EW Skyrmions in the HEFT

HEFT 2022 / All Things EFT

JCC, M. Spannowsky, V. Khoze: 2012.07694, 2109.01596

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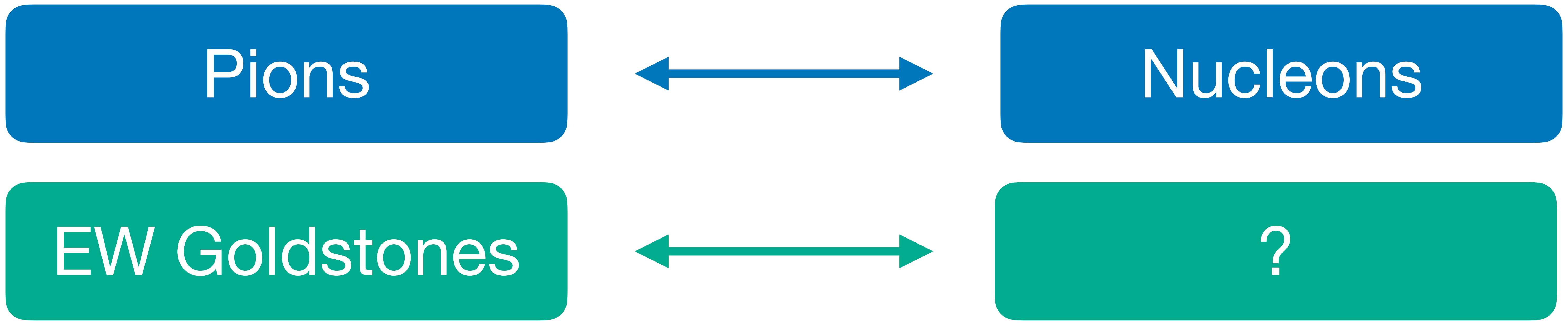
Motivation



- Hidden states around 1 TeV in the EFTs for the SM?
- **Dark matter** from the SM fields only?
- Information about **SMEFT vs HEFT**
- Machine learning for non-perturbative QFT

The Skyrme model

Symmetry

The pions are the pseudo-Goldstones of the breaking:

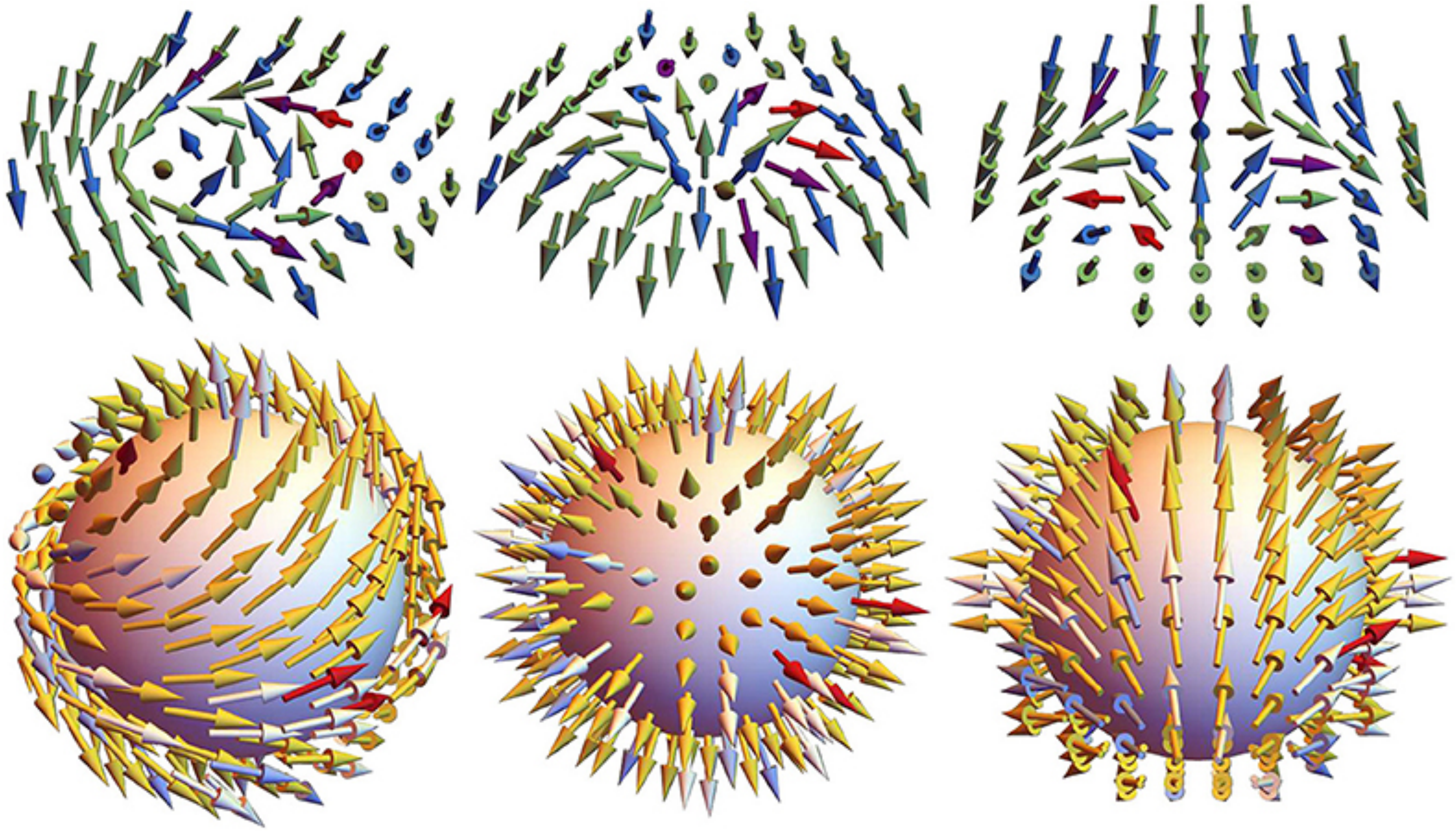
$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$$

⬇

Non-linear realisation $U \rightarrow LUR^\dagger$, with $U = e^{i\sigma^a \pi^a/f}$

The Skyrme model
Topology

- Static solutions $U(\mathbf{x})$ depend on spatial coordinates $\mathbf{x} \in \mathbb{R}^3$
- Finite energy implies $U(\mathbf{x}) \overset{\mathbf{x}\to\infty}{=} U_\infty$
- $U : \mathbb{R}^3 \cup \{\infty\} \cong S^3 \longrightarrow S^3$, classified by a integer winding number



Kovalev and Sandhoefner, Front. Phys. 6 (2018)

The Skyrme model

Interactions

Non-trivial stable solutions: **local minima** of the energy functional

Derrick's theorem

Perform a rescaling $\mathbf{x} \to \mathbf{x}/\lambda$ in a given solution

- Only 2-derivative terms: $E \sim d^3x\partial^2 \to \lambda E \implies$ no local minima
- Both 2- and 4-derivative terms: $E \sim d^3x(\partial^2 + \partial^4) \to (\lambda + C/\lambda)E$

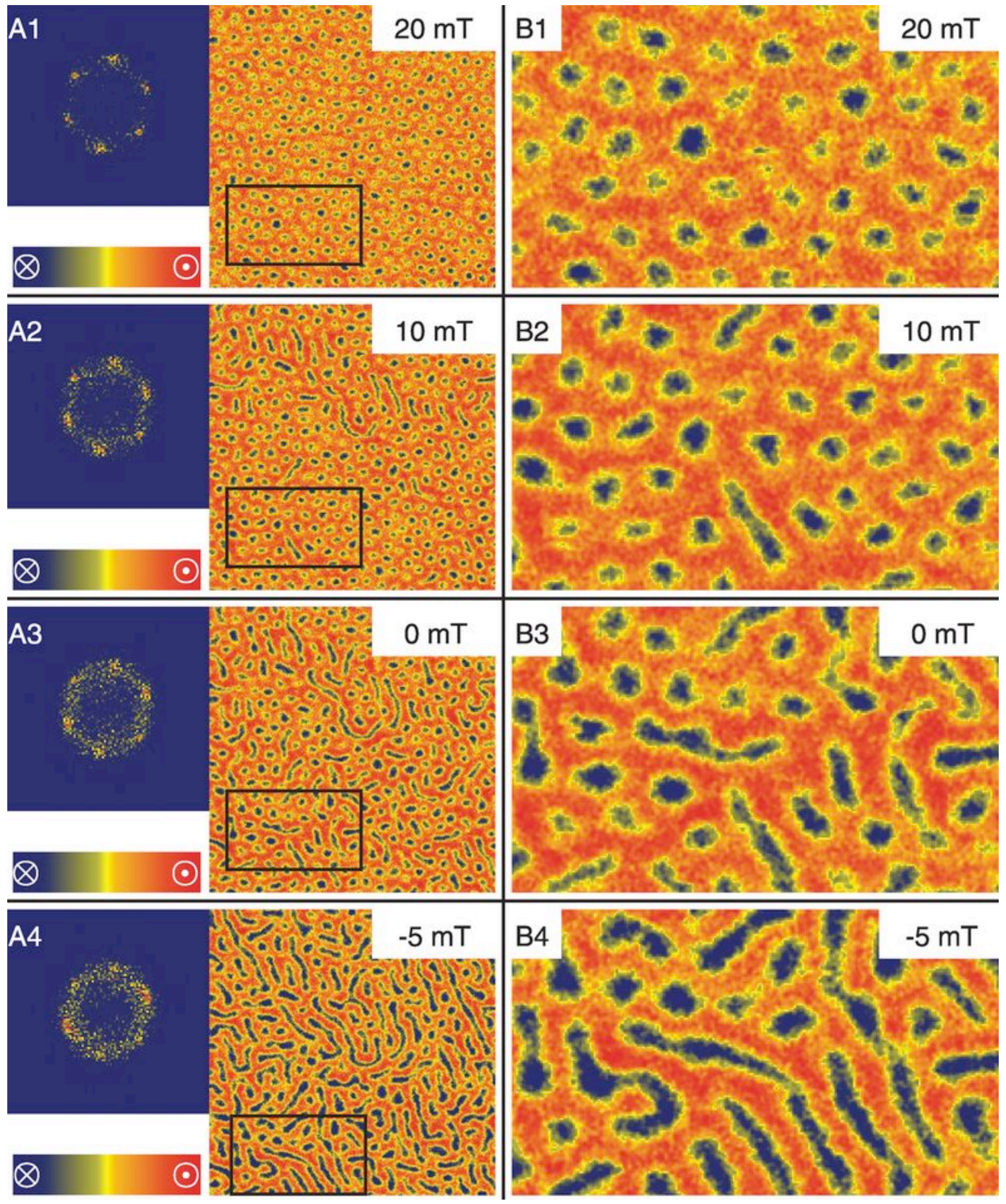
$$\mathscr{L} = \frac{f_\pi^2}{4}\left\langle \partial_\mu U \partial^\mu U^\dagger \right\rangle + \frac{1}{32e^2}\left\langle \left[U\partial_\mu U^\dagger, U\partial_\nu U^\dagger\right]^2 \right\rangle$$

The Skyrme model

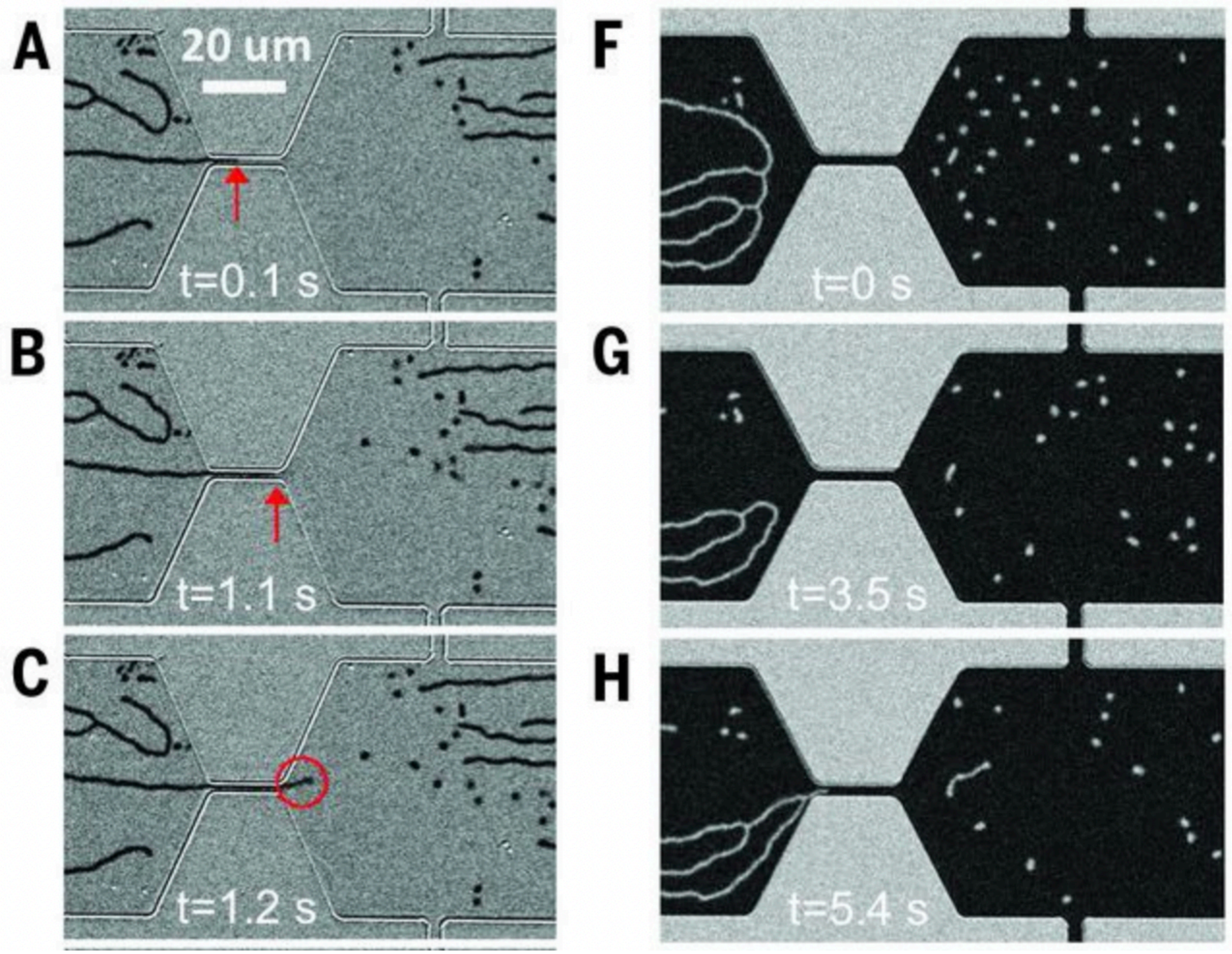
Skyrmions = baryons

[Skyrme 1961], [Witten 1973], [Adkins, Nappi, Witten 1983], …

Skyrmions in condensed matter physics



[Milde et al. 2013]



[Hoffman et al. 2015]

Electroweak skyrmions

Electroweak skyrmions

Electroweak vs pion EFT

3 would-be Goldstone bosons collected in $U = e^{i\sigma^a G^a/f} \in S^3$

Extra degrees of freedom: gauge and radial Higgs component

Fields

$$U = \exp \frac{i\sigma^a G^a}{\sqrt{2}v}, \qquad h, \qquad W_\mu^a$$

EFT for EW skyrmions

SMEFT vs HEFT topology

HEFT

Well-defined $U \in S^3$ for any h

example: $(h, U) \in \mathbb{R} \times S^3$

SMEFT

S^3 collapses to a point at $h = -v$,

since we have $\phi \sim (h + v)U = 0$

$$\left[\phi = \frac{v + h}{\sqrt{2}} U \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]$$

Electroweak skyrmions

Limits

Frozen Higgs (technicolor)	$m_h \to \infty \implies h = 0$ Gauge fields + Goldstones Meta-stable skyrmions, only for $e > e_{crit}$
Decoupled gauge fields	$g \to 0$ Higgs + Goldstones Stable skyrmions
Both (≅ pion EFT)	$m_h \to \infty, g \to 0$ Goldstones Stable skyrmions

Topological charges

Scalar winding number: $$n_U = \frac{1}{24\pi^2}\epsilon_{ijk}\int d^3x \left\langle L_i L_j L_k \right\rangle \in \mathbb{Z} \qquad \left(L_\mu = iU\partial_\mu U^\dagger\right)$$

Chern-Simons number: $$n_{CS} = \frac{1}{16\pi^2}\epsilon_{ijk}\int d^3x \left\langle W_i W_{jk} + \frac{2i}{3} W_i W_j W_k \right\rangle \in \mathbb{R}$$

$W_\mu = \mathcal{U}\partial_\mu \mathcal{U}^\dagger \implies n_{CS} \in \mathbb{Z}$ is the winding number for $\mathcal{U}$

Skyrmion number

Under large gauge transformations:

$$n_U \to n_U + N, \quad n_{CS} \to n_{CS} + N$$

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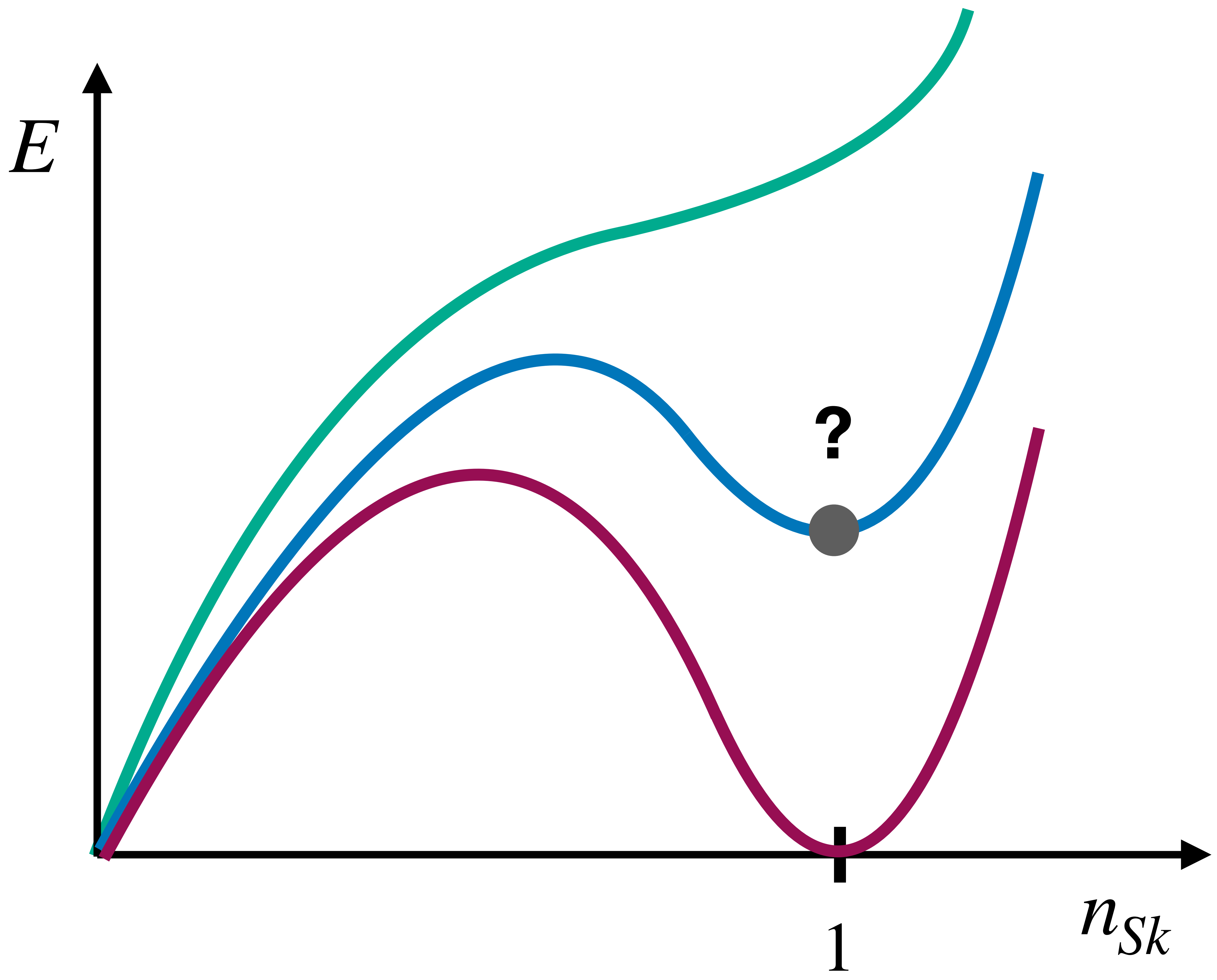
Define gauge-invariant number: $n_{Sk} = n_{CS} - n_U$

We'll look for local minima of the energy with $n_{Sk} \simeq 1$

Potential energy profile



HEFT Lagrangian

$$\mathscr{L} = \sum_i F_i(h/v)\mathcal{Q}_i$$

$$\left(F_i(\eta) = \sum_{n=0}^{\infty} c_{i,n}\eta^n\right)$$

Name	Operator
$\mathcal{Q}_1$	λ
$\mathcal{Q}_h$	$\partial_\mu h \partial^\mu h$
$\mathcal{Q}_U$	$\left\langle D_\mu U^\dagger D^\mu U \right\rangle$
$\mathcal{Q}_{Xh2}$	$\frac{1}{g^2}\langle W_{\mu\nu}W^{\mu\nu}\rangle$
$\mathcal{Q}_{Xh5}$	$\epsilon^{\mu\nu\rho\sigma}\langle W_{\mu\nu}W_{\rho\sigma}\rangle$
$\mathcal{Q}_{XU8}$	$i\langle W_{\mu\nu}[L^\mu, L^\nu]\rangle$
$\mathcal{Q}_{XU11}$	$i\epsilon^{\mu\nu\rho\sigma}\langle W_{\mu\nu}[L_\rho, L_\sigma]\rangle$
$\mathcal{Q}_{D1}$	$\langle L_\mu L^\mu\rangle^2$
$\mathcal{Q}_{D2}$	$\langle L_\mu L_\nu\rangle\langle L^\mu L^\nu\rangle$
$\mathcal{Q}_{D7}$	$\langle L_\mu L^\mu\rangle \partial_\nu h \partial^\nu h$
$\mathcal{Q}_{D8}$	$\langle L_\mu L_\nu\rangle \partial^\mu h \partial^\nu h$
$\mathcal{Q}_{D11}$	$(\partial_\mu h \partial^\mu h)^2$

Chiral dim. 2 (SM) — $\mathcal{Q}_1$ to $\mathcal{Q}_{Xh2}$

Chiral dim. 4 — $\mathcal{Q}_{Xh5}$ to $\mathcal{Q}_{D11}$

(at least one needed for Skyrmions)

$$\left(L_\mu = iU\partial_\mu U^\dagger\right)$$

The Skyrme term in the HEFT

$$\mathscr{L}_{Sk} = -\frac{1}{16e^2}\left(\mathcal{Q}_{D1} - \mathcal{Q}_{D2}\right)$$

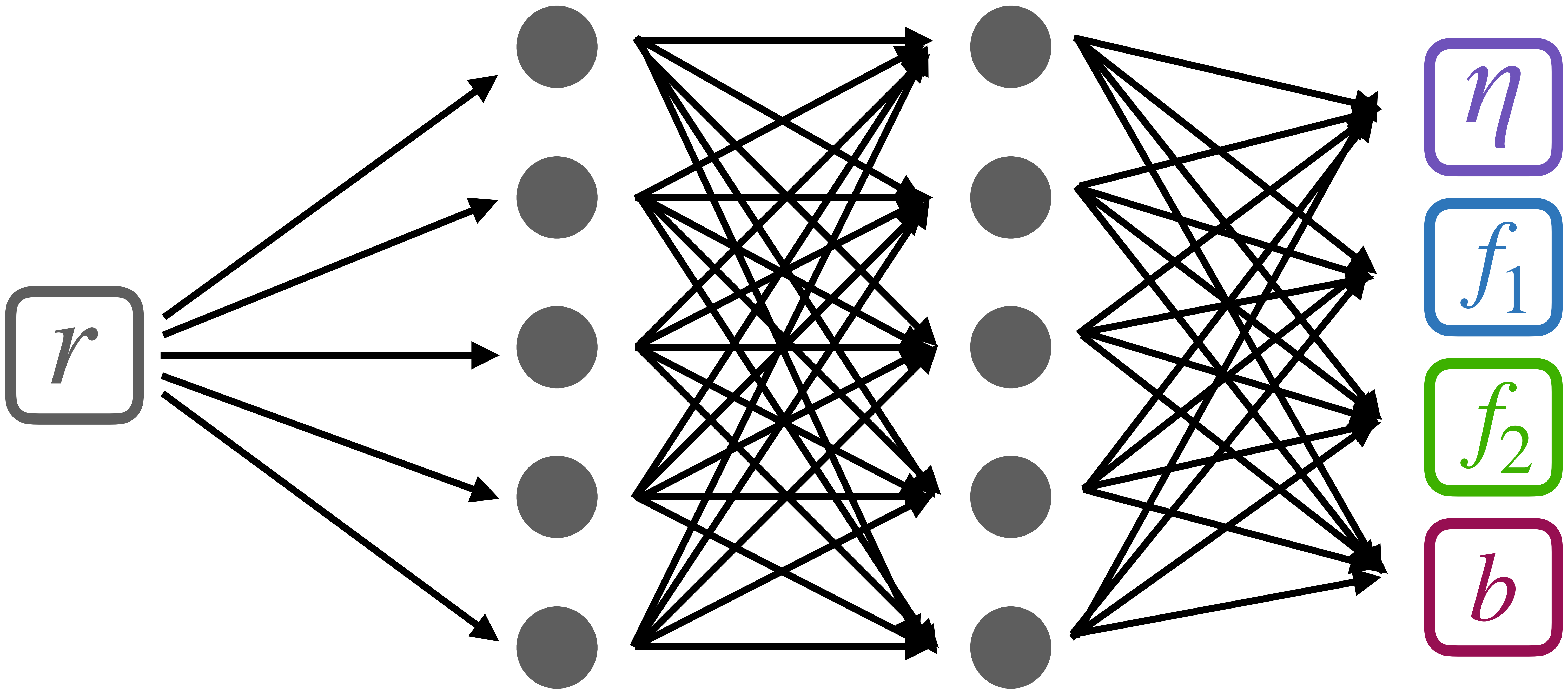
The power counting dictates that: $e \sim \dfrac{\Lambda}{4v}$

Computing the skyrmion solution

Ansatz

Unitary gauge	$U(\mathbf{x}) = \mathbf{1}_{2\times2}$
Spherically symmetric Higgs	$h(\mathbf{x}) = \frac{v}{\sqrt{2}} \boxed{\eta(r)}$
Gauge fields invariant under $SU(2)_V \subset SU(2)_{space} \times SU(2)_L$	$W_i(\mathbf{x}) = ve\sigma_a \left(\epsilon_{ija} n_j \frac{\boxed{f_1(r)}}{r} + (\delta_{ia} - n_i n_a) \frac{\boxed{f_2(r)}}{r} + n_i n_a \boxed{b(r)} \right)$

Neural network parametrisation



$$\eta = \sigma(w^{(3)}_{0j} x^{(2)}_j + b^{(3)}_i)$$

$$f_1 = \sigma(w^{(3)}_{1j} x^{(2)}_j + b^{(3)}_i)$$

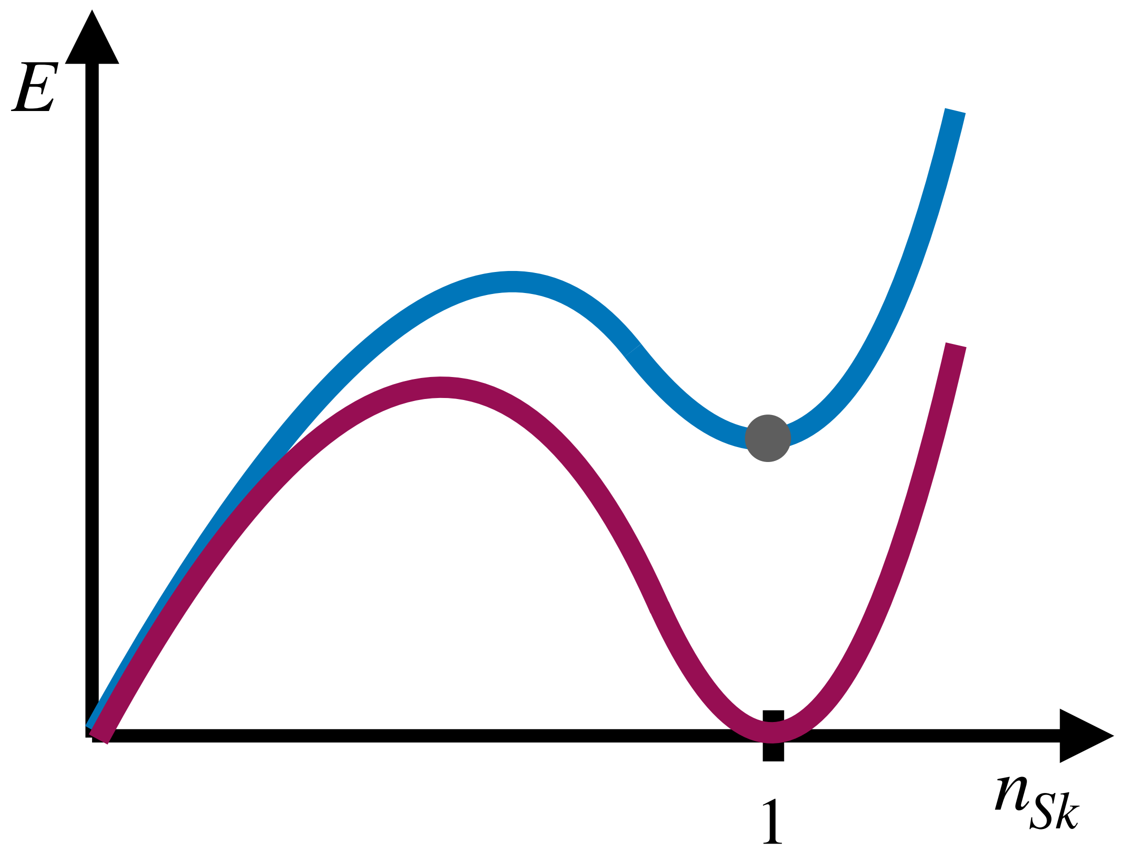
$$f_2 = \sigma(w^{(3)}_{2j} x^{(2)}_j + b^{(3)}_i)$$

$$b = \sigma(w^{(3)}_{3j} x^{(2)}_j + b^{(3)}_i)$$

$$x^{(1)}_i = \sigma(w^{(1)}_i r + b^{(1)}_i) \qquad x^{(2)}_i = \sigma(w^{(2)}_{ij} x^{(1)}_j + b^{(2)}_i)$$

Neural Network Training

= gradient descent



The problem

Minimize the energy $E[\eta, f_1, f_2, b]$, while statisfying

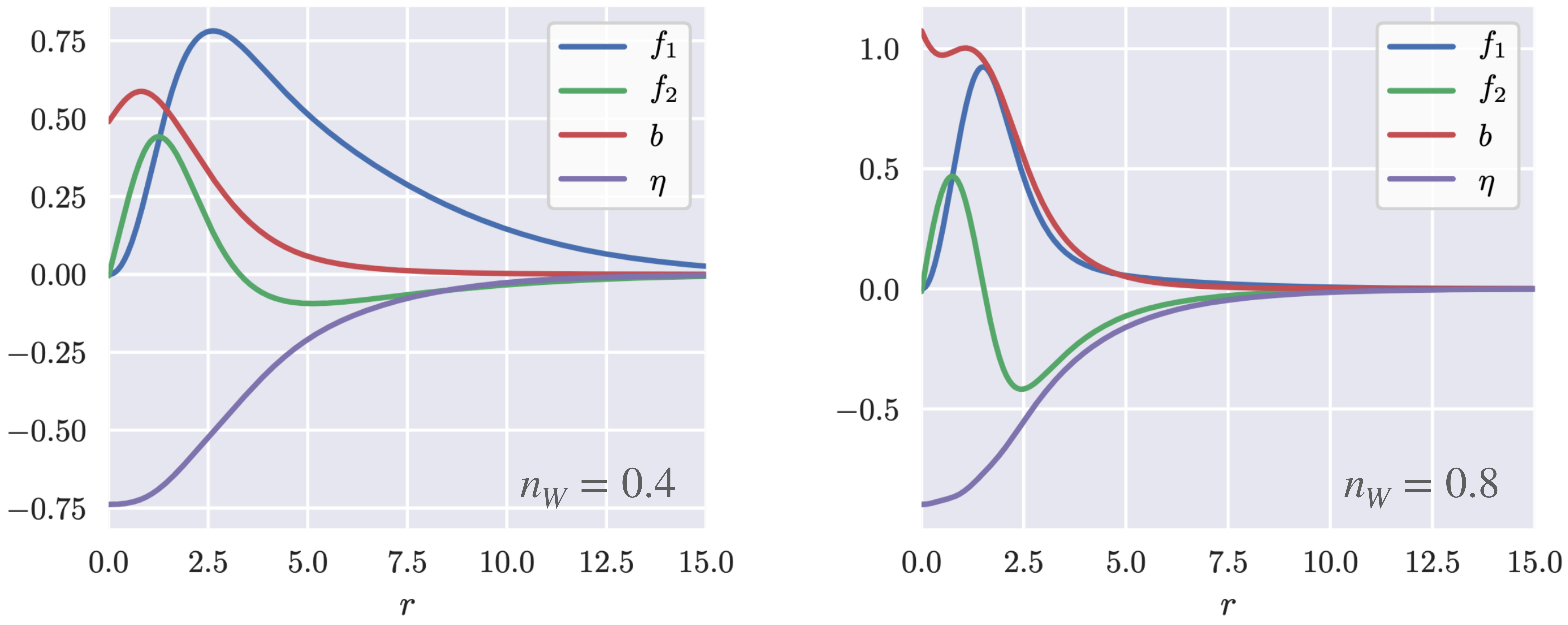
- $n_{Sk}[\eta, f_1, f_2, b] = n_W$
- The boundary conditions $B[\eta, f_1, f_2, b] = 0$

$$L[\eta, f_1, f_2, b] = E[\eta, f_1, f_2, b] + \omega_W \left(n_{SK}[\eta, f_1, f_2, b] - n_W\right)^2 + \omega_B B[\eta, f_1, f_2, b]^2$$

Elvet: https://gitlab.com/elvet/elvet

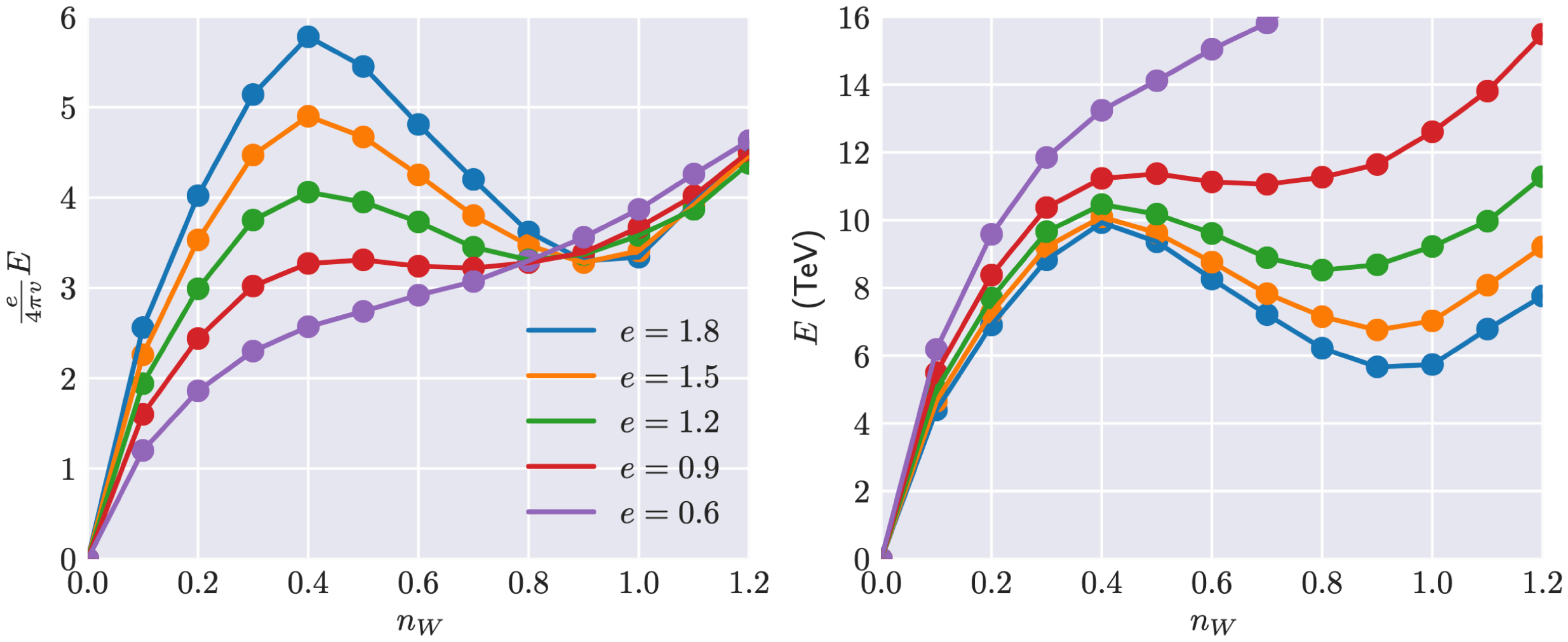
J. Araz, JCC, M. Spannowsky, [2103.14575]

Example solutions



Skyrme term only, $e = 1.2$

Potential energy profiles



$M_{Sk} \simeq \dfrac{10\,\text{TeV}}{e}$ $\quad E_{barrier} \simeq 11\,\text{TeV}$ $\quad e_{crit} \simeq 0.9$

Skyrmion radius

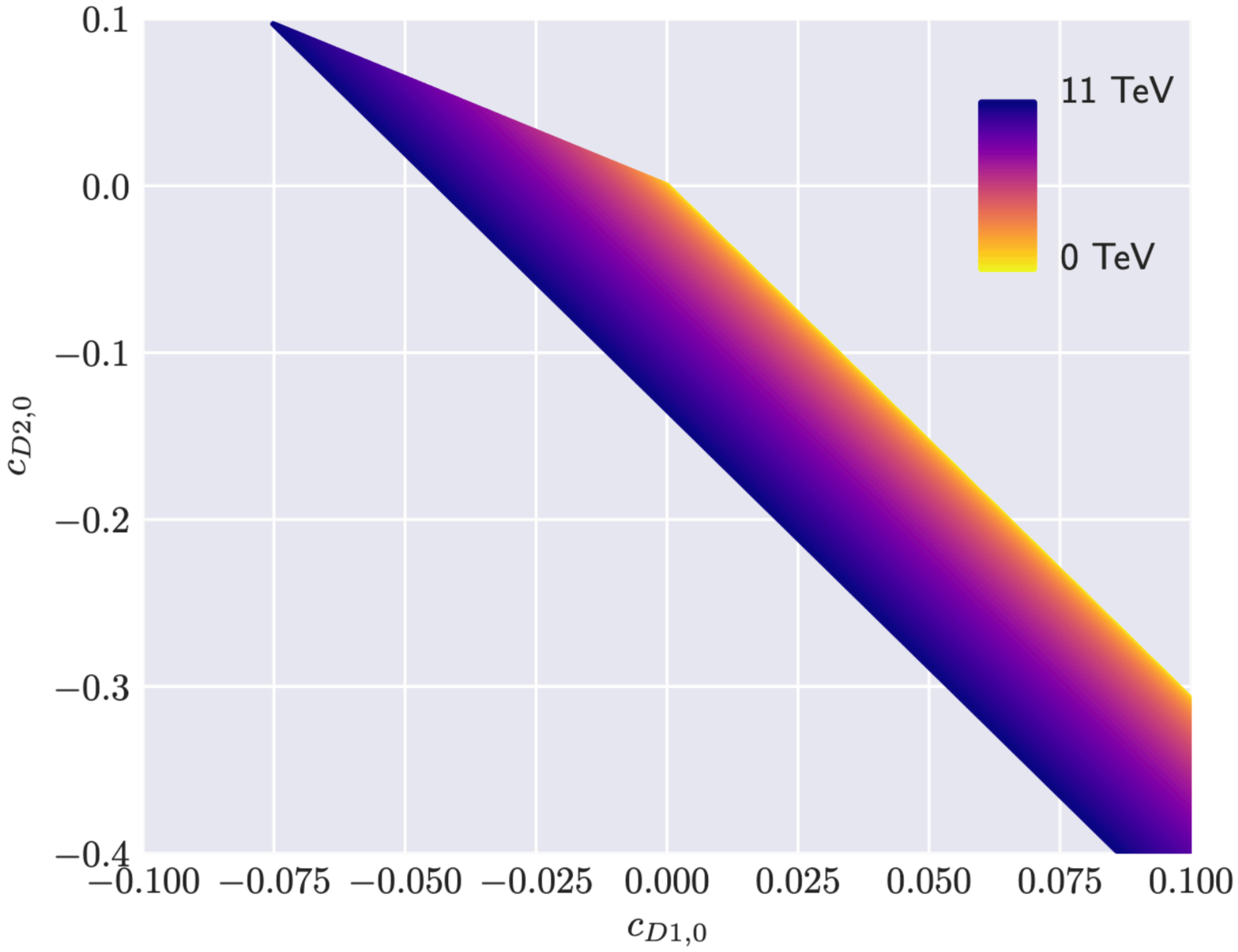
$$R_{Sk}^2 = \frac{i}{24\pi^2}\epsilon_{ijk}\int d^3x\,|\,\mathbf{x}\,|^2\left\langle W_i W_j W_k\right\rangle = \frac{2}{\pi(ve)^2}\int dr\; r^2\, b\,(f_1^2 + f_2^2)$$

$$\implies R_{Sk} \simeq \frac{1.6}{ve}$$

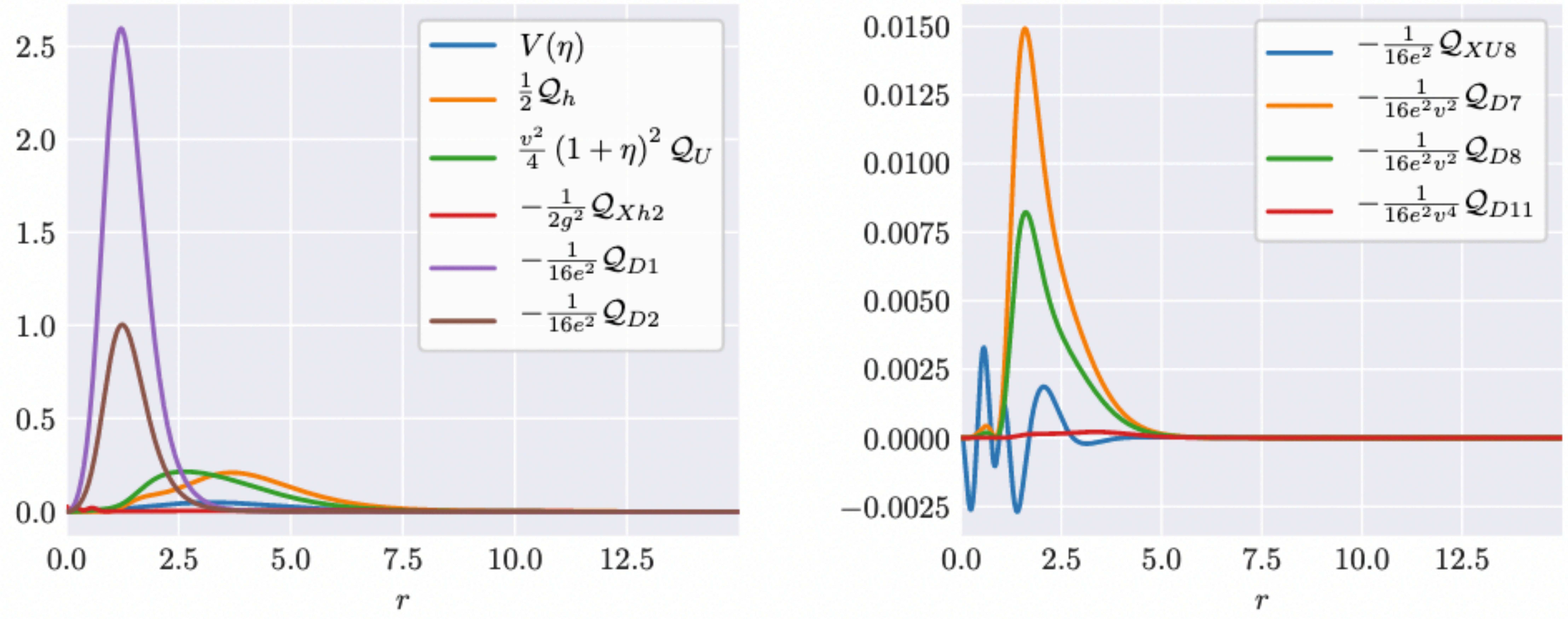
Other operators

Name	Operator	Radial energy density ρ_i in spherical ansatz	
$\mathcal{Q}_1$	λ	$-\frac{r^2}{e^2 v^4}$	**Standard Model**
$\mathcal{Q}_h$	$\partial_\mu h \partial^\mu h$	$\frac{r^2}{2}(\eta')^2$	
$\mathcal{Q}_U$	$\left\langle D_\mu U^\dagger D^\mu U \right\rangle$	$\frac{2}{v^2}\left(f_1^2 + f_2^2 + \frac{r^2}{2} b^2\right)$	
$\mathcal{Q}_{Xh2}$	$\frac{1}{g^2}\langle W_{\mu\nu} W^{\mu\nu} \rangle$	$-8e^2\left[(f_1' - 2bf_2)^2 + (f_2' + (2f_1 - 1)b)^2 + \frac{2}{r^2}(f_1^2 + f_2^2 - f_1)^2\right]$	
$\mathcal{Q}_{Xh5}$	$\epsilon^{\mu\nu\rho\sigma}\langle W_{\mu\nu} W_{\rho\sigma} \rangle$	0	**Vanish in pure gauge**
$\mathcal{Q}_{XU8}$	$i\langle W_{\mu\nu}[L^\mu, L^\nu] \rangle$	$\frac{16e^2}{2r^2}\left[(f_1^2 + f_2^2)(f_1^2 + f_2^2 - f_1 + 2r^2 b^2) - br^2(f_2 f_1' - f_1 f_2' + bf_1)\right]$	
$\mathcal{Q}_{XU11}$	$i\epsilon^{\mu\nu\rho\sigma}\langle W_{\mu\nu}[L_\rho, L_\sigma] \rangle$	0	
$\mathcal{Q}_{D1}$	$\langle L_\mu L^\mu \rangle^2$	$-\frac{4e^2}{r^2}\left[2(f_1^2 + f_2^2) + r^2 b^2\right]^2$	**Support skyrmions**
$\mathcal{Q}_{D2}$	$\langle L_\mu L_\nu \rangle \langle L^\mu L^\nu \rangle$	$-\frac{4e^2}{r^2}\left[2(f_1^2 + f_2^2)^2 + r^4 b^4\right]$	
$\mathcal{Q}_{D7}$	$\langle L_\mu L^\mu \rangle \partial_\nu h \partial^\nu h$	$-e^2 v^2 (\eta')^2\left[2(f_1^2 + f_2^2) + r^2 b^2\right]$	**(Numerically) do not support skyrmions**
$\mathcal{Q}_{D8}$	$\langle L_\mu L_\nu \rangle \partial^\mu h \partial^\nu h$	$-e^2 v^2 (\eta')^2 r^2 b^2$	
$\mathcal{Q}_{D11}$	$(\partial_\mu h \partial^\mu h)^2$	$-\frac{e^2 v^4}{4}(\eta')^4 r^2$	

The skyrmion region

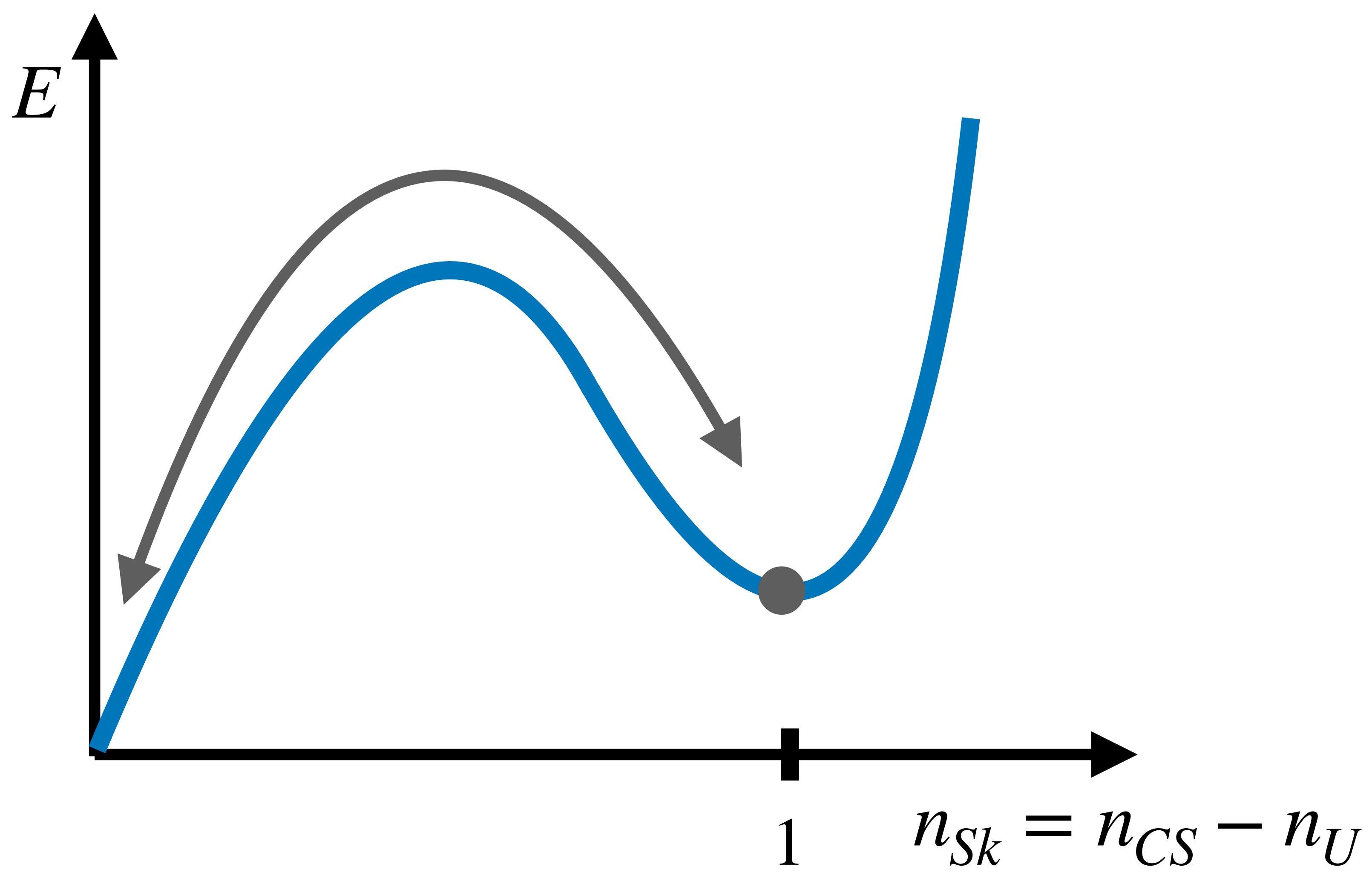


“Non-skyrmion” operators



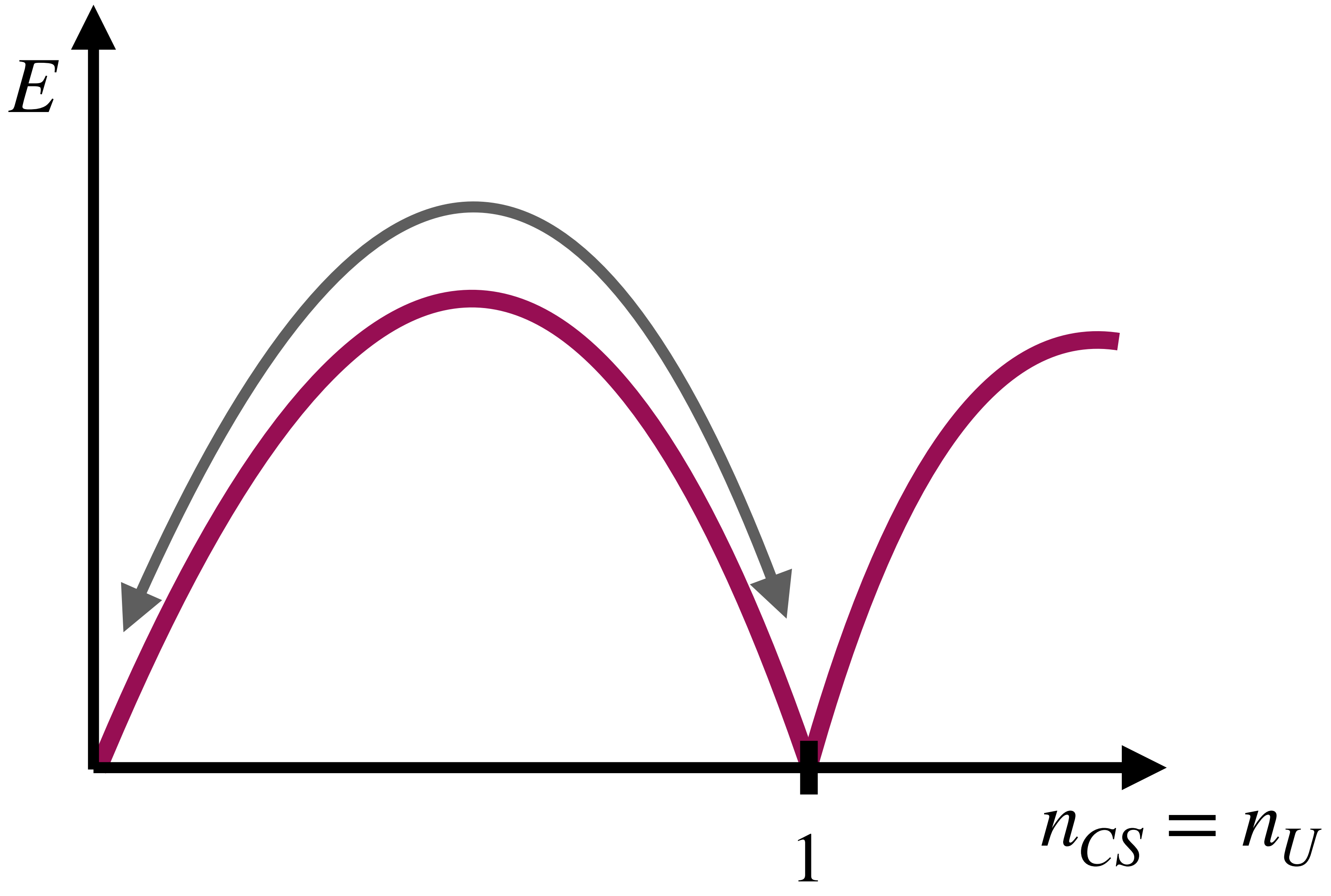
Phenomenology

Skyrmion production/decay



Skyrmion creation/decay

$\simeq$



Instanton/sphaleron

$\sim e^{-1/g^2}$ suppression $\Longrightarrow$

- Unlikely production at colliders.
- Stable $\rightarrow$ dark matter?

Limits on the skyrmion operators

Limits from LHC on aQGC

$$-2.7 \times 10^{-3} \leq 2c_{D1,0} + c_{D2,0} \leq 2.9 \times 10^{-3}$$

$$-8.2 \times 10^{-3} \leq c_{D2,0} \leq 8.9 \times 10^{-3}$$

$\dots$

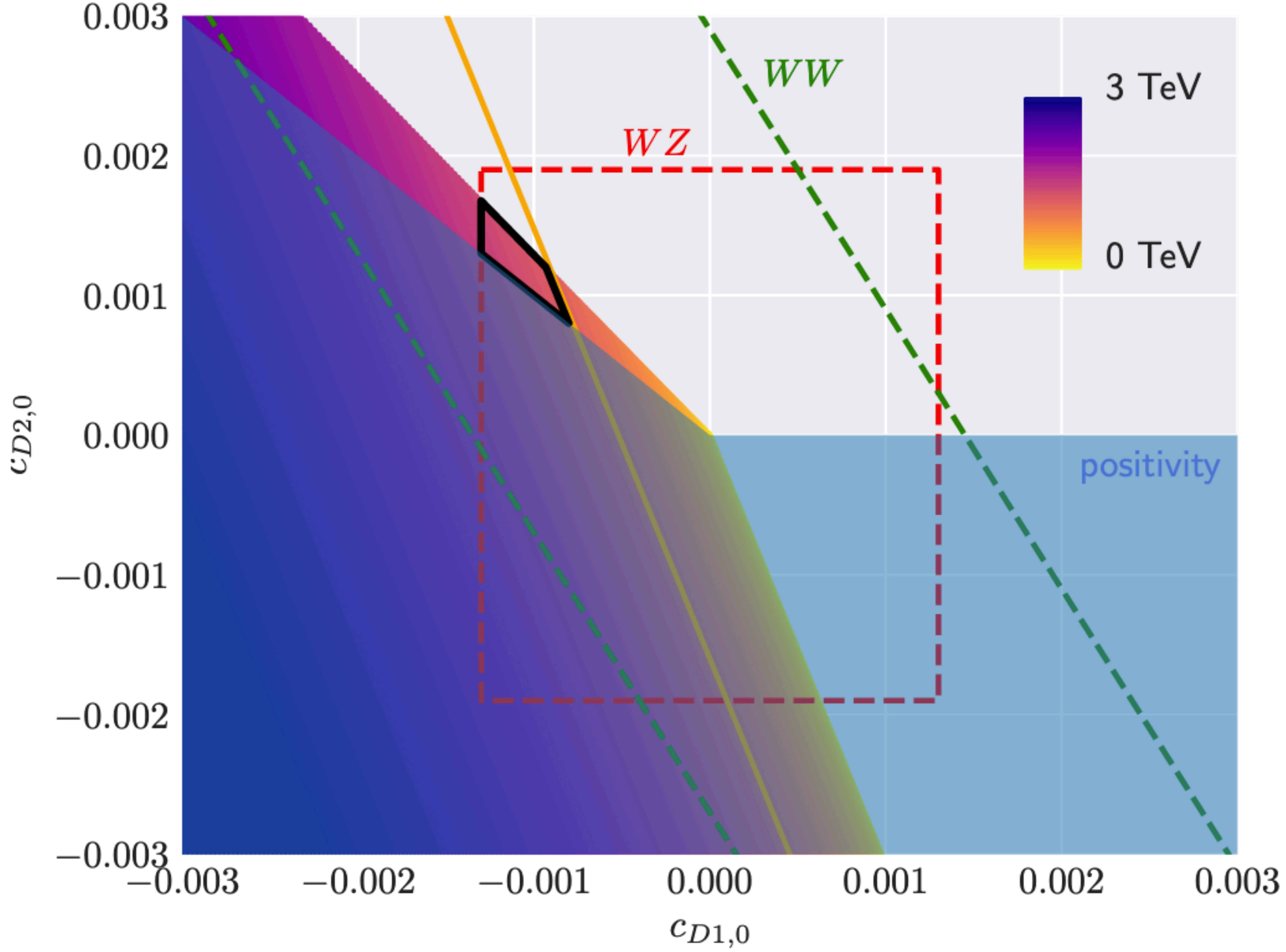
Positivity

$$c_{D1,0} + c_{D2,0} > 0, \qquad c_{D2} > 0.$$

Behaviour as a classical particle

$$R_{Sk} \gtrsim 1/M_{Sk}$$

Limits on the skyrmion operators



$M_{Sk} \simeq 1-2$ TeV

Skyrmion dark matter

$$0.1 \simeq \Omega_{Sk} h^2 \simeq \frac{3 \times 10^{-27}\,\text{cm}^3\text{s}^{-1}}{\langle \sigma_{ann} \text{v} \rangle}$$

$$\sigma_{ann} \simeq \pi R_{Sk}^2$$

$$\Longrightarrow \quad M_{Sk} \simeq 60\,\text{GeV}$$

- Finite temperature corrections on the shape of the potential
- More accurate computations of the annihilation cross section
- …

Summary

- **Machine learning** techniques for non-perturbative QFT solutions

- There are meta-stable **skyrmions in the HEFT**
- We have identified which Wilson coefficients generate them
- Direct production at LHC unlikely $\Longrightarrow$ indirect limits
- EW Skyrmions have mass $M_{Sk} \simeq 1-2$ TeV and barrier $E_{barrier} \simeq 11$ TeV
- **Dark matter** candidates?