Gravitational Effective Field Theory Islands and the Four-Graviton Amplitude

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2103.12728 & 2205.01655 & WiP

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Gravitational theories beyond General Relativity

What is the complete theory of gravity that describes the universe?

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Modest step towards the answer:

Set of low-energy theories that could originate in complete gravitational theories

[Adams, Arkani-Hamed, Bellazzini, Camanho, Caron-Huot, Cheung, Chiang, de Rham, Dubovsky, Edelstein, Huang, Huang, Li, Maldecena, Melville, Miró, Nicolis, Rastelli, Rattazzi, Remmen, Riembau, Riva, Rodina, Simmons-Duffin, Tolley, Van Duong, Weng, Zhiboedov, Zhou, ...]

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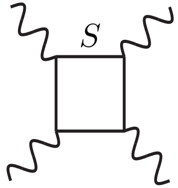
Parallel modest step towards the answer:

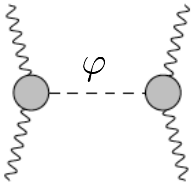
Study of examples of low-energy theories that originate in healthy gravitational theories

- String theory at tree level
- Minimally coupled massive spinning matter at loop level (New!)

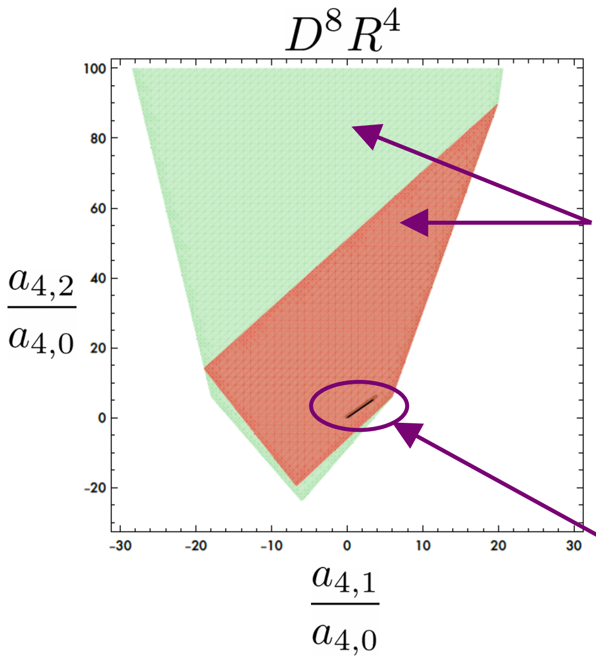
$$\left(S = 0,\ \frac{1}{2},\ 1,\ \frac{3}{2},\ 2\right)$$



- Supersymmetric field theory in the confined phase (New!)



Preview of Results



$a_{k,j} \leftrightarrow$ Wilson coefficients of low energy gravitational EFT

Green region: unitarity, causality, Lorentz invariance

Red region: unitarity, causality, Lorentz inv. and crossing

[Arkani-Hamed, Huang, Huang]

Black region: theory island

[Bern, DK, Zhiboedov; Bern, Herrmann, DK, Roiban]

Known examples populate tiny islands

Outline

- Bounds on low-energy effective field theories (EFTs)
- Calculation of explicit examples of gravitational EFTs
- Study of tiny islands in allowed regions
- Recent progress: Better bounds and higher dimensions
- Outlook

Low-Energy Gravitational EFT

Parametrize low-energy gravitational EFT:

$$S_{\rm EFT} = \int d^4x\sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{2\beta_\phi}{\kappa^2}\phi\, C + \frac{8}{\kappa^3}\frac{\beta_{R^3}}{3!}R^3 + \frac{2\beta_{R^4}}{\kappa^4}C^2 + \frac{2\tilde{\beta}_{R^4}}{\kappa^4}\tilde{C}^2 + \dots\right]$$

$$R^3 \equiv R^{\mu\nu\kappa\lambda}R_{\kappa\lambda\alpha\gamma}R^{\alpha\gamma}{}_{\mu\nu}\,, \qquad C \equiv R^{\mu\nu\kappa\lambda}R_{\mu\nu\kappa\lambda}\,, \qquad \tilde{C} \equiv \frac{1}{2}R^{\mu\nu\alpha\beta}\epsilon_{\alpha\beta}{}^{\gamma\delta}R_{\gamma\delta\mu\nu}$$

We obtain bounds via the 2 → 2 scattering amplitude

- Physical principles naturally encoded in amplitude
- No gauge/field-basis dependence

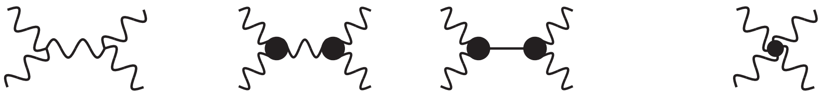
We want bounds on the Wilson coefficients β_i

Low-Energy Gravitational Amplitude

$$S_{\mathrm{EFT}} = \int d^4x\sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{2\beta_\phi}{\kappa^2}\phi\, C + \frac{8}{\kappa^3}\frac{\beta_{R^3}}{3!}\,R^3 + \frac{2\beta_{R^4}}{\kappa^4}C^2 + \frac{2\tilde{\beta}_{R^4}}{\kappa^4}\tilde{C}^2 + \ldots\right]$$

$$\mathcal{M}(1^-,2^-,3^+,4^+) = (\langle 12\rangle[34])^4\ f(t,u)$$

$$f(t,u) = \left(\frac{\kappa}{2}\right)^2\frac{1}{stu} + (\beta_{R^3})^2\frac{tu}{s} - (\beta_\phi)^2\frac{1}{s} + \sum_{k\geq j\geq 0} a_{k,j}\,s^{k-j}t^j$$



Bounds on $a_{k,j} \Leftrightarrow$ *bounds on the Wilson coefficients* β_i

$\langle 12\rangle \sim [34] \sim \sqrt{s}$

High-Energy Amplitude

In the center-of-mass frame:

[Arkani-Hamed, Huang, Huang; Hebbar, Karateev, Penedones]

$$\mathcal{M}(1^{h_1}, 2^{h_2}, 3^{h_3}, 4^{h_4}) = \sum_{J=0}^{\infty} f_J^{h_{12},h_{34}}(s)\, d^J_{h_{12},h_{34}}(\cos\theta)\,, \quad h_{ij} = h_i - h_j$$

Form of amplitude fixed by Lorentz invariance and unitarity

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Kinematics: Known polynomials,
fixed by Lorentz invariance

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Kinematics: Known polynomials, fixed by Lorentz invariance

Dynamics: Unknown

Optical Theorem (Unitarity): $\rho_J^{h_{12},h_{34}}(s) = \mathrm{Im} f_J^{h_{12},h_{34}}(s) \geq 0\,, \quad \text{if} \quad h_{12} = h_{34}$

Form of amplitude fixed by Lorentz invariance and unitarity

Matching Low-Energy and High-Energy Amplitudes

Low-energy amplitude:

$$\mathcal{M}(1^-,2^-,3^+,4^+) = (\langle 12\rangle[34])^4\ f(t,u)$$

$$f(t,u) = \left(\frac{\kappa}{2}\right)^2 \frac{1}{stu} + (\beta_{R^3})^2 \frac{tu}{s} - (\beta_\phi)^2 \frac{1}{s} + \sum_{k\geq j\geq 0} a_{k,j} s^{k-j} t^j$$

High-energy amplitude:

$$\mathcal{M}(1^-,2^-,3^+,4^+) = \sum_{J=0}^{\infty} f_J^{++}(s)\, d_{00}^J(1+\frac{2t}{s})\,, \quad \rho_J^{++} = \mathrm{Im} f_J^{++} \geq 0$$

$$\mathcal{M}(1^-,2^-,3^+,4^+) = \sum_{J=4}^{\infty} f_J^{+-}(u)\, d_{44}^J(1+\frac{2t}{u})\,, \quad \rho_J^{+-} = \mathrm{Im} f_J^{+-} \geq 0$$

Wilson coefficients in terms of spectral densities

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$$\boxed{a_{k,j} = \sum_{J \text{ even}} \langle \rho_J^{++}\rangle_k P_{++}^{j}(\mathcal{J}^2) + \sum_{J=4}^{\infty} \langle \rho_J^{+-}\rangle_k P_{+-}^{k,j}(\mathcal{J}^2)\,, \quad \mathcal{J}^2 = J(J+1)}$$

$$\langle \rho_J \rangle_k \equiv \frac{1}{\pi} \int_{m_{gap}^2}^{\infty} \frac{dm^2}{m^{2k+10}} \rho_J(m^2)$$

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Dynamics: Unknown, but positive

Wilson coefficients in terms of spectral densities

Obtaining Bounds

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- Starting point for obtaining bounds
- Numerical approaches:

  [Caron-Huot, Van Duong; Caron-Huot, Mazac, Rastelli, Simmons-Duffin; …]

- Analytical approaches:

  [Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi; Bellazzini, Miró, Rattazzi, Riembau, Riva; Arkani-Hamed, Huang, Huang; …]

We obtain analytical bounds on the Wilson coefficients

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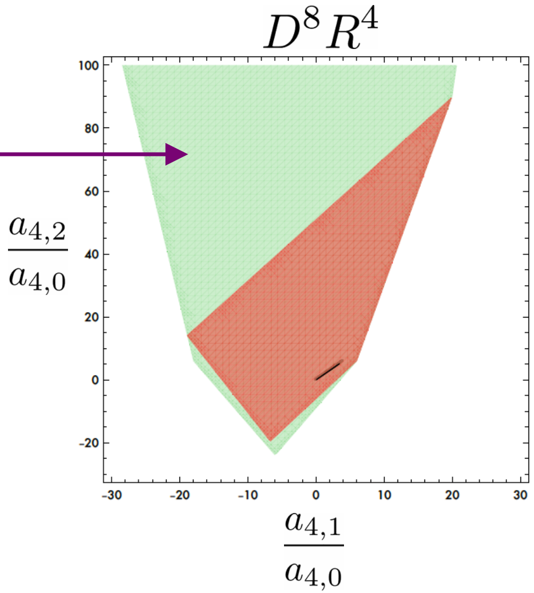

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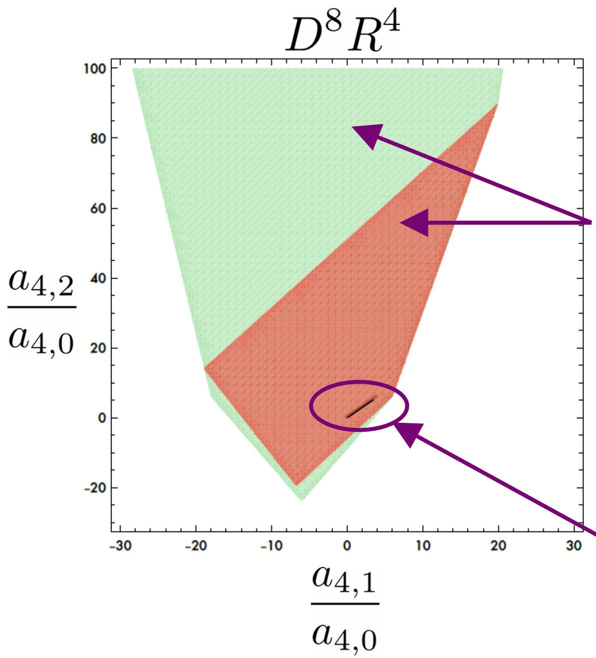
Crossing



$a_{k,j} \leftrightarrow$ Wilson coefficients of low energy gravitational EFT

Green region: unitarity, causality, Lorentz invariance ✔

Red region: unitarity, causality, Lorentz inv. and crossing ✔

[Arkani-Hamed, Huang, Huang]

Black region: theory island

[Bern, DK, Zhiboedov; Bern, Herrmann, DK, Roiban]

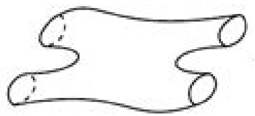
Crossing: $a_{4,3} = 2(a_{4,2} - a_{4,1})\,, \quad a_{4,4} = a_{4,2} - a_{4,1}$

[Caron-Huot, Van Duong]

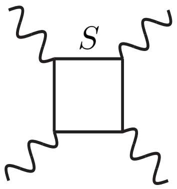
Crossing further constrains allowed region

Theoretical Data

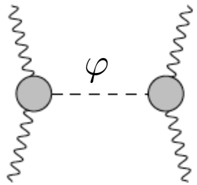
- String theory (tree level):
  - Superstring
  - Heterotic String
  - Bosonic String



- Field theory (one loop):
  - Minimally coupled massive matter (New!) $\left(S = 0,\ \frac{1}{2},\ 1,\ \frac{3}{2},\ 2\right)$



- Field theory in the confined phase (non perturbative):
  - $\mathcal{N} = 1$ supersymmetric gauge theory bellow confinement scale (New!)



String Theory Amplitudes

$$\mathcal{M}^{(\mathrm{st})}(1^-,2^-,3^+,4^+) = \left(\frac{\kappa}{2}\right)^2 (\langle 12\rangle[34])^4 f^{(\mathrm{st})}(t,u)\,, \quad \alpha' = 4$$

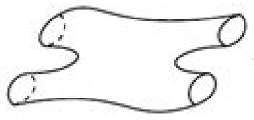
$$f^{(\mathrm{ss})}(t,u) = -\frac{\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)}$$
$$f^{(\mathrm{hs})}(t,u) = -\frac{\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)}\left(1-\frac{tu}{s+1}\right)$$
$$f^{(\mathrm{bs})}(t,u) = -\frac{\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)}\left(1-\frac{tu}{s+1}\right)^2$$

st: String Theory
ss: Superstring
hs: Heterotic String
bs: Bosonic String

Low-energy expansion example: $$f^{(\mathrm{ss})}(t,u) = \frac{1}{stu} + 2\zeta_3 + 2\zeta_5(t^2+u^2+tu) + \ldots$$

Amplitude coefficients $a_{k,j}$

Ex: $a_{0,0}^{(\mathrm{ss})} = 2\zeta_3$



Gravitational EFT data from String Theory

Amplitudes Methods for Loop Calculations

- Double Copy: Gravity = (Gauge theory) x (Gauge theory)

  [Kawai, Lewellen, Tye; Bern, Carrasco, Johansson]

- Generalized Unitarity: Tree amplitudes → Loop integrand

  [Bern, Dixon, Dunbar, Kosower]

- Supersymmetric decomposition: Break problem into simpler pieces

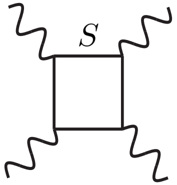
  [Bern, Dixon, Kosower; Dunbar, Norridge; Bern, Morgan]

- Integration-by-parts identities: Complicated → Simple loop integrands

  [Passarino, Veltman; Smirnov; Smirnov, Chuharev]

- Dimension shifting: Manifest simple structure

  [Bern, Dixon, Kosower]



Field Theory One Loop Amplitudes

$$\mathcal{M}^{\{S\}}(1^-,2^-,3^+,4^+) = \mathcal{K}\left(\langle 12\rangle[34]\right)^4 f^{\{S\}}(t,u)\,, \quad \mathcal{K} = \frac{1}{(4\pi)^2}\left(\frac{\kappa}{2}\right)^4$$

[Bern, DK, Zhiboedov]

Particle in the loop

$$f^{\{0\}} = \frac{1}{6300m^4} + \frac{s}{41580m^6} + \frac{81(t^2+u^2)+155tu}{15135120m^8} + \frac{\left(161(t^2+u^2)+324tu\right)s}{151351200m^{10}} + \cdots$$

$$f^{\{1/2\}} = \frac{1}{1120m^4} + \frac{s}{8400m^6} + \frac{15(t^2+u^2)+28tu}{554400m^8} + \frac{\left(153(t^2+u^2)+313tu\right)s}{30270240m^{10}} + \cdots$$

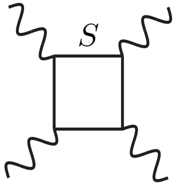
$$f^{\{1\}} = \frac{1}{180m^4} + \frac{s}{1680m^6} + \frac{22(t^2+u^2)+39tu}{151200m^8} + \frac{\left(20(t^2+u^2)+43tu\right)s}{831600m^{10}} + \cdots$$

$$f^{\{3/2\}} = \frac{1}{24m^4} + \frac{s}{360m^6} + \frac{9(t^2+u^2)+14tu}{10080m^8} + \frac{\left(8(t^2+u^2)+21tu\right)s}{75600m^{10}} + \cdots$$

$$f^{\{2\}} = \frac{1}{2m^4} + \frac{t^2+tu+u^2}{120m^8} + \frac{stu}{504m^{10}} + \cdots$$

Amplitude coefficients $\quad a_{k,j}$

Ex: $\quad a^{\{0\}}_{0,0} = \dfrac{1}{6300m^4}$



Gravitational EFT data from minimally coupled particles

Non Perturbative Field Theory Amplitudes

[Bern, Herrmann, DK, Roiban]

Energy

$\mathcal{N}=1$ gauge theory minimally coupled to
$\mathcal{N}=1$ gravity

Λ Confinement

m_S Glueball: Lightest massive state

$\mathcal{N}=1$ gravitational EFT

Gravitational EFT data from a non-perturbative model

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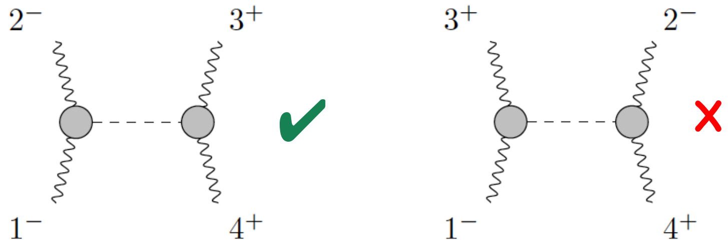
Glueball coupling to gravitons



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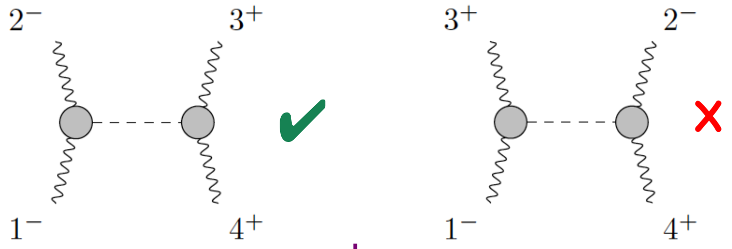

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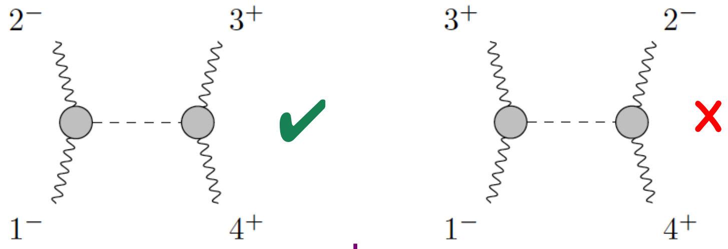

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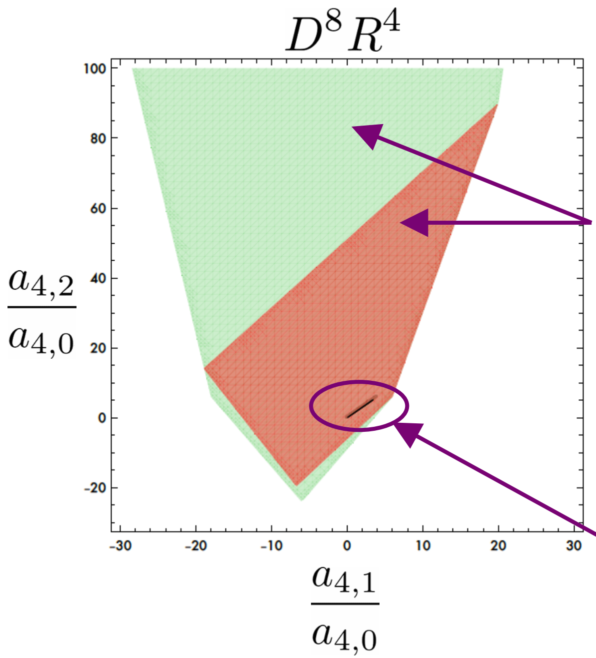
$$\Rightarrow \frac{a_{k,j}}{a_{k,0}} = 0\,, \quad j>0$$

Gravitational EFT data from a non-perturbative model

Theory Island



$a_{k,j} \leftrightarrow$ Wilson coefficients of low energy gravitational EFT

Green region: unitarity, causality, Lorentz invariance ✔

Red region: unitarity, causality, Lorentz inv. and crossing ✔

[Arkani-Hamed, Huang, Huang]

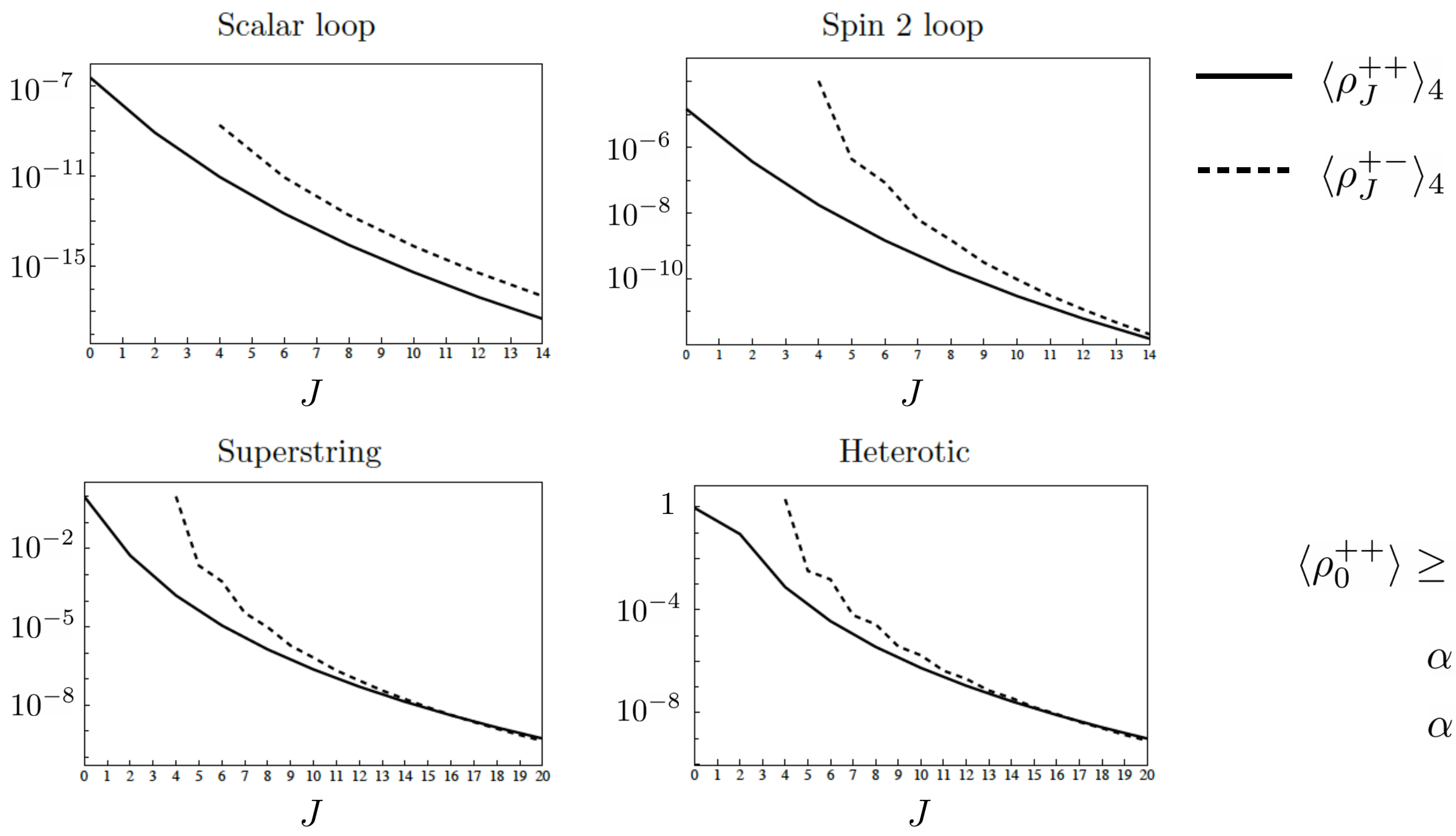
Black region: theory island ✔

[Bern, DK, Zhiboedov; Bern, Herrmann, DK, Roiban]

$$\mathcal{M}_{\text{theory}} = c_{(\text{ss})}\mathcal{M}^{(\text{ss})}_{\text{tree}} + c_{(\text{hs})}\mathcal{M}^{(\text{hs})}_{\text{tree}} + c_{(\text{bs})}\mathcal{M}^{(\text{bs})}_{\text{tree}} + \sum_{S=0,\frac{1}{2},1,\frac{3}{2},2} c_S \mathcal{M}_S^{1-\text{loop}} + c_{\text{NP}}\mathcal{M}^{\text{NP}}\,, \quad c_i \geq 0$$

Theory island = arbitrary combination of known theories

Spectral densities



Input so far: $\langle \rho_J \rangle \geq 0$

Observation:

$$\langle \rho_0^{++} \rangle \geq \alpha \langle \rho_{J>0}^{++} \rangle, \quad \langle \rho_4^{+-} \rangle \geq \alpha \langle \rho_{J>4}^{+-} \rangle$$

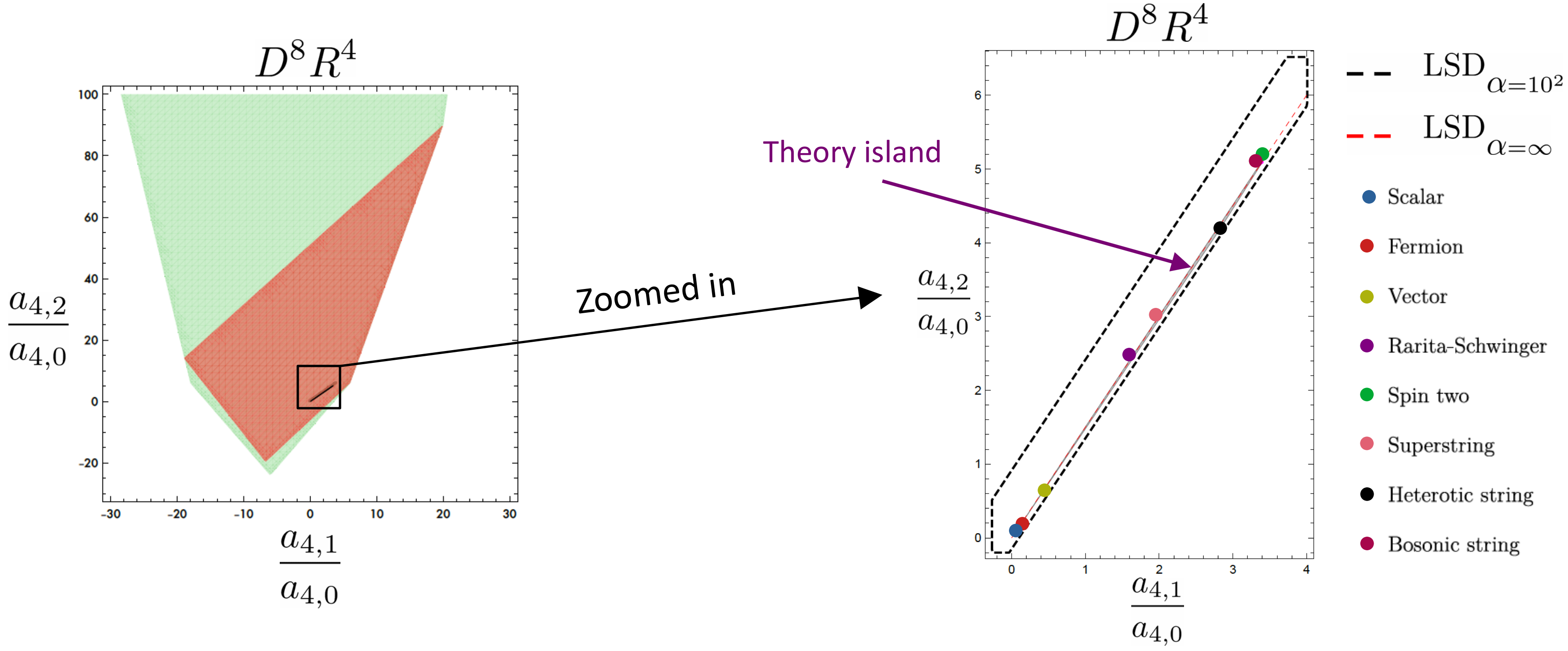
$$\alpha \sim 10\,, \quad \text{Heterotic String}$$

$$\alpha \sim 100\,, \quad \text{All other theories}$$

Spectral densities highly restricted

[Bern, DK, Zhiboedov]

Low-Spin Dominance



Additional qualitative assumption – Low-Spin Dominance: $\langle \rho_0^{++} \rangle \geq \alpha \langle \rho_{J>0}^{++} \rangle, \quad \langle \rho_4^{+-} \rangle \geq \alpha \langle \rho_{J>4}^{+-} \rangle$

(Red → Black) from Low-Spin Dominance. Underlying physical principle?

Low-Spin Dominance bounds capture the data

[Bern, DK, Zhiboedov]

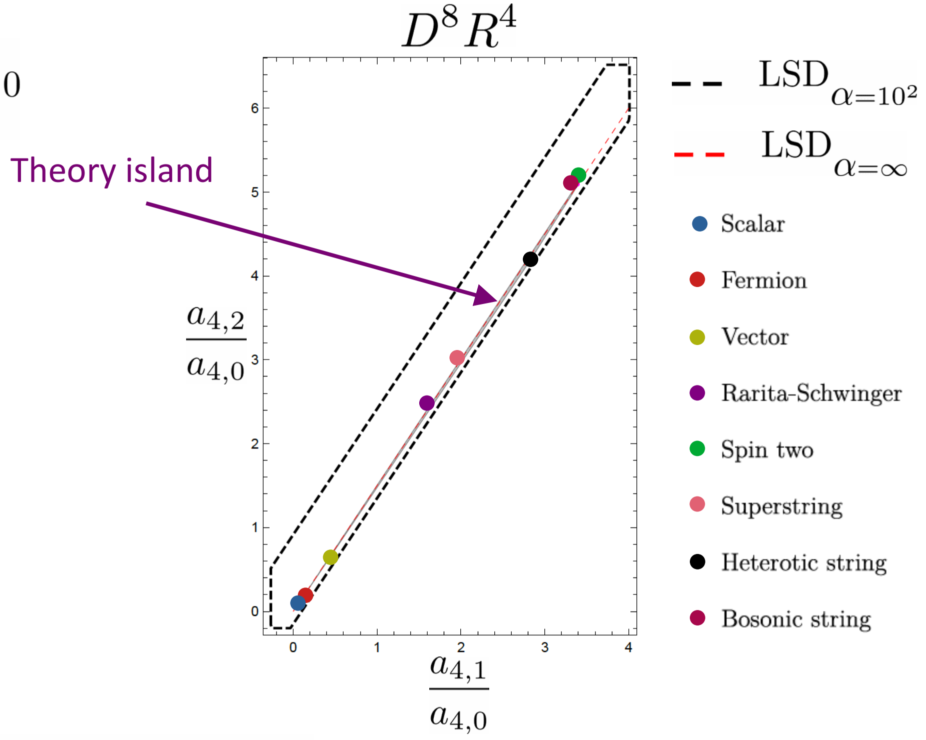
Position of the Theory Island

$$a_{k,j} = \sum_{J \text{ even}} \langle \rho_J^{++} \rangle_k P^j_{++}(\mathcal{J}^2) + \sum_{J=4}^{\infty} \langle \rho_J^{+-} \rangle_k P^{k,j}_{+-}(\mathcal{J}^2)\,, \quad \langle \rho_J \rangle_k \geq 0$$

$$a_{4,2} = 6\langle \rho_4^{+-} \rangle_4 + \text{higher spin}$$

$$a_{4,1} = 4\langle \rho_4^{+-} \rangle_4 + \text{higher spin}$$

$$a_{4,0} = \langle \rho_0^{++} \rangle_4 + \langle \rho_4^{+-} \rangle_4 + \text{higher spin}$$



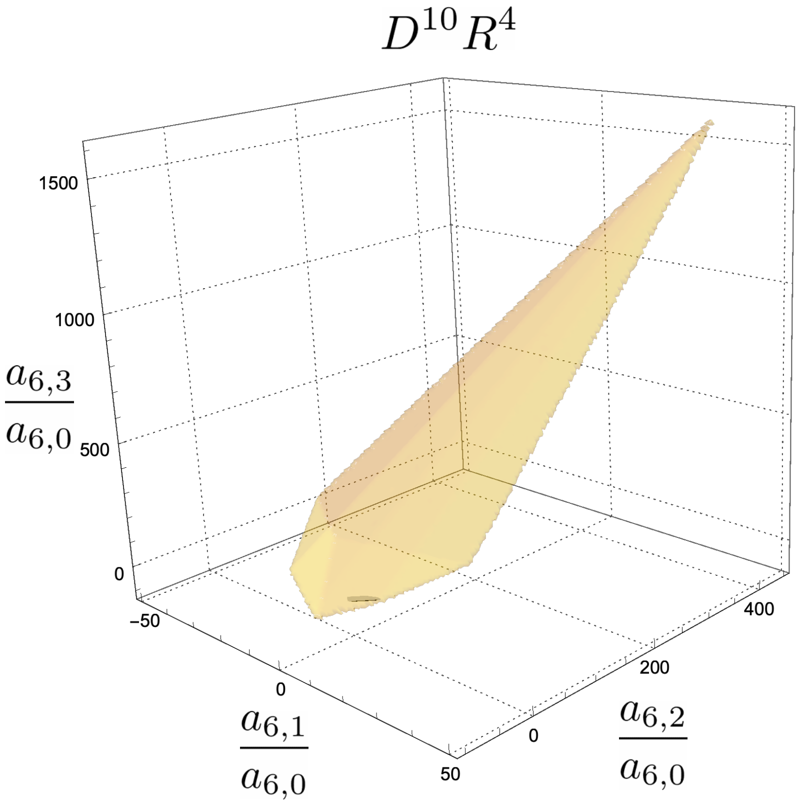
Theory island well approximated by:

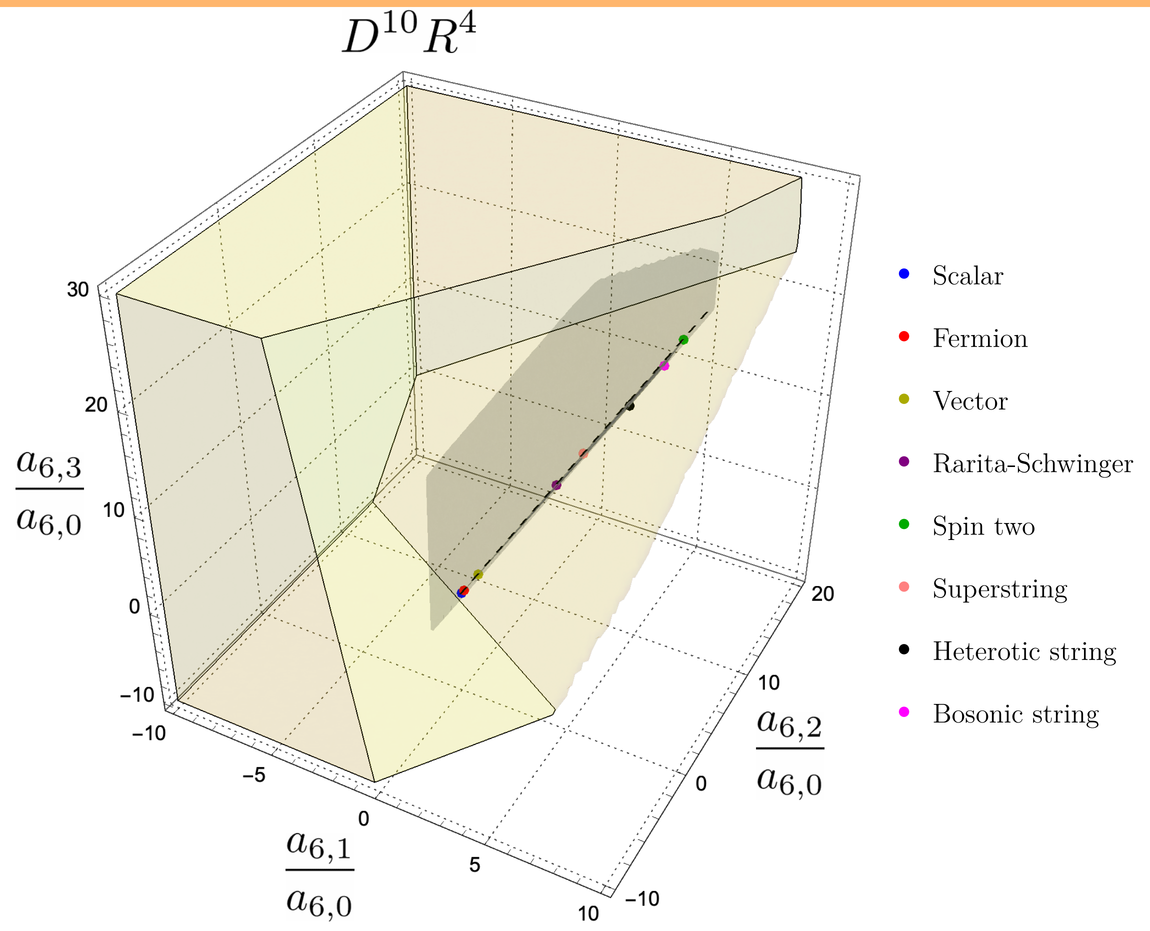
$$\frac{a_{4,2}}{a_{4,0}} = \frac{3}{2}\frac{a_{4,1}}{a_{4,0}}\,, \quad 0 \leq \frac{a_{4,1}}{a_{4,0}} \leq 4$$

Low-Spin Dominance predicts position of theory island

[Bern, DK, Zhiboedov]

A Different Example





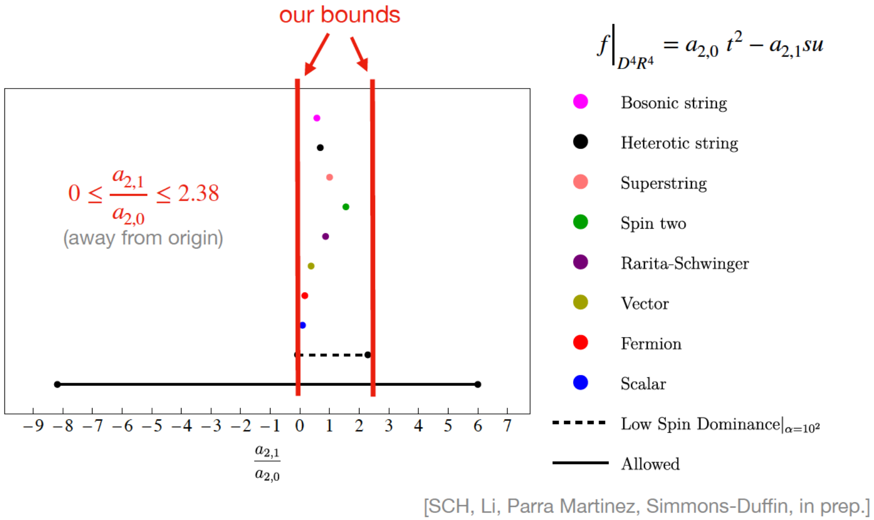
Shaded region: $\alpha = 10^2$

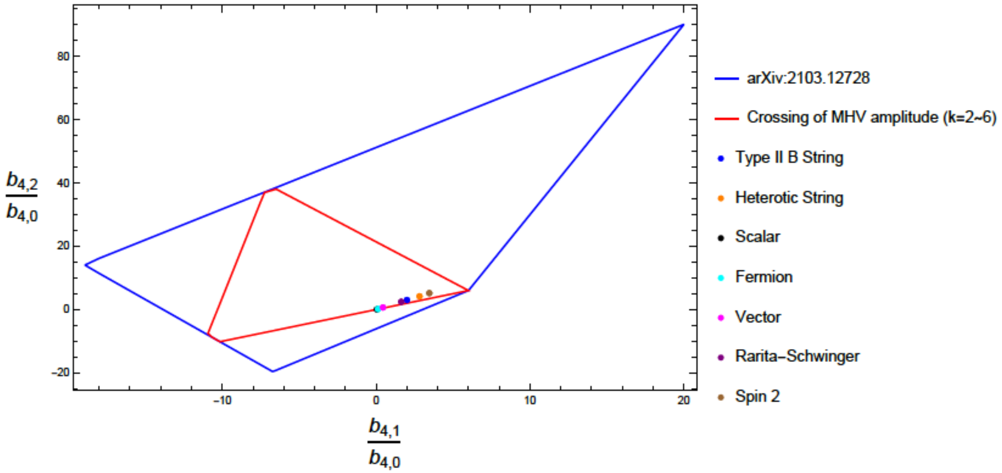
Dashed line: $\alpha = \infty$

All examples share same qualitative features

[Bern, DK, Zhiboedov; Bern, Herrmann, DK, Roiban]

Sharper bounds



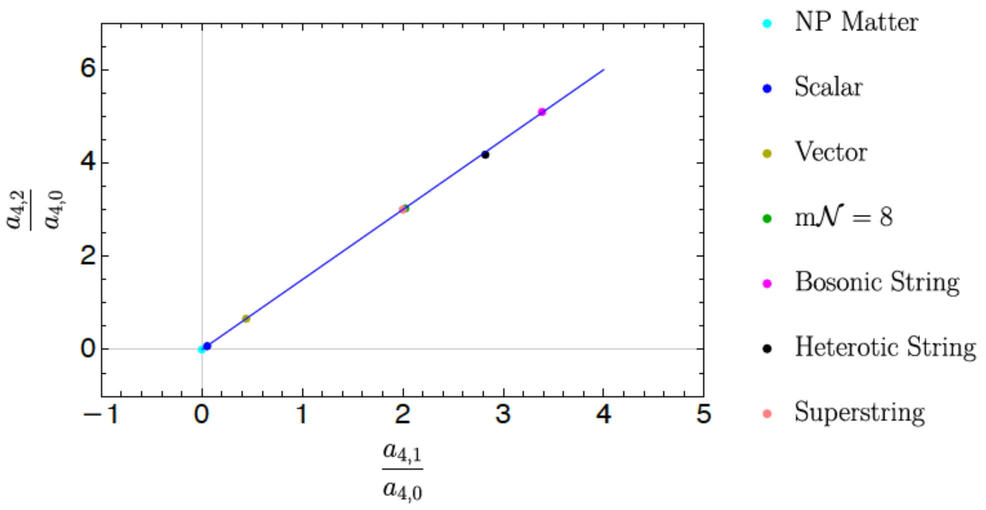


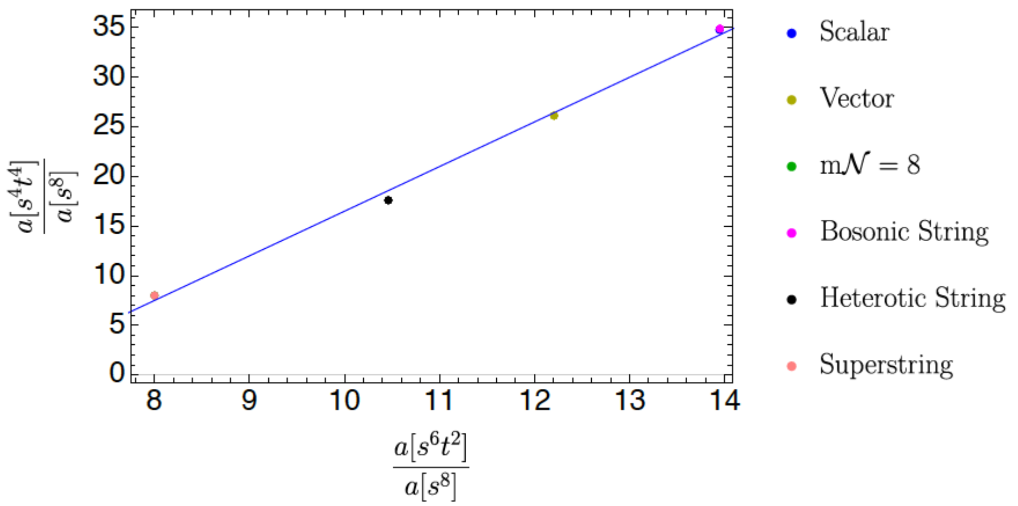
[Caron-Huot @ QCD Meets Gravity 2021]
Original plot: [Bern, DK, Zhiboedov]

[Chiang, Huang, Li, Rodina, Weng]

Theory island still smaller than state-of-the-art regions

Islands in $D \geq 4$





$$\mathcal{M}(1_{4^-}, 2_{4^-}, 3_{4^+}, 4_{4^+})$$

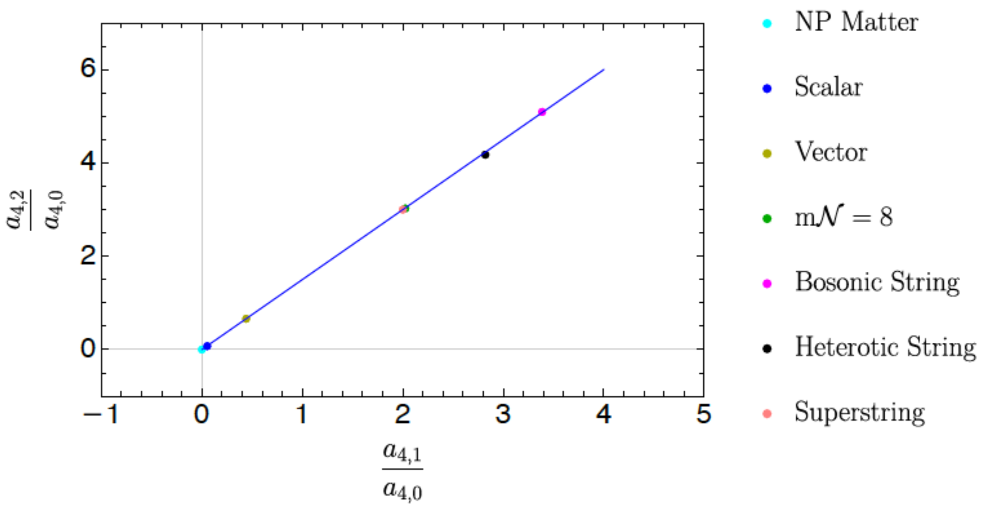
$$\mathcal{M}(1_{6^-}, 2_{10^-}, 3_{10^+}, 4_{6^+})$$

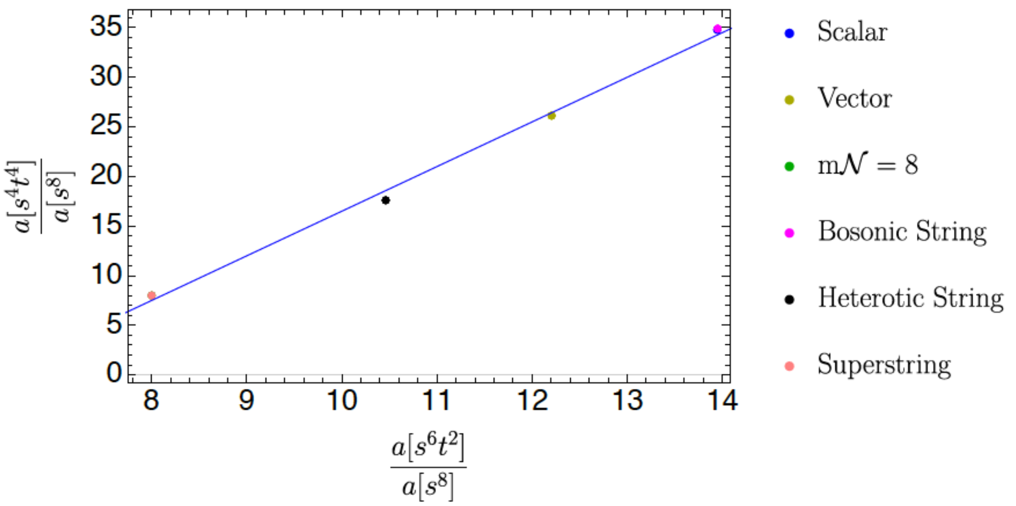
Islands are similar in $D \geq 4$

$$\varepsilon^{\mu}_{6\pm} = (0, 0, 0, 0, 1, \pm i, 0, \ldots, 0)/\sqrt{2}$$

[Bern, Herrmann, DK, Roiban]

Islands in $D \geq 4$





$$\mathcal{M}(1_{4^-}, 2_{4^-}, 3_{4^+}, 4_{4^+})$$

$$\mathcal{M}(1_{6^-}, 2_{10^-}, 3_{10^+}, 4_{6^+})$$

For the regions:
[Caron-Huot, Li, Parra-Martinez, Simmons-Duffin;
Herrmann, DK, Kravchuk (WiP)]

Islands are similar in $D \geq 4$

$$\varepsilon^{\mu}_{6\pm} = (0,0,0,0,1,\pm i,0,\ldots,0)/\sqrt{2}$$

[Bern, Herrmann, DK, Roiban]

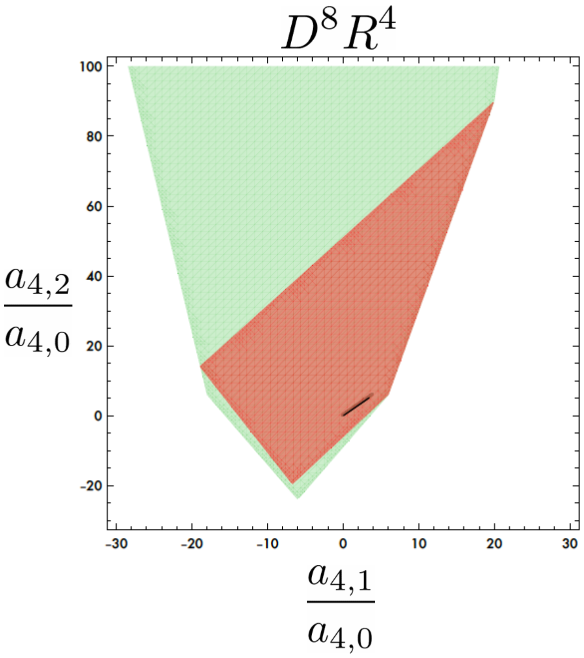
Future Directions

- Islands in other cases?
  - Strongly coupled matter (e.g. Large-*N* QCD, AdS/CFT) [Kaplan, Kundu]
  - Non-gravitational theories (e.g. Gauge theory) [Henriksson, McPeak, Russo, Vichi; Häring, Hebbar, Karateev, Meineri, Penedones]
- Running in the EFT: Graviton loops [Bellazzini, Miró, Rattazzi, Riembau, Riva; Bellazzini, Riembau, Riva]
- n→m scattering ⇒ stronger 2→2 bounds? [Shu, Xiao, Zheng]

Can we shrink down to the islands?

Conclusions

- Obtained bounds on gravitational EFT coefficients
- Compared with explicit examples
  - Calculated 1-loop 4-graviton amplitudes with massive matter up to *S=2* in the loop
  - Studied simple non-perturbative model
- Physical theories occupy tiny island in allowed space
  - Tiny islands seem to obey Low-Spin Dominance. Underlying physical principle?
  - Islands seem to be a robust feature of gravitational theories



Thank you!

Future bounds = Black region?

Backup Material

Avoiding IR Obstructions: Graviton Pole

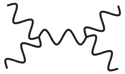
Low-energy expansion

$$\mathcal{M}(1^-,2^-,3^+,4^+) = (\langle 12\rangle[34])^4 \; f(t,u)$$

$$f(t,u) = \left(\frac{\kappa}{2}\right)^2 \frac{1}{stu} + (\beta_{R^3})^2 \frac{tu}{s} - (\beta_\phi)^2 \frac{1}{s} + \sum_{k\geq j\geq 0} a_{k,j}\, s^{k-j} t^j$$

Forward limit

$$\mathcal{M}(s,t) = \left(\frac{\kappa}{2}\right)^2 \frac{s^2}{-t} + \dots \, , \qquad s \gg -t > 0$$



Dispersion relation
$$\oint_{s_0} \frac{ds}{2\pi i} \frac{1}{s} \frac{\mathcal{M}}{(s(s+t))^k} = \int_{m_{\rm gap}^2}^{\infty} \frac{ds}{\pi} \frac{1}{(s(s+t))^k} \left(\frac{\mathrm{Disc}_s \mathcal{M}}{s} + \frac{\mathrm{Disc}_u \mathcal{M}}{s+t} \right) \, , \qquad k \geq 2$$

insensitive to graviton pole

Dispersion relation avoids graviton pole

$$\langle 12\rangle \sim [34] \sim \sqrt{s}$$

Avoiding IR Obstructions: Graviton Pole

Low-energy expansion

$$\mathcal{M}(1^-,2^-,3^+,4^+) = (\langle 12\rangle[34])^4\ f(t,u)$$

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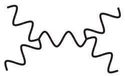

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insensitive to graviton pole

Alternative: Impact parameter approach
[Caron-Huot, Mazac, Rastelli, Simmons-Duffin;
Caron-Huot, Li, Parra-Martinez, Simmons-Duffin]

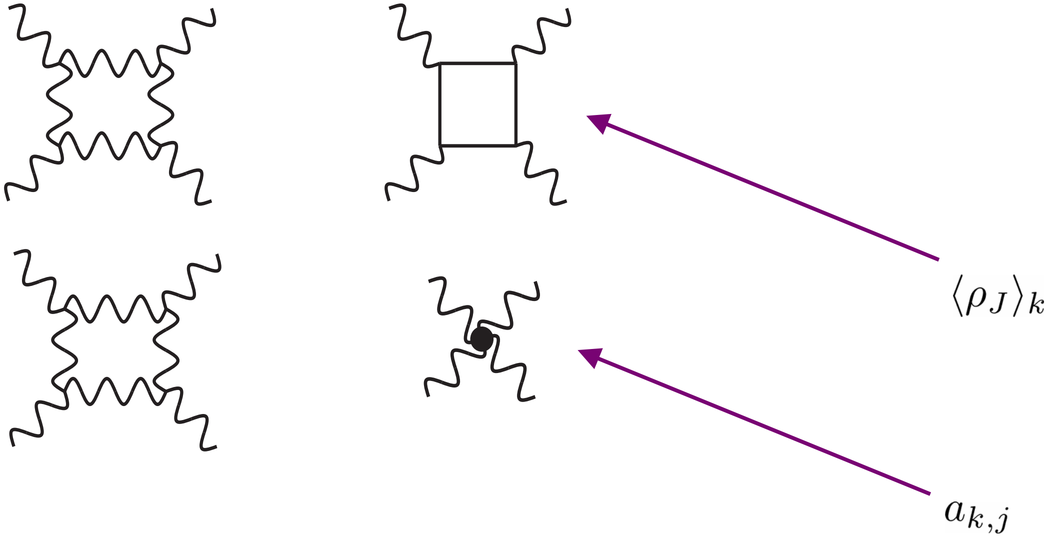
Dispersion relation avoids graviton pole

$$\langle 12\rangle \sim [34] \sim \sqrt{s}$$

Avoiding IR Obstructions: Graviton Loops

High-energy theory:

Low-energy EFT:

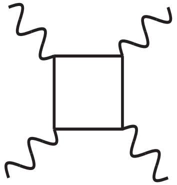


$$a_{k,j} = \sum_{J\ \mathrm{even}} \langle \rho_J^{++} \rangle_k P^{j}_{++}(\mathcal{J}^2) + \sum_{J=4}^{\infty} \langle \rho_J^{+-} \rangle_k P^{k,j}_{+-}(\mathcal{J}^2)$$

Massive loops have by themselves all properties assumed

Consistency Checks

- No infrared divergence as $m \to 0$, except for the spin-2 particle in the loop.
- Match the literature in the massless limit, accounting for different state counts [Dunbar, Norridge]
- Vanishing of the UV divergences in $D = 4$
- UV divergences local in $D > 4$
- Decoupling in the $m \to \infty$ limit
- Consistency with all the derived EFT bounds



Confident we have the correct answer

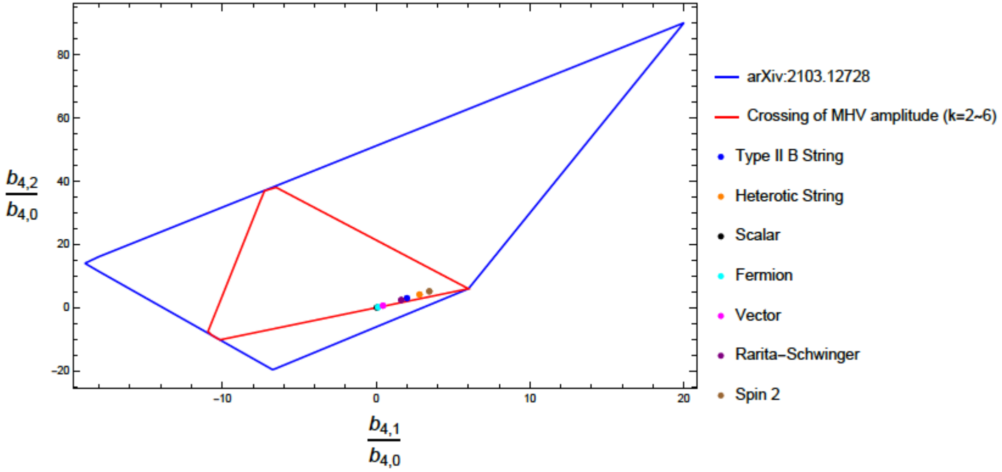
What takes us off the island?

Example of accumulation-point model
for gravitons [Bern, DK, Zhiboedov]
based on one for scalars [Caron-Huot, Van Duong]

$$\mathcal{M}(1^-,2^-,3^+,4^+) = (\langle 12\rangle[34])^4\ f(t,u)$$

$$f(t,u) = -\frac{1}{(t-m_1^2)(u-m_1^2)(s-m_2^2)}$$

$$\mathrm{Res} f(t,u)\Big|_{s=m_2^2} = \sum_{J=0}^{\infty} \rho_J^{++}(m_2^2)\, d^J_{0,0}(1+\frac{2t}{m_2^2})$$



[Chiang, Huang, Li, Rodina, Weng]:
"we conjecture that the area outside of known theories are populated by spectrums with an accumulation point"

See also: [Figueroa, Tourkine; Huang, Remmen; Geiser, Lindwasser]

Accumulation-point models may lie off the islands