Automatic generation of EFT operators

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Tony Bowden, www.flickr.com/photos/tm-tm/5998492383

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Counting operators

Tony Bowden, www.flickr.com/photos/tm-tm/5998492383

Rapid progress in recent years

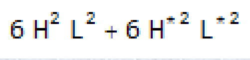
Using **the Hilbert series**, it became possible to count all SMEFT operators up to very high dimensions

!

Benvenuti, Feng, Hanany, He hep-th/0608050
Feng, Hanany, He hep-th/0701063
Hanany, Jenkins, Manohar, Torri 1010.3161
Lehman, Martin 1503.07537, 1510.00372
Henning, Lu, Melia, Murayama 1512.03433
...

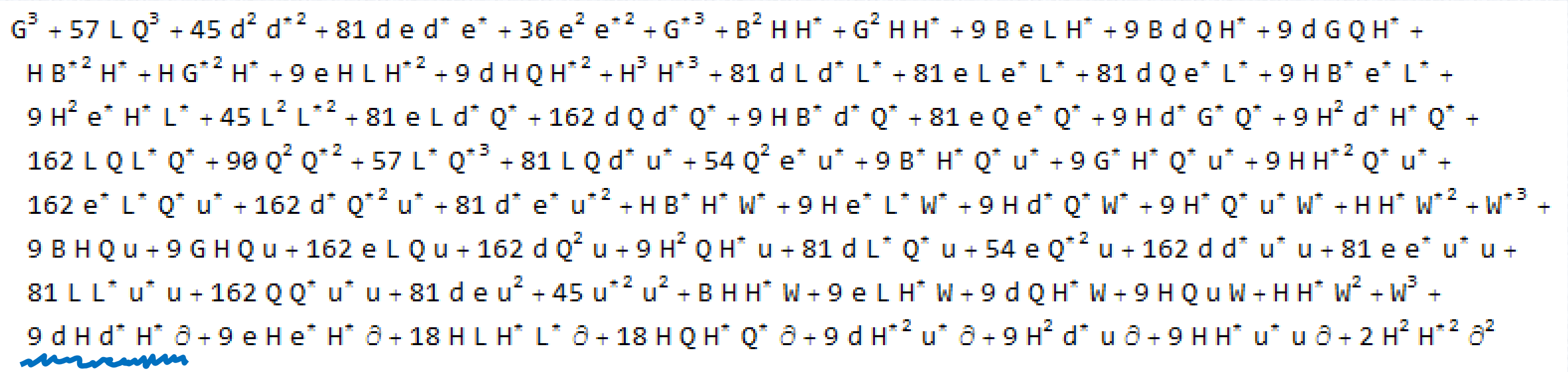
Sample

Dim 5



$$6\,H^{2}\,L^{2}+6\,H^{*2}\,L^{*2}$$

Dim 6

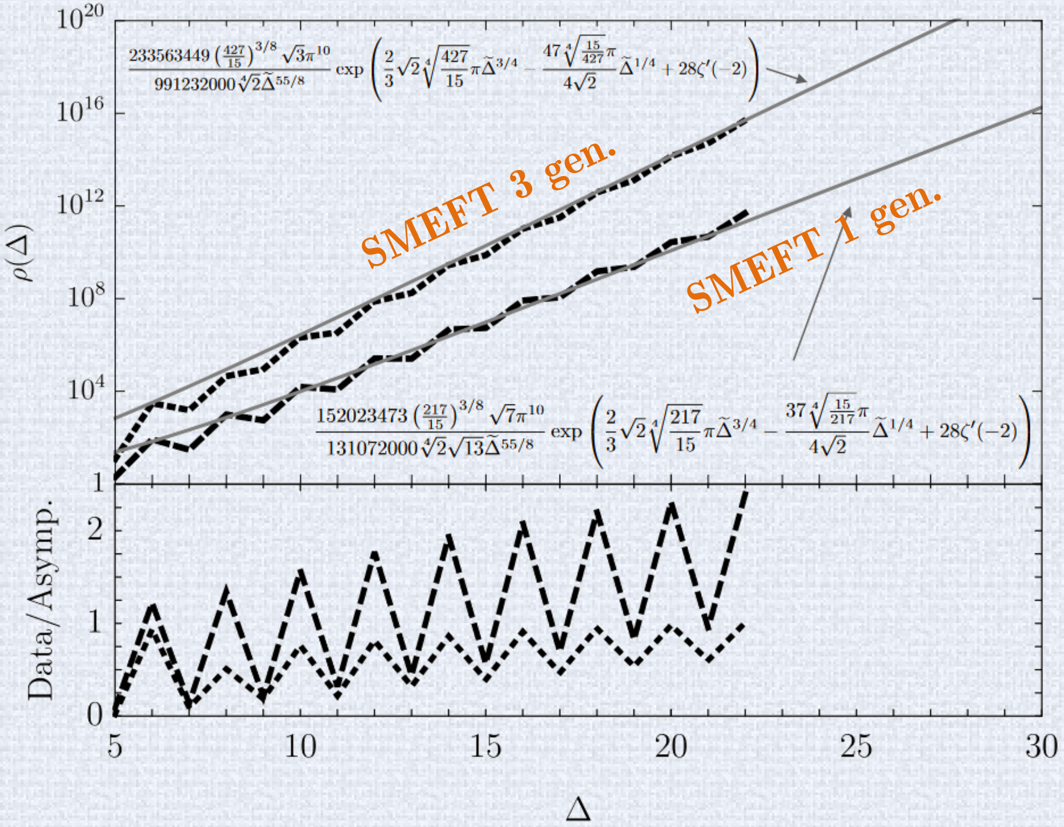


$$\begin{gathered}
G^{3}+57\,L\,Q^{3}+45\,d^{2}\,d^{*2}+81\,d\,e\,d^{*}\,e^{*}+36\,e^{2}\,e^{*2}+G^{*3}+B^{2}\,H\,H^{*}+G^{2}\,H\,H^{*}+9\,B\,e\,L\,H^{*}+9\,B\,d\,Q\,H^{*}+9\,d\,G\,Q\,H^{*}+\\
H\,B^{*2}\,H^{*}+H\,G^{*2}\,H^{*}+9\,e\,H\,L\,H^{*2}+9\,d\,H\,Q\,H^{*2}+H^{3}\,H^{*3}+81\,d\,L\,d^{*}\,L^{*}+81\,e\,L\,e^{*}\,L^{*}+81\,d\,Q\,e^{*}\,L^{*}+9\,H\,B^{*}\,e^{*}\,L^{*}+\\
9\,H^{2}\,e^{*}\,H^{*}\,L^{*}+45\,L^{2}\,L^{*2}+81\,e\,L\,d^{*}\,Q^{*}+162\,d\,Q\,d^{*}\,Q^{*}+9\,H\,B^{*}\,d^{*}\,Q^{*}+81\,e\,Q\,e^{*}\,Q^{*}+9\,H\,d^{*}\,G^{*}\,Q^{*}+9\,H^{2}\,d^{*}\,H^{*}\,Q^{*}+\\
162\,L\,Q\,L^{*}\,Q^{*}+90\,Q^{2}\,Q^{*2}+57\,L^{*}\,Q^{*3}+81\,L\,Q\,d^{*}\,u^{*}+54\,Q^{2}\,e^{*}\,u^{*}+9\,B^{*}\,H^{*}\,Q^{*}\,u^{*}+9\,G^{*}\,H^{*}\,Q^{*}\,u^{*}+9\,H\,H^{*2}\,Q^{*}\,u^{*}+\\
162\,e^{*}\,L^{*}\,Q^{*}\,u^{*}+162\,d^{*}\,Q^{*2}\,u^{*}+81\,d^{*}\,e^{*}\,u^{*2}+H\,B^{*}\,H^{*}\,W^{*}+9\,H\,e^{*}\,L^{*}\,W^{*}+9\,H\,d^{*}\,Q^{*}\,W^{*}+9\,H^{*}\,Q^{*}\,u^{*}\,W^{*}+H\,H^{*}\,W^{*2}+W^{*3}+\\
9\,B\,H\,Q\,u+9\,G\,H\,Q\,u+162\,e\,L\,Q\,u+162\,d\,Q^{2}\,u+9\,H^{2}\,Q\,H^{*}\,u+81\,d\,L^{*}\,Q^{*}\,u+54\,e\,Q^{*2}\,u+162\,d\,d^{*}\,u^{*}\,u+81\,e\,e^{*}\,u^{*}\,u+\\
81\,L\,L^{*}\,u^{*}\,u+162\,Q\,Q^{*}\,u^{*}\,u+81\,d\,e\,u^{2}+45\,u^{*2}\,u^{2}+B\,H\,H^{*}\,W+9\,e\,L\,H^{*}\,W+9\,d\,Q\,H^{*}\,W+9\,H\,Q\,u\,W+H\,H^{*}\,W^{2}+W^{3}+\\
9\,d\,H\,d^{*}\,H^{*}\,\partial+9\,e\,H\,e^{*}\,H^{*}\,\partial+18\,H\,L\,H^{*}\,L^{*}\,\partial+18\,H\,Q\,H^{*}\,Q^{*}\,\partial+9\,d\,H^{*2}\,u^{*}\,\partial+9\,H^{2}\,d^{*}\,u\,\partial+9\,H\,H^{*}\,u^{*}\,u\,\partial+2\,H^{2}\,H^{*2}\,\partial^{2}
\end{gathered}$$

Format of each term: (#operators) x (field combinations)

- The **Hilbert series method counts operators**
  It does not build them explicitly
- This method also does not indicate where to apply the **derivatives**

Rapid progress in recent years



Number of operators

Operator dimension

Melia, Pal 2010.08560

Eco

Marinissen, Rahn, Waalewijn 2004.09521

The traditional way

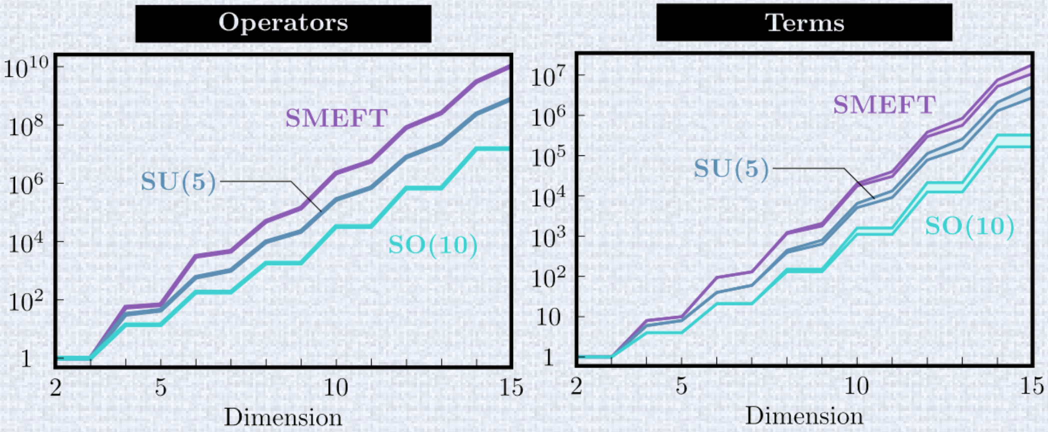
The Hilbert series (HS) gained prominence only in recent years

For decades, physicists have been building models and listing operators taking all combinations of fields, and picking out the ones which are gauge and Lorentz invariant (the *traditional method*)

Can it be used to reproduce the Hilbert series counting?
Yes. There are programs doing that.

BasisGen Criado 1901.03501

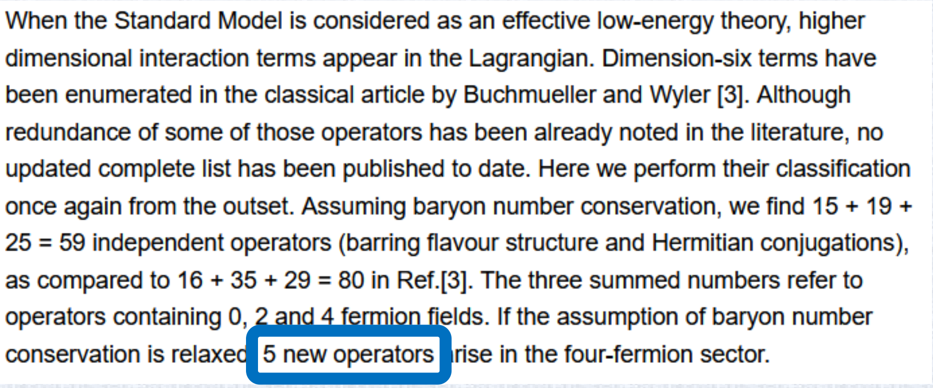
Sym2Int RF 1703.05221, 1907.12584
more on it later



RF 1907.12584

- Viable to high dimensions
- Works out of the box with any group, representations
- Yields more information than just the number of operators, namely permutation symmetries of flavor indices
- Can't tell where to apply derivatives (same as HS method)

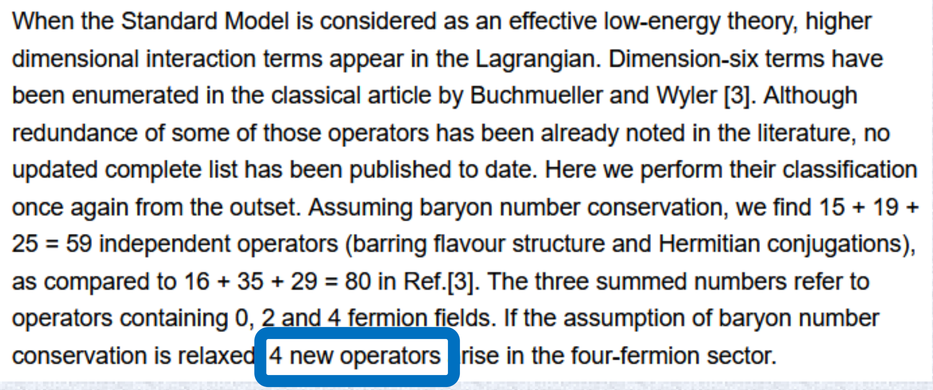
$QQQL$ in SMEFT



When the Standard Model is considered as an effective low-energy theory, higher dimensional interaction terms appear in the Lagrangian. Dimension-six terms have been enumerated in the classical article by Buchmueller and Wyler [3]. Although redundance of some of those operators has been already noted in the literature, no updated complete list has been published to date. Here we perform their classification once again from the outset. Assuming baryon number conservation, we find 15 + 19 + 25 = 59 independent operators (barring flavour structure and Hermitian conjugations), as compared to 16 + 35 + 29 = 80 in Ref.[3]. The three summed numbers refer to operators containing 0, 2 and 4 fermion fields. If the assumption of baryon number conservation is relaxed 5 new operators arise in the four-fermion sector.

Grzadkowski, Iskrzyński, Misiak, Rosiek, 1008.4884
(a.k.a. the "*Warsaw paper*")

7 years later (2017)
v3 in arXiv of the same work



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The culprit

Easy to tackle this kind of problem systematically

QQQL in SMEFT

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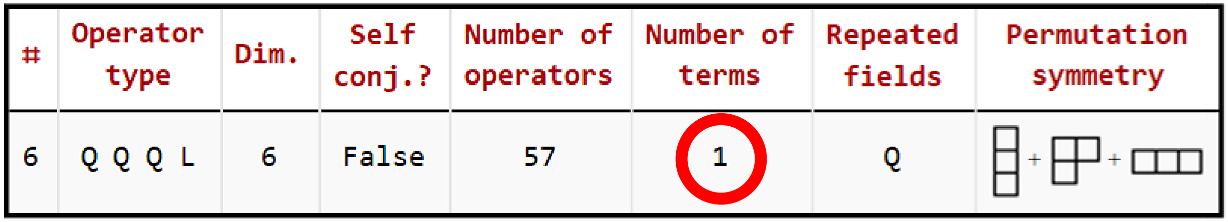

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
6	Q Q Q L	6	False	57	1	Q	$\yng(1,1,1) + \yng(2,1) + \yng(3)$

Grzadkowski, Iskrzyński, Misiak, Rosiek, 1008.4884
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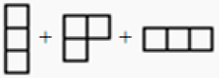
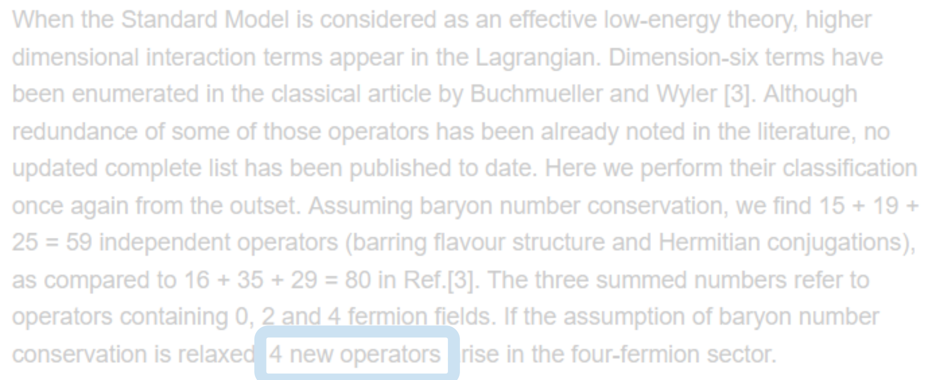
6	Q Q Q L	6	False	57	1	Q	

Grzadkowski, Iskrzyński, Misiak, Rosiek 1008.4884
(a.k.a. the "Warsaw paper")

Let's square it: *QQQQQQLL*. How many terms in the Lagrangian? The answer is still straightforward to calculate with a computer

7 years later (2017)
v3 in arXiv of the same work



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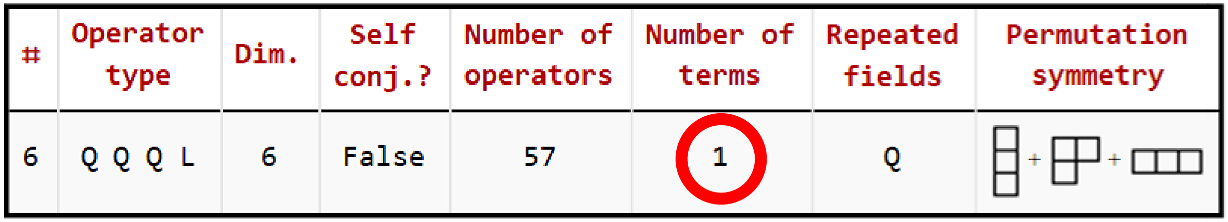

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6	Q Q Q L	6	False	57	1	Q	

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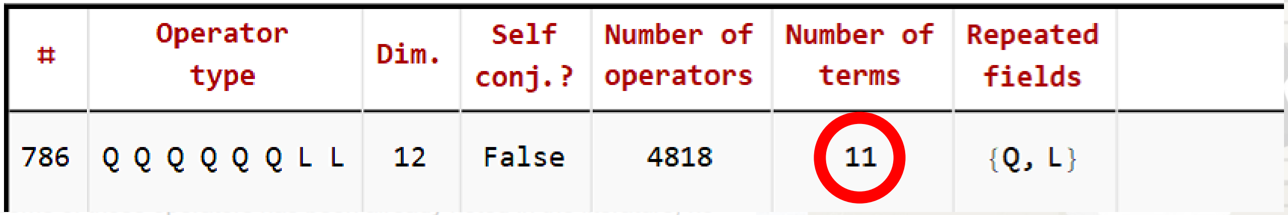

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields
786	Q Q Q Q Q Q L L	12	False	4818	11	{Q, L}

When the Star

dimensional in

been enumera

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Easy to tackle this kind of problem systematically (see extra slides)

Sym2Int

«Symmetries to Interactions»

GroupMath

A Mathematica package for the
group theory computations

RF 2011.01764

Basis-independent functions

Adjoint | Casimir | ConjugateIrrep | DynkinIndex | DimR | PermutationSymmetryOfInvariants | ReduceRepProduct | RepName | RepsUpToDimN | Weights | TriangularAnomalyValue | ...

Basis-dependent functions

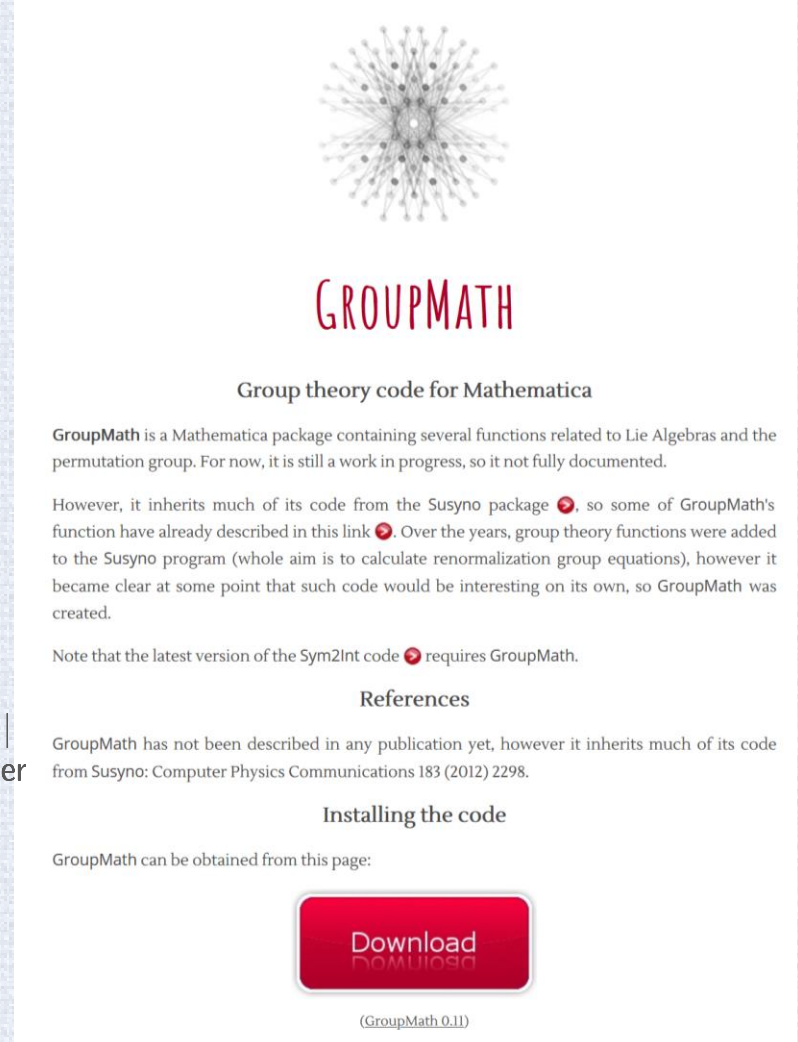
IrrepInProduct | RepMatrices | Invariants

Permutation group functions

DecomposeSnProduct | DrawYoungDiagram | GenerateStandardTableaux | HookContentFormula | LittlewoodRichardsonCoefficients | SnClassCharacter | SnClassOrder | SnIrrepDim | SnIrrepGenerators | ...

Symmetry breaking functions

DecomposeRep | FindAllEmbeddings | MaximalSubgroups | RegularSubgroupProjectionMatrix | SubgroupEmbeddingCoefficients



GROUPMATH

Group theory code for Mathematica

GroupMath is a Mathematica package containing several functions related to Lie Algebras and the permutation group. For now, it is still a work in progress, so it not fully documented.

However, it inherits much of its code from the Susyno package, so some of GroupMath's function have already described in this link. Over the years, group theory functions were added to the Susyno program (whole aim is to calculate renormalization group equations), however it became clear at some point that such code would be interesting on its own, so GroupMath was created.

Note that the latest version of the Sym2Int code requires GroupMath.

References

GroupMath has not been described in any publication yet, however it inherits much of its code from Susyno: Computer Physics Communications 183 (2012) 2298.

Installing the code

GroupMath can be obtained from this page:

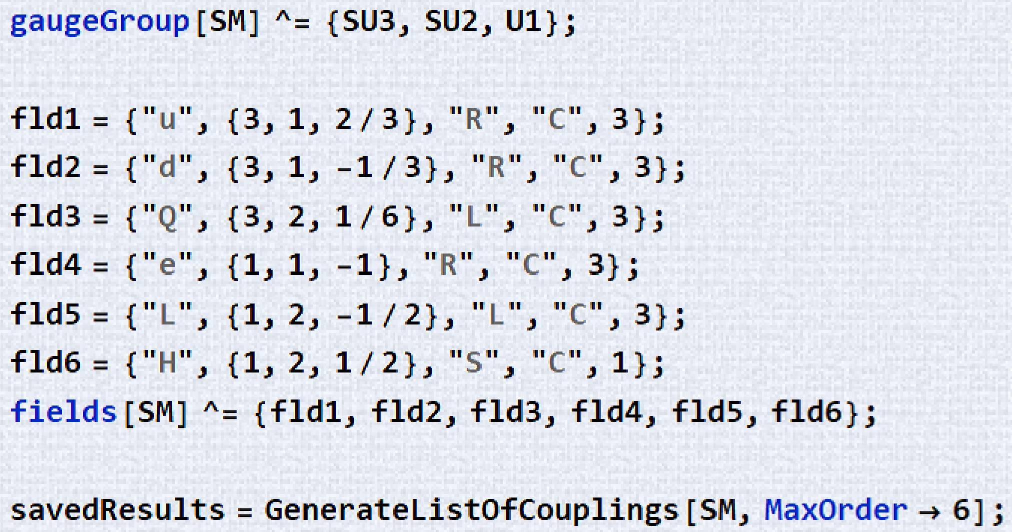
(GroupMath 0.11)

Sym2Int

«Symmetries to Interactions»

A Mathematica package to list the operators in a model
Works out of the box for any gauge group and representations

RF 1703.05221, 1907.12584



```
gaugeGroup[SM] ^= {SU3, SU2, U1};

fld1 = {"u", {3, 1, 2/3}, "R", "C", 3};
fld2 = {"d", {3, 1, -1/3}, "R", "C", 3};
fld3 = {"Q", {3, 2, 1/6}, "L", "C", 3};
fld4 = {"e", {1, 1, -1}, "R", "C", 3};
fld5 = {"L", {1, 2, -1/2}, "L", "C", 3};
fld6 = {"H", {1, 2, 1/2}, "S", "C", 1};
fields[SM] ^= {fld1, fld2, fld3, fld4, fld5, fld6};

savedResults = GenerateListOfCouplings[SM, MaxOrder → 6];
```

Sym2Int

«Symmetries to Interactions»

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A name to the model
(e.g. SM)

```
gaugeGroup[SM] ^= {SU3, SU2, U1};
```

The gauge group
(e.g. SU(3) x SU(2) x U(1))

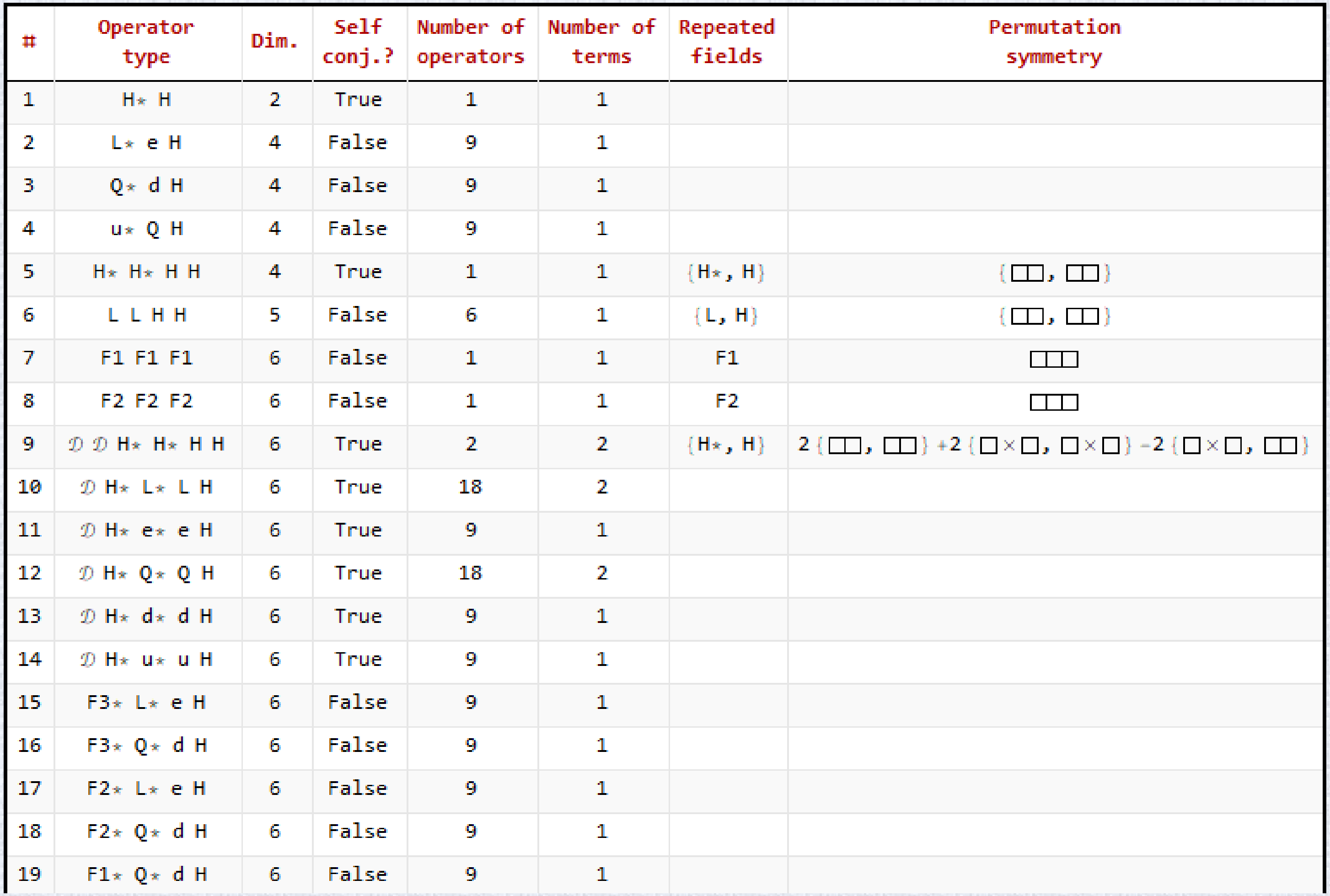
```
fld1 = {"u", {3, 1, 2 / 3}, "R", "C", 3};
fld2 = {"d", {3, 1, -1 / 3}, "R", "C", 3};
fld3 = {"Q", {3, 2, 1 / 6}, "L", "C", 3};
fld4 = {"e", {1, 1, -1}, "R", "C", 3};
fld5 = {"L", {1, 2, -1 / 2}, "L", "C", 3};
fld6 = {"H", {1, 2, 1 / 2}, "S", "C", 1};
fields[SM] ^= {fld1, fld2, fld3, fld4, fld5, fld6};
```

The fields, i.e. the irreps under the gauge and Lorentz groups, including #flavors

```
savedResults = GenerateListOfCouplings[SM, MaxOrder → 6];
```

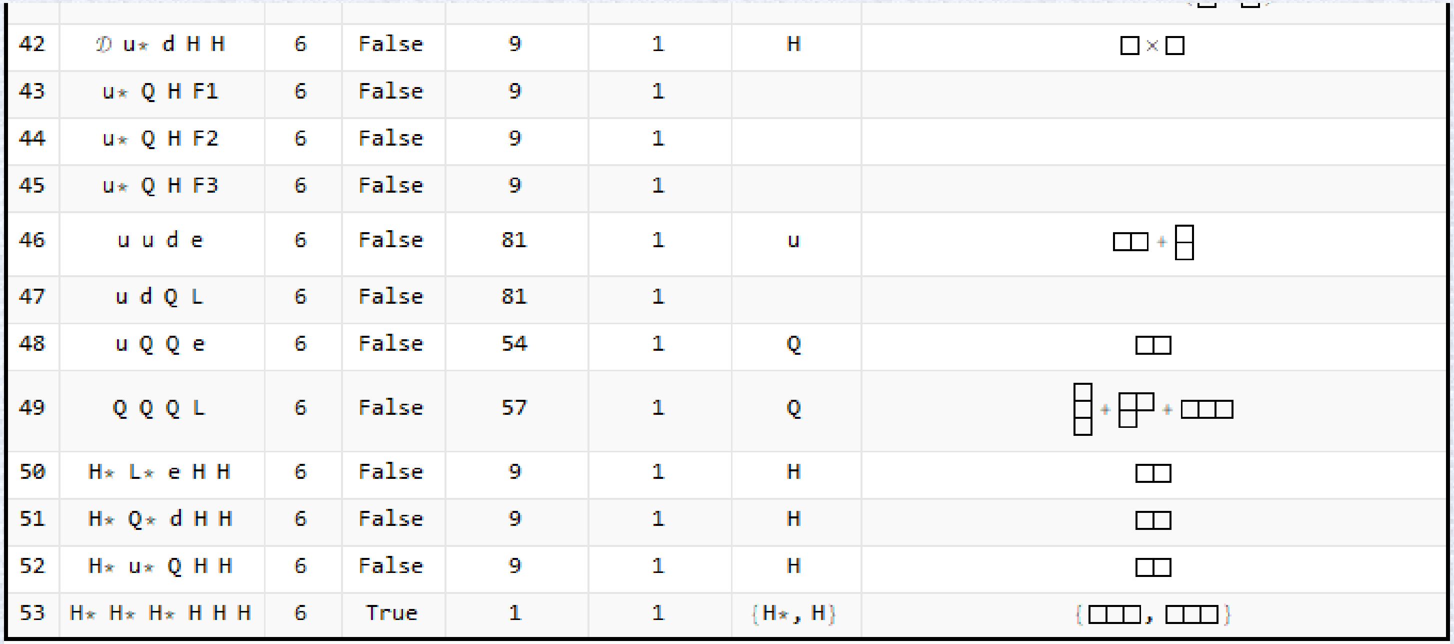
Max dimension of interactions
(e.g.: 6)

Example: SMEFT up to dim 6

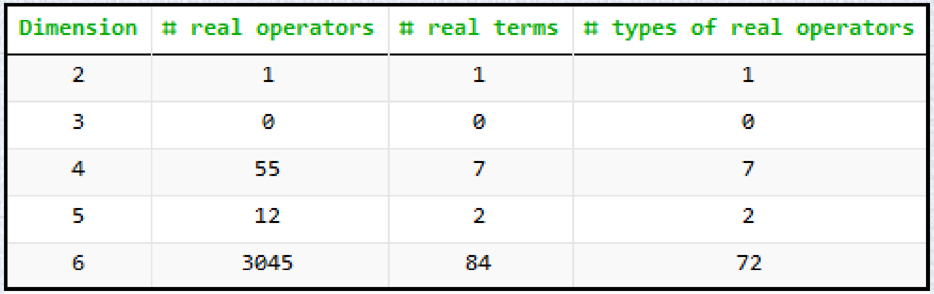


#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
1	H* H	2	True	1	1		
2	L* e H	4	False	9	1		
3	Q* d H	4	False	9	1		
4	u* Q H	4	False	9	1		
5	H* H* H H	4	True	1	1	{H*, H}	{□□, □□}
6	L L H H	5	False	6	1	{L, H}	{□□, □□}
7	F1 F1 F1	6	False	1	1	F1	□□□
8	F2 F2 F2	6	False	1	1	F2	□□□
9	𝒟 𝒟 H* H* H H	6	True	2	2	{H*, H}	2 {□□, □□} +2 {□×□, □×□} -2 {□×□, □□}
10	𝒟 H* L* L H	6	True	18	2		
11	𝒟 H* e* e H	6	True	9	1		
12	𝒟 H* Q* Q H	6	True	18	2		
13	𝒟 H* d* d H	6	True	9	1		
14	𝒟 H* u* u H	6	True	9	1		
15	F3* L* e H	6	False	9	1		
16	F3* Q* d H	6	False	9	1		
17	F2* L* e H	6	False	9	1		
18	F2* Q* d H	6	False	9	1		
19	F1* Q* d H	6	False	9	1		

Example: SMEFT up to dim 6



42	𝒟 u⋆ d H H	6	False	9	1	H	□ × □
43	u⋆ Q H F1	6	False	9	1		
44	u⋆ Q H F2	6	False	9	1		
45	u⋆ Q H F3	6	False	9	1		
46	u u d e	6	False	81	1	u	□□ + ⊟
47	u d Q L	6	False	81	1		
48	u Q Q e	6	False	54	1	Q	□□
49	Q Q Q L	6	False	57	1	Q	⊟ + ⊞ + □□□
50	H⋆ L⋆ e H H	6	False	9	1	H	□□
51	H⋆ Q⋆ d H H	6	False	9	1	H	□□
52	H⋆ u⋆ Q H H	6	False	9	1	H	□□
53	H⋆ H⋆ H⋆ H H H	6	True	1	1	{H⋆, H}	{□□□, □□□}



Dimension	# real operators	# real terms	# types of real operators
2	1	1	1
3	0	0	0
4	55	7	7
5	12	2	2
6	3045	84	72

Extending Sym2Int

Building operators explicitly

Known results for SMEFT

SMEFT dim 6

1986-2017

Buchmüller, Wyler NPB 268 (1986) 621
Grzadkowski, Iskrzyński, Misiak, Rosiek, 1008.4884

SMEFT dim 7

2014

Lehman 1410.4193

SMEFT dim 8

2020

Murphy 2005.00059
Li, Ren, Shu, Xiao, Yu, Zheng, 2005.00008

SMEFT dim 9

2020

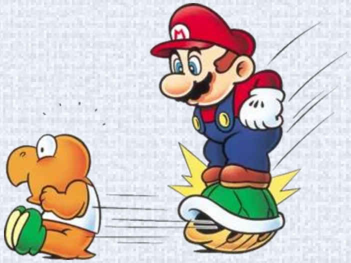
Li, Ren, Xiao, Yu, Zheng, 2007.07899

DEFT Gripaios, Sutherland 1807.07546

ABC4EFT Li, Ren, Xiao, Yu, Zheng 2201.04639

Off shell

EOMs are not used (Green basis)



SMEFT dim 6

Gherardi, Marzocca, Venturini, 2003.12525

SMEFT dim 8 (bosons)

Chala, Díaz-Carmona, Guedes 2112.12724

Operators = polynomials in many variables

Operators are just **homogenous polynomials in many variables**

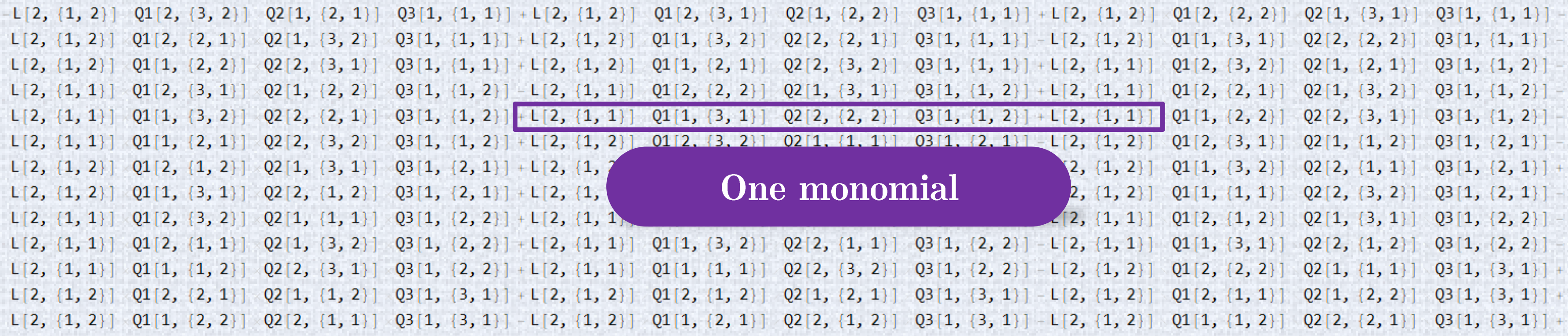
The **variables are field components**

Once we have a (potential over-complete) basis of operators of some kind, we can take each monomial to be a basis of a vector space and **covert operators into vectors**

At this stage we have a **Linear Algebra problem**

EOMs and IBPs are linear relations among the operators; they define **directions (vectors) in this vector space**

E.g.: Q1 Q2 Q3 L



```
-L[2, {1, 2}] Q1[2, {3, 2}] Q2[1, {2, 1}] Q3[1, {1, 1}] + L[2, {1, 2}] Q1[2, {3, 1}] Q2[1, {2, 2}] Q3[1, {1, 1}] + L[2, {1, 2}] Q1[2, {2, 2}] Q2[1, {3, 1}] Q3[1, {1, 1}] -
 L[2, {1, 2}] Q1[2, {2, 1}] Q2[1, {3, 2}] Q3[1, {1, 1}] + L[2, {1, 2}] Q1[1, {3, 2}] Q2[2, {2, 1}] Q3[1, {1, 1}] - L[2, {1, 2}] Q1[1, {3, 1}] Q2[2, {2, 2}] Q3[1, {1, 1}] -
 L[2, {1, 2}] Q1[1, {2, 2}] Q2[2, {3, 1}] Q3[1, {1, 1}] + L[2, {1, 2}] Q1[1, {2, 1}] Q2[2, {3, 2}] Q3[1, {1, 1}] + L[2, {1, 1}] Q1[2, {3, 2}] Q2[1, {2, 1}] Q3[1, {1, 2}] -
 L[2, {1, 1}] Q1[2, {3, 1}] Q2[1, {2, 2}] Q3[1, {1, 2}] - L[2, {1, 1}] Q1[2, {2, 2}] Q2[1, {3, 1}] Q3[1, {1, 2}] + L[2, {1, 1}] Q1[2, {2, 1}] Q2[1, {3, 2}] Q3[1, {1, 2}] -
 L[2, {1, 1}] Q1[1, {3, 2}] Q2[2, {2, 1}] Q3[1, {1, 2}] + L[2, {1, 1}] Q1[1, {3, 1}] Q2[2, {2, 2}] Q3[1, {1, 2}] + L[2, {1, 1}] Q1[1, {2, 2}] Q2[2, {3, 1}] Q3[1, {1, 2}] -
 L[2, {1, 1}] Q1[1, {2, 1}] Q2[2, {3, 2}] Q3[1, {1, 2}] + L[2, {1, 2}] Q1[2, {3, 2}] Q2[1, {1, 1}] Q3[1, {2, 1}] - L[2, {1, 2}] Q1[2, {3, 1}] Q2[1, {1, 2}] Q3[1, {2, 1}] -
 L[2, {1, 2}] Q1[2, {1, 2}] Q2[1, {3, 1}] Q3[1, {2, 1}] + L[2, {1, [illegible] 2, {1, 2}] Q1[1, {3, 2}] Q2[2, {1, 1}] Q3[1, {2, 1}] +
 L[2, {1, 2}] Q1[1, {3, 1}] Q2[2, {1, 2}] Q3[1, {2, 1}] + L[2, {1, [illegible] 2, {1, 2}] Q1[1, {1, 1}] Q2[2, {3, 2}] Q3[1, {2, 1}] -
 L[2, {1, 1}] Q1[2, {3, 2}] Q2[1, {1, 1}] Q3[1, {2, 2}] + L[2, {1, 1 [illegible] [2, {1, 1}] Q1[2, {1, 2}] Q2[1, {3, 1}] Q3[1, {2, 2}] -
 L[2, {1, 1}] Q1[2, {1, 1}] Q2[1, {3, 2}] Q3[1, {2, 2}] + L[2, {1, 1}] Q1[1, {3, 2}] Q2[2, {1, 1}] Q3[1, {2, 2}] - L[2, {1, 1}] Q1[1, {3, 1}] Q2[2, {1, 2}] Q3[1, {2, 2}] -
 L[2, {1, 1}] Q1[1, {1, 2}] Q2[2, {3, 1}] Q3[1, {2, 2}] + L[2, {1, 1}] Q1[1, {1, 1}] Q2[2, {3, 2}] Q3[1, {2, 2}] - L[2, {1, 2}] Q1[2, {2, 2}] Q2[1, {1, 1}] Q3[1, {3, 1}] +
 L[2, {1, 2}] Q1[2, {2, 1}] Q2[1, {1, 2}] Q3[1, {3, 1}] + L[2, {1, 2}] Q1[2, {1, 2}] Q2[1, {2, 1}] Q3[1, {3, 1}] - L[2, {1, 2}] Q1[2, {1, 1}] Q2[1, {2, 2}] Q3[1, {3, 1}] +
 L[2, {1, 2}] Q1[1, {2, 2}] Q2[2, {1, 1}] Q3[1, {3, 1}] - L[2, {1, 2}] Q1[1, {2, 1}] Q2[2, {1, 2}] Q3[1, {3, 1}] - L[2, {1, 2}] Q1[1, {1, 2}] Q2[2, {2, 1}] Q3[1, {3, 1}] +
```

One monomial

$+ \ldots$

Segregate Lorentz and gauge contractions

Handle the possible contractions of the Lorentz indices

Includes distributing derivatives by the fields

Handle the possible contractions of the gauge indices

Why? Convenience/elegance and speed.

It should be possible to sort out what is happening to the Lorentz indices, independently of what is happening to the gauge indices (and vice-versa). [Spoiler: this is not true]

Consider the gluon field strength tensor: it is faster to handle separately the 8 color indices, and the 6 Lorentz components, than to handle polynomials in $6 \times 8 = 48$ variables

Lorentz contractions

Distribute the derivatives by the fields in all possible ways

Bosons

Vector indices: contract them in all possible ways with g's and ε's

Explicitly build the expressions and **check for redundancies**

Fermions

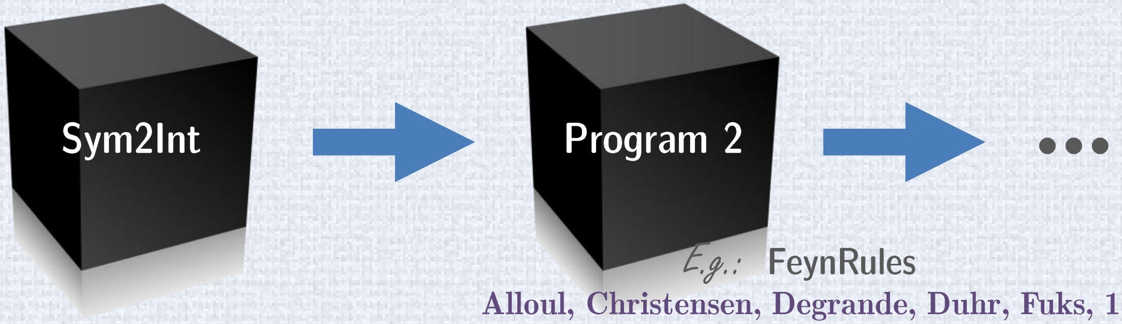
- Place Weyl spinors in 4-D **Dirac spinors**
- Form **fermion bilinears**
- Use Dirac gamma matrices and C to **convert spinor indices into vector indices**

$$\left(\begin{array}{c:c} \boxed{L^* L} & \boxed{L^* R} \\ & \gamma^0 \\ \gamma^0\gamma^\mu & \gamma^0\left[\gamma^\mu,\gamma^\nu\right] \\ \hdashline \boxed{R^* L} & \boxed{R^* R} \\ \gamma^0 & \\ \gamma^0\left[\gamma^\mu,\gamma^\nu\right] & \gamma^0\gamma^\mu \end{array}\right) \qquad \left(\begin{array}{c:c} \boxed{LL} & \boxed{LR} \\ C & \\ C\left[\gamma^\mu,\gamma^\nu\right] & C\gamma^\mu \\ \hdashline \boxed{RL} & \boxed{RR} \\ & C \\ C\gamma^\mu & C\left[\gamma^\mu,\gamma^\nu\right] \end{array}\right)$$

Gauge contractions (#1)

GroupMath can find the explicit gauge invariant contractions of a set of representations of arbitrary Lie algebras

It works fine. However ... it might not be ideal.



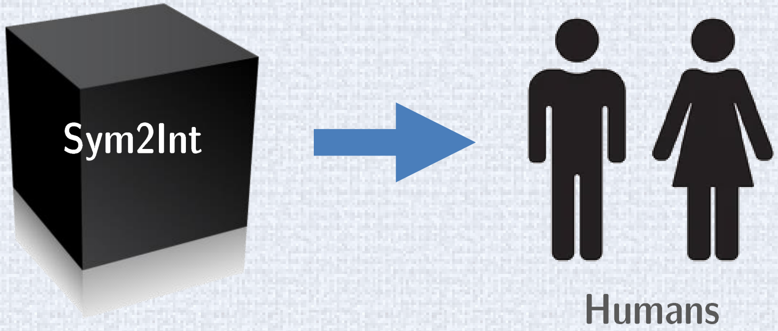
Large tensors are passed to other programs. Output of Sym2Int does not need to be easy to read

E.g.: FeynRules

Alloul, Christensen, Degrande, Duhr, Fuks, 1310.1921

MatchMakereft

Carmona, Lazopoulos, Olgoso, Santiago, 2112.10787



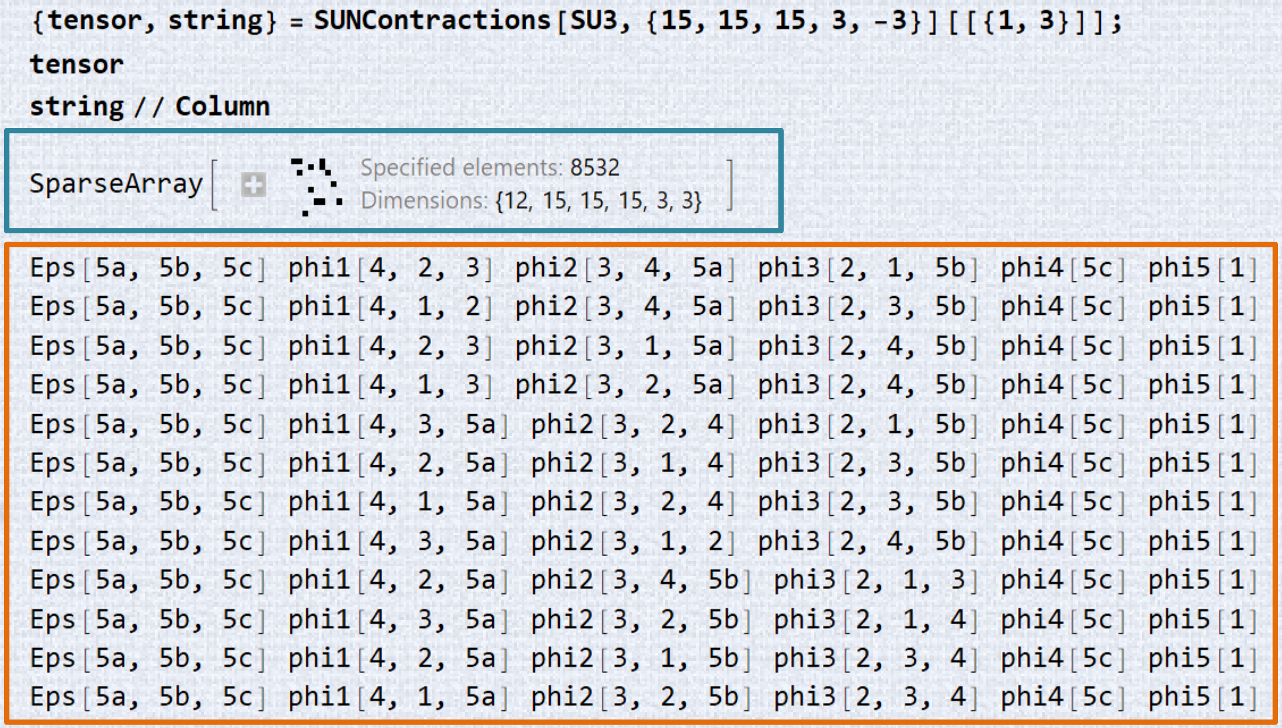
Output of Sym2Int should be **human readable**

No right/wrong answers here. But in the end, in both cases it is **convenient that the gauge contractions used are similar to what a human would write**

Gauge contractions (#2)

To this of end, I've been extending GroupMath so that in the case of *SU(n)* groups contractions are done via the tensor method.

The program outputs a tensor with the result, but also a string identifying which type of contraction was made



```
{tensor, string} = SUNContractions[SU3, {15, 15, 15, 3, -3}][[{1, 3}]];
tensor
string // Column
```

The tensor

```
SparseArray[ Specified elements: 8532
             Dimensions: {12, 15, 15, 15, 3, 3} ]
```

The contractions

```
Eps[5a, 5b, 5c] phi1[4, 2, 3] phi2[3, 4, 5a] phi3[2, 1, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 1, 2] phi2[3, 4, 5a] phi3[2, 3, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 2, 3] phi2[3, 1, 5a] phi3[2, 4, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 1, 3] phi2[3, 2, 5a] phi3[2, 4, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 3, 5a] phi2[3, 2, 4] phi3[2, 1, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 2, 5a] phi2[3, 1, 4] phi3[2, 3, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 1, 5a] phi2[3, 2, 4] phi3[2, 3, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 3, 5a] phi2[3, 1, 2] phi3[2, 4, 5b] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 2, 5a] phi2[3, 4, 5b] phi3[2, 1, 3] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 3, 5a] phi2[3, 2, 5b] phi3[2, 1, 4] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 2, 5a] phi2[3, 1, 5b] phi3[2, 3, 4] phi4[5c] phi5[1]
Eps[5a, 5b, 5c] phi1[4, 1, 5a] phi2[3, 2, 5b] phi3[2, 3, 4] phi4[5c] phi5[1]
```

Assumed indices:

$15^{*}{}_{**}$

3_{*}

$\overline{3}^{*}$

EOM relations

Replace in each operators $\mathcal{O}$ the expression $\partial^n\Phi$ by a new expression where the part removable by EOMs is segregated

$$\partial_\mu\partial_\nu\phi = \begin{pmatrix} \partial_0\partial_0\phi & \partial_0\partial_1\phi & \partial_0\partial_2\phi & \partial_0\partial_3\phi \\ \partial_0\partial_1\phi & \partial_1\partial_1\phi & \partial_1\partial_2\phi & \partial_1\partial_3\phi \\ \partial_0\partial_2\phi & \partial_1\partial_2\phi & \partial_2\partial_2\phi & \partial_2\partial_3\phi \\ \partial_0\partial_3\phi & \partial_1\partial_3\phi & \partial_2\partial_3\phi & \partial_3\partial_3\phi \end{pmatrix}$$

Change variables

$$\begin{pmatrix} \frac{K[1]}{4}+\frac{K[3]}{4}+\frac{K[6]}{4}+\frac{R[1]}{4} & K[9] & K[8] & K[7] \\ K[9] & -\frac{K[1]}{4}+\frac{3K[6]}{4}-\frac{K[3]}{4}-\frac{R[1]}{4} & K[5] & K[4] \\ K[8] & K[5] & -\frac{K[1]}{4}+\frac{3K[3]}{4}-\frac{K[6]}{4}-\frac{R[1]}{4} & K[2] \\ K[7] & K[4] & K[2] & \frac{3K[1]}{4}-\frac{K[3]}{4}-\frac{K[6]}{4}-\frac{R[1]}{4} \end{pmatrix}$$

The EOM removes the R[...] components and leaves all the K[...] components:

$$\boxed{\partial_\mu\partial^\mu\phi = \mathrm{R}[1]}$$

So in this case we can just set R[1]=0 and see what relations appear between the operators in the "maximal basis"

In the end: **EOMs = vectors (linear relations among operators)**

IBP relations

Not complicated if things are done explicitly. In short:

- **Leave one of the derivatives free** (don't apply it to any field). For all purposes it is a standard 4-vector field.
- Using the **Leibniz rule, apply the free derivative** to the remaining fields
- We get in each case an expression, which must be a **linear combination of the basis of operators previously derived**

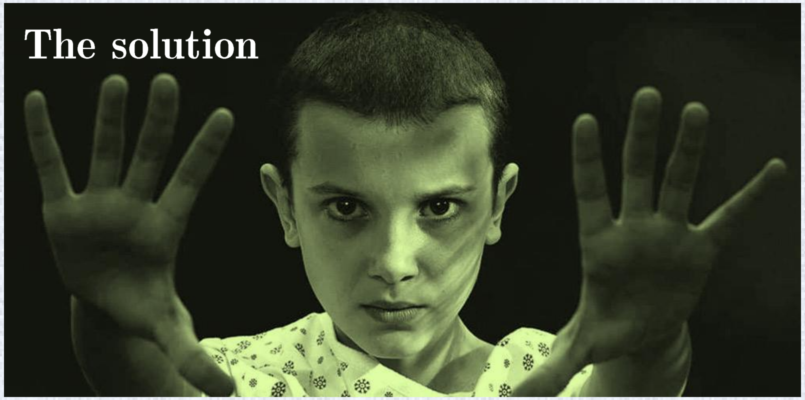
In the end: **IBPs = vectors (linear relations among operators)**

A major problem ... and its solution

Repeated fields

Operators with repeated fields (such as $LLHH$) are much harder to handle.

Even ignoring derivatives, just consider that (# contractions) $\neq$ (# gauge contr.) × (# Lorentz contr.)



Differentiate fields

$$LLHH \rightarrow L_1 L_2 H_1 H_2$$

Obtain a "*super basis*" of operators

Permutations of equal fields = redundancies of the "super basis"

Contractions $\neq$ gauge $\times$ Lorentz

Let's simply life

Consider that both the gauge group and the Lorentz group are SU(2)

Gauge ↘ ↙ Lorentz

E.g. $\Phi = (2,2)$

How many $\Phi\Phi\Phi\Phi$ independent contractions?

Four doublets contract into 2 singlets: $2\times2\times2\times2 = 1 + 1 + \cdots$

So we might think that there are two × two = 4 contractions. This is not the case, because there is a single Φ

$$\kappa^{(\alpha)}_{g_1g_2g_3g_4}\, c^{(\beta)}_{l_1l_2l_3l_4}\, \Phi_{g_1l_1}\Phi'_{g_2l_2}\Phi''_{g_3l_3}\Phi'''_{g_4l_4}$$

2 possib. 2 possib.

✓ 4 contractions

$$\kappa^{(\alpha)}_{g_1g_2g_3g_4}\, c^{(\beta)}_{l_1l_2l_3l_4}\, \Phi_{g_1l_1}\Phi_{g_2l_2}\Phi_{g_3l_3}\Phi_{g_4l_4}$$

2 possib. 2 possib.

✗ 1 contractions

So in these cases I distinguish the fields: this gives rise to an excess of operators: a "super basis". But it is easy to study EOM and IBM relations for such a set of operators. All that is left is to study the relations among the operators in the super basis, imposed by the existence of repeated fields

Grid of *super basis* of operators

I think it is very useful to picture all operators in a grid

		Lorentz contractions								
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**...**
Gauge contractions	**1**	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
	2	$\mathcal{O}_{2,1}$	$\mathcal{O}_{2,2}$	$\mathcal{O}_{2,3}$	$\mathcal{O}_{2,4}$	$\mathcal{O}_{2,5}$	$\mathcal{O}_{2,6}$	$\mathcal{O}_{2,7}$	$\mathcal{O}_{2,8}$	$\cdots$
	3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
	4	$\mathcal{O}_{4,1}$	$\mathcal{O}_{4,2}$	$\mathcal{O}_{4,3}$	$\mathcal{O}_{4,4}$	$\mathcal{O}_{4,5}$	$\mathcal{O}_{4,6}$	$\mathcal{O}_{4,7}$	$\mathcal{O}_{4,8}$	$\cdots$
	5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
	...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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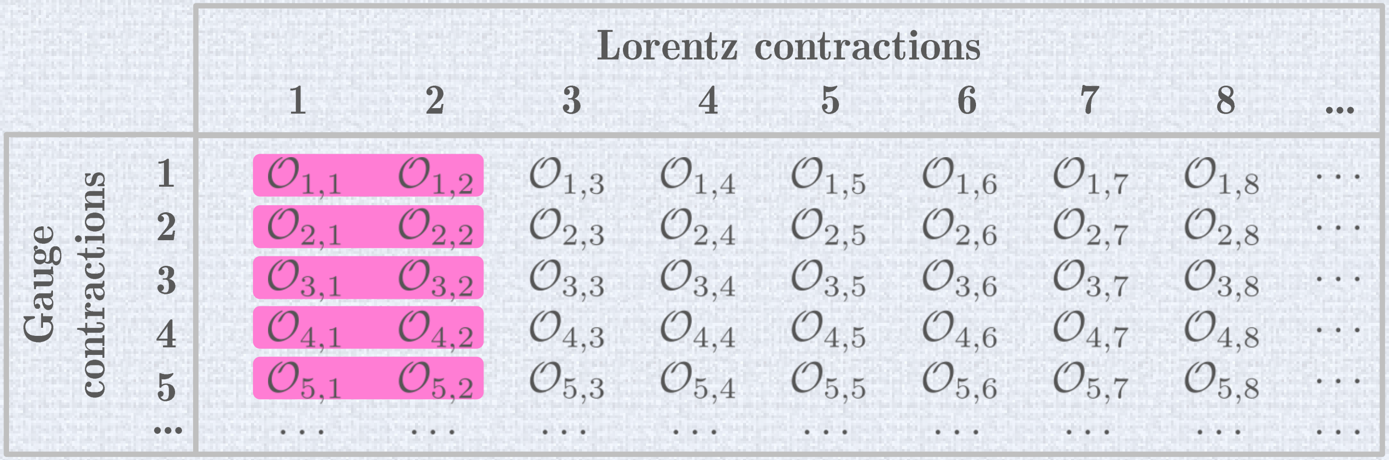

		Lorentz contractions								
		1	2	3	4	5	6	7	8	...
Gauge contractions	1	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
	2	$\mathcal{O}_{2,1}$	$\mathcal{O}_{2,2}$	$\mathcal{O}_{2,3}$	$\mathcal{O}_{2,4}$	$\mathcal{O}_{2,5}$	$\mathcal{O}_{2,6}$	$\mathcal{O}_{2,7}$	$\mathcal{O}_{2,8}$	$\cdots$
	3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
	4	$\mathcal{O}_{4,1}$	$\mathcal{O}_{4,2}$	$\mathcal{O}_{4,3}$	$\mathcal{O}_{4,4}$	$\mathcal{O}_{4,5}$	$\mathcal{O}_{4,6}$	$\mathcal{O}_{4,7}$	$\mathcal{O}_{4,8}$	$\cdots$
	5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
	...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

EOM's

Horizontal relations; the **same for all rows** (i.e. all gauge contractions)

Grid of *super basis* of operators

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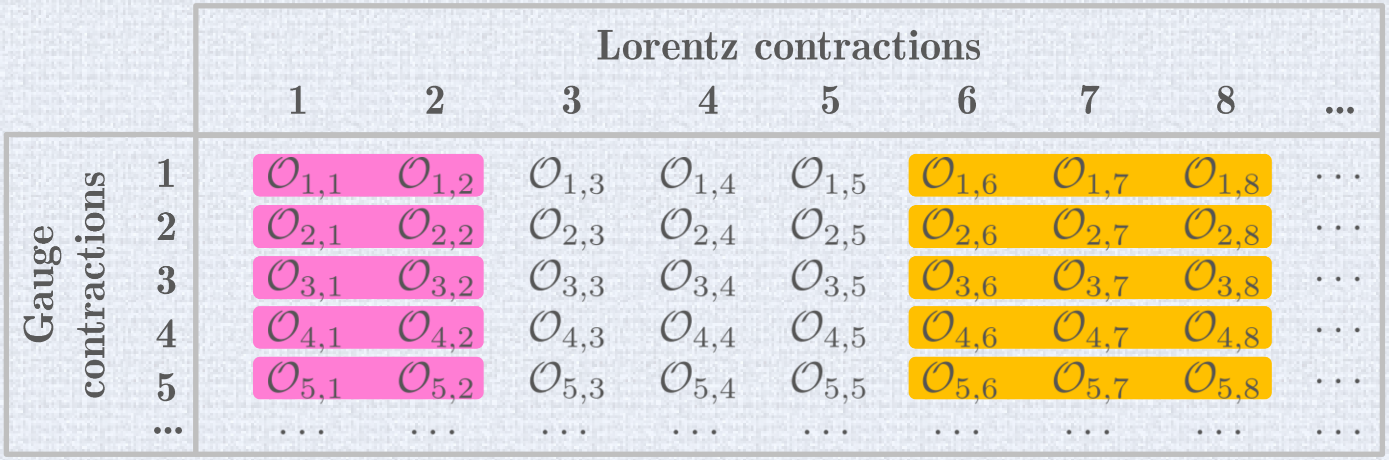

		Lorentz contractions								
		1	2	3	4	5	6	7	8	...
Gauge contractions	1	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
	2	$\mathcal{O}_{2,1}$	$\mathcal{O}_{2,2}$	$\mathcal{O}_{2,3}$	$\mathcal{O}_{2,4}$	$\mathcal{O}_{2,5}$	$\mathcal{O}_{2,6}$	$\mathcal{O}_{2,7}$	$\mathcal{O}_{2,8}$	$\cdots$
	3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
	4	$\mathcal{O}_{4,1}$	$\mathcal{O}_{4,2}$	$\mathcal{O}_{4,3}$	$\mathcal{O}_{4,4}$	$\mathcal{O}_{4,5}$	$\mathcal{O}_{4,6}$	$\mathcal{O}_{4,7}$	$\mathcal{O}_{4,8}$	$\cdots$
	5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
	...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

EOM's Horizontal relations; the same for all rows (i.e. all gauge contractions)

IBP's Horizontal relations; the same for all rows (i.e. all gauge contractions)

Grid of *super basis* of operators

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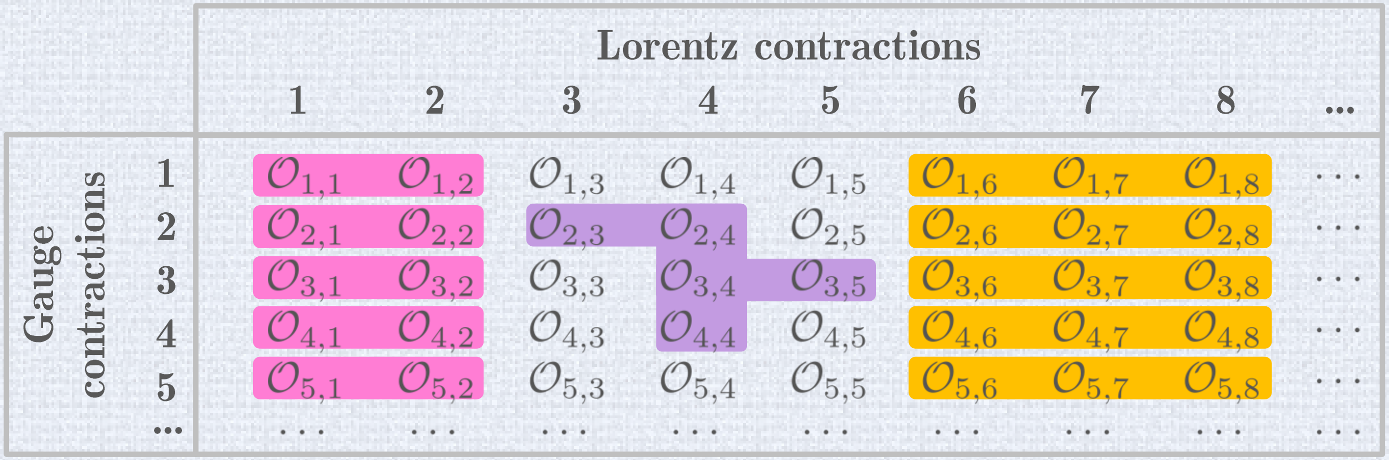


		Lorentz contractions							
		1	2	3	4	5	6	7	8	...
Gauge contractions	1	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
	2	$\mathcal{O}_{2,1}$	$\mathcal{O}_{2,2}$	$\mathcal{O}_{2,3}$	$\mathcal{O}_{2,4}$	$\mathcal{O}_{2,5}$	$\mathcal{O}_{2,6}$	$\mathcal{O}_{2,7}$	$\mathcal{O}_{2,8}$	$\cdots$
	3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
	4	$\mathcal{O}_{4,1}$	$\mathcal{O}_{4,2}$	$\mathcal{O}_{4,3}$	$\mathcal{O}_{4,4}$	$\mathcal{O}_{4,5}$	$\mathcal{O}_{4,6}$	$\mathcal{O}_{4,7}$	$\mathcal{O}_{4,8}$	$\cdots$
	5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
	...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

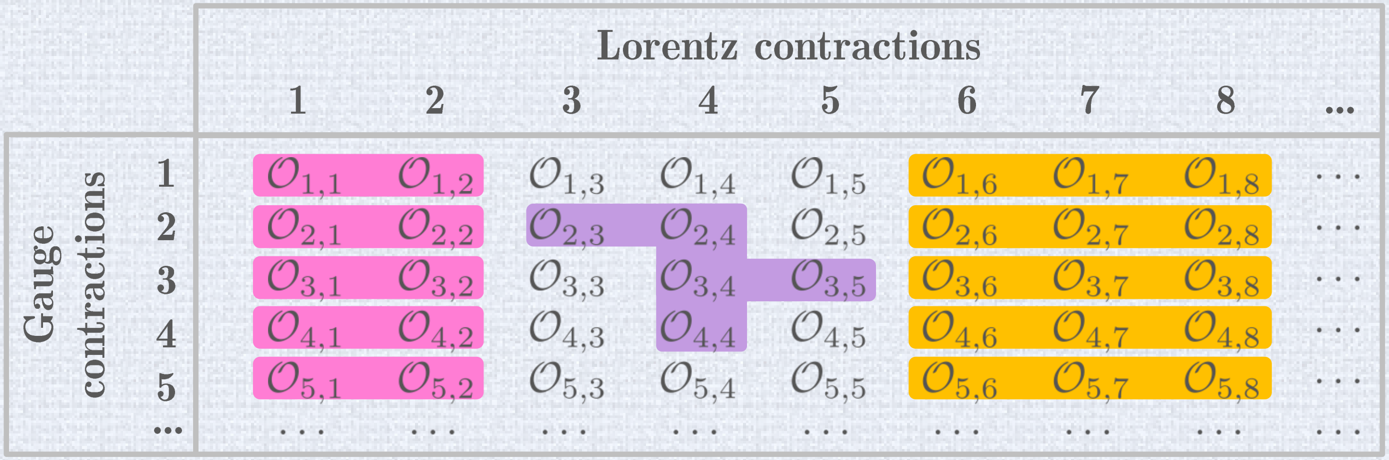
EOM’s Horizontal relations; the same for all rows (i.e. all gauge contractions)

IBP’s Horizontal relations; the same for all rows (i.e. all gauge contractions)

Repeated fields Oblique relations in general! Not the same for each row

Grid of *super basis* of operators

I think it is very useful to picture all operators in a grid



Lorentz contractions

Gauge contractions	1	2	3	4	5	6	7	8	...
1	$\mathcal{O}_{1,1}$	$\mathcal{O}_{1,2}$	$\mathcal{O}_{1,3}$	$\mathcal{O}_{1,4}$	$\mathcal{O}_{1,5}$	$\mathcal{O}_{1,6}$	$\mathcal{O}_{1,7}$	$\mathcal{O}_{1,8}$	$\cdots$
2	$\mathcal{O}_{2,1}$	$\mathcal{O}_{2,2}$	$\mathcal{O}_{2,3}$	$\mathcal{O}_{2,4}$	$\mathcal{O}_{2,5}$	$\mathcal{O}_{2,6}$	$\mathcal{O}_{2,7}$	$\mathcal{O}_{2,8}$	$\cdots$
3	$\mathcal{O}_{3,1}$	$\mathcal{O}_{3,2}$	$\mathcal{O}_{3,3}$	$\mathcal{O}_{3,4}$	$\mathcal{O}_{3,5}$	$\mathcal{O}_{3,6}$	$\mathcal{O}_{3,7}$	$\mathcal{O}_{3,8}$	$\cdots$
4	$\mathcal{O}_{4,1}$	$\mathcal{O}_{4,2}$	$\mathcal{O}_{4,3}$	$\mathcal{O}_{4,4}$	$\mathcal{O}_{4,5}$	$\mathcal{O}_{4,6}$	$\mathcal{O}_{4,7}$	$\mathcal{O}_{4,8}$	$\cdots$
5	$\mathcal{O}_{5,1}$	$\mathcal{O}_{5,2}$	$\mathcal{O}_{5,3}$	$\mathcal{O}_{5,4}$	$\mathcal{O}_{5,5}$	$\mathcal{O}_{5,6}$	$\mathcal{O}_{5,7}$	$\mathcal{O}_{5,8}$	$\cdots$
...	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

EOM's Horizontal relations; the same for all rows (i.e. all gauge contractions)

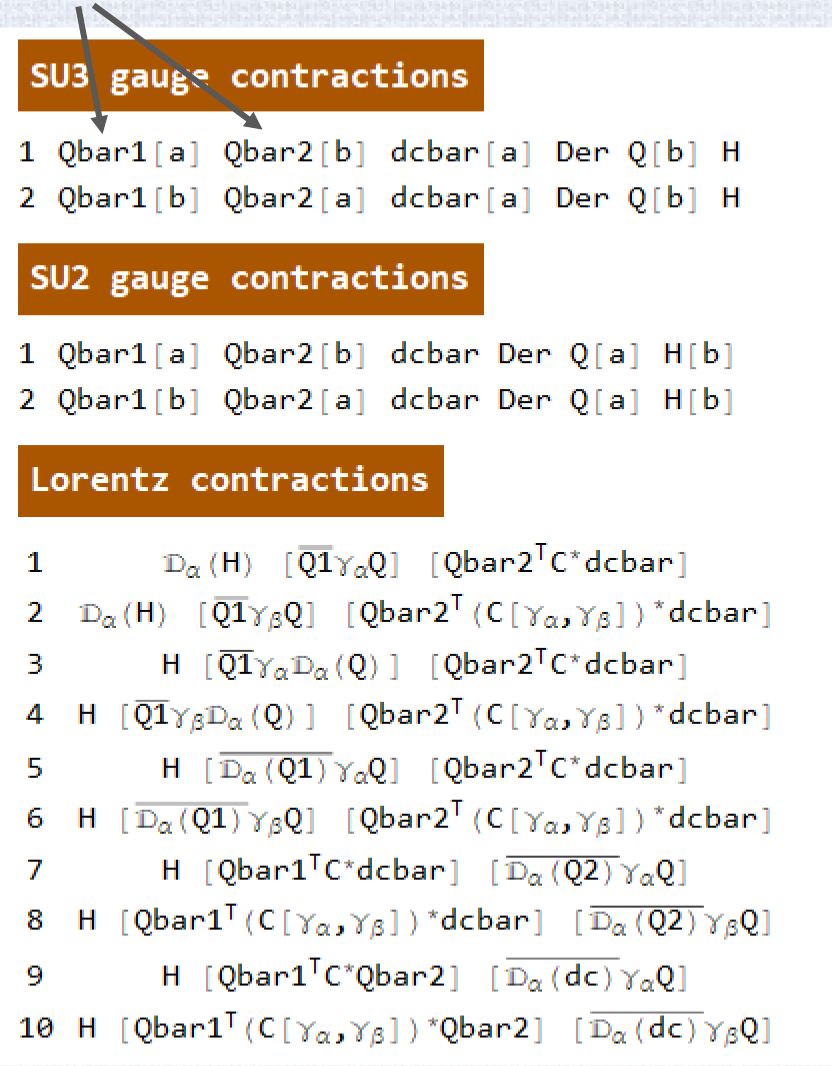
IBP's Horizontal relations; the same for all rows (i.e. all gauge contractions)

Repeated fields Oblique relations in general! Not the same for each row

A nice fact: in order to know the "repeated field redundancies" it is not necessary to know the details of the gauge contractions – only how permutation symmetries act on them (elegant; one can change the group/reps and still reuse results)

Example: $D_\mu\overline{Q}\overline{Q}Q\overline{d^c}H$

Discriminate
the $\overline{Q}$'s



SU3 gauge contractions

1 Qbar1[a] Qbar2[b] dcbar[a] Der Q[b] H
2 Qbar1[b] Qbar2[a] dcbar[a] Der Q[b] H

SU2 gauge contractions

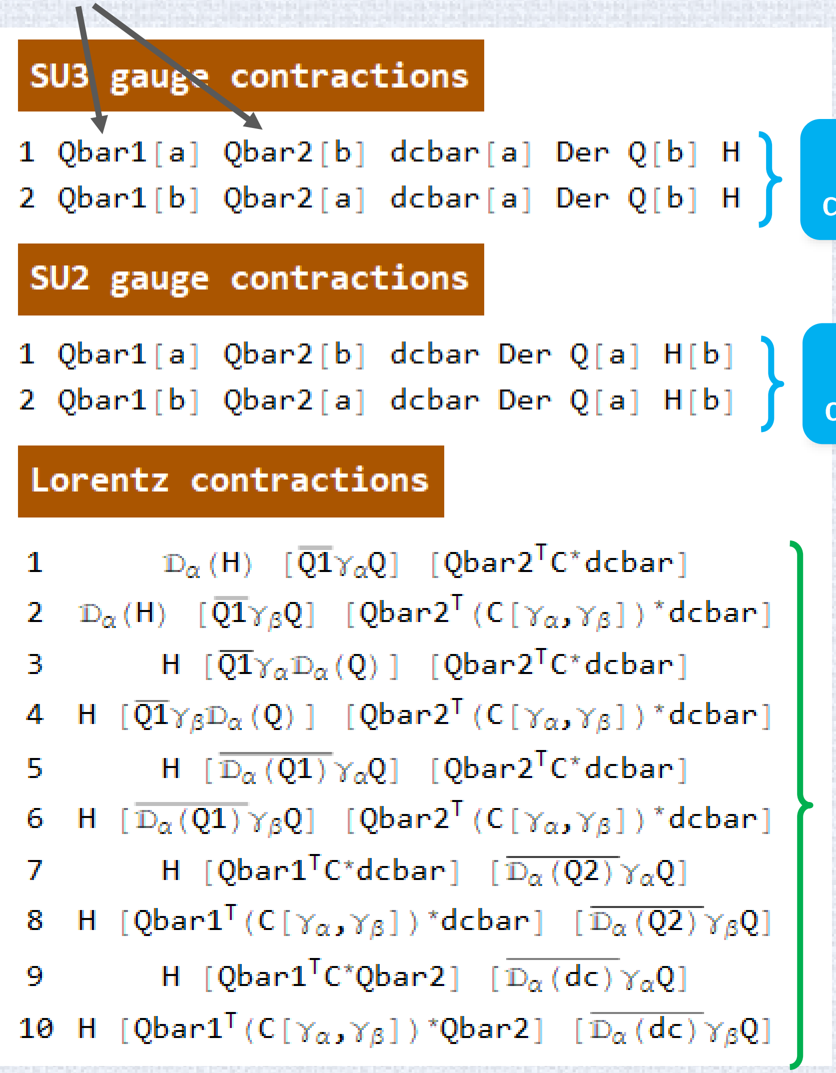
1 Qbar1[a] Qbar2[b] dcbar Der Q[a] H[b]
2 Qbar1[b] Qbar2[a] dcbar Der Q[a] H[b]

Lorentz contractions

1 $\mathbb{D}_\alpha(\text{H})\ [\overline{\text{Q1}}\gamma_\alpha\text{Q}]\ [\text{Qbar2}^\text{T}\text{C}^*\text{dcbar}]$
2 $\mathbb{D}_\alpha(\text{H})\ [\overline{\text{Q1}}\gamma_\beta\text{Q}]\ [\text{Qbar2}^\text{T}(\text{C}[\gamma_\alpha,\gamma_\beta])^*\text{dcbar}]$
3 $\text{H}\ [\overline{\text{Q1}}\gamma_\alpha\mathbb{D}_\alpha(\text{Q})]\ [\text{Qbar2}^\text{T}\text{C}^*\text{dcbar}]$
4 $\text{H}\ [\overline{\text{Q1}}\gamma_\beta\mathbb{D}_\alpha(\text{Q})]\ [\text{Qbar2}^\text{T}(\text{C}[\gamma_\alpha,\gamma_\beta])^*\text{dcbar}]$
5 $\text{H}\ [\overline{\mathbb{D}_\alpha(\text{Q1})}\gamma_\alpha\text{Q}]\ [\text{Qbar2}^\text{T}\text{C}^*\text{dcbar}]$
6 $\text{H}\ [\overline{\mathbb{D}_\alpha(\text{Q1})}\gamma_\beta\text{Q}]\ [\text{Qbar2}^\text{T}(\text{C}[\gamma_\alpha,\gamma_\beta])^*\text{dcbar}]$
7 $\text{H}\ [\text{Qbar1}^\text{T}\text{C}^*\text{dcbar}]\ [\overline{\mathbb{D}_\alpha(\text{Q2})}\gamma_\alpha\text{Q}]$
8 $\text{H}\ [\text{Qbar1}^\text{T}(\text{C}[\gamma_\alpha,\gamma_\beta])^*\text{dcbar}]\ [\overline{\mathbb{D}_\alpha(\text{Q2})}\gamma_\beta\text{Q}]$
9 $\text{H}\ [\text{Qbar1}^\text{T}\text{C}^*\text{Qbar2}]\ [\overline{\mathbb{D}_\alpha(\text{dc})}\gamma_\alpha\text{Q}]$
10 $\text{H}\ [\text{Qbar1}^\text{T}(\text{C}[\gamma_\alpha,\gamma_\beta])^*\text{Qbar2}]\ [\overline{\mathbb{D}_\alpha(\text{dc})}\gamma_\beta\text{Q}]$

Example: $D_\mu\overline{Q}\overline{Q}Q\overline{d^c}H$

Discriminate
the $\overline{Q}$'s



SU3 gauge contractions

```
1 Qbar1[a] Qbar2[b] dcbar[a] Der Q[b] H
2 Qbar1[b] Qbar2[a] dcbar[a] Der Q[b] H
```

2 SU(3) contractions

SU2 gauge contractions

```
1 Qbar1[a] Qbar2[b] dcbar Der Q[a] H[b]
2 Qbar1[b] Qbar2[a] dcbar Der Q[a] H[b]
```

2 SU(2) contractions

Lorentz contractions

1 $\mathbb{D}_\alpha(\mathrm{H})\ [\overline{\mathrm{Q1}}\gamma_\alpha\mathrm{Q}]\ [\mathrm{Qbar2}^\mathrm{T}\mathrm{C}^*\mathrm{dcbar}]$

2 $\mathbb{D}_\alpha(\mathrm{H})\ [\overline{\mathrm{Q1}}\gamma_\beta\mathrm{Q}]\ [\mathrm{Qbar2}^\mathrm{T}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]$

3 $\mathrm{H}\ [\overline{\mathrm{Q1}}\gamma_\alpha\mathbb{D}_\alpha(\mathrm{Q})]\ [\mathrm{Qbar2}^\mathrm{T}\mathrm{C}^*\mathrm{dcbar}]$

4 $\mathrm{H}\ [\overline{\mathrm{Q1}}\gamma_\beta\mathbb{D}_\alpha(\mathrm{Q})]\ [\mathrm{Qbar2}^\mathrm{T}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]$

5 $\mathrm{H}\ [\overline{\mathbb{D}_\alpha(\mathrm{Q1})}\gamma_\alpha\mathrm{Q}]\ [\mathrm{Qbar2}^\mathrm{T}\mathrm{C}^*\mathrm{dcbar}]$

6 $\mathrm{H}\ [\overline{\mathbb{D}_\alpha(\mathrm{Q1})}\gamma_\beta\mathrm{Q}]\ [\mathrm{Qbar2}^\mathrm{T}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]$

7 $\mathrm{H}\ [\mathrm{Qbar1}^\mathrm{T}\mathrm{C}^*\mathrm{dcbar}]\ [\overline{\mathbb{D}_\alpha(\mathrm{Q2})}\gamma_\alpha\mathrm{Q}]$

8 $\mathrm{H}\ [\mathrm{Qbar1}^\mathrm{T}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{dcbar}]\ [\overline{\mathbb{D}_\alpha(\mathrm{Q2})}\gamma_\beta\mathrm{Q}]$

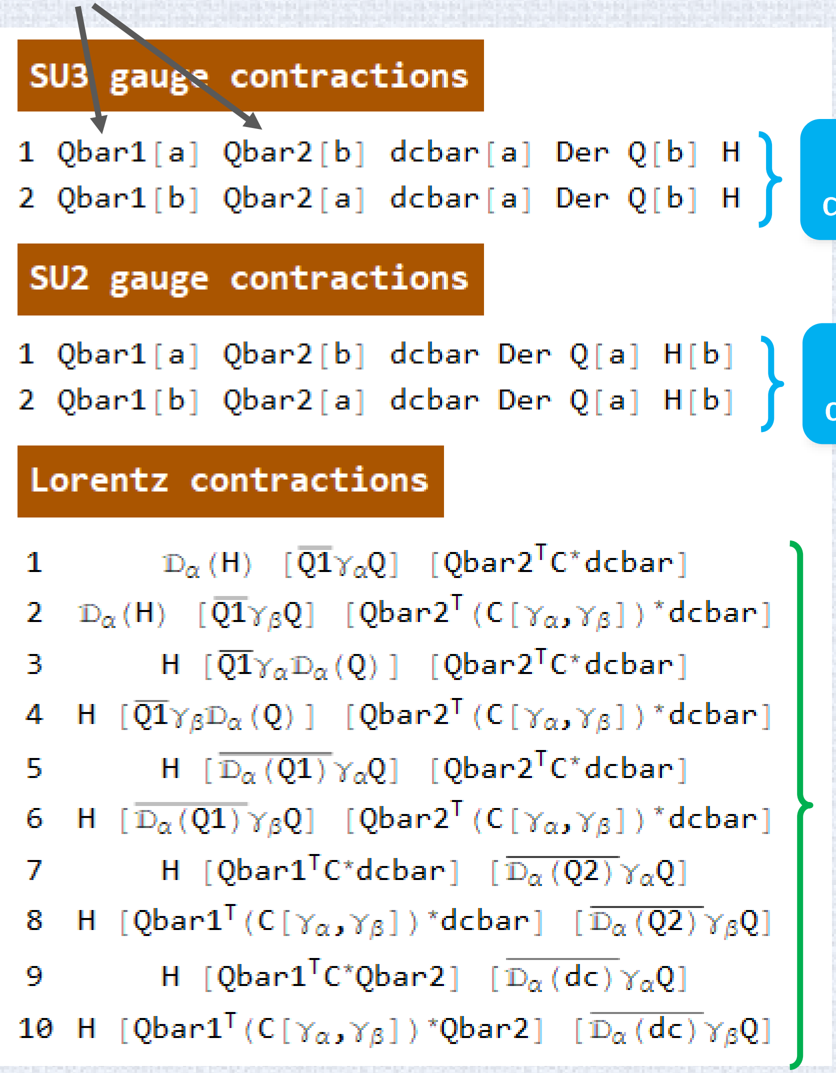
9 $\mathrm{H}\ [\mathrm{Qbar1}^\mathrm{T}\mathrm{C}^*\mathrm{Qbar2}]\ [\overline{\mathbb{D}_\alpha(\mathrm{dc})}\gamma_\alpha\mathrm{Q}]$

10 $\mathrm{H}\ [\mathrm{Qbar1}^\mathrm{T}(\mathrm{C}[\gamma_\alpha,\gamma_\beta])^*\mathrm{Qbar2}]\ [\overline{\mathbb{D}_\alpha(\mathrm{dc})}\gamma_\beta\mathrm{Q}]$

10 Lorentz contractions

Example: $D_\mu \overline{Q}\,\overline{Q} Q \overline{d^c} H$

Discriminate
the $\overline{Q}$'s



SU3 gauge contractions

1 Qbar1[a] Qbar2[b] dcbar[a] Der Q[b] H
2 Qbar1[b] Qbar2[a] dcbar[a] Der Q[b] H

SU2 gauge contractions

1 Qbar1[a] Qbar2[b] dcbar Der Q[a] H[b]
2 Qbar1[b] Qbar2[a] dcbar Der Q[a] H[b]

Lorentz contractions

1 $\mathbb{D}_\alpha(H)\ [\overline{Q1}\gamma_\alpha Q]\ [\mathrm{Qbar2}^T C^* \mathrm{dcbar}]$
2 $\mathbb{D}_\alpha(H)\ [\overline{Q1}\gamma_\beta Q]\ [\mathrm{Qbar2}^T (C[\gamma_\alpha,\gamma_\beta])^* \mathrm{dcbar}]$
3 $H\ [\overline{Q1}\gamma_\alpha \mathbb{D}_\alpha(Q)]\ [\mathrm{Qbar2}^T C^* \mathrm{dcbar}]$
4 $H\ [\overline{Q1}\gamma_\beta \mathbb{D}_\alpha(Q)]\ [\mathrm{Qbar2}^T (C[\gamma_\alpha,\gamma_\beta])^* \mathrm{dcbar}]$
5 $H\ [\overline{\mathbb{D}_\alpha(Q1)}\gamma_\alpha Q]\ [\mathrm{Qbar2}^T C^* \mathrm{dcbar}]$
6 $H\ [\overline{\mathbb{D}_\alpha(Q1)}\gamma_\beta Q]\ [\mathrm{Qbar2}^T (C[\gamma_\alpha,\gamma_\beta])^* \mathrm{dcbar}]$
7 $H\ [\mathrm{Qbar1}^T C^* \mathrm{dcbar}]\ [\overline{\mathbb{D}_\alpha(Q2)}\gamma_\alpha Q]$
8 $H\ [\mathrm{Qbar1}^T (C[\gamma_\alpha,\gamma_\beta])^* \mathrm{dcbar}]\ [\overline{\mathbb{D}_\alpha(Q2)}\gamma_\beta Q]$
9 $H\ [\mathrm{Qbar1}^T C^* \mathrm{Qbar2}]\ [\overline{\mathbb{D}_\alpha(dc)}\gamma_\alpha Q]$
10 $H\ [\mathrm{Qbar1}^T (C[\gamma_\alpha,\gamma_\beta])^* \mathrm{Qbar2}]\ [\overline{\mathbb{D}_\alpha(dc)}\gamma_\beta Q]$

2 SU(3) contractions

2 SU(2) contractions

10 Lorentz contractions

Same-field redundancies

$$\begin{array}{c|cccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \hline (1,1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (1,2) & \frac{1}{2} & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (2,1) & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (2,2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

+19 others

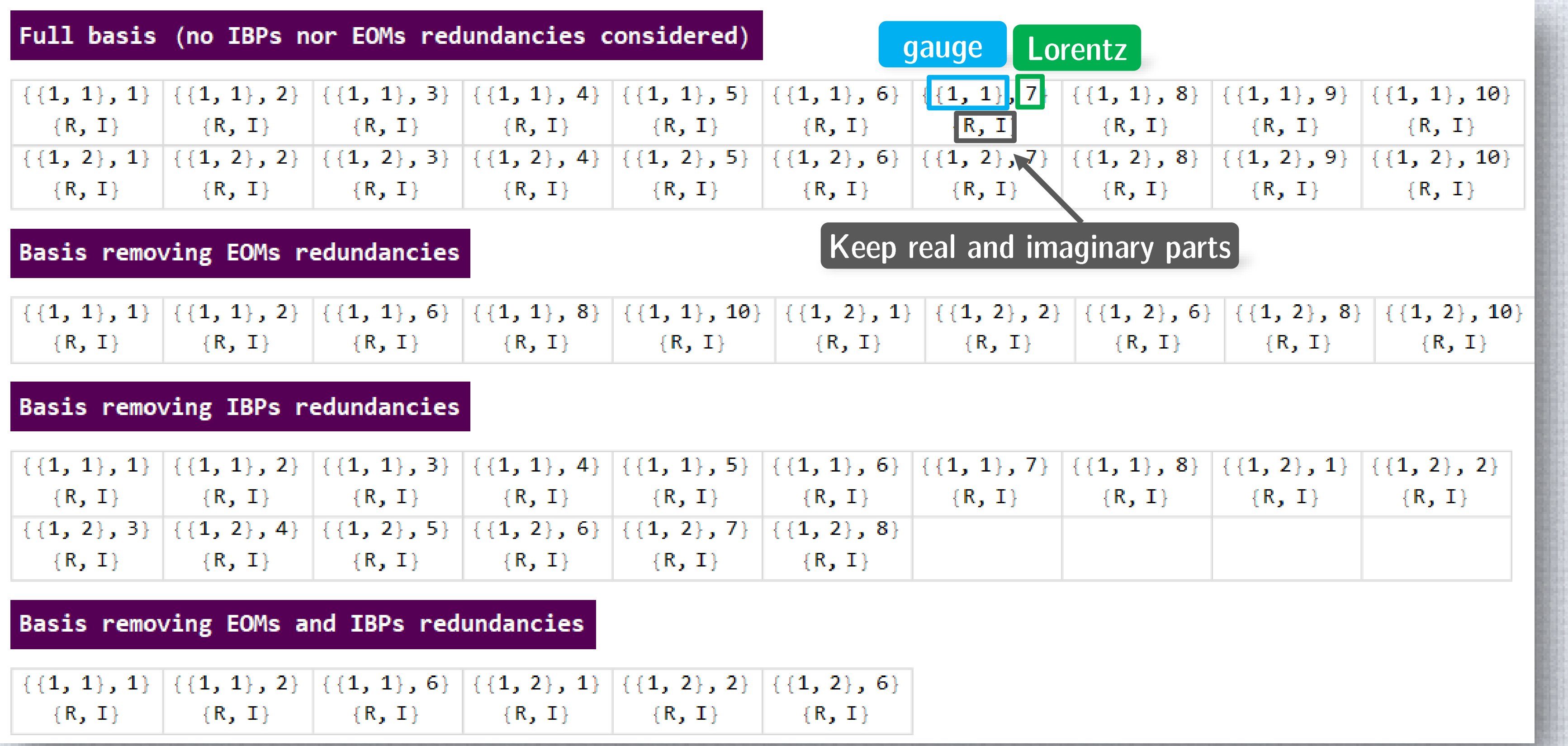
IBP redundancies

$$\begin{array}{c} \text{for each (i,j)} \\ \text{for each (i,j)} \end{array} \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & 1 & 0 & 1 & -3 & -\frac{1}{2} & -3 & -\frac{1}{2} \end{pmatrix}$$

EOM redundancies

$$\text{for each (i,j)} \left\{ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right.$$

Example: $D_\mu \overline{QQ} Q \overline{d^c} H$



gauge

Lorentz

Keep real and imaginary parts

Full basis (no IBPs nor EOMs redundancies considered)

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R, I}	{{1, 1}, 5} {R, I}	{{1, 1}, 6} {R, I}	{{1, 1}, 7} {R, I}	{{1, 1}, 8} {R, I}	{{1, 1}, 9} {R, I}	{{1, 1}, 10} {R, I}
{{1, 2}, 1} {R, I}	{{1, 2}, 2} {R, I}	{{1, 2}, 3} {R, I}	{{1, 2}, 4} {R, I}	{{1, 2}, 5} {R, I}	{{1, 2}, 6} {R, I}	{{1, 2}, 7} {R, I}	{{1, 2}, 8} {R, I}	{{1, 2}, 9} {R, I}	{{1, 2}, 10} {R, I}

Basis removing EOMs redundancies

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 6} {R, I}	{{1, 1}, 8} {R, I}	{{1, 1}, 10} {R, I}	{{1, 2}, 1} {R, I}	{{1, 2}, 2} {R, I}	{{1, 2}, 6} {R, I}	{{1, 2}, 8} {R, I}	{{1, 2}, 10} {R, I}

Basis removing IBPs redundancies

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R, I}	{{1, 1}, 5} {R, I}	{{1, 1}, 6} {R, I}	{{1, 1}, 7} {R, I}	{{1, 1}, 8} {R, I}	{{1, 2}, 1} {R, I}	{{1, 2}, 2} {R, I}
{{1, 2}, 3} {R, I}	{{1, 2}, 4} {R, I}	{{1, 2}, 5} {R, I}	{{1, 2}, 6} {R, I}	{{1, 2}, 7} {R, I}	{{1, 2}, 8} {R, I}				

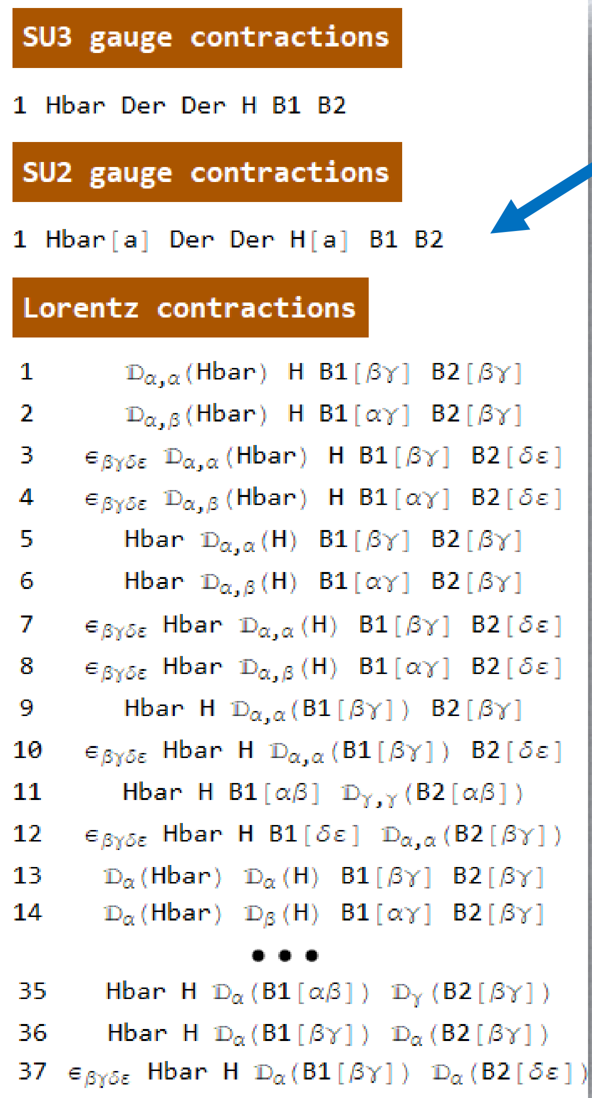
Basis removing EOMs and IBPs redundancies

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 6} {R, I}	{{1, 2}, 1} {R, I}	{{1, 2}, 2} {R, I}	{{1, 2}, 6} {R, I}

This is one possibility: sets of operators that work are picked automatically. With the redundancies calculated, another conceivable scenario is to allow the user to ask the code "Do the operators A,B,C form a basis?".

Interface & output format require thinking (work in progress)

Example: $D_\mu D_\nu B B \overline{H} H$



SU3 gauge contractions

1 Hbar Der Der H B1 B2

SU2 gauge contractions

1 Hbar[a] Der Der H[a] B1 B2

Lorentz contractions

1 $\mathbb{D}_{\alpha,\alpha}$(Hbar) H B1[$\beta\gamma$] B2[$\beta\gamma$]

2 $\mathbb{D}_{\alpha,\beta}$(Hbar) H B1[$\alpha\gamma$] B2[$\beta\gamma$]

3 $\epsilon_{\beta\gamma\delta\varepsilon}$ $\mathbb{D}_{\alpha,\alpha}$(Hbar) H B1[$\beta\gamma$] B2[$\delta\varepsilon$]

4 $\epsilon_{\beta\gamma\delta\varepsilon}$ $\mathbb{D}_{\alpha,\beta}$(Hbar) H B1[$\alpha\gamma$] B2[$\delta\varepsilon$]

5 Hbar $\mathbb{D}_{\alpha,\alpha}$(H) B1[$\beta\gamma$] B2[$\beta\gamma$]

6 Hbar $\mathbb{D}_{\alpha,\beta}$(H) B1[$\alpha\gamma$] B2[$\beta\gamma$]

7 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar $\mathbb{D}_{\alpha,\alpha}$(H) B1[$\beta\gamma$] B2[$\delta\varepsilon$]

8 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar $\mathbb{D}_{\alpha,\beta}$(H) B1[$\alpha\gamma$] B2[$\delta\varepsilon$]

9 Hbar H $\mathbb{D}_{\alpha,\alpha}$(B1[$\beta\gamma$]) B2[$\beta\gamma$]

10 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar H $\mathbb{D}_{\alpha,\alpha}$(B1[$\beta\gamma$]) B2[$\delta\varepsilon$]

11 Hbar H B1[$\alpha\beta$] $\mathbb{D}_{\gamma,\gamma}$(B2[$\alpha\beta$])

12 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar H B1[$\delta\varepsilon$] $\mathbb{D}_{\alpha,\alpha}$(B2[$\beta\gamma$])

13 $\mathbb{D}_{\alpha}$(Hbar) $\mathbb{D}_{\alpha}$(H) B1[$\beta\gamma$] B2[$\beta\gamma$]

14 $\mathbb{D}_{\alpha}$(Hbar) $\mathbb{D}_{\beta}$(H) B1[$\alpha\gamma$] B2[$\beta\gamma$]

• • •

35 Hbar H $\mathbb{D}_{\alpha}$(B1[$\alpha\beta$]) $\mathbb{D}_{\gamma}$(B2[$\beta\gamma$])

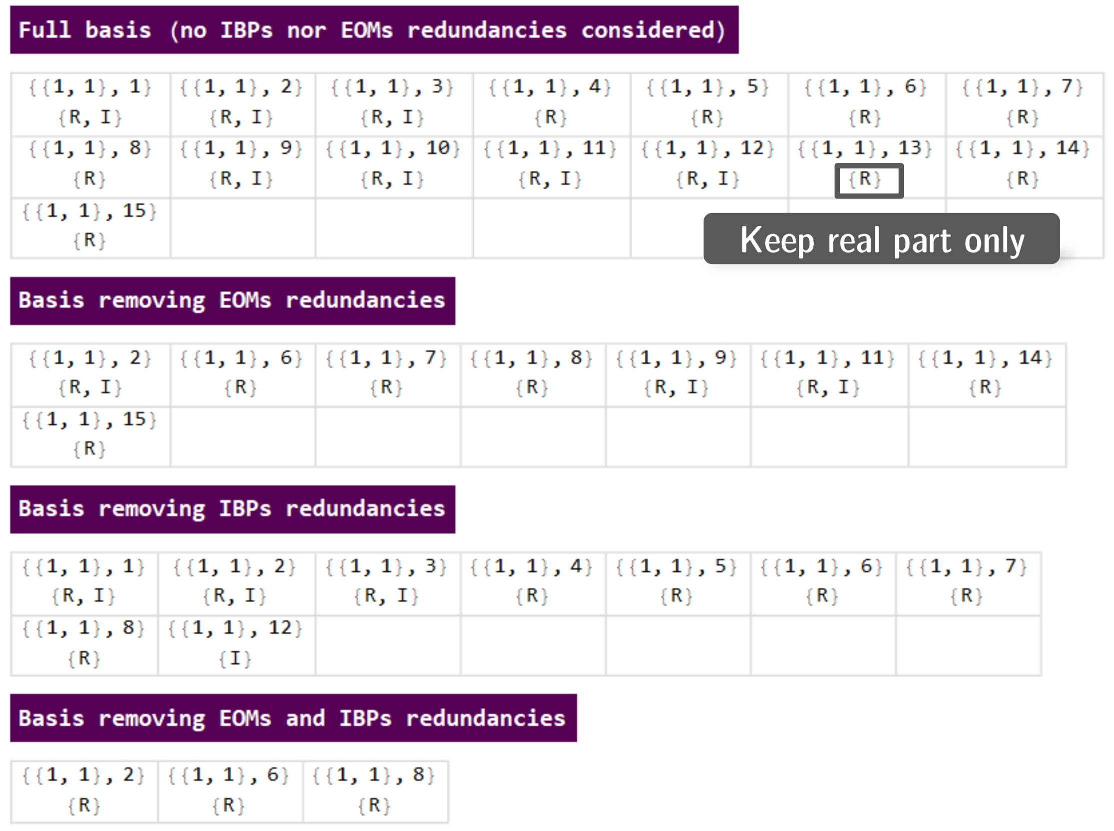
36 Hbar H $\mathbb{D}_{\alpha}$(B1[$\beta\gamma$]) $\mathbb{D}_{\alpha}$(B2[$\beta\gamma$])

37 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar H $\mathbb{D}_{\alpha}$(B1[$\beta\gamma$]) $\mathbb{D}_{\alpha}$(B2[$\delta\varepsilon$])

Symmetric under exchange of B1 and B2

This is all that matters

H, B could transform differently (even under some different group), but the results would be the same as long as B1 and B2 are symmetrically contracted



Full basis (no IBPs nor EOMs redundancies considered)

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R}	{{1, 1}, 5} {R}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}
{{1, 1}, 8} {R}	{{1, 1}, 9} {R, I}	{{1, 1}, 10} {R, I}	{{1, 1}, 11} {R, I}	{{1, 1}, 12} {R, I}	{{1, 1}, 13} {R}	{{1, 1}, 14} {R}
{{1, 1}, 15} {R}						

Keep real part only

Basis removing EOMs redundancies

{{1, 1}, 2} {R, I}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}	{{1, 1}, 8} {R}	{{1, 1}, 9} {R, I}	{{1, 1}, 11} {R, I}	{{1, 1}, 14} {R}
{{1, 1}, 15} {R}						

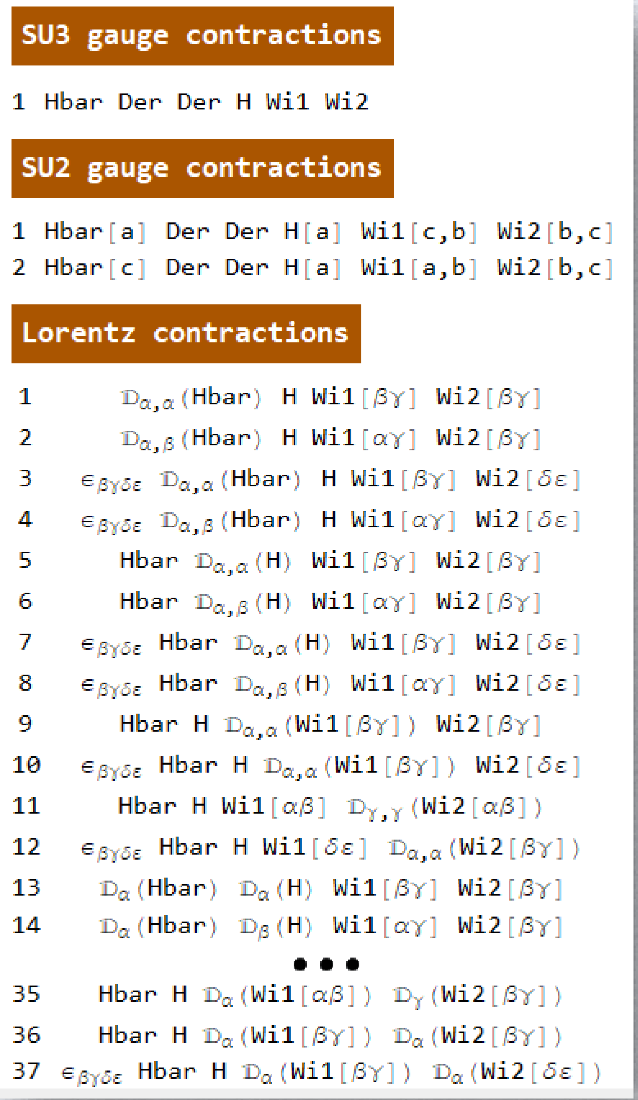
Basis removing IBPs redundancies

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R}	{{1, 1}, 5} {R}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}
{{1, 1}, 8} {R}	{{1, 1}, 12} {I}					

Basis removing EOMs and IBPs redundancies

{{1, 1}, 2} {R}	{{1, 1}, 6} {R}	{{1, 1}, 8} {R}

Example: $D_\mu D_\nu W W \overline{H} H$



SU3 gauge contractions

1 Hbar Der Der H Wi1 Wi2

SU2 gauge contractions

1 Hbar[a] Der Der H[a] Wi1[c,b] Wi2[b,c]
2 Hbar[c] Der Der H[a] Wi1[a,b] Wi2[b,c]

Lorentz contractions

1 $\mathbb{D}_{\alpha,\alpha}$(Hbar) H Wi1[$\beta\gamma$] Wi2[$\beta\gamma$]
2 $\mathbb{D}_{\alpha,\beta}$(Hbar) H Wi1[$\alpha\gamma$] Wi2[$\beta\gamma$]
3 $\epsilon_{\beta\gamma\delta\varepsilon}$ $\mathbb{D}_{\alpha,\alpha}$(Hbar) H Wi1[$\beta\gamma$] Wi2[$\delta\varepsilon$]
4 $\epsilon_{\beta\gamma\delta\varepsilon}$ $\mathbb{D}_{\alpha,\beta}$(Hbar) H Wi1[$\alpha\gamma$] Wi2[$\delta\varepsilon$]
5 Hbar $\mathbb{D}_{\alpha,\alpha}$(H) Wi1[$\beta\gamma$] Wi2[$\beta\gamma$]
6 Hbar $\mathbb{D}_{\alpha,\beta}$(H) Wi1[$\alpha\gamma$] Wi2[$\beta\gamma$]
7 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar $\mathbb{D}_{\alpha,\alpha}$(H) Wi1[$\beta\gamma$] Wi2[$\delta\varepsilon$]
8 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar $\mathbb{D}_{\alpha,\beta}$(H) Wi1[$\alpha\gamma$] Wi2[$\delta\varepsilon$]
9 Hbar H $\mathbb{D}_{\alpha,\alpha}$(Wi1[$\beta\gamma$]) Wi2[$\beta\gamma$]
10 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar H $\mathbb{D}_{\alpha,\alpha}$(Wi1[$\beta\gamma$]) Wi2[$\delta\varepsilon$]
11 Hbar H Wi1[$\alpha\beta$] $\mathbb{D}_{\gamma,\gamma}$(Wi2[$\alpha\beta$])
12 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar H Wi1[$\delta\varepsilon$] $\mathbb{D}_{\alpha,\alpha}$(Wi2[$\beta\gamma$])
13 $\mathbb{D}_{\alpha}$(Hbar) $\mathbb{D}_{\alpha}$(H) Wi1[$\beta\gamma$] Wi2[$\beta\gamma$]
14 $\mathbb{D}_{\alpha}$(Hbar) $\mathbb{D}_{\beta}$(H) Wi1[$\alpha\gamma$] Wi2[$\beta\gamma$]

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35 Hbar H $\mathbb{D}_{\alpha}$(Wi1[$\alpha\beta$]) $\mathbb{D}_{\gamma}$(Wi2[$\beta\gamma$])
36 Hbar H $\mathbb{D}_{\alpha}$(Wi1[$\beta\gamma$]) $\mathbb{D}_{\alpha}$(Wi2[$\beta\gamma$])
37 $\epsilon_{\beta\gamma\delta\varepsilon}$ Hbar H $\mathbb{D}_{\alpha}$(Wi1[$\beta\gamma$]) $\mathbb{D}_{\alpha}$(Wi2[$\delta\varepsilon$])

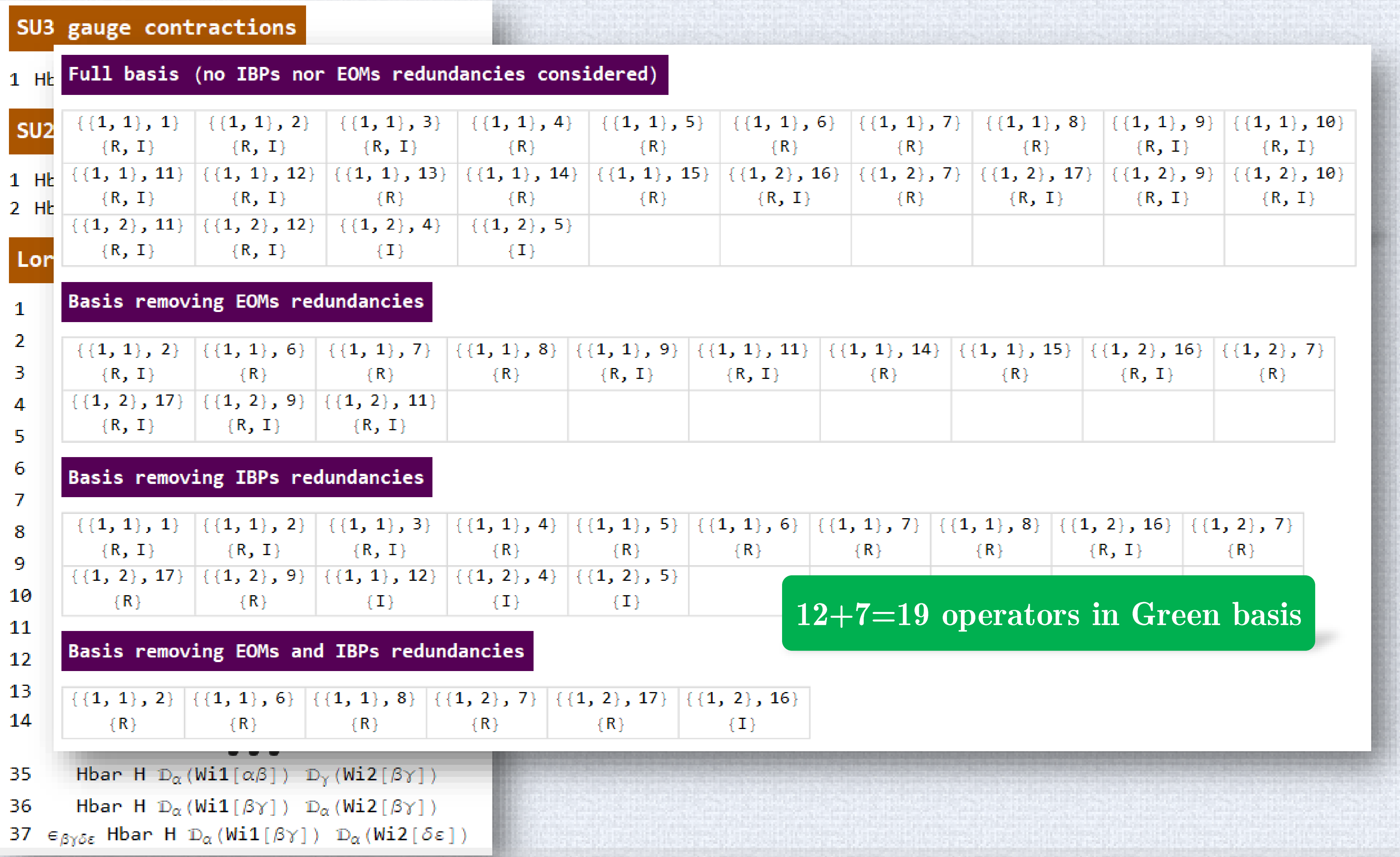
One contraction is symmetric (S) under exchange of W1 and W2, and the other is anti-symmetric (A)

Written in this form, the S and A are mixed (they are not cleanly separated)

For the symmetric (S) contraction the results on the previous slide apply! For example, there are 12 operators after application of IBPs

For the anti-symmetric (A) gauge contraction, there are an addition 7 operators in the Green basis. Total: 12+7=19

Example: $D_\mu D_\nu W W \overline{H} H$



```
SU3 gauge contractions

1  Hb

SU2

1  Hb
2  Hb

Lor

1
2
3
4
5
6
7
8
9
10
11
12
13
14
           …
35     Hbar H 𝔻α(Wi1[αβ]) 𝔻γ(Wi2[βγ])
36     Hbar H 𝔻α(Wi1[βγ]) 𝔻α(Wi2[βγ])
37 ϵβγδε Hbar H 𝔻α(Wi1[βγ]) 𝔻α(Wi2[δε])
```

Full basis (no IBPs nor EOMs redundancies considered)

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R}	{{1, 1}, 5} {R}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}	{{1, 1}, 8} {R}	{{1, 1}, 9} {R, I}	{{1, 1}, 10} {R, I}
{{1, 1}, 11} {R, I}	{{1, 1}, 12} {R, I}	{{1, 1}, 13} {R}	{{1, 1}, 14} {R}	{{1, 1}, 15} {R}	{{1, 2}, 16} {R, I}	{{1, 2}, 7} {R}	{{1, 2}, 17} {R, I}	{{1, 2}, 9} {R, I}	{{1, 2}, 10} {R, I}
{{1, 2}, 11} {R, I}	{{1, 2}, 12} {R, I}	{{1, 2}, 4} {I}	{{1, 2}, 5} {I}						

Basis removing EOMs redundancies

{{1, 1}, 2} {R, I}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}	{{1, 1}, 8} {R}	{{1, 1}, 9} {R, I}	{{1, 1}, 11} {R, I}	{{1, 1}, 14} {R}	{{1, 1}, 15} {R}	{{1, 2}, 16} {R, I}	{{1, 2}, 7} {R}
{{1, 2}, 17} {R, I}	{{1, 2}, 9} {R, I}	{{1, 2}, 11} {R, I}							

Basis removing IBPs redundancies

{{1, 1}, 1} {R, I}	{{1, 1}, 2} {R, I}	{{1, 1}, 3} {R, I}	{{1, 1}, 4} {R}	{{1, 1}, 5} {R}	{{1, 1}, 6} {R}	{{1, 1}, 7} {R}	{{1, 1}, 8} {R}	{{1, 2}, 16} {R, I}	{{1, 2}, 7} {R}
{{1, 2}, 17} {R}	{{1, 2}, 9} {R}	{{1, 1}, 12} {I}	{{1, 2}, 4} {I}	{{1, 2}, 5} {I}					

Basis removing EOMs and IBPs redundancies

{{1, 1}, 2} {R}	{{1, 1}, 6} {R}	{{1, 1}, 8} {R}	{{1, 2}, 7} {R}	{{1, 2}, 17} {R}	{{1, 2}, 16} {I}

12+7=19 operators in Green basis

Flavor

Ongoing work

Possible solution: run the same code multiple times, with slightly different input

This is not so inefficient: total computation time does not scale with the number of flavors/dimension of operator as badly was you might think!

Examples:

$L_i L_j H H$	Run “flavorless” code 2 times:	`{L,L,H,H}` and `{L,L’,H,H}`

$Q_i Q_j Q_k L_l$	Run “flavorless” code 3 times:	`{Q,Q,Q,L}`
		`{Q,Q,Q’,L}`
		`{Q,Q’,Q’’,L}`

$Q^6 L^2$	Run “flavorless” code 22 times (not $3^8 = 6561$ times)	

Somehow use this information to populate coupling matrices/tensors in flavor space

Summary

Tony Bowden, www.flickr.com/photos/tm-tm/5998492383

Summary


Pass it on to ...

From a list of fields and some symmetries,
we want to get a basis of EFT operators.
Maybe also tweak them (change basis)

I've described the possibility of making
GroupMath + **Sym2Int** not just list,
but also **build explicitly EFT operators**

The good news: building such a code seems doable.
All SMEFT operators, with 3 generations, can be computed up to dimension 10 in a couple of hours

Ongoing work. Hopefully on the www sometime soon

Thank you