Moments for positivity

Using Drell-Yan data to test positivity bounds & reverse-engineer new physics

X. Li, KM, K. Yamashita, C. Yang, C. Zhang, S.-Y. Zhou; arXiv:2204.13121

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Outline

Drell-Yan (DY) as a probe for SMEFT at the LHC

- Beyond dim-6: higher ($l \geq 3$) angular moments

Positivity bounds

- Elastic positivity ↔ higher moments in DY
- Beyond elastic: the positivity cone

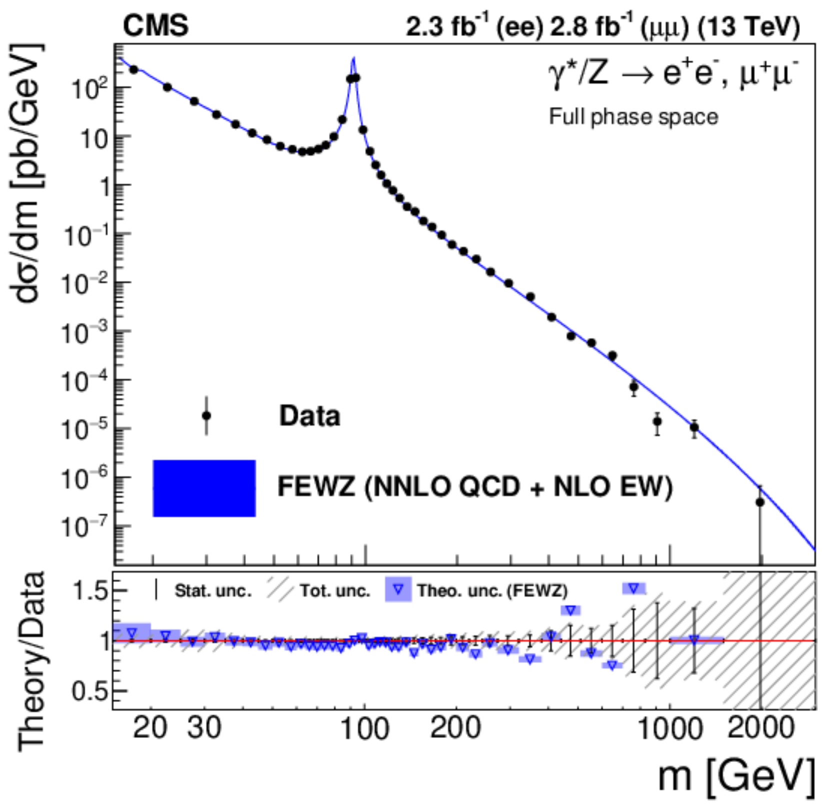
LHC sensitivity study to higher moment observables

- Testing positivity (fundamental principles of QFT) at colliders

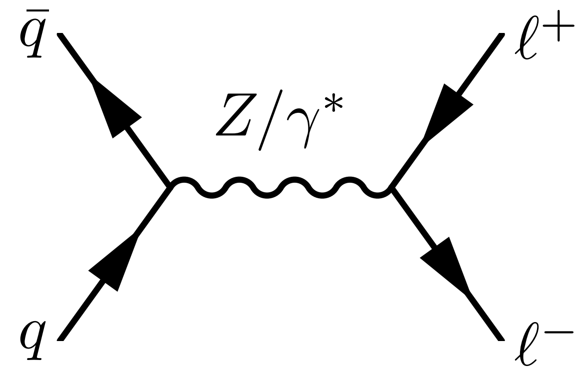
Combining experimental bounds with positivity

- Interesting bounds on new physics scale
- Inferring the existence/nature of UV states

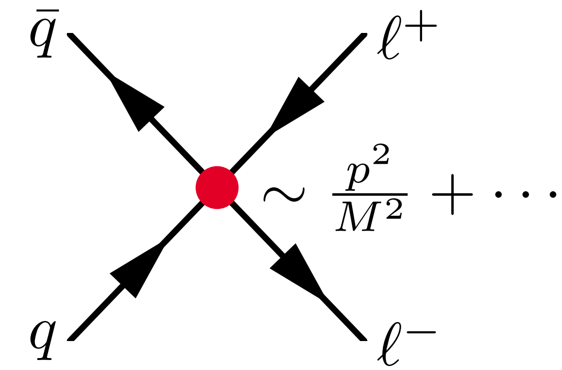
Drell Yan at the LHC



$pp \to \ell^+\ell^-$ at high mass



SM



EFT

Clean, high-energy probe of 2 → 2 scattering

- Strong bounds on new resonances
- Sensitive to energy-growing contact interactions: 4F operators
- Complete reconstruction of final state: fully differential cross section

Angular dependence

$$\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}d\Omega_\ell}=\frac{3}{16\pi}\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}}\Bigg[(1+c_\theta^2)+\frac{\tilde{A}_0}{2}(1-3c_\theta^2)+\tilde{A}_1 s_{2\theta}c_\phi$$
$$+\frac{\tilde{A}_2}{2}s_\theta^2 c_{2\phi}+\tilde{A}_3 s_\theta c_\phi+\tilde{A}_4 c_\theta+\tilde{A}_5 s_\theta^2 s_{2\phi}+\tilde{A}_6 s_{2\theta}s_\phi+\tilde{A}_7 s_\theta s_\phi\Bigg]$$

Factorised leptonic angular dependence

- Polar (θ) & azimuthal (ϕ) scattering angle in the Collins Soper frame
- $\ell^+\ell^-$ rest frame where $\hat{z}$ bisects the angle between proton beams
- LO: $\hat{z}$ = longitudinal direction of the $\ell^+\ell^-$ system

$$\cos\theta_{CS}=\frac{p^z_{\ell^-}E_{\ell^+}-p^z_{\ell^+}E_{\ell^-}}{m_{\ell\ell}\sqrt{(m_{\ell\ell})^2+(p^T_{\ell\ell})^2}}$$

SM: dependence holds to all orders in QCD

- Spin-1 photon & Z-boson $\to l\leq 2$ angular dependence
- LO is ϕ symmetric: $\tilde{A}_{1,4}\neq 0$, NLO: $\tilde{A}_{1-7}\neq 0$

Angular dependence

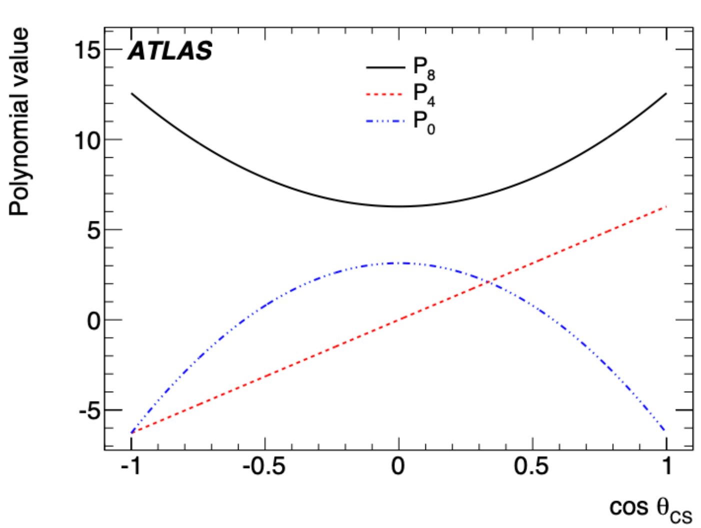
Extracting the $\tilde{A}_i$: moments of spherical harmonics *

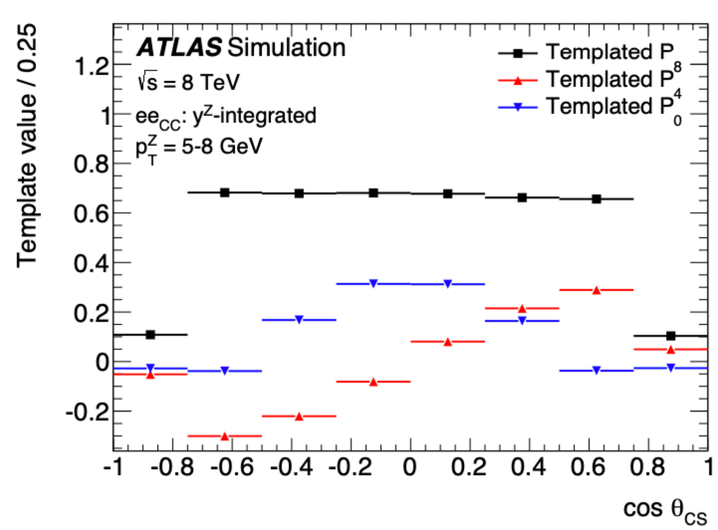
$$\langle f(\theta,\phi)\rangle \equiv \left(\frac{d\sigma}{dm\,d\eta\,d\Omega}\right)^{-1}\int d\Omega_\ell \,\frac{d\sigma}{dm\,d\eta\,d\Omega}\, f(\theta,\phi) \qquad f(\theta,\phi)\propto \left\{Y_{0,0}, Y_{1,0}, Y_{1,\pm1}, Y_{2,0}, Y_{2,\pm1}, Y_{2,\pm2}\right\}$$

- A.K.A. weighted sum of the basis functions over event sample
- $\tilde{A}_i$'s are linear functions of the $\langle Y_{l,m}\rangle$

* In practice, finite experimental acceptance

- Spoils the orthonormality of spherical harmonics

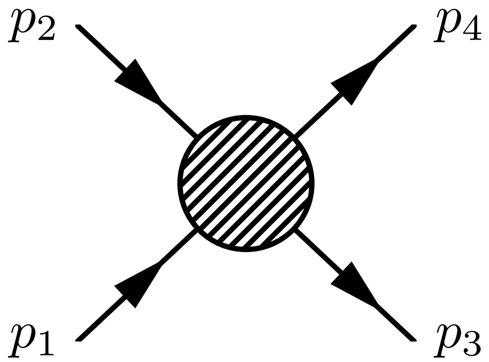




Extracted by fit to signal templates

[CMS; PLB 750 (2015) 154-175]
[ATLAS; JHEP 08 (2016) 159]

New angular dependence



$\rightarrow \mathscr{A}(s,t) \qquad \cos\theta \sim 1 - \frac{2t}{s}$

$\mathscr{A}_{SM}$: spin-1 $\rightarrow \propto \cos\theta \sim \frac{t}{s}$

- Differential cross section $|\mathscr{A}|^2 \sim t, t^2$: $Y_{l\leq 2,m}$
- QCD corrections factorise, $l \leq 2$ unchanged
- Leading higher l contribution: NLL EW Sudakov $\sim \frac{\alpha^2}{16\pi^2}\log\frac{t}{m_W^2}$

$\mathscr{A}_{BSM}$: new Lorentz structures

- Higher spin states or contact interactions (4F operators)

Dim 6

$\mathscr{A} \sim s, t \Rightarrow |\mathscr{A}|^2 : \; l \leq 2$

Dim 8

$\mathscr{A} \sim s^2, t^2 \Rightarrow \mathscr{A}_{SM}\mathscr{A}_{EFT} : \; l \leq 3$

[Alioli et al.; PLB 809 (2020) 135703]
See also:
[Banerjee et al.; JHEP 09 (2020) 170
& JHEP 06 (2021) 013]

Higher moments

$$\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}d\Omega_\ell}=\frac{3}{16\pi}\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}}\Bigg[(1+c_\theta^2)+\frac{\tilde{A}_0}{2}(1-3c_\theta^2)+\tilde{A}_1 s_{2\theta}c_\phi$$

$l \leq 2$

$$+\frac{\tilde{A}_2}{2}s_\theta^2 c_{2\phi}+\tilde{A}_3 s_\theta c_\phi+\tilde{A}_4 c_\theta+\tilde{A}_5 s_\theta^2 s_{2\phi}+\tilde{A}_6 s_{2\theta}s_\phi+\tilde{A}_7 s_\theta s_\phi$$

$l = 3$

$$+\frac{\tilde{B}_1^e}{2}s_\theta\left(5c_\theta^2-1\right)c_\phi+\frac{\tilde{B}_1^o}{2}s_\theta\left(5c_\theta^2-1\right)s_\phi+\boxed{\frac{\tilde{B}_0}{2}\left(5c_\theta^3-3c_\theta\right)}$$

$$+\tilde{B}_3^e s_\theta^3 c_{3\phi}+\tilde{B}_3^o s_\theta^3 s_{3\phi}+\tilde{B}_2^e s_\theta^2 c_\theta c_{2\phi}+\tilde{B}_2^o s_\theta^2 c_\theta s_{2\phi}\Bigg]$$

$\tilde{B}_i$ coefficients: $Y_{3,m}$ spherical harmonics

- Only populated by certain class of dim-8 4F operators $\mathcal{A}_{SM}\mathcal{A}_{EFT}\sim\cos^3\theta$
- At LO, no SM or dim-6 contribution
- Dominant moment $\tilde{B}_0$: $Y_{3,0}$ spherical harmonic (no ϕ dependence)
- Clean probe of dim-8 effects in Drell Yan

Which operators

Relevant dim-8 operators: $\mathscr{A}(q\bar{q} \to \ell^+\ell^-) \sim t^2$

- Two-derivative 4F operators, $\psi^4 D^2$
- No additional Higgs fields (powers of E, not v)

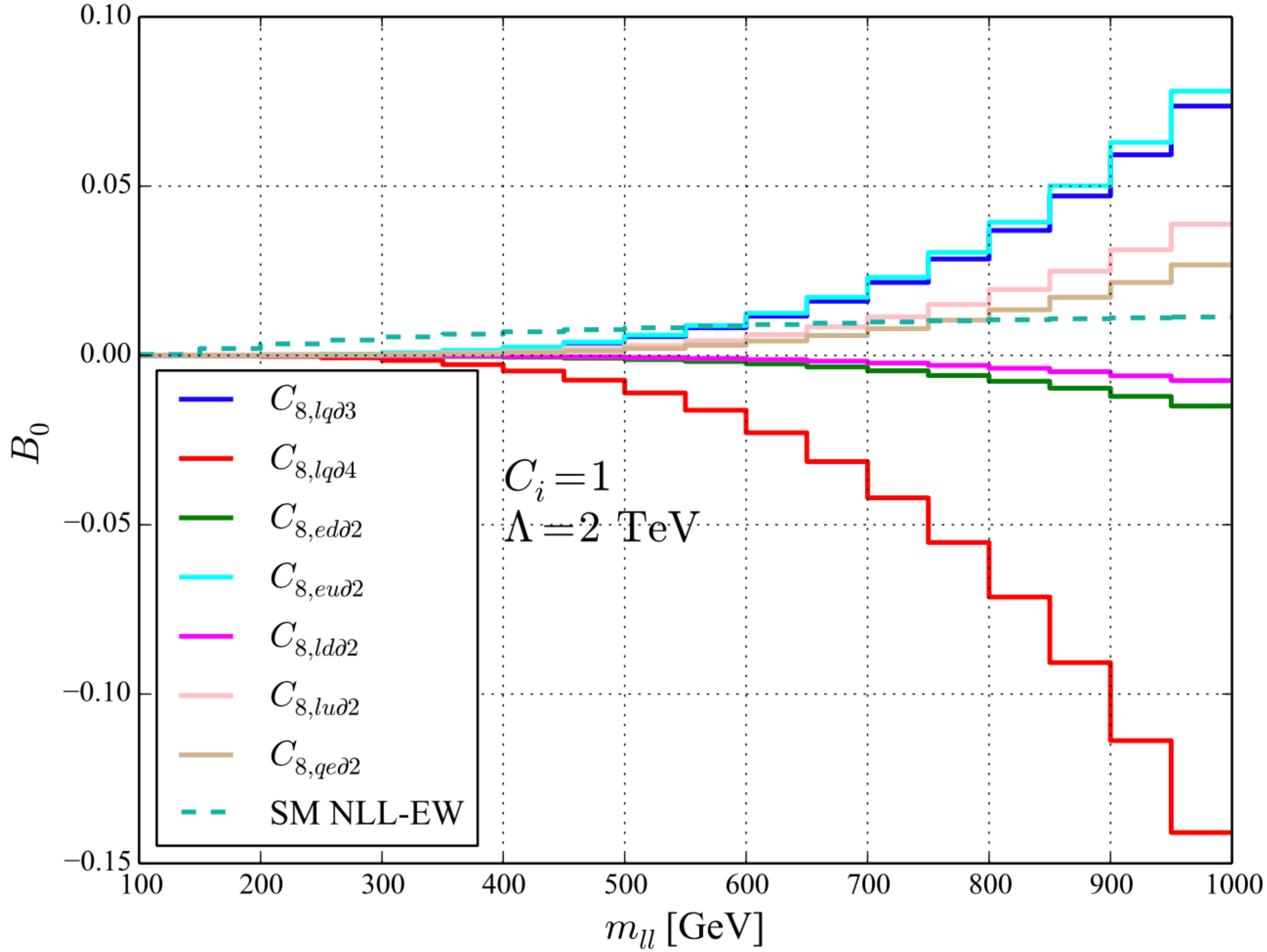
$$O_{8,lq\partial 3} = (\bar{\ell}\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{q}\gamma^\mu \overleftrightarrow{D}^\nu q) \qquad O_{8,lq\partial 4} = (\bar{\ell}\tau^I\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{q}\tau^I\gamma^\mu \overleftrightarrow{D}^\nu q)$$
$$O_{8,ed\partial 2} = (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{d}\gamma^\mu \overleftrightarrow{D}^\nu d) \qquad O_{8,eu\partial 2} = (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)$$
$$O_{8,ld\partial 2} = (\bar{\ell}\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{d}\gamma^\mu \overleftrightarrow{D}^\nu d) \qquad O_{8,lu\partial 2} = (\bar{\ell}\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)$$
$$O_{8,qe\partial 2} = (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{q}\gamma^\mu \overleftrightarrow{D}^\nu q)$$

- Other class of $\psi^4 D^2$: $(\bar{\ell}\gamma_\mu \ell)\partial^2(\bar{q}\gamma^\mu q) \Rightarrow \mathscr{A}(q\bar{q} \to \ell^+\ell^-) \sim s^2$

No new angular dependence

Higher moments in $q\bar{q} \to \ell^+\ell^-$ $\Leftrightarrow$ Positivity bounds

$\tilde{B}_0(m_{\ell^+\ell^-})$



Positivity in a nutshell

Set of theoretical constraints on scattering amplitudes

- Apply to a subset of $D \geq 8$ Wilson coefficients

Result from basic assumptions about UV QFT/S-matrix

- Lorentz invariance, unitarity, causality & locality

  [Pham & Troung; PRD 31 (1985 3027)]
  [Anathanarayan et al.; PRD 51 (1995) 1093-1100]
  [Adams et al.; JHEP 10 (2006) 014]

  + many more in recent years…

A. Generalised optical theorem for scattering amplitude $\mathscr{M}_{ij\to kl}$ satisfies

B. Twice subtracted, forward dispersion relation by analytic continuation in complex s: $M_{ijkl}(s, t=0)$ $\quad M_{ijkl} \equiv \mathscr{M}_{ij\to kl} - (\text{low energy poles})$

$$\frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi i \mu^3}\left(m_{ij}m^*_{kl} + m_{i\tilde{l}}m^*_{k\tilde{j}}\right) \qquad m_{ij} \equiv M_{ij\to X}(\mu)$$

[Zhang & Zhou; PRL 125 (2020) 201601]
[Li et al.; PRL 127 (2021) 121601]

Positivity cone

$$\frac{1}{2}\frac{d^2M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi i \mu^3}\left(m_{ij}m_{kl}^* + m_{i\tilde{l}}m_{k\tilde{j}}^*\right) \qquad m_{ij} \equiv M_{ij\to X}(\mu)$$

$2 \to 1$ amplitudes m_{ij} are unknown complex functions of μ

- Encode masses of new states at & above Λ^2

Elastic ($ij = kl$): $$\frac{1}{2}\frac{d^2M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi i \mu^3}\left(|m_{ij}|^2 + |m_{i\tilde{j}}|^2\right) \geq 0$$

Inelastic ($ij \neq kl$): more information

- $m_{ij}m_{kl}^*$ are not positive-definite. However, RHS is a **positive sum** (integral)

$$\frac{1}{2}\frac{d^2M_{ijkl}(0)}{ds^2}$$ forms a **convex cone**

- Constraints the Wilson coefficients to a non-trivial, conical subspace

Connecting to DY

Relevant dim-8 operators: $\mathscr{A}(q\bar{q} \to \ell^+\ell^-) \sim t^2$

- By crossing symmetry, elastic amplitude: $\mathscr{A}(q\ell \to q\ell) \sim s^2!$
- Novel angular dependence in Drell Yan ⇔ positivity bounds from $ql \to ql$

Positivity bound	channel: $\|1\rangle + \|2\rangle \to \|1\rangle + \|2\rangle$
$-4C_{8,lq\partial 3} + 4C_{8,lq\partial 4} \geq 0$	$\|1\rangle = \|e_L^-\rangle\,,\quad \|2\rangle = \|u_L\rangle$
$-4C_{8,lq\partial 3} - 4C_{8,lq\partial 4} \geq 0$	$\|1\rangle = \|e_L^-\rangle\,,\quad \|2\rangle = \|d_L\rangle$
$-4C_{8,ed\partial 2} \geq 0$	$\|1\rangle = \|e_R^-\rangle\,,\quad \|2\rangle = \|d_R\rangle$
$-4C_{8,eu\partial 2} \geq 0$	$\|1\rangle = \|e_R^-\rangle\,,\quad \|2\rangle = \|u_R\rangle$
$-4C_{8,ld\partial 2} \geq 0$	$\|1\rangle = \|e_L^-\rangle\,,\quad \|2\rangle = \|d_R\rangle$
$-4C_{8,lu\partial 2} \geq 0$	$\|1\rangle = \|e_L^-\rangle\,,\quad \|2\rangle = \|u_R\rangle$
$-4C_{8,qe\partial 2} \geq 0$	$\|1\rangle = \|e_R^-\rangle\,,\quad \|2\rangle = \|u_L\rangle$

- Use higher angular moments in DY to test positivity ⇒ Fundamental properties of QFT in the UV

DY matrix elements

Squared amplitude for $l \geq 3$ angular moments

$$|M|^2 = |M_{\rm SM\,NLL\,EW}|^2 + \sum_i \frac{C_i^{(8)}}{\Lambda^4} \Delta M_i^{(8)} + \sum_{i \geq j} \frac{C_i^{(8)} C_j^{(8)}}{\Lambda^8} |M_{ij}^{(8)}|^2 \qquad \Delta M_i^{(8)} \equiv 2\mathrm{Re}[M_{SM}^* M_i^{(8)}]$$

LO SM: up to $\cos^2\theta$

$$|\mathcal{M}_{\rm SM}(q_L\bar{q}_R \to e^-_R e^+_L)|^2 = 16\pi^2\alpha^2(1-\hat{c}_\theta)^2 \cdot \left| Q_f - \frac{(I_3 - s_W^2 Q_f)\hat{s}}{c_W^2(\hat{s} - M_Z^2)} \right|^2,$$

$$|\mathcal{M}_{\rm SM}(q_R\bar{q}_L \to e^-_R e^+_L)|^2 = 16\pi^2\alpha^2(1+\hat{c}_\theta)^2 \cdot \left| Q_f + \frac{s_W^2 Q_f \hat{s}}{c_W^2(\hat{s} - M_Z^2)} \right|^2,$$

$$|\mathcal{M}_{\rm SM}(q_L\bar{q}_R \to e^-_L e^+_R)|^2 = 16\pi^2\alpha^2(1+\hat{c}_\theta)^2 \cdot \left| Q_f - \frac{(I_3 - s_W^2 Q_f)(-1/2 + s_W^2)\hat{s}}{c_W^2 s_W^2(\hat{s} - M_Z^2)} \right|^2,$$

$$|\mathcal{M}_{\rm SM}(q_R\bar{q}_L \to e^-_L e^+_R)|^2 = 16\pi^2\alpha^2(1-\hat{c}_\theta)^2 \cdot \left| Q_f + \frac{Q_f(-1/2 + s_W^2)\hat{s}}{c_W^2(\hat{s} - M_Z^2)} \right|^2.$$

DY matrix elements

LO dimension-8

$O(\Lambda^{-4})$

$$\Delta\mathcal{M}_{8,lq\partial 3} = -\frac{C_{8,lq\partial 3}}{\Lambda^4} 8\pi\alpha\,\hat{c}_\theta(1+\hat{c}_\theta)^2\hat{s}^2 \cdot \left(Q_f - \frac{(I_3 - s_W^2 Q_f)(-1/2+s_W^2)\hat{s}}{c_W^2 s_W^2(\hat{s}-M_Z^2)}\right),$$

$$\Delta\mathcal{M}_{8,lq\partial 4} = \frac{C_{8,lq\partial 4}}{\Lambda^4} 8\pi\alpha\,\hat{c}_\theta(1+\hat{c}_\theta)^2\hat{s}^2(2I_3) \cdot \left(Q_f - \frac{(I_3 - s_W^2 Q_f)(-1/2+s_W^2)\hat{s}}{c_W^2 s_W^2(\hat{s}-M_Z^2)}\right),$$

$$\Delta\mathcal{M}_{8,ed\partial 2} = \frac{C_{8,ed\partial 2}}{\Lambda^4} 8\pi\alpha\,\hat{c}_\theta(1+\hat{c}_\theta)^2\hat{s}^2 \cdot \frac{1}{3}\left(1 + \frac{s_W^2\hat{s}}{c_W^2(\hat{s}-M_Z^2)}\right),$$

$$\Delta\mathcal{M}_{8,eu\partial 2} = -\frac{C_{8,eu\partial 2}}{\Lambda^4} 8\pi\alpha\,\hat{c}_\theta(1+\hat{c}_\theta)^2\hat{s}^2 \cdot \frac{2}{3}\left(1 + \frac{s_W^2\hat{s}}{c_W^2(\hat{s}-M_Z^2)}\right),$$

$$\Delta\mathcal{M}_{8,ld\partial 2} = \frac{C_{8,ld\partial 2}}{\Lambda^4} 8\pi\alpha\,\hat{c}_\theta(1-\hat{c}_\theta)^2\hat{s}^2 \cdot \frac{1}{3}\left(1 + \frac{(-1/2+s_W^2)\hat{s}}{c_W^2(\hat{s}-M_Z^2)}\right),$$

$$\Delta\mathcal{M}_{8,lu\partial 2} = -\frac{C_{8,lu\partial 2}}{\Lambda^4} 8\pi\alpha\,\hat{c}_\theta(1-\hat{c}_\theta)^2\hat{s}^2 \cdot \frac{2}{3}\left(1 + \frac{(-1/2+s_W^2)\hat{s}}{c_W^2(\hat{s}-M_Z^2)}\right),$$

$$\Delta\mathcal{M}_{8,qe\partial 2} = -\frac{C_{8,qe\partial 2}}{\Lambda^4} 8\pi\alpha\,\hat{c}_\theta(1-\hat{c}_\theta)^2\hat{s}^2 \cdot \left(Q_f - \frac{(I_3 - s_W^2 Q_f)\hat{s}}{c_W^2(\hat{s}-M_Z^2)}\right).$$

$\cos^{n\leq 3}\theta$ dependence $\Leftrightarrow$ $l \leq 3$

$O(\Lambda^{-8})$

$$|\mathcal{M}_{8,lq\partial 3}|^2 = \frac{C_{8,lq\partial 3}^2}{\Lambda^8}\hat{c}_\theta^2(1+\hat{c}_\theta)^2\hat{s}^4,$$

$$|\mathcal{M}_{8,lq\partial 4}|^2 = \frac{C_{8,lq\partial 4}^2}{\Lambda^8}\hat{c}_\theta^2(1+\hat{c}_\theta)^2\hat{s}^4,$$

$$|\mathcal{M}_{8,ed\partial 2}|^2 = \frac{C_{8,ed\partial 2}^2}{\Lambda^8}\hat{c}_\theta^2(1+\hat{c}_\theta)^2\hat{s}^4,$$

$$|\mathcal{M}_{8,eu\partial 2}|^2 = \frac{C_{8,eu\partial 2}^2}{\Lambda^8}\hat{c}_\theta^2(1+\hat{c}_\theta)^2\hat{s}^4,$$

$$|\mathcal{M}_{8,ld\partial 2}|^2 = \frac{C_{8,ld\partial 2}^2}{\Lambda^8}\hat{c}_\theta^2(1-\hat{c}_\theta)^2\hat{s}^4,$$

$$|\mathcal{M}_{8,lu\partial 2}|^2 = \frac{C_{8,lu\partial 2}^2}{\Lambda^8}\hat{c}_\theta^2(1-\hat{c}_\theta)^2\hat{s}^4,$$

$$|\mathcal{M}_{8,qe\partial 2}|^2 = \frac{C_{8,qe\partial 2}^2}{\Lambda^8}\hat{c}_\theta^2(1-\hat{c}_\theta)^2\hat{s}^4$$

$$|\mathcal{M}_{8,lq\partial 3,lq\partial 4}|^2 = \frac{C_{8,lq\partial 3}C_{8,lq\partial 4}}{\Lambda^8}\hat{c}_\theta^2(1+\hat{c}_\theta)^2\hat{s}^4.$$

$\cos^{n\leq 4}\theta$ dependence $\Leftrightarrow$ $l \leq 4$

Even higher moments

Exploit the full information of D8 amplitude: $l = 4$ moments

$$\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}d\Omega_\ell} = \frac{3}{16\pi}\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}}\Bigg[(1+c_\theta^2) + \frac{\tilde{A}_0}{2}(1-3c_\theta^2) + \tilde{A}_1 s_{2\theta}c_\phi$$

$l \leq 2$

$$+\frac{\tilde{A}_2}{2}s_\theta^2 c_{2\phi} + \tilde{A}_3 s_\theta c_\phi + \tilde{A}_4 c_\theta + \tilde{A}_5 s_\theta^2 s_{2\phi} + \tilde{A}_6 s_{2\theta}s_\phi + \tilde{A}_7 s_\theta s_\phi$$

$l = 3$

$$+\frac{\tilde{B}_1^e}{2}s_\theta\left(5c_\theta^2-1\right)c_\phi + \frac{\tilde{B}_1^o}{2}s_\theta\left(5c_\theta^2-1\right)s_\phi + \boxed{\frac{\tilde{B}_0}{2}\left(5c_\theta^3-3c_\theta\right)}$$

$$+\tilde{B}_3^e s_\theta^3 c_{3\phi} + \tilde{B}_3^o s_\theta^3 s_{3\phi} + \tilde{B}_2^e s_\theta^2 c_\theta c_{2\phi} + \tilde{B}_2^o s_\theta^2 c_\theta s_{2\phi}$$

$l = 4$

$$+\tilde{D}_4^e s_\theta^4 c_{4\phi} + \tilde{D}_4^o s_\theta^4 s_{4\phi} + \tilde{D}_3^e s_\theta^3 c_\theta c_{3\phi} + \tilde{D}_3^o s_\theta^3 c_\theta s_{3\phi}$$

$$+\tilde{D}_2^e s_\theta^2(7c_\theta^2-1)c_{2\phi} + \tilde{D}_2^o s_\theta^2(7c_\theta^2-1)s_{2\phi} + \tilde{D}_1^e s_\theta(7c_\theta^3-3c_\theta)c_\phi$$

$$+\tilde{D}_1^o s_\theta(7c_\theta^3-3c_\theta)s_\phi + \boxed{\frac{\tilde{D}_0}{2}(35c_\theta^4-30c_\theta^2+3)}\Bigg]$$

Use $(\tilde{B}_0, \tilde{D}_0)$ to constrain the space of dim-8 WCs

- Quantify the ability of the LHC to test positivity in $q\ell$ scattering

Higher orders

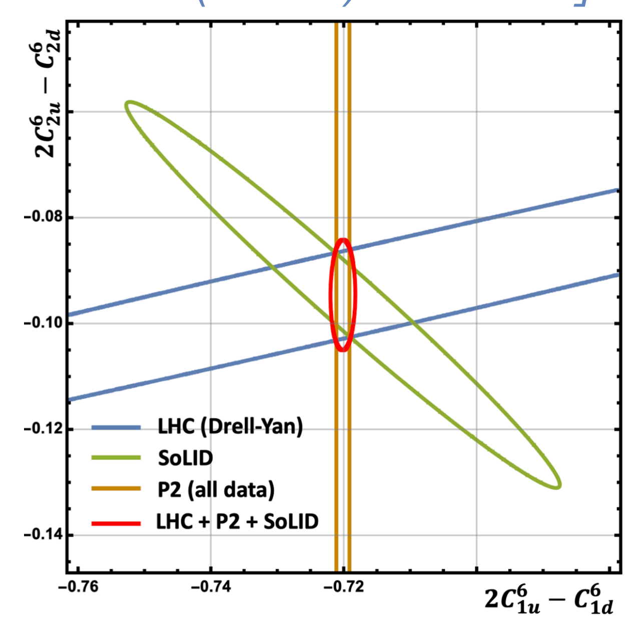
$l \le n, O(\Lambda^{-m})$ A_i B_i D_i

D=	n,m	1,0	1,2	2,4	3,6	…
4	1,0	2,0				
6	1,2	2,2	2,4			
8	2,4	3,4	3,6	4,8		
10	3,6	4,6	4,8	5,10	6,12	
…	…					

$M_{SM,D=6} \Leftrightarrow l = 1$ & $M_{D=8} \Leftrightarrow l = 2, \ldots$

- $M_{D=6} \times M_{D=8}$ populates $l = 3$ at $O(\Lambda^{-6})$
- $M_{SM} \times M_{D=10}$ populates $l = 4$ at $O(\Lambda^{-8})$
- $M_{D=6} \times M_{D=10}$ populates $l = 4$ at $O(\Lambda^{-10})$
- …

[Boughezal et al.; PRD 104 (2021) 016005]



Our assumption: neglect possible $D = 6$

- Constrained elsewhere: A_0 moments, APV, β-decays,…

$l = 4$ have other possible contributions

- We currently neglect them, try to mitigate impact

Ultimately, more complete (higher l) & global ($D = 6,8,\ldots$) analysis needed

Higher moments at LHC

Define dimensionful coefficients $\tilde{B} \rightarrow B\,[\mathrm{fb}]$

- Encode the WC dependence on the $(m_{\ell\ell}, \eta_{\ell\ell})$ differential cross section

$$\frac{dB_0}{dm_{\ell\ell}\, d\eta_{\ell\ell}} = \frac{4\sqrt{7\pi}}{9} \cdot \frac{\xi m_{\ell\ell}}{s} \int_{-1}^{1} dc_\theta\, Y_3^0(c_\theta) \left[G_{q\bar{q}}(m_{\ell\ell}, \eta_{\ell\ell}, Q^2) - G_{\bar{q}q}(m_{\ell\ell}, \eta_{\ell\ell}, Q^2)\right] \frac{d\hat{\sigma}}{d\hat{c}_\theta}(c_\theta, m_{\ell\ell})$$

$$\frac{dD_0}{dm_{\ell\ell}\, d\eta_{\ell\ell}} = \frac{\sqrt{\pi}}{3} \cdot \frac{m_{\ell\ell}}{s} \int_{-1}^{1} dc_\theta\, Y_4^0(c_\theta) \left[G_{q\bar{q}}(m_{\ell\ell}, \eta_{\ell\ell}, Q^2) + G_{\bar{q}q}(m_{\ell\ell}, \eta_{\ell\ell}, Q^2)\right] \frac{d\hat{\sigma}}{d\hat{c}_\theta}(c_\theta, m_{\ell\ell})$$

$$c_\theta = c_\theta^* \frac{\eta}{|\eta|} \equiv c_\theta^* \xi, \qquad G_{ij}(x_1, x_2, Q^2) = f_{i/p}(x_1, Q^2) \cdot f_{j/p}(x_2, Q^2)$$

CS frame (c_θ) — CM frame w.r.t beam (c_θ^*) — (anti)quark PDFs

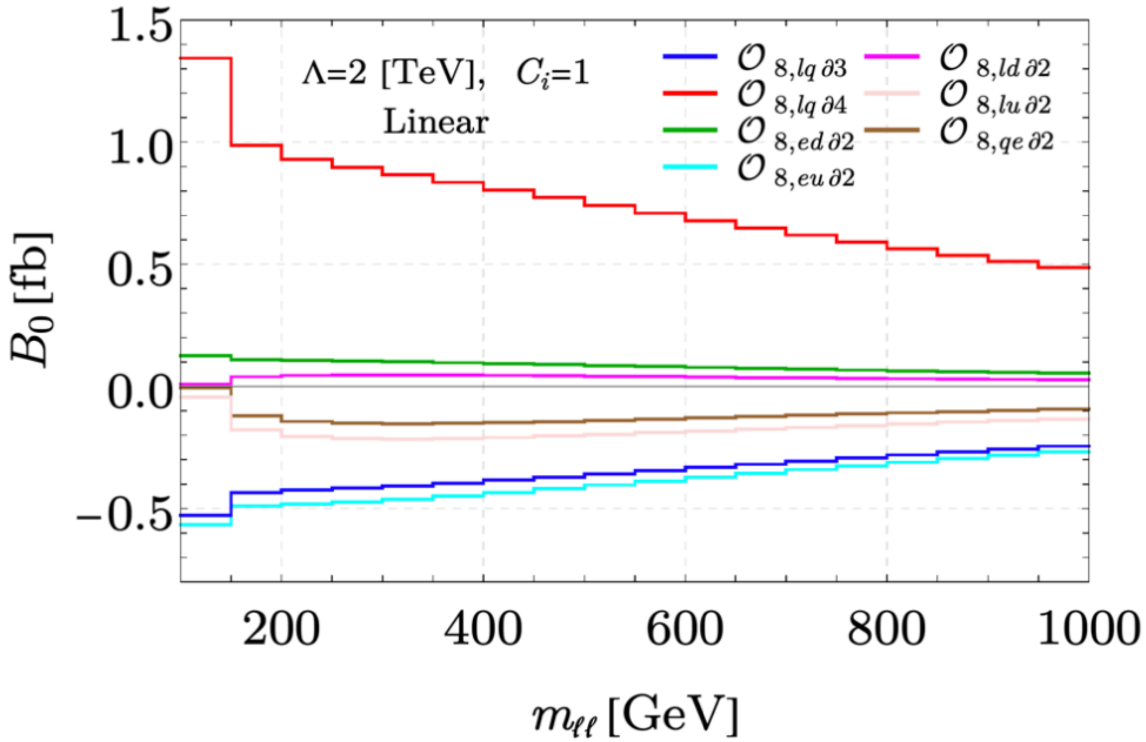
Double differential moments bring maximal information

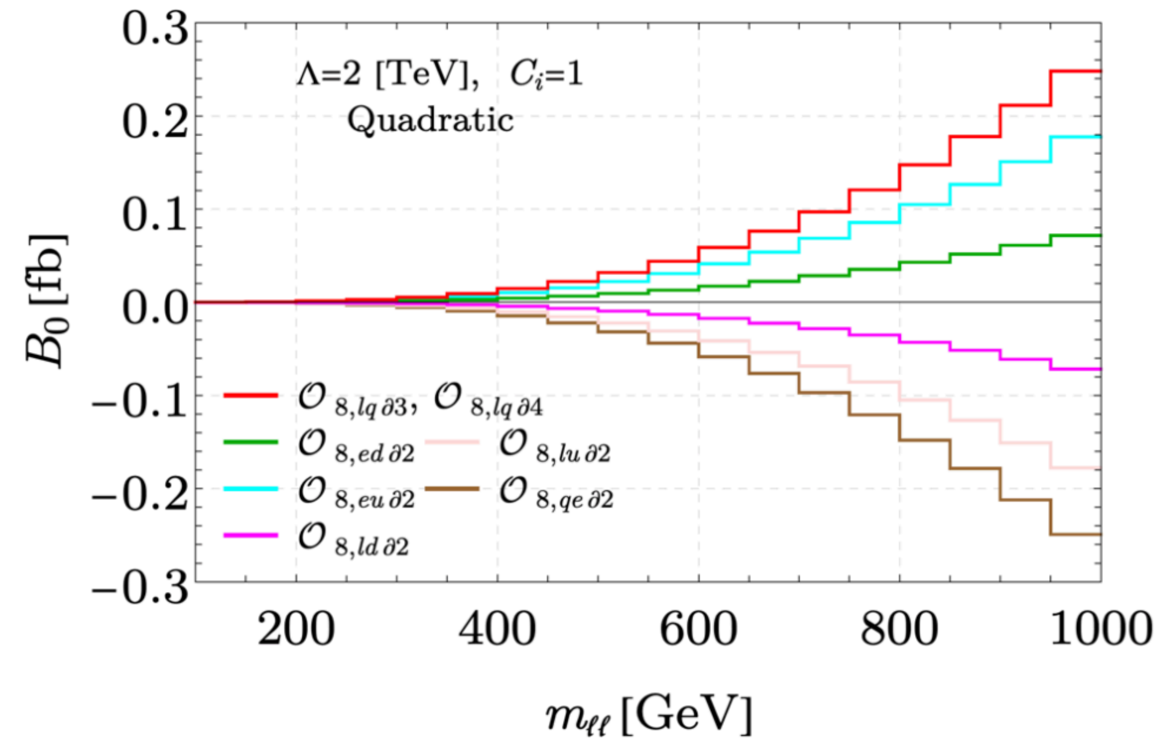
- $m_{\ell\ell}$: energy growth
- $\eta_{\ell\ell}$: partonic initial state $u\bar{u}$ vs. $d\bar{d}$

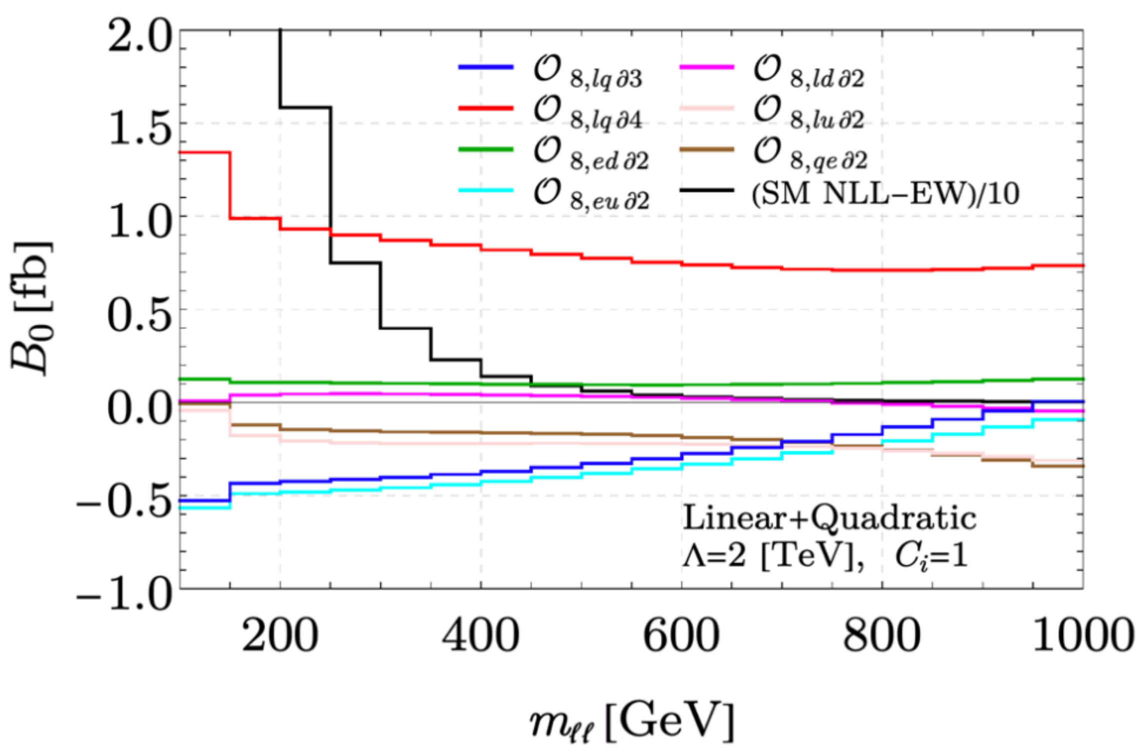
PDF: `NNPDF_nlo_as_0118_luxqed`

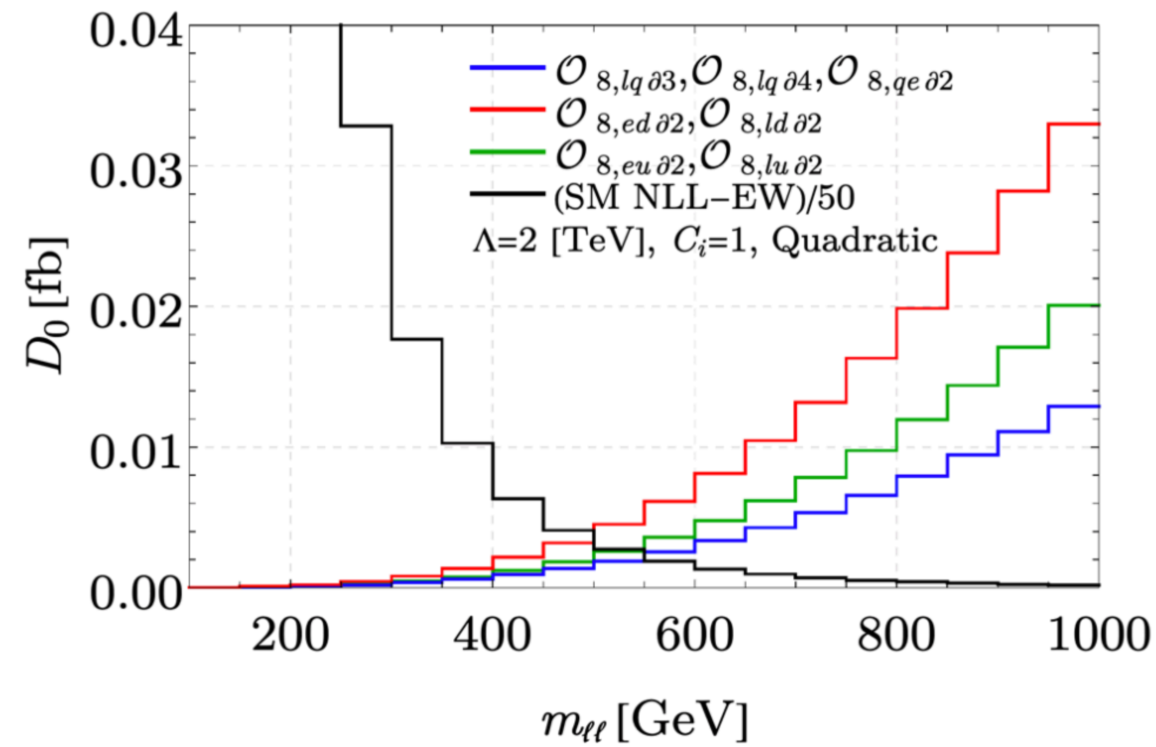
LHC predictions

$\sqrt{s} = 14\,\mathrm{TeV}$









Disentangling operators

Leading, $O(\Lambda^{-4})$ interference effects are degenerate

- Modulo $\eta_{\ell\ell}$ sensitivity to the 3 different partonic initial state combinations
- Mild differences at low energies near Z-pole

Operators	Quark	$\Delta\|\mathcal{M}^{(8)}\|^2$	$\|\mathcal{M}^{(8)}\|^2$	B_0(lin.)	B_0(quad.)	D_0(quad.)
$\mathcal{O}_{8,lq\partial 3} + \mathcal{O}_{8,lq\partial 4}$	$d\bar{d}$	$1.9 \cdot c_\theta(1+c_\theta)^2$	$c_\theta^2(1+c_\theta)^2$	$+$	$+$	$+$
$\mathcal{O}_{8,lq\partial 3} - \mathcal{O}_{8,lq\partial 4}$	$u\bar{u}$	$-2.4 \cdot c_\theta(1+c_\theta)^2$	$c_\theta^2(1+c_\theta)^2$	$-$	$+$	$+$
$\mathcal{O}_{8,ed\partial 2}$	$d\bar{d}$	$0.43 \cdot c_\theta(1+c_\theta)^2$	$c_\theta^2(1+c_\theta)^2$	$+$	$+$	$+$
$\mathcal{O}_{8,eu\partial 2}$	$u\bar{u}$	$-0.87 \cdot c_\theta(1+c_\theta)^2$	$c_\theta^2(1+c_\theta)^2$	$-$	$+$	$+$
$\mathcal{O}_{8,ld\partial 2}$	$d\bar{d}$	$0.22 \cdot c_\theta(1-c_\theta)^2$	$c_\theta^2(1-c_\theta)^2$	$+$	$-$	$+$
$\mathcal{O}_{8,lu\partial 2}$	$u\bar{u}$	$-0.43 \cdot c_\theta(1-c_\theta)^2$	$c_\theta^2(1-c_\theta)^2$	$-$	$-$	$+$
$\mathcal{O}_{8,qe\partial 2}$	$u\bar{u}$ or $d\bar{d}$	$-0.22 \cdot c_\theta(1-c_\theta)^2$	$c_\theta^2(1-c_\theta)^2$	$-$	$-$	$+$

Additional information from $|M^{(8)}|^2$

- Relative size & sign between linear & quadratic B_0
- D_0: quadratic only ($l = 4$)

LHC sensitivity

1 TeV cut to mitigate impact of quadratics

Consider 10 × 10 square binning:

$$m_{\ell\ell}: \{100,\ 190,\ 280,\ 370,\ 460,\ 550,\ 640,\ 730,\ 820,\ 910,\ 1000\}\ \mathrm{GeV},$$
$$\eta_{\ell\ell}: \{-5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5\},$$

Binned $\Delta\chi^2$, combining (B_0, D_0), for $L_{\mathrm{int.}} = 3000\ \mathrm{fb}^{-1}$

$$\chi^2(C_i) \equiv \Delta\chi^2(C_i) = \sum_i \left(B_0^i(\vec{C}), D_0^i(\vec{C})\right) \cdot \mathbf{V}^{-1} \cdot \left(B_0^i(\vec{C}), D_0^i(\vec{C})\right) \leq 3.84,$$

- B_0 & D_0 are correlated: statistical covariance matrix $\mathbf{V}$

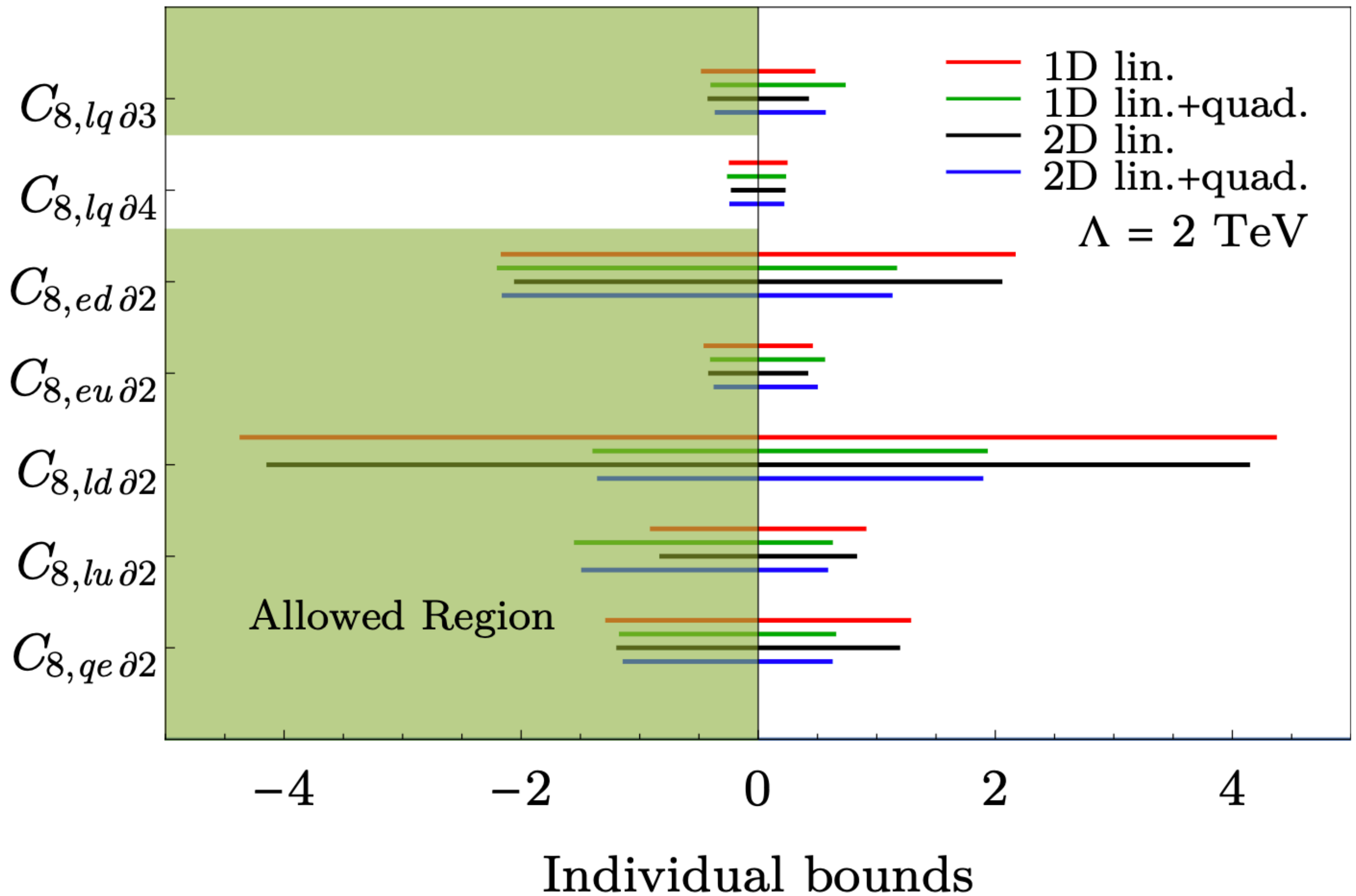
$$V_{ij} = \frac{1}{L}\int_{m_{\mathrm{min.}}}^{m_{\mathrm{max.}}} dm_{\ell\ell} \int_{\eta_{\mathrm{min.}}}^{\eta_{\mathrm{max.}}} d\eta_{\ell\ell} \int_{-1}^{1} dc_\theta \frac{d\sigma_{pp\to\ell^-\ell^+}}{d\eta_{\ell\ell}\, dm_{\ell\ell}\, dc_\theta} \cdot F_{ij}(c_\theta),$$

(co)variance of weighted average(s)

$$F_{11} = \frac{448\pi}{9}\left(Y_3^0(c_\theta)\right)^2; \quad F_{22} = \frac{36\pi^3}{49}\left(Y_4^0(c_\theta)\right)^2; \quad F_{12} = F_{21} = \sqrt{\frac{16}{7}} 4\pi^2 Y_3^0(c_\theta) Y_4^0(c_\theta)$$

- Variances dominated by SM, computed @ NLO QCD with `mg5`

Individual bounds on C_i



Observe the SM: $O(0.5-4)$ sensitivity for $\Lambda = 2\,\text{TeV}$

- $\eta_{\ell\ell}$ (2D) information not important for individual bounds (expected)
- Quadratic effects significant for weakly constrained direction(s)

Profiled bounds on C_i

Closed fit requires full information (quadratic $B_0 + D_0$)

- Linearised $\Delta\chi^2$ has 2(1) well(moderately) constrained eigenvectors
- $\delta C_i = \{0.15, 2.5, 40\}$
- Full information yields $O(1)$ bounds for $\Lambda = 2\,\mathrm{TeV}$

Coeff.	$C_{8,lq\partial 3}$	$C_{8,lq\partial 4}$	$C_{8,ed\partial 2}$	$C_{8,eu\partial 2}$	$C_{8,ld\partial 2}$	$C_{8,lu\partial 2}$	$C_{8,qe\partial 2}$
$B_0 + D_0$	$(-1.1, 1.1)$	$(-0.95, 0.85)$	$(-1.8, 1.8)$	$(-1.2, 1.3)$	$(-1.7, 1.7)$	$(-1.2, 1.2)$	$(-1.0, 1.0)$

Flat directions expected from previous discussion

- Reliance on $O(\Lambda^{-8})$ effects means results should be interpreted with care
- Up to dim-12(!) interference with SM
- Our 1 TeV cut will partially mitigate them
- They contribute to higher moments

More complete analysis required

Testing positivity

No concrete reason to expect violation of positivity

- Nature (data) should have the last word
- Probe the violation of positivity to test the axiomatic principles of QFT

Define "distance" from region allowed by elastic positivity

$$-\Delta^{-4} \equiv \min\left[\min_{\text{processes}} \frac{1}{2}\frac{d^2M(0)}{ds^2}, 0\right] = \frac{\delta(\vec{C}_0)}{\Lambda^4},$$

$$\delta(\vec{C}_0) \equiv \min\Big[-4C_{8,lq\partial 3} + 4C_{8,lq\partial 4}, -4C_{8,lq\partial 3} - 4C_{8,lq\partial 4}, -4C_{8,ed\partial 2}, -4C_{8,eu\partial 2}, -4C_{8,ld\partial 2}, -4C_{8,lu\partial 2}, -4C_{8,qe\partial 2}, 0\Big]$$

elastic ql scatterings

"most non-positive" direction

Associates a scale, Δ, to positivity violation

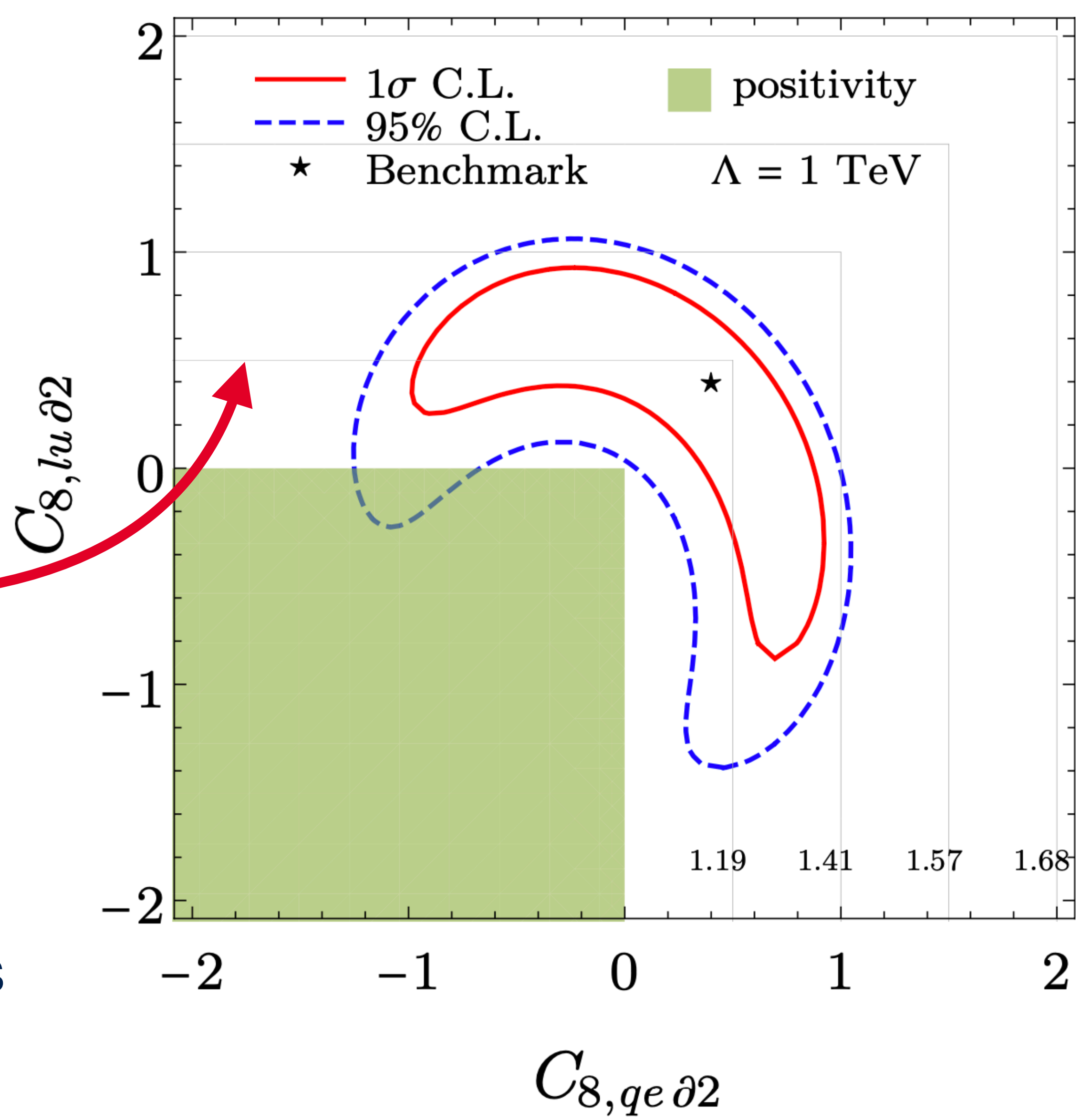
Satisfied: $\Delta = \infty$

Violated: $\Delta = \dfrac{\Lambda}{\sqrt[4]{\delta C_{\text{min.}}}}$

Testing positivity

Suppose we observe some non-zero WCs, $\vec{C}_0$

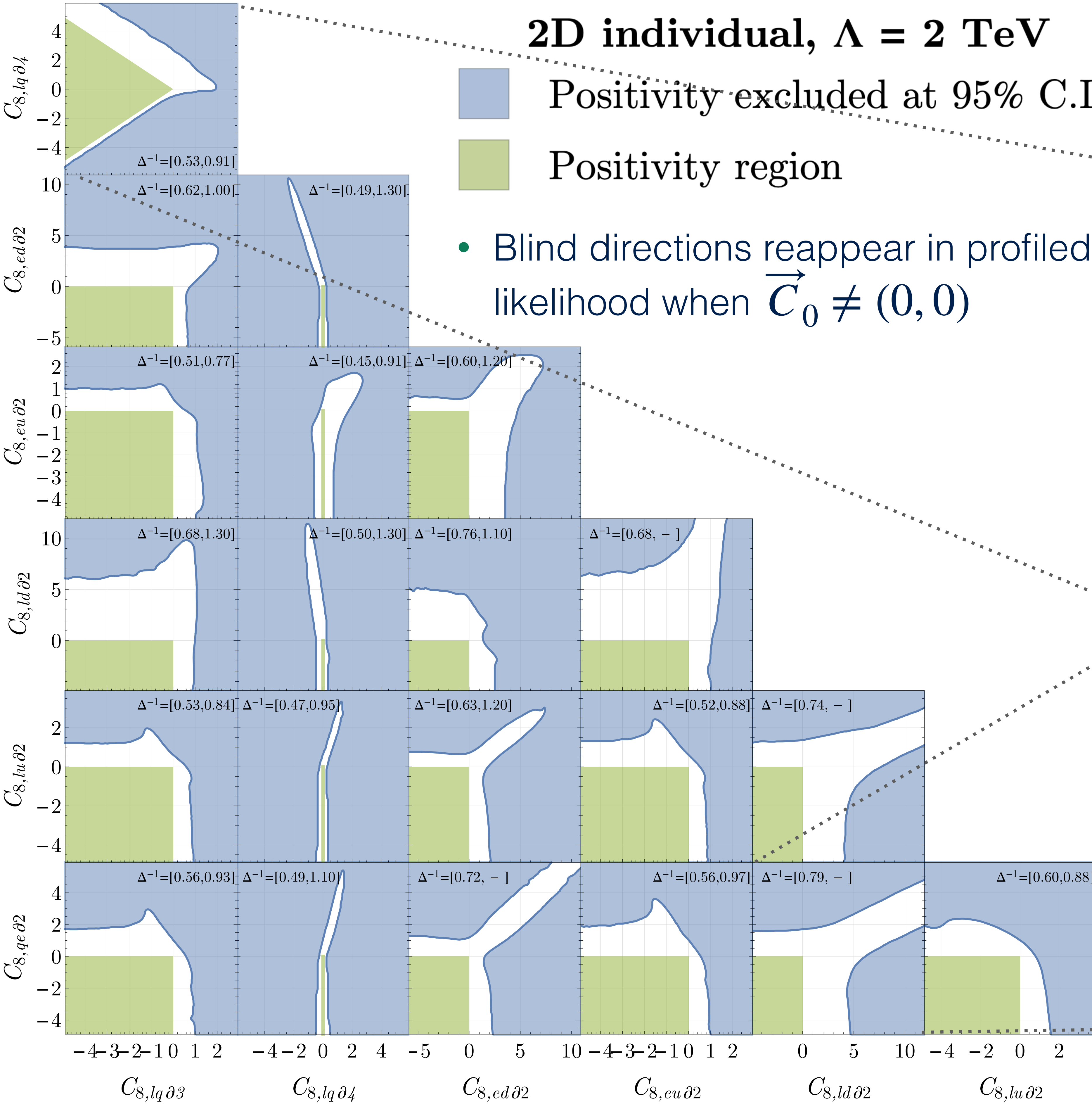
- Uncertainty crucial to determine whether we claim evidence for positivity violation: $\Delta\chi^2$
- If 95% confidence region overlaps with positivity allowed region, cannot rule out positivity
- Δ values shown in TeV
- Cannot exclude positivity at 95% C.L in this case: $\vec{C}_0 = (0.4, 0.4)$
- $\Delta^{-1} = [\Delta^{-1}_{\text{low}}, \Delta^{-1}_{\text{high}}]$, Δ_{low} gives conservative estimate

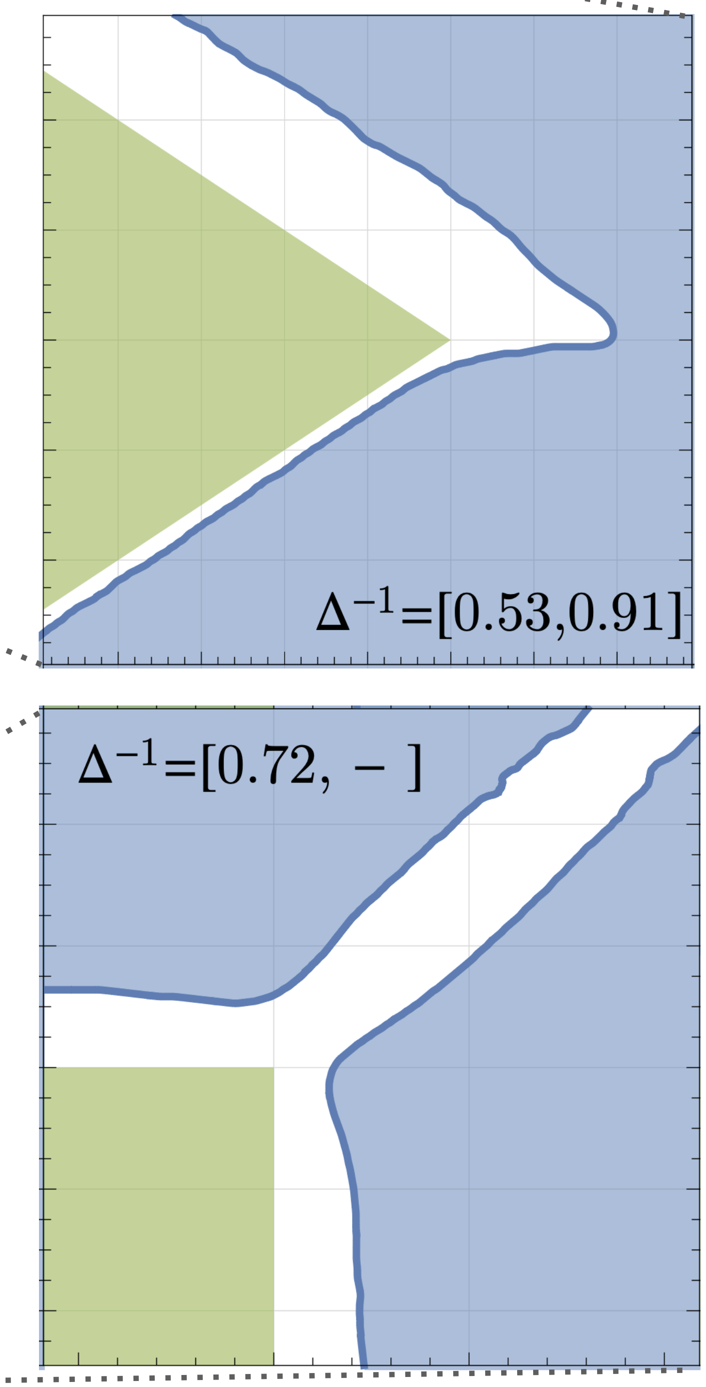


2D individual, Λ = 2 TeV

- Positivity excluded at 95% C.L. (blue)
- Positivity region (green)

- Blind directions reappear in profiled likelihood when $\overrightarrow{C}_0 \neq (0,0)$





[Zhang & Zhou; PRL 125 (2020) 201601]
[Li et al.; PRL 127 (2021) 121601]

Beyond elastic positivity

Optimal forward positivity bounds: positivity cone

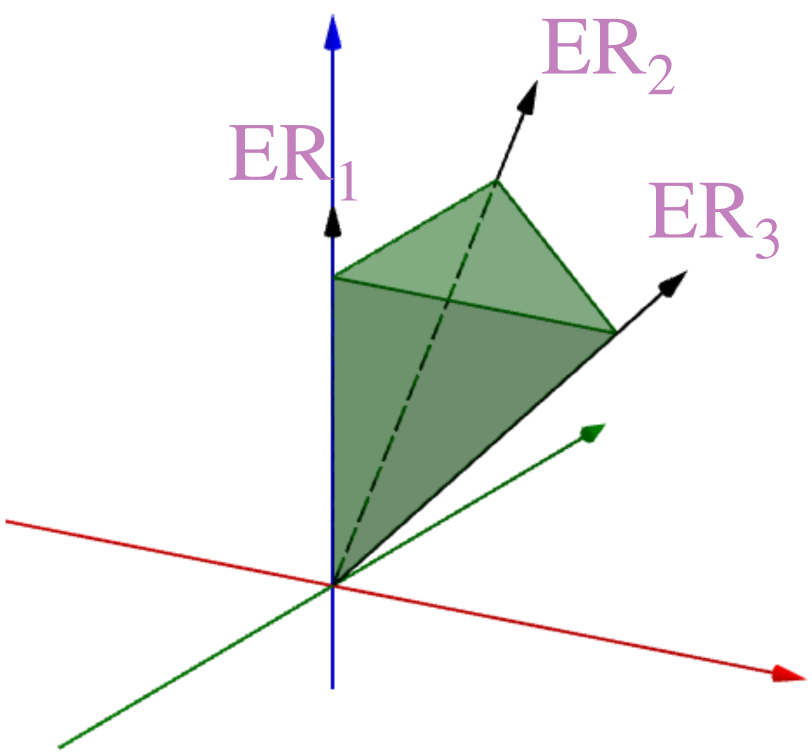
$$\frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi i \mu^3}\left(m_{ij}m_{kl}^* + m_{i\tilde{l}}m_{k\tilde{j}}^*\right) \qquad m_{ij} \equiv M_{ij\to X}(\mu)$$

$\frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2}$ forms a salient **convex cone** (Cone that does not contain a straight line, only half lines)

Generated by all conical hulls (+ve linear combinations) of its **Extremal Rays** (ERs)

ER: cannot be expressed as a +ve sum of other elements

$m_{ij}m_{kl}^* + m_{i\tilde{l}}m_{k\tilde{j}}^*$ are ERs of the positivity cone



Constructing the cone

$m_{ij} \equiv M_{ij\to X}$: ER $\Rightarrow$ m_{ij} cannot be decomposed into other m'_{ij}

- X is an irrep. in the decomposition of the product of i, j's symmetry reps
- Construct (potential) ERs from CG coefficients of $r_i \otimes r_j$

OR enumerate all possible tree-level completions

- Integrate them out to get the WC vectors for the pERs

Simplified example: new physics only couples to e_R, u_R

$$C_1 : (\bar{e}\gamma_\mu e)\,\partial^2\,(\bar{e}\gamma^\mu e) \qquad C_4 : (\bar{u}\gamma_\mu \overleftrightarrow{D}_\nu u)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)$$

$$C_2 : (\bar{e}\gamma_\mu e)\,\partial^2\,(\bar{u}\gamma^\mu u) \qquad C_5 : (\bar{u}\gamma_\mu u)\,\partial^2\,(\bar{u}\gamma^\mu u)$$

$$C_3 : (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u) \equiv C_{8,eu\partial 2}$$

$$\vec{C} = (C_1, C_2, C_3, C_4, C_5)$$

One state $\quad \dfrac{\vec{C}_X}{\Lambda^4} = w_X \vec{c}_X, \quad w_X = \dfrac{g_X^2}{M_X^4} \geq 0 \quad \Rightarrow \quad$ Generic UV theory $\quad \dfrac{\vec{C}}{\Lambda^4} = \sum_X \dfrac{\vec{C}_X}{\Lambda^4} = \sum_X w_X \vec{c}_X$

UV states & new bounds

Notation of [de Blas et al.; JHEP 03 (2018) 109]

UV interaction	(SU(3), SU(2))$^{\text{spin}}_{\text{U(1)}}$	dim-8 EFT coefficients ($\vec{c}_X$)	ER?
$\bar{e^c} e \mathcal{S}_2 + h.c.$	$\mathcal{S}_2$: $(1,1)^0_2$	$(1,0,0,0,0)$	✔
$\frac{1}{M_{UV}} \bar{u^c}_i \overleftrightarrow{D}^\mu u_j \epsilon_{ijk} \mathcal{U}^{\dagger k}_{4\mu} + h.c.$	$\mathcal{U}^k_{4\mu}$: $(\bar{3},1)^1_{4/3}$	$(0,0,0,-\frac{1}{2},-\frac{3}{2})$	✔
$(\bar{u^c}_i u_j \Omega^{\dagger ij}_4 + sym.) + h.c.$	Ω^{ij}_4: $(6,1)^0_{4/3}$	$(0,0,0,-\frac{1}{4},\frac{1}{4})$	✗
$\bar{e^c} u_i \omega^{\dagger i}_1 + h.c.$	ω^i_1: $(3,1)^0_{1/3}$	$(0,\frac{1}{4},-\frac{1}{4},0,0)$	✔
$\bar{e}\gamma_\mu u_i \mathcal{U}^{\dagger i\mu}_5 + h.c.$	$\mathcal{U}^{i\mu}_5$: $(3,1)^1_{5/3}$	$(0,-\frac{1}{2},-\frac{1}{2},0,0)$	✗
$(\sin\theta\, \bar{e}\gamma_\mu e + \cos\theta\, \bar{u}_i\gamma_\mu u_i)\mathcal{B}^\mu$	$\mathcal{B}^\mu$: $(1,1)^1_0$	$(\sin^2\theta, 2\cos\theta\sin\theta, 0, 0, \cos^2\theta)$	✔
$\bar{u}_i\gamma_\mu u_j T^a_{ij} \mathcal{G}^{\dagger a\mu}$	$\mathcal{G}^{a\mu}$: $(8,1)^1_0$	$(0,0,0,-\frac{1}{4},-\frac{5}{12})$	✗
$\frac{1}{M_{UV}} i\bar{e^c} \overleftrightarrow{D}^\mu u^i \mathcal{U}^{\dagger i}_{1\mu} + h.c.$	$\mathcal{U}^i_{1\mu}$: $(3,1)^1_{1/3}$	$(0,-\frac{3}{4},-\frac{1}{4},0,0)$	✔

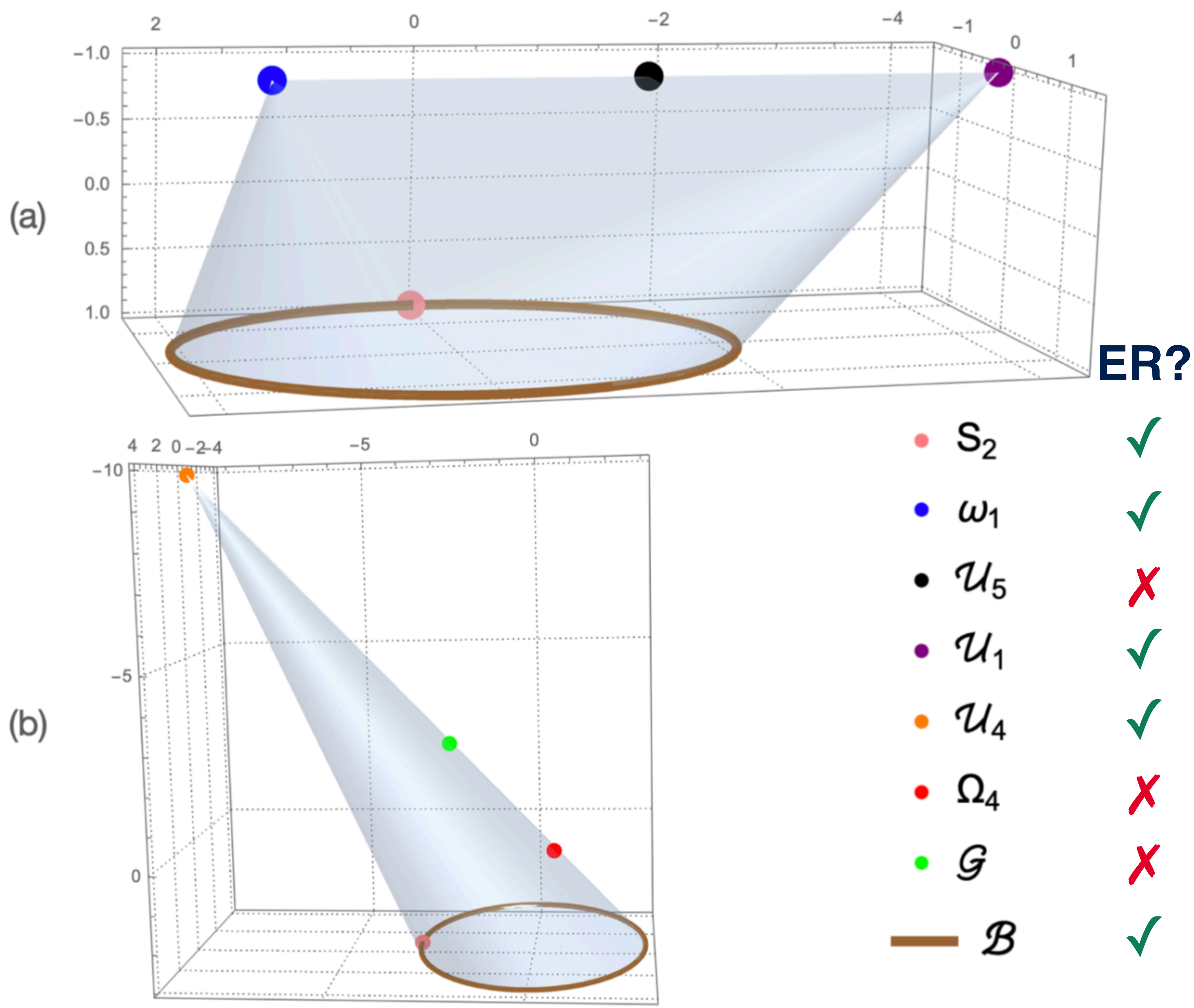
- Here, tree-level pERs span the full space of ERs, not generally the case
- One ER (B^μ) has a free parameter, it is continuous, must be eliminated

$$\Rightarrow \quad \begin{gathered} C_3 \le 0, \quad -3C_4 + C_5 \ge 0, \quad C_4 \le 0, \quad C_1 \ge 0, \\ -(2\sqrt{C_1(-3C_4+C_5)} - 3C_3) \le C_2 \le 2\sqrt{C_1(-3C_4+C_5)} - C_3 \end{gathered}$$

Visualising the cone

5D salient cone
w/ 6 4D faces

Need to project
down to $\leq$ 3D



Reverse-engineering the UV

Given a measurement of $\overrightarrow{C}_0$, what more can we say

- Combine the experimental likelihood with the positivity cone
- If $\overrightarrow{C}_0$ is extremal, we can pinpoint the UV origin
- What is the maximum allowed value of $w_H = g_H^2/M_H^4$ given the data?
- Lower bound on $M_H/\sqrt{g_h}$

$$\text{maximize}: \quad \lambda$$
$$\text{subject to}: \quad \vec{C} - \lambda \vec{c}_H \in \mathcal{C} \quad \text{and} \quad \chi^2(\vec{C}, \vec{C}_0) \leq \chi^2_c$$

Positivity + Likelihood

$$w_H \leq \lambda_{\text{max}}$$

Removing the H contribution, we should still remain in the positivity cone

- The value of λ that pushes $\overrightarrow{C}$ outside of the cone $\mathscr{C}$
- Since $\overrightarrow{c}_H$ is an ER of the cone, it provides a robust bound:

Independent of the presence/absence of other states!

[Zhang; arXiv:2112.11665]

Reverse-engineering the UV

Finite sensitivity to the WC space (uncertainties)

Take phenomenologically motivated sensitivities to $\vec{C}$

- $C_1 : (\bar{e}\gamma_\mu e)\,\partial^2\left(\bar{e}\gamma^\mu e\right)$, ILC 250 projection [Fuks et al.; Chin. phys. C 45 (2021) 023108]
- $C_2 : (\bar{e}\gamma_\mu e)\partial^2(\bar{u}\gamma^\mu u)$, LHC 8 TeV Drell Yan [Boughezal et al.; PRD 104 (2021) 095022]
- $C_3 : (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)$, our work (DY moments)

$$C_1 = 0 \pm 0.024, \quad C_2 = 0 \pm 0.45, \quad C_3 = 0 \pm 0.37$$

- $C_{4,5}$ no bounds available, take ± 0, ± 10

$$B : (\sin^2\theta, 2\cos\theta\sin\theta, 0, 0, \cos^2\theta) \qquad U_1 : (0, -\tfrac{3}{4}, -\tfrac{1}{4}, 0, 0)$$

$$S_2 : (1,0,0,0,0) \qquad \Omega_4 : (0,0,0,-\tfrac{1}{4},\tfrac{1}{4}) \qquad U_5 : (0,-\tfrac{1}{2},-\tfrac{1}{2},0,0)$$

$$U_4 : (0,0,0,-\tfrac{1}{2},-\tfrac{3}{2}) \qquad \omega_1 : (0,\tfrac{1}{4},-\tfrac{1}{4},0,0) \qquad \mathcal{G} : (0,0,0,-\tfrac{1}{4},-\tfrac{5}{12})$$

Reverse-engineering the UV

$\overrightarrow{C}_0 = \overrightarrow{0}$, observe SM:

$\delta C_{4,5} = 0$

UV particle H	$\lambda_{\max}$ [TeV^{-4}]	$\frac{M_H}{\sqrt{g_H}}$ [TeV])
$\mathcal{S}_2$	0.0015	≥ 5.1
$\mathcal{U}_4$	0	∞
Ω_4	0	∞
ω_1	0.090	≥ 1.8
$\mathcal{U}_5$	0.045	≥ 2.2
$\mathcal{B}$	0	∞
$\mathcal{G}$	0	∞
$\mathcal{U}_1$	0.049	≥ 2.1

$\delta C_{4,5} = 10$

UV particle H	$\lambda_{\max}$ [TeV^{-4}]	$\frac{M_H}{\sqrt{g_H}}$ [TeV]
$\mathcal{S}_2$	0.0015	≥ 5.1
$\mathcal{U}_4$	1.2	≥ 0.95
Ω_4	1.1	≥ 0.97
ω_1	0.092	≥ 1.8
$\mathcal{U}_5$	0.046	≥ 2.2
$\mathcal{B}$	0.00075	≥ 6.1
$\mathcal{G}$	2.5	≥ 0.80
$\mathcal{U}_1$	0.092	≥ 1.8

Inject $U_1, \Lambda = 2\,\text{TeV}, g_U = 1$

UV particle H	$\lambda_{\max}$ [TeV^{-4}]	$\frac{M_H}{\sqrt{g_H}}$ [TeV]
$\mathcal{S}_2$	0.0015	≥ 5.1
$\mathcal{U}_4$	0	∞
Ω_4	0	∞
ω_1	0.10	≥ 1.7
$\mathcal{U}_5$	0.10	≥ 1.7
$\mathcal{B}$	0	∞
$\mathcal{G}$	0	∞
$\mathcal{U}_1$	0.17	≥ 1.5

Inject $\omega_1, \Lambda = 2\,\text{TeV}, g_\omega = 1$

UV particle H	$\lambda_{\max}$ [TeV^{-4}]	$\frac{M_H}{\sqrt{g_H}}$ [TeV]
$\mathcal{S}_2$	0.0015	≥ 5.1
$\mathcal{U}_4$	0	∞
Ω_4	0	∞
ω_1	0.22	≥ 1.5
$\mathcal{U}_5$	0.053	≥ 2.1
$\mathcal{B}$	0	∞
$\mathcal{G}$	0	∞
$\mathcal{U}_1$	0.053	≥ 2.1

Conclusions

$l \geq 3$ moments in Drell Yan probe interesting Dim-8 effects

- Connected to elastic/extremal positivity bounds
- We can use them to experimentally test axiomatic principles of QFT in the UV

$O(1)$ sensitivity to relevant WCs for $\Lambda = 2\,\text{TeV}$

- Full information of dim-8 amplitudes encoded in $l = 4$ moments
- Required for closed fit when profiling over d.o.f.
- Possible higher order effects should be investigated carefully

Sensitivity to positivity violation around $\Delta \sim 2\,\text{TeV}$

- Cannot conclusively observe violation in full generality (7D)

Positivity cone + data $\Rightarrow$ interesting bounds on UV models

Backup

Positivity for pedestrians

Set of theoretical constraints on scattering amplitudes

- Apply to a subset of $D \geq 8$ Wilson coefficients

Result from basic assumptions about UV QFT/S-matrix

- Lorentz invariance, unitarity, causality & locality

  [Pham & Troung; PRD 31 (1985 3027)]
  [Anathanarayan et al.; PRD 51 (1995) 1093-1100]
  [Adams et al.; JHEP 10 (2006) 014]

  + many more in recent years…

Unitarity ⇔ conservation of probability in full theory

- Generalised optical theorem: scattering amplitude $\mathscr{M}_{ij\to kl}$ satisfies

$$\frac{1}{2i}\left(\mathscr{M}_{ij\to kl} - \mathscr{M}^*_{kl\to ij}\right) = \frac{1}{2}\sum_X \int d\Pi_X \, \mathscr{M}_{ij\to X}\mathscr{M}^*_{kl\to X}$$

Forward limit: $\mathrm{Im}\mathscr{M}_{ij\to ij} = \frac{1}{2}\sum_X \int d\Pi_X \, |\mathscr{M}_{ij\to X}|^2 > 0$ ⇒ Elastic positivity bounds

Positivity for pedestrians

Tree-level dimension 8 $\Rightarrow$ highest growth: s^2, st, t^2

- We will be taking 2 derivatives of w.r.t. $s \Rightarrow$ set $t = 0$ w.l.o.g.

Causality $\Rightarrow \mathcal{M}(s, t = 0)$ analytic in the complex s plane

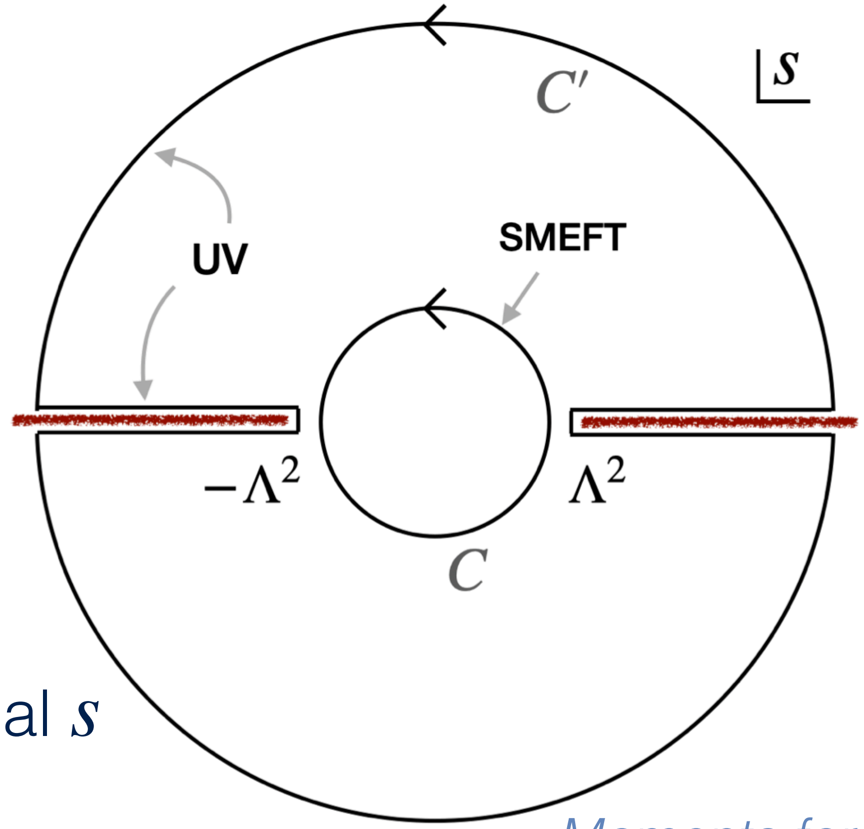
- Only poles & branch cuts on the real axis
- Define a pole subtracted amplitude: $M_{ijkl} \equiv \mathcal{M}_{ij\to kl} - (\text{low energy poles})$

Cauchy's intergral formula

$$\frac{1}{2}\frac{d^2 M_{ijkl}(s)}{ds^2} = \oint_C \frac{d\mu}{2\pi i}\frac{M_{ijkl}(\mu)}{(\mu - s)^3}$$

Avoiding UV branch cuts

- Deform contour to infinity $\Rightarrow C'$
- C' = 2 semi-circles + discontinuities along real s



Positivity for pedestrians

Infinite semi-circle contributions vanish

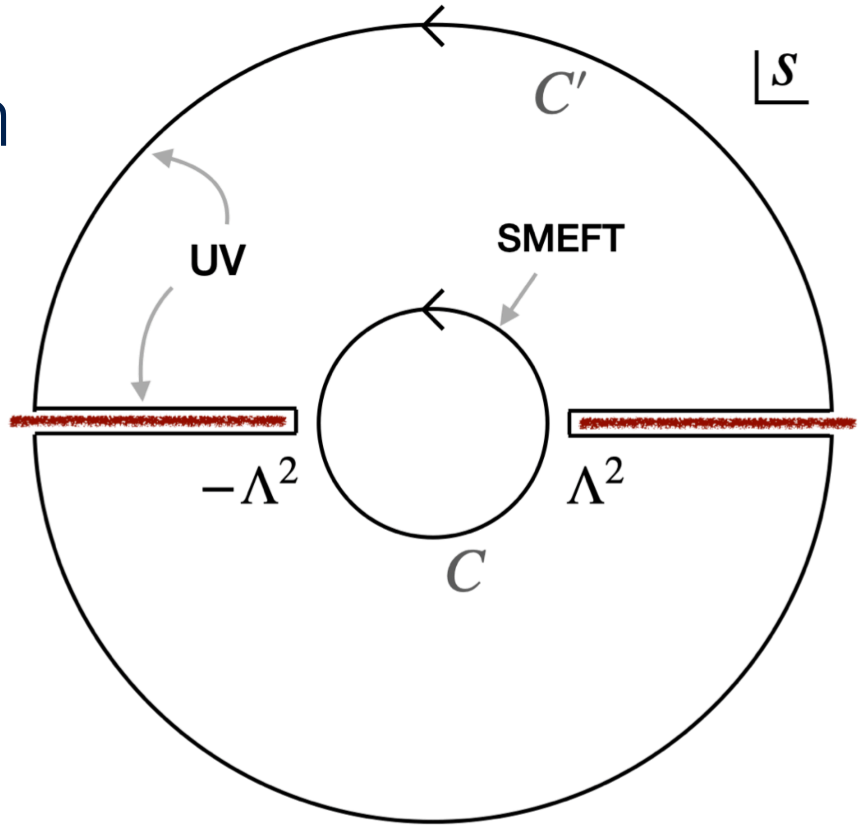
- Froissart bound for unitary & local amplitude

$$M \lesssim s \log^2 s, \;\; s \to \infty$$

[Froissart; Phys. Rev. 123 (1961) 1053-1057]

$$\Rightarrow \frac{1}{2}\frac{d^2 M_{ijkl}(s)}{ds^2} = \int_{-\infty}^{\infty} \frac{d\mu}{2\pi i} \frac{\mathrm{Disc}[M_{ijkl}(\mu)]}{(\mu - s)^3}$$

$$\mathrm{Disc}[f(s)] \equiv f(s + i\epsilon) - f(s - i\epsilon)$$



"Twice subtracted" dispersion relation

For small s:

LHS: approximated by EFT amplitude (IR)

RHS: integral to infinity: full amplitude (UV)

Positivity for pedestrians

$\Lambda \gg v$: take SM particles massless

- Crossing symmetry in massless, forward limit: $M_{ijkl}(\mu) = M_{i\tilde{l}k\tilde{j}}(-\mu)$
- No discontinuities in M below $s = \Lambda^2$

$$\frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2} = \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi i \mu^3}\Big(\mathrm{Disc}[M_{ijkl}(\mu)] + \mathrm{Disc}[M_{i\tilde{l}k\tilde{j}}(\mu)]\Big)$$

Recall:

$$\frac{1}{2i}\left(\mathcal{M}_{ij\to kl} - \mathcal{M}^*_{kl\to ij}\right) = \frac{1}{2}\sum_X \int d\Pi_X\, \mathcal{M}_{ij\to X}\mathcal{M}^*_{kl\to X}$$

$$\mathcal{M}^*_{kl\to ij}(s,t) = \mathcal{M}_{ij\to kl}(s^*,t) \quad \Rightarrow \quad \frac{1}{2i}\mathrm{Disc}[\mathcal{M}_{ij\to kl}] = \frac{1}{2}\sum_X \int d\Pi_X\, \mathcal{M}_{ij\to X}\mathcal{M}^*_{kl\to X}$$

- Generalised optical theorem + twice subtracted dispersion relation:

$$\frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi i \mu^3}\left(m_{ij}m^*_{kl} + m_{i\tilde{l}}m^*_{k\tilde{j}}\right) \qquad m_{ij} \equiv M_{ij\to X}(\mu)$$

Individual bounds on C_i

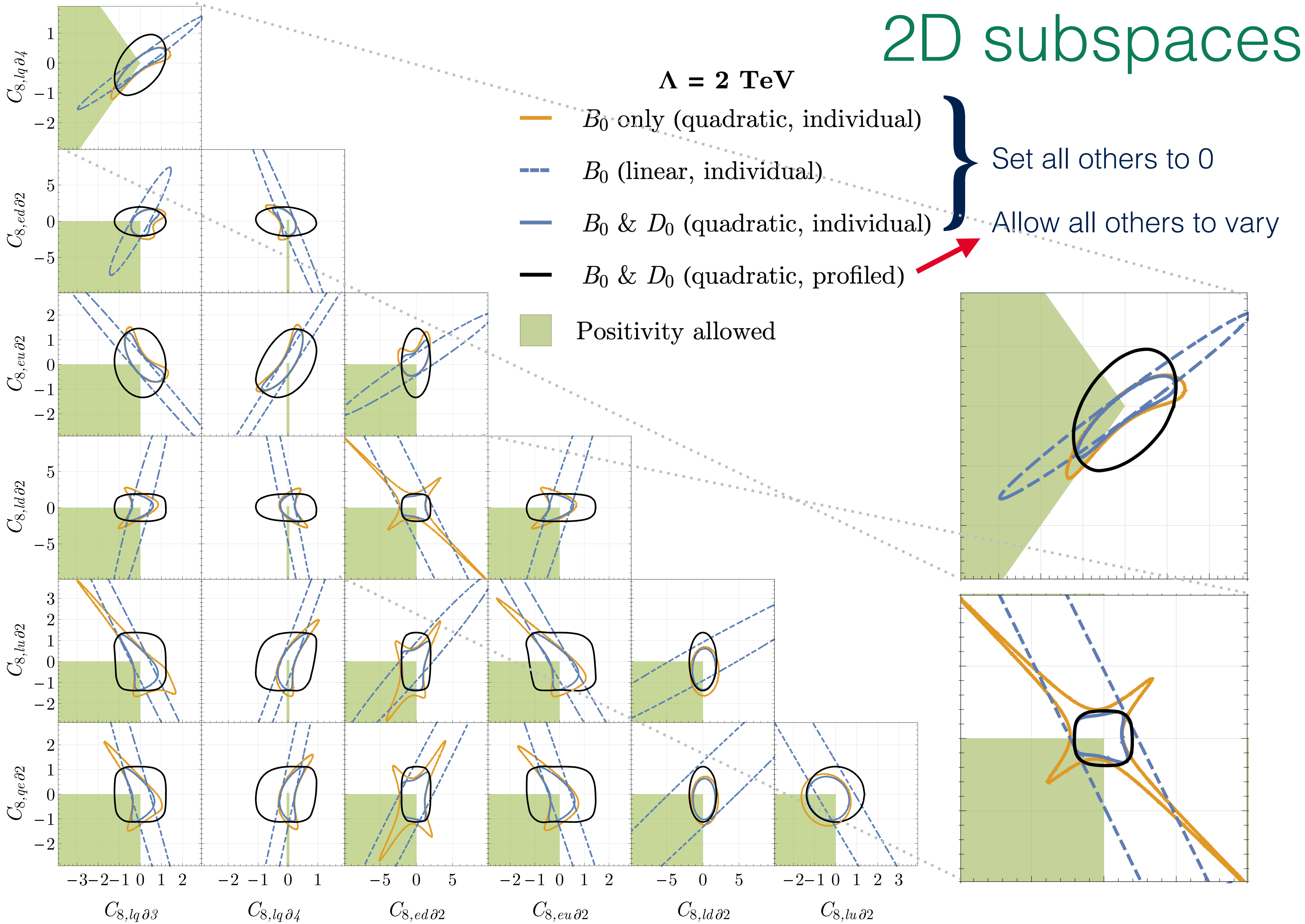
Breakdown:

Coeff.	1D lin.	1D lin.+quad.	2D lin.	2D lin.+quad.	$+D_0$
$C_{8,lq\partial 3}$	$(-0.46, 0.46)$	$(-0.38, 0.72)$	$(-0.41, 0.41)$	$(-0.35, 0.55)$	$(-0.32, 0.44)$
$C_{8,lq\partial 4}$	$(-0.23, 0.23)$	$(-0.24, 0.22)$	$(-0.21, 0.21)$	$(-0.22, 0.20)$	$(-0.20, 0.18)$
$C_{8,ed\partial 2}$	$(-2.2, 2.2)$	$(-2.2, 1.2)$	$(-2.0, 2.0)$	$(-2.1, 1.1)$	$(-1.8, 1.1)$
$C_{8,eu\partial 2}$	$(-0.44, 0.44)$	$(-0.39, 0.55)$	$(-0.40, 0.40)$	$(-0.36, 0.48)$	$(-0.33, 0.41)$
$C_{8,ld\partial 2}$	$(-4.4, 4.4)$	$(-1.4, 1.9)$	$(-4.1, 4.1)$	$(-1.3, 1.9)$	$(-1.1, 1.5)$
$C_{8,lu\partial 2}$	$(-0.89, 0.89)$	$(-1.5, 0.61)$	$(-0.81, 0.81)$	$(-1.5, 0.57)$	$(-1.2, 0.5)$
$C_{8,qe\partial 2}$	$(-1.3, 1.3)$	$(-1.2, 0.64)$	$(-1.2, 1.2)$	$(-1.1, 0.61)$	$(-0.93, 0.54)$

- Weakest constraints for operators that mediate $d\bar{d}$, luminosity suppressed
- For others, quadratic impact below 50%
- D_0 inclusion leads to 10-20% improvement

Taking $C_i = 1$, scales probed: $1.7 - 3\,\mathrm{TeV}$

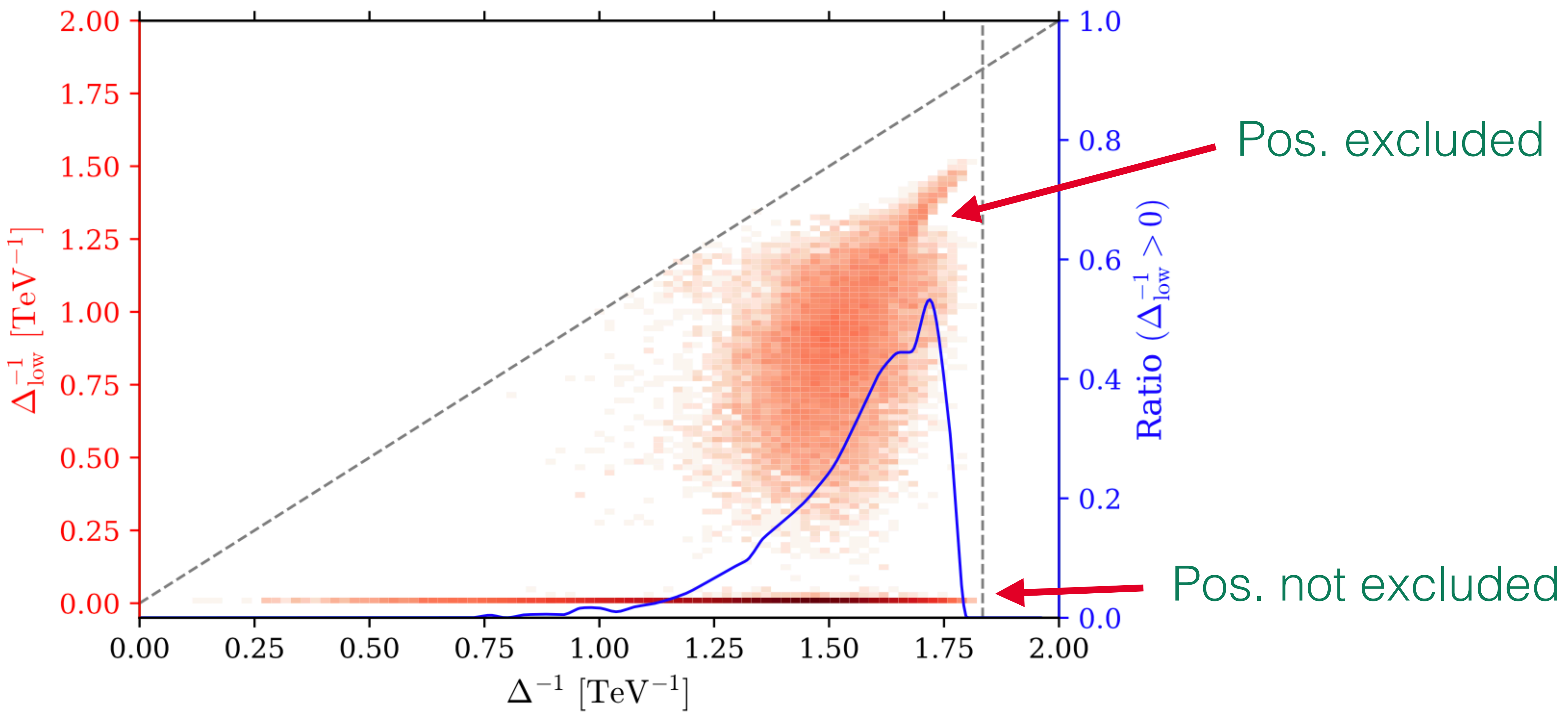
2D subspaces



Testing positivity

7D case: does the allowed region intersect positivity region?

- $\Delta^{-1} = [\Delta^{-1}_{\rm low}, \Delta^{-1}_{\rm high}]$, $\Delta_{\rm low}$ gives conservative estimate (highest scale)
- Uniformly sample a ball of radius 2, with $\Lambda = 1\,\mathrm{TeV}$



Constructing the cone

$m_{ij} \equiv M_{ij \to X}$: ER $\Rightarrow$ m_{ij} cannot be decomposed into other m'_{ij}

- X is an irrep. in the product decomposition of i, j's symmetry reps.
- Lorentz & internal symmetries
- Construct ERs from Glebsch-Gordan coefficients $\quad m_{ij} \propto C^{r,\alpha}_{i,j}$

projector: $$m_{ij}m^*_{kl} \propto P^r_{ijkl} = \sum_\alpha C^{r,\alpha}_{i,j}(C^{r,\alpha}_{k,l})^*$$

- $m_{ij}m^*_{kl} + m_{i\tilde{l}}m^*_{k\tilde{j}}$ $\Rightarrow$ take j, l symmetric projector
- Some ERs become non-ERs

Find potential ERs (pERs) and geometrically pick out ERs

$$(m_{ij}m^*_{kl} + m_{i\tilde{l}}m^*_{k\tilde{j}}) \propto P^r_{i(j|k|l)} = \sum_\alpha C^{r,\alpha}_{i,(j|}(C^{r,\alpha}_{k,|l)})^*$$

Constructing the cone

Alternative: enumerate all possible tree-level completions

- Integrate them out to get the WC vectors for the pERs
- Often easier in practice

Simplified example: new physics only couples to e_R, u_R

$$C_1 : (\bar{e}\gamma_\mu e)\,\partial^2\,(\bar{e}\gamma^\mu e) \qquad C_4 : (\bar{u}\gamma_\mu \overleftrightarrow{D}_\nu u)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)$$

$$C_2 : (\bar{e}\gamma_\mu e)\,\partial^2\,(\bar{u}\gamma^\mu u) \qquad C_5 : (\bar{u}\gamma_\mu u)\,\partial^2\,(\bar{u}\gamma^\mu u)$$

$$C_3 : (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u) \equiv C_{8,eu\partial 2}$$

$$\vec{C} = (C_1, C_2, C_3, C_4, C_5)$$

Integrate out a state, X $\Rightarrow$ $\dfrac{\vec{C}_X}{\Lambda^4} = w_X \vec{c}_X, \quad w_X = \dfrac{g_X^2}{M_X^4} \geq 0$

Generic UV completion: $\dfrac{\vec{C}}{\Lambda^4} = \sum_X \dfrac{\vec{C}_X}{\Lambda^4} = \sum_X w_X \vec{c}_X$