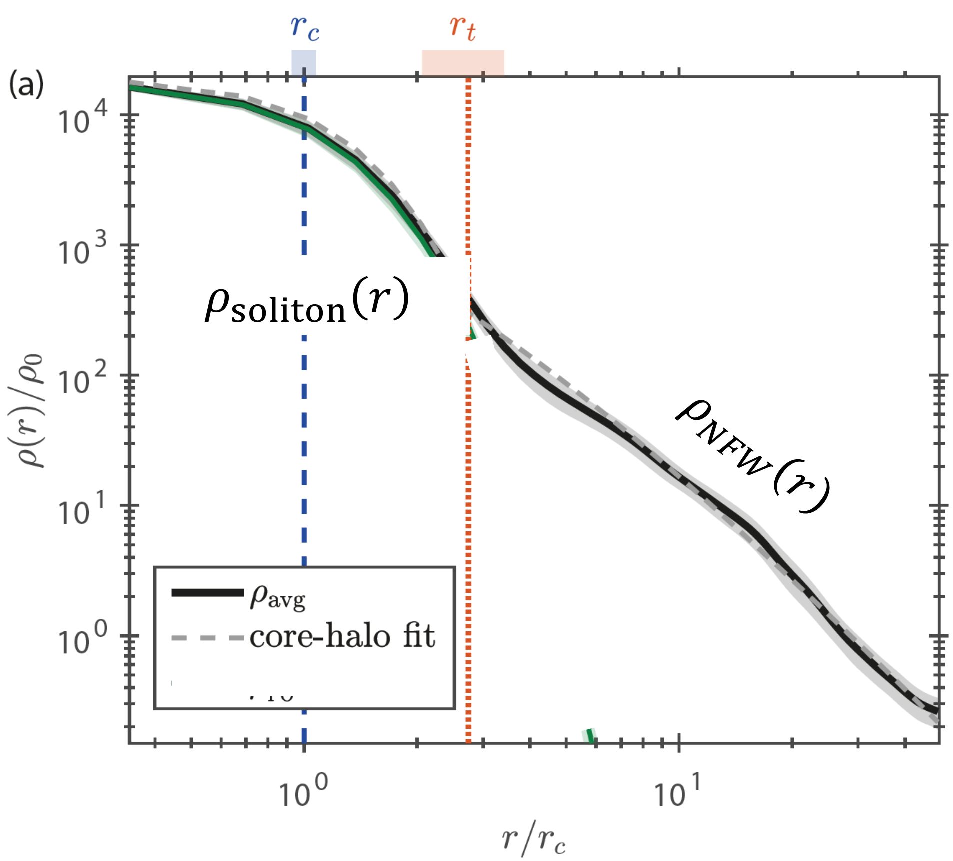


Bimodal Core-Halo Profile

$$\rho_{\text{cNFW}}(r) = \begin{cases} \rho_{\text{soliton}}(r) & , r \le r_t \\ \rho_{\text{NFW}}(r) & , r > r_t \end{cases}$$

$$\rho_{\text{soliton}}(r) = \rho_c \left[1 - \lambda \left(\frac{r}{r_c}\right)^2\right]^{-8}$$

$$\rho_{\text{NFW}}(r) = \rho_h \left(\frac{r}{r_h}\right)^{-1} \left(1 + \frac{r}{r_h}\right)^{-2}$$



Let us now
Analyze
the Coherence Properties
of this State

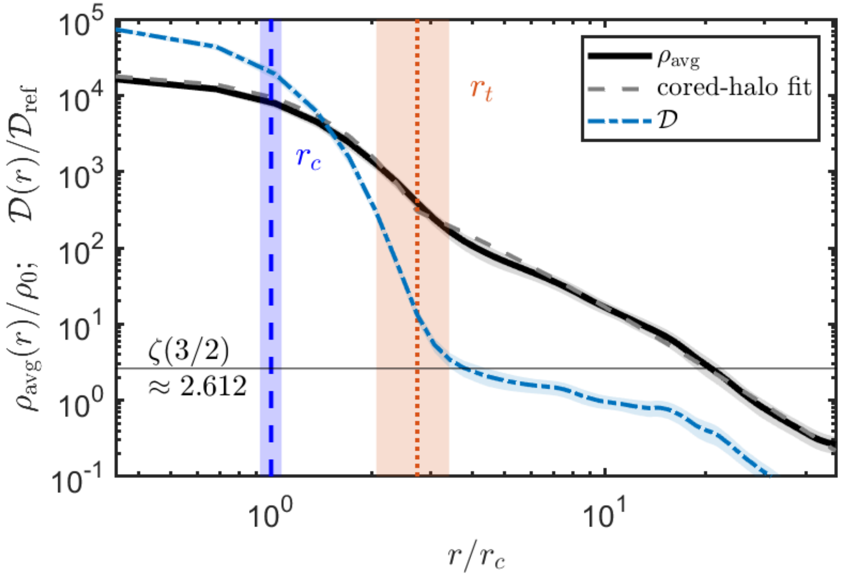
ANALYSIS OF ISOLATED VIRIALIZED FDM CORE + HALO

Phase-Space Density

$$\mathcal{D}(r) = n\,\lambda_{\rm dB}^3 = \frac{\langle \rho'(r) \rangle}{\langle v'(r) \rangle^3}\mathcal{D}_{\rm ref}$$

Significant Change over $0 < r < r_t$

$$\mathcal{D}_{\rm ref} = \left(\frac{\hbar^3 \rho_{\rm ref}}{m^4 v_{\rm ref}^3}\right) \approx 2.1 \times 10^{106}$$



ANALYSIS OF ISOLATED VIRIALIZED FDM CORE + HALO

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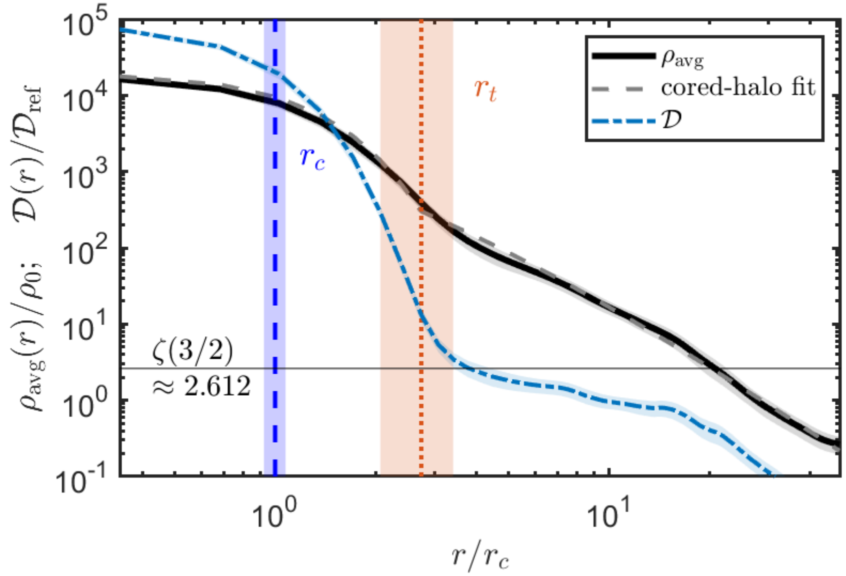
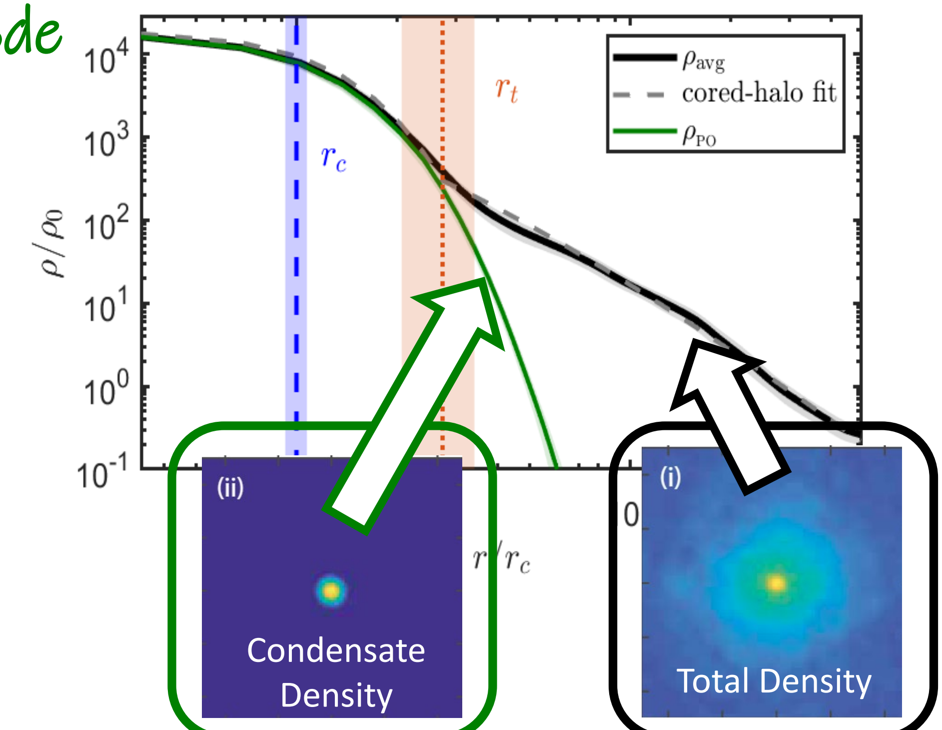


Condensate (Penrose-Onsager) Mode

$$\int \hat{\varrho}(\mathbf{r},\mathbf{r}')\Psi_{\rm PO}(\mathbf{r}) = N_{\rm PO}\Psi_{\rm PO}(\mathbf{r})$$

$$\hat{\varrho}(\mathbf{r},\mathbf{r}') = \langle \Psi^*(\mathbf{r}')\Psi(\mathbf{r})\rangle$$

Density Dominated by Condensate over $0 < r < r_t$



Liu, Proukakis, Rigopoulos, arXiv:2211.02565

ANALYSIS OF ISOLATED VIRIALIZED FDM CORE + HALO

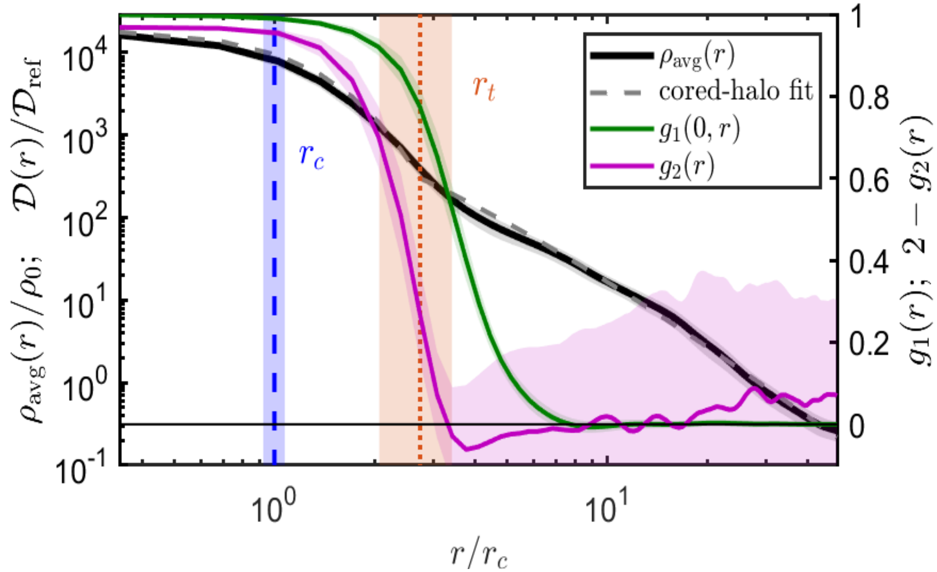
(Non-Local) Phase Correlations

$$g^{(1)}(r,r') = \frac{\langle \Phi^*(\mathrm{r})\Phi(\mathrm{r}')\rangle}{\sqrt{\langle|\Phi(r)|^2\rangle}\sqrt{\langle|\Phi(r')|^2\rangle}}$$

(Local) Density-Density Correlations

$$g^{(2)}(r) = \frac{\langle|\Phi(\mathrm{r})|^4\rangle}{\langle|\Phi(\mathrm{r})|^2\rangle^2}$$

$g^{(1)}(r) \approx g^{(2)}(r) \approx 1$ over $0 < r < r_c$



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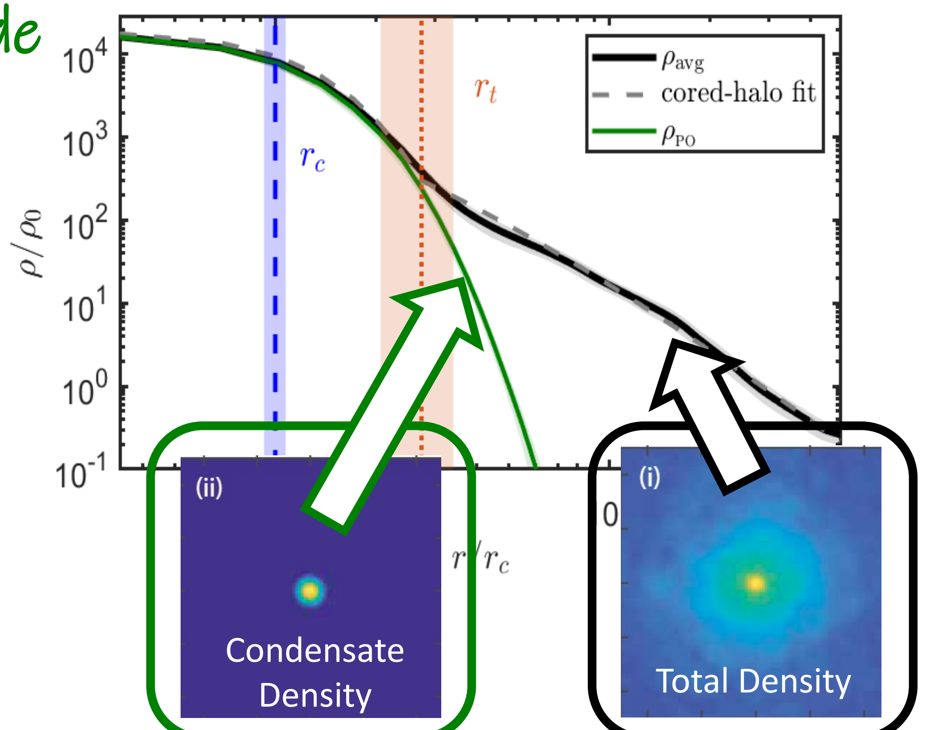


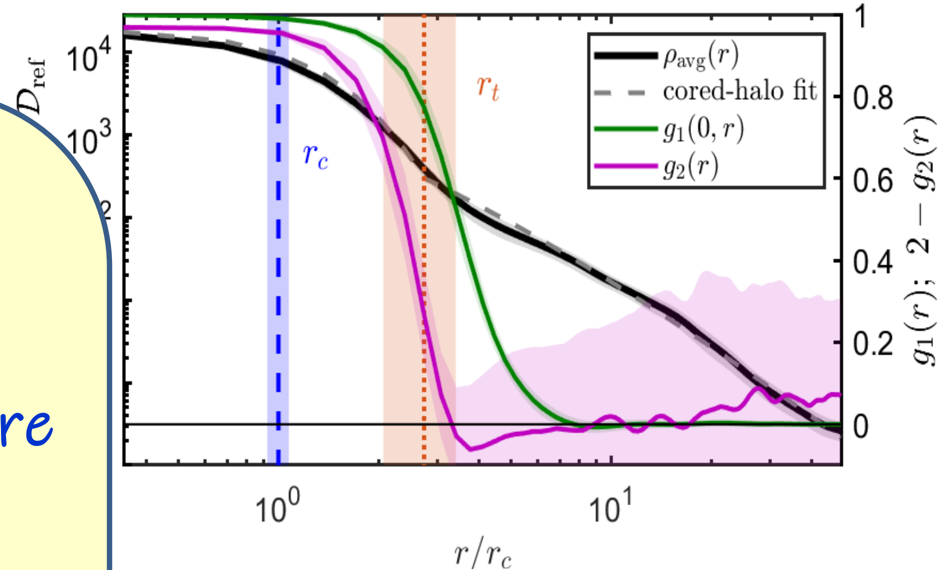
Liu, Proukakis, Rigopoulos, arXiv:2211.02565

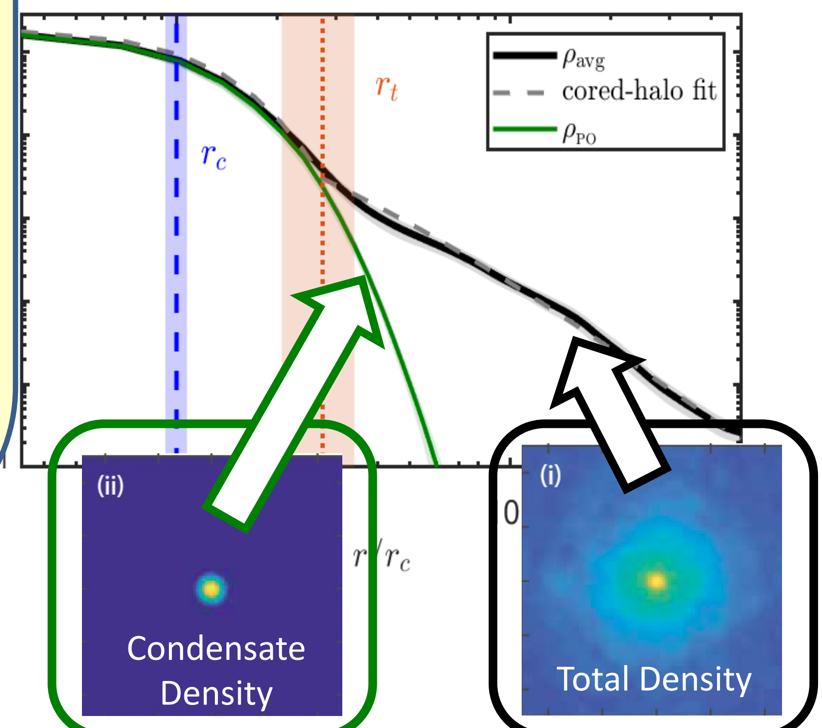
ANALYSIS OF ISOLATED VIRIALIZED FDM CORE + HALO

(Non-Local) Phase Co

$g^{(1)}(r,r') = \frac{\langle \Phi^*(r)\Phi(r') \rangle}{\ldots}$

i.e.
Galaxy-Sized Profile
Consists of
Purely Coherent High-Density Core
(A Bose-Einstein Condensate)
surrounded by an
Incoherent Lower-Density Halo





Liu, Proukakis, Rigopoulos,
arXiv:2211.02565

ANALYSIS OF ISOLATED VIRIALIZED FDM CORE + HALO

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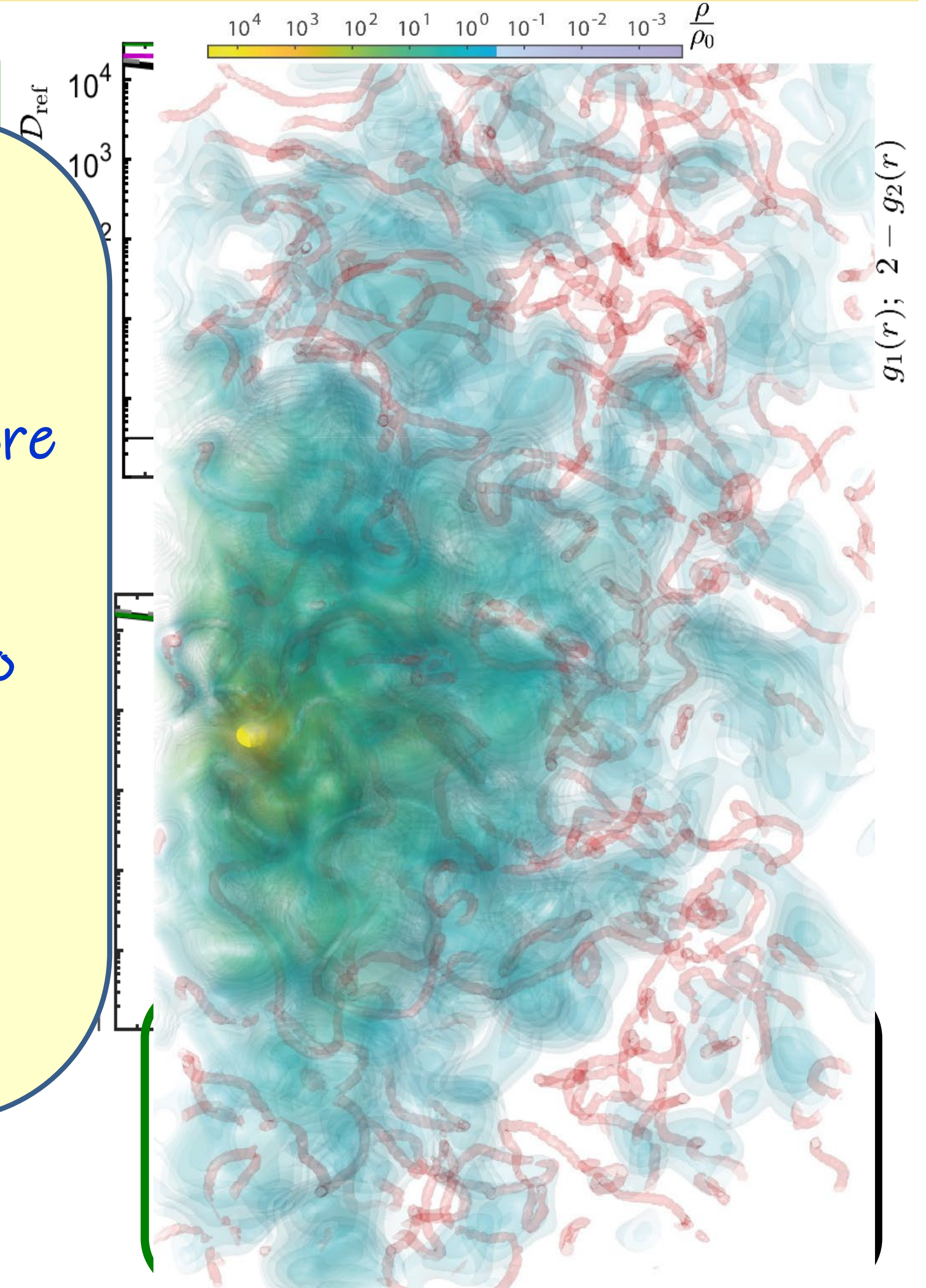
Galaxy-Sized Profile
Consists of
Purely Coherent High-Density Core
(A Bose-Einstein Condensate)
surrounded by an
Incoherent Lower-Density Halo
Filled with
Tangled / Turbulent
Quantum Vortices
Separating Quasi-Coherent
`Density Granules'



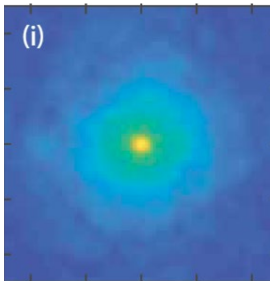
Liu, Proukakis, Rigopoulos,
arXiv:2211.02565

ANALYSIS OF ISOLATED VIRIALIZED FDM CORE + HALO

Fuzzy Dark Matter

Self-Trapping Provided

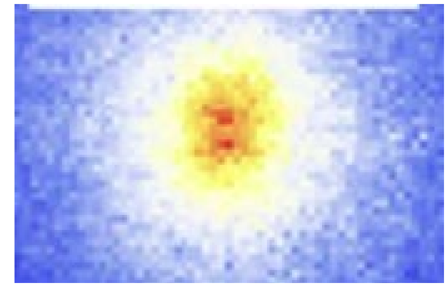
by Gravitational Potential



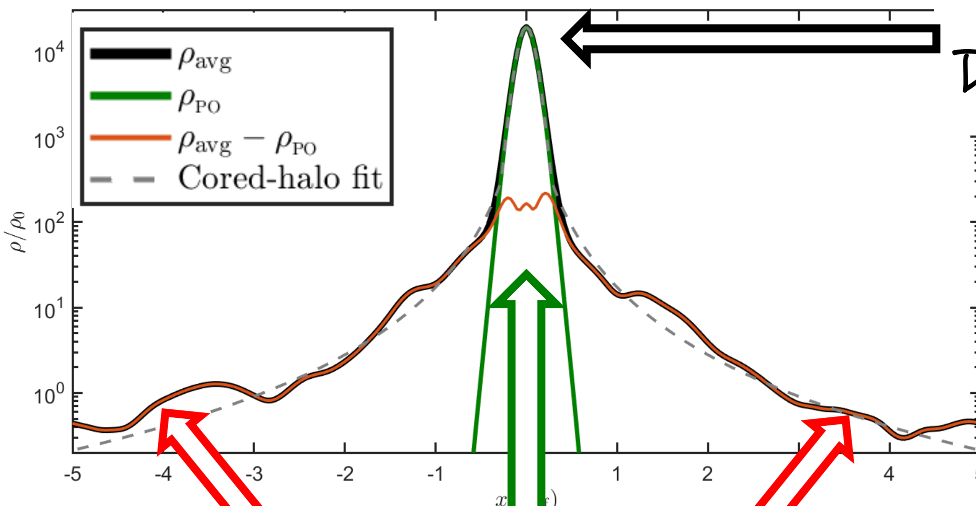
Ultracold Atomic Gas

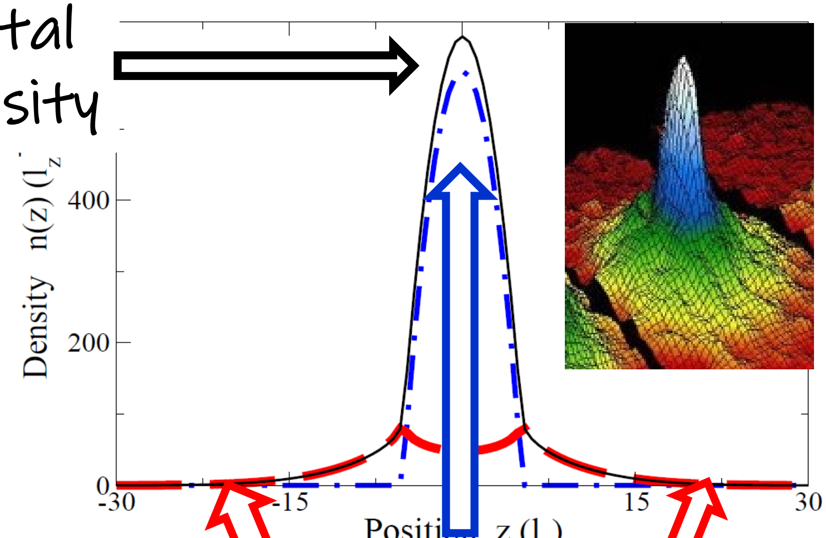
Self-Trapping Provided

By Harmonic Trap



Total Density





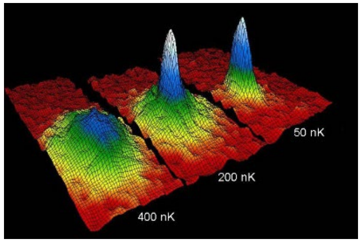
Condensate

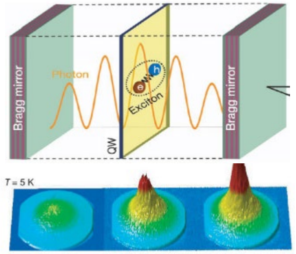
Incoherent Component

Atomic Condensate

Thermal Cloud

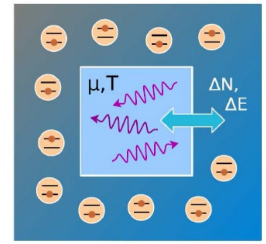
SUMMARY

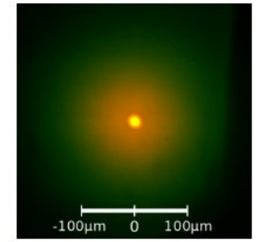


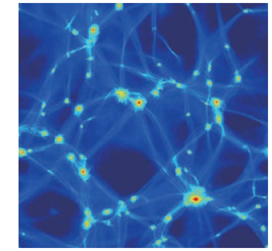


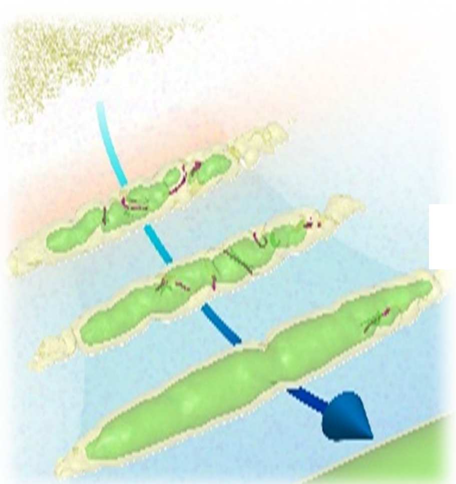
Bose-Einstein Condensation Arises Across Vastly Different Scales

$n\lambda^3 \sim 1$

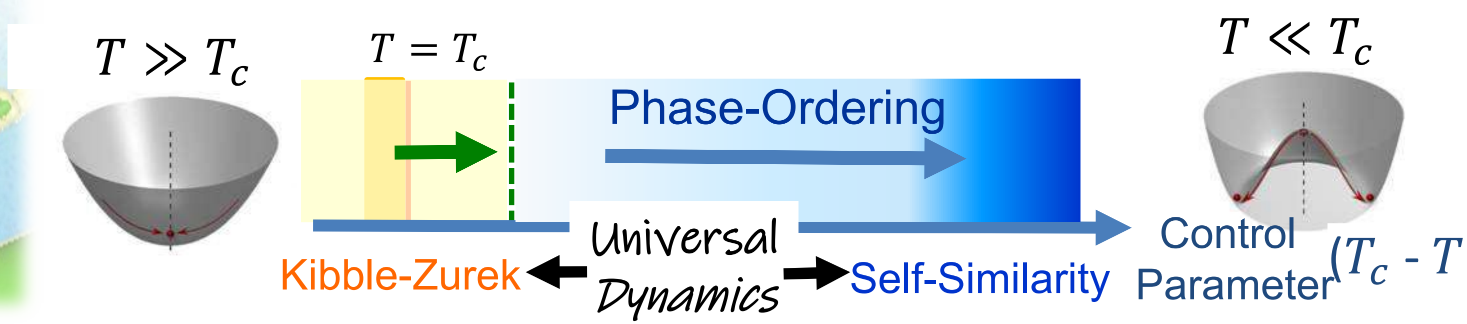








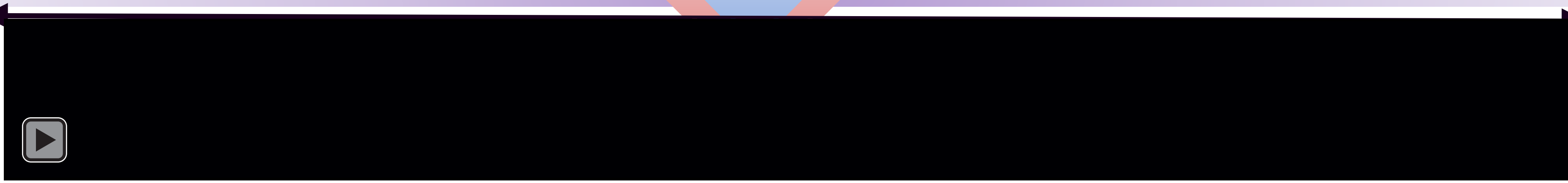
Laboratory Quantum Gases
(Ultracold Atoms / Exciton-Polariton Condensates)
are *Ideal Systems for Universal Dynamical Studies*



Incoherent NFW outer halo
Fluctuating Phase & Density via Vortices

Coherent soliton core

Incoherent NFW outer halo
Quasi-condensate & Turbulent State



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Ultracold Atomic Gases

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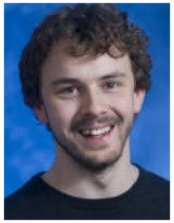
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TP Billam

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University of Trento


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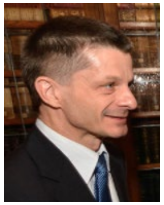
S Donadello


G Lamporesi


G Ferrari

Uniwersytet Jagielloński w Krakowie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UltraCold Atom Theory UCAT


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Exciton-Polariton Condensates

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Newcastle University


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A. Soto


M. Indjin

PhD Positions Available September 2023

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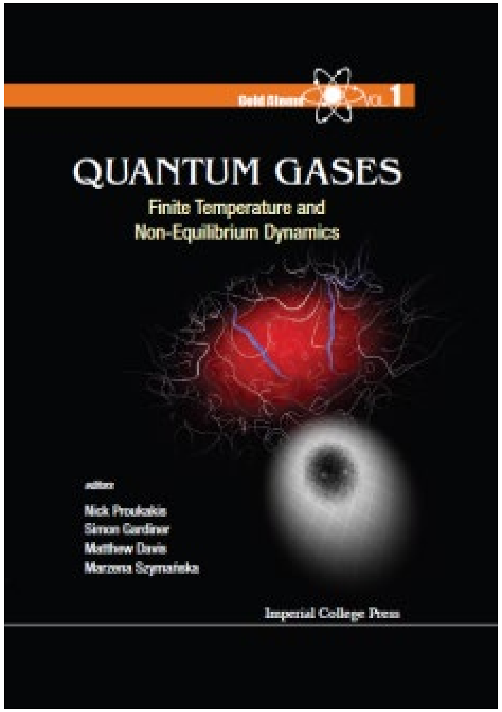
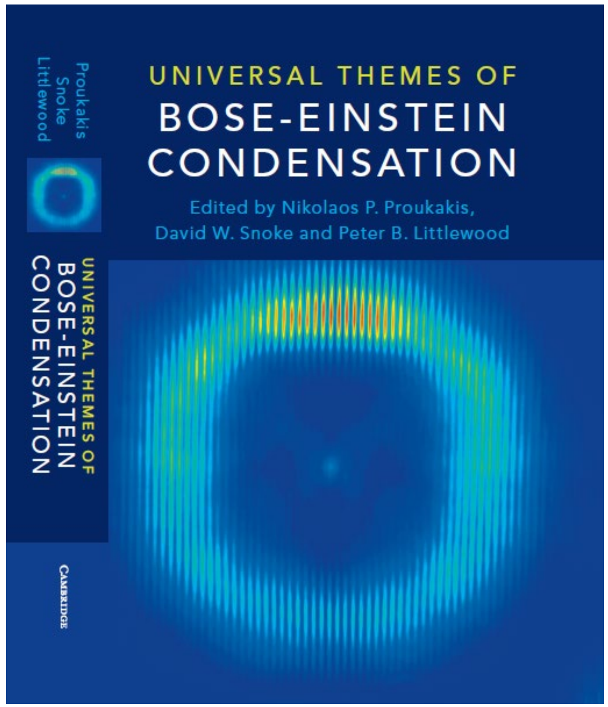
FURTHER READING

MODELLING REVIEWS:

Proukakis & Jackson, J Phys B 41, 203002 (2008)
Berloff, Brachet & Proukakis, PNAS 111 (Suppl. 1) 4675 (2014)
Blakie, Bradley, Davis, Ballagh & Gardiner, Adv. Phys. 57, 363 (2008)

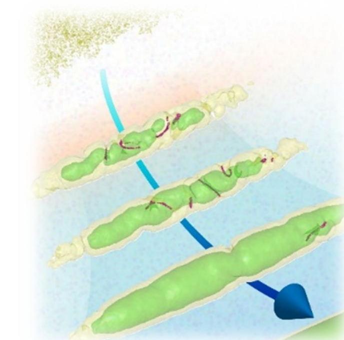


Quantum Gases: Finite Temperature & Dynamics
(World Scientific, 2013)
A methodology book
Edited by
Proukakis, Gardiner, Davis & Szymanska

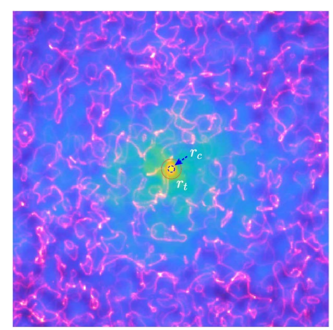


Universal Themes of Bose-Einstein Condensation
(Cambridge University Press, 2017)
BEC in different fields of physics
Edited by
Proukakis, Snoke & Littlewood

Discussed Research Papers:



Comms.Phys. (Nature) 1, 24 (2018)
PRR 2, 033183 (2020)
JPB 53, 115301 (2019)
PRR 3, 013097 (2021)
PRR 013212 (2021)
PRL 121, 095302 (2018)
PRL 125, 095301 (2020)



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