Towards Automatic Matching with Functional Methods

Anders Eller Thomsen

Based on work with J. Fuentes-Martín, M. König, J. Pagès, and F. Wilsch

UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Higgs and Effective Field Theory,
Granada 17–19 June 2022

MATCHETE

EFT matching

EFTs are used to interpret experiments and quantify observations

$$\mathcal{L}_{\mathrm{EFT}}(\eta_{\mathrm{L}}) = \mathcal{L}^{d=4}(\eta_{\mathrm{L}}) + \sum_{n=5}^{\infty} \frac{C_{n,i}}{M_{\mathrm{H}}^{n-4}} \mathcal{O}_{n,i}(\eta_{\mathrm{L}}) \quad \longrightarrow \quad \text{UV physics}$$

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NP models have to be analyzed one by one

$$\mathcal{L}_{\text{UV}}(\eta_{\text{H}}, \eta_{\text{L}}) \quad \xrightarrow{\text{Matching}} \quad \mathcal{L}_{\text{EFT}}(\eta_{\text{L}})$$

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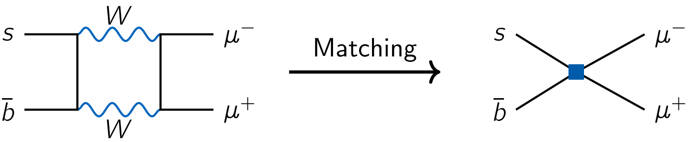
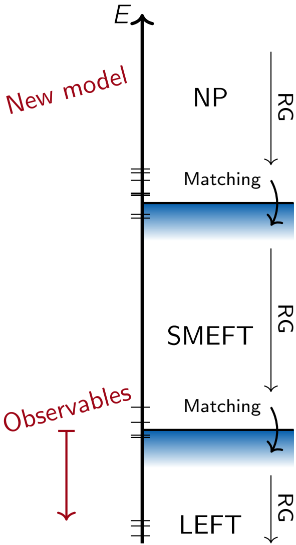
Loop-level matching is required for many processes, e.g.,



EFT-workflow



Jenkins, Manohar, Trott [1308.2627]
Jenkins, Manohar, Trott [1310.4838]
Alonso *et al.* [1312.2014]

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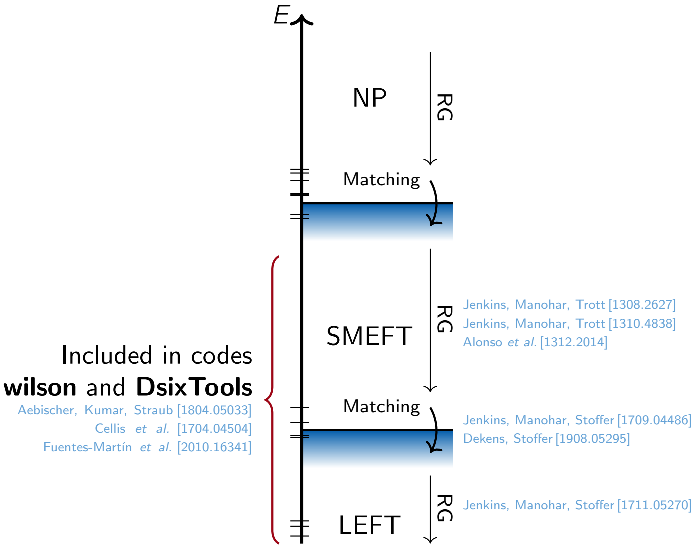

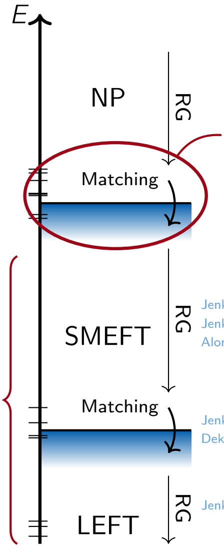

Recent progress with partial automation and **Matchmakereft**

Criado [1710.06445]; Das Bakshi *et al.* [1808.04403] Cohen, Lu, Zhang [2012.07851]; Fuentes-Martín *et al.* [2012.08506]; Carmona *et al.* [2112.10787]

Included in codes **wilson** and **DsixTools**

Aebischer, Kumar, Straub [1804.05033]
Cellis *et al.* [1704.04504]
Fuentes-Martín *et al.* [2010.16341]

Jenkins, Manohar, Trott [1308.2627]
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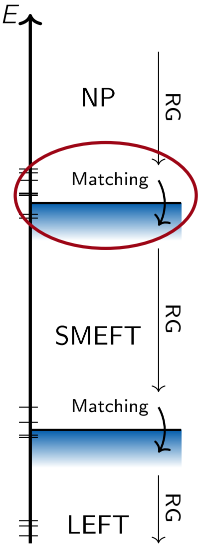
Matching weakly coupled theories



$$\mathcal{L}_{\mathrm{EFT}}(\eta_{\mathrm{L}}) = \mathcal{L}_{\mathrm{kin}}(\eta_{\mathrm{L}}) + \sum_{n=2}^{\infty}\sum_{\ell=0}^{\infty} \frac{C_{n,i}^{(\ell)}}{(4\pi)^{2\ell} M_{\mathrm{H}}^{n-4}} \mathcal{O}_{n,i}^{(\ell)}(\eta_{\mathrm{L}})$$

$$\mathcal{L}_{\mathrm{UV}}(\eta_{\mathrm{L}}, \eta_{\mathrm{H}}) \xrightarrow{\text{Matching}} \mathcal{L}_{\mathrm{EFT}}(\eta_{\mathrm{L}})$$

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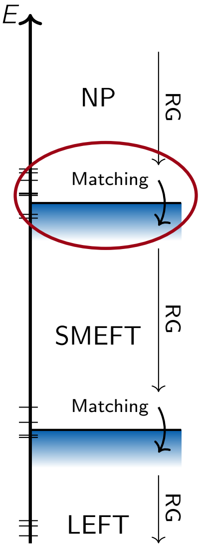

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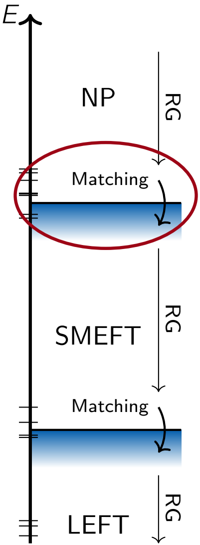
$$\begin{array}{ccc} \mathcal{L}_{\mathrm{UV}}(\eta_{\mathrm{L}}, \eta_{\mathrm{H}}) & \xrightarrow{\text{Matching}} & \mathcal{L}_{\mathrm{EFT}}(\eta_{\mathrm{L}}) \\ \downarrow & & \downarrow \\ \Gamma_{\mathrm{UV}}[\hat{\eta}_{\mathrm{L}}, \hat{\eta}_{\mathrm{H}}(\hat{\eta}_{\mathrm{L}})] & \simeq & \Gamma_{\mathrm{EFT}}[\hat{\eta}_{\mathrm{L}}] \end{array}$$

Matching weakly coupled theories



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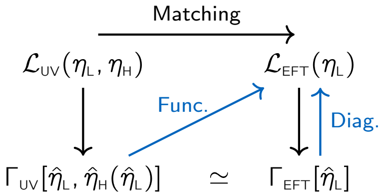

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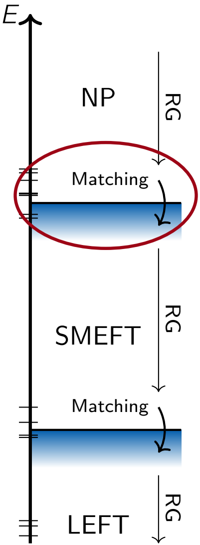

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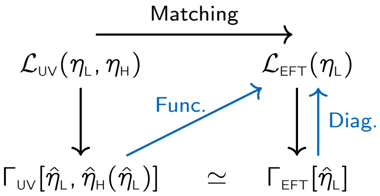

Advantages of functional matching:

- Does not require knowledge of EFT basis
- Well-suited for algorithmic approach
- Computations are manifestly gauge covariant

Separation of scales

Expansion by region allows for separating scales in dimensional regularization:

Beneke, Smirnov [hep-ph/9711391]; Jantzen [1111.2589]

$$\Gamma^{(1)}_{\mathrm{UV}} = \Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{hard}} + \Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{soft}}$$

Heavy field loops

Mixed field loops

Light field loops

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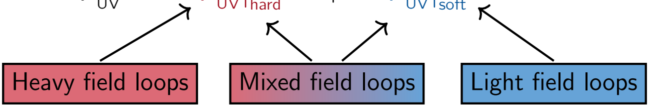
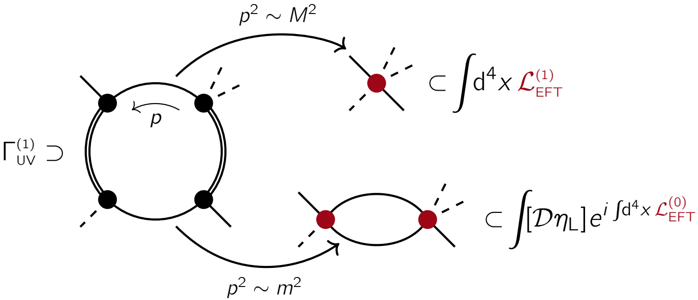


Mixed (heavy–light) loop example:



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Heavy field loops → $\Gamma^{(1)}_{\mathrm{UV}}\big|_{\mathrm{hard}}$ | Mixed field loops → $\Gamma^{(1)}_{\mathrm{UV}}\big|_{\mathrm{hard}}$, $\Gamma^{(1)}_{\mathrm{UV}}\big|_{\mathrm{soft}}$ | Light field loops → $\Gamma^{(1)}_{\mathrm{UV}}\big|_{\mathrm{soft}}$

- $\Gamma^{(1)}_{\mathrm{UV}}\big|_{\mathrm{soft}}$: long-distance contributions included in 1-loop matrix elements of tree-level EFT operators
- $\Gamma^{(1)}_{\mathrm{UV}}\big|_{\mathrm{hard}}$: short-distance contributions going into the EFT operators

$$\int \mathrm{d}^d x\, \mathcal{L}^{(1)}_{\mathrm{EFT}} = \Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{hard}}$$

Functional Matching

Functional matching relies on Henning, Lu, Murayama [1412.1837]; Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]

$$e^{i\,\Gamma_{\text{UV}}[\hat{\eta}]} = \int \mathcal{D}\eta \;\exp\left(i \int \mathrm{d}^d x \;\mathcal{L}_{\text{UV}}[\eta + \hat{\eta}] \right)$$

By saddle point approximation

Covariant derivatives

$$\Gamma^{(1)}_{\text{UV}} = \frac{i}{2}\,\text{STr}\,\ln\left(\Delta^{-1} - X\right), \qquad \delta_{ij}\,\Delta^{-1}_i(\hat{P},\,M_i) - X_{ij}(\hat{P},\,\hat{\eta}) \equiv \left.\frac{\delta^2 \mathcal{L}_{\text{UV}}}{\delta\eta_j \delta\bar{\eta}_i}\right|_{\eta=\hat{\eta}}$$

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The master formula for 1-loop matching: (evaluated with CDE) Cohen, Lu, Zhang [2011.02484]

$$\int \mathrm{d}^d x \;\mathcal{L}^{(1)}_{\text{EFT}} = \underbrace{\frac{i}{2}\,\text{STr}\ln \Delta^{-1}\Big|_{\text{hard}}}_{\text{Log term}} - \underbrace{\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\text{STr}\big[(\Delta X)^n\big]\Big|_{\text{hard}}}_{\text{Power-type terms}}$$

STrEAM

Cohen, Lu, Zhang [2012.07851]

SUPER TRACER

Fuentes-Martín, König, Pagès, AET, Wilsch [2012.08506]

Evanescent operators

Using 4-dimensional Fierz identities and gamma reduction for basis reduction introduces evanescent contributions

Buras, Weisz '90; Herrlich, Nierste [hep-ph/9412375];...

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In $d = 4$ dimensions

$$\Big[\mathcal{L}_{\text{EFT}} \supset C_{lqde}^{prst}(\bar{\ell}^p \gamma_\mu q^t)(\bar{d}^s \gamma^\mu e^r)\Big] \quad = \quad \Big[\tilde{\mathcal{L}}_{\text{EFT}} \supset -2C_{lqde}^{prst}(\bar{\ell}^p e^r)(\bar{d}^s q^t)\Big]$$

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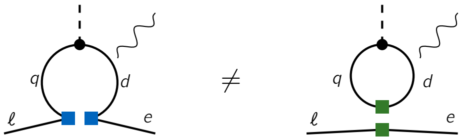
But the 1-loop effective action

$$\Gamma^{(1)}_{\text{EFT}} \quad \neq \quad \tilde{\Gamma}^{(1)}_{\text{EFT}}$$



Computing evanescent contributions

In $d = 4 - 2\epsilon$ dimensions

$\mathcal{O}(\epsilon)$

$$C_{lqde}^{prst}(\bar{\ell}^p \gamma_\mu q^t)(\bar{d}^s \gamma^\mu e^r) \quad = \quad -2C_{lqde}^{prst}(\bar{\ell}^p e^r)(\bar{d}^s q^t) + C_{lqde}^{prst} E_{lqde}^{prst}$$

But the 1-loop effective action

local

$$\Gamma_{\text{EFT}}^{(1)} \quad = \quad \tilde{\Gamma}_{\text{EFT}}^{(1)} + S_E^{(1)}, \qquad\qquad \tilde{\mathcal{L}}_{\text{EFT}}^{(1)} \to \tilde{\mathcal{L}}_{\text{EFT}}^{(1)} + \mathcal{L}_E^{(1)}$$

Computing evanescent contributions

In $d = 4 - 2\epsilon$ dimensions

$$C^{prst}_{lqde} (\bar{\ell}^p \gamma_\mu q^t)(\bar{d}^s \gamma^\mu e^r) \quad = \quad -2 C^{prst}_{lqde} (\bar{\ell}^p e^r)(\bar{d}^s q^t) + C^{prst}_{lqde} E^{prst}_{lqde}$$

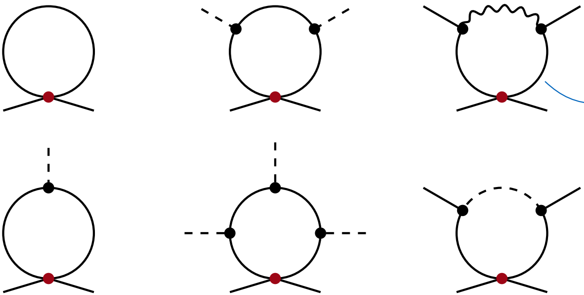
$\mathcal{O}(\epsilon)$

But the 1-loop effective action

$$\Gamma^{(1)}_{\text{EFT}} \quad = \quad \tilde{\Gamma}^{(1)}_{\text{EFT}} + S^{(1)}_E, \qquad\qquad \tilde{\mathcal{L}}^{(1)}_{\text{EFT}} \to \tilde{\mathcal{L}}^{(1)}_{\text{EFT}} + \mathcal{L}^{(1)}_E$$

local

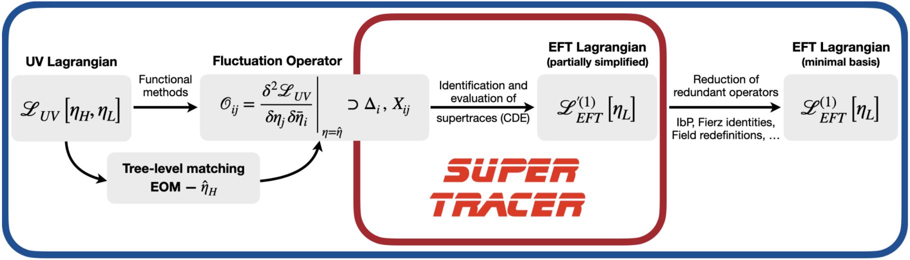
For dimension-6 SMEFT, 6 covariant trace topologies contribute:



From the $\mathcal{O}(1/\epsilon)$ pieces of the loops

Fuentes-Martín, König, Pagès, AET, Wilsch [w.i.p.]

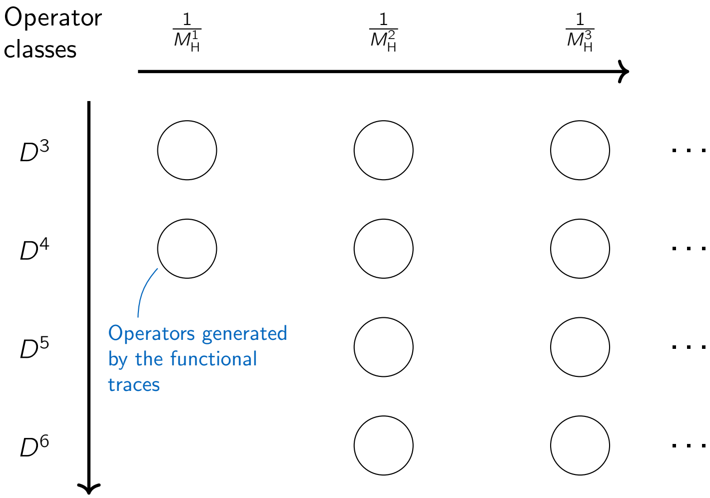
Matching Effective Theories Efficiently



Fuentes-Martín, König,
Pagès, AET, Wilsch [w.i.p.]

MATCHETE

Simplifications



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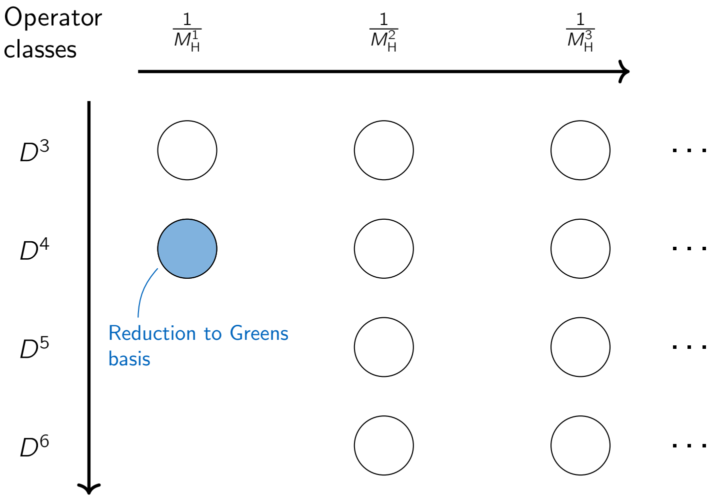

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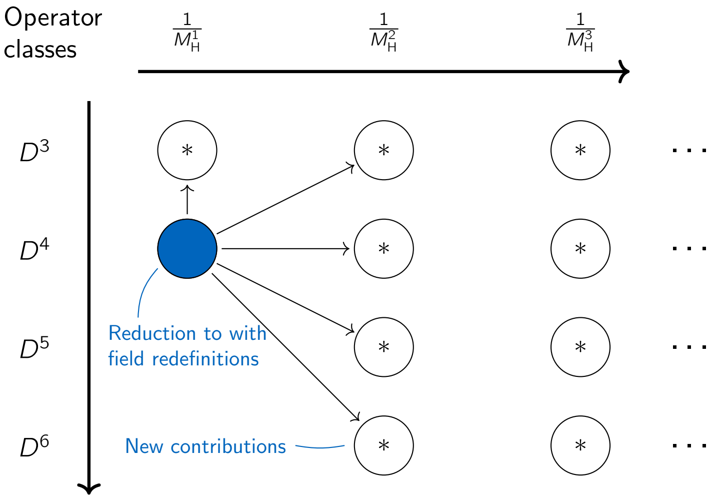

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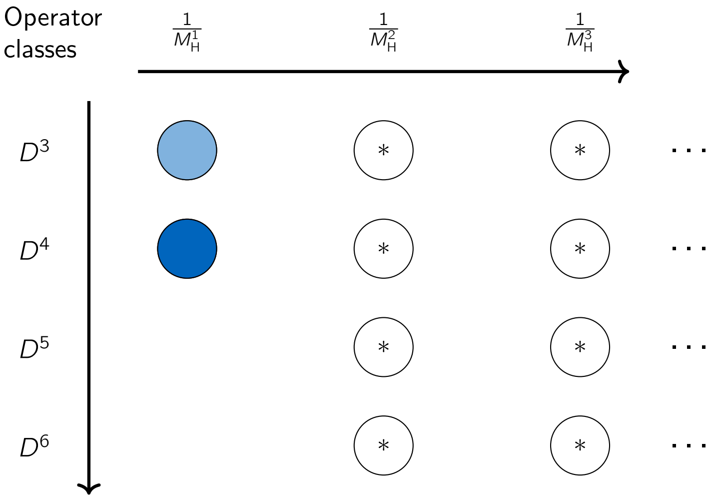

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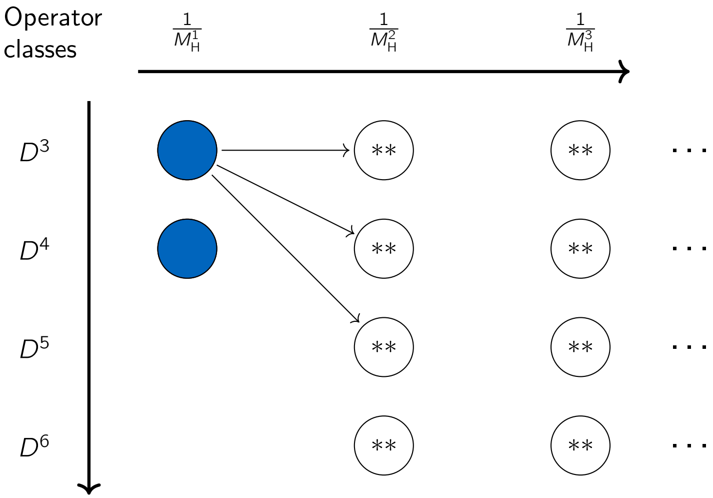

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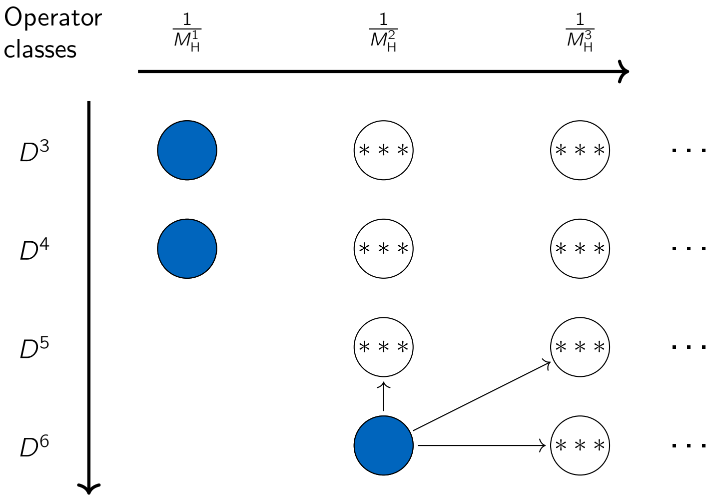

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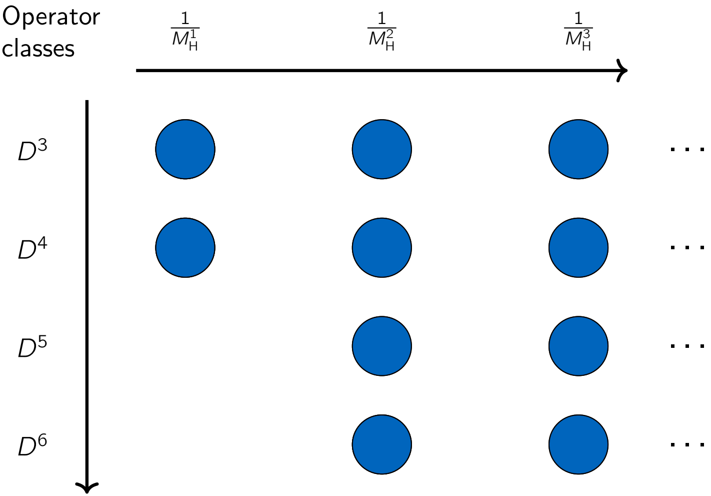

$+$ evanescent operators

Matching with Matchete

1-loop matching of singlet extension of the SM:[1]

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \tfrac{1}{2}(\partial_\mu S)^2 - \tfrac{1}{2}M_S^2 S^2 - \tfrac{1}{24}\lambda_S\, S^4 - \tfrac{1}{2}\lambda_{SH}(H^\dagger H)S^2 - \kappa\,(H^\dagger H)S,$$

at a scale $M_S \gg v_{\text{EW}}$

Outlook

- Functional matching is well-suited for automatic matching
- Progress on the development of **Matchete**; work is being done on the reduction to EFT basis
- Functional methods can be extended to β-functions and evanescent contributions

Backup

Matchete demonstration (SM implementation)

Gauge Groups

```
DefineGaugeGroup[SU3c, SU@3, gs, G,
    FundAlphabet → CharacterRange["a", "f"],
    AdjAlphabet → CharacterRange["A", "F"]]
DefineGaugeGroup[SU2L, SU@2, gL, W,
    FundAlphabet → CharacterRange["i", "n"],
    AdjAlphabet → CharacterRange["I", "N"]]
DefineGaugeGroup[U1Y, U@1, gY, B]
```

Generation index

```
DefineFlavorIndex[Flavor, 3,
    IndexAlphabet → {"p", "r", "s", "t", "u", "v"}]
```

Fermions

```
DefineField[q, Fermion,
    Indices → {SU3c@fund, SU2L@fund, Flavor},
    Charges → {U1Y[1/6]},
    Chiral → LeftHanded,
    Mass → 0]
DefineField[u, Fermion,
    Indices → {SU3c@fund, Flavor},
    Charges → {U1Y[2/3]},
    Chiral → RightHanded,
    Mass → 0]
DefineField[d, Fermion,
    Indices → {SU3c@fund, Flavor},
    Charges → {U1Y[-1/3]},
    Chiral → RightHanded,
    Mass → 0]
```

```
DefineField[l, Fermion,
    Indices → {SU2L@fund, Flavor},
    Charges → {U1Y[-1/2]},
    Chiral → LeftHanded,
    Mass → 0]
DefineField[e, Fermion,
    Indices → {Flavor},
    Charges → {U1Y[-1]},
    Chiral → RightHanded,
    Mass → 0]
```

Higgs

```
DefineField[H, Scalar,
    Indices → {SU2L@fund},
    Charges → {U1Y[1/2]},
    Mass → 0]
```

Couplings

```
DefineCoupling[λ, SelfConjugate → True]
DefineCoupling[μ, SelfConjugate → True,
  EFTorder → 1];
DefineCoupling[Ye,
    Indices → {Flavor, Flavor}]
DefineCoupling[Yu,
    Indices → {Flavor, Flavor}]
DefineCoupling[Yd,
    Indices → {Flavor, Flavor}]
```

Matchete demonstration (SM implementation)

Lagrangian

```
ℒSM = FreeLag[] +
    -μ[]^2 Bar@H[i] × H[i] -
    λ[]/2 Bar@H[i] × H[i] × Bar@H[j] × H[j] +
    PlusHc[
        -Yu[p, r] × CG[eps@SU2L, {i, j}] ×
        Bar@H@i × Bar@q[a, j, p] ** u[a, r]
        - Yd[p, r] × H@i × Bar@q[a, i, p] ** d[a, r]
        - Ye[p, r] × H@i × Bar@l[i, p] ** e[r]
    ] // RelabelIndices;
```

```
ℒSM // NiceForm
```

Form=

$$
\begin{aligned}
&-\frac{1}{4} B^{\mu\nu 2} - \frac{1}{4} G^{\mu\nu A 2} - \frac{1}{4} W^{\mu\nu I 2} + D_\mu \bar{H}_i D_\mu H^i - \\
&\mu^2 \bar{H}_i H^i + i \left(\bar{d}^p_a \cdot \gamma_\mu P_R \cdot D_\mu d^{ap}\right) + i \left(\bar{e}^p \cdot \gamma_\mu P_R \cdot D_\mu e^p\right) + \\
&i \left(\bar{l}^p_i \cdot \gamma_\mu P_L \cdot D_\mu l^{ip}\right) + i \left(\bar{q}^p_{ai} \cdot \gamma_\mu P_L \cdot D_\mu q^{aip}\right) + \\
&i \left(\bar{u}^p_a \cdot \gamma_\mu P_R \cdot D_\mu u^{ap}\right) - \frac{1}{2} \lambda \bar{H}_i \bar{H}_j H^i H^j - \\
&\overline{Yd}^{pr} \bar{H}_i \left(\bar{d}^r_a \cdot P_L \cdot q^{aip}\right) - \overline{Ye}^{pr} \bar{H}_i \left(\bar{e}^r \cdot P_L \cdot l^{ip}\right) - \\
&Ye^{pr} H^i \left(\bar{l}^p_i \cdot P_R \cdot e^r\right) - Yd^{pr} H^i \left(\bar{q}^p_{ai} \cdot P_R \cdot d^{ar}\right) - \\
&Yu^{pr} \bar{H}_i \left(\bar{q}^p_{aj} \cdot P_R \cdot u^{ar}\right) \varepsilon^{ij} - \overline{Yu}^{pr} H^i \left(\bar{u}^r_a \cdot P_L \cdot q^{ajp}\right) \bar{\varepsilon}_{ij}
\end{aligned}
$$

Matchete demonstration (singlet extension)

Field

```
DefineField[S, Scalar,
    SelfConjugate → True,
    Mass → Heavy]
```

New couplings

```
DefineCoupling[λSH,
    SelfConjugate → True]
DefineCoupling[λS,
    SelfConjugate → True]
DefineCoupling[κ,
    SelfConjugate → True]
```

NP Lagrangian

```
ℒNP = (FreeLag@S
    - λSH[]/2 Bar@H@i × H@i S[]^2
    - λS[]/24 S[]^4
    - κ[] Bar@H@i × H@i × S[]) //
  RelabelIndices;
```

```
ℒNP // NiceForm
```

Form=

$$\frac{1}{2}\left(D_\mu S\right)^2 - \frac{1}{2} MS^2\, S^2 - \kappa\, \bar{H}_i\, H^i\, S - \frac{1}{2}\lambda SH\, \bar{H}_i\, H^i\, S^2 - \frac{1}{24}\lambda S\, S^4$$

Matchete demonstration (tree-level matching)

At tree level

```
ℒEFT0 = Match[ℒUV, LoopOrder → 0];
ℒEFT0 - ℒSM // RelabelIndices // NiceForm
```

eForm=

$$\frac{1}{2}\,\frac{1}{MS^2}\,\kappa^2\,\bar{H}_i\,\bar{H}_j\,H^i\,H^j - \frac{1}{2}\,\lambda SH\,\frac{1}{MS^4}\,\kappa^2\,\bar{H}_i\,\bar{H}_j\,\bar{H}_k\,H^i\,H^j\,H^k + \frac{1}{2}\,\frac{1}{MS^4}\,\kappa^2\,D_\mu\bar{H}_i\,D_\mu\bar{H}_j\,H^i\,H^j + \frac{1}{2}\,\frac{1}{MS^4}\,\kappa^2\,\bar{H}_i\,D_\mu\bar{H}_j\,D_\mu H^i\,H^j + \frac{1}{2}\,\frac{1}{MS^4}\,\kappa^2\,D_\mu\bar{H}_i\,\bar{H}_j\,H^i\,D_\mu H^j + \frac{1}{2}\,\frac{1}{MS^4}\,\kappa^2\,\bar{H}_i\,\bar{H}_j\,D_\mu H^i\,D_\mu H^j$$

```
HcSimplify@GreensSimplify@% // NiceForm
```

eForm=

$$\frac{1}{2}\,\frac{1}{MS^2}\,\kappa^2\,\bar{H}_i\,\bar{H}_j\,H^i\,H^j - \frac{1}{2}\,\lambda SH\,\frac{1}{MS^4}\,\kappa^2\,\bar{H}_i\,\bar{H}_j\,\bar{H}_k\,H^i\,H^j\,H^k - \frac{1}{MS^4}\,\kappa^2\,\bar{H}_i\,D_\mu\bar{H}_j\,H^i\,D_\mu H^j + \left(-\frac{1}{2}\,\frac{1}{MS^4}\,\kappa^2\,\bar{H}_i\,\bar{H}_j\,D^2H^i\,H^j + \mathrm{H.c.}\right)$$

Matchete demonstration (1-loop matching)

At loop level

```
(ℒEFT = EchoTiming@GreensSimplify@ Match[ℒUV]) // NiceForm
```

23.4272

eForm=

$$-\frac{1}{4} B^{\mu\nu 2} - \frac{1}{4} G^{\mu\nu A 2} - \frac{1}{4} W^{\mu\nu I 2} + i \left(\bar{d}^{p}_{a} \cdot \gamma_\mu P_R \cdot D_\mu d^{ar}\right) \delta^{pr} +$$
$$i \left(\bar{e}^{p} \cdot \gamma_\mu P_R \cdot D_\mu e^{r}\right) \delta^{pr} + i \left(\bar{l}^{p}_{i} \cdot \gamma_\mu P_L \cdot D_\mu l^{ir}\right) \delta^{pr} +$$
$$i \left(\bar{q}^{p}_{ai} \cdot \gamma_\mu P_L \cdot D_\mu q^{air}\right) \delta^{pr} + i \left(\bar{u}^{p}_{a} \cdot \gamma_\mu P_R \cdot D_\mu u^{ar}\right) \delta^{pr} + \frac{1}{6} \hbar \frac{1}{MS^4} \kappa^2 D^2\bar{H}_i D^2 H^i +$$
$$\left(-\mu^2 + \frac{1}{2} \hbar \frac{1}{MS^4} \left(2\, MS^4 \kappa^2 + \lambda SH\, MS^6 + 2\, MS^2 \kappa^2 \mu^2 + 2 \kappa^2 \mu^4\right) \left(1 + \mathrm{Log}\left[\bar{\mu}^2 \frac{1}{MS^2}\right]\right)\right)$$
$$\bar{H}_i H^i +$$
$$\left(\frac{1}{2} \left(\frac{1}{MS^2} \kappa^2 - \lambda\right) + \frac{1}{4} \hbar \left(\lambda SH^2\, \mathrm{Log}\left[\bar{\mu}^2 \frac{1}{MS^2}\right] - 8 \frac{1}{MS^6} \kappa^4 \mu^2 \left(4 + 3\, \mathrm{Log}\left[\bar{\mu}^2 \frac{1}{MS^2}\right]\right) +\right.\right.$$

*For a total of 58 terms