How to measure the W Mass: A Theory Perspective
Joshua Isaacson
In Collaboration with: Yao Fu and C.-P. Yuan
Based on: arxiv:2005.02788
CERN Theory Seminar
23 May 2022
Standard Model: W Mass

**Standard Model EW Fit**

\[
M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_F} (1 + \Delta r)
\]

\[
\Delta r = \Delta \alpha - \frac{c_W^2}{s_W^2} \Delta \rho + \Delta r_{\text{rem}} (M_H),
\]

where \(s_W^2\) is the Weinberg angle, \(\Delta \alpha\) is the correction to \(\alpha\) from the light fermions, \(\Delta \rho\) is the correction to the \(\rho\) parameter, and \(\Delta r_{\text{rem}}\) contains all corrections containing the Higgs mass.

**Parameter Fit Result**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(G_\mu) ([\text{GeV}^{-2}])</td>
<td>(1.1663787 \times 10^{-5})</td>
</tr>
<tr>
<td>(\alpha(0)^{-1})</td>
<td>137.035999139</td>
</tr>
<tr>
<td>(\Delta \alpha_{\text{had}}(M_Z^2))</td>
<td>0.027627 ± 0.000096</td>
</tr>
<tr>
<td>(M_Z) ([\text{GeV}])</td>
<td>91.1883 ± 0.0021</td>
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<tr>
<td>(M_H) ([\text{GeV}])</td>
<td>125.21 ± 0.12</td>
</tr>
<tr>
<td>(m_t) ([\text{GeV}])</td>
<td>172.75 ± 0.44</td>
</tr>
<tr>
<td>(M_W) ([\text{GeV}])</td>
<td>80.3591 ± 0.0052</td>
</tr>
</tbody>
</table>

Table reproduced from: HEPFit Group (2112.07274).
**Experimental Measurements**

- CDF Run II results most precise
- $7\sigma$ tension with SM
- $3\sigma$ tension between CDF-II and ATLAS result
- Missing LHCb result: $80,354 \pm 32$ MeV

Figure reproduced from CDF-II measurement (Science 376, 170).
Extracting W Mass from Data

- Can’t measure invariant mass directly due to neutrino
- Look at sensitive observables
  - $M_T = \sqrt{2 \left( p_T^\ell p_T^{\nu} - \vec{p}_T^\ell \cdot \vec{p}_T^{\nu} \right)}$
  - $p_T^\ell$
  - $p_T^{\nu}$ with $(\vec{p}_T^{\nu} = -\vec{p}_T^\ell - \vec{u}_T)$
- Requires precise theory calculation
- Fit theory templates with varying $M_W$

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Theory Calculation
Breakdown of Fixed Order

- Perturbative series has terms proportional to $\alpha_s^n \log^m \left( \frac{p_T^2}{M_W^2} \right)$, $m \leq 2n$
- As $p_T^W \rightarrow 0$ the series no longer converges
- Need to include corrections to all orders by resumming the series
Analytic vs. Numeric Resummation

**Analytic:**
- Formal resummation (focus here on $b$-space CSS resummation)
- Pros:
  - High precision and accuracy
- Cons:
  - Inclusive only
  - Numerically expensive
- Used by CDF to obtain $M_W$

**Numerical**
- Parton Showers (Pythia, Sherpa, Herwig, Dire, Vincia)
- Pros:
  - Exclusive final states
  - Quick
- Cons:
  - Currently only LL with some subleading effects included
- Used by ATLAS to obtain $M_W$
Evolution Equation

\[
\frac{df_a(x,t)}{d\ln t} = \sum_{b=q,g} \int_0^1 dz \frac{\alpha_s}{2\pi} [P_{ab}(z)] + f_b\left(\frac{x}{z}, t\right)
\]

- \(f_a(x,t)\) is the observable being evolved
- \(P_{ab}(z)\) is the evolution (splitting) kernel
- Solve using Markovian Monte-Carlo algorithms
- Treat \(P_{ab}\) as a probability
- Virtual corrections defined at kinematic endpoints by \(+\) prescription
Collins-Soper-Sterman Formalism

Resummation

\[
\frac{d\sigma_{\text{res}}}{dQ^2 d^2 q_T d\Omega} = \sigma \int \frac{d^2 b}{(2\pi)^2} e^{i q_T \cdot \vec{b}} \tilde{W},
\]

\[
\tilde{W} = e^{-S(b)} \ C \otimes f(x_A, C_3/b) C \otimes f(x_B, C_3/b)
\]

\[
S(b) = \int_{C_1^2/b^2}^{C_2^2 Q^2} \frac{d\mu^2}{\mu^2} \left[ \ln \left( \frac{C_2^2 Q^2}{\mu^2} \right) A(\bar{\mu}) + B(\bar{\mu}) \right]
\]

[Collins, Soper, Sterman, ’85] […]
Collins-Soper-Sterman Formalism

Resummation

\[ \frac{d\sigma_{\text{res}}}{dQ^2 d^2 q_T dy d\Omega} = \sigma \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}} \tilde{W}, \]

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- Electroweak cross section

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- Electroweak cross section
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J. Isaacson  W Mass: A Theory Overview  8 / 28  Fermilab
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- Sudakov factor
- Collinear factors

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J. Isaacson

W Mass: A Theory Overview
Collins-Soper-Sterman Formalism

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S(b) = \int_0^{C_2^2 Q^2} \frac{d\ln^2 \mu^2}{\mu^2} \left[ \ln \left( \frac{C_2^2 Q^2}{\mu^2} \right) A(\mu) + B(\mu) \right]
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- Electroweak cross section
- Sudakov factor
- Collinear factors
- Perturbative Coefficients \((A, B, C)\)

[Collins, Soper, Sterman, '85] [...]

J. Isaacson  W Mass: A Theory Overview  8 / 28  Fermilab
### Order Definitions

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Accuracy used by CDF
- Current accuracy available in ResBos code
  - All terms known to this accuracy
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Non-Perturbative Fit

\[ S(b) = \int_{C_1^2/b^2}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[ \ln \left( \frac{C_2^2 Q^2}{\bar{\mu}^2} \right) A(\bar{\mu}) + B(\bar{\mu}) \right] \]

- Lower limit goes to zero as \( b \) goes to infinity
- Requires evaluation of \( \alpha_s(C_1/b) \) which is non-perturbative
- Need to introduce a non-perturbative cutoff (\( b^* \)-prescription):

\[ b^* = \frac{b}{\sqrt{1 + \frac{b^2}{b_{\text{max}}^2}}} \]
**BLNY Form**

\[ S_{NP}(b) = -b^2 \left( g_1 + g_2 \log\left( \frac{Q}{2Q_0} \right) + g_1 g_3 \log(100x_1x_2) \right) \]

- \( g_1 \) and \( g_3 \) extracted from global fit
- \( g_2 \) tuned to reproduce CDF-II \( p_T^Z \)
- \( M_W \) vs. \( M_Z \) captured in \( Q \) dependence
- No flavor dependence included
- No consideration of uncertainty from changing form, but expected to be small

**NOTE:** SIYY2 is the same functional form as BLNY, but with \( b_{max} = 1.5 \text{ GeV}^{-1} \)
Flavor Dependence

- Study on flavor dependence for $\sqrt{s} = 7$ TeV LHC
- $S_{NP}(b) = -b^2 (g_a + g_{evo} \log(Q^2/Q_0^2))$, where $g_a$ is the flavor dependent piece
- Found shift could be up to 10 MeV
- Additional studies are required to validate
- Unclear what the global shift would be

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<tr>
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<td>0</td>
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Table reproduced from: Phys. Letters B 788 (2019) 542-545
Speed Improvements
Ogata Integration

\[
\int_0^\infty dxf(x)J_n(x) = \pi \sum_{j=1}^{N} w_{nj} f \left( \frac{\pi}{h} \psi(x_{nj}) \right) J_n \left( \frac{\pi}{h} \psi(x_{nj}) \right) \psi'(x_{nj})
\]

\[
+ \left[ I_{nN+1} + \mathcal{O} \left( e^{-c/h} \right) \right],
\]

\[
\psi(t) = t \tanh \left( \frac{\pi}{2} \sinh(t) \right), \quad x_{nj} = h\xi_{nj}, \quad w_{nj} = \frac{2}{\pi^2 \xi_{nj} J_{n+1} (\pi \xi_{nj})}
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Ogata Integration

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\]

- Truncation error

[1906.05949]
Ogata Integration

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- Truncation error
- Finite step size error

[1906.05949]
Ogata Integration

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- Truncation error
- Finite step size error
- Evaluation points

\[\text{[1906.05949]}\]
Ogata Integration

\[\int_{0}^{\infty} dx f(x) J_n(x) = \pi \sum_{j=1}^{N} w_{nj} f \left( \frac{\pi}{h} \psi (x_{nj}) \right) J_n \left( \frac{\pi}{h} \psi (x_{nj}) \right) \psi' (x_{nj}) \]

\[\psi(t) = t \tanh \left( \frac{\pi}{2} \sinh (t) \right), \quad \psi'(t) = \frac{\pi}{2} \cosh (t) \tanh (t) \]

- Truncation error
- Finite step size error
- Evaluation points
- Quadrature weights

\[w_{nj} = \frac{2}{\pi^2 \xi_{nj} J_{n+1} (\pi \xi_{nj})} \]
Ogata Quadrature

Optimization Procedure described in [1906.05949] to minimize function calls.

\[
\langle N_{\text{tot}} \rangle = 107
\]
\[
\langle N_{\text{tot}} \rangle = 379
\]
\[
\langle N_{\text{tot}} \rangle = 944
\]

\[
\langle N_{\text{tot}} \rangle = 1352
\]
\[
\langle N_{\text{tot}} \rangle = 2780
\]
\[
\langle N_{\text{tot}} \rangle = 5671
\]

\[|M_h^+ (q_{\perp}, x_{bj}, z, Q)|\]
Angular Coefficients
Angular Coefficients

\[
\frac{\text{d} \sigma}{\text{d} p_T^2 \, \text{d} y \, \text{d} \Omega \, \text{d} \cos \theta \, \text{d} \phi} = \frac{3}{16\pi} \frac{\text{d} \sigma^{u+L}}{\text{d} p_T^2 \, \text{d} y \, \text{d} \Omega} \left\{ (1 + \cos^2 \theta) + \frac{1}{2} A_0 (1 - 3 \cos^2 \theta) + A_1 \sin 2\theta \cos \phi \\
+ \frac{1}{2} A_2 \sin^2 \theta \cos 2\phi + A_3 \sin \theta \cos \phi + A_4 \cos \theta \\
+ A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi \right\}.
\]

\[
\langle P(\cos \theta, \phi) \rangle = \frac{\int P(\cos \theta, \phi) \text{d}\sigma(\cos \theta, \phi) \text{d} \cos \theta \text{d}\phi} {\int \text{d}\sigma(\cos \theta, \phi) \text{d} \cos \theta \text{d}\phi}.
\]

\[
\begin{align*}
\langle 1/2 (1 - 3 \cos^2 \theta) \rangle &= \frac{3}{20} (A_0 - \frac{2}{3}); \\
\langle \sin 2\theta \cos \phi \rangle &= \frac{1}{5} A_1; \\
\langle \sin^2 \theta \cos 2\phi \rangle &= \frac{1}{10} A_2; \\
\langle \sin \theta \cos \phi \rangle &= \frac{1}{4} A_3; \\
\langle \cos \theta \rangle &= \frac{1}{4} A_4; \\
\langle \sin^2 \theta \sin 2\phi \rangle &= \frac{1}{5} A_5; \\
\langle \sin 2\theta \sin \phi \rangle &= \frac{1}{5} A_6; \\
\langle \sin \theta \sin \phi \rangle &= \frac{1}{4} A_7.
\end{align*}
\]
NNLO Angular Coefficients

- Well known issue with angular coefficients in the ResBos code at NNLO (No issue with matching to NLO)
- CDF-II only used the NLO so the angular functions are exact to that order
- ResBos only included NNLO corrections to the total rate, but not to the angular functions
- This is an issue with matching to an incomplete NNLO calculation, and not an issue with the resummation or the matching to fixed order
- Only effects larger $p_T$ ($p_T > 30$ GeV, CDF has a cut of $p_T < 15$ GeV)
- Has been resolved via matching to MCFM (preliminary results next slides)
NOTE: Uncertainties are purely statistical for ResBos + MCFM
NNLO Angular Coefficients

NOTE: Uncertainties are purely statistical for ResBos + MCFM
NNLO Angular Coefficients

NOTE: Uncertainties are purely statistical for ResBos + MCFM
Results
\[ \frac{P_T(Z)}{P_T(W)} \]

- Ratio is stable to higher order corrections at small $p_T$
- Scale uncertainty only using correlated prediction
- Need to investigate the CDF estimated uncertainty from this ratio
Methodology

Our Procedure:

- Generate pseudodata using $N^3LL$+NNLO prediction
- Tune NNLL+NLO prediction to reproduce $p_T(Z)$ data
- Validate tuned result against $p_T(W)$ data
- Use tuned result to generate mass templates
- Extract $W$ mass from template fit for each observable
- Calculate the mass shift from the input value for pseudodata

Details:

- Pseudodata $M_W = 80, 358$ MeV
- Cuts:
  - $p_T(Z) < 15$ GeV, $p_T(W) < 15$ GeV
  - $30 < p_T(\ell) < 55$ GeV, $30 < p_T(\nu) < 55$ GeV
  - $|\eta(\ell)| < 1$
  - $66 < M_{\ell\ell} < 116$ GeV ($Z$ events), $60 < m_T < 100$ GeV ($W$ events)
- Number of Events:
  - $1,811,700$ $W \rightarrow e\nu$
  - $66,180$ $Z \rightarrow ee$
  - $2,424,486$ $W \rightarrow \mu\nu$
  - $238,534$ $Z \rightarrow \mu\mu$
Tuning to Pseudodata

Tuned result:

- Fit to $p_T(Z) < 15$ GeV
- $g_2 = 0.662$ GeV$^2$

- $\alpha_S(M_Z) = 0.120$
- Tuned PDF set: CT18NNLO_as_120
Results

<table>
<thead>
<tr>
<th>Observable</th>
<th>RESBos2</th>
<th>+Detector Effect+FSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_T$</td>
<td>1.5 ± 0.5</td>
<td>0.2 ± 1.8 ± 1.0</td>
</tr>
<tr>
<td>$p_T(\ell)$</td>
<td>3.1 ± 2.1</td>
<td>4.3 ± 2.7 ± 1.3</td>
</tr>
<tr>
<td>$p_T(\nu)$</td>
<td>4.5 ± 2.1</td>
<td>3.0 ± 3.4 ± 2.2</td>
</tr>
</tbody>
</table>

Best Fit: $M_W = 80,386$ MeV

Best Fit: $M_W = 80,388$ MeV

Best Fit: $M_W = 80,389$ MeV
Future Studies

- Investigate effect of non-perturbative functional form on $M_W$
- Investigate flavor dependence effects on $M_W$ extraction
- Perform detailed study on the $p_T(Z)/p_T(W)$ ratio and its impact on the $M_W$ uncertainty
- Work with experimentalists to better understand detector smearing
- Understand how to properly combine the three observables
CDF used ResBos code at NNLL+NLO accuracy
- ResBos v2 is able to go to $N^3\text{LL}+\text{NNLO}$ accuracy
- ResBos2 corrected major criticism of incorrect angular functions in the ResBos code
- Mimic CDF analysis using pseudoexperiments at $N^3\text{LL}+\text{NNLO}$ accuracy
- Find shift to be consistent with 0 MeV and up to 10 MeV ($2\sigma$).
Backup
Detector Smearing:

- **Fit functional form (Smearing 1):**
  \[ \frac{\sigma}{E} = a \oplus \frac{b}{\sqrt{E}} \oplus c \]

- Use gaussian with 5%(11%) width for \( \ell(\nu) \) (Smearing 2)

- Note results not sensitive to smearing effect if data and theory smeared identically

<table>
<thead>
<tr>
<th>Observable</th>
<th>Smearing 1</th>
<th>Smearing 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_T )</td>
<td>( 0.2 \pm 1.8 \pm 1.0 )</td>
<td>( 1.0 \pm 2.1 \pm 1.3 )</td>
</tr>
<tr>
<td>( p_T(\ell) )</td>
<td>( 4.3 \pm 2.7 \pm 1.3 )</td>
<td>( 4.5 \pm 2.6 \pm 1.4 )</td>
</tr>
<tr>
<td>( p_T(\nu) )</td>
<td>( 3.0 \pm 3.4 \pm 2.2 )</td>
<td>( 3.8 \pm 4 \pm 2.7 )</td>
</tr>
</tbody>
</table>
Width Effect:

- Central width: \( \Gamma_W = 2.0895 \text{ GeV} \)
- NLO width proportional to \( M_W^3 \)
- Negligible shift

<table>
<thead>
<tr>
<th>Width</th>
<th>Mass Shift [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0475 GeV</td>
<td>2.0 ± 0.5</td>
</tr>
<tr>
<td>2.1315 GeV</td>
<td>0.3 ± 0.5</td>
</tr>
<tr>
<td>NLO</td>
<td>1.2 ± 0.5</td>
</tr>
</tbody>
</table>
## PDF Uncertainties

<table>
<thead>
<tr>
<th>PDF Set</th>
<th>( m_T )</th>
<th>( p_T(\ell) )</th>
<th>( p_T(\nu) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NNLO</td>
<td>NLO</td>
<td>NNLO</td>
</tr>
<tr>
<td>CT18</td>
<td>0.0 (\pm) 1.3</td>
<td>1.8 (\pm) 1.2</td>
<td>0.0 (\pm) 15.9</td>
</tr>
<tr>
<td>MMHT2014</td>
<td>1.0 (\pm) 0.6</td>
<td>2.6 (\pm) 0.6</td>
<td>6.2 (\pm) 7.8</td>
</tr>
<tr>
<td>NNPDF3.1</td>
<td>1.1 (\pm) 0.3</td>
<td>2.1 (\pm) 0.4</td>
<td>2.1 (\pm) 3.8</td>
</tr>
<tr>
<td>CTEQ6M</td>
<td>N/A</td>
<td>2.8 (\pm) 0.9</td>
<td>N/A</td>
</tr>
</tbody>
</table>

- Central value is shift from 80,385 MeV
- Uncertainty is the PDF uncertainty for the given set
- Need to combine to compare to 3.9 MeV from CDF
- Rough estimates say it is consistent with CDF
PDF Correlations

- PDF-induced correlation of $M_W$ and CT18 NNLO error set vs. $x$ at $Q = 100$ GeV
- Region around $x = \frac{M_W}{\sqrt{s}}$ dominated by $\bar{d}/\bar{u}$, $d/u$ and $d$ PDFs