Gauge Invariance and Finite Counterterms in Chiral Gauge Theories

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Gauge Anomaly Cancellation

- Self-consistency (unitarity, physical dof, renormalizability) ➔ anomaly cancellation:

$$D^{abc} = \mathrm{tr}(T^a_L\{T^b_L, T^c_L\}) - \mathrm{tr}(T^a_R\{T^b_R, T^c_R\}) = 0\,.$$ Georgi-Glashow (1972)

- No new **relevant** anomalies emerge at non-renormalizable level. See, e.g., Gomis-Weinberg (1995)

In practical perturbative calculations, however, Gauge Invariance is <u>explicitly</u> broken:

- ❋ by **gauge fixing**
- ❋ by **regularization** (action and/or measure not invariant)

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- ❋ by **gauge fixing:** BRST symmetry + Cohomology ➔ self-consistency

- ❋ by **regularization:** unphysical ➔ <u>must be</u> removable by counterterms

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↕

Sometimes it is not possible to find a regulator that respects all symmetries.
In Dimensional-Regularization breaking is unavoidable if the theory is chiral (Standard Model).

$\Longrightarrow$ **we must add a counterterm to amplitudes!**

In this talk we discuss the Gauge-Restoring Counterterm for a renormalizable theory with arbitrary gauge group and (chiral) fermion (reducible) representation.

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Why renormalizable and no Yukawa?
Simplicity, result missing.
Inclusion of Yukawa and higher-dim. operators leads to no conceptual difficulty

The counterterm depends on the regularization and renormalization scheme.
We use:
(i) Background field method
(ii) Dimensional regularization
(iii) Renormalization via minimal subtraction
(iv) Breitenlohner-Maison-'t Hooft-Veltman prescription for γ_5

Why background field method?

The gauge symmetry acts linearly on the 1PI effective action: easier.

1PI effective action

$$\delta_\alpha \Gamma = \int d^4x \ \alpha_a(x) L_a(x) \Gamma$$

Gauge parameter

Generator of infinitesimal gauge transformations of fields

The previous literature impose Slavnov-Taylor:

Martin-SanchezRuiz (2000), SanchezRuiz (2003) BeluscaMaito et al. (20202021) 't Hooft-Veltman (1972)

Dimensional Regularization

— Coordinates split into 4-dimensional and (d-4)-dimensional: $\mu = \bar{\mu} \oplus \hat{\mu}$

$$\underbrace{\text{4-dimensional}}_{\bar{\mu} \in SO(1,3)} \quad \underbrace{\text{(d-4)-dimensional}}_{\hat{\mu} \in O(d-4)}$$

— PROBLEM: **Fermion chirality does not exist in arbitrary d: ɣ_5 problem…**

γ_5?

There is no way to simultaneously respect:

$$\begin{cases} \{\gamma^\mu, \gamma_5\} = 0 \\ \mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) = 4i\epsilon^{\mu\nu\rho\sigma} \\ \text{Cyclicity of the trace} \end{cases}$$

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't Hooft-Veltman: chirality is a purely 4-dimensional concept.

$$\gamma_5 = \frac{i}{4!}\epsilon_{\bar\mu\bar\nu\bar\alpha\bar\beta}\gamma^{\bar\mu}\gamma^{\bar\nu}\gamma^{\bar\alpha}\gamma^{\bar\beta}$$

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Breitenlohner-Maison proved consistency <u>at all orders</u> (necessary).
No alternative prescription has been proven consistent (yet?).

Our regularization scheme (continued…)

— Kinetic terms and space-time measure are continued to arbitrary (complex) d.
A lot of freedom in the interactions → **additional regularization scheme-dependence**.

— Bosonic sector can be continued without violating any of the 4-dimensional symmetries
We choose natural extension to d dimensions. Then $L_a S^{(d)}_{\rm Bosons} = 0$

$$\mathcal{L}_{\rm Fermions} = i\bar{f}\gamma^\mu\partial_\mu f - A^a_\mu\;\bar{f}\left(\gamma^\mu P_L T^a_L + \gamma^\mu P_R T^a_R\right) f$$

P_R=(1+ɣ_5)/2

$$\mathcal{L}^{(d)}_{\rm Fermions} = i\bar{f}\gamma^\mu\partial_\mu f - A^a_{\bar\mu}\;\bar{f}\left(P_R\gamma^{\bar\mu} P_L T^a_L + P_L\gamma^{\bar\mu} P_R T^a_R\right) f$$

Fixed

Scheme-dependent

Our regularization scheme (continued…)

— Regularization of Fermionic Action:
Kinetic term must be d-dimensional and mixes L with R → **explicit breaking!**

$$\mathcal{L}^{(d)}_{\text{Fermions}} = i\bar{f}\gamma^{\mu}\partial_{\mu}f - A^{a}_{\bar{\mu}}\ \bar{f}\left(P_R\gamma^{\bar{\mu}}P_L T^a_L + P_L\gamma^{\bar{\mu}}P_R T^a_R\right)f$$

$$\Longrightarrow\ L_a S^{(d)} = \underbrace{2\alpha_a \overline{f} T^a_A \gamma^{\hat{\mu}}\gamma_5 \partial_{\hat{\mu}} f + \partial_{\hat{\mu}}\alpha_a \overline{f} T^a \gamma^{\hat{\mu}} f}_{\text{Evanescent}}$$

The fermionic action is not invariant unless vector-like (T_A=0).

$$T^a_A = T^a_R - T^a_L$$
$$T^a = T^a_R P_R + T^a_L P_L$$

The calculation

In Dim-Reg the path integral measure is invariant. This is true for local transformations since δ(0)=0.
The anomaly arises because the regularized action is not invariant.

Regularized action
(includes gauge-fixing)

$$e^{i\Gamma^{(d)}[\phi]} = \int_{1\mathrm{PI}} \mathcal{D}\tilde{\phi}\; e^{iS^{(d)}_{\mathrm{full}}[\phi+\tilde{\phi}]}$$

Only from fermions
(In our scheme)

4-dimensional
gauge transformation

$$L_a\Gamma^{(d)}[\phi] = \frac{\int_{1\mathrm{PI}} \mathcal{D}\tilde{\phi}\; e^{iS^{(d)}_{\mathrm{full}}[\phi+\tilde{\phi}]}\, L_a S^{(d)}_{\mathrm{full}}[\phi+\tilde{\phi}]}{\int_{1\mathrm{PI}} \mathcal{D}\tilde{\phi}\; e^{iS^{(d)}_{\mathrm{full}}[\phi+\phi]}}$$

In the minimal subtraction scheme the anomaly is the finite part of this variation
UV-divergent diagrams combine with the evanescent operator to give a finite effect.

$$\lim_{d\to4} L_a\Gamma^{(d)}_{\text{Gauge}}\Big|_{(1)} = -\text{tr}\left[T^a_A(a^{\epsilon}_2(x) + a^{\not\epsilon}_2(x))\right]$$

$$a^{\epsilon}_2 = \frac{\epsilon^{\mu\nu\alpha\beta}}{16\pi^2}\left[\mathcal{V}_{\mu\nu}\mathcal{V}_{\alpha\beta} + \frac{1}{3}\mathcal{A}_{\mu\nu}\mathcal{A}_{\alpha\beta} - \frac{8}{3}i\left(\mathcal{A}_\alpha\mathcal{A}_\beta\mathcal{V}_{\mu\nu} + \mathcal{A}_\alpha\mathcal{V}_{\mu\nu}\mathcal{A}_\beta + \mathcal{V}_{\mu\nu}\mathcal{A}_\alpha\mathcal{A}_\beta\right) - \frac{32}{3}\mathcal{A}_\mu\mathcal{A}_\nu\mathcal{A}_\alpha\mathcal{A}_\beta\right]$$

$$a^{\not\epsilon}_2 = \frac{1}{16\pi^2}\left[\frac{4}{3}D^{\mathcal{V}}_\nu D^{\mathcal{V}}_\nu D^{\mathcal{V}}_\mu\mathcal{A}_\mu + \frac{8}{3}i[\mathcal{A}_\mu, D^{\mathcal{V}}_\nu\mathcal{V}_{\mu\nu}] - \frac{2}{3}i[\mathcal{A}_{\mu\nu}, \mathcal{V}_{\mu\nu}]\right]$$
$$+\frac{1}{16\pi^2}\left[-8\mathcal{A}_\mu(D^{\mathcal{V}}_\nu\mathcal{A}^\nu)\mathcal{A}_\mu - \frac{8}{3}\left\{D^{\mathcal{V}}_\mu\mathcal{A}_\nu + D^{\mathcal{V}}_\nu\mathcal{A}_\mu, \mathcal{A}_\mu\mathcal{A}_\nu\right\} + \frac{4}{3}\left\{D^{\mathcal{V}}_\mu\mathcal{A}^\mu, \mathcal{A}_\nu\mathcal{A}_\nu\right\}\right]$$

$$\lim_{d\to4} L_c\Gamma^{(d)}_{\text{Fermions}}\Big|_{(1)} = -\frac{2G_{aa}}{16\pi^2}\left(1 + \frac{\xi-1}{6}\right)\bar{f}\gamma_5\left(\overrightarrow{\not\partial} + \overleftarrow{\not\partial}\right)T^aT^c_AT^af$$
$$+\frac{2G_{aa}}{16\pi^2}\left(1 + \frac{\xi-1}{6}\right)\bar{f}\gamma_5\gamma^\mu\left\{[T^c, T^aT^m_AT^a] - if^{cmn}T^aT^n_AT^a\right\}f\,A^m_\mu$$

— Calculation performed via path integral (heat kernel)
— Multiple checks using Feynman diagrams
— Compatible with Wess-Zumino consistency conditions
— Satisfies spurious P and CP

Do not commute

$$\mathcal{V}_\mu = \frac{1}{2}(T^a_R + T^a_L)A^a_\mu \qquad T^a = T^a_RP_R + T^a_LP_L$$
$$\mathcal{A}_\mu = \frac{1}{2}(T^a_R - T^a_L)A^a_\mu \qquad G_{aa} = \delta^G_{ab}g^2_G$$

Consistently, the “regularization anomaly” is the variation of a local counterterm <u>only</u> if the theory is anomaly free (D=0).

Spurious anomaly

$$\left.\begin{aligned} L_a\Gamma &= \Delta_a \\ \Delta_a &= -L_a S_{\rm ct} \end{aligned}\right\} \Longrightarrow \Gamma_{\rm inv} = \Gamma + S_{\rm ct}$$

Invariant 1PI action: renormalization and regularization dependent

The explicit result (up to gauge-invariant terms) is:

$$\mathcal{L}_{\rm ct}\big|_{(1)} = \frac{\epsilon^{\mu\nu\alpha\beta}}{16\pi^2}\mathrm{Tr}\left\{\frac{8}{3}\partial_\mu \mathcal{V}_\nu \left\{\mathcal{V}_\alpha, \mathcal{A}_\beta\right\} + 4i\mathcal{V}_\mu\mathcal{V}_\nu\mathcal{V}_\alpha\mathcal{A}_\beta + \frac{4}{3}i\mathcal{V}_\mu\mathcal{A}_\nu\mathcal{A}_\alpha\mathcal{A}_\beta\right\}$$

$$+\frac{1}{16\pi^2}\mathrm{Tr}\left\{-\frac{4}{3}(D^{\mathcal{V}}_\mu\mathcal{A}_\nu)^2 + 2(D^{\mathcal{V}}_\mu\mathcal{A}^\mu)^2 - \frac{4}{3}[\mathcal{A}_\mu,\mathcal{A}_\nu]^2 + \frac{4}{3}(\mathcal{A}_\mu\mathcal{A}_\nu)^2 + \mathcal{A}^2_{\mu\nu}\right\}$$

$$-\frac{2}{16\pi^2}\left(1+\frac{\xi-1}{6}\right)G_{aa}\bar{f}\gamma_5\gamma^\mu T^a\mathcal{A}_\mu T^a f$$

Vanishes in the vector-like limit, as expected.
Invariant under spurious P & CP.

Cornella-Feruglio-LV (2022)

In the Standard Model:
— only electroweak vertices, no gluon couplings
— no terms with Levi-Civita, peculiarity of SU(2)xU(1)
— QCD & QED are vector-like and manifest

VVDD: $D_\mu W^-_\nu D^\mu W^{+\nu}$ $\quad$ $\partial_\mu Z_\nu \partial^\mu Z^\nu$

VVVD: $iF^{\mu\nu} W^+_\mu W^-_\nu$ $\quad$ $iD^\mu W^-_\mu W^+_\nu Z^\nu$ $\quad$ $iD^\nu W^-_\mu W^+_\nu Z^\mu$ $\quad$ $iD_\nu W^-_\mu W^{+\mu} Z^\nu$ $\quad$ $+\mathrm{hc}$

VVVV: $(W^-_\mu W^{+\mu})^2$ $\quad$ $(W^-_\mu W^{-\mu})(W^+_\nu W^{+\nu})$ $\quad$ $(Z_\mu Z^\mu)^2$ $\quad$ $(W^+_\mu Z^\mu)(W^-_\nu Z^\nu)$ $\quad$ $(W^+_\mu W^{-\mu})(Z_\nu Z^\nu)$

ffW: $W^+_\mu \overline{f_u} \gamma^\mu P_L f_d$ $\quad$ $W^+_\mu \overline{f_u} \gamma^\mu P_R f_d$ $\quad$ $+\mathrm{hc}$

ffZ: $Z_\mu \overline{f} \gamma^\mu P_L f$ $\quad$ $Z_\mu \overline{f} \gamma^\mu P_R f$ $\quad$ $+\mathrm{hc}$

Our calculation was performed in a specific scheme.

Scheme-independent mapping "Regularization Anomaly" $\Longrightarrow$ Counterterm

I_a^0	$\Box\partial^\mu A_{a\mu}$
I_{ab}^1	$\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha A_{a\mu})(\partial_\beta A_{b\nu})$
I_{ab}^2	$A_{a\mu}(\partial^\mu\partial^\nu - \Box g^{\mu\nu})A_{b\nu}$
I_{ab}^3	$A_{a\mu}\Box A_b^\mu$
I_{ab}^4	$(\partial_\nu A_{a\mu})(\partial^\nu A_b^\mu)$
I_{ab}^5	$(\partial_\nu A_{a\mu})(\partial^\mu A_b^\nu)$
I_{ab}^6	$(\partial^\mu A_{a\mu})(\partial^\nu A_{b\nu})$
I_{abd}^7	$(\partial_\mu A_a^\mu)A_{b\nu}A_d^\nu$
I_{abd}^8	$(\partial_\mu A_a^\nu)A_{b\mu}A_d^\nu$
I_{abd}^9	$\epsilon^{\mu\nu\alpha\beta}(\partial_\beta A_{a\mu})A_{b\nu}A_{d\alpha}$
I_{abde}^{10}	$A_{a\mu}A_b^\mu A_{d\nu}A_e^\nu$
I_{abde}^{11}	$\epsilon^{\mu\nu\rho\sigma}A_{a\mu}A_{b\nu}A_{d\rho}A_{e\sigma}$
I_{Xij}^{12}	$\bar{f}_{Xi}\overrightarrow{\partial\!\!\!/} f_{Xj}$
I_{Xij}^{13}	$\bar{f}_{Xi}\overleftarrow{\partial\!\!\!/} f_{Xj}$
I_{Xaij}^{14}	$\bar{f}_{Xi}A\!\!\!/_a f_{Xj}$

$\Longrightarrow$

$\mathcal{I}_{ghl}^1$	$(\partial^\nu A_g^\mu)A_{h\nu}A_{l\mu}$
$\mathcal{I}_{gh}^2$	$A_{g\mu}\Box A_h^\mu$
$\mathcal{I}_{gh}^3$	$A_{g\mu}\partial^\mu\partial^\nu A_{h\nu}$
$\mathcal{I}_{ghl}^4$	$\epsilon^{\mu\nu\rho\sigma}A_{g\mu}A_{h\nu}(\partial_\rho A_{l\sigma})$
$\mathcal{I}_{ghlm}^5$	$\epsilon^{\mu\nu\rho\sigma}A_{g\mu}A_{h\nu}\ A_{l\rho}A_{m\sigma}$
$\mathcal{I}_{ghlm}^6$	$A_{g\mu}A_h^\mu\ A_{l\nu}A_m^\nu$
$\mathcal{I}_{Xij}^7$	$\bar{f}_{Xi}\overrightarrow{\partial\!\!\!/} f_{Xj}$
$\mathcal{I}_{Xaij}^8$	$\bar{f}_{Xi}A\!\!\!/_a f_{Xj}$

Require 4-dim. Lorentz, vectorial gauge, spurious P&CP

Impose Wess-Zumino consistency conditions

Given the "regularization anomaly", the Counterterm is fixed up to gauge invariant terms.

Conclusions

☑ In concrete calculations, counterterms needed to restore gauge invariance

☑ **Explicit 1-loop in dim-reg with MS & BMHV for general fermionic reps**

(i) Useful for automation (rather than adding "by hand")
(ii) At the very least, non-trivial check of explicit calculations

☑ **General map anomaly → counterterm, valid for any regularization that**

(i) respects vectorial symmetry
(ii) 4-dimensional Lorentz
(iii) Spurionic CP and P

☐ **Extendable to Yukawa sector and SM-EFT**