Flavour of the SMEFT

Admir Greljo

AG, Thomsen, Palavric; 2203.09561

UNIVERSITÄT
BERN

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The SM

- Basic rules:

1. **Symmetries**: Spacetime & Gauge: Poincaré + $SU(3) \times SU(2) \times U(1)$

2. **Field Content** (d.o.f): 5 fermionic representations in *3 generations* + 1 scalar rep.

$$q_i, \ell_i, u_i, d_i, e_i \quad i = 1,2,3$$

3. **Renormalisability**

parameters:

- Gauge and Higgs sector: 5

- Yukawa sector: 13

*Would be 3 for a single generation

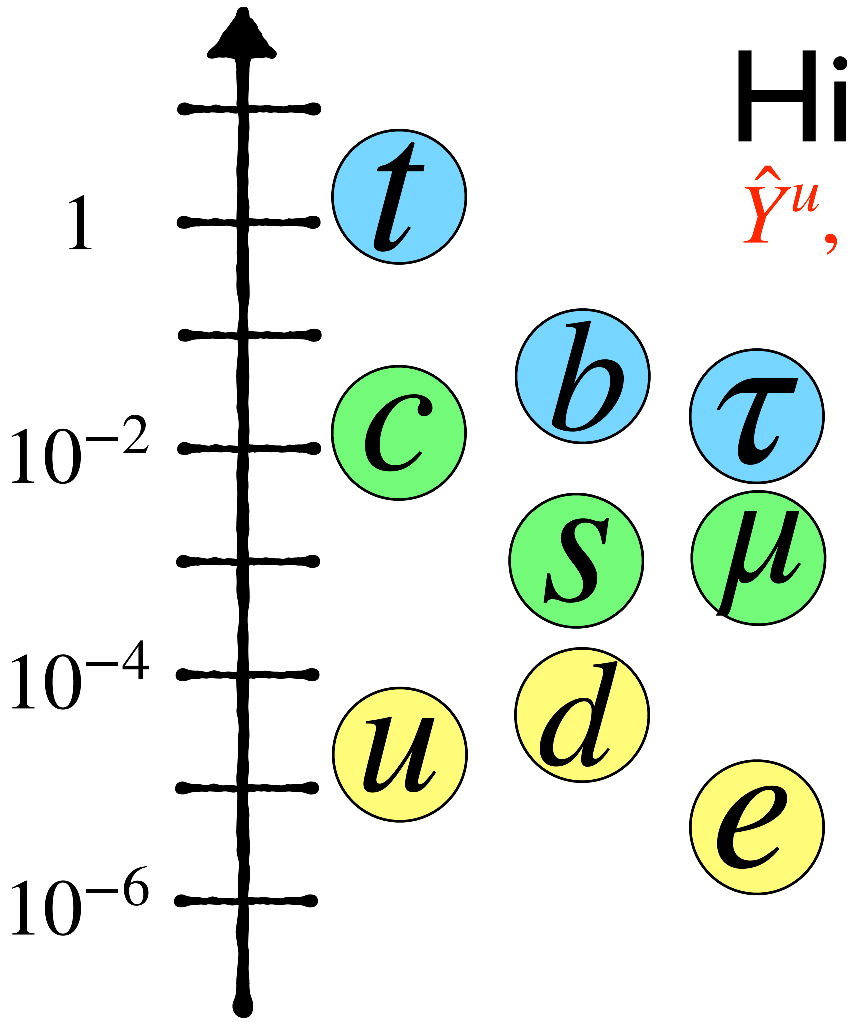
- The fermionic kinetic term has $U(3)^5$ global (flavour) symmetry
- Yukawa matrices explicitly break this down to $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$

The flavour puzzle

- Peculiar flavour patterns observed in $\mathbf{dim}[\mathcal{O}] = 4$ Yukawa interactions

$$-\mathscr{L}_{\text{Yuk}} = \bar{q} V^\dagger \hat{Y}^u \tilde{H} u + \bar{q} \hat{Y}^d H d + \bar{\ell} \hat{Y}^e H e$$

$[U(3)^5$ transformation and a singular value decomposition theorem$]$



Hierarchy

$\hat{Y}^u, \hat{Y}^d, \hat{Y}^e$

Alignment

Y^u / Y^d

$$V \sim \begin{bmatrix} 1 & 0.2 & 0.2^3 \\ 0.2 & 1 & 0.2^2 \\ 0.2^3 & 0.2^2 & 1 \end{bmatrix}$$

The CKM mixing

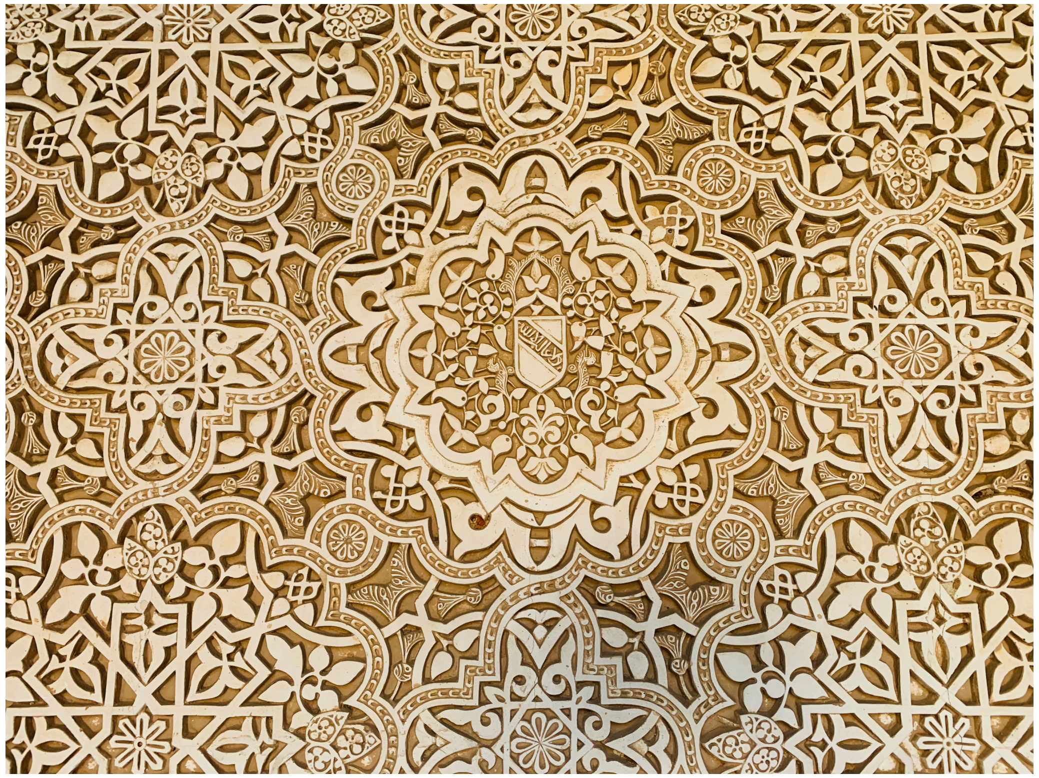
$\Longrightarrow$ No reason to expect the flavour anarchy in the $\mathbf{dim}[\mathcal{O}] = 6$

*We neglect neutrino masses and mixings in this work

Patterns indicate symmetries

- Peculiar flavour patterns observed in $\dim[\mathcal{O}] = 4$ Yukawa interactions
  $\Longrightarrow$ *Approximate flavour symmetries in the SM*

  (The largest parameter $y_t = Y^u_{33} \sim 1$ breaks $U(3)_q \times U(3)_u \to U(2)^2 \times U(1)$, etc…)



Alhambra of **Granada**

- Flavour symmetries are important to understand the phenomenology:
  isospin, SU(3), heavy-quark symmetries,
  suppressions in the FCNC, etc.

SMEFT

*optimally summarise the knowledge about the microscopic physics accessible at low-energies.

- Basic rules:

  1. **Symmetries**: Spacetime & Gauge: Poincaré + $SU(3) \times SU(2) \times U(1)$

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  $$q_i, \ell_i, u_i, d_i, e_i \quad i = 1,2,3$$

- Lagrangian(x) = infinite series

$$\mathscr{L} = \sum c_{\mathcal{O}} \Lambda_{\mathcal{O}}^{4-\dim \mathcal{O}} \mathcal{O}$$

Parameter

The cutoff scale

Local operator:
monomial in fields and derivatives

- Truncation at $\dim \mathcal{O} \leq N$
  $\Longrightarrow$ *finite number of parameters*

Physical effects $\sim \left(\frac{E}{\Lambda_{\mathcal{O}}}\right)^{\dim \mathcal{O}-4}$

SMEFT

- Challenge: ***a large number of independent parameters!***
- ***2499*** $\dim[\mathcal{O}] = 6$ ***operators*** ($\Delta B = \Delta L = 0$) Alonso et al; 1312.2014
- Why?
  FLAVOUR
- For a single generation, this would be 59

Charting the space of SMEFT with flavour symmetries

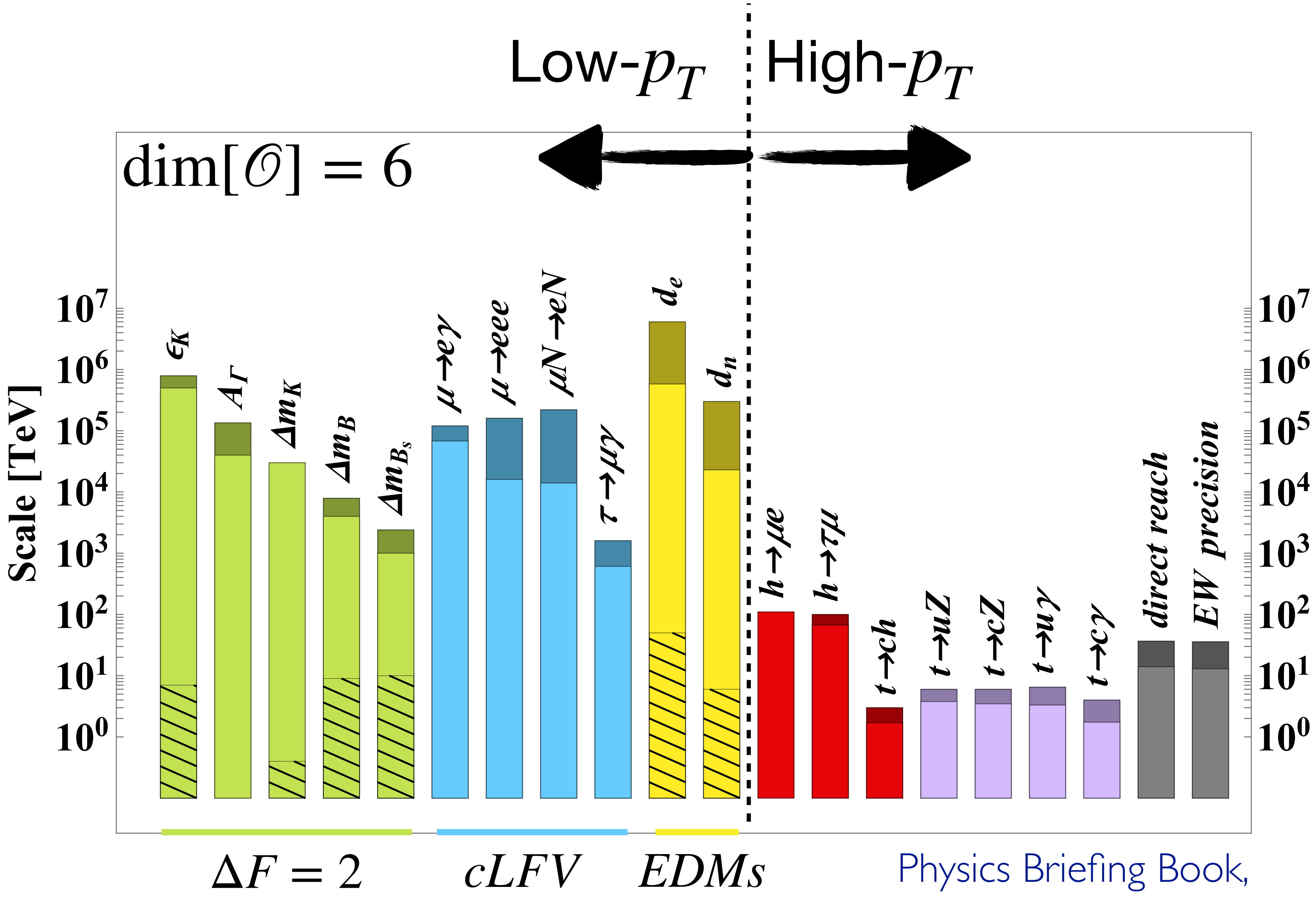
AG, Thomsen, Palavric; 2203.09561
see also Faroughy et al; 2005.05366

Organising principle:
extra global symmetries (flavour symmetries and breaking patterns) to reduce the number of independent parameters at $\dim[\mathcal{O}] = 6$.

Motivation:
- facilitate global SMEFT fits,
- chart the BSM space into universality classes.

Flavour physics constraints

- $\dim[\mathcal{O}] = 6$ - new sources of violation of approximate flavour symmetries.
- Strong constraints from flavour physics experiments!
- An anarchic flavour $\Longrightarrow$ no hope for deviations in Top/Higgs/EW



Physics Briefing Book, 1910.11775

Scope

AG, Thomsen, Palavric; 2203.09561

2 Quark Sector
- 2.1 $\mathrm{U}(2)^3$ symmetry
- 2.2 $\mathrm{U}(2)^3 \times \mathrm{U}(1)_{d_3}$ symmetry
- 2.3 $\mathrm{U}(2)^2 \times \mathrm{U}(3)_d$ symmetry
- 2.4 MFV_Q symmetry

3 Lepton Sector
- 3.1 $\mathrm{U}(1)^3$ vectorial symmetry
- 3.2 $\mathrm{U}(1)^6$ symmetry
- 3.3 $\mathrm{U}(2)$ vectorial symmetry
- 3.4 $\mathrm{U}(2)^2$ symmetry
- 3.5 $\mathrm{U}(2)^2 \times \mathrm{U}(1)^2$ symmetry
- 3.6 $\mathrm{U}(3)$ vectorial symmetry
- 3.7 MFV_L symmetry

C Mixed quark-lepton operators

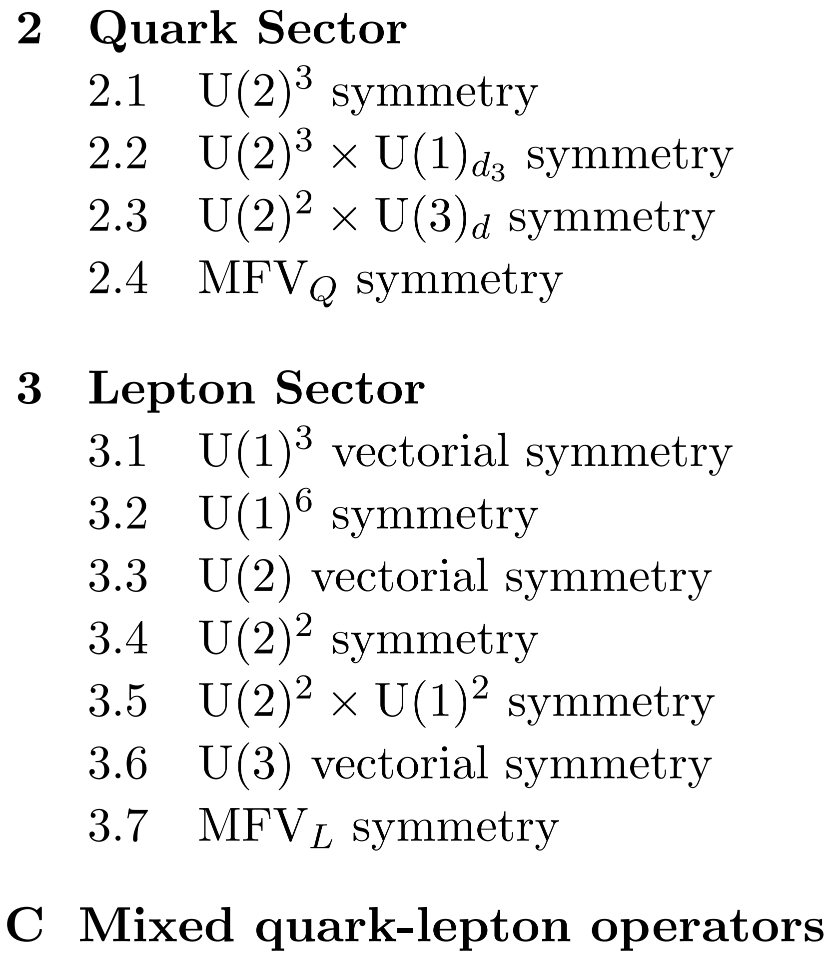
- 28 different hypotheses for a flavour symmetry + symmetry-breaking spurions*

  *A flavor spurion can be viewed as a non-dynamical (spurious) field transforming under a nontrivial representation of the flavor group and whose background value breaks the flavor symmetry.

- Flavor-breaking spurions needed to reproduce the observed charged fermion masses and mixings.
- Systematically moving from MFV towards anarchy
- Keep (some) suppression in FCNCs to allow for NP not far above the TeV scale (i.e. motivate global fits)

Scope

AG, Thomsen, Palavric; 2203.09561



- Work in the Warsaw basis
- Construct operator bases order by order in the spurion expansion.
- Operators without spurion insertion relevant for *Top/Higgs/EW physics*
- Operators with spurion insertion relevant for *Flavour physics*
- Flavour-symmetric operators will be "protected" from flavour physics constraints. Not a theorem!
- Systematic approach: $U(3) \supset U(2) \supset U(1)$ (smaller symmetry $\Longrightarrow$ more terms)

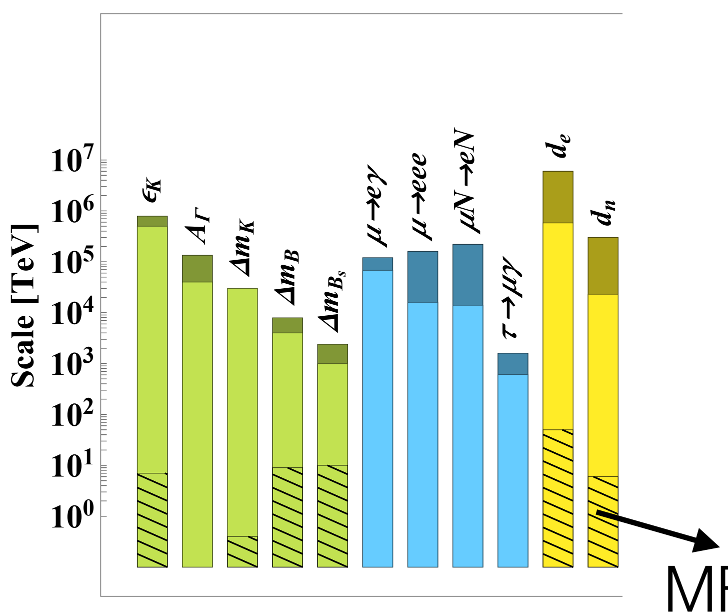
MFV

- The flavour breaking in the NP sector is also from the Yukawa matrices

$$G_Q = \mathrm{U}(3)_q \times \mathrm{U}(3)_u \times \mathrm{U}(3)_d$$

$$Y_u \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}), \quad Y_d \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}).$$

- The MFV allows for the NP cutoff as low as the TeV scale!



MFV

D'Ambrosio et al; hep-ph/0207036

MFV

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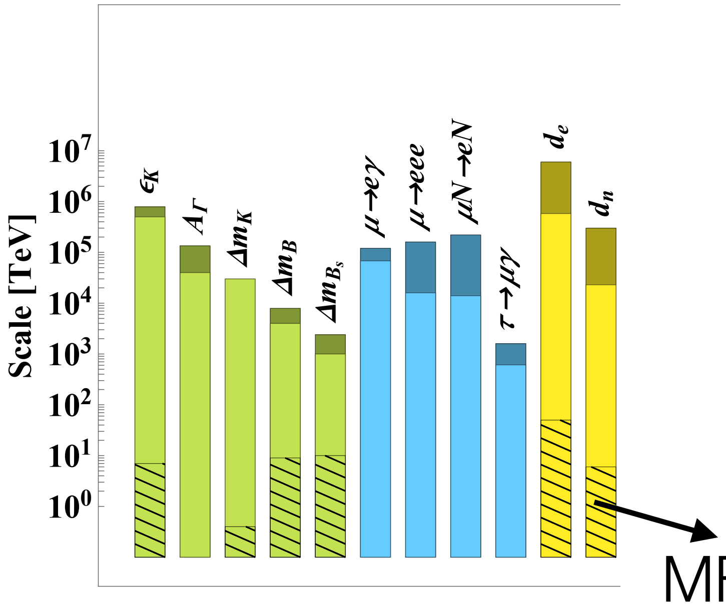


MFV

D'Ambrosio et al; hep-ph/0207036

U(2)

- Approximate symmetry of the SM
- Small breaking spurions (well-defined power counting)
- Also protects against dangerous FCNC but less restrictive than the MFV

$$G = \mathrm{U}(2)_q \times \mathrm{U}(2)_u \times \mathrm{U}(2)_d$$

$$V_q \sim (\mathbf{2}, \mathbf{1}, \mathbf{1})\ , \qquad \Delta_u \sim (\mathbf{2}, \bar{\mathbf{2}}, \mathbf{1})\ , \qquad \Delta_d \sim (\mathbf{2}, \mathbf{1}, \bar{\mathbf{2}})$$

Barbieri et at; 1105.2296

$$Y_{u,d} \sim \begin{pmatrix} \boxed{\Delta_{u,d}} & \boxed{V_q} \\ 0 \quad 0 & \textcircled{1} \end{pmatrix}$$

$$\Delta \ll V \ll 1 \qquad V^\dagger \propto (V_{td}, V_{ts})$$

$U(2)^3$ quark flavour

- Examples of bilinear structures

$\boxed{(\bar{q}q)}$

$$\begin{aligned}&\mathcal{O}(1):\ (\bar{q}q)\ ,\quad (\bar{q}_3q_3)\ ,\qquad \mathcal{O}(V):\ (\bar{q}V_qq_3)\ ,\quad {V_q}^a\varepsilon_{ab}(\bar{q}_3q^b)\ ,\quad \text{H.c.}\ ,\\ &\mathcal{O}\big(V^2\big):\ (\bar{q}V_qV_q^\dagger q)\ ,\quad \Big[\epsilon_{bc}(\bar{q}V_qV_q^cq^b)\ ,\quad \text{H.c.}\Big]\ .\end{aligned}\tag{2.12}$$

$\boxed{(\bar{u}u)}$

$$\begin{aligned}\mathcal{O}(1):&\quad (\bar{u}u)\ ,\quad (\bar{u}_3u_3)\ ,\\ \mathcal{O}(\Delta V):&\quad (\bar{u}\Delta_u^\dagger V_qu_3)\ ,\quad (\bar{u}_au_3)\varepsilon^{ab}(V_q^\dagger\Delta_u)_b\ ,\quad \epsilon^{ad}\epsilon_{bc}[\bar{u}^aV_q^b(\Delta_u)^c{}_du_3]\ ,\quad \text{H.c.}\ ,\\ &\quad \epsilon_{bc}[\bar{u}_3V_q^b(\Delta_u)^c{}_au^a]\ ,\quad \text{H.c.}\ .\end{aligned}\tag{2.13}$$

$\boxed{(\bar{d}d)}$

$$\begin{aligned}\mathcal{O}(1):&\quad (\bar{d}d)\ ,\quad (\bar{d}_3d_3)\ ,\\ \mathcal{O}(\Delta V):&\quad (\bar{d}\Delta_d^\dagger V_qd_3)\ ,\quad (\bar{d}_ad_3)\varepsilon^{ab}(V_q^\dagger\Delta_d)_b\ ,\quad \epsilon^{ad}\epsilon_{bc}[\bar{d}^aV_q^b(\Delta_d)^c{}_dd_3]\ ,\quad \text{H.c.}\ ,\\ &\quad \epsilon_{bc}[\bar{d}_3V_q^b(\Delta_d)^c{}_ad^a]\ ,\quad \text{H.c.}\ .\end{aligned}\tag{2.14}$$

*the new structures that appear in case of SU(2)^3 symmetry are denoted in blue

Watch out redundancies
$\varepsilon^{ij}\varepsilon_{k\ell}={\delta^i}_\ell{\delta^j}_k-{\delta^i}_k{\delta^j}_\ell$

- Examples of quartic structures

$\boxed{(\bar{q}q)(\bar{q}q)}$

$$\begin{aligned}\mathcal{O}(1):&\quad (\bar{q}_aq^b)(\bar{q}_bq^a)\ ,\quad (\bar{q}_aq_3)(\bar{q}_3q^a)\ ,\\ \mathcal{O}(V):&\quad (\bar{q}_aq_3)(\bar{q}V_qq^a)\ ,\quad (\bar{q}_3q^a)(\bar{q}_a\epsilon_{bc}V_q^cq^b)\ ,\quad (\bar{q}_3q^a)(\bar{q}V_q\epsilon_{ac}q^c)\ ,\quad \text{H.c.}\ ,\\ \mathcal{O}\big(V^2\big):&\ (\bar{q}_aV_q^\dagger q)(\bar{q}V_qq^a)\ .\end{aligned}\tag{2.18}$$

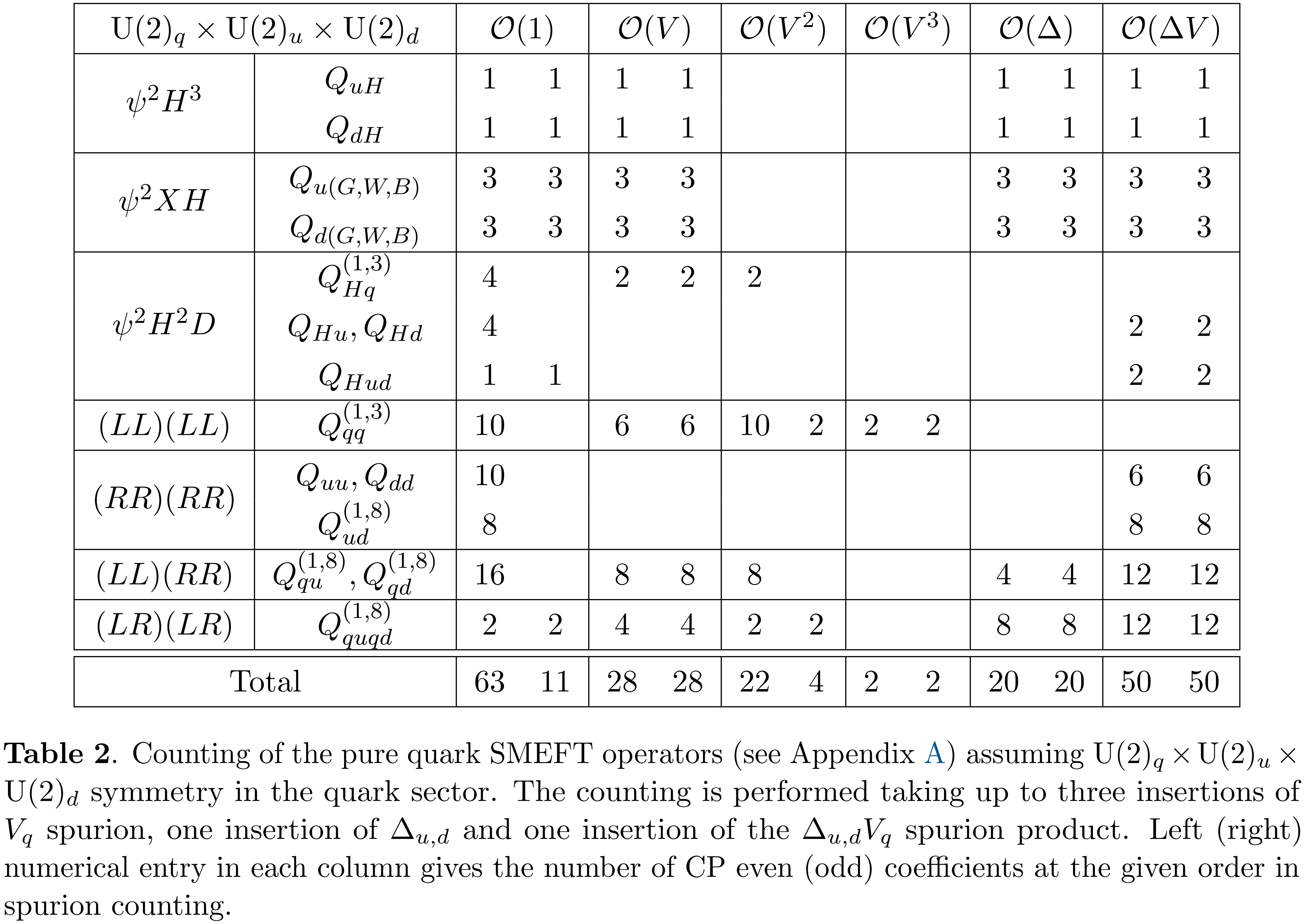
$\boxed{(\bar{u}u)(\bar{u}u)}$

$$\begin{aligned}\mathcal{O}(1):&\quad (\bar{u}_au^b)(\bar{u}_bu^a)\ ,\quad (\bar{u}_au_3)(\bar{u}_3u^a)\ ,\\ \mathcal{O}(\Delta V):&\ (\bar{u}_au_3)(\bar{u}\Delta_u^\dagger V_qu^a)\ ,\quad (\bar{u}_au_3)\epsilon^{ab}\epsilon_{de}[\bar{u}_bV_q^d(\Delta_u)^e{}_cu^c]\ ,\quad \epsilon^{be}\epsilon_{cd}(\bar{u}_au_3)[\bar{u}_bV_q^c(\Delta_u)^d{}_eu^a]\ ,\quad \text{H.c.}\ ,\\ &\ (\bar{u}_3u^a)[\bar{u}_aV_q^c\epsilon_{cd}(\Delta_u)^d{}_bu^b]\ ,\quad (\bar{u}_3u^a)[\bar{u}_a\epsilon_{bd}V_q^c(\Delta_u^*)_c{}^du^b]\ ,\quad \epsilon_{ac}(\bar{u}_3u^a)[\bar{u}_bV_q^d(\Delta_u^*)_d{}^bu^c]\ ,\quad \text{H.c.}\ .\end{aligned}\tag{2.19}$$

$\boxed{(\bar{d}d)(\bar{d}d)}$

$$\begin{aligned}\mathcal{O}(1):&\quad (\bar{d}_ad^b)(\bar{d}_bd^a)\ ,\quad (\bar{d}_ad_3)(\bar{d}_3d^a)\ ,\\ \mathcal{O}(\Delta V):&\ (\bar{d}_ad_3)(\bar{d}\Delta_d^\dagger V_qd^a)\ ,\quad (\bar{d}_ad_3)\epsilon^{ab}\epsilon_{de}[\bar{d}_bV_q^d(\Delta_d)^e{}_cd^c]\ ,\quad \epsilon^{be}\epsilon_{cd}(\bar{d}_ad_3)[\bar{d}_bV_q^c(\Delta_d)^d{}_ed^a]\ ,\quad \text{H.c.}\ ,\\ &\ (\bar{d}_3d^a)[\bar{d}_aV_q^c\epsilon_{cd}(\Delta_d)^d{}_bd^b]\ ,\quad (\bar{d}_3d^a)[\bar{d}_a\epsilon_{bd}V_q^c(\Delta_d^*)_c{}^dd^b]\ ,\quad \epsilon_{ac}(\bar{d}_3d^a)[\bar{d}_bV_q^d(\Delta_d^*)_d{}^bd^c]\ ,\quad \text{H.c.}\ .\end{aligned}\tag{2.20}$$

$U(2)^3$ quark flavour



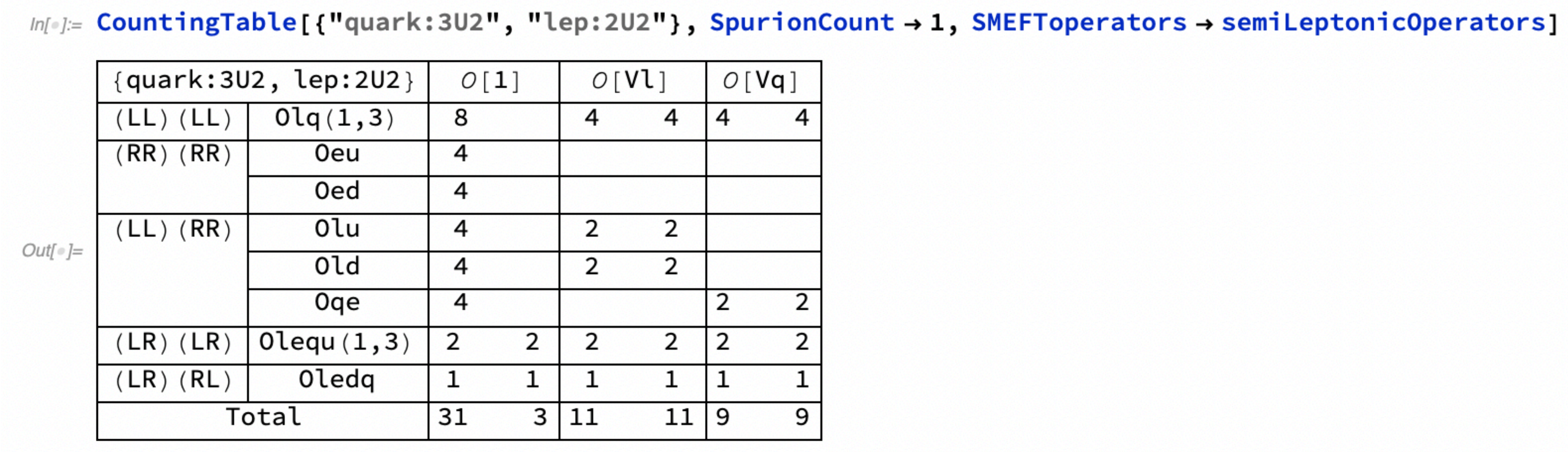
$U(2)_q \times U(2)_u \times U(2)_d$		$\mathcal{O}(1)$		$\mathcal{O}(V)$		$\mathcal{O}(V^2)$		$\mathcal{O}(V^3)$		$\mathcal{O}(\Delta)$		$\mathcal{O}(\Delta V)$	
$\psi^2 H^3$	Q_{uH}	1	1	1	1					1	1	1	1
	Q_{dH}	1	1	1	1					1	1	1	1
$\psi^2 XH$	$Q_{u(G,W,B)}$	3	3	3	3					3	3	3	3
	$Q_{d(G,W,B)}$	3	3	3	3					3	3	3	3
$\psi^2 H^2 D$	$Q_{Hq}^{(1,3)}$	4		2	2	2							
	Q_{Hu}, Q_{Hd}	4										2	2
	Q_{Hud}	1	1									2	2
$(LL)(LL)$	$Q_{qq}^{(1,3)}$	10		6	6	10	2	2	2				
$(RR)(RR)$	Q_{uu}, Q_{dd}	10										6	6
	$Q_{ud}^{(1,8)}$	8										8	8
$(LL)(RR)$	$Q_{qu}^{(1,8)}, Q_{qd}^{(1,8)}$	16		8	8	8				4	4	12	12
$(LR)(LR)$	$Q_{quqd}^{(1,8)}$	2	2	4	4	2	2			8	8	12	12
Total		63	11	28	28	22	4	2	2	20	20	50	50

Table 2. Counting of the pure quark SMEFT operators (see Appendix A) assuming $U(2)_q \times U(2)_u \times U(2)_d$ symmetry in the quark sector. The counting is performed taking up to three insertions of V_q spurion, one insertion of $\Delta_{u,d}$ and one insertion of the $\Delta_{u,d} V_q$ spurion product. Left (right) numerical entry in each column gives the number of CP even (odd) coefficients at the given order in spurion counting.

Tools

- Mathematica package **SMEFTflavor** to facilitate the use of flavor symmetries

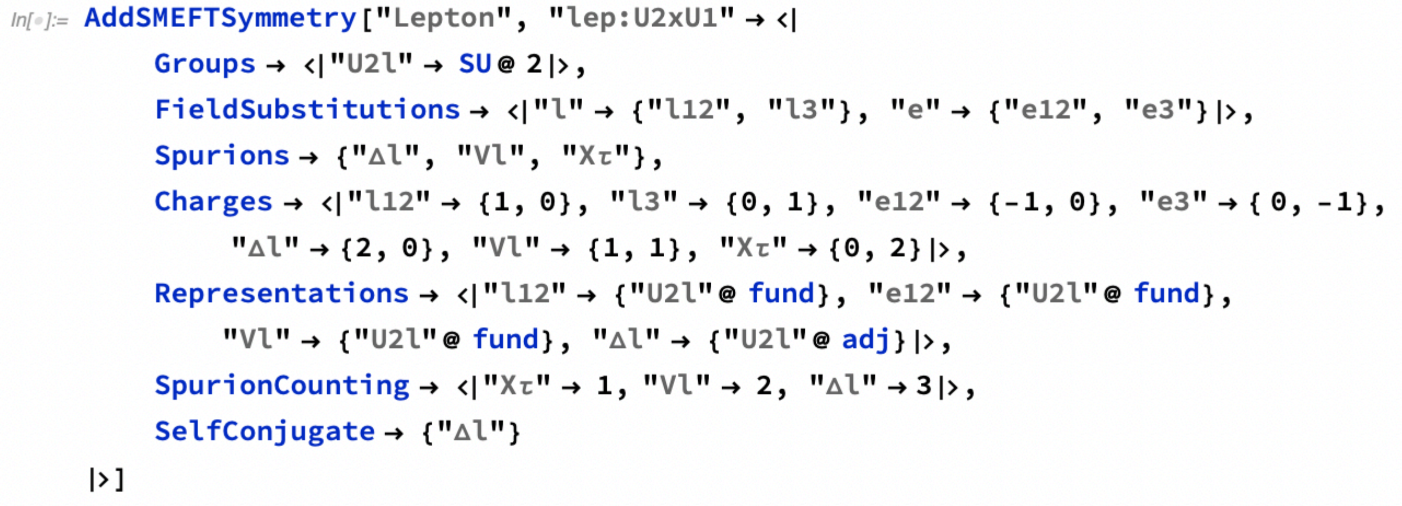
https://github.com/aethomsen/SMEFTflavor



```
In[∘]:= CountingTable[{"quark:3U2", "lep:2U2"}, SpurionCount → 1, SMEFToperators → semiLeptonicOperators]
```

Out[∘]=

{quark:3U2, lep:2U2}		O[1]		O[Vl]		O[Vq]	
(LL)(LL)	Olq(1,3)	8		4	4	4	4
(RR)(RR)	Oeu	4					
	Oed	4					
(LL)(RR)	Olu	4		2	2		
	Old	4		2	2		
	Oqe	4				2	2
(LR)(LR)	Olequ(1,3)	2	2	2	2	2	2
(LR)(RL)	Oledq	1	1	1	1	1	1
Total		31	3	11	11	9	9

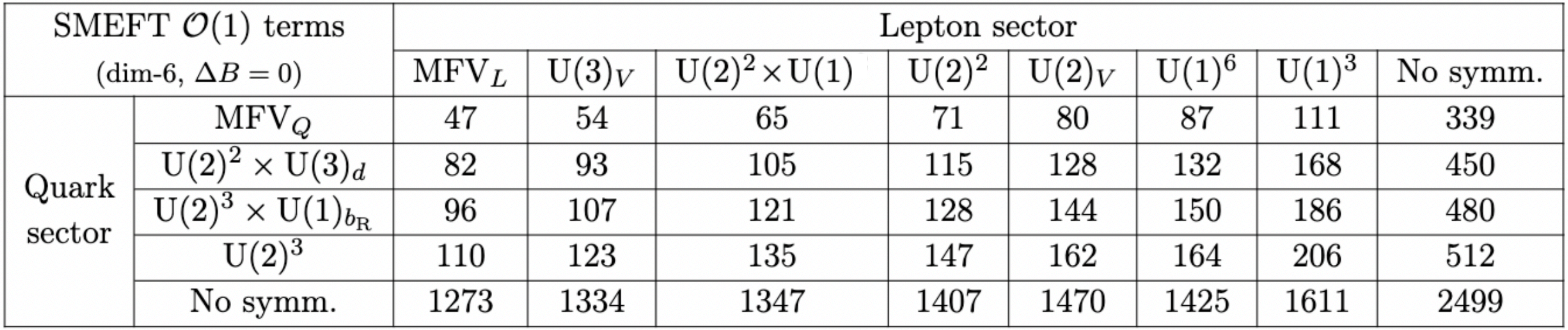


```
In[∘]:= AddSMEFTSymmetry["Lepton", "lep:U2xU1" → <|
          Groups → <|"U2l" → SU@2|>,
          FieldSubstitutions → <|"l" → {"l12", "l3"}, "e" → {"e12", "e3"}|>,
          Spurions → {"Δl", "Vl", "Xτ"},
          Charges → <|"l12" → {1, 0}, "l3" → {0, 1}, "e12" → {-1, 0}, "e3" → { 0, -1},
              "Δl" → {2, 0}, "Vl" → {1, 1}, "Xτ" → {0, 2}|>,
          Representations → <|"l12" → {"U2l"@ fund}, "e12" → {"U2l"@ fund},
              "Vl" → {"U2l"@ fund}, "Δl" → {"U2l"@ adj}|>,
          SpurionCounting → <|"Xτ" → 1, "Vl" → 2, "Δl" → 3|>,
          SelfConjugate → {"Δl"}
        |>]
```

AG, Thomsen, Palavric; 2203.09561

Flavour-symmetric bases: Summary

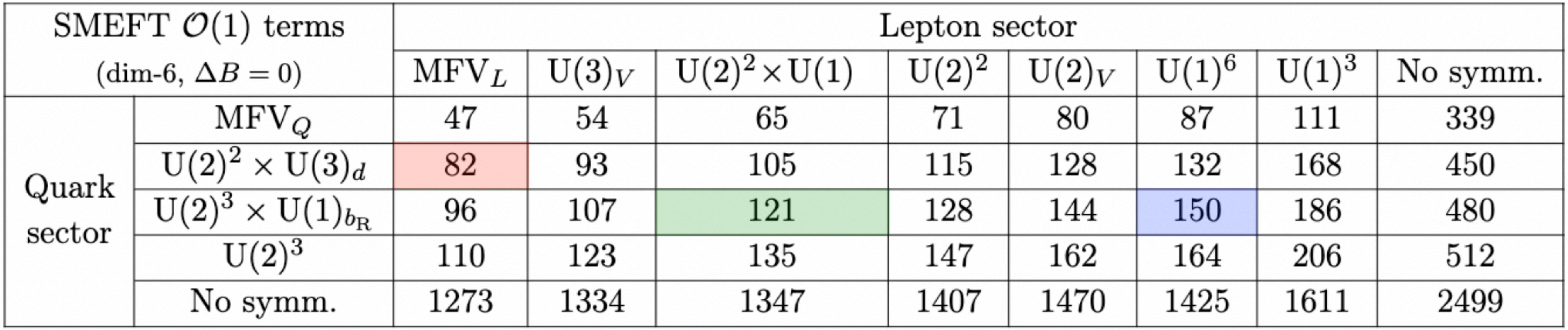
AG, Thomsen, Palavric; 2203.09561



SMEFT $\mathcal{O}(1)$ terms (dim-6, $\Delta B = 0$)		Lepton sector							
		MFV_L	$\mathrm{U}(3)_V$	$\mathrm{U}(2)^2 \times \mathrm{U}(1)$	$\mathrm{U}(2)^2$	$\mathrm{U}(2)_V$	$\mathrm{U}(1)^6$	$\mathrm{U}(1)^3$	No symm.
Quark sector	MFV_Q	47	54	65	71	80	87	111	339
	$\mathrm{U}(2)^2 \times \mathrm{U}(3)_d$	82	93	105	115	128	132	168	450
	$\mathrm{U}(2)^3 \times \mathrm{U}(1)_{b_\mathrm{R}}$	96	107	121	128	144	150	186	480
	$\mathrm{U}(2)^3$	110	123	135	147	162	164	206	512
	No symm.	1273	1334	1347	1407	1470	1425	1611	2499

*Terms relevant for the high-p_T physics: Top, Higgs & electroweak

Some comments



SMEFT $\mathcal{O}(1)$ terms (dim-6, $\Delta B = 0$)		Lepton sector							
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	No symm.	1273	1334	1347	1407	1470	1425	1611	2499

🟥	MFV with all breakings neglected apart from y_t. Radiatively stable (approximate symmetry of $\dim[\mathcal{O}] = 4$)
🟩	Third-family specific. Discriminates t and b from light jets, and τ from μ/e (experimentally possible).
🟦	Allows for LFUV between e and μ which is experimentally accessible.

- Moving away from the MFV, many four-fermion operators!
- Add high-mass Drell-Yan and multi-jet to the EFT program
- Interplay with flavour physics

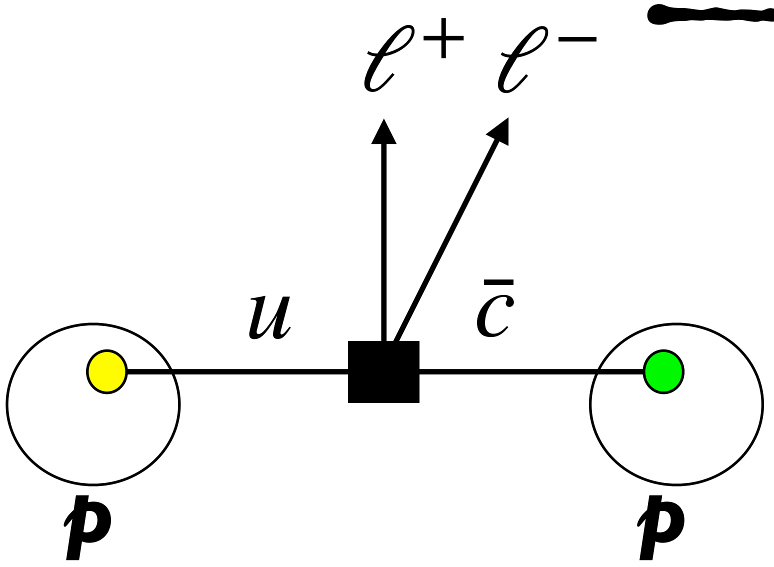
*see also the LHC EFT WG

Flavour Physics in High-p_T Tails

Example: Rare $c \to u\ell^+\ell^-$ decays

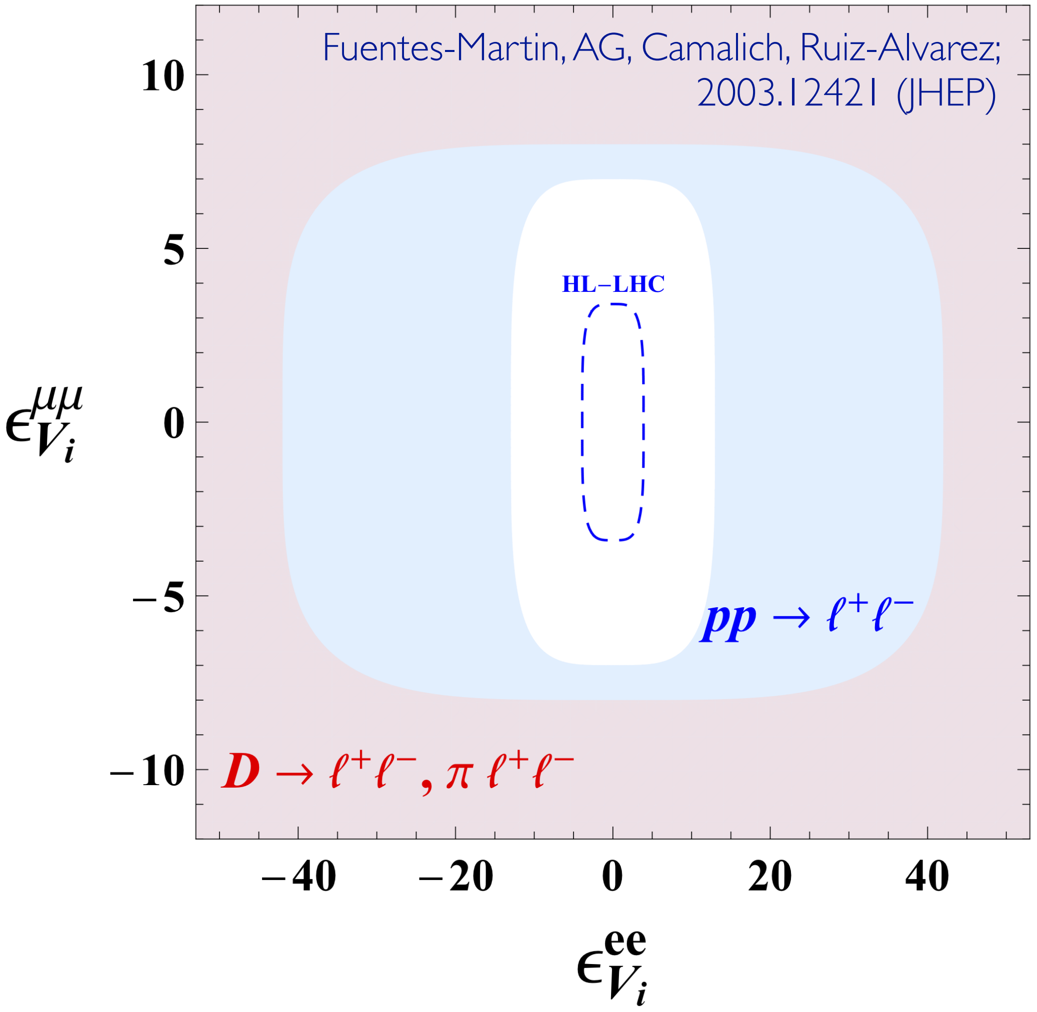
- Tiny SM rates: $BR(D^0 \to \mu^+\mu^-) \sim \mathcal{O}(10^{-13})$
  short-distance contribution negligible (efficient GIM suppression), long-distance dominated
- Already strong experimental upper limits: $BR(D^0 \to \mu^+\mu^-) \lesssim 6 \times 10^{-9}$ LHCb, 1305.5059
- **Null test of the SM** sensitive to New Physics

$$\mathcal{L}_{NP} \approx \frac{\epsilon_V^{\ell\ell}}{(15\,\mathrm{TeV})^2}(\bar{u}_R\gamma^\mu c_R)(\bar{\ell}_R\gamma^\mu\ell_R)$$



Implementation
in Flavio & Smelli
AG, Salko, Smolkovic,
Stangl; w.i.p

See talk by Stangl
See talk by Wilsch
See talk by Madigan


Fuentes-Martin, AG, Camalich, Ruiz-Alvarez; 2003.12421 (JHEP)

Conclusions

- A UV theory will leave imprints on the flavor structure of the SMEFT.
- The selection rules implied have the advantage of reducing the number of important SMEFT operators by truncating the flavor-spurion expansion.
- Global flavor symmetries and their breaking patterns, thus, provide a good organizing principle for the SMEFT, mapping the space of theories beyond the SM into universality classes.
- We constructed operator bases order by order in the spurion expansion for 28 different flavour symmetries supporting TeV-scale NP.
- Ready-for-use setups for phenomenological studies and global fits.

AG, Thomsen, Palavric; 2203.09561

https://github.com/aethomsen/SMEFTflavor

The End

Alhambra of **Granada**

A Warsaw basis

Here we list the $\Delta B = 0$ dimension-6 fermionic SMEFT operators in the Warsaw basis [13] with division into classes as presented in [14].

5–7: Fermion Bilinears

non-hermitian $(\bar{L}R)$							
5: $\psi^2 H^3$		6: $\psi^2 XH$					
Q_{eH}	$(H^\dagger H)(\bar{\ell}_p e_r H)$	Q_{eW}	$(\bar{\ell}_p \sigma^{\mu\nu} e_r)\tau^I H W^I_{\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\tilde{H} G^A_{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G^A_{\mu\nu}$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$	Q_{eB}	$(\bar{\ell}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \tilde{H} W^I_{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H W^I_{\mu\nu}$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$			Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tilde{H} B_{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$

hermitian $(+\, Q_{Hud})$ $\sim$ 7: $\psi^2 H^2 D$					
$(\bar{L}L)$		$(\bar{R}R)$		$(\bar{R}R')$	
$Q^{(1)}_{H\ell}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\ell}_p \gamma^\mu \ell_r)$	Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$	Q_{Hud}	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$
$Q^{(3)}_{H\ell}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{\ell}_p \tau^I \gamma^\mu \ell_r)$	Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$		
$Q^{(1)}_{Hq}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$	Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$		
$Q^{(3)}_{Hq}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r)$				

8: Fermion Quadrilinears

hermitian					
$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$Q_{\ell\ell}$	$(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{\ell}_s \gamma^\mu \ell_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	$Q_{\ell e}$	$(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{e}_s \gamma^\mu e_t)$
$Q^{(1)}_{qq}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	$Q_{\ell u}$	$(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{u}_s \gamma^\mu u_t)$
$Q^{(3)}_{qq}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{\ell d}$	$(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{d}_s \gamma^\mu d_t)$
$Q^{(1)}_{\ell q}$	$(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q^{(3)}_{\ell q}$	$(\bar{\ell}_p \gamma_\mu \tau^I \ell_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q^{(1)}_{qu}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q^{(1)}_{ud}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q^{(8)}_{qu}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q^{(8)}_{ud}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q^{(1)}_{qd}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$Q^{(8)}_{qd}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$

non-hermitian			
$(\bar{L}R)(\bar{R}L)$		$(\bar{L}R)(\bar{L}R)$	
$Q_{\ell edq}$	$(\bar{\ell}^j_p e_r)(\bar{d}_s q_{tj})$	$Q^{(1)}_{quqd}$	$(\bar{q}^j_p u_r)\epsilon_{jk}(\bar{q}^k_s d_t)$
		$Q^{(8)}_{quqd}$	$(\bar{q}^j_p T^A u_r)\epsilon_{jk}(\bar{q}^k_s T^A d_t)$
		$Q^{(1)}_{\ell equ}$	$(\bar{\ell}^j_p e_r)\epsilon_{jk}(\bar{q}^k_s u_t)$
		$Q^{(3)}_{\ell equ}$	$(\bar{\ell}^j_p \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t)$