SMEFT at higher orders through geometry

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Based on

- Gauge fixing the Standard Model Effective Field Theory, AH, Michael Paraskevas, Michael Trott, 1803.08001
- Ward Identities for the Standard Model Effective Field Theory, Tyler Corbett, AH, Michael Trott, 1909.08470
- The Geometric Standard Model Effective Field Theory, AH, Adam Martin, Michael Trott, 2001.01453
- Exact SMEFT formulation and expansion to $\mathcal{O}(v^4/\Lambda^4)$, Chris Hays, AH, Adam Martin, Michael Trott, 2007.00565
- EWPD in the SMEFT to dimension eight, Tyler Corbett, AH, Adam Martin, Michael Trott, 2102.02819
- Geometric soft theorems , Clifford Cheung, AH, Julio Parra-Martinez, 2111.03045

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What are the building blocks?

We can define a Lagrangian that consists of fields. But fields can be redefined, while physical observables are unchanged.

$$\phi \to \phi + c\phi^2 + \dots$$

We could start with on-shell scattering amplitudes; e.g., the non-factorizable part of the four-point amplitude between two gluons and two Higgs bosons [Shadmi,Weiss]

$$\mathcal{M}(g^{a+}(p_1)g^{b+}(p_2)hh) = \delta_{ab}\frac{[12]^2}{\Lambda^2}\left[a_1 + \frac{(s_{13}+s_{23})a_2}{\Lambda^2} + \frac{s_{12}a_3}{\Lambda^2} + \dots\right]$$

What is a_1?

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Geometric field space — and what it can do for you

Questions to answer:

- What is a geometric field space?
- What does it do?
- How does this help?

Geometric field space – scalars

Kinetic terms for the Higgs field

$$\begin{aligned}\mathcal{L} =& (D_\mu H)^\dagger (D^\mu H) \\ & + C_{H\Box}(H^\dagger H)\Box(H^\dagger H) + C_{HD}(H^\dagger D_\mu H)^*(H^\dagger D^\mu H) + \dots \\ =& \frac{1}{2} h_{IJ}(\phi)(D_\mu \phi)^I (D^\mu \phi)^J\end{aligned}$$

where $I, J = 1 \dots 4$ and

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix}$$

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Field redefinition

A scalar field redefinition is a coordinate change in field space

$$\phi^I \to \varphi^I(\phi)$$
$$D_\mu \phi^I \to \frac{\delta \varphi^I}{\delta \phi^J} D_\mu \phi^J$$
$$h_{IJ} \to \frac{\delta \phi^K}{\delta \varphi^I} \frac{\delta \phi^L}{\delta \varphi^J} h_{KL}$$

ϕ^I is the coordinate, $D_\mu \phi^I$ is a vector, h_{IJ} is a tensor (metric)

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All-order form of metric

From Hilbert series counting, the number of new operators contributing to metrics saturates.

Scalar field space metric

$$h_{IJ} = \left[1 + \phi^2 C^{(6)}_{H\Box} + \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2}\right)^{n+2} \left(C^{(8+2n)}_{HD} - C^{(8+2n)}_{H,D2}\right)\right] \delta_{IJ}$$
$$+ \frac{\Gamma^I_{A,J}\phi_K \Gamma^K_{A,L}\phi^L}{2} \left(\frac{C^{(6)}_{HD}}{2} + \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2}\right)^{n+1} C^{(8+2n)}_{H,D2}\right)$$

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From metric to curvature

We calculate the Christoffel symbol

$$\Gamma^{I}_{JK} = \frac{1}{2} h^{IL} \left(h_{LK,J} + h_{JL,K} - h_{JK,L} \right)$$

and Riemann curvature

$$R_{IJKL} = h_{IM} \left(\Gamma^{M}_{LJ,K} - \Gamma^{M}_{KJ,L} + \Gamma^{M}_{KN} \Gamma^{N}_{LJ} - \Gamma^{M}_{LN} \Gamma^{N}_{KJ} \right)$$

Geometric field space – scalars

Scattering amplitude for 4 scalars

$$\mathcal{A}^{ijkl} = R^{ikjl} s_{34} + R^{ijkl} s_{24}$$

Scattering amplitude for 5 scalars

$$\mathcal{A}^{ijklm} = \nabla^k R^{iljm} s_{45} + \nabla^l R^{ikjm} s_{35} + \nabla^l R^{ijkm} s_{25} + \nabla^m R^{ikjl} s_{34} + \nabla^m R^{ijkl} (s_{24} + s_{45})$$

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Fields vs. particles

The tetrad flattens the metric

$$g_{IJ}(v)e_i^I(v)e_j^J(v) = \delta_{ij}$$

Tetrad relates fields to states

$$\langle p^j|\phi^J(x)|0\rangle = e^{jJ}(v)e^{ip\cdot x}$$

Automatically takes into account canonical normalization and rotation to mass eigenstates

Geometric field space - gauge bosons

Let's combine the terms

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + C_{HB}(H^\dagger H)B_{\mu\nu}B^{\mu\nu} \\ &+ C_{HW}(H^\dagger H)W^a_{\mu\nu}W^{a\mu\nu} + C_{HWB}(H^\dagger\sigma^a H)W^a_{\mu\nu}B^{\mu\nu} + \dots \\ = &-\frac{1}{4}g_{AB}(H)W^A_{\mu\nu}W^{B\mu\nu}\end{aligned}$$

where $A, B = 1\dots4$ and $W^4_{\mu\nu} = B_{\mu\nu}$.

$$g_{AB}(H): \text{ gauge field space metric}$$

All-order form of metrics

From Hilbert series counting, the number of new operators contributing to metrics saturates.

The gauge field space metric is given by

$$
\begin{aligned}
g_{AB}(\phi_I) = & \left[1 - 4\sum_{n=0}^{\infty}\left(C_{HW}^{(6+2n)}(1-\delta_{A4}) + C_{HB}^{(6+2n)}\delta_{A4}\right)\left(\frac{\phi^2}{2}\right)^{n+1}\right]\delta_{AB} \\
& - \sum_{n=0}^{\infty} C_{HW,2}^{(8+2n)}\left(\frac{\phi^2}{2}\right)^{n}\left(\phi_I \Gamma^I_{A,J}\phi^J\right)\left(\phi_L \Gamma^L_{B,K}\phi^K\right)(1-\delta_{A4})(1-\delta_{B4}) \\
& + \left[\sum_{n=0}^{\infty} C_{HWB}^{(6+2n)}\left(\frac{\phi^2}{2}\right)^{n}\right]\left[\left(\phi_I \Gamma^I_{A,J}\phi^J\right)(1-\delta_{A4})\delta_{B4} + (A \leftrightarrow B)\right]
\end{aligned}
$$

Geometric field space – mass-eigenstate basis

$$W_\mu^A = e_c^A \mathcal{A}_\mu^c, \qquad e_c^A e_d^B \delta^{cd} = g^{AB}, \qquad e_c^A = \sqrt{g}^{AB} \delta^{Bb} U_{bc}$$

$$U_{bc} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \cos(\bar{\theta}) & \sin(\bar{\theta}) \\ 0 & 0 & -\sin(\bar{\theta}) & \cos(\bar{\theta}) \end{pmatrix}$$

W_μ^A : weak eigenstate

$\mathcal{A}_\mu^a$: mass eigenstate

$\sqrt{g}^{AB}$: matrix square root + vacuum expectation value

Geometric definition of Lagrangian parameters

Consistency dictates that

$$s^2_{\theta_Z} = \frac{g_1\left(\sqrt{g}^{44} s_{\bar\theta} - \sqrt{g}^{34} c_{\bar\theta}\right)}{g_2\left(\sqrt{g}^{33} c_{\bar\theta} - \sqrt{g}^{34} s_{\bar\theta}\right) + g_1\left(\sqrt{g}^{44} s_{\bar\theta} - \sqrt{g}^{34} c_{\bar\theta}\right)}$$

$$s^2_{\bar\theta} = \frac{\left(g_1\sqrt{g}^{44} - g_2\sqrt{g}^{34}\right)^2}{g_1^2\left[g^{34} + g^{44}\right] + g_2^2\left[g^{33} + g^{34}\right] - 2g_1 g_2\sqrt{g}^{34}\left(\sqrt{g}^{33} + \sqrt{g}^{44}\right)}$$

$$\bar g_2 = g_2\sqrt{g}^{11}$$

$$\bar g_Z = \frac{g_2}{c^2_{\theta_Z}}\left(c_{\bar\theta}\sqrt{g}^{33} - s_{\bar\theta}\sqrt{g}^{34}\right)$$

$$\bar e = g_2\left(s_{\bar\theta}\sqrt{g}^{33} + c_{\bar\theta}\sqrt{g}^{34}\right)$$

$$\bar M_W^2 = \frac{\bar g_2^2}{4}\sqrt{h_{11}}^2 v_T^2$$

$$\bar M_Z^2 = \frac{\bar g_Z^2}{4}\sqrt{h_{33}}^2 v_T^2$$

$$\bar M_A^2 = 0$$

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On-shell amplitudes – photons and Higgs boson

Consider the amplitude for Higgs decay to 2 photons.

$$\mathcal{M}(\gamma^+(p_1)\gamma^+(p_2)h) = \frac{[12]^2}{\Lambda^2} b_1$$

Dropping the CP odd operators, the coefficient b_1 is related to the gauge field metric

$$b_1 = \frac{1}{2}\nabla_{i=h} g_{a=\gamma,b=\gamma} = \#\sqrt{h}^{44}\left[\langle\frac{\delta g_{33}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_2^2} + 2\langle\frac{\delta g_{34}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1 g_2} + \langle\frac{\delta g_{44}(\phi)}{\delta\phi_4}\rangle\frac{\bar{e}^2}{g_1^2}\right]$$

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Partial width for Z decay

The partial width for Z going to two fermions of the same chirality is

$$\Gamma_{Z\to\bar{\psi}\psi} = \sum_{\psi} \frac{N_c^{\psi}}{24\pi}\sqrt{\bar{m}_Z^2}|g_{\text{eff}}^{Z,\psi}|^2 \left(1 - \frac{4\bar{m}_{\psi}^2}{\bar{m}_Z^2}\right)^{3/2}$$

where

$$g_{\text{eff}}^{Z,\psi} = \frac{\bar{g}_Z}{2}\left[(2s_{\theta_Z}Q_{\psi} - \sigma_3)\delta_{pr} + \bar{v}_T\langle L_{3,4}^{\psi,pr}\rangle + \sigma_3\bar{v}_T\langle L_{3,3}^{\psi,pr}\rangle\right]$$

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Partial width for Z decay

The field space connection is

$$L^{\psi,pr}_{J,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_p\gamma_\mu\sigma_A\psi_r)$$

which takes the all-order form

$$\begin{aligned}
L^{\psi,pr}_{J,A} = &-(\phi\gamma_4)_J\delta_{A4}\sum_{n=0}^{\infty} C^{1,(6+2n)}_{\substack{H\psi \\ pr}}\left(\frac{\phi^2}{2}\right)^n - (\phi\gamma_A)_J(1-\delta_{A4})\sum_{n=0}^{\infty} C^{3,(6+2n)}_{\substack{H\psi_L \\ pr}}\left(\frac{\phi^2}{2}\right)^n \\
&+\frac{1}{2}(\phi\gamma_4)_J(1-\delta_{A4})(\phi_K\Gamma^K_{A,L}\phi^L)\sum_{n=0}^{\infty} C^{2,(8+2n)}_{\substack{H\psi_L \\ pr}}\left(\frac{\phi^2}{2}\right)^n \\
&+\frac{\epsilon^A_{BC}}{2}(\phi\gamma_B)(\phi_K\Gamma^K_{C,L}\phi^L)\sum_{n=0}^{\infty} C^{\epsilon,(8+2n)}_{\substack{H\psi_L \\ pr}}\left(\frac{\phi^2}{2}\right)^n
\end{aligned}$$

Expand to $\mathcal{O}(v^4/\Lambda^4)$

Issue: only results for dimension-6. Should we include (dimension-6)^2 in the decay rates/cross sections?

Expand to $\mathcal{O}(v^4/\Lambda^4)$

Issue: only results for dimension-6. Should we include (dimension-6)^2 in the decay rates/cross sections?

We can now answer this question by comparing! Expanding $g_{\rm eff}$

$$
\begin{aligned}
\langle g^{\mathcal{Z},u_L}_{\rm SM,pp}\rangle &= -0.26, \\
\langle g^{\mathcal{Z},u_L}_{\rm eff,pp}\rangle_{\mathcal{O}(v^2/\Lambda^2)} &= -0.13\,\tilde{C}^{(6)}_{HD} - 0.21\,\tilde{C}^{(6)}_{HWB} + 0.18\,\delta G^{(6)}_F + 0.37\,(\tilde{C}^{(6)}_{\substack{Hq\\pp}} - \tilde{C}^{3,(6)}_{\substack{Hq\\pp}}), \\
\langle g^{\mathcal{Z},u_L}_{\rm eff,pp}\rangle_{\mathcal{O}(v^4/\Lambda^4)} &= -\left(\frac{\tilde{C}^{(6)}_{HD}}{4} + \frac{\delta G^{(6)}_F}{\sqrt{2}}\right)\langle g^{\mathcal{Z},u_L}_{\rm eff,pp}\rangle_{\mathcal{O}(v^2/\Lambda^2)} \\
&\quad + \tilde{C}^{(6)}_{HWB}\left(0.13\,\tilde{C}^{(6)}_{HD} - 0.21(\tilde{C}^{(6)}_{HB} + \tilde{C}^{(6)}_{HW})\right) \\
&\quad - 0.01(\tilde{C}^{(6)}_{HD})^2 + 0.05\,\tilde{C}^{(6)}_{HD}\,\delta G^{(6)}_F + 0.03\,\tilde{C}^{(8)}_{HD} - 0.16\,\tilde{C}^{(8)}_{H,D2} - 0.10\,\tilde{C}^{(8)}_{HWB} \\
&\quad - 0.38\,\tilde{C}^{(8)}_{HW,2} - \frac{0.37}{2}(\tilde{C}^{2,(8)}_{\substack{Hq\\pp}} + \tilde{C}^{3,(8)}_{\substack{Hq\\pp}} - \tilde{C}^{(8)}_{\substack{Hq\\pp}}) - 0.07(\delta G^{(6)}_F)^2 + 0.18\,\delta G^{(8)}_F
\end{aligned}
$$

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Summary

The geometric Standard Model Effective Field Theory is useful:

- field redefinitions are coordinate changes
- define all-order field space connections
- relate Lagrangian parameters to field space metrics
- relate input parameters to field space metrics
- gauge fixing and simple Ward identities
- coefficients of on-shell amplitudes
- simplifies calculations
- relevant for phenomenology

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Thank you!

Standard Model Effective Field Theory (SM EFT)

SM EFT is an *effective field theory* based on SM field content and symmetries:

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \ldots$$

$$\mathcal{L}^{(d)} = \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)}, \qquad d > 4$$

$C_i^{(d)}$: Wilson coefficient

$Q_i^{(d)}$: Effective operator

On-shell amplitudes – gluons and Higgs bosons

Let's again look at the non-factorizable part of the four-point amplitude between two gluons and two Higgs bosons [Shadmi,Weiss]

$$\mathcal{M}(g^{a+}(p_1)g^{b+}(p_2)hh) = \delta_{ab}\frac{[12]^2}{\Lambda^2}\left[a_1 + \frac{(s_{13}+s_{23})a_2}{\Lambda^2} + \frac{s_{12}a_3}{\Lambda^2} + \dots\right]$$

The relevant field space connections for the first term are

$$k_{ab}(\phi)G^a_{\mu\nu}G^{a\mu\nu} \qquad \tilde{k}_{ab}(\phi)G^a_{\mu\nu}\tilde{G}^{b\mu\nu}$$

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On-shell amplitudes – gluons and Higgs bosons

The all-order form of the field space connections are

$$k_{ab}(\phi) = \left(1 - 4\sum_{n=0}^{\infty} C_{HG}^{(6+2n)} \left(\frac{\phi^2}{2}\right)^n\right)\delta_{ab}$$

$$\tilde{k}_{ab}(\phi) = \left(1 - 4\sum_{n=0}^{\infty} \tilde{C}_{HG}^{(6+2n)} \left(\frac{\phi^2}{2}\right)^n\right)\delta_{ab}$$

The coefficient of the on-shell amplitude can be expressed in terms of the field space connections:

$$a_1 = \# \left(\sqrt{h}^{44}\right)^2 \langle \frac{\delta^2 k_{ab}(\phi)}{\delta\phi_4 \delta\phi_4} \rangle + \# \left(\sqrt{h}^{44}\right)^2 \langle \frac{\delta^2 \tilde{k}_{ab}(\phi)}{\delta\phi_4 \delta\phi_4} \rangle$$

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Generalization to other connections

CP even electroweak bosonic two- and three-point connections

$$V(\phi), \quad h_{IJ}(\phi)(D_\mu\phi)^I(D_\mu\phi)^J, \quad g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu},$$
$$k^A_{IJ}(\phi)(D_\mu\phi)^I(D_\nu\phi)^J\,\mathcal{W}^{\mu\nu}_A, \quad f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho.$$

Connections with fermions

$$Y(\phi)\bar{\psi}_1\psi_2, \quad L_{I,A}(\phi)\bar{\psi}_1\gamma^\mu\sigma_A\psi_2(D_\mu\phi)^I, \quad d_A(\phi)\bar{\psi}_1\sigma^{\mu\nu}\psi_2\mathcal{W}^A_{\mu\nu}.$$

Small set of connections!

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Hilbert series counting for connections

The number of operators saturates!

	Mass Dimension				
Field space connection	6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u + \text{h.c.}$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d + \text{h.c.}$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e + \text{h.c.}$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A + \text{h.c.}$	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A + \text{h.c.}$	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A + \text{h.c.}$	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$

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Real representation of the scalar field

We use the real representation

$$\gamma^I_{1,J} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \gamma^I_{2,J} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\gamma^I_{3,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \gamma^I_{4,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

We have that

$$[\gamma_a, \gamma_b] = 2\epsilon^c_{ab}\gamma_c, \qquad \Gamma^I_{A,K} = \gamma^I_{A,J}\gamma^J_{4,K}$$

$$[\gamma_a, \gamma_4] = 0,$$

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Mass eigenstate generators

$$\gamma^{I}_{C,J} = \frac{1}{2}\tilde{\gamma}^{I}_{A,J}\sqrt{g}^{AB}U_{BC}$$

$$\gamma^{I}_{1,J} = \frac{\overline{g}_2}{2\sqrt{2}}\begin{bmatrix} 0 & 0 & i & -1 \\ 0 & 0 & -1 & -i \\ -i & 1 & 0 & 0 \\ 1 & i & 0 & 0 \end{bmatrix}, \qquad \gamma^{I}_{2,J} = \frac{\overline{g}_2}{2\sqrt{2}}\begin{bmatrix} 0 & 0 & -i & -1 \\ 0 & 0 & -1 & i \\ i & 1 & 0 & 0 \\ 1 & -i & 0 & 0 \end{bmatrix},$$

$$\gamma^{I}_{3,J} = \frac{\overline{g}_Z}{2}\begin{bmatrix} 0 & -(c^2_{\theta_Z} - s^2_{\theta_Z}) & 0 & 0 \\ (c^2_{\theta_Z} - s^2_{\theta_Z}) & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad \gamma^{I}_{4,J} = \overline{e}\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$