Damping of Pseudo-Goldstone Fields

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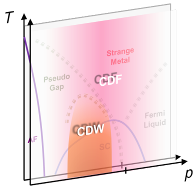
Motivation & preview

- Spontaneous symmetry breaking leads to massless Goldstone fields, included in low-energy effective field theories

- Typically symmetries are **approximate**, and also involve some small explicit breaking $\Rightarrow$ **pinned** Goldstone fields with **small mass**

- Zero temperature EFTs well studied [Weinberg], but crucial to include finite temperature dissipative effects

- Understand the structure of **hydrodynamic EFTs with pseudo-spontaneous symmetry breaking**, particulary systems with broken **translations**



Temperature vs doping
[Arpaia,Ghiringhelli]

- Dynamical charge fluctuations with translational order in phase diagram of cuprate High-T_c Superconductors [Seibold et al.]

Motivation & preview

- Pinning leads to **damping** of Goldstones, i.e. Josephson relation takes the form

$$\boxed{\dot{\phi} = -\mu - \Omega\phi + \cdots}$$

- Typically comes from topological defects, which relax winding of phase [Anderson; Delacrétaz,Goutéraux,Hartnoll,Karlsson]
  - Here focus only on phase relaxation due to **pinning**

Main result

Damping rate $\sim$ (Pinning mass)$^2 \times$ Diffusivity

Motivation & preview

- Initially observed in:
  - Various holographic models of pseudo-spontaneous breaking of translations [Amoretti,Areán,Goutéraux,Musso; Ammon,Baggioli,Jiménez-Alba; Donos,Martin,Pantelidou,VZ]
  - Holographic superfluids [Donos,Kailidis,Pantelidou; Ammon,Aréan,Baggioli,Gray,Grieninger]
  - QCD with approximate chiral symmetry due to quark masses $\Rightarrow$ constraints between pion thermal mass, diffusivity, and relaxation rate [Grossi,Soloviev,Teaney,Yan]
- We now understand it from various points of view
  - **Locality of hydrodynamics**
  - **Schwinger-Keldysh finite temperature EFTs** for hydrodynamics
  - Second law of thermodynamics [Armas,Jain,Lier]
- Not a coincidence or artifact: **consistency of effective field theory**
- Practical application of holography!

Overview

Motivation

Hydrodynamics
- Superfluids

Schwinger-Keldysh EFT
- Holographic construction

Outloook

Hydrodynamics

- Hydrodynamics describes **late-time, long wavelength** behavior of thermalizing systems compared to local equilibration scale $\ell_{th} \sim T^{-1}$ [Landau,Lifshitz; Kovtun]
  - QGP, Heavy-Ion collisions, neutron star mergers, early universe, high-temperature superconductors, strange metals, graphene, charge density waves, Wigner crystals,...

- (i) **Identify slow modes** (conserved charges, Goldstone modes, order parameters near criticality) and corresponding **conservation laws/Josephson-type relations**

$$\dot{n}_a + \nabla j_a = 0$$

- (ii) **Constitutive relations** for currents in **derivative expansion**

$$j_a = \alpha_{ab} n_b + \sigma_{ab} \nabla n_b + \lambda_{ab} \nabla^2 n_b + \cdots$$

with transport coefficients determined by UV theory

Hydrodynamics

- (iii) Equations of motion

$$\dot{n}_a(q,t) + M_{ab}(q) n_b(q,t) = 0$$

- (iv) (Phenomenological) **restrictions** on M:
  - Isotropy, Galilean/Lorentzian boosts,...
  - Time-reversal $\Rightarrow$ Onsager relations $G_{ab}(\omega, q; B) = \eta_a \eta_b G_{ba}(\omega, -q; -B)$
  - Positivity of entropy production $\nabla_\mu J^\mu_S \geq 0$
  - Existence of equilibrium on arbitrary backgrounds
  - ...

  $\Longrightarrow$ (in)equality conditions between transport coefficients

- (v) Retarded Green's functions [Kadanoff,Martin]

$$G_{ab} = M_{ac}(-i\omega + M)^{-1}_{cd} \chi_{db} \, , \qquad \chi_{ab} = -\frac{\delta^2 f}{\delta \mu_a \delta \mu_b}$$

- Physical modes correspond to poles of Green's functions

$$\det(-i\omega + M) = 0$$

Locality of hydrodynamics

- Introducing external sources deforms the Hamiltonian

$$\delta H(t) = -\delta\mu_a(q,t) n_a(-q,t)$$

leads to modified equations of motion

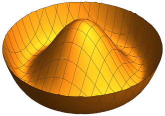
$$\dot{n}_a(q,t) + M_{ab}(q)\left[n_b(q,t) - \chi_{bc}(q)\delta\mu_c(q,t)\right] = 0$$

- Usual restrictions on M, χ
- ... but also **locality**!

- For (pseudo-)Goldstone modes, thermal correlation length ξ is (parametrically) large, so locality of $M \cdot \chi$ is **not automatic** and leads to **constrains on transport coefficients**

Superfluid hydrodynamics

- **Isolate condensate** $\Rightarrow$ Hydrodynamic dofs: $U(1)$ charge density n, conjugate phase (Goldstone) ϕ



- ϕ shifts under the symmetry $\Rightarrow$ only **gradients** appear in f

$$f = \frac{f_s}{2}(\nabla\phi)^2 - \frac{\chi_{nn}}{2}\delta\mu^2 + \cdots$$

  - Constitutive relation

$$j = f_s \nabla\phi - D_n \nabla n$$

  - Current conservation & Josephson relation

$$\dot{n} + \nabla \cdot j = 0\,, \qquad \dot{\phi} = -\frac{1}{\chi_{nn}} n + D_\phi \nabla^2 \phi$$

  - Read off $M \cdot \chi \Longrightarrow$ **local**

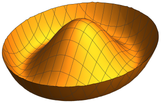
$$M \cdot \chi = \begin{pmatrix} \chi_{nn} D_n q^2 & -1 \\ 1 & D_\phi / f_s \end{pmatrix}$$

  - Second sound mode

$$\omega = \pm c_s q - \frac{i}{2}(D_n + D_\phi) q^2\,, \qquad c_s^2 = \frac{f_s}{\chi_{nn}}$$

Pinning the Goldstone field

- **Break the symmetry weakly** $\Longrightarrow$ (lower-gradient **mass**) term breaking shift symmetry and introducing new length scale $1/q_o$



$$f = \frac{f_s}{2}[(\nabla\phi)^2 + q_o^2\phi^2] - \frac{\chi_{nn}}{2}\delta\mu^2 + \cdots$$

  - Susceptibility matrix becomes

$$\chi(q) \simeq \begin{pmatrix} \chi_{nn} & 0 \\ 0 & \frac{1}{f_s(q^2+q_o^2)} \end{pmatrix}$$

  - Charge conservation is also **weakly broken**

$$\dot{n} + \nabla \cdot j = -\Gamma n + f_s q_o^2 \phi + \cdots$$

  - Josephson relation gets **phase relaxation** term

$$\dot{\phi} \simeq -\Omega\phi - \frac{1}{\chi_{nn}} n + D_\phi \nabla^2\phi + \cdots$$

Pinning the Goldstone field

- Now $M \cdot \chi$ is generically **not local**

$$M \cdot \chi \simeq \begin{pmatrix} \chi_{nn}(\Gamma + D_n q^2) & -1 \\ 1 & \frac{\Omega + D_\phi q^2}{f_s(q_o^2 + q^2)} \end{pmatrix}$$

- Locality only **restored** if the transport parameters satisfy

$$\boxed{\Omega \simeq q_o^2 D_\phi}$$

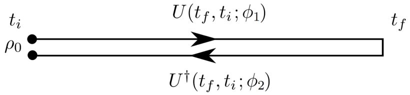
- Sound mode acquires gap and resonance

$$\omega = \pm c_s q_o - \frac{i}{2}(\Gamma + \Omega) + \cdots = \pm c_s q_o - \frac{i}{2}\left(\Gamma + q_o^2 D_\phi\right) + \cdots$$

- **Applications**: QCD, nematic/hexatic liquid crystals, (anti-)ferromagnets, Wigner crystal/Charge density waves, Strange metallic transport, ...

Schwinger-Keldysh EFT

- **Action principle** for hydrodynamics on **SK closed time path contour**
  [Crossley,Glorioso,Liu; Haehl,Loganayagam,Rangamani; Liu,Glorioso]



- Compute *n*-point correlators from generating functional

$$e^{W[s_1,s_2]} = \int D\psi_1 D\psi_2 e^{iS[\psi_1,s_1]-iS[\psi_2,s_2]} \simeq \int D\phi_1 D\phi_2 e^{iS_{EFT}[\phi_1,s_1,\phi_2,s_2]}$$

- **Doubling** of the fields $\Rightarrow$ ϕ_r: **physical**, ϕ_a: **stochastic**

$$\phi_r = \frac{1}{2}(\phi_1 + \phi_2) \qquad \phi_a = \phi_1 - \phi_2$$

Schwinger-Keldysh EFT

Systematic procedure for constructing S_{EFT}: [Liu,Glorioso]

- (i) **Identify** the low-energy dynamical **dofs** and write down the effective action in **derivative expansion**
- (ii) Impose **unitarity constraints**
  - $S^*_{EFT}[\phi_r, \phi_a] = -S_{EFT}[\phi_r, -\phi_a]$
  - $\mathrm{Im} S_{EFT} \geq 0$
  - $S_{EFT}[\phi_r, \phi_a = 0] = 0$
- (iii) Impose **dynamical KMS condition**
  - $S_{EFT}[\phi_r, \phi_a] = S_{EFT}[\Theta\phi_r, \Theta\phi_a + i\Theta\beta\dot{\phi}_r]$

Symmetry principles $\Rightarrow$ Inclusion of stochastic fluctuations, interactions between hydro modes, manifestly local and consistent, implies Onsager relations and 2nd law, ...

- Beyond linear response, there exist stochastic coefficients appearing in higher-point correlators and loop corrections invisible to classical hydrodynamics [Jain,Kovtun]

Schwinger-Keldysh EFT

Write down **superfluid SK effective action**:

- Shift symmetry $\Rightarrow \phi_{a,r}$ enter with derivatives

$$S_{eff} = \chi_{nn} \int \left(\dot{\phi}_a \dot{\phi}_r - c_s^2 \nabla \phi_a \nabla \phi_r \right) + \left(D_n \nabla^2 \phi_a \dot{\phi}_r + \frac{D_\phi}{c_s^2} \ddot{\phi}_a \dot{\phi}_r \right) + \mathcal{O}(\phi_a^2, \partial^4)$$

- Current conservation derived as **eom** of S_{eff}

$$J^\mu \equiv \frac{\delta S_{eff}}{\delta(\partial_\mu \phi_a)} \qquad\qquad \partial_\mu J^\mu = 0$$

- Retarded Green's function

$$G^R_{\phi\phi} \sim \left(c_s^2 q^2 - \omega^2 - i D_n \omega q^2 - i \frac{D_\phi}{c_s^2} \omega^3 \right)^{-1}$$

Schwinger-Keldysh EFT

Now **explicitly break** the symmetry

- We can write **only two** new terms

$$\delta S_{eff} = -\chi_{nn} \int q_o^2 c_s^2 \phi_a \phi_r + \Gamma \phi_a \dot{\phi}_r + \cdots$$

giving the pinned sound mode

$$G^R_{\phi\phi}(\omega, q = 0) \sim \left(q_o^2 c_s^2 - \omega^2 - i\omega\Gamma - i\frac{D_\phi}{c_s^2}\omega^3 \right)^{-1}$$

- Note that SK effective action **manifestly local up to hydrodynamic UV cutoff!**

  - Coupling to external gauge fields is ambiguous when the symmetry is not exact $\Rightarrow$ further coefficients which do not affect linearized modes, but may affect Green's functions [Armas,Jain,Lier; Delacrétaz,Goutéraux,VZ]

Holographic construction

- Bulk Maxwell field $\Leftrightarrow$ dual QFT with conserved $U(1)$ current

$$\partial_\mu j^\mu = 0$$

- Softly **break bulk gauge symmetry** $\Leftrightarrow$ QFT with charge relaxation

$$\partial_\mu j^\mu \simeq -\Gamma n$$

$\Rightarrow$ We consider bulk **Proca theory** [ongoing work w/ Baggioli,Bu]

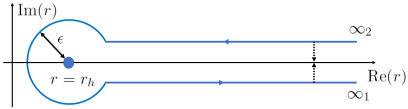
$$S_{\text{bulk}} = -\int d^5x\sqrt{-g}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu\right]$$

Holography for broken $U(1)$ symmetry

- Schwarzschild-AdS bulk geometry $\Leftrightarrow$ neutral, thermal state in QFT

$$ds^2 = 2drdv - r^2\left(1 - r_h^4/r^4\right)dv^2 + r^2\delta_{ij}dx^i dx^j$$

- Holographic prescription for SK contour [Skenderis,van Rees; de Boer,Heller,Pinzani-Fokeeva]
  - **Complexify radial coordinate** and **analytically continue** around the horizon [Glorioso,Crossley,Liu]



- We then (partially) solve the bulk eoms, and derive the **finite temperature SK action** for the hydrodynamics of broken $U(1)$ symmetry

- Transport coefficients given by **horizon quantities** $\Rightarrow$ horizon encodes dissipation [Kovtun,Son,Starinets; Iqbal,Liu; Donos,Gauntlett]

Outlook

Future directions:

- **Holographic superfluids**, with charge and phase relaxation
- **Beyond linear response** with SK action
- Explore consequences of locality for
  - **order parameter** fluctuations near phase transitions
  - **higher-form symmetries**
- (Pseudo-)spontaneous **translational symmetry breaking**
- Include **dynamical topological defects**
- Implications for **strange metal phenomenology**

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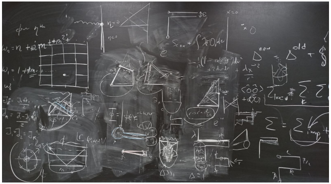
Thank You!

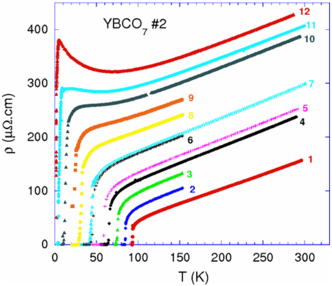


- Expect diffusivities in strongly-correlated materials to saturate **Planckian bound** [Hartnoll]

$$D \simeq \frac{\hbar}{k_B T} c_s^2$$

- Resistivity for CDWs

$$\rho_{\rm dc} = \frac{m^\star}{ne^2}\left(\Gamma_\pi + \frac{q_o^2 c_s^2}{\Omega}\right) \simeq \frac{m^\star}{ne^2}\left(\Gamma_\pi + \frac{k_B T}{\hbar}\right)$$



Resistivity vs temperature for irradiated single crystal $YBCO_7$ [Rullier-Albenque,Alloul,Tourbot]

- Γ_π from conventional scattering (Umklapp, disorder, el-ph interactions)
- Slope of the linear term **independent** from disorder
- Natural mechanism for T-**linear** resistivity