High Energy Theory

Higgs and Effective Field Theory HEFT 2022

Institut de Física d'Altes Energies

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Gearing up for the next generation of LFV experiments, via on-shell methods

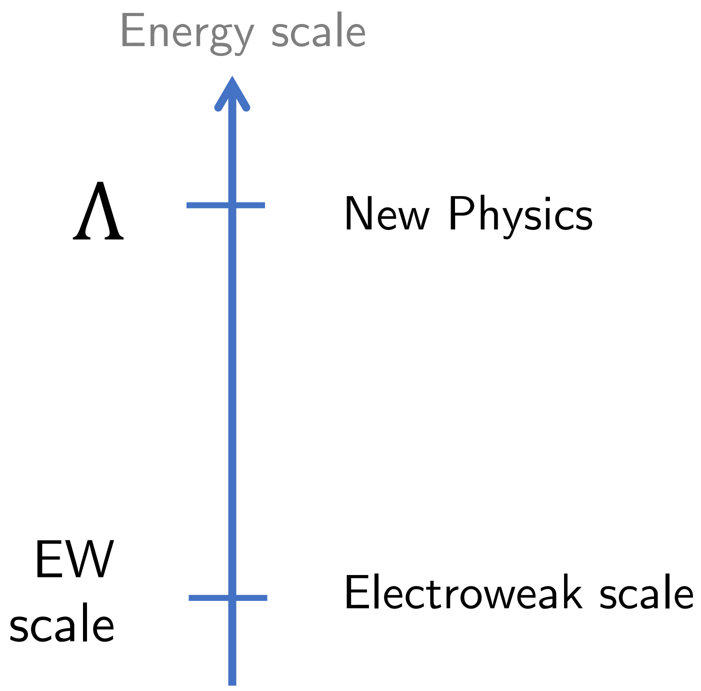
Clara Fernández Castañer

Based on 2112.12131 (J. Elias-Miró, CF, M. A. Gumus, A. Pomarol)

OUTLINE

- Anomalous dimension matrix from on-shell amplitude methods
- Application to LFV observables in the SMEFT.

Motivation



$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{\mathcal{L}_6}{\Lambda^2} + \frac{\mathcal{L}_8}{\Lambda^4} + \mathcal{O}\left(\frac{1}{\Lambda^6}\right)$$

- Expansion of operators of increasing mass dimension d.
- Small corrections to the Standard Model at low energies.
- The operator mixing is important to understand all the BSM contributions to a given observable.

- On-shell amplitude methods are useful. We can easily implement generalized unitarity methods to compute the anomalous dimension matrix without explicit loop calculations.
- The calculations are simplified and we get new insight into the mixing patterns.

Anomalous dimension matrix from on-shell amplitude methods

- The renormalization scale dependence of an amplitude is encoded in its logarithms, which can be detected with **unitary cuts**.

1607.06448 (S. Caron-Huot, M. Wilhelm)
2005.06983 (J. Elias-Miró, J. Ingoldby, M. Riembau)
2005.07129 (P. Baratella, CF, A. Pomarol)
2005.12917 (Z. Bern, J. Parra-Martinez, E. Sawyer)

- Main formula to get the anomalous dimension matrix:

$$e^{-i\pi D} F_O^*(\vec{n}) = \sum_{\vec{m}} S_{nm} F_O^*(\vec{m})$$

Dilatation operator

$$D \equiv \sum_{\text{all particles}} p_i \frac{\partial}{\partial p_i}$$

Form factor

$$F_O(\vec{n}) \equiv {}_{\text{out}}\langle \vec{n}| O(0) |0\rangle$$

S-matrix

$$S_{nm} \equiv \langle \vec{n}| \hat{S} |\vec{m}\rangle = {}_{\text{out}}\langle \vec{n}| \vec{m}\rangle_{\text{in}}$$

Anomalous dimension matrix from on-shell amplitude methods

- Regularization of the IR and UV divergences of the form factors. In dimensional regularization with the MS scheme, the Callan-Symanzik equation is

$$\left(\mu\partial_\mu + \gamma - \gamma_{\rm IR} + \beta_g \partial_g\right) F_O(\vec{n};\mu) = 0$$

- The dilatation operator is related to the renormalization scale by: $D = -\mu\partial_\mu$

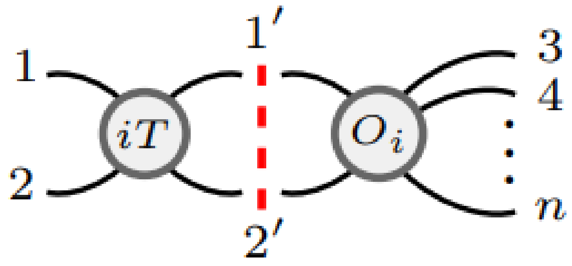
- Acting on the form factor, $DF_O \approx -\mu\partial_\mu F_O^{(1)} = (\gamma - \gamma_{\rm IR} + \beta_g\partial_g)^{(1)} F_O^{(0)}$

- Now we can get the form factor anomalous dimensions from $e^{-i\pi D} F_O^*(\vec{n}) = \sum_{\vec{m}} S_{nm} F_O^*(\vec{m})$

Expand in perturbation theory to get order-by-order expressions for γ

Anomalous dimension matrix from on-shell amplitude methods

- Using $S = \mathbb{1} + i\mathcal{M}$, at first order we obtain, for a minimal form factor,



$$\langle \vec{n}|\, O_j\, |0\rangle^{(0)} \,(\gamma_{ji} - \gamma_{\mathrm{IR}}^{i}\delta_{ij})^{(1)} = -\frac{1}{\pi}\, \langle \vec{n}|\, \mathcal{M} \otimes O_i\, |0\rangle^{(0)}$$

- M is a tree-level 2 → 2 S-matrix and ⊗ denotes a phase-space integration over intermediate particle states.

- Phase-space parametrization using spinors: $\begin{pmatrix} \lambda_1' \\ \lambda_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta\, e^{i\phi} \\ \sin\theta\, e^{-i\phi} & \cos\theta \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$

- Finally,

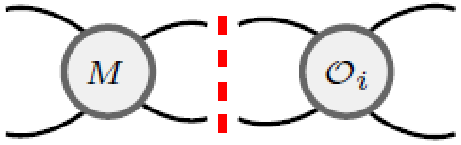
$$F_{O_j}(12\ldots n)\,(\gamma_{ji} - \gamma_{\mathrm{IR}}^{i}\delta_{ij}) = -\frac{1}{16\pi^2}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^{\pi/2} 2\sin\theta\cos\theta\, d\theta\, M(1,2;1'2')\, F_{O_i}(1'2'3\ldots n)$$

Example: $O_{LuQe} \rightarrow O_{DB}$ at 1-loop

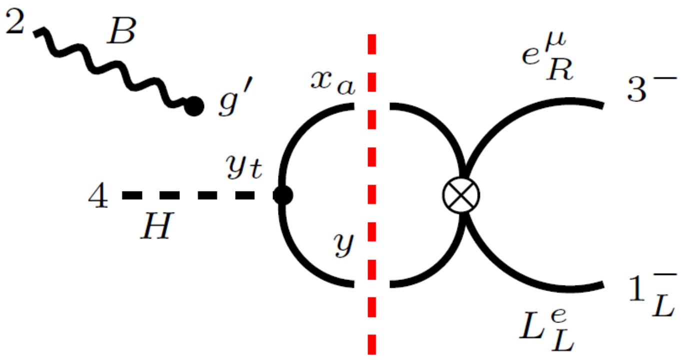
- 1-loop renormalization of O_{DB} by O_{LuQe} in the SMEFT

$$\begin{cases} \mathcal{O}_{DB}^{\mu e} = y_\mu g' \bar{L}_L^{(2)} \sigma^{\mu\nu} e_R^{(1)} H B_{\mu\nu} \\ \mathcal{O}_{LuQe}^{\mu eqq} = y_\mu (\bar{L}_L^{(2)} u_R)(\bar{Q}_L e_R^{(1)}) \end{cases}$$

- There are no IR divergences, so the general formula reads



$$= -\frac{1}{8\pi^2} \int d\Omega_2\, M(12; xy)\, F_{\mathcal{O}_i}(xy34)$$

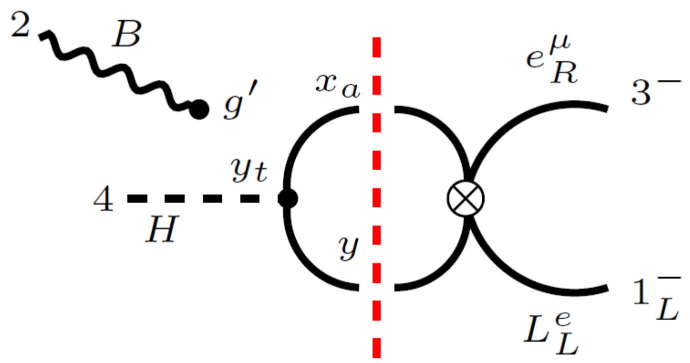


Dipole form factor: $\langle 1^-_{L_l} 2^-_B 3^-_e 4_{H_k} | \mathcal{O}_{DB} | 0 \rangle^{(0)} = 2\sqrt{2} g' y_\mu \langle 12 \rangle \langle 23 \rangle \epsilon_{lk}\,,$

4-fermion form factor: $F_{\mathcal{O}_{LuQe}^{e\mu tt}}(x_a y 1_l 3) = -y_\mu \langle 1y \rangle \langle x3 \rangle \epsilon_{la}$

SM amplitude: $M(24_k; x_a y) = -\sqrt{2} y_t \left(Y_{t_R} \frac{[xy]^2}{[x2][y2]} + Y_H \frac{[xy][4x]}{[42][x2]} \right)$

Example: $\mathcal{O}_{\psi^4} \to \mathcal{O}_{DB}$ at 1-loop



Phase-space parametrization:

$$\begin{pmatrix} |x\rangle \\ |y\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -e^{i\phi}\sin\theta \\ e^{-i\phi}\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix}$$

- With a few elementary integrals, we obtain

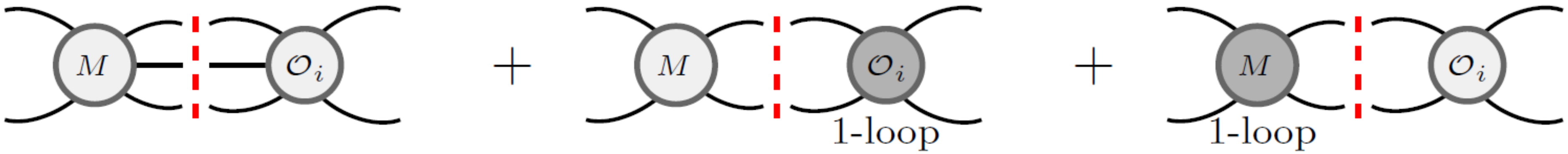
$$\underbrace{2\sqrt{2}g' y_\mu \langle 12\rangle\langle 23\rangle \epsilon_{lk}}\,(-y_t)\frac{N_c/2}{(16\pi^2)}(Y_H - 2Y_{t_R})$$

- The anomalous dimension is $\boxed{\gamma_{DB} = \frac{y_t}{2}\frac{N_c}{16\pi^2}(Y_{Q_L} + Y_{t_R})}$

  → We have obtained it from a product of tree-level amplitudes and a phase-space integration

Anomalous dimensions at 2-loops

- When there is no 1-loop contribution, the 2-loop anomalous dimension is particularly easy to obtain. One has to consider the following unitary cuts



Application: LFV processes

- Lepton number is conserved in the SM, but not when we consider higher dimension operators.
- Processes where the relative lepton numbers are violated, but the total lepton number is preserved are among the best indirect probes for new physics at the TeV.
- We will consider in particular LFV processes with $\Delta L_e = \Delta L_\mu = 1$.
  The most competitive experimental measurements come from

$$\left\{ \begin{array}{l} \mu \to e\gamma \\ \mu \to eee \\ \mu N \to eN \end{array} \right.$$

	$\mathrm{BR}(\mu \to e\gamma)$	$\mathrm{BR}(\mu \to eee)$	$R(\mu N \to eN)$	$\mathrm{BR}(h \to \mu e)$
Current	$4.2 \cdot 10^{-13}$ [33]	$1 \cdot 10^{-12}$ [34]	$7 \cdot 10^{-13}$ [35]	$6.1 \cdot 10^{-5}$ [36]
Future	$6.0 \cdot 10^{-14}$ [37]	$1 \cdot 10^{-16}$ [38]	$8 \cdot 10^{-17}$ [39]	

Table 1: Current and near future upper bounds on $\Delta L_e = \Delta L_\mu = 1$ processes.

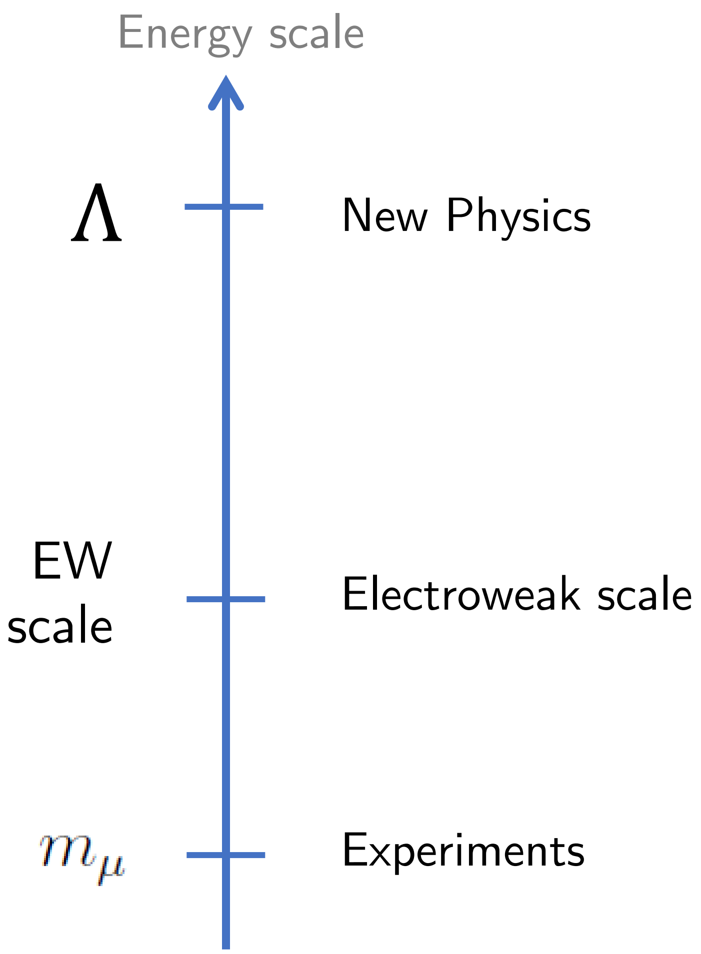
Application: LFV processes

- We can characterize the BSM contributions to the LFV processes in the SMEFT with dimension-six operators.

- The goal is to understand at what loop order the Wilson coefficients enter into the considered observables. Then, we can use the experimental constraints on the branching ratios to bound the Wilson coefficients.

- We need to compute anomalous dimensions at 1 and 2 loops. Using on-shell amplitude methods is an efficient way to do it.

Application: LFV processes



Effects to be considered in the operator mixing:

i. Finite matching contributions from the scale Λ
ii. RG mixing from Λ to the electroweak scale mW
iii. finite threshold corrections from integrating out the W, Z, H and the heavy SM fermions
iv. RG mixing from the electroweak scale mW to mμ

↓

We will study the RG mixing from Λ to the electroweak scale at leading order

$\mu \to e\gamma$ at tree-level

- The process arises from $$-\frac{4G_F}{\sqrt{2}} m_\mu \left[d_{\mu e}\, \bar{\mu}_L \sigma^{\mu\nu} e_R F_{\mu\nu} + d_{e\mu}\, \bar{e}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu} + \text{h.c.} \right]$$

- The branching ratio is given by $$\mathrm{BR}\,(\mu \to e\gamma) = 384\pi^2 \left(|d_{\mu e}|^2 + |d_{e\mu}|^2 \right)$$

- At tree level the only SMEFT operators entering $d_{\mu e}$ are the dipoles, which give $$d_{\mu e} = \frac{e\, v^2}{2\Lambda^2} \left(C_{DW}^{\mu e} - C_{DB}^{\mu e} \right)$$ $$\begin{cases} \mathcal{O}_{DB}^{\mu e} = y_\mu g' \bar{L}_L^{(2)} \sigma^{\mu\nu} e_R^{(1)} H B_{\mu\nu} \\ \mathcal{O}_{DW}^{\mu e} = y_\mu g \bar{L}_L^{(2)} \tau^a \sigma^{\mu\nu} e_R^{(1)} H W_{\mu\nu}^a \end{cases}$$

- From the experimental constraints, we can bound C/Λ^2. Assuming $(C_{DW} - C_{DB})$ is of order 1, we obtain the following energy bounds

	$\mathrm{BR}(\mu \to e\gamma)$
Current	$4.2 \cdot 10^{-13}$ [33]
Future	$6.0 \cdot 10^{-14}$ [37]

→

Current: $\Lambda \gtrsim 950$ TeV
Future: $\Lambda \gtrsim 1550$ TeV

$\mu \to e\gamma$ at 1-loop

1505.01844(C. Cheung, C.-H. Shen)

- At one-loop, helicity selection rules dictate the anomalous dimension mixing

$$\boxed{\Delta n \geqslant |\Delta h|} \quad \text{where} \quad \begin{cases} \Delta n = n_f - n_i \\ \Delta h = h_f - h_i \end{cases} \longrightarrow \text{Exception:} \quad \bar{\psi}^2\psi^2 \leftrightarrow \psi^4$$

- The dimension-6 operators outside of the grey cells cannot mix with the dipole ($F\psi^2\phi$)

- The only non-vanishing mixings are

$$\begin{cases} C_{DW} + C_{DB} \to C_{DW} - C_{DB} \\ C_{LuQe} \longrightarrow C_{DW,DB} \end{cases}$$

(we already saw it)

$$\mathcal{O}_{LuQe}^{\mu eqq} = y_\mu(\bar{L}_L^{(2)} u_R)(\bar{Q}_L e_R^{(1)})$$

n							
6				ϕ^6			
5			$\bar{\psi}^2\phi^3$		$\psi^2\phi^3$		
4		$\bar{F}^2\phi^2$ $\bar{F}\bar{\psi}^2\phi$ $\bar{\psi}^4$		$\phi^4 D^2$ $\psi^2\bar{\psi}^2$ $\psi\bar{\psi}\phi^2 D$		$F^2\phi^2$ $F\psi^2\phi$ ψ^4	
3	$\bar{F}^3$						F^3
	-3	-2	-1	0	1	2	3

h

- We can get energy bounds, turning on 1 operator at a time

$\mu \to e\gamma$ at 2-loops

- 2-loop double-log contributions proportional to $\frac{C_i C_j}{(16\pi^2)^2} \ln^2(\Lambda/m_W)\,, \quad i \neq j\,.$

$$\mathcal{O}_i \xrightarrow{\text{1-loop}} \mathcal{O}_j \xrightarrow{\text{1-loop}} \mathcal{O}_{DW,DB} \longrightarrow$$

1-loop mixings with LuQe

$$\begin{cases} \mathcal{O}_{LeQu}^{\mu eqq} = y_\mu (\bar{L}_L^{(2)} e_R^{(1)})(\bar{Q}_L u_R) \\ \mathcal{O}_{LL}^{\mu eff} = (\bar{L}_L^{(2)} \gamma_\mu L_L^{(1)})(\bar{F}_L \gamma^\mu F_L) \\ \mathcal{O}_{RR}^{\mu eff} = (\bar{e}_R^{(2)} \gamma_\mu e_R^{(1)})(\bar{f}_R \gamma^\mu f_R) \\ \cdots \end{cases}$$

- 2-loop single-log contributions proportional to $\frac{C_i}{(16\pi^2)^2} \ln(\Lambda/m_W)$

$$\mathcal{O}_i \xrightarrow{\text{2-loop}} \mathcal{O}_{DW,DB}$$

4-fermion operators 1810.09413 $\longrightarrow$

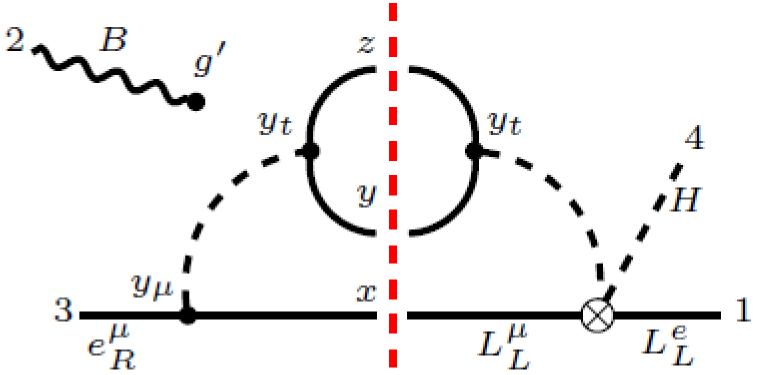
$$\mathcal{O}_{led\bar{q}} = (\bar{L}_L e_R)(\bar{d}_R Q_L)$$
$$\mathcal{O}_{le\bar{e}'\bar{l}'} = (\bar{L}_L e_R)(\bar{e}'_R L'_L)$$

$\mathcal{O}_y^{\mu e} = y_\mu \left(H^\dagger H\right)\left(\bar{L}_L^{(2)} e_R^{(1)} H\right)$ 2005.06983

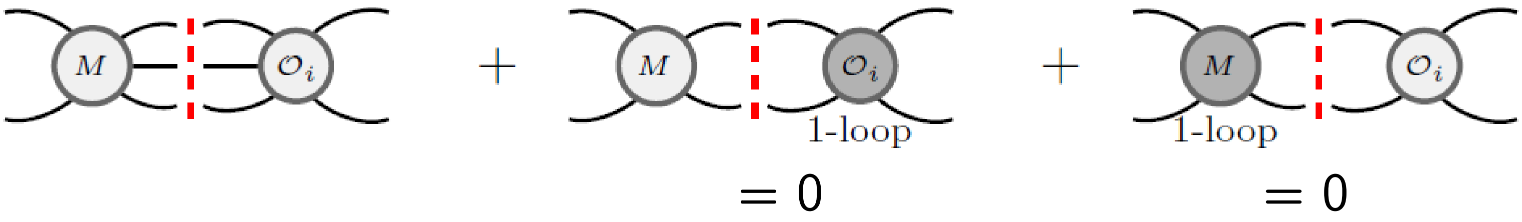
2-fermion, 2-scalar operators $\longrightarrow$

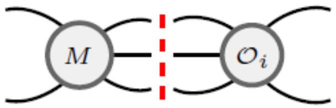
$$\mathcal{O}_L^{\mu e} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{L}_L^{(2)} \gamma^\mu L_L^{(1)})$$
$$\mathcal{O}_{L3}^{\mu e} = (H^\dagger i\overleftrightarrow{D}_\mu^a H)(\bar{L}_L^{(2)} \tau^a \gamma^\mu L_L^{(1)})$$
$$\mathcal{O}_R^{\mu e} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_R^{(2)} \gamma^\mu e_R^{(1)})$$

$O_L \to O_{DB}$ at 2-loops (y_t^2)



- We only have to consider the 3-particle cut



- The cut is  $= \dfrac{\langle 12\rangle[12]}{(16\pi^2)^2}\displaystyle\int d\Omega_3\, M(12; xyz)\, F_{\mathcal{O}_i}(xyz34)$

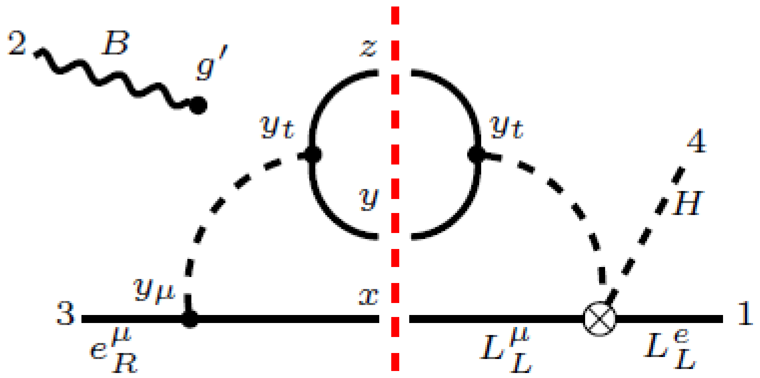
- We can parametrize

$$\begin{pmatrix}|x\rangle\\ |y\rangle\\ |z\rangle\end{pmatrix} = \begin{pmatrix}\cos\theta_1 & -e^{i\phi}\cos\theta_3\sin\theta_1\\ \cos\theta_2\sin\theta_1 & e^{i\phi}\left(\cos\theta_1\cos\theta_2\cos\theta_3 - e^{i\delta}\sin\theta_2\sin\theta_3\right)\\ \sin\theta_1\sin\theta_2 & e^{i\phi}\left(\cos\theta_1\cos\theta_3\sin\theta_2 + e^{i\delta}\cos\theta_2\sin\theta_3\right)\end{pmatrix}\begin{pmatrix}|1\rangle\\ |2\rangle\end{pmatrix}$$

- And the measure is

$$d\Omega_3 = 4\cos\theta_1\sin^3\theta_1 d\theta_1 2\cos\theta_2\sin\theta_2 d\theta_2 2\cos\theta_3\sin\theta_3 d\theta_3 \frac{d\delta}{2\pi}\frac{d\phi}{2\pi}.$$

$O_L \to O_{DB}$ at 2-loops (y_t^2)



Dipole FF: $\langle 1^-_{L_l} 2^-_B 3^-_e 4_{H_k} | \mathcal{O}_{DB} | 0 \rangle^{(0)} = 2\sqrt{2} g' y_\mu \langle 12 \rangle \langle 23 \rangle \epsilon_{lk}$,

O_L FF: $F_{\mathcal{O}_L^{e\mu}}(x_a y_b z 1_l 4_k) = -2 y_t \frac{\langle 14 \rangle [4x]}{\langle yz \rangle} \mathcal{B}^{ba}_{kl}$

M: $M_1(32; x_a y_b z) = \sqrt{2} y_t y_\mu \left(Y_{\mu_R} \frac{\langle yz \rangle}{[x2][32]} - Y_{t_R} \frac{\langle x3 \rangle}{[y2][z2]} - Y_H \frac{\langle z3 \rangle}{[y2][x2]} \right) \mathcal{A}_{ba}$

- After the phase-space integration, we obtain

$$\underbrace{2\sqrt{2} y_\mu g' \langle 12 \rangle \langle 23 \rangle \epsilon_{lk}} \frac{-N_c y_t^2}{(16\pi^2)^2} (Y_{\mu_R} + 2Y_H)$$

- The anomalous dimension is $\gamma_{DB} = \frac{-N_c y_t^2}{(16\pi^2)^2} (Y_{\mu_R} + 2Y_H)$

$\mu \to e\gamma$ at 2-loops

- Similarly, one can compute the mixing with O_R and O_{L3} and for the dipole O_{DW}.

$$(\gamma_{C_{DB}^{e\mu}}, \gamma_{C_{DW}^{e\mu}})^T = \gamma_{D1} \cdot (C_L^{e\mu}, C_{L3}^{e\mu}, C_R^{e\mu})^T \qquad \text{with} \qquad \gamma_{D1} = \frac{N_c y_t^2}{(16\pi^2)^2} \begin{pmatrix} 0 & 0 & -3y_e/(2y_\mu) \\ 1 & -1 & y_e/(2y_\mu) \end{pmatrix}$$

Combination $C_L - C_{L3}$

$\mu \to eee$

- The process arises from $$-\frac{4G_F}{\sqrt{2}}\Big[g_1(\bar{\mu}_R e_L)(\bar{e}_R e_L) + g_2(\bar{\mu}_L e_R)(\bar{e}_L e_R) + g_3(\bar{\mu}_R \gamma_\mu e_R)(\bar{e}_R \gamma_\mu e_R) + g_4(\bar{\mu}_L \gamma^\mu e_L)(\bar{e}_L \gamma_\mu e_L) + g_5(\bar{\mu}_R \gamma^\mu e_R)(\bar{e}_L \gamma_\mu e_L) + g_6(\bar{\mu}_L \gamma^\mu e_L)(\bar{e}_R \gamma_\mu e_R)\Big] + \text{h.c.},$$

- The branching ratio is $$\mathrm{BR}\,(\mu \to eee) = 2\left(|g_3|^2 + |g_4|^2\right) + |g_5|^2 + |g_6|^2 + 32e^2\left(\ln(\frac{m_\mu^2}{m_e^2}) - \frac{11}{4}\right)\left(|d_{\mu e}|^2 + |d_{e\mu}|^2\right) + 8e\,\mathrm{Re}\left(d_{e\mu}^* g_6^* + d_{\mu e} g_5^*\right) + 16e\,\mathrm{Re}\left(d_{e\mu}^* g_4^* + d_{\mu e} g_3^*\right) + \frac{1}{8}\left(|g_1|^2 + |g_2|^2\right)\,.$$

- At tree level
  - Dipole operators O_{DW}, O_{DB}
  - 4-fermion operators $O_{LL}, O_{RR}, O_{LR}, O_{RL}$
  - 2-fermion, 2-scalar operators O_R, $O_L + O_{L3}$

- Then we can get the 1-loop mixings to these operators

Bounds comparison

	$\mu \to e\gamma$	$\mu \to eee$
$C_{DB}^{\mu e} - C_{DW}^{\mu e}$	951 TeV (1547 TeV)	218 TeV (2183 TeV)
$C_{DB}^{\mu e} + C_{DW}^{\mu e}$	127 TeV (214 TeV)	26 TeV (309 TeV)
$C_{R}^{\mu e}$	35 TeV (59 TeV)	160 TeV (1602 TeV)
$C_{L}^{\mu e} + C_{L3}^{\mu e}$	4 TeV (7 TeV)	164 TeV (1642 TeV)
$C_{L}^{\mu e} - C_{L3}^{\mu e}$	24 TeV (41 TeV)	35 TeV (421 TeV)

Tree level

1 loop

2 loops

CONCLUSIONS

- Using on-shell amplitude methods, it is possible to extract the anomalous dimensions of SMEFT operators. The 1-loop contributions are obtained from a product of tree-level amplitudes integrated over a phase-space. Some 2-loop contributions can be retrieved in the same way.

- On-shell amplitude methods allow us to simplify the calculation of the anomalous dimension matrix and they can also give us a better understanding of its structure.

- These techniques are very useful in the context of the next generation of LFV precision measurements, which require the knowledge of renormalization effects at higher orders. We have seen an example where the2-loop mixings are obtained from a single 3-particle cut.

- In particular, the bound on $(C_L - C_{L3})$ from $\mu \to e\gamma$ is competitive with the bound coming from $\mu \to eee$, even if the former comes from a 2-loop process.

Gearing up for the next generation of LFV experiments, via on-shell methods

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Anomalous dimension matrix from on-shell amplitude methods

- The renormalization scale dependence of an amplitude is encoded in its logarithms, which can be detected with **unitary cuts**.

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- Main formula to get the anomalous dimension matrix:

$$e^{-i\pi D} F_O^*(\vec{n}) = \sum_{\vec{m}} S_{nm} F_O^*(\vec{m})$$

Dilatation operator

$$D \equiv \sum_{\text{all particles}} p_i \frac{\partial}{\partial p_i}$$

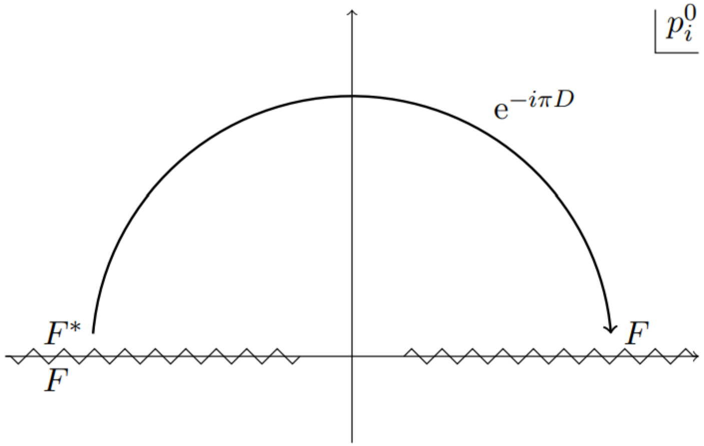
Form factor

$$F_O(\vec{n}) \equiv {}_{\text{out}}\langle \vec{n}|\, O(0)\, |0\rangle$$

S-matrix

$$S_{nm} \equiv \langle \vec{n}|\, \hat{S}\, |\vec{m}\rangle = {}_{\text{out}}\langle \vec{n}|\, \vec{m}\rangle_{\text{in}}$$

Anomalous dimension matrix from on-shell amplitude methods



- Action of the dilatation operator on a form factor:

$$F = e^{-i\pi D} F^*$$

- By unitarity, $\mathbb{1} = \sum_{\vec{m}} |\vec{m}\rangle_{\text{in in}}\langle\vec{m}|$, so we can write:

$$F_O(\vec{n}) \equiv {}_{\text{out}}\langle\vec{n}|\, O(0)\, |0\rangle = \sum_{\vec{m}} {}_{\text{out}}\langle\vec{n}|\, \vec{m}\rangle_{\text{in in}}\langle\vec{m}|\, O(0)\, |0\rangle.$$

- And finally we obtain:

$$e^{-i\pi D}\, F_O^*(\vec{n}) = \sum_{\vec{m}} S_{nm}\, F_O^*(\vec{m})$$

Anomalous dimension matrix from on-shell amplitude methods

- Regularization of the IR and UV divergences of the form factors. In dimensional regularization with the MS scheme, the Callan-Symanzik equation is

$$\left(\mu\partial_\mu + \gamma - \gamma_{\text{IR}} + \beta_g\partial_g\right) F_O(\vec{n};\mu) = 0$$

- The dilatation operator is related to the renormalization scale by: $D = -\mu\partial_\mu$

- Acting on the form factor, $DF_O \approx -\mu\partial_\mu F_O^{(1)} = (\gamma - \gamma_{\text{IR}} + \beta_g\partial_g)^{(1)} F_O^{(0)}$

- Now we can get the form factor anomalous dimensions from $e^{-i\pi D}\, F_O^*(\vec{n}) = \sum_{\vec{m}} S_{nm}\, F_O^*(\vec{m})$

Expand in perturbation theory to get order-by-order expressions for γ