Rambling about $1/\Lambda^4$

(and sorta $\sum_n^\infty 1/\Lambda^n$)

Tyler Corbett

Niels Bohr Institute

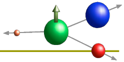
geoSMEFT: Corbett, Helset, Martin, Trott, ...

arXiv:2001.01453, 2007.00565, 2102.02819 2106.10284, 2107.07470

Outline

1. D6 squared
2. geoSMEFT
3. HEFT vs SMEFT

Which EFT?



- How to compare different nuclear beta decays?
  - ➔ Effective Lagrangian at the **hadron** level!

[Lee & Yang'1956]

$$\begin{aligned}\mathcal{L}_{\text{Lee-Yang}} = & -\bar{p}\gamma^{\mu}n\left(C_V^{+}\bar{e}\gamma_{\mu}\nu_L + C_V^{-}\bar{e}\gamma_{\mu}\nu_R\right) - \bar{p}\gamma^{\mu}\gamma_5 n\left(C_A^{+}\bar{e}\gamma_{\mu}\nu_L - C_A^{-}\bar{e}\gamma_{\mu}\nu_R\right) \\ & - \bar{p}n\left(C_S^{+}\bar{e}\nu_L + C_S^{-}\bar{e}\nu_R\right) - \frac{1}{2}\bar{p}\sigma^{\mu\nu}n\left(C_T^{+}\bar{e}\sigma_{\mu\nu}\nu_L + C_T^{-}\bar{e}\sigma_{\mu\nu}\nu_R\right) \\ & + \bar{p}\gamma_5 n\left(C_P^{+}\bar{e}\nu_L - C_P^{-}\bar{e}\nu_R\right) + \text{h.c.}\end{aligned}$$

$$d\Gamma \approx f(C_i, M_F, M_{GT})$$

For some transitions and observables:

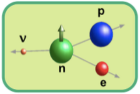
$\mathcal{O} \approx f(C_i)$ + small corrections

High precision measurements

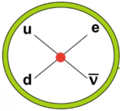
UV meaning of the C coefficients?
(within & beyond the SM)
(hadronization, RC, EFT, ...)

SM fit

$$\begin{pmatrix} v^2 C_V^+ \\ v^2 C_A^+ \\ \rho_F \\ \rho_{Ne} \\ \rho_{Na} \\ \rho_P \\ \rho_{Ar} \\ \rho_K \end{pmatrix} = \begin{pmatrix} 0.98576(22) \\ -1.25754(39) \\ -1.2955(13) \\ 1.60157(75) \\ -0\cdots127(11) \\ -0\cdots380(21) \\ -\cdots34(25) \\ 0.5787(20) \end{pmatrix}$$



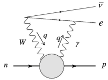
$$V_{ud} = 0.97382(24)$$
$$g_A = 1.27553(45)$$



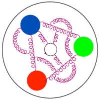
$$C_V^+ = \frac{V_{ud}}{v^2} g_V \sqrt{1 + \Delta_R^V}$$
$$C_A^+ = -\frac{V_{ud}}{v^2} g_A \sqrt{1 + \Delta_R^A}$$

Inner RC:

[Seng et al., PRL121 (2018)]
[Gorchtein & Seng, JHEP10 (2021)]



NEW: missed % level corrections?
Cirigliano et al., 2202.10439



Axial charge

$$\langle p | \bar{u} \gamma_\mu \gamma_5 d | n \rangle$$

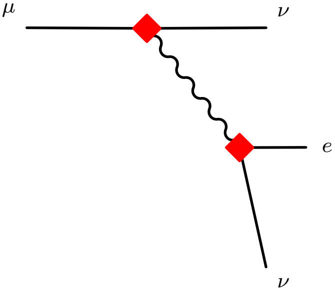
g_A = 1.2642(93) CalLat, Nature'18 + update

g_A = 1.218(39) PNDME, PRD'18

g_A = 1.246(28) FLAG'21

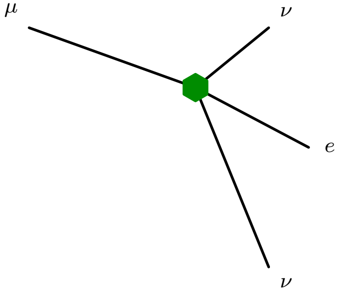
The Fermi-theory example

In the SM



$$\mathcal{M} \sim \frac{g_{\mathrm{W}}^2}{2} \frac{(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e)}{k^2 - M_W^2}$$

In the Fermi theory



$\frac{1}{M_W^2}(\bar{\psi}\gamma^\mu P_L \psi)^2$

$\frac{1}{M_W^4}\partial^2(\bar{\psi}\gamma^\mu P_L \psi)^2$

$$\mathcal{M} \sim -\frac{g_{\mathrm{W}}^2}{2M_W^2}(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e) - \frac{g_{\mathrm{W}}^2 k^2}{2M_W^4}(\bar{\nu}_\mu \gamma^\mu P_L \mu)(\bar{e}\gamma^\mu P_L \nu_e) + \cdots$$

Fetscher & Gerber PDG review of muon decay

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\chi=S,V,T,\\ \epsilon,\mu=L,R}} g^{\chi}_{\epsilon,\mu} \langle \bar{e}_\epsilon | \Gamma^\chi | \nu_e \rangle \langle \bar{\nu}_\mu | \Gamma^\chi | \mu_\mu \rangle$$

Table 57.1: Coupling constants $g^{\gamma}_{\varepsilon\mu}$ and some combinations of them. Ninety-percent confidence level experimental limits. The limits on $|g^S_{LL}|$ and $|g^V_{LL}|$ are from [8–10], and the others from a general analysis of muon decay measurements. Top three rows: [11], fourth row: [12], next three rows: [13], last row: [14]. The experimental uncertainty on the muon polarization in pion decay is included. Note that, by definition, $|g^S_{\varepsilon\mu}| \leq 2$, $|g^V_{\varepsilon\mu}| \leq 1$ and $|g^T_{\varepsilon\mu}| \leq 1/\sqrt{3}$.

$\lvert g^S_{RR}\rvert < 0.035$	$\lvert g^V_{RR}\rvert < 0.017$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.050$	$\lvert g^V_{LR}\rvert < 0.023$	$\lvert g^T_{LR}\rvert < 0.015$
$\lvert g^S_{RL}\rvert < 0.420$	$\lvert g^V_{RL}\rvert < 0.105$	$\lvert g^T_{RL}\rvert < 0.105$
$\lvert g^S_{LL}\rvert < 0.550$	$\lvert g^V_{LL}\rvert > 0.960$	$\lvert g^T_{LL}\rvert \equiv 0$
$\lvert g^S_{LR} + 6\, g^T_{LR}\rvert < 0.143$	$\lvert g^S_{RL} + 6\, g^T_{RL}\rvert < 0.418$	
$\lvert g^S_{LR} + 2\, g^T_{LR}\rvert < 0.108$	$\lvert g^S_{RL} + 2\, g^T_{RL}\rvert < 0.417$	
$\lvert g^S_{LR} - 2\, g^T_{LR}\rvert < 0.070$	$\lvert g^S_{RL} - 2\, g^T_{RL}\rvert < 0.418$	
$Q_{RR} + Q_{LR} < 8.2\times 10^{-4}$		

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The "canonical" example of an EFT

$\Rightarrow$ leading order contribution at $|\mathcal{M}_{\mathrm{D6}}|^2$

$|g^V_{\varepsilon\mu}| \leq 1$ and $|g^T_{\varepsilon\mu}| \leq 1/\sqrt{3}$.

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Ambiguities of $(\text{D6})^2$

Imagine we match a UV model ⇒ this IR Lagrangian: (top-down)

$$\begin{aligned}\mathcal{L}_{\rm IR} &= \mathcal{L}_{\rm SM} + \left(\frac{c_{eH}^{(6)}}{\Lambda^2}(H^\dagger H)\bar{L}eH + \frac{c_{eH}^{(8)}}{\Lambda^4}(H^\dagger H)^2\bar{L}eH + h.c.\right) \\ &\quad + c_{Hl}^{1,(6)}\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{L}\gamma_\mu L\right) + c_{Hl}^{1,(8)}(H^\dagger H)(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{L}\gamma_\mu L)\end{aligned}$$

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Equivalence theorem $\Rightarrow$ can xform fields (consistently) & the S-matrix remains invariant:

$$L \to L + \alpha(H^\dagger H)L$$

Ambiguities of (D6)^2

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$$\begin{aligned}\mathcal{L}_{\rm IR} \to\ & i\bar{L}\not{D}L - Y(\bar{L}eH + \bar{e}LH^\dagger) + \frac{i\alpha}{\Lambda^2}(H^\dagger H)\bar{L}\overleftrightarrow{D_\mu}\gamma_\mu L + \frac{i\alpha^2}{2\Lambda^4}(H^\dagger H)^2\bar{L}\overleftrightarrow{D_\mu}\gamma_\mu L \\ & + \frac{\left(c_{Hl}^{1,(6)}\right)}{\Lambda^2}(H^\dagger i\overleftrightarrow{D_\mu}H)(\bar{L}\gamma_\mu L) + \frac{c_{Hl}^{1,(8)} + 2\alpha c_{Hl}^{1,(6)}}{\Lambda^4}(H^\dagger H)(H^\dagger i\overleftrightarrow{D_\mu}H)(\bar{L}\gamma_\mu L) \\ & + \frac{c_{eH}^{(6)} - Y\alpha}{\Lambda^2}(H^\dagger H)(\bar{L}eH + \bar{e}LH^\dagger) + \frac{c_{eH}^{(8)} + c_{eH}^{(6)}\alpha}{\Lambda^4}(H^\dagger H)^2(\bar{L}eH + \bar{e}LH^\dagger) \\ & + \frac{c_{Hl}^{1,(6)}\alpha^2 + 2c_{Hl}^{1,(8)}\alpha}{\Lambda^6}Q_{Hl}^{1,(10)} + \frac{c_{eH}^{(8)}\alpha}{\Lambda^6}Q_{eH}^{(10)} + \frac{\alpha^2 c_{Hl}^{1,(8)}}{\Lambda^8}Q_{Hl}^{1,(12)}\end{aligned}$$

Ambiguities of $(\mathrm{D6})^2$ II

So we have two different bases of operators:

$$
\begin{aligned}
\mathcal{L}_{\rm IR}^{(0)} &= \mathcal{L}_{\rm SM} + c_{Hl}^{1,(6)}\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{L}\gamma_\mu L\right) + c_{Hl}^{1,(8)}(H^\dagger H)\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{L}\gamma_\mu L\right) \\
&\quad + c_{eH}^{(6)}(H^\dagger H)(\bar{L}eH + \bar{e}LH^\dagger) + c_{eH}^{(8)}(H^\dagger H)^2(\bar{L}eH + \bar{e}LH^\dagger) \\
\mathcal{L}_{\rm IR}^{(1)} &= \mathcal{L}_{\rm SM} + i\kappa^{(6)}(H^\dagger H)\left(\bar{L}\overleftrightarrow{D}_\mu\gamma_\mu L\right) + i\kappa^{(8)}(H^\dagger H)^2\left(\bar{L}\overleftrightarrow{D}_\mu\gamma_\mu L\right) \\
&\quad + \kappa_{Hl}^{1,(6)}\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{L}\gamma_\mu L\right) + \kappa_{Hl}^{1,(8)}(H^\dagger H)\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{L}\gamma_\mu L\right) \\
&\quad + \kappa_{eH}^{(6)}(H^\dagger H)(\bar{L}eH + \bar{e}LH^\dagger) + \kappa_{eH}^{(8)}(H^\dagger H)^2(\bar{L}eH + \bar{e}LH^\dagger)
\end{aligned}
$$

Ambiguities of $(\mathrm{D6})^2$ II

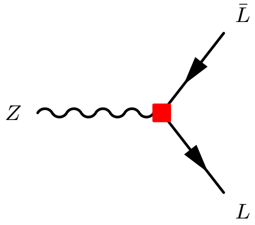
So we have two different bases of operators:

$$\mathcal{L}_{\mathrm{IR}}^{(0)} = \mathcal{L}_{\mathrm{SM}} + c_{Hl}^{1,(6)}\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)(\bar{L}\gamma_\mu L) + c_{Hl}^{1,(8)}(H^\dagger H)\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)(\bar{L}\gamma_\mu L)$$
$$+c_{eH}^{(6)}(H^\dagger H)(\bar{L}eH + \bar{e}LH^\dagger) + c_{eH}^{(8)}(H^\dagger H)^2(\bar{L}eH + \bar{e}LH^\dagger)$$
$$\mathcal{L}_{\mathrm{IR}}^{(1)} = \mathcal{L}_{\mathrm{SM}} + i\kappa^{(6)}(H^\dagger H)\left(\bar{L}\overleftrightarrow{D}_\mu\gamma_\mu L\right) + i\kappa^{(8)}(H^\dagger H)^2\left(\bar{L}\overleftrightarrow{D}_\mu\gamma_\mu L\right)$$
$$+\kappa_{Hl}^{1,(6)}\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)(\bar{L}\gamma_\mu L) + \kappa_{Hl}^{1,(8)}(H^\dagger H)\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)(\bar{L}\gamma_\mu L)$$
$$+\kappa_{eH}^{(6)}(H^\dagger H)(\bar{L}eH + \bar{e}LH^\dagger) + \kappa_{eH}^{(8)}(H^\dagger H)^2(\bar{L}eH + \bar{e}LH^\dagger)$$

Which are related by:

$$\kappa^{(6)} = \alpha \qquad \kappa^{(8)} = \frac{\alpha^2}{2}$$
$$\kappa_{Hl}^{1,(6)} = c_{Hl}^{1,(6)} \qquad \kappa_{Hl}^{1,(8)} = c_{Hl}^{1,(8)} + 2\alpha c_{Hl}^{1,(6)}$$
$$\kappa_{He}^{(6)} = c_{eH}^{(6)} - Y\alpha \qquad \kappa_{He}^{(8)} = c_{eH}^{(8)} + \alpha c_{eH}^{(6)}$$

Ambiguities of $(\mathrm{D6})^2$, $Z \to \bar{L}L$



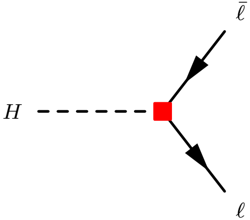
$$\mathcal{M}^{(0)} = \frac{g_Z}{2}\left[\text{(SM)} + c_{HL}^{1,(6)} v^2 + c_{HL}^{1,(8)} \frac{v^4}{2}\right] \bar{u}\not{\epsilon} P_L v$$

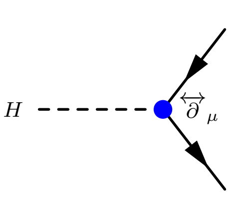
$$\mathcal{M}^{(1)} = \frac{g_Z}{2}\left[\text{(SM)} + \kappa_{HL}^{1,(6)} v^2 + \left(\kappa_{HL}^{1,(8)} - 2\kappa^{(6)}\kappa_{HL}^{1,(6)}\right) \frac{v^4}{2}\right] \bar{u}\not{\epsilon} P_L v$$

$$|\mathcal{M}^{(1)}|^2 \propto \frac{g_Z^2}{4}\left[\text{(SM)}^2 + \underbrace{2v^2\,\text{(SM)}\,\kappa_{Hl}^{1,(6)}}_{\text{Invariant}} + \underbrace{\left(\underbrace{(\kappa_{Hl}^{1,(6)})^2 - 2\,\text{(SM)}\,\kappa^6\kappa_{Hl}^{1,(6)}}_{\text{Not Invariant!}} + \text{(SM)}\kappa_{Hl}^{1,(8)}\right)}_{\text{Invariant}}\right]$$

$$\text{(SM)} = c_W^2 - s_W^2$$

Ambiguities of $(\mathrm{D6})^2$, $H \to \bar{\ell}\ell$





$$\mathcal{M}^{(0)} = -\frac{1}{\sqrt{2}}\left[Y - \frac{3v^2}{2}c^{(6)}_{eH} - \frac{5v^4}{4}c^{(8)}_{eH}\right](\bar{u}v)$$

$$\mathcal{M}^{(1)} = -\frac{1}{\sqrt{2}}\left[Y - \frac{v^2}{2}(3\kappa^{(6)}_{eH} + Y\kappa^{(6)}) + \frac{v^4}{8}\left(6\kappa^{(6)}\kappa^{(6)}_{eH} + 3(\kappa^{(6)})^2)Y - 2(5\kappa^{(8)}_{eH} + Y\kappa^{(8)})\right)\right]$$
$$+ v\left[\kappa^{(6)} + \left(\kappa^{(8)} - [\kappa^{(6)}]^2\right)v^2\right]\bar{m}(\bar{u}v) \qquad (\not{p}\psi = \pm\bar{m}\psi)$$

$$|\mathcal{M}^{(1)}|^2 \propto \frac{Y^2}{2} - \frac{3v^2}{2}Y\overbrace{\left(\kappa^{(6)}_{eH} + Y\kappa^{(6)}\right)}^{\to(c^{(6)}_{eH} - Y\alpha) + Y\alpha}$$
$$+ \frac{v^4}{8}\underbrace{\left[\underbrace{9(\kappa^{(6)})^2 + 28Y\kappa^{(6)}\kappa^{(6)}_{eH} + 24Y^2(\kappa^{(6)})^2}_{\text{Not Invariant!}} - 10Y\left(Y\kappa^{(8)} + \kappa^{(8)}_{eH}\right)\right]}_{\text{Invariant}}$$

Ambiguities of $(\text{D6})^2$, III

Let's denote:

$$\frac{1}{\Lambda^2}\mathcal{M}_6 \quad = \quad \text{the amplitude at } 1/\Lambda^2$$

$$\frac{1}{\Lambda^4}\mathcal{M}_{6^2} \quad = \quad \text{the amplitude with two insertions of D6 operators}$$

$$\frac{1}{\Lambda^4}\mathcal{M}_8 \quad = \quad \text{the amplitude with one insertion of D8 operators}$$

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For the coupling $Z\bar{L}L$, and after a bit of simplifying, $H\bar{\ell}\ell$:

$$\mathcal{M} = \mathcal{M}_{\text{SM}} + \underbrace{\frac{1}{\Lambda^2}\mathcal{M}_6}_{\text{Invariant}} + \underbrace{\frac{1}{\Lambda^4}\mathcal{M}_{6^2} + \frac{1}{\Lambda^4}\mathcal{M}_8}_{\text{Invariant}}$$

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So:

$$|\mathcal{M}_6|^2 \quad = \quad \text{Invariant!}$$

$$(\mathcal{M}_{6^2} + \mathcal{M}_8) \times \mathcal{M}_{\mathrm{SM}} \quad = \quad \text{Separately Invariant!}$$

Ambiguities of (D6)^2, the Equivalence Theorem

The equivalence theorem, per I.V. Tyutin hep-th/0001050

The equivalence theorem states "The independence of physical observables, in particular, the S–matrix, in quantum theory on changes of variables in the classical Lagrangian, i.e. on the choice of parametrization of the classical action"

Schwartz's textbook:

$$\mathcal{S} = \mathbb{1} + (2\pi)^4 \delta^4 \left(\Sigma p_i\right) i\mathcal{M}$$

$$\langle f|S - \mathbb{1}|i\rangle = i(2\pi)^4 \delta^4 \left(\Sigma p_i\right) \langle f|\mathcal{M}|i\rangle$$

If $\mathcal{S}$ is invariant then so is $\mathcal{M}$

- Implies that $\mathcal{M}_6$ and associated $|\mathcal{M}_6|^2$ term are well defined & and can be freely translated between bases (in the SMEFT)
- This doesn't apply to the corresponding $\mathcal{M}_8$ term, only $\mathcal{M}_{6^2} + \mathcal{M}_8$ is well defined

What are we actually measuring? (In Warsaw)

Say we can actually measure the $H\ell\ell$ process at the LHC:

In the SM we measure:

$$\mathcal{M}^2 \propto \frac{\bar{m}^2}{v^2}$$

In the SMEFT to $1/\Lambda^2$ we measure

$$\mathcal{M}^2 \propto \frac{\bar{m}^2}{v^2} + \left[\bar{m}^2\left(2c^{(6)}_{H\Box} - \frac{1}{2}c^{(6)}_{HD}\right) - \sqrt{2}\bar{m}vc^{(6)}_{eH}\right]\frac{1}{\Lambda^2}$$

In the SMEFT to $1/\Lambda^4$ we measure

$$\begin{aligned}\mathcal{M}^2 \propto \tfrac{\bar{m}^2}{v^2} &+ \left[\bar{m}^2\left(2c^{(6)}_{H\Box} - \tfrac{1}{2}c^{(6)}_{HD}\right) - \sqrt{2}\bar{m}vc^{(6)}_{eH}\right]\tfrac{1}{\Lambda^2} \\ &+ \tfrac{v^2}{4}\underbrace{\left[\bar{m}^2\left([4c^{(6)}_{H\Box} + c^{(6)}_{HD}]^2 - c^{(8)}_{HD} - c^{(8)}_{HD,2}\right) + 2\sqrt{2}\left(c^{(8)}_{HD} - 4c^{(6)}_{H\Box}\right)\bar{m}vc^{(6)}_{eH} + 2v^2\left(c^{(6)}_{eH}\right)^2\right]}_{\text{Inseparable}\heartsuit}\tfrac{1}{\Lambda^4}\end{aligned}$$

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In the SM we measure:

In the SMEFT

¡Baloney/Bologna!

In the SMEFT

$$\mathcal{M}^2 \propto \frac{\bar{m}^2}{v^2} + \Big[\bar{m}^2$$

$$+ \frac{v^2}{4} \underbrace{\left[\bar{m}^2 \left([4c^{(6)}_{H\Box} + c^{(6)}_{HD}]^2 - c^{(8)}_{HD} - c^{(8)}_{HD,2}\right) + 2\sqrt{2}\left(c^{(8)}_{HD} - 4c^{(6)}_{H\Box}\right)\bar{m}vc^{(6)}_{eH} + 2v^2\left(c^{(6)}_{eH}\right)^2\right]}_{\text{Inseparable}\heartsuit} \frac{1}{\Lambda^4}$$

What are we actually measuring? (In Warsaw)

$$-\frac{Y}{\sqrt{2}}$$

$$+\frac{v^2\left(3(c^{(6)}_{eH})-4(c^{(6)}_{H\Box})Y+(c^{(6)}_{HD})Y\right)}{2\sqrt{2}}\frac{1}{\Lambda^2}$$

$$+\frac{v^4\left(Y\left((c^{(8)}_{HD2})-((c^{(6)}_{HD})-4(c^{(6)}_{H\Box}))^2\right)+3(c^{(6)}_{eH})(4(c^{(6)}_{H\Box})-(c^{(6)}_{HD}))+5(c^{(8)}_{eH})\right)}{4\sqrt{2}}\frac{1}{\Lambda^4}$$

$$+\frac{v^6 \big(2(c^{(10)}_{HD2})Y+(c^{(8)}_{HD2})\left(-3(c^{(6)}_{eH})+8(c^{(6)}_{H\Box})Y-2(c^{(6)}_{HD})Y\right)+(4(c^{(6)}_{H\Box})-(c^{(6)}_{HD}))\big(5(c^{(8)}_{eH})-(4(c^{(6)}_{H\Box})-(c^{(6)}_{HD}))\big(-3}{8\sqrt{2}}$$

$$+ v^8 \big(-6(c^{(10)}_{HD2})(c^{(6)}_{eH})+16(c^{(10)}_{HD2})(c^{(6)}_{H\Box})Y-4(c^{(10)}_{HD2})(c^{(6)}_{HD})Y+4(c^{(12)}_{HD2})Y-(c^{(8)}_{HD2})^2Y+3(c^{(8)}_{HD2})(4(c^{(6)}_{H\Box})-(c^{(6)}_{H}$$

$$+ v^{10}\big(-2(c^{(10)}_{HD2})\left(Y\left(2(c^{(8)}_{HD2})-3((c^{(6)}_{HD})-4(c^{(6)}_{H\Box}))^2\right)+6(c^{(6)}_{eH})(4(c^{(6)}_{H\Box})-(c^{(6)}_{HD}))+5(c^{(8)}_{eH})\right)-12(c^{(12)}_{HD2})(c^{(6)}_{eH})+3$$

$$+\cdots$$

What are we actually measuring?

The $h\bar{L}e$ correlation function is (más o menos):

$$\begin{aligned}
\langle h\bar{L}e\rangle &\sim \left\langle 0\right| T\left\{h(x_1)\bar{L}(x_2)e(x_3)\right\}\left|0\right\rangle \\
&\sim \frac{\delta^3}{\delta h\delta\bar{L}\delta e}\int\mathcal{D}(\text{fields})e^{iS_{\text{SMEFT}}} \\
&\sim \left\langle\frac{\delta}{\delta h}\frac{\delta^2}{\delta\bar{L}\delta e}\mathcal{L}_{\text{SMEFT}}\right\rangle
\end{aligned}$$

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Simplifying:

$$\begin{aligned}\frac{\delta}{\delta h}\frac{\delta^2}{\delta\bar{L}\delta e}\mathcal{L}_{\text{SMEFT}} &\Leftrightarrow \frac{\delta}{\delta h}\frac{\delta^2}{\delta\bar{L}\delta e}\Big[\underbrace{(\text{something})}_{\text{Not } W,B,\psi}\bar{L}e\Big] \\ &= \frac{\delta}{\delta h}\Big[(\text{something})\Big] \\ &\equiv \frac{\delta}{\delta h}\mathcal{Y}\end{aligned}$$

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Simplifying:

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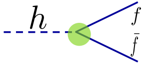
So two important quantities are:

$$\bar{m} = \langle\mathcal{Y}\rangle \qquad\qquad \langle h\bar{L}e\rangle = \left\langle\frac{\delta\mathcal{Y}}{\delta h}\right\rangle$$

Higgs characterization model

Consider also simpler description of effective Higgs coupling modifiers (kappa framework)

$$\mathcal{L}_{\rm Yuk} = -\sum_f \frac{y_f}{\sqrt{2}} \bar{f} \left(c_f + i\gamma_5 \tilde{c}_f\right) f h,$$



Translate kappa SMEFT: $g_f = c_f + i\tilde{c}_f = 3 - \frac{2}{1 + T_f^R + iT_f^I}$ with $T_f^{R,I} \equiv \frac{v^2}{2\Lambda^2}\frac{X_f^{R,I}}{y_f}$

Allow also modifications of real parts of HVV couplings $\mathcal{L}_V = c_V H \left(\frac{M_Z^2}{v} Z_\mu Z^\mu + 2\frac{M_W^2}{v} W_\mu^+ W^{-\mu}\right)$

Capture BSM effects in effective Hgg and Hγγ couplings: $c_g, \tilde{c}_g, c_\gamma, \tilde{c}_\gamma$

Let's try another example:

Higgs two point function:

$$\begin{aligned}\langle hh\rangle &\sim \langle 0|T\left\{h(x_1)h(x_2)\right\}|0\rangle \\ &\sim \frac{\delta^2}{\delta h^2}\int \mathcal{D}(\text{fields})e^{iS_{\text{SMEFT}}} \\ &\sim \left\langle \frac{\delta^2}{\delta h^2}\mathcal{L}_{\text{SMEFT}}\right\rangle + \left\langle \frac{\delta^2}{\delta (DH)^2}\mathcal{L}_{\text{SMEFT}}\right\rangle\end{aligned}$$

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Higgs two point function:

$$
\begin{aligned}
\langle hh \rangle &\sim \langle 0 | T\{h(x_1)h(x_2)\} | 0 \rangle \\
&\sim \frac{\delta^2}{\delta h^2} \int \mathcal{D}(\text{fields}) e^{iS_{\text{SMEFT}}} \\
&\sim \left\langle \frac{\delta^2}{\delta h^2} \mathcal{L}_{\text{SMEFT}} \right\rangle + \left\langle \frac{\delta^2}{\delta (DH)^2} \mathcal{L}_{\text{SMEFT}} \right\rangle
\end{aligned}
$$

Consider:

$$
\begin{aligned}
\left\langle \frac{\delta^2}{\delta (DH)^2} \mathcal{L}_{\text{SMEFT}} \right\rangle &\sim \left\langle \frac{\delta^2}{\delta (DH)^2} (\text{something})(DH)^2 \right\rangle \\
&= \left\langle \text{something} \right\rangle
\end{aligned}
$$

Let's try another example:

Higgs two point function:

$$\begin{aligned}\langle hh\rangle &\sim \langle 0|T\{h(x_1)h(x_2)\}|0\rangle \\ &\sim \frac{\delta^2}{\delta h^2}\int \mathcal{D}(\text{fields})e^{iS_{\text{SMEFT}}} \\ &\sim \left\langle \frac{\delta^2}{\delta h^2}\mathcal{L}_{\text{SMEFT}}\right\rangle + \left\langle \frac{\delta^2}{\delta (DH)^2}\mathcal{L}_{\text{SMEFT}}\right\rangle\end{aligned}$$

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This $\langle$something$\rangle$ is the finite renormalization factor for the scalar fields we know and hate:

$$\langle\text{something}\rangle = 1 + \frac{v^2}{4\Lambda^2}\left(c_{HD}^{(6)} - 4c_{H\Box}^{(6)}\right) + \frac{v^4}{32\Lambda^4}\left[4c_{HD}^{(8)} + 4c_{HD,2}^{(8)} - \left(c_{HD}^{(6)} - 4c_{H\Box}^{(6)}\right)^2\right] + \cdots$$

Let's formalize a bit: (AKA geoSMEFT)

Take four-component real scalar and vector fields:

$$\phi^I \quad \Leftrightarrow \quad H = \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix} \qquad W^A \quad = \quad \begin{pmatrix} W^1 \\ W^2 \\ W^3 \\ B \end{pmatrix}$$

Then all two-point functions can be defined with just:

$$h_{IJ} = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\text{SMEFT}}}{\delta(D_\mu \phi)^I \delta(D_\nu \phi)^J}\bigg|_{\text{things}\to 0} \quad \Leftrightarrow \quad h_{IJ}(D_\mu\phi)^I(D_\nu\phi)^J$$

$$g_{AB} = \frac{-2g^{\mu\nu}g^{\sigma\rho}}{d^2} \frac{\delta^2 \mathcal{L}_{\text{SMEFT}}}{\delta W^A_{\mu\sigma} \delta W^B_{\nu\rho}}\bigg|_{\text{things}\to 0} \quad \Leftrightarrow \quad g_{AB} W^A_{\mu\nu} W^B_{\mu\nu}$$

$$\mathcal{Y} = \frac{\delta \mathcal{L}_{\text{SMEFT}}}{\delta(\bar{\Psi}\psi)}\bigg|_{\text{things}\to 0} \quad \Leftrightarrow \quad Y\bar{\Psi}\psi$$

(that's it, at least Warsaw style)

the geoSMEFT (op forms $\mathcal{W}^{A,\mu\nu}\mathcal{W}^{B}_{\mu\nu}$)

Consider all operators involving two of $\{W^{A,\mu\nu}, B^{\mu\nu}\}$ and many $\{H,\ H^\dagger, \tau^A\}$:

keep in mind, $\tau^A\tau^B = \frac{1}{4}\left(\delta^{AB} + 2i\epsilon^{ABC}\tau^J\right)$

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D4: $-\frac{1}{4}B^{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^{A,\mu\nu}W^{A,\mu\nu}$

D6: $c_{HB}(H^\dagger H)B^{\mu\nu}B^{\mu\nu} + c_{HW}(H^\dagger H)W^{A,\mu\nu}W^{A,\mu\nu} + c_{HWB}(H^\dagger\tau^A H)W^{A,\mu\nu}B^{\mu\nu}$

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D8: $c^{(8)}_{HB}(H^\dagger H)^2 B^{\mu\nu}B^{\mu\nu} + c^{(8)}_{HW}(H^\dagger H)^2 W^{A,\mu\nu}W^{A,\mu\nu}$

$+ c^{(8)}_{HWB}(H^\dagger H)(H^\dagger\tau^A H)W^{A,\mu\nu}B^{\mu\nu} + c^{(8)}_{HW,2}(H^\dagger\tau^A H)(H^\dagger\tau^B H)W^{A,\mu\nu}W^{B,\mu\nu}$

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D(8+2n): $c^{(8+2n)}_{HB}(H^\dagger H)^{2n+4}B^{\mu\nu}B^{\mu\nu} + c^{(8+2n)}_{HW}(H^\dagger H)^{2n+4}W^{A,\mu\nu}W^{A,\mu\nu}$
$+ c^{(8+2n)}_{HWB}(H^\dagger H)^{2n+2}(H^\dagger\tau^A H)W^{A,\mu\nu}B^{\mu\nu}$
$+ c^{(8+2n)}_{HW,2}(H^\dagger H)^{2n}(H^\dagger\tau^A H)(H^\dagger\tau^B H)W^{A,\mu\nu}W^{B,\mu\nu}$

the geoSMEFT (op forms $\mathcal{W}^{A,\mu\nu}\mathcal{W}^{B}_{\mu\nu}$)

Consider all operators involving two of $\{W^{A,\mu\nu}, B^{\mu\nu}\}$ and many $\{H,\ H^\dagger, \tau^A\}$:

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$$
\begin{aligned}
g_{AB} \quad = \quad & \left[1 - 4\sum\left(c^{(6+2n)}_{HW}(1-\delta_{A4}) + c^{(6+2n)}_{HB}\delta_{A4}\right)\left(\frac{\phi^2}{2}\right)^{n+1}\right]\delta_{AB} \\
& - \sum c^{(8+2n)}_{HW,2}\left(\frac{\phi^2}{2}\right)^n (\phi_I\Gamma^I_{A,J}\phi^J)(\phi_L\Gamma^L_{B,K}\phi^K)(1-\delta_{A4})(1-\delta_{B4}) \\
& + \left[\sum c^{(6+2n)}_{HWB}\left(\frac{\phi^2}{2}\right)^n\right]\left[(\phi_I\Gamma^I_{A,J}\phi^J)(1-\delta_{A4})\delta_{B4} + (A\leftrightarrow B)\right]
\end{aligned}
$$

All orders finite field/mass renormalizations:

Again, the fermion mass is just:

$$\bar{m} = \langle \mathcal{Y} \rangle$$

The **shifts** to the mass basis are trivial:

$$\begin{aligned} \phi^J &= \sqrt{h}^{JK} V_{KL} \Phi^L \\ W_\nu^A &= \sqrt{g}^{AB} U_{BC} A_\nu^C \end{aligned}$$

where

$$\begin{aligned} U &= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{I}{\sqrt{2}} & -\frac{I}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \bar{c} & \bar{s} \\ 0 & 0 & -\bar{s} & \bar{c} \end{pmatrix} \\ \bar{s} &= f(\sqrt{g}^{AB}, g_1, g_2) \end{aligned}$$

The vector masses are:

$$\begin{aligned} \bar{m}_W^2 &= \frac{g_2^2}{4} \left(\sqrt{g}^{11}\right)^2 \left(\sqrt{h}_{11}\right)^2 v^2 \\ \bar{m}_Z^2 &= \frac{g_2^2}{4c_Z^2} \left(\bar{c}\sqrt{g}^{33} - \bar{s}\sqrt{g}^{34}\right)^2 \left(\sqrt{h}_{33}\right)^2 v^2 \end{aligned}$$

All orders hVV couplings:

Hays, Helset, Martin, Trott, arXiv:2007.00565

$$
\begin{aligned}
\langle h Z_\mu Z^\mu \rangle &= \frac{\bar{m}_Z^2}{2v} \sqrt{h}_{44} \left[\left\langle \frac{\delta h_{33}}{\delta h} \right\rangle \frac{v^2}{4} + \langle h_{33} \rangle \frac{v}{2} \right] \\
\sqrt{h}_{44} &= 1 + \frac{v^2}{4} \left(c_{HD} - 4 c_{H\Box} \right) + \frac{v^4}{32} \left(4 c_{HD}^{(8)} + 4 c_{HD,2}^{(8)} - \left[c_{HD}^{(6)} - 4 c_{H\Box}^{(6)} \right]^2 \right) \\
\langle h W_\mu W^\mu \rangle &\sim 2 \frac{\bar{m}_W^2}{2v} \sqrt{h}_{11} \left[\left\langle \frac{\delta h_{11}}{\delta h} \right\rangle \frac{v^2}{4} + \langle h_{11} \rangle \frac{v}{2} \right] \\
\sqrt{h}_{11} &= 1 + \frac{v^4}{8} \left(c_{HD}^{(8)} - c_{HD,2}^{(8)} \right)
\end{aligned}
$$

κ framework:

Allow also modifications of real parts of HVV couplings $\mathcal{L}_V = c_V H \left(\frac{M_Z^2}{v} Z_\mu Z^\mu + 2 \frac{M_W^2}{v} W_\mu^+ W^{-\mu} \right)$

Also misses:

$$\langle h Z^{\mu\nu} Z_{\mu\nu} \rangle \qquad \langle \partial_\nu h Z_\mu Z^{\mu\nu} \rangle$$

(which are known to all orders in geoSMEFT)

The geoSMEFT

Helset, Martin, Trott, arXiv:2001.01453

We can define all two-point functions in the SMEFT with just:

$$g_{AB}\mathcal{W}^A_{\mu\nu}\mathcal{W}^B_{\mu\nu} \quad\Leftrightarrow\quad g_{AB} = \frac{-2g^{\mu\nu}g^{\sigma\rho}}{d^2}\frac{\delta^2\mathcal{L}}{\delta\mathcal{W}^A_{\mu\sigma}\delta\mathcal{W}^B_{\nu\rho}}$$

$$h_{IJ}(D^\mu\phi)^I(D_\mu\phi)^J \quad\Leftrightarrow\quad h_{IJ} = \frac{g^{\mu\nu}}{d}\frac{\delta^2\mathcal{L}}{\delta(D_\mu\phi)^I\delta(D_\nu\phi)^J}$$

$$Y\bar{\Psi}\psi \quad\Leftrightarrow\quad Y(\phi) = \frac{\delta\mathcal{L}}{\delta(\bar{\Psi}\psi)}$$

Further we can define all three-point functions in the SMEFT with just:

$$L^\psi_J(D^\mu\phi)^J(\bar{\psi}\Gamma_\mu\psi) \quad\Leftrightarrow\quad L^\psi_J = \frac{\delta^2\mathcal{L}}{\delta(D^\mu\phi)^J\delta(\bar{\psi}\Gamma_\mu\sigma\psi)}$$

$$d^\psi_A(\bar{\psi}\sigma^{\mu\nu}\psi)\mathcal{W}^A_{\mu\nu} \quad\Leftrightarrow\quad d^\psi_A = \frac{\delta^2\mathcal{L}}{\delta(\bar{\psi}\sigma^{\mu\nu}\psi)\delta\mathcal{W}^A_{\mu\nu}}$$

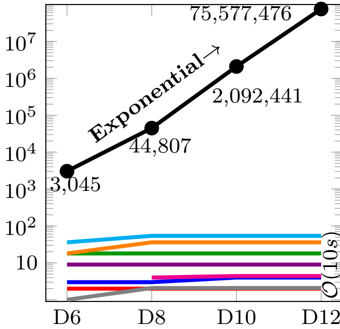
$$f_{ABC}W^{A,\mu\nu}W^B_{\nu\rho}W^{C,\rho}_\mu \quad\Leftrightarrow\quad f_{ABC} = \frac{g^{\nu\rho}g^{\sigma\alpha}g^{\beta\mu}}{3!d^3}\frac{\delta^3\mathcal{L}}{\delta\mathcal{W}^A_{\mu\nu}\delta\mathcal{W}^B_{\rho\sigma}\delta\mathcal{W}^C_{\alpha\beta}}$$

$$k^A_{IJ}(D_\mu\phi)^I(D_\nu\phi)^JW^A_{\mu\nu} \quad\Leftrightarrow\quad k^A_{IJ} = \frac{g^{\mu\rho}g^{\nu\sigma}}{2d^2}\frac{\delta^3\mathcal{L}}{\delta(D_\mu\phi)^I\delta(D_\nu\phi)^J\delta\mathcal{W}^A_{\rho\sigma}}$$

Saturation of number of operators

(This information is contained in the Hilbert Series)
(see e.g. Lehman & Martin 2015, Henning et al. 2015)

	Mass Dimension		
Operator form:	6	8	10
$h_{IJ}(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2
$g_{AB}W^A_{\mu\nu}W^{B,\mu\nu}$	3	4	4
$k_{IJA}(D^\mu\phi)^I(D^\nu\phi)^J W^A_{\mu\nu}$	0	3	4
$f_{ABC}W^A_{\mu\nu}W^{B,\nu\rho}W^{C,\mu}_\rho$	1	2	2
$Y^\psi_{pr}\bar{\Psi}_L\psi_R + h.c.$	$2N_f^2$	$2N_f^2$	$2N_f^2$
$d^{\psi,pr}_A\bar{\Psi}_L\sigma_{\mu\nu}\psi_R W^{\mu\nu}_A + h.c.$	$4N_f^2$	$6N_f^2$	$6N_f^2$
$L^{\psi_R}_{pr,J,A}(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2
$L^{\Psi_L}_{pr,J,A}(D^\mu\phi)^J(\bar{\Psi}_{p,L}\gamma_\mu\sigma_A\Psi_{r,L})$	$2N_f^2$	$4N_f^2$	$4N_f^2$



Beyond 3pt functions:

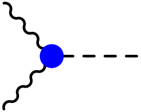
For 2 point functions, think: $p_i^2 = m^2$

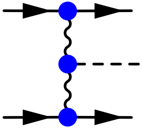
For 3 point functions, think: $p_i \cdot p_j = \frac{1}{2}(m_k^2 - m_i^2 - m_j^2)$

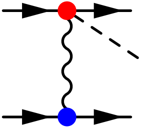
For 4 point fucntions, think: s^n, t^n, and u^n...

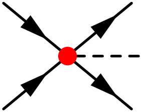
we cannot sum the momentum expansion beyond 3-point functions...

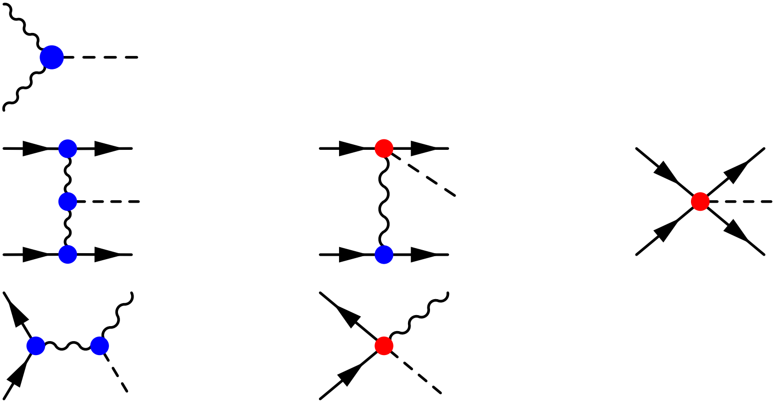
but we can still do the vev expansion as discussed before

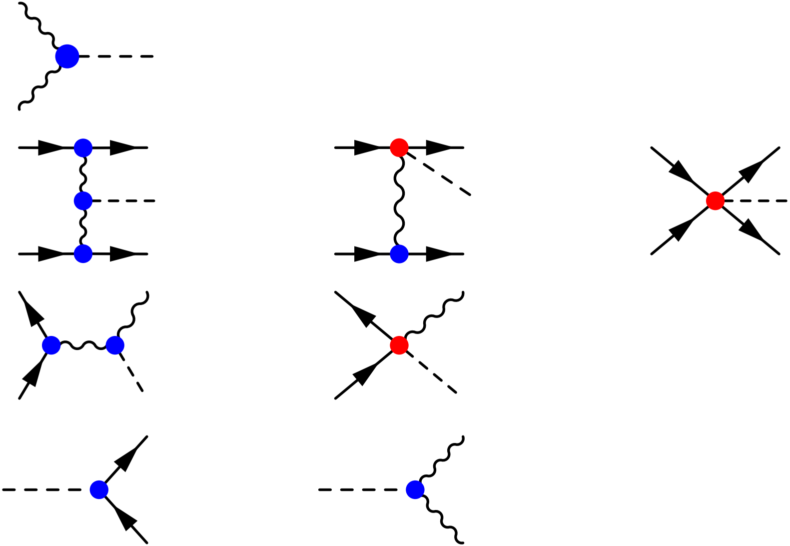


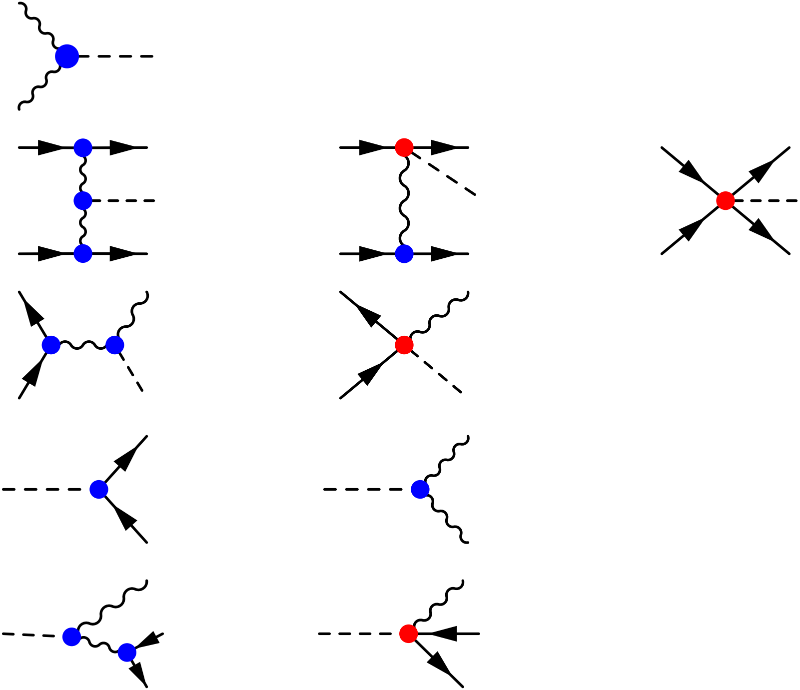


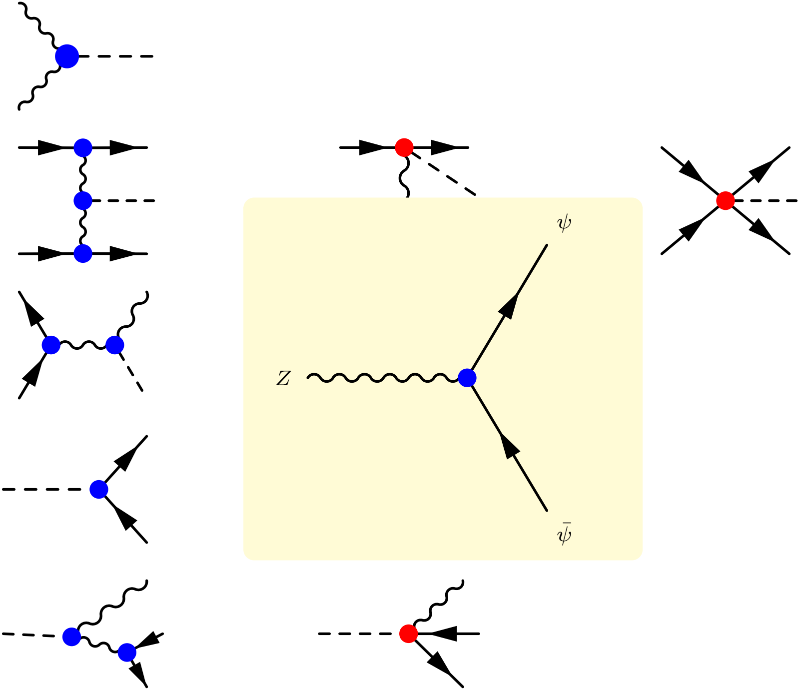








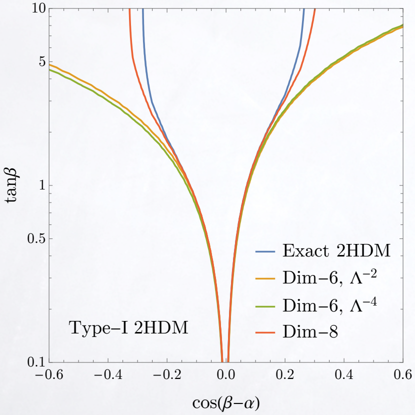



Z
ψ
$\bar{\psi}$

Why $1/\Lambda^4$? (why geoSMEFT?)

- As discussed, we're not measuring $|\mathcal{M}_6|^2$ separately from $\mathcal{M}_{SM} \times (\mathcal{M}_{6^2} + \mathcal{M}_8)$
- Cen Zhang, SMEFTs living on the edge, arXiv:2112.11665
  "Our results indicate that the dimension-8 operators encode much more information about the UV than one would naively expect, which can be used to reverse engineer the UV physics from the SMEFT."
  (See also Ken's talk)
- If we can do the calculations, why not do them to the best of our abilities?
- Duarte & Felix say it's a good idea

- Now, the fits. Type I:



- For high $\tan\beta$, the dim-6 results are poorly constrained
  - the only WCs are the Yukawa ones
  - $\lambda_f^{(2)} = \dfrac{\eta_f}{\tan\beta}\lambda_f^{(1)}$

	Type-I
η_u	1
η_d	1
η_l	1

- Obviously, this does not change with the squared terms
- The exact 2HDM has more info than Yukawas

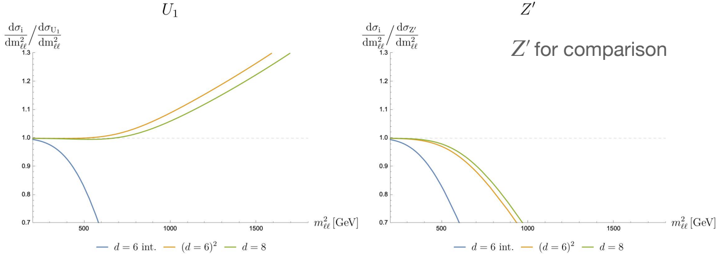
  ⇢ gauge-Higgs interactions
- But that info is contained in the dim-8 results

$$S_{\text{all},8} \ni C^{(1)}_{\mathcal{H}^6}(\mathcal{H}^\dagger\mathcal{H})^2\,(D_\mu\mathcal{H})^\dagger\,(D^\mu\mathcal{H})$$

- The dim-8 EFT is thus a good reproduction of the exact model – whereas dim-6 is clearly insufficient for some regions

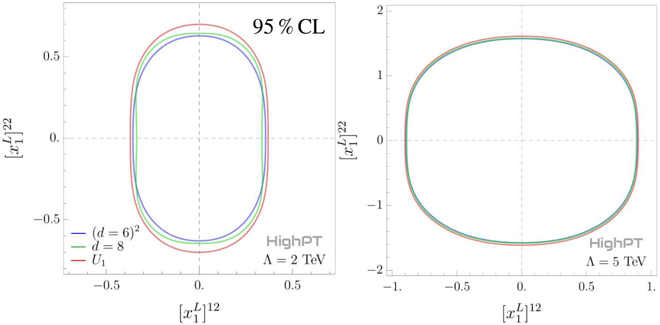
$d = 8$ effects for the U_1 leptoquark

Universität Zürich[UZH]



Matching the U_1 LQ to the SMEFT at $d = 8$

Compare effects of: $d = 6$, $d = 8$, model



Why $1/\Lambda^4$? (why geoSMEFT?)

- As discussed, we're not measuring $|\mathcal{M}_6|^2$ separately from $\mathcal{M}_{SM} \times (\mathcal{M}_{6^2} + \mathcal{M}_8)$
- Cen Zhang, SMEFTs living on the edge, arXiv:2112.11665
  "Our results indicate that the dimension-8 operators encode much more information about the UV than one would naively expect, which can be used to reverse engineer the UV physics from the SMEFT."
- If we can do the calculations, why not do them to the best of our abilities?
- Duarte says it's a good idea

Can we constrain the full parameter space at D8? Probably not at the moment...
(what about in the context the the geoSMEFT-y quantities like $\langle \delta M \rangle$)

Can we differentiate $c_{HW}^{(6)}$ from $c_{HW}^{(8)}$ without multi-Higgs? Nop...

What about the HEFT at the HEFT workshop?

(I know it's Higgs and EFT)
(Ilaria did mention it a bit)
Corbett, Éboli, Gonzalez-Garcia arXiv:1509.01585

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Consider the scalar Singlet extension:

$$\begin{aligned}\mathcal{L} &= (D^\mu H)^\dagger (D_\mu H) + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) + \mu_H^2 |H|^2 - \lambda |H|^4 + \frac{\mu_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 + \frac{\lambda_m}{2}|H|^2 S^2 \\ &\to \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{m_H^2}{2} h^2 + \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - \frac{m_s^2}{2} S^2 + \frac{(v+h)^2}{4} \mathrm{Tr}\left[(D^\mu U)(D_\mu U)^\dagger\right] \\ &\quad -\lambda_m v v_s H S - \cdots\end{aligned}$$

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Integrating out S/performing the matching we get the following operators:
(h' is after rotating to the mass basis):

$$\begin{aligned}\mathcal{P}_C &= \tfrac{v^2}{4}\mathrm{Tr}[(D^\mu U)(D_\mu U)^\dagger]\mathcal{F}_C(h') &\qquad \mathcal{P}_H &= \tfrac{1}{2}(\partial^\mu h')(\partial_\mu h')\mathcal{F}_H(h') \\ \mathcal{P}_6 &= \mathrm{Tr}[(D^\mu U)(D_\mu U)^\dagger]^2 \mathcal{F}_6(h') & \mathcal{P}_7 &= \mathrm{Tr}[(D^\mu U)(D_\mu U)^\dagger]\Box\mathcal{F}_7(h') \\ \mathcal{F}_i(h) &= c_i + a_i \tfrac{h'}{v} + b_i \tfrac{(h')^2}{v^2} + \cdots\end{aligned}$$

Matching HEFT vs SMEFT

If I do it above EWSB, I instead get:

$$Q_{H,2} = \frac{1}{2}\partial_\mu |H|^2 \partial^\mu |H|^2 \qquad Q_{H,3} = \frac{1}{3}|H|^6$$

$$Q_{H,4} = |H|^2 (D^\mu H)^\dagger (D_\mu H) \qquad Q_{H,5} = \frac{1}{4}|H|^8$$

$$Q_{H,6} = \frac{1}{2}|H|^2 \partial_\mu |H|^2 \partial^\mu |H|^2$$

$$Q_{H,7} = |H|^4 (D^\mu H)^\dagger (D_\mu H)$$

$$Q_{S,1} = (D^\mu H)^\dagger (D_\mu H)(D^\nu H)^\dagger (D_\nu H)$$

Matching HEFT vs SMEFT

The doublet predicts $(h+v)^{2n}$, or for hWW vs h^2WW:

$$2nv^{2n-1} \qquad \text{vs.} \qquad n(2n-1)v^{2n-1}$$

In HEFT:

$$2-X \qquad \text{vs.} \qquad 1-2X \qquad\qquad X \equiv \frac{\lambda_m^2 v^2}{2\lambda_S M_S^2}$$

Conclusions

- $|\mathcal{M}_6|^2$ is theoretically well defined
- But we only ever measure something like $\delta^n \mathcal{L}_{\rm SMEFT}$
- The geoSMEFT lets us calculate $\delta^n \mathcal{L}_{\rm SMEFT}$ (sometimes)
- we can supplement the geoSMEFT to get complete $1/\Lambda^4$ results
- we should be able to use the geoSMEFT to sum the v expansion
  $\rightarrow$ we only have the p expansion left
- (generally?) integrating out particles gives the HEFT
  $\rightarrow$ at least, if $\exists$ mixing between the 'heavy' state and the SM states below EWSB
  $\rightarrow$ it's really not measurable at a hadron collider (prop to the mixing parameter)