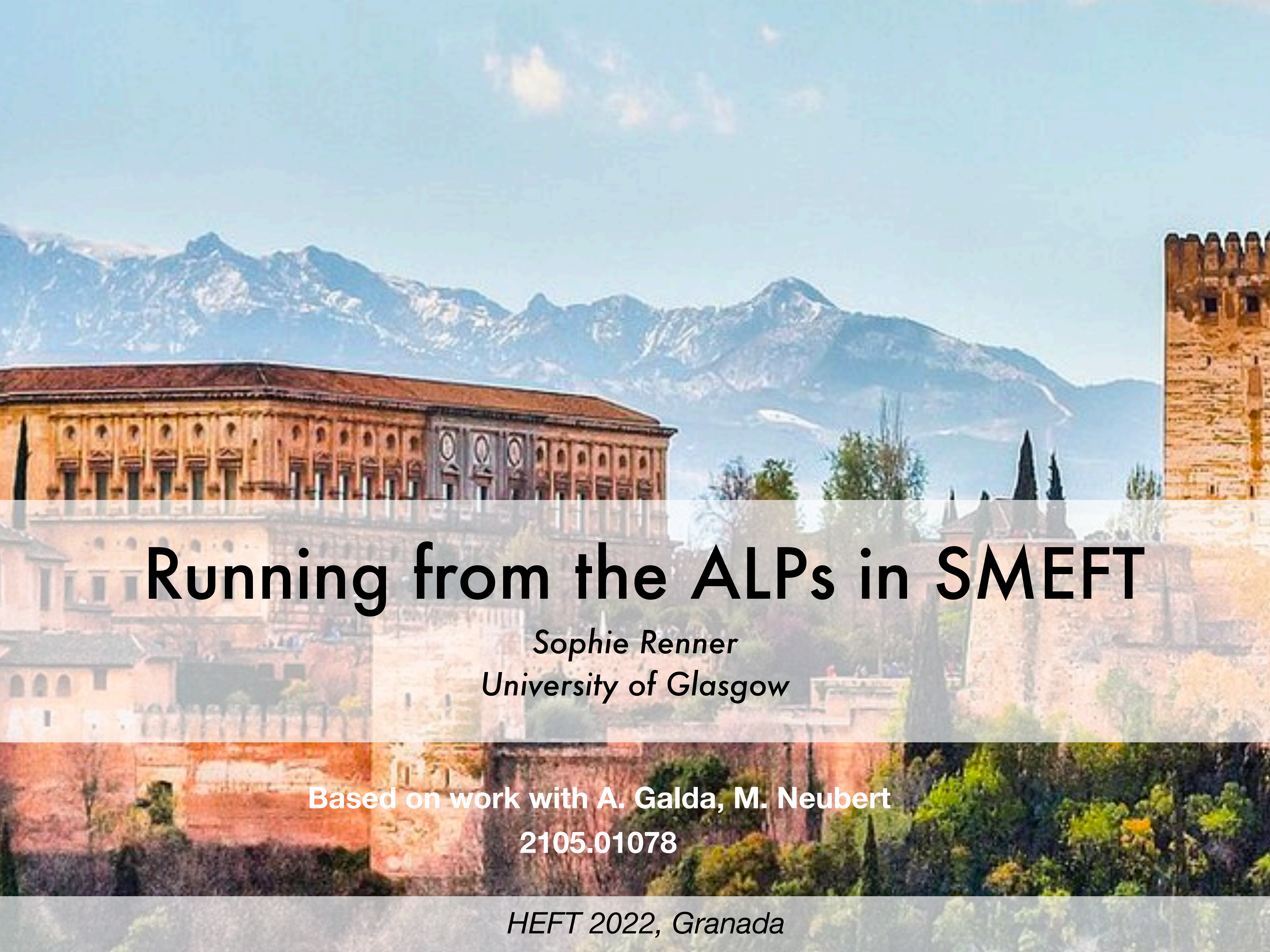


Running from the ALPs in SMEFT

Sophie Renner
University of Glasgow

Based on work with A. Galda, M. Neubert
2105.01078

HEFT 2022, Granada

Why axion like particles (ALPs)?

MODEL-BUILDING MOTIVATIONS:

Any dynamics with a spontaneously broken approximate global symmetry will produce light spinless particles

Analogy: QCD pions

$\Lambda_{\rm QCD}$ $\sim$ GeV — $p, n, \ldots$

m_π — π

Pions are pseudo goldstone bosons of an approximate spontaneously broken symmetry

BSM physics

Λ_{UV} $\gtrsim$ TeV — ??

m_a — a

ALP is a pseudo-goldstone boson

CP-odd gauge singlet

Mass much below scale of BSM physics

<u>Many motivated explicit models:</u> e.g. QCD axion, dark sector models, flavon models, composite Higgs models, ...

Why ALPs? Motivations II

MODEL-INDEPENDENT MOTIVATIONS:

All new particles are heavy ($m \gg v$)? → SM EFT (or similar)

One or more light ($m \lesssim v$) BSM particles?

+ possibly higher spin...

spin 0

ALP (Goldstone boson explains lightness)

spin 1/2

RH neutrino/heavy neutral lepton

spin 1

Dark photon/Z'

spin 3/2

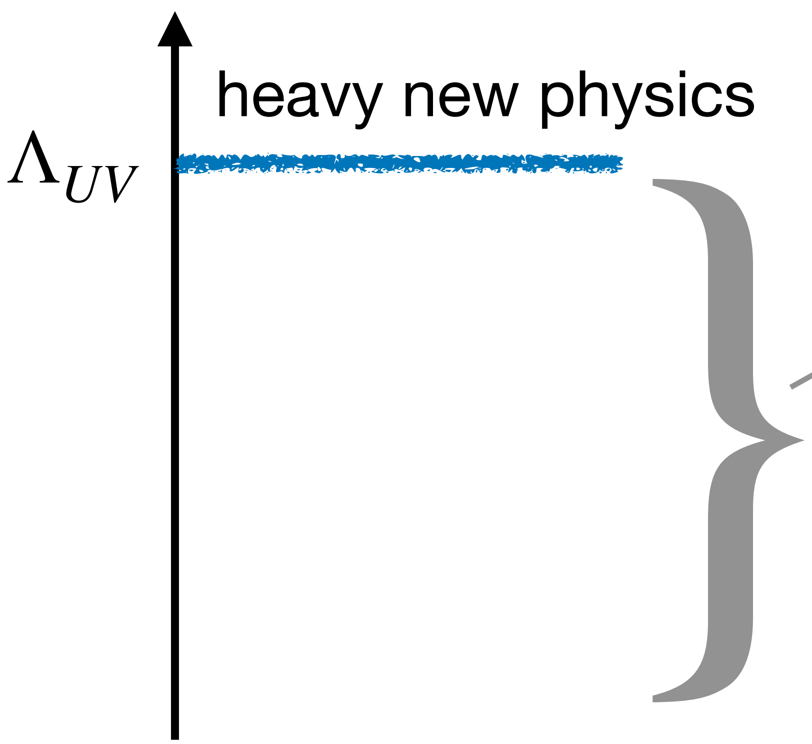
e.g. gravitino/ composite dark sector resonance

Outline of talk

- ALP EFT at dimension five and six
- How ALP loops can affect the RGEs of SMEFT-like operators
- Overview and features of new source terms for the RGEs
- Directions for applications and phenomenology

ALP EFT to dimension 5

Don't need to know the details of the UV physics to study the ALP



Here particle content is SM+ALP

1) ALP is pseudoscalar under CP, and its interactions preserve CP

2) Invariant under ALP shift symmetry, $a \to a + \sigma$, broken only by ALP mass term

$F = Q, u, d, L, e$

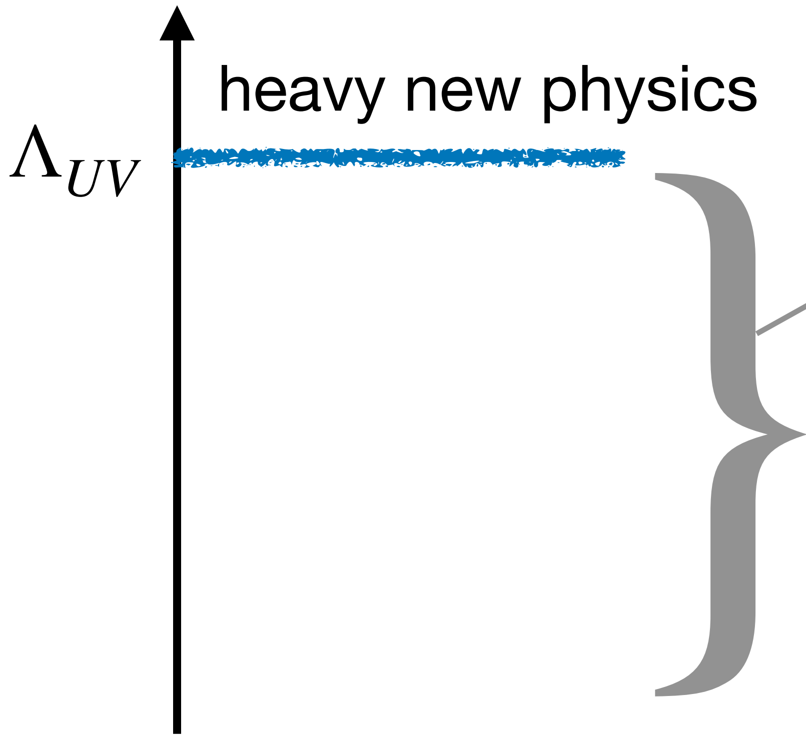
$$\mathcal{L}_{\rm eff}^{D\leq 5} = \frac{1}{2}\,(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 + \frac{\partial^\mu a}{f}\,\sum_F \bar{\psi}_F\, \boldsymbol{c}_F\, \gamma_\mu \psi_F$$
$$+\, c_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde{G}^{\mu\nu,a} + c_{WW}\,\frac{\alpha_2}{4\pi}\,\frac{a}{f}\,W_{\mu\nu}^A\,\tilde{W}^{\mu\nu,A} + c_{BB}\,\frac{\alpha_1}{4\pi}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde{B}^{\mu\nu}$$

$\Lambda_{UV} = 4\pi f$

Then the parameter space of the model depends on $m_a, f, \mathbf{c}_F, c_{XX}$

hermitian matrices in flavour space

ALP EFT vs SMEFT



Here particle content is SM+ALP

Only difference between SMEFT and ALP EFT is the addition of ONE more degree of freedom below Λ

<u>This extra d.o.f has large effects at dim 5:</u>

SMEFT at dim 5:

$$\mathscr{L}^{(d=5)}_{SMEFT} = \frac{c_5}{\Lambda} HHLL + h.c.$$

ALP at dim 5:

+ ALP couplings to all SM fields (prev. slide)

<u>...But doesn't change much at dim 6:</u>

SMEFT at dim 6:

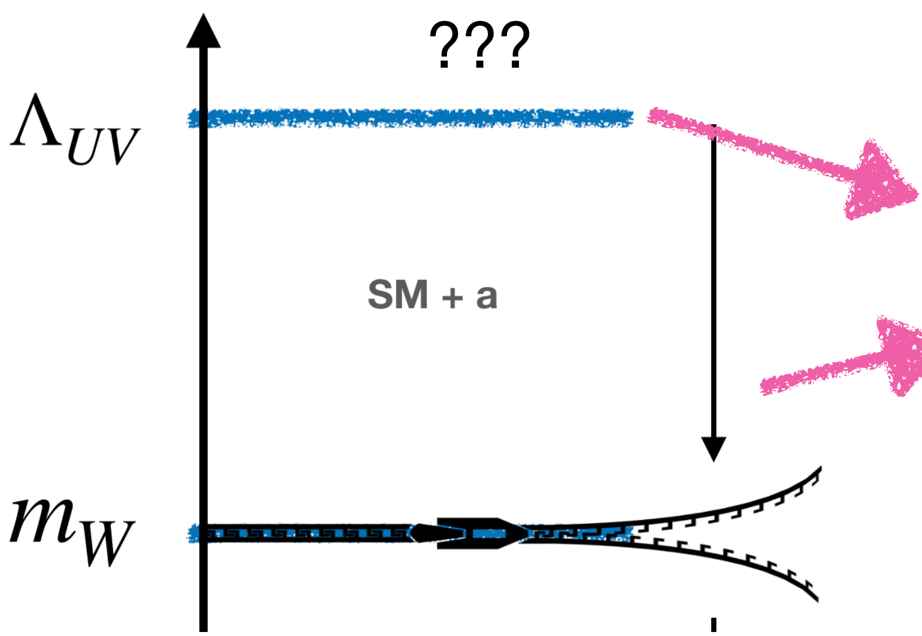
2499 parameters (for B and L conserving)

ALP at dim 6:

$$+ c_{aH}(H^\dagger H)(\partial_\mu a)(\partial^\mu a)$$

Seeing the ALP in SMEFT-like operators

Within an ALP theory, different ways of getting contributions to dim 6 operators:



1) Contributions from integrating out unknown fields at Λ

2) Contributions from ALP loops:

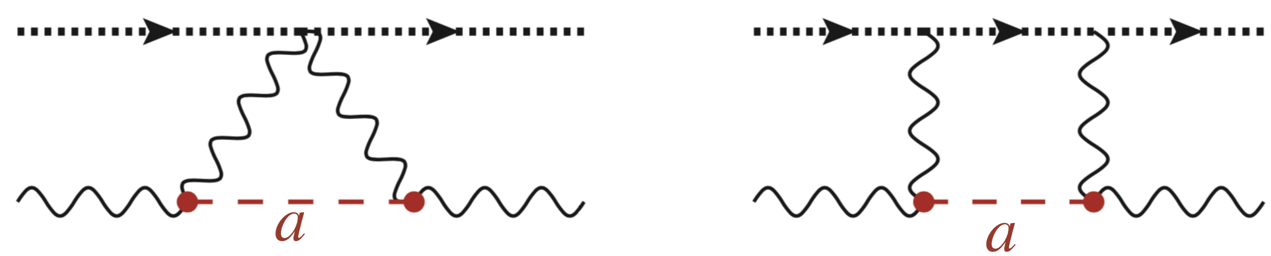
Source terms quadratic in dim-5 ALP couplings

$$\frac{d}{d\ln\mu}C_i^{\text{SMEFT}} - \gamma_{ji}^{\text{SMEFT}}\,C_j^{\text{SMEFT}} = \frac{S_i}{(4\pi f)^2}$$

Galda, SR, Neubert, 2105.01078

e.g. contributions to RG of

$$H^\dagger H\, W^I_{\mu\nu} W^{I\mu\nu}$$



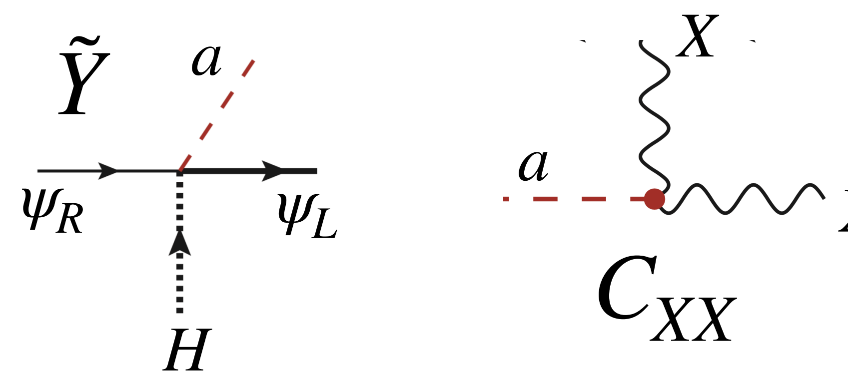
Calculating the RG source terms

We use the equivalent Lagrangian

$$
\mathcal{L}^{D=5\,\prime}_{\mathrm{SM+ALP}} = C_{GG}\,\frac{a}{f}\,G^a_{\mu\nu}\,\tilde{G}^{\mu\nu,a} + C_{WW}\,\frac{a}{f}\,W^I_{\mu\nu}\,\tilde{W}^{\mu\nu,I} + C_{BB}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde{B}^{\mu\nu}
- \frac{a}{f}\left(\bar{Q}\,\tilde{H}\,\widetilde{\boldsymbol{Y}}_u\,u_R + \bar{Q}\,H\,\widetilde{\boldsymbol{Y}}_d\,d_R + \bar{L}\,H\,\widetilde{\boldsymbol{Y}}_e\,e_R + \mathrm{h.c.}\right),
$$

$$
\widetilde{\boldsymbol{Y}}_u = i\left(\boldsymbol{Y}_u\,\boldsymbol{c}_u - \boldsymbol{c}_Q\boldsymbol{Y}_u\right), \qquad \widetilde{\boldsymbol{Y}}_d = i\left(\boldsymbol{Y}_d\,\boldsymbol{c}_d - \boldsymbol{c}_Q\boldsymbol{Y}_d\right), \qquad \widetilde{\boldsymbol{Y}}_e = i\left(\boldsymbol{Y}_e\,\boldsymbol{c}_e - \boldsymbol{c}_L\boldsymbol{Y}_e\right)
$$

ALP vertices are of the form:

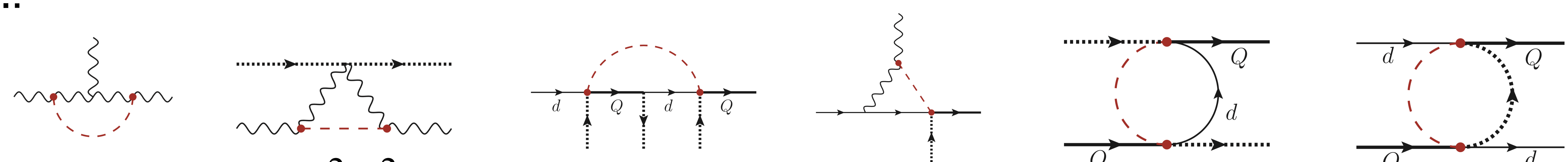


$X = \{B, W, G\}$

Use basis of dim 6 operators which is complete *before* using EOM relations ("Green's Basis")

Then can calculate 1PI diagrams to find the counterterms

e.g.:



X^3 $\quad H^2X^2$ $\quad \psi^2H^3$ $\quad \psi^2XH$ $\quad \psi^2H^2D$ $\quad \psi^4$

In a final step, use EOMs to get back to the Warsaw basis

Green's basis and all EOM relations: Gherardi, Marzocca, Venturini 2008.09548

$\propto C_{GG}^2$

$\propto C_{WW}^2$

$\propto C_{BB}^2$

$\propto \tilde{Y}^2$

$\propto \tilde{Y}\, C_{XX}$

$\propto C_{WW} C_{BB}$

Almost every operator in Warsaw basis is sourced by the ALP!

Galda, SR, Neubert, 2105.01078

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

$8: (\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$

$8: (\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$

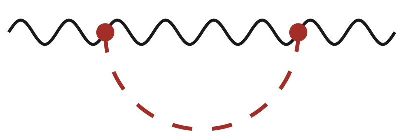
$8: (\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$8: (\bar{L}R)(\bar{R}L) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8: (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

Patterns and structure

ALP-boson interactions appear in many places via EOMs



$\Downarrow$

$$\widehat{Q}_{G,2} = (D^\rho G_{\rho\mu})^a (D_\omega G^{\omega\mu})^a\,,$$
$$\widehat{Q}_{W,2} = (D^\rho W_{\rho\mu})^I (D_\omega W^{\omega\mu})^I\,,$$
$$\widehat{Q}_{B,2} = (D^\rho B_{\rho\mu}) (D_\omega B^{\omega\mu})\,.$$

$$\begin{aligned}
\widehat{Q}_{W,2} &\cong \frac{g_2^2}{4}\left(H^\dagger i\overleftrightarrow{D}^I_\mu H + \bar{Q}\gamma_\mu\sigma^I Q + \bar{L}\gamma_\mu\sigma^I L\right)^2 \\
&= \frac{g_2^2}{4}\Big[-4m_H^2\,(H^\dagger H)^2 + 4\lambda\, Q_H + 3Q_{H\Box} + 2\left([Q^{(3)}_{Hl}]_{pp} + [Q^{(3)}_{Hq}]_{pp}\right) \\
&\quad + 2\left[(Y_u)_{pr}\,[Q_{uH}]_{pr} + (Y_d)_{pr}\,[Q_{dH}]_{pr} + (Y_e)_{pr}\,[Q_{eH}]_{pr} + \text{h.c.}\right] \\
&\quad + 2\,[Q^{(3)}_{lq}]_{pprr} + 2\,[Q_{ll}]_{prrp} - [Q_{ll}]_{pprr} + [Q^{(3)}_{qq}]_{pprr}\Big]\,,
\end{aligned}$$

$\Longrightarrow$ correlated effects in many operators

Also sum rules on $\tilde{Y}$ contributions from ALP shift symmetry:
Bonnefoy, Grojean, Kley 2206.04182

The presence of the ALP can change conclusions c.f. SMEFT-only running

e.g. <u>in SMEFT</u>, X^3 operators are only *self-renormalised:*

Helicity argument for why:
Cheung, Shen, 1505.01844

$$\dot{C}_G = (12c_{A,3} - 3b_{0,3})\,g_3^2 C_G$$
$$\dot{C}_W = (12c_{A,2} - 3b_{0,2})\,g_2^2 C_W$$

$\Longrightarrow$ if zero at Λ, zero at m_W (to 1-loop)

Alonso, Jenkins, Manohar, Trott 1312.2014

But <u>in ALP EFT</u>, same operators are renormalised by ALP-boson interactions:

e.g. $\dfrac{d}{d\ln\mu}C_G(\mu) \ni \dfrac{8g_s}{(4\pi f)^2}C_{GG}^2$ $\Longrightarrow$ unavoidable in an ALP theory if $C_{GG} \neq 0$

Searching for new physics

Where will the effects of ALP-induced running be important?

▶ **Many SMEFT operators only generated *at loop level*** by weakly coupled UV completions

Einhorn, Wudka 1307.0478
Craig, Jiang, Li, Sutherland 2001.00017

e.g. dipole operators, 3 field strength operators

see Chala, Guedes, Ramos, Santiago 2012.09017 for ALP-induced RG of dipole operators below EW scale

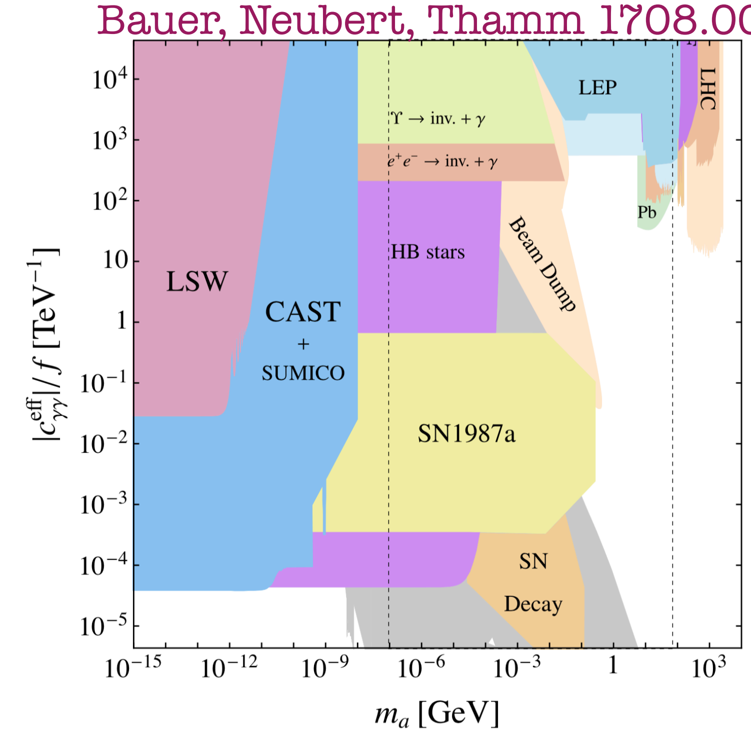
▶ **Best way to discover ALPs: when they can be produced *on-shell***

Constraints found in this way depend strongly on ALP mass and decay modes/width

But contributions to SMEFT operators are ~mass independent (if $m_a < m_W$)

Maybe can use them to fill in gaps?

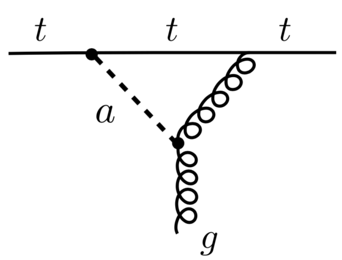
Bauer, Neubert, Thamm 1708.00443



Other ALP mass & width independent constraints from non-resonant searches:
Gavela, No, Sanz, de Trocóniz 1905.12953,
Carrà et al. 2106.10085,
Bonilla, Brivio, Machado-Rodríguez, de Trocóniz 2202.03450

Pheno and applications: $\hat{\mu}_t$

Top chromomagnetic dipole moment



$$\hat{\mu}_t = \frac{y_t v^2}{g_s} \Re e\, C_{uG}^{33}$$

$$-0.014 < \hat{\mu}_t < 0.004$$

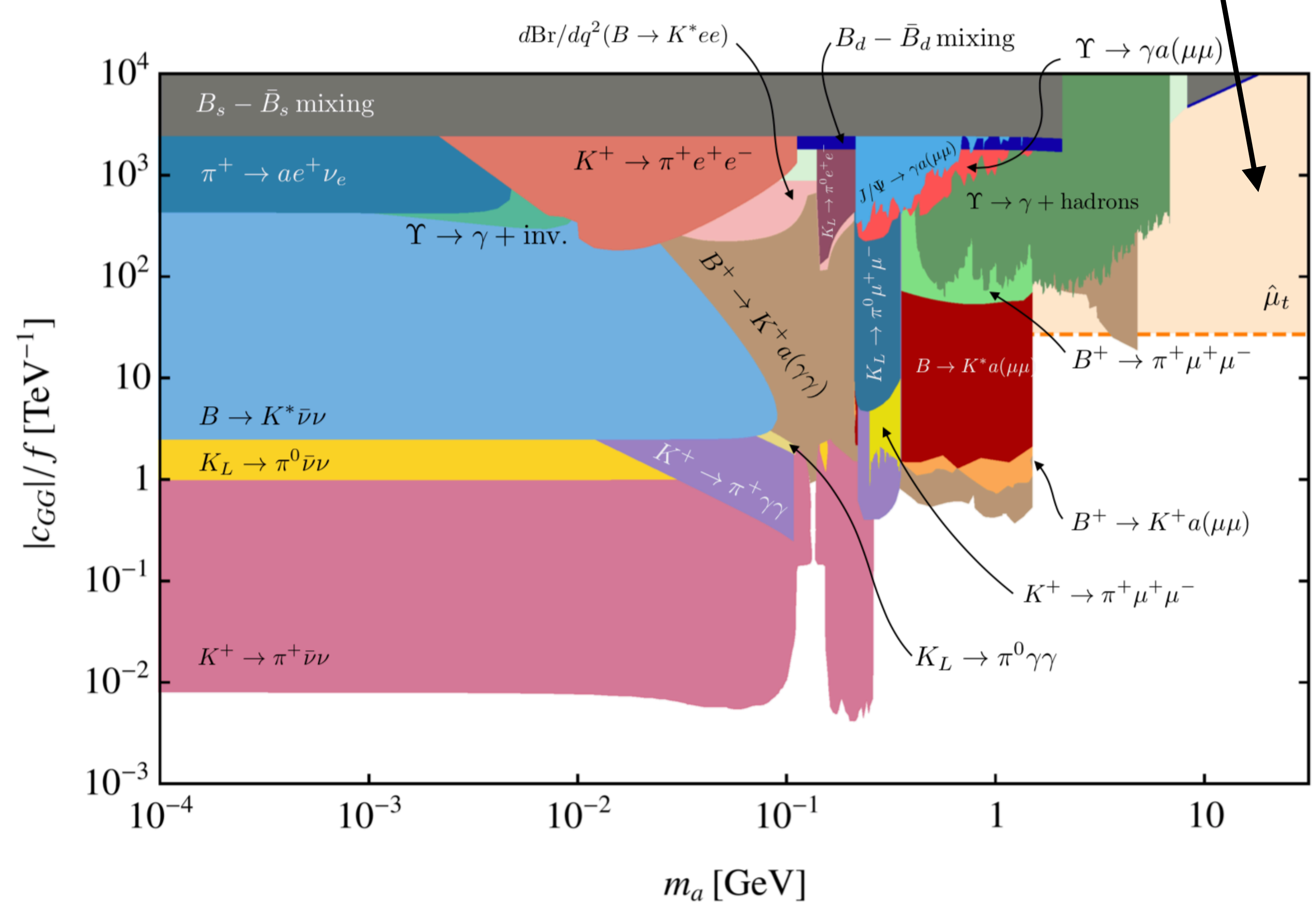
CMS, 1907.03729

ALP contribution:

$$\hat{\mu}_t \approx -\frac{8 m_t^2}{(4\pi f)^2}\left[c_{tt}\, C_{GG}\, \ln\frac{4\pi f}{m_t} - \frac{25\alpha_s}{4\pi}\, C_{GG}^2\, \ln^2\frac{4\pi f}{m_t}\right]$$

$$-0.68 < \left(c_{tt}\, C_{GG} - 0.94\, C_{GG}^2\right) \times \left[\frac{1\,\text{TeV}}{f}\right]^2 < 2.38 \qquad (95\%\ \text{CL})$$

~ALP mass independent bound



Caveat:

CMS bound found from distributions of $t\bar{t}$...

ALP emission from $t\bar{t}$ pair?

Production via an ALP?

$pp \to a \to \bar{t}t$ for heavy ALPs studied in Bonilla, Brivio, Gavela, Sanz 2107.11392

Pheno and applications: EWPTs

Z-pole measurements constrain SMEFT very strongly...

e.g. (first 2 generations): $-0.11\,\mathrm{TeV}^{-2} < C_{Hq}^{(3)} < 0.012\,\mathrm{TeV}^{-2} \qquad (95\%\ \mathrm{CL})$

marginalised bound from Ellis, Madigan, Mimasu, Sanz, You 2012.02779

ALP contribution:

$$C_{Hq}^{(3)} \approx -\frac{\alpha}{3\pi \sin^2\theta_w}\frac{C_{WW}^2}{f^2}\ln\frac{4\pi f}{m_Z} \quad \Longrightarrow \quad |C_{WW}| \times \left[\frac{1\,\mathrm{TeV}}{f}\right] < 2.50 \qquad (95\%\ \mathrm{CL})$$

Competitive with LEP diphoton constraints for ALPs between 10 MeV - 10 GeV

A combined/global fit should do much better

All these operators receive contributions $\propto C_{WW}^2$:

Q_{HW}	$H^\dagger H\, W^I_{\mu\nu} W^{I\mu\nu}$
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_H	$(H^\dagger H)^3$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p\tau^I\gamma^\mu q_r)$
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$

Conclusions & thoughts

- ALPs are well-motivated and well studied options for BSM physics

- ALP EFT includes SMEFT-like operators, and ALP loops affect their RGEs

- In general, very few options for a light neutral particle (RH neutrinos, ALPs, dark photon, ...)

  By including them in the EFT and calculating their effects, gain:
  a) More robust/general EFT
  b) New indirect probes of light new physics through dim 6 operators

Backup

Lagrangians

$$\mathcal{L}_{\rm SM+ALP}^{D=5} = c_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde G^{\mu\nu,a} + c_{WW}\,\frac{\alpha_2}{4\pi}\,\frac{a}{f}\,W_{\mu\nu}^I\,\tilde W^{\mu\nu,I} + c_{BB}\,\frac{\alpha_1}{4\pi}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde B^{\mu\nu} + \frac{\partial^\mu a}{f}\sum_F \bar\psi_F\,\boldsymbol{c}_F\,\gamma_\mu\psi_F\,.$$

$$\mathcal{L}_{\rm SM+ALP}^{D=5\,\prime} = C_{GG}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde G^{\mu\nu,a} + C_{WW}\,\frac{a}{f}\,W_{\mu\nu}^I\,\tilde W^{\mu\nu,I} + C_{BB}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde B^{\mu\nu} - \frac{a}{f}\left(\bar Q\,\tilde H\,\widetilde{\boldsymbol{Y}}_u u_R + \bar Q\,H\,\widetilde{\boldsymbol{Y}}_d\,d_R + \bar L\,H\,\widetilde{\boldsymbol{Y}}_e e_R + \text{h.c.}\right),$$

$$\widetilde{\boldsymbol{Y}}_u = i\left(\boldsymbol{Y}_u\,\boldsymbol{c}_u - \boldsymbol{c}_Q\boldsymbol{Y}_u\right), \qquad \widetilde{\boldsymbol{Y}}_d = i\left(\boldsymbol{Y}_d\,\boldsymbol{c}_d - \boldsymbol{c}_Q\boldsymbol{Y}_d\right), \qquad \widetilde{\boldsymbol{Y}}_e = i\left(\boldsymbol{Y}_e\,\boldsymbol{c}_e - \boldsymbol{c}_L\boldsymbol{Y}_e\right). \qquad (5)$$

Note the important fact that the ALP–boson couplings in (3) are also affected by the field redefinitions. One finds

$$\begin{aligned}
C_{GG} &= \frac{\alpha_s}{4\pi}\left[c_{GG} + \frac{1}{2}\,\text{Tr}\left(\boldsymbol{c}_d + \boldsymbol{c}_u - 2\,\boldsymbol{c}_Q\right)\right] \equiv \frac{\alpha_s}{4\pi}\,\tilde c_{GG}\,,\\
C_{WW} &= \frac{\alpha_2}{4\pi}\left[c_{WW} - \frac{1}{2}\,\text{Tr}\left(N_c\,\boldsymbol{c}_Q + \boldsymbol{c}_L\right)\right] \equiv \frac{\alpha_2}{4\pi}\,\tilde c_{WW}\,, \qquad (6)\\
C_{BB} &= \frac{\alpha_1}{4\pi}\left[c_{BB} + \text{Tr}\left[N_c\left(\mathcal{Y}_d^2\,\boldsymbol{c}_d + \mathcal{Y}_u^2\,\boldsymbol{c}_u - 2\,\mathcal{Y}_Q^2\,\boldsymbol{c}_Q\right) + \mathcal{Y}_e^2\,\boldsymbol{c}_e - 2\,\mathcal{Y}_L^2\,\boldsymbol{c}_L\right]\right] \equiv \frac{\alpha_1}{4\pi}\,\tilde c_{BB}\,,
\end{aligned}$$

ALP affecting dim 4 running

$$\frac{d\lambda}{d\ln\mu} = -\frac{16g_2^2}{3}\,\frac{m_H^2}{(4\pi f)^2}\,C_{WW}^2 + \text{SM contribution}$$

$$\beta^{(3)}(\{\alpha_i\}) = \beta^{(3)}_{\rm SM}(\{\alpha_i\}) + \frac{8m_a^2}{(4\pi f)^2}\,C_{GG}^2\,,$$

$$\beta^{(2)}(\{\alpha_i\}) = \beta^{(2)}_{\rm SM}(\{\alpha_i\}) + \frac{8m_a^2}{(4\pi f)^2}\,C_{WW}^2\,,$$

$$\beta^{(1)}(\{\alpha_i\}) = \beta^{(1)}_{\rm SM}(\{\alpha_i\}) + \frac{8m_a^2}{(4\pi f)^2}\,C_{BB}^2\,.$$

1 loop RG above EW scale

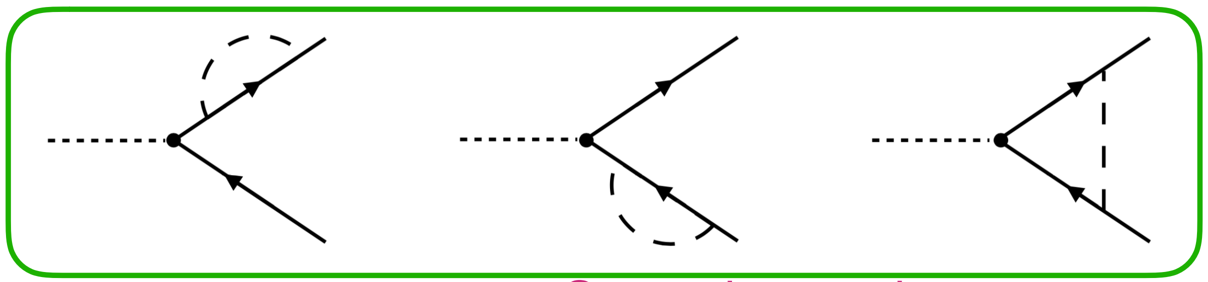
No running for gauge couplings

$$\frac{d}{d\ln\mu}\, c_{VV}(\mu) = 0\,; \quad V = G, W, B$$

Chetyrkin, Kniehl, Steinhauser, Bardeen 1998

Fermion couplings:

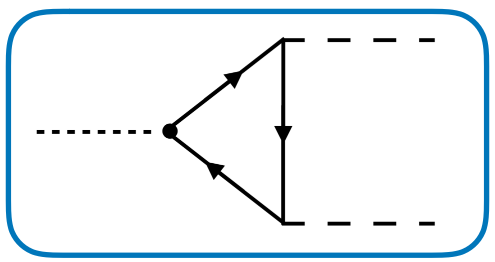
Yukawa interactions



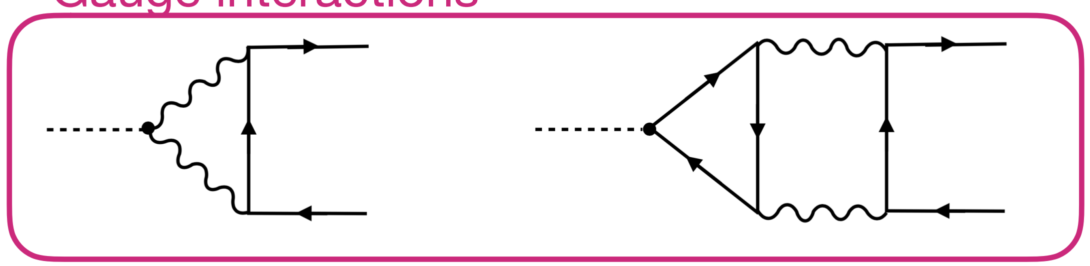
$$\tilde{c}_{GG} = c_{GG} + T_F\, \mathrm{Tr}\left(\boldsymbol{c}_u + \boldsymbol{c}_d - N_L\, \boldsymbol{c}_Q\right),$$

$$\tilde{c}_{WW} = c_{WW} - T_F\, \mathrm{Tr}\left(N_c\, \boldsymbol{c}_Q + \boldsymbol{c}_L\right),$$

$$\tilde{c}_{BB} = c_{BB} + \mathrm{Tr}\left[N_c\left(\mathcal{Y}_u^2\, \boldsymbol{c}_u + \mathcal{Y}_d^2\, \boldsymbol{c}_d - N_L\, \mathcal{Y}_Q^2\, \boldsymbol{c}_Q\right) + \mathcal{Y}_e^2\, \boldsymbol{c}_e - N_L\, \mathcal{Y}_L^2\, \boldsymbol{c}_L\right]$$



Gauge interactions



2 loop diagram included because it can be of the same order as the 1 loop diagram

$$c_{GG}\, \frac{\alpha_s}{4\pi}\, \frac{a}{f}\, G^a_{\mu\nu}\, \tilde{G}^{\mu\nu,a}$$

$$\tilde{c}_{GG} = c_{GG} + \frac{1}{2}\, \mathrm{Tr}\left(\boldsymbol{c}_u + \boldsymbol{c}_d - 2\boldsymbol{c}_Q\right)$$

$$\frac{d}{d\ln\mu}\, \boldsymbol{c}_Q(\mu) = \frac{1}{32\pi^2}\left\{\boldsymbol{Y}_u \boldsymbol{Y}_u^\dagger + \boldsymbol{Y}_d \boldsymbol{Y}_d^\dagger, \boldsymbol{c}_Q\right\} - \frac{1}{16\pi^2}\left(\boldsymbol{Y}_u\, \boldsymbol{c}_u \boldsymbol{Y}_u^\dagger + \boldsymbol{Y}_d\, \boldsymbol{c}_d \boldsymbol{Y}_d^\dagger\right)$$
$$+ \left[\frac{\beta_Q}{8\pi^2}\, X - \frac{3\alpha_s^2}{4\pi^2}\, C_F^{(3)}\, \tilde{c}_{GG} - \frac{3\alpha_2^2}{4\pi^2}\, C_F^{(2)}\, \tilde{c}_{WW} - \frac{3\alpha_1^2}{4\pi^2}\, \mathcal{Y}_Q^2\, \tilde{c}_{BB}\right] \mathbb{1}\,,$$

$q = u, d$

$$\frac{d}{d\ln\mu}\, \boldsymbol{c}_q(\mu) = \frac{1}{16\pi^2}\left\{\boldsymbol{Y}_q^\dagger \boldsymbol{Y}_q, \boldsymbol{c}_q\right\} - \frac{1}{8\pi^2}\, \boldsymbol{Y}_q^\dagger \boldsymbol{c}_Q \boldsymbol{Y}_q + \left[\frac{\beta_q}{8\pi^2}\, X + \frac{3\alpha_s^2}{4\pi^2}\, C_F^{(3)}\, \tilde{c}_{GG} + \frac{3\alpha_1^2}{4\pi^2}\, \mathcal{Y}_q^2\, \tilde{c}_{BB}\right] \mathbb{1}$$

$$X = \mathrm{Tr}\left[3\boldsymbol{c}_Q\left(\boldsymbol{Y}_u \boldsymbol{Y}_u^\dagger - \boldsymbol{Y}_d \boldsymbol{Y}_d^\dagger\right) - 3\boldsymbol{c}_u \boldsymbol{Y}_u^\dagger \boldsymbol{Y}_u + 3\boldsymbol{c}_d \boldsymbol{Y}_d^\dagger \boldsymbol{Y}_d - \boldsymbol{c}_L \boldsymbol{Y}_e \boldsymbol{Y}_e^\dagger + \boldsymbol{c}_e \boldsymbol{Y}_e^\dagger \boldsymbol{Y}_e\right]$$

RGEs for the lepton couplings are highly analogous!