Extreme Black Holes: Anabasis and Accidental Symmetry

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Why extreme black holes?

Two reasons:

- They are observationally relevant:

  Many accreting black holes are found to be spinning very rapidly
- They are theoretically manageable:

  Near the horizon of (near-)extreme black holes spacetime is AdS-like

Rapidly spinning black holes

Many accreting black holes are found to be spinning very rapidly

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Observational Constraints on Black Hole Spin

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Keywords

active galactic nuclei, accretion disks, general relativity, gravitational waves, jets

Abstract

The spin of a black hole is an important quantity to study, providing a window into the processes by which a black hole was born and grew. Furthermore, spin can be a potent energy source for powering relativistic jets and energetic particle acceleration. In this review, I describe the techniques currently used to detect and measure the spins of black holes. It is shown that:

- Two well-understood techniques, X-ray reflection spectroscopy and thermal continuum fitting, can be used to measure the spins of black holes that are accreting at moderate rates. There is a rich set of other electromagnetic techniques allowing us to extend spin measurements to lower accretion rates.
- Many accreting supermassive black holes are found to be rapidly spinning, although a population of more slowly spinning black holes emerges at masses above $M > 3 \times 10^7\,M_\odot$ as expected from recent structure formation models.
- Many accreting stellar-mass black holes in X-ray binary systems are rapidly spinning and must have been born in this state.

Rapidly spinning black holes

Many accreting black holes are found to be spinning very rapidly

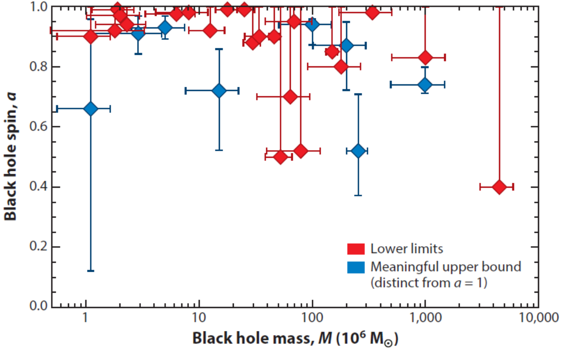

Figure 6

SMBH spins as a function of mass for the 32 objects in **Table 1** that have available mass estimators. All spin measurements reported here are from the X-ray reflection method. Lower limits are reported in red, and measurements that include a meaningful upper bound (distinct from $a = 1$) are reported in blue. Following the convention of the relevant primary literature, error bars in spin show the 90% confidence range. The error bars in mass are the 1σ errors from **Table 1** or, where that is not available, we assume a ±50% error. Abbreviation: SMBH, supermassive black hole.

AdS_2 and near-extreme black holes

Near the horizon of (near-)extreme black holes spacetime is AdS_2-like

Extreme Reissner-Nordstrom; Bertotti-Robinson: [Bertotti, Robinson (1959)]

$$ds^2 = M^2\left[-r^2dt^2 + \frac{dr^2}{r^2} + d\Omega^2\right], \qquad A_t = Mr$$

- Applies for a wide class of theories in any D [Kunduri, Lucietti, Reall (2007)]
    - e.g. extreme Kerr in 4D pure Einstein GR [Bardeen, Horowitz (1999)]
- Near-horizon approximations *and* Exact solutions

1. Anabasis:

Backreaction that destroys the AdS_2 boundary and builds the asymptotically flat region of (near-)extreme BHs.

2012.06562 [JHEP 2103] *with* S. Hadar, A. Lupsasca

"AdS_2 has no dynamics"

Anti-de Sitter fragmentation

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ABSTRACT: Low-energy, near-horizon scaling limits of black holes which lead to string theory on $AdS_2 \times S^2$ are described. Unlike the higher-dimensional cases, in the simplest approach all finite-energy excitations of $AdS_2 \times S^2$ are suppressed. Surviving zero-energy configurations are described. These can include tree-like structures in which the $AdS_2 \times S^2$ throat branches as the horizon is approached, as well as disconnected $AdS_2 \times S^2$ universes. In principle, the black hole entropy counts the quantum ground states on the moduli space of such configurations. In a nonsupersymmetric context AdS_D for general D can be unstable against instanton-mediated fragmentation into disconnected universes. Several examples are given.

KEYWORDS: Black Holes in String Theory, Conformal Field Models in String Theory, Supersymmetry and Duality.

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“AdS_2 has no dynamics”

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Rigidity of asymptotically $AdS_2 \times S^2$ spacetimes

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Abstract

The spacetime $AdS_2 \times S^2$ is well known to arise as the ‘near horizon’ geometry of the extremal Reissner–Nordstrom solution, and for that reason it has been studied in connection with the AdS/CFT correspondence. Here we consider asymptotically $AdS_2 \times S^2$ spacetimes that obey the null energy condition (or a certain averaged version thereof). Supporting a conjectural viewpoint of Juan Maldacena, we show that any such spacetime must have a special geometry similar in various respects to $AdS_2 \times S^2$, and under certain circumstances must be isometric to $AdS_2 \times S^2$.

Wider picture on AdS_2 dynamics

- Backreaction in *asymptotically AdS_2 spacetimes* is problematic.
  - Q: Starting with a linear solution for a scalar ϕ on $AdS_2 \times S^2$, does it extend to a non-linear solution of Einstein-Maxwell-Scalar?
  - A: Not if we insist on an asymptotically AdS_2 solution. E.g. if we impose Dirichlet boundary conditions on the AdS_2 boundary then backreaction of the scalar on the geometry destroys them.
- Backreaction in *asymptotically flat spacetimes* makes perfect sense.
  - Q: Starting with a linear solution for a scalar $\phi \sim \sqrt{\epsilon}$ on ERN, does it extend to a non-linear solution of Einstein-Maxwell-Scalar?
  - A: Yes. Generically the fully backreacted nonlinear endpoint is a near-extreme RN with $Q = M\sqrt{1 - \mathcal{O}(\epsilon)}$. [Murata, Reall, Tanahashi (2013)]

The connection of AdS_2 with the asymptotically flat region of BHs allows for consistent backreaction. How? What are the correct boundary conditions?

Perturbations of Bertotti-Robinson

- Backgound:

$$ds^2 = M^2\left[-r^2 dt^2 + \frac{dr^2}{r^2} + d\Omega^2\right], \qquad A_t = Mr$$

- Spherically symmetric perturbations $(h_{\mu\nu}, a_\mu)$ fully characterized by:

$$h_{\theta\theta} = \Phi_0 + ar + brt + cr\left(t^2 - 1/r^2\right)$$

Comments:

- $h_{\theta\theta}$ is gauge invariant under $h_{\mu\nu} \to h_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu}$.
- 4-parameter (Φ_0, a, b, c) family of solutions.
- Φ_0 parameterizes overall rescaling $M \to M + \delta M$ with $\Phi_0 = 2M\,\delta M$.
- Focus on the remaining triplet:

$$\Phi = ar + brt + cr\left(t^2 - 1/r^2\right)$$

$SL(2)$ transformation properties

$$\Phi = ar + brt + cr\left(t^2 - 1/r^2\right)$$

- The background is invariant under the $SL(2)$ isometries of AdS_2:

$$\begin{aligned}
H:&\quad t \to t + \alpha \\
D:&\quad t \to t/\beta\,, \quad r \to \beta r \\
K:&\quad t \to \frac{t - \gamma\left(t^2 - 1/r^2\right)}{1 - 2\gamma t + \gamma^2\left(t^2 - 1/r^2\right)}\,, \quad r \to r\left[1 - 2\gamma t + \gamma^2\left(t^2 - 1/r^2\right)\right]
\end{aligned}$$

- Φ is $SL(2)$-breaking: (a, b, c) get rotated by the above transformations.
- However,

$$\mu = b^2 - 4ac \qquad \text{is} \quad SL(2)\text{-invariant}$$

- Using $SL(2)$ transformations one may set

$$\begin{aligned}
\Phi &= 2r\,, && \text{when} \quad \mu = 0\,,\ \mathrm{sgn}(a+c) = 1 \\
\Phi &= -\sqrt{\mu}\, rt\,, && \text{when} \quad \mu > 0
\end{aligned}$$

- $SL(2)$-breaking solutions Φ are *not* asymptotically $AdS_2 \times S^2$

Anabasis perturbations

Bertotti-Robinson arises from two physically distinct
near-horizon near-extremality scaling limits, $\lambda \to 0$, of Reissner-Nordstrom

- Limit #1: Begin with $Q = M$ and put the BH horizon at $r = 0$ (set $M = 1$):

$$ds^2 = -\left(\frac{r}{1+\lambda r}\right)^2 dt^2 + \left(\frac{r}{1+\lambda r}\right)^{-2} dr^2 + (1+\lambda r)^2\, d\Omega^2\,, \quad A_t = \frac{r}{1+\lambda r}$$

At $\mathcal{O}(1)$ we get Bertotti-Robinson in Poincare coordinates

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + d\Omega^2\,, \qquad A_t = r$$

At $\mathcal{O}(\lambda)$ we get, by definition, a linear solution around the above.

$$h_{\theta\theta} = 2r$$

This is the $SL(2)$-breaking $\mu = 0$ solution $\Phi = 2r$ —Poincare *anabasis solution*

Begins to build the asymptotically flat region of an extreme Reissner-Nordstrom

The nonlinear solution obtained from the $\mu = 0$ perturbation of $AdS_2 \times S^2$, when backreaction is fully taken into account in the Einstein-Maxwell theory, is the extreme Reissner-Nordström black hole.

Anabasis perturbations

- Limit #2: Begin with $Q = M\sqrt{1-\lambda^2\kappa^2}$ and put the BH horizon at $\rho = 0$:

$$
\begin{aligned}
ds^2 =& -\frac{\rho(\rho+2\kappa+\lambda\kappa\rho)}{(1+\lambda\kappa)(1+\lambda\rho)^2}d\tau^2 + \frac{(1+\lambda\kappa)^3(1+\lambda\rho)^2}{\rho(\rho+2\kappa+\lambda\kappa\rho)}d\rho^2 \\
&+ (1+\lambda\kappa)^2(1+\lambda\rho)^2 d\Omega^2 \\
A_\tau =& \frac{1}{\lambda}\left(1-\sqrt{\frac{1-\lambda\kappa}{1+\lambda\kappa}}\frac{1}{1+\lambda\rho}\right)
\end{aligned}
$$

At $\mathcal{O}(1)$ we get Bertotti-Robinson in Rindler coordinates

$$
ds^2 = -\rho(\rho+2\kappa)d\tau^2 + \frac{d\rho^2}{\rho(\rho+2\kappa)} + d\Omega^2\,, \quad A_\tau = M(\rho+\kappa)
$$

At $\mathcal{O}(\lambda)$ we get, by definition, a linear solution around the above.

$$
h_{\theta\theta} = 2(\rho+\kappa)
$$

Anabasis perturbations

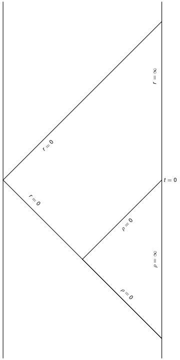
- Rindler to Poincare transformation for the Bertotti-Robinson:

$$\tau = -\frac{1}{2\kappa} \ln \left(t^2 - 1/r^2 \right)$$
$$\rho = -\kappa(1 + rt)$$
$$A \to A + d\Lambda \, , \Lambda = \frac{1}{2} \ln \frac{\rho}{\rho + 2\kappa}$$

Transforms the Rindler anabasis solution to

$$h_{\theta\theta} = 2(\rho + \kappa) = -2\kappa rt$$

This is the $SL(2)$-breaking $\sqrt{\mu} = 2\kappa$ solution $\Phi = -2\kappa rt$.
Begins to build the asymptotically flat region of a near-extreme RN
In general, $\Phi = ar + brt + cr(t^2 - 1/r^2)$ with $\mu > 0$, leads to
Rindler anabasis with $\sqrt{\mu} = \sqrt{b^2 - 4ac} = 2\kappa$



The nonlinear solution obtained from the $\mu > 0$ perturbation of $AdS_2 \times S^2$, when backreaction is fully taken into account in the Einstein-Maxwell theory, is the near-extreme Reissner-Nordström black hole with $Q = M\sqrt{1 - \mu/4}$.

Summary

Anabasis: Backreaction that destroys the AdS_2 boundary and builds the asymptotically flat region of (near-)extreme BHs.

Remarks

- Q: What is the dual of anabasis in AdS/CFT?

  A: Following an inverse RG, from IR to UV, along an irrelevant deformation of the boundary field theory that does *not* respect AdS boundary conditions (e.g. the single-trace $T\overline{T}$ deformation of CFT_2 studied by [Giveon, Itzhaki, Kutasov, et al 2017–])
- Q: What about JT gravity?

  A: $\Phi = \Phi_{JT}$ solves the JT eom $\nabla_\mu \nabla_\nu \Phi_{JT} - g_{\mu\nu} \nabla^2 \Phi_{JT} + g_{\mu\nu} \Phi_{JT} = 0$ on AdS_2.

  $\mu =$ ADM mass of the 2D black holes in JT gravity.

  Connected AdS_2 is a "nearly-AdS_2" with $SL(2)$ broken to maintain connection.

2. Accidental Symmetry:

Coordinate transformation that acts on the perturbative solutions of Einstein equation near extreme black hole horizon

2112.13853 [JHEP 2203] *with* G. Remmen

The linearized Einstein equation

Schematic notation:

- Background geometry $\bar{g}$ —the Bertotti-Robinson spacetime
- Metric perturbation h —the Φ solution
- The linearized Einstein equation as a linear differential operator

$$\mathcal{E}(\bar{g}, h) = 0$$

Consider a finite diffeomorphism

$$(t, r) \to (t, r) + \lambda \left(\xi^t(t, r), \xi^r(t, r)\right)$$

which transforms both $\bar{g} \to \bar{g}(\lambda)$ and $h \to h(\lambda)$.

By general covariance, for *arbitrary* λ and ξ^μ, we have:

$$\mathcal{E}(\bar{g}(\lambda), h(\lambda)) = 0$$

Expanding in λ, we have

$$\mathcal{E}(\bar{g}(0), h(0)) + \lambda \frac{\delta}{\delta\lambda} \mathcal{E}(\bar{g}(\lambda), h(0)) + \lambda \frac{\delta}{\delta\lambda} \mathcal{E}(\bar{g}(0), h(\lambda)) + \mathcal{O}(\lambda^2) = 0$$

Accidental symmetry: definition

Starting with a solution to the linearized Einstein equations around the original background, $\mathcal{E}(\bar{g}(0), h(0)) = 0$, we have

$$\lim_{\lambda\to 0}\left[\partial_\lambda \mathcal{E}(\bar{g}(\lambda), h(0)) + \partial_\lambda \mathcal{E}(\bar{g}(0), h(\lambda))\right] = 0 \tag{1}$$

- 1^{st} term: hold perturbation fixed, act with a linearized diffeo on the background
- 2^{nd} term: on fixed background, transform perturbation using linearized diffeo

Equation (1) is valid for any diffeo, i.e. for any ξ^μ.

What if we impose the strong requirement that each term in (1) vanishes individually?

$$\lim_{\lambda\to 0}\partial_\lambda \mathcal{E}(\bar{g}(0), h(\lambda)) = 0 \tag{2}$$

- Trivial solutions: Isometries of the background $\bar{g}(\lambda) = \bar{g}(0)$
- Other solution: *accidental symmetry* —transforms solns h among themselves

Accidental symmetry: electrovacuum case

$$\begin{aligned} &\mathcal{E} \text{ : linearized Einstein-Maxwell equations (electrovacuum)} \\ &\bar{g}(0) \text{ : Bertotti-Robinson} \\ &h(0) \text{ : } \Phi = ar \;\; (\mu = 0 \text{ solution}) \end{aligned}$$

the solution of $\lim_{\lambda\to 0} \partial_\lambda \mathcal{E}(\bar{g}(0), h(\lambda)) = 0$ is given by

$$\xi = -\left[\epsilon(t) + \frac{\epsilon''(t)}{2r^2} + \frac{t\epsilon'''(t)}{r^2}\right]\partial_t + \left[r\epsilon'(t) - \frac{\epsilon'''(t)}{2r}\right]\partial_r ,$$

where $\epsilon(t)$ is an arbitrary cubic polynomial in t,

$$\epsilon(t) = e_0 + e_1 t + e_2 t^2 + e_3 t^3 .$$

- $\xi_{0,1,2}$: SL(2) Killing vectors of AdS_2

$$\xi_0 = -(1,0)\,, \quad \xi_1 = -(t,-r)\,, \quad \xi_2 = -\left(t^2 + \frac{1}{r^2}, -2rt\right)$$

- ξ_3: non-trivial accidental symmetry

$$\xi_3 = -\left(t^3 + \frac{9t}{r^2}, \frac{3}{r} - 3rt^2\right)$$

Accidental symmetry: electrovacuum equations

Question: What does ξ_3 do?

Answer: Relates $\mu = 0$ to $\mu \neq 0$. Indeed, we have

$$\Delta\mu = -4a\Delta c = -12\lambda e_3 a^2$$

Accidental symmetries enlarge the possible mappings among solutions to include those beyond the SL(2) isometries, thereby allowing to move from one μ orbit to another.

In spherical symmetry the electrovacuum solutions are constrained by Birkhoff's theorem to the non-propagating degrees of freedom that we have discussed so far.

Can accidental symmetries also turn on propagating d.o.f.?

Accidental symmetry: adding matter

$$\lim_{\lambda\to 0}\partial_\lambda \mathcal{E}(\bar{g}(0), h(\lambda)) = T \tag{3}$$

Source T must satisfy equations of motion. We consider Klein-Gordon scalar $\Box\phi = 0$ s.t. the most general spherically symmetric solution is ($u = t - 1/r$, $v = t + 1/r$)

$$\phi = f_+(v) + f_-(u)$$

Can get solution to (3) from the electrovacuum $\Phi = r$ using the transformation

$$\begin{aligned}\xi^t = \; & \frac{3}{2r}[F'_+(v) + F'_-(u)] - \frac{3}{2r^2}[F''_+(v) - F''_-(u)] \\ & + \frac{3}{r^3}\left[\int^v \frac{F_+(t_0)}{(t-t_0)^4}\,\mathrm{d}t_0 + \int^u \frac{F_-(t_0)}{(t-t_0)^4}\,\mathrm{d}t_0\right] \\ & - \frac{1}{r^3}\int^r\int^t \frac{f'_+\left(\hat{t}+\frac{1}{\hat{r}}\right) f'_-\left(\hat{t}-\frac{1}{\hat{r}}\right)}{\hat{r}}\,\mathrm{d}\hat{t}\,\mathrm{d}\hat{r}\end{aligned}$$

$$\xi^r = r[F'_+(v) - F'_-(u)] - [F''_+(v) + F''_-(u)],$$

where $F''''_+(v) = [f'_+(v)]^2$ and $F''''_-(u) = [f'_-(u)]^2$.

Summary

Accidental Symmetry: Coordinate transformation that acts on the perturbative solutions of Einstein equation near extreme black hole horizon and maps them among themselves.

- Electrovacuum eqs: turn on deviation from extremality
- Adding KG matter: turn on arbitrary KG source

Remark

Accidental symmetries are "on-shell large diffeomorphisms of AdS_2"

This is made precise in JT gravity below

Note:

- In AdS/CFT one rarely puts large diffeos on-shell.
- For good reason: main attraction of AdS/CFT is that the gravitational theory in the bulk may be defined from an *independent* prescription of observables on the boundary.

Summary

Putting on-shell the large diffeomorphisms of AdS_2 in JT gravity

- The large diffeomorphisms of AdS_2, in FG gauge, are given by

$$t \to f(t) + \frac{2f''(t)f'(t)^2}{4r^2f'(t)^2 - f''(t)^2}, \qquad r \to \frac{4r^2f'(t)^2 - f''(t)^2}{4rf'(t)^3}$$

$$\mathrm{d}s_2^2 \to -r^2\left(1 + \frac{\mathrm{Sch}(f,t)}{2r^2}\right)^2 \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2} \qquad \text{and} \qquad \Phi \to \phi_0(t)r + \frac{v(t)}{r},$$

  with $\phi_0(t) = [a + bf(t) + cf(t)^2]/f'(t)$ and $v(t) = -[\phi_0''(t) + \mathrm{Sch}(f,t)\phi_0(t)]/2$.

- For *arbitrary* f, this source satisfies the Schwarzian equation of motion

$$\left[\frac{1}{f'}\left(\frac{(f'\phi_0)'}{f'}\right)'\right]' = 0$$

- If one imposes that $\phi_0(t) = \mathrm{constant}$, before as well as after acting with the large diffeo, then for infinitesimal diffeo $f(t) = t + \epsilon(t)$, the Schwarzian eom reduces to

$$\epsilon''''(t) = 0$$

  with its cubic solution $\epsilon(t) = e_0 + e_1t + e_2t^2 + e_3t^3$. ✓

Summary

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Thank you