

# Exploding operators for Majorana neutrino masses

HEFT 2022, Granada

Mainly based on JHEP 01, 074 (2021) (arXiv: 2009.13537) with Raymond Volkas

**Dr John Gargalionis**

 <https://github.com/johngarg/neutrinomass>

arXiv <https://arxiv.org/abs/2009.13537>



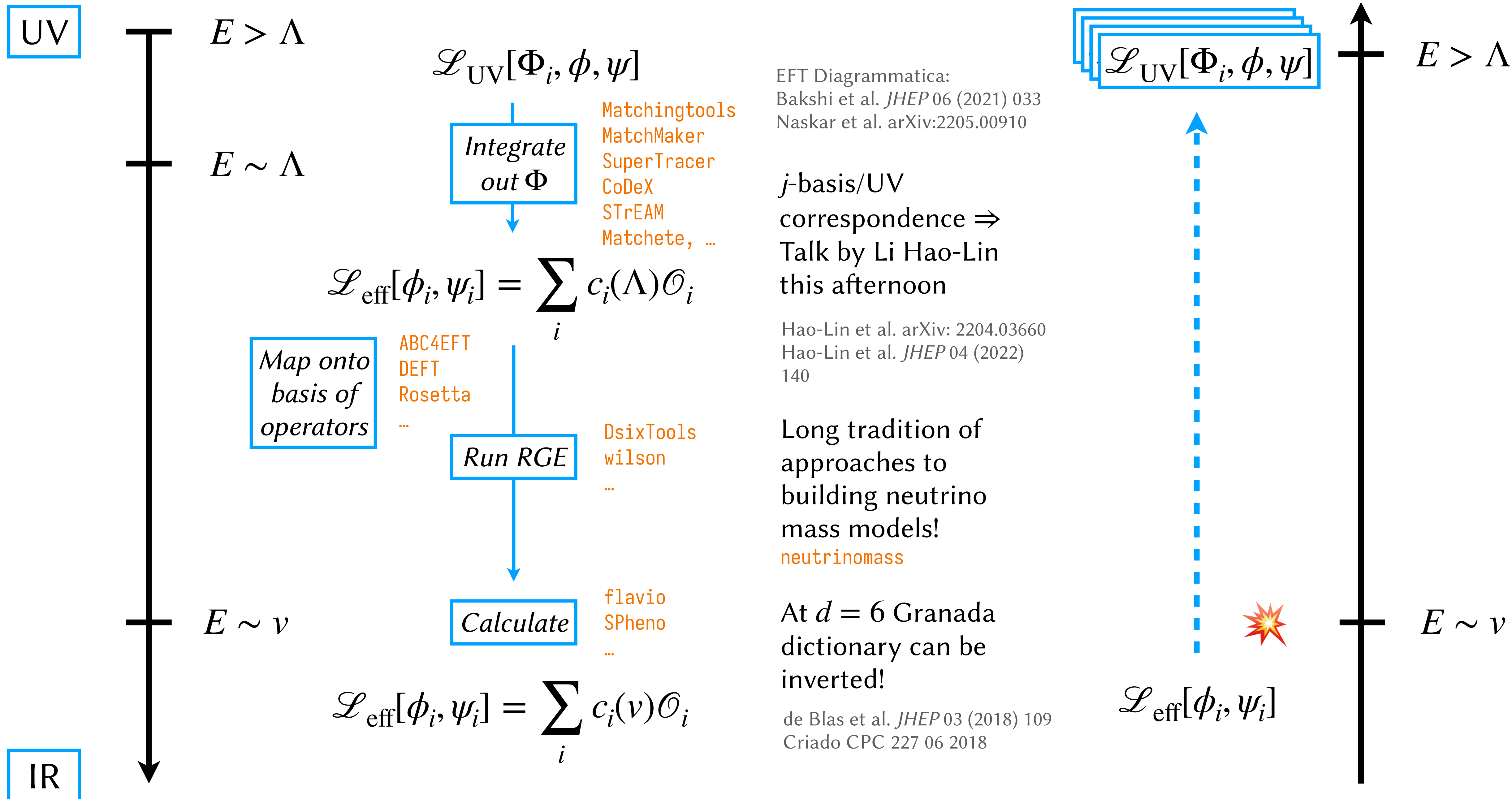
THE UNIVERSITY OF  
**MELBOURNE**



# Outline

- I. Motivation and introduction
- II. Automated model building from EFT
- III. Code and model database
- IV. Closing thoughts and future directions

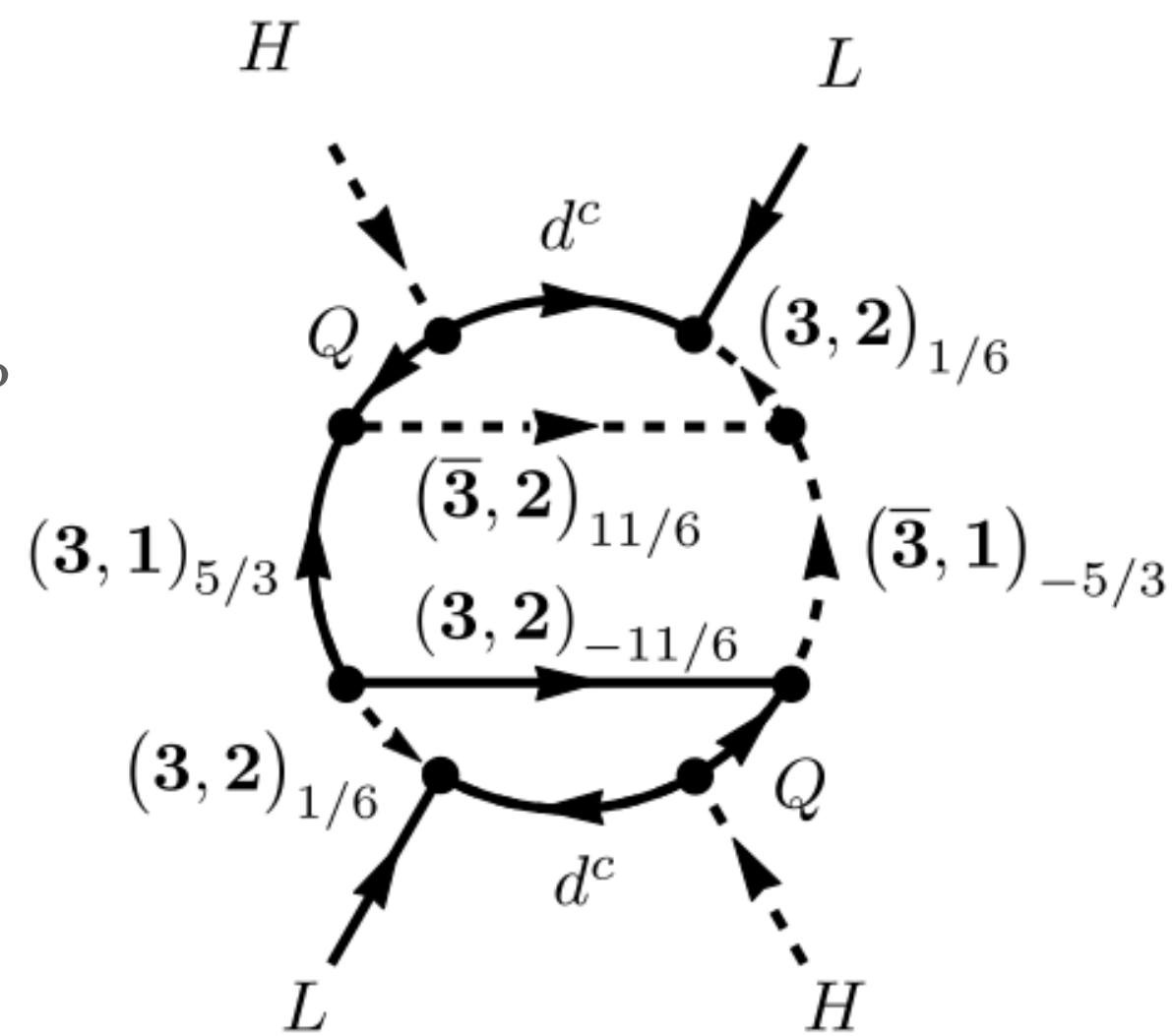
# **Motivation and introduction**



## Tree- and loop-level completions of the Weinberg-like operators

$$\mathcal{L}_W = \sum_n \frac{C_{5+2n}}{\Lambda^{2n+1}} \cdot LLHH(H^\dagger H)^n$$

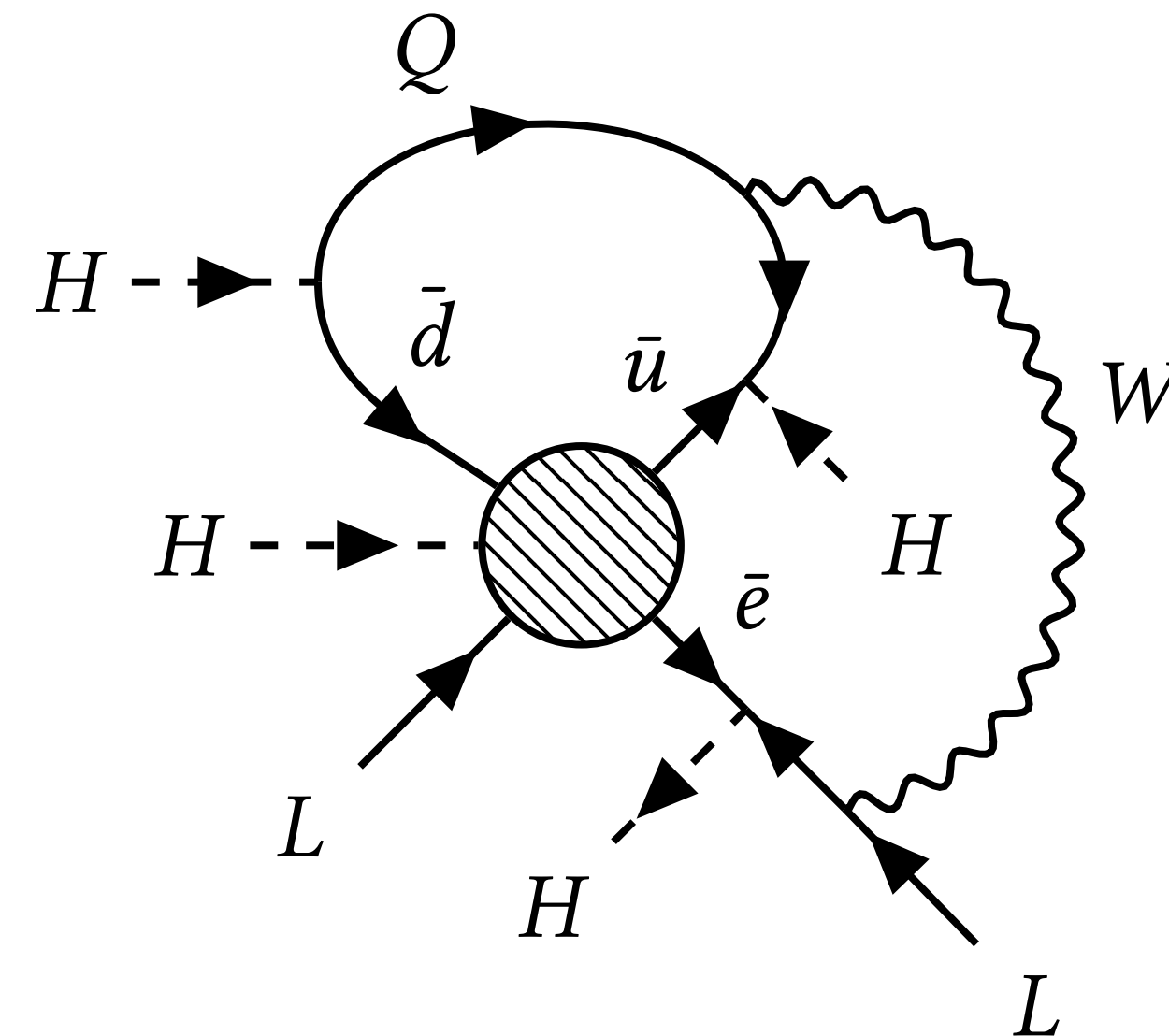
Bonnet et al. JHEP 07, 153 (2012);  
 Cepedello, Hirsch, Helo JHEP 07, 079 (2017); JHEP 01, 009 (2018)  
 Cepedello, Fonseca, Hirsch JHEP 10, 197 (2018);  
 Anamiati et al. JHEP 12, 066 (2018)



## Tree-level completions of $\Delta L = 2$ operators in the SMEFT

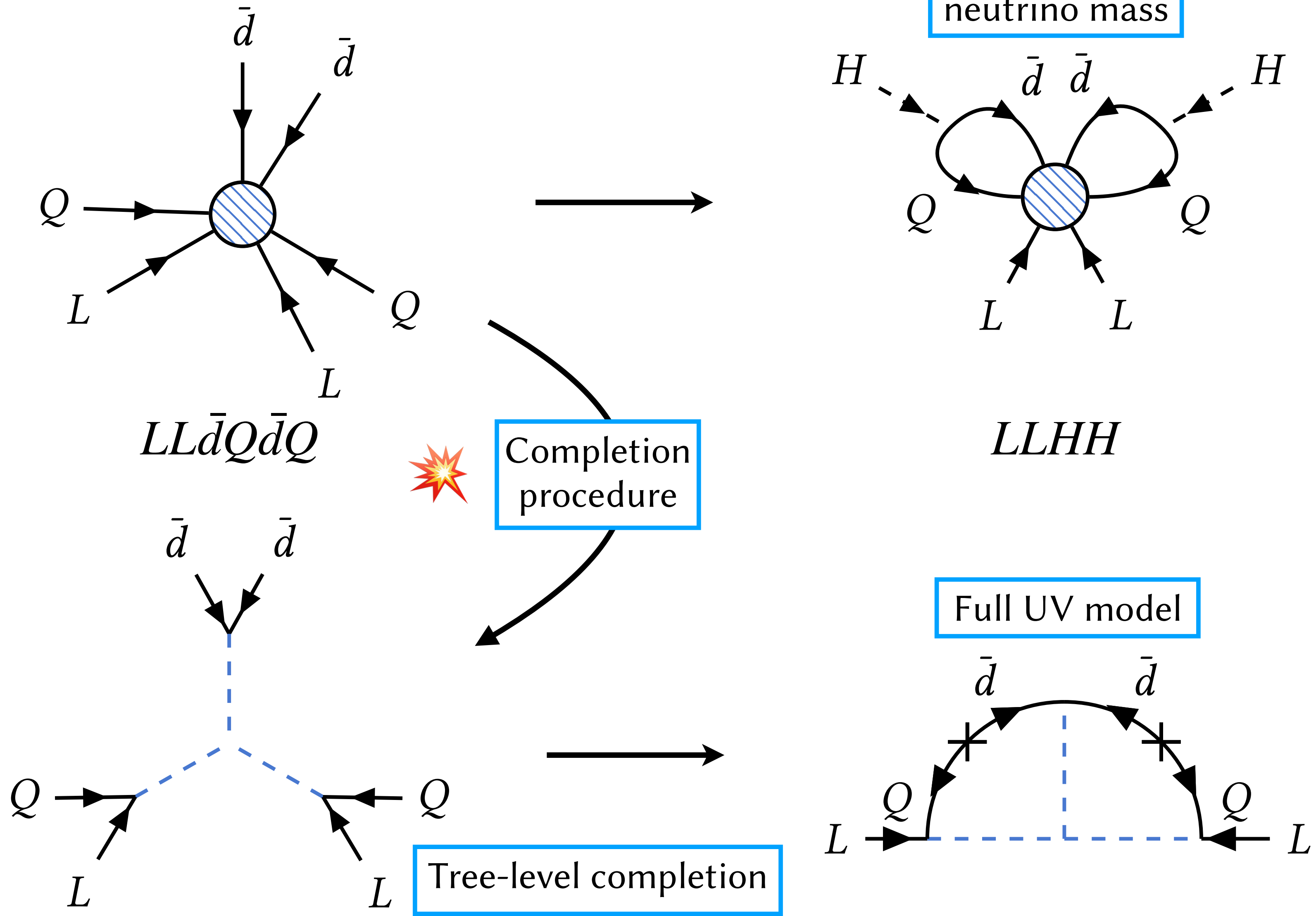
$$\mathcal{L}_{\Delta L=2} = \frac{C_5}{\Lambda} \cdot LLHH + \frac{C_7^{(1)}}{\Lambda^3} \cdot LLQ\bar{d}H + \frac{C_7^{(2)}}{\Lambda^3} \cdot L\bar{d}\bar{u}^\dagger \bar{e}^\dagger H + \dots$$

Kobach PLB 758 (2016)  
 Lehman PRD 90, 125023 (2014)  
 Ma, Liao JHEP 11 (2020) 152, ...



e.g.  
 Babu, Leung NPD 619, 667 (2001)  
 de Gouvêa, Jenkins PRD 77, 013008 (2008)  
 del Aguila et al. JHEP 06 146 (2012)  
 Herrero-García et al. JHEP 11 084 (2016)  
 Angel, Rodd, Volkas PRD 87, 073007 (2013)  
 Cai et al. JHEP 02 161 (2015)  
**Gargalionis, Volkas JHEP 01 074 (2021)**

Tree-level completion paradigm



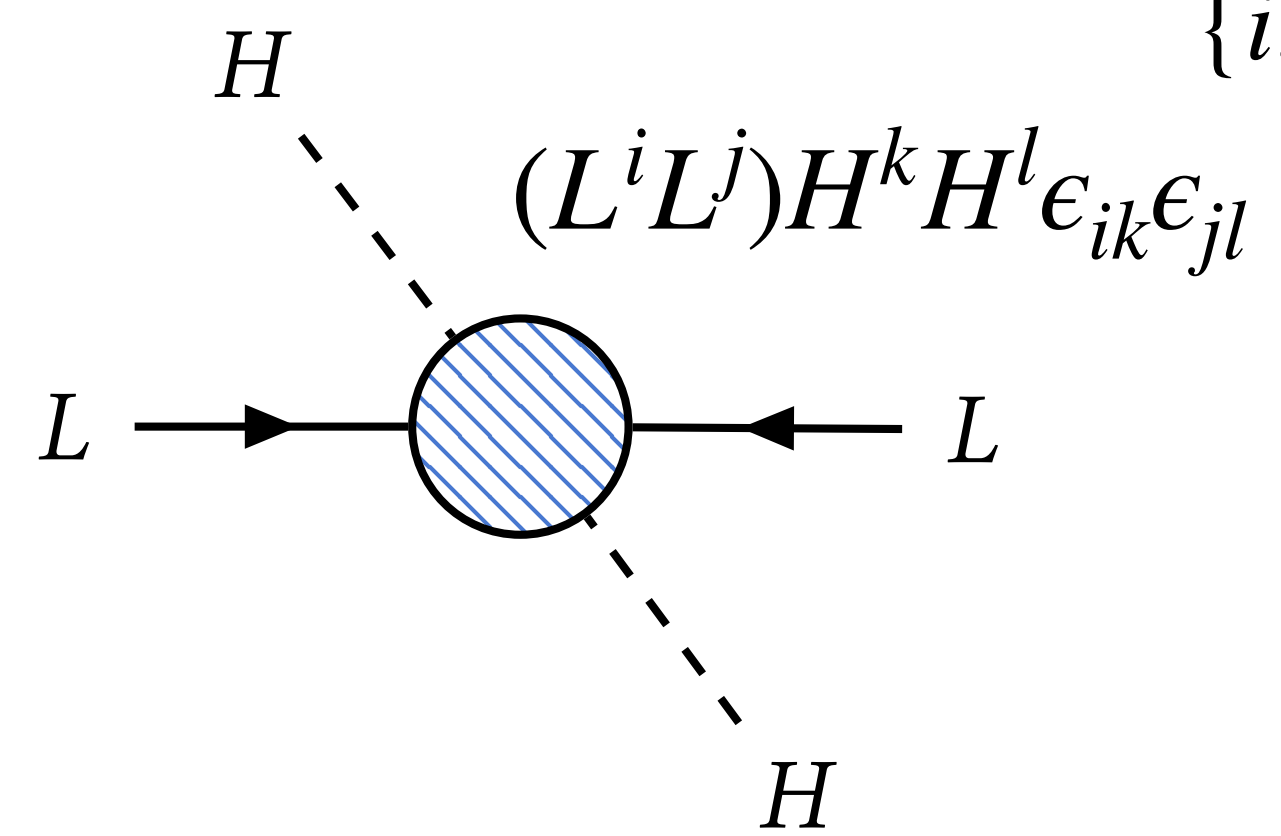
# **Automated model building from EFT**

# What does the UV physics look like?

P. Minkowski (1977)  
 T. Yanagida (1979)  
 M. Gell-Mann, P. Ramond, R. Slansky (1979)  
 R. Mohapatra, G. Senjanović (1980)  
 S. Glashow (1980)

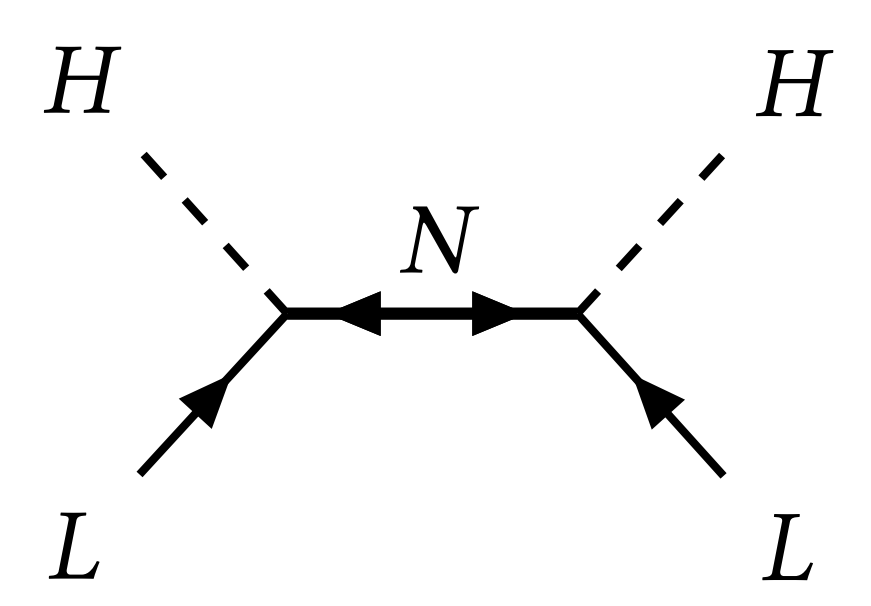
<i>Gauge</i>	<i>Lorentz</i>
$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	$SU(2)_+ \otimes SU(2)_-$
$\{i, j, \dots\}$	$\{\alpha, \beta, \dots\} \quad \{\dot{\alpha}, \dot{\beta}, \dots\}$

$$(\psi\chi) \equiv \psi^\alpha \chi^\beta \epsilon_{\alpha\beta}$$



$$N \sim (1, 1, 0)_{(2,1)}$$

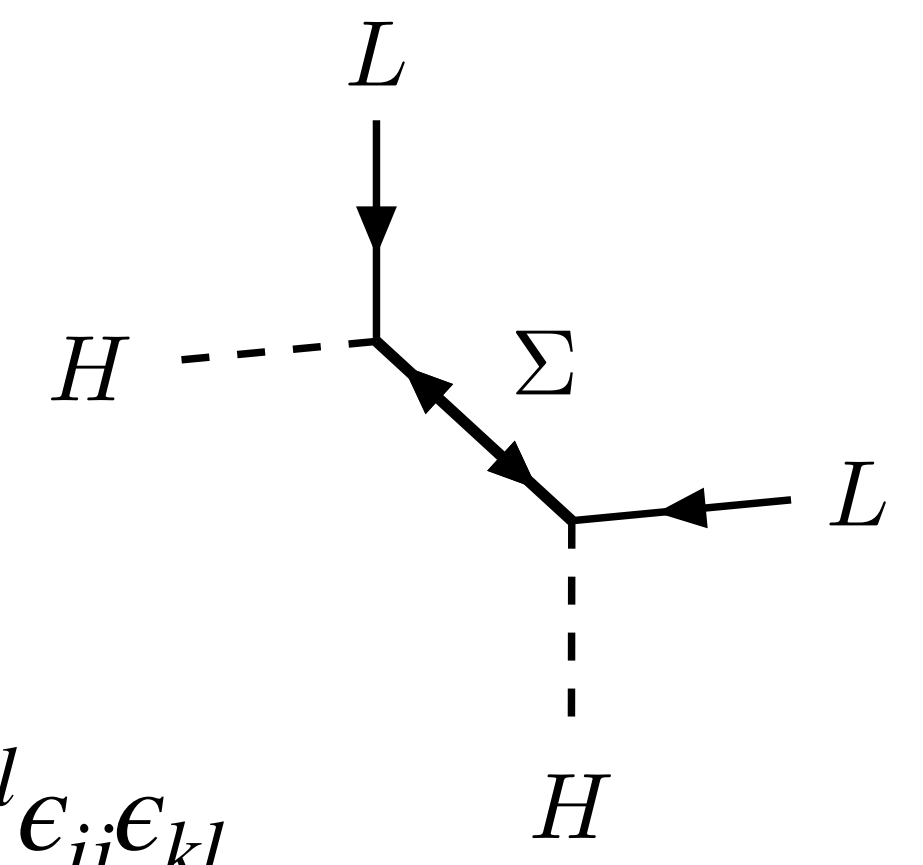
$$\mathcal{L}_N = y_N (L^i N) H^j \epsilon_{ij}$$



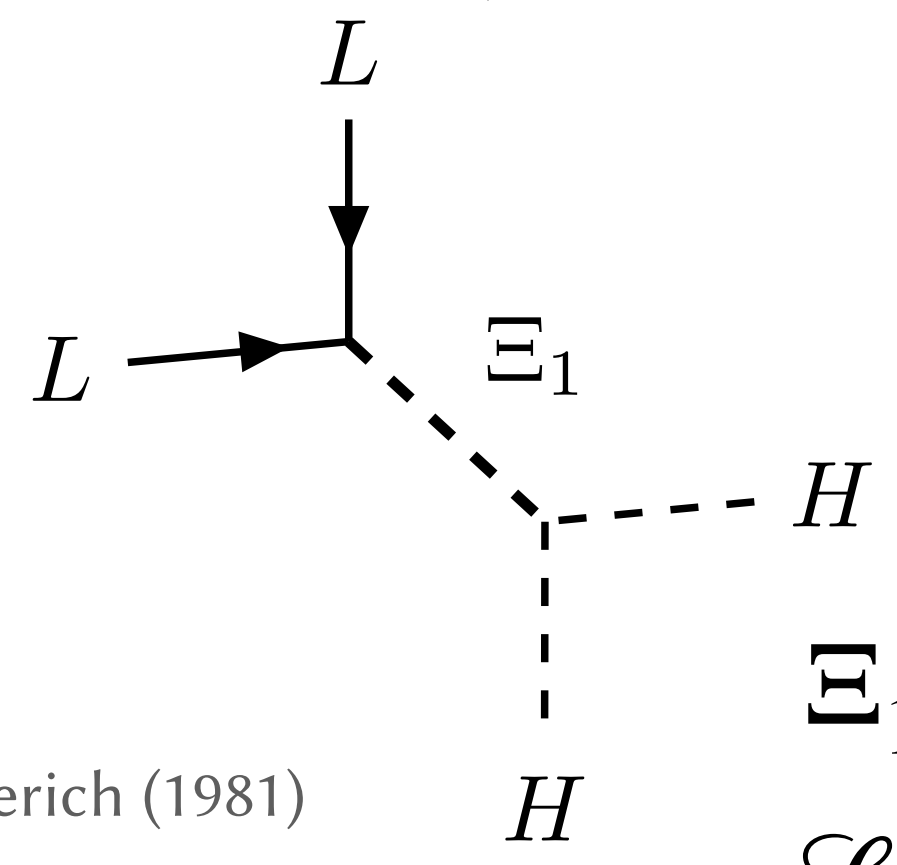
M. Magg, C. Wetterich (1980)  
 J. Schechter, J. Valle (1980)  
 T.-P. Cheng, L.-F. Li (1980)  
 G. Lazarides, Q. Shafi, C. Wetterich (1981)  
 C. Wetterich (1981)  
 R. Mohapatra, G. Senjanović (1981)

$$\Sigma \sim (1, 3, 0)_{(2,1)}$$

$$\mathcal{L}_\Sigma = y_\Sigma (L^i \Sigma^{jk}) H^l \epsilon_{ij} \epsilon_{kl}$$



R. Foot, X.-G. He, H. Lew, G. Joshi (1989)



$$\Xi_1 \sim (1, 3, 1)_{(1,1)}$$

$$\mathcal{L}_{\Xi_1} = y_{\Xi_1} (L^i L^j) \Xi_1^{kl} \epsilon_{ik} \epsilon_{jl} + \kappa_{\Xi_1} H^i H^j \Xi_1^\dagger_{ij}$$



# What does the UV physics look like?

Gauge

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

$\{i, j, \dots\}$

$$H \quad (L^i L^j) H^k H^l \epsilon_{ik} \epsilon_{jl}$$

Lorentz

$$+ SU(2)_+ \otimes SU(2)_-$$

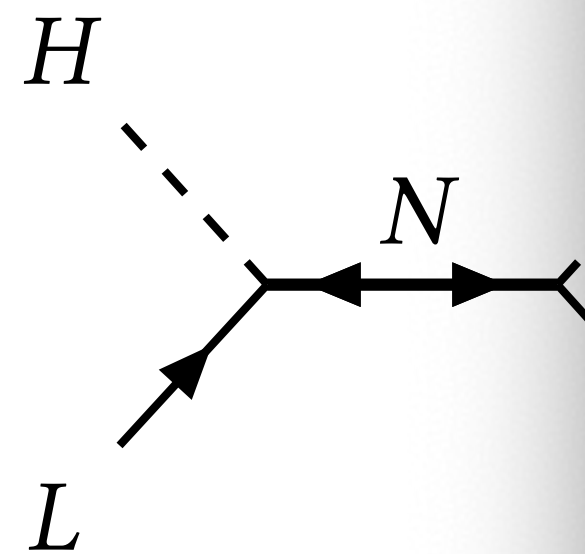
$\{\alpha, \beta, \dots\} \quad \{\dot{\alpha}, \dot{\beta}, \dots\}$

$$(\psi\chi) \equiv \psi^\alpha \chi^\beta \epsilon_{\alpha\beta}$$

- P. Minkowski (1977)
- T. Yanagida (1979)
- M. Gell-Mann, P. Ramond, R. Slansky (1979)
- R. Mohapatra, G. Senjanović
- S. Glashow (1980)

$$N \sim (1, 1, 0)_{(2,1)}$$

$$\mathcal{L}_N = y_N (L^i N)$$



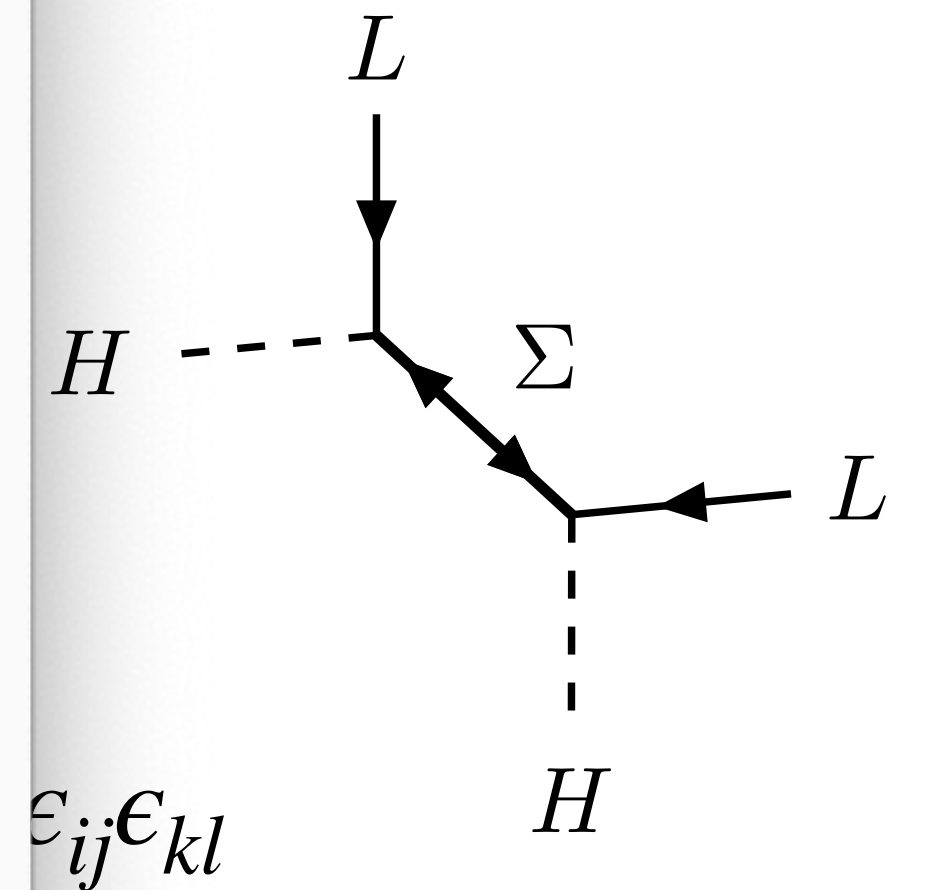
```
#!/usr/bin/env python

from neutrinomass.tensormethod import L, H, eps
from neutrinomass.completions import completions

op = L("u0 i0") * L("u1 i1") * H("i2") * H("i3") \
     * eps("-u0 -u1") * eps("-i0 -i2") * eps("-i1 -i3")

models = completions(op)
# => [Model(S(1, 3, 1)(0)), Model(F(1, 1, 0)(0)), Model(F(1, 3, 0)(0))]

# models[0].lagrangian
# models[0].effective_lagrangian
# models[0].symmetries
# models[0].diagram
```



H. Lew, G. Joshi (1989)

$$+ \kappa_{\Xi_1} H^i H^j \Xi_{1ij}^\dagger$$

Tree-level matching backwards

$\Phi \rightarrow$  heavy field  
 $\phi, \psi \rightarrow$  light fields

$$\mathcal{L}_{\text{HE}}[\Phi, \phi, \psi]$$

$E > M$

$$\mathcal{L}_{\Phi} = -\Phi^{\dagger}(D^2 + M^2)\Phi + \Phi^{\dagger} \cdot \frac{\partial \mathcal{L}}{\partial \Phi^{\dagger}} + \mathcal{O}(\Phi^3)$$

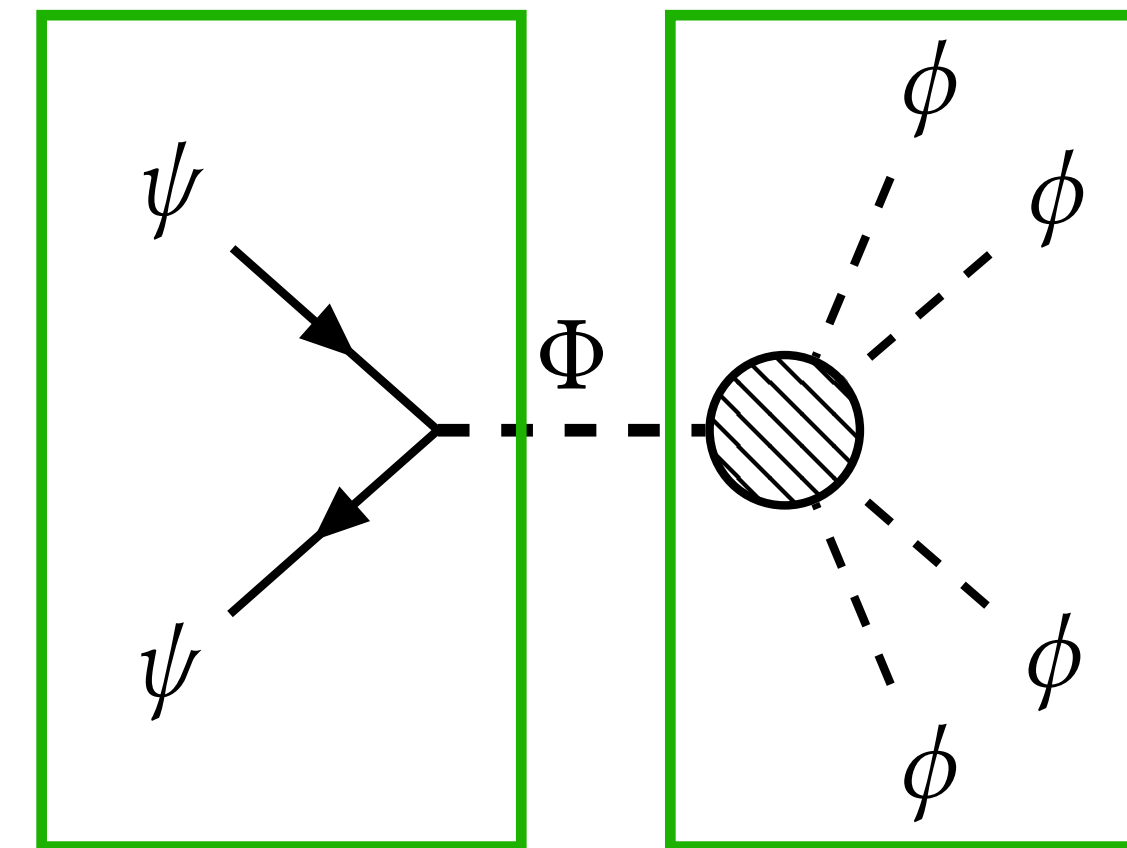
Replace  
by  
EOM

$E < M$

$$\mathcal{L}_{\text{eff}} = \frac{\partial \mathcal{L}}{\partial \Phi} \cdot \frac{1}{M^2} \left[ 1 + \frac{(iD)^2}{M^2} + \dots \right] \cdot \frac{\partial \mathcal{L}}{\partial \Phi^{\dagger}}$$

Fierz  
EOM  
IBP

$$\sum_i c_i \mathcal{O}_i$$



One cannot talk about UV completions unambiguously **without defining some kind of a basis of operators**

Use a general spanning set of operators:  
**Green's basis**, implicit Lorentz structure...

Tree-level matching backwards

$\Phi \rightarrow$  heavy field  
 $\phi, \psi \rightarrow$  light fields

$E > M$

$$\mathcal{L}_{\text{HE}}[\Phi, \phi, \psi]$$

$E < M$

$$\mathcal{L}_{\Phi} = -\Phi^\dagger(D^2 + M^2)\Phi + \Phi^\dagger \cdot \frac{\partial \mathcal{L}}{\partial \Phi^\dagger} + \mathcal{O}(\Phi^3)$$

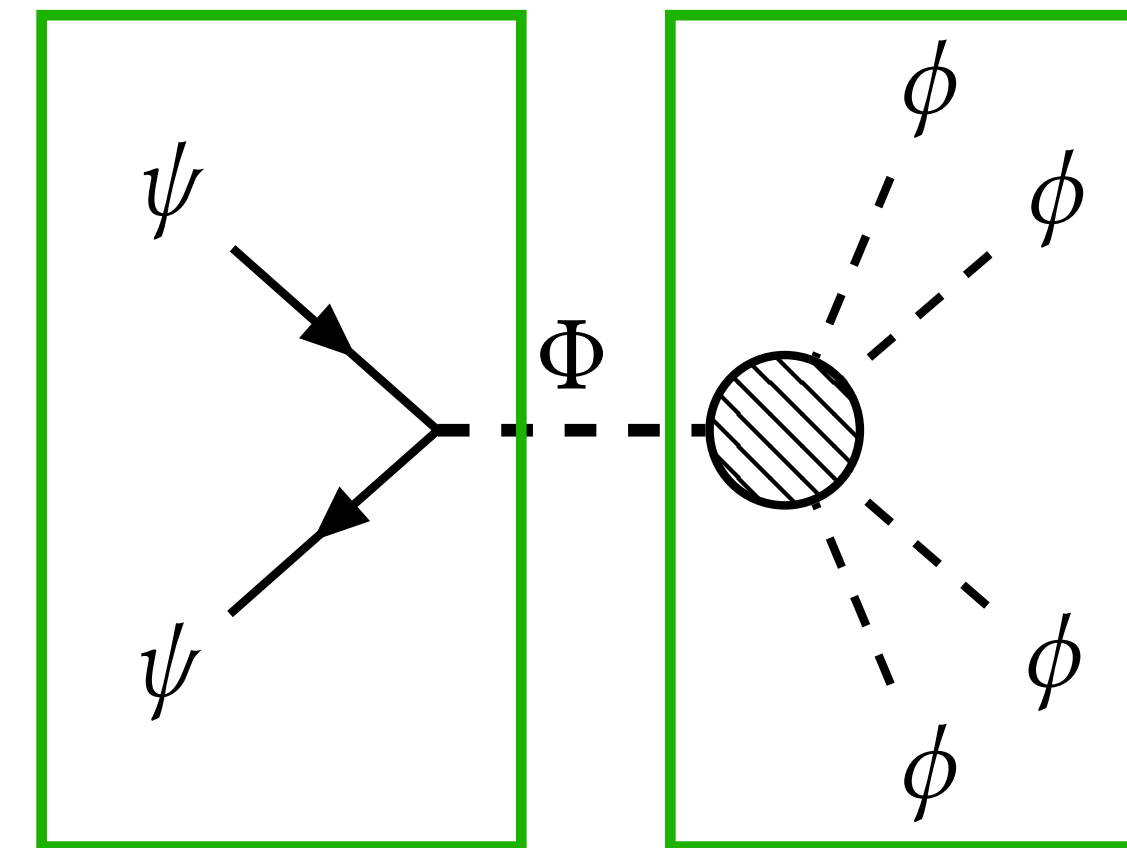
$$\mathcal{L}_{\text{eff}} = \frac{\partial \mathcal{L}}{\partial \Phi} \cdot \frac{1}{M^2} \left[ 1 + \frac{(iD)^2}{M^2} + \dots \right] \cdot \frac{\partial \mathcal{L}}{\partial \Phi^\dagger}$$

Fill in all possible  $\frac{\partial \mathcal{L}}{\partial \Phi^\dagger}$

$+ \mathcal{O}_2 + \dots$

$$\sum_i c_i \mathcal{O}_i$$

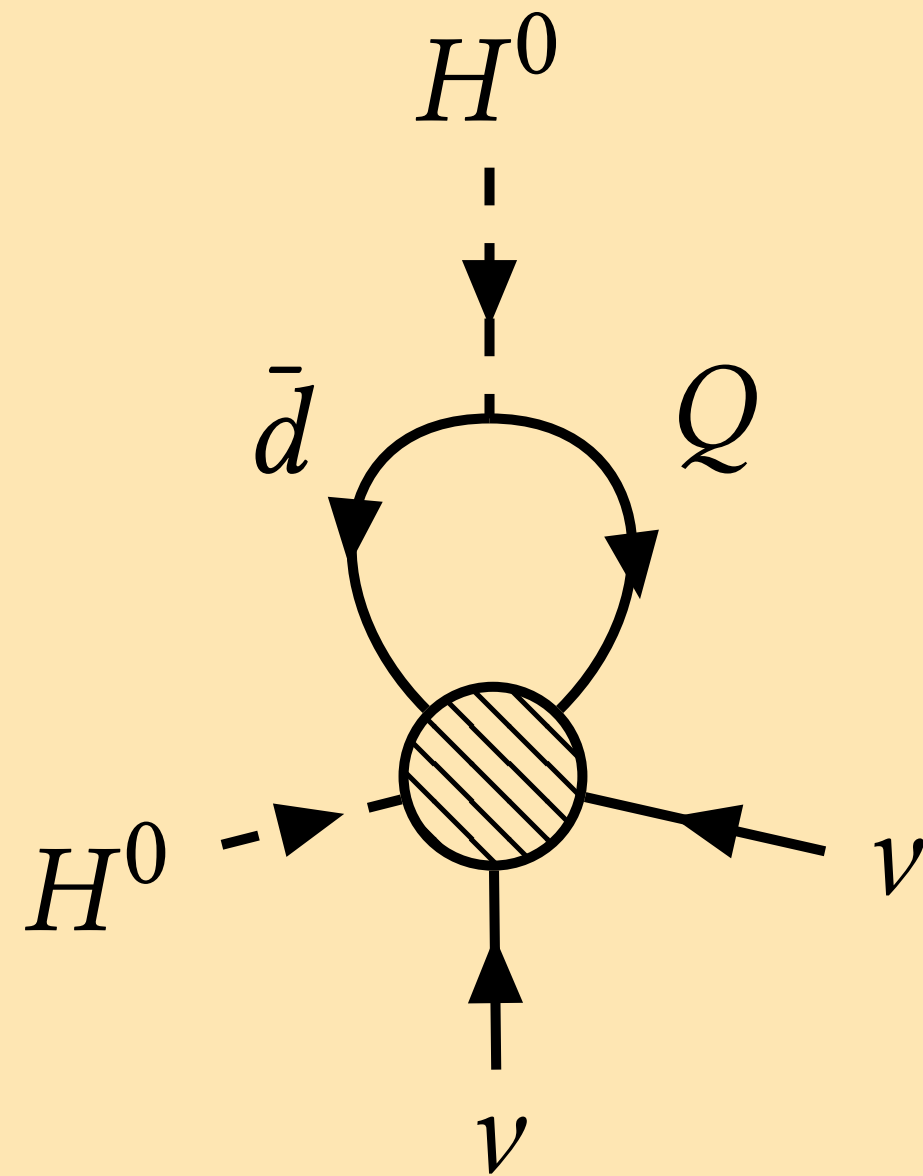
$\mathcal{O}_1$



One cannot talk about UV completions unambiguously **without defining some kind of a basis of operators**

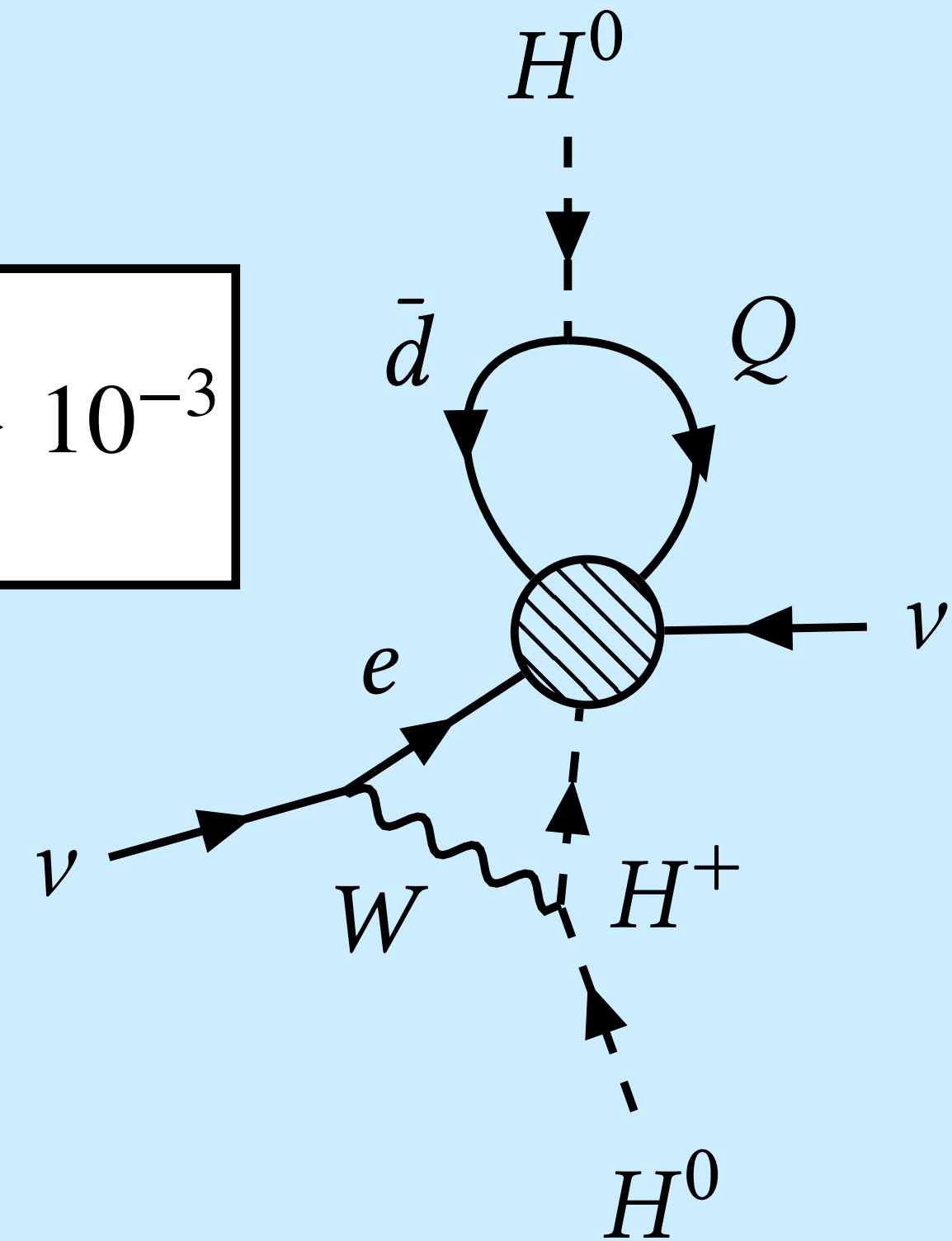
Use a general spanning set of operators:  
**Green's basis**, implicit Lorentz structure...

Want general spanning set of operators, for  $m_\nu$  we care most about distinguishing  $SU(2)_L$  structures. E.g.  $\mathcal{O}_3 = LLQ\bar{d}H$



$$\mathcal{O}_{3b} = (L^i Q^k)(L^j \bar{d})H^l \cdot \epsilon_{ik}\epsilon_{jl}$$

$$\frac{m_\nu^{(3a)}}{m_\nu^{(3b)}} \sim \frac{C_{3a}}{C_{3b}} \cdot g^2 \frac{1}{16\pi^2} \rightarrow 10^{-3}$$

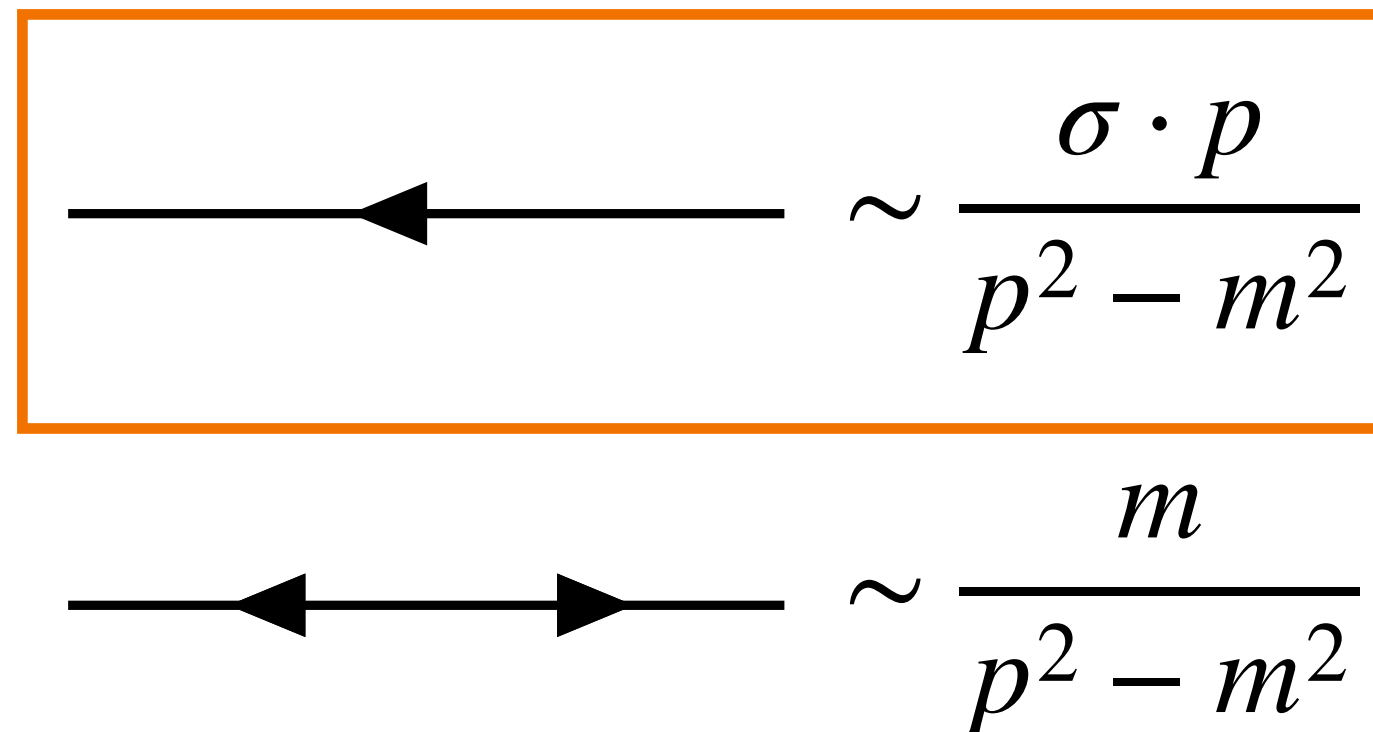


$$\mathcal{O}_{3a} = (L^i Q^k)(L^j \bar{d})H^l \cdot \epsilon_{ij}\epsilon_{kl}$$

# Operators

Updated list of  $\sim 150$  distinguished operators up to  $d = 11$  including those **with derivatives**, kept when:

- Tree-level topology can accommodate as many **arrow-preserving** fermion propagators
- Structure like  $H^i(DH)^j\epsilon_{ij}$  present



Labels	Operator	Models	Filtered	Loops	$\Lambda$ [TeV]
1	$L^i L^j H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	3	3	0	$6 \cdot 10^{11}$
2	$L^i L^j L^k \bar{e} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	8	2	1	$4 \cdot 10^7$
3a	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ij} \epsilon_{kl}$	9	2	2	$2 \cdot 10^5$
3b	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	14	5	1	$9 \cdot 10^7$
4a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ik} \epsilon_{jl}$	5	0	1	$4 \cdot 10^9$
4b	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ij} \epsilon_{kl}$	4	2	2	$10 \cdot 10^6$
5a	$L^i L^j Q^k \bar{d} H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	790	36	2	$6 \cdot 10^5$
5b	$\mathcal{O}_1 \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	492	14	1,2	$6 \cdot 10^5$
5c	$\mathcal{O}_{3a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	509	0	2,3	$1 \cdot 10^3$
5d	$\mathcal{O}_{3b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	799	16	1,2	$6 \cdot 10^5$
6a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	289	14	2	$2 \cdot 10^7$
6b	$\mathcal{O}_1 \cdot \tilde{Q}^i \bar{u}^\dagger \tilde{H}^j \cdot \epsilon_{ij}$	177	0	1,2	$2 \cdot 10^7$
6c	$\mathcal{O}_{4a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	262	0	1,2	$2 \cdot 10^7$
6d	$\mathcal{O}_{4b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	208	0	2,3	$6 \cdot 10^4$
7	$L^i \bar{e}^\dagger Q^j \tilde{Q}^k H^l H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	240	15	2	$2 \cdot 10^5$
8	$L^i \bar{e}^\dagger \bar{u}^\dagger \bar{d} H^j \cdot \epsilon_{ij}$	5	1	2,3	$2 \cdot 10^1$
D16a	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	58	8	2	$2 \cdot 10^5$
D16b	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{il} \epsilon_{jk}$	58	8	2	$2 \cdot 10^5$
D16c	$L^i \bar{e}^\dagger \tilde{Q}^j \bar{u}^\dagger (DH)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	27	4	2	$2 \cdot 10^5$
D17	$\bar{e}^\dagger \bar{e}^\dagger \bar{u}^\dagger \bar{d} (DH)^i H^j \cdot \epsilon_{ij}$	16	7	3,4	$2 \cdot 10^{-1}$
D18a	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{ik} \epsilon_{jm} \epsilon_{ln}$	53	1	0,1	$4 \cdot 10^9$
D18b	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{ik} \epsilon_{jl} \epsilon_{mn}$	53	1	0,1	$4 \cdot 10^9$
D18c	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{im} \epsilon_{jl} \epsilon_{kn}$	53	1	0,1	$4 \cdot 10^9$
D18d	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{ij} \epsilon_{km} \epsilon_{ln}$	24	1	1,2	$10 \cdot 10^6$
D18e	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{in} \epsilon_{jl} \epsilon_{km}$	34	0	1	$4 \cdot 10^9$
D18f	$(DL)^i L^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	34	0	1	$4 \cdot 10^9$
D19a	$(D^2 L)^i L^j (D^2 H)^k H^l \cdot \epsilon_{ij} \epsilon_{kl}$	1	0	1	$2 \cdot 10^9$
D19b	$(D^2 L)^i L^j (D^2 H)^k H^l \cdot \epsilon_{il} \epsilon_{jk}$	3	3	0	$6 \cdot 10^{11}$
D19c	$(D^2 L)^i L^j (D^2 H)^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	3	3	0	$6 \cdot 10^{11}$
D20	$L^i \bar{e}^\dagger H^j H^k H^l (DH)^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	129	0	1,2	$2 \cdot 10^5$
D21	$(DL)^i (D\bar{e}^\dagger) H^j H^k (DH)^l \cdot \epsilon_{ik} \epsilon_{jl}$	2	0	1	$4 \cdot 10^7$
D22	$\bar{e}^\dagger \bar{e}^\dagger (DH)^i (DH)^j H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	9	0	2	$3 \cdot 10^3$

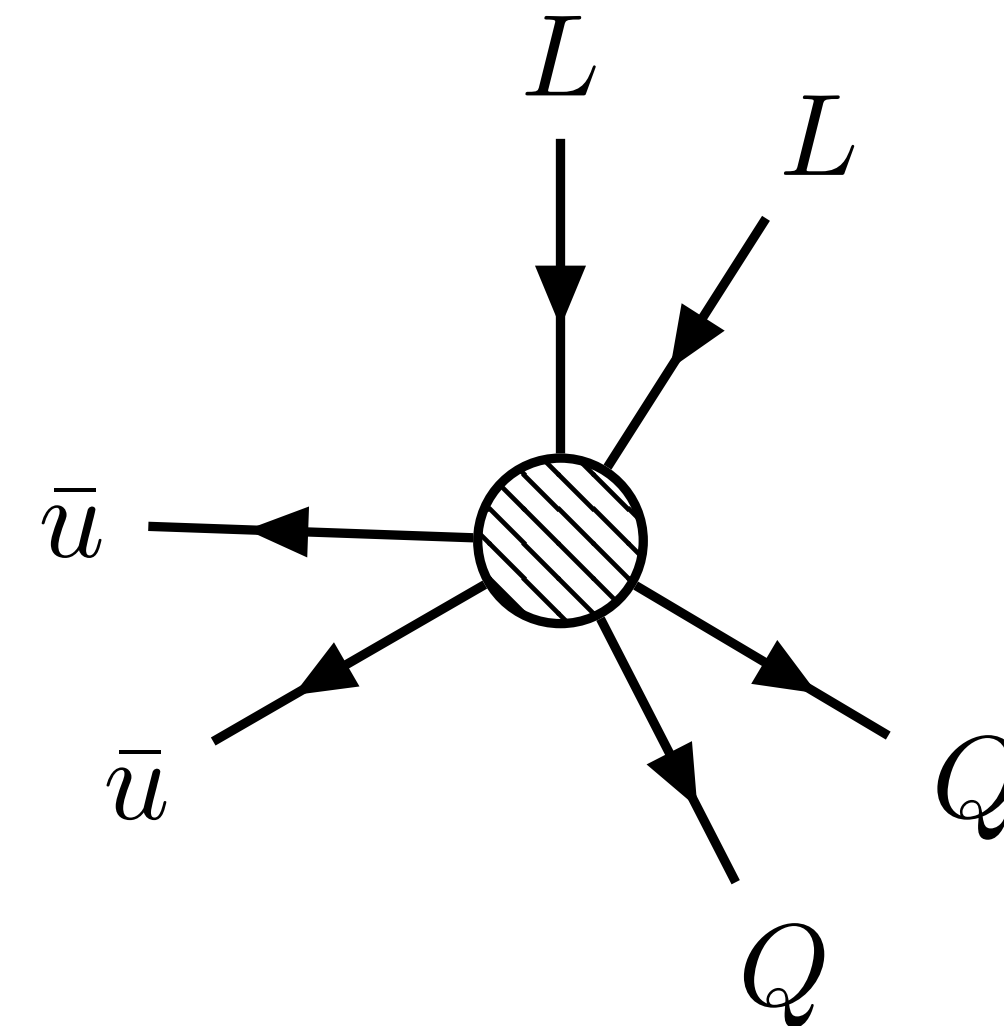
Example of completion algorithm

<i>Gauge</i>		<i>Lorentz</i>	
$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	$+$	$SU(2)_+ \otimes SU(2)_-$	
$\{a, b, \dots\}$	$\{i, j, \dots\}$	$\{\alpha, \beta, \dots\}$	$\{\dot{\alpha}, \dot{\beta}, \dots\}$

The procedure can be phrased as a kind of **abstract term-rewriting system**

$$\mathcal{L} = C \cdot L^i L^j Q_{ia}^\dagger Q_{jb}^\dagger \bar{u}^{\dagger a} \bar{u}^{\dagger b}$$

$$L^{\alpha i} L^{\beta j} Q_{ia}^{\dagger \dot{\alpha}} Q_{jb}^{\dagger \dot{\beta}} \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \dot{\delta} b}$$



Example of completion algorithm

<i>Gauge</i>	<i>Lorentz</i>
$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	$SU(2)_+ \otimes SU(2)_-$
$\{i, j, \dots\}$	$\{\alpha, \beta, \dots\} \quad \{\dot{\alpha}, \dot{\beta}, \dots\}$

The procedure can be phrased as a kind of **abstract term-rewriting system**

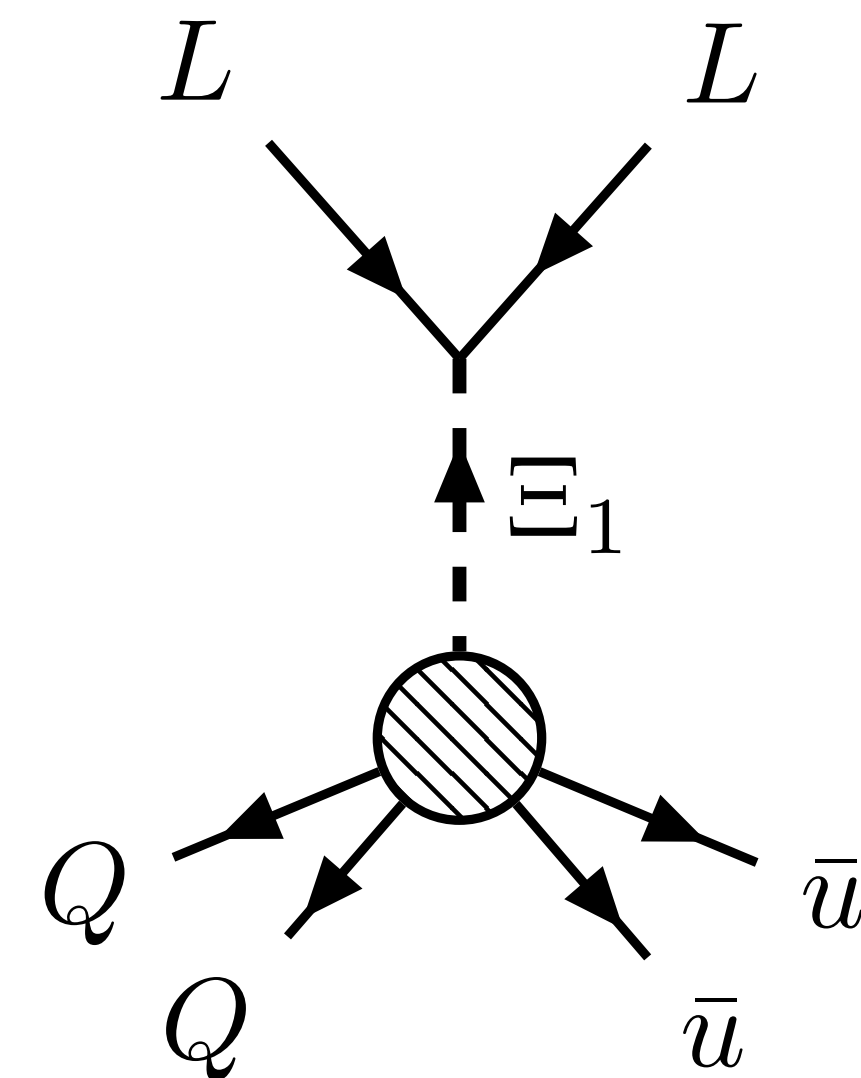
$$\mathcal{L} = y_{LL} \Xi_{1\{ij\}}^\dagger (L^i L^j) + C' \cdot \Xi_1^{\{ij\}} Q_{ia}^\dagger Q_{jb}^\dagger \bar{u}^{\dagger a} \bar{u}^{\dagger b}$$

$$L^{\alpha i} L^{\beta j} Q_{ia}^\dagger Q_{jb}^\dagger \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \delta b}$$

$$\rightarrow L^{\alpha i} L^{\beta j} Q_{ia}^\dagger Q_{jb}^\dagger \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \delta b} \quad + (i \leftrightarrow j)$$

$$\quad \quad \quad \cdot \epsilon_{\alpha\beta}$$

$\square$   
 $\Xi_1 \sim (1, 3, -1)_{(1,1)}$



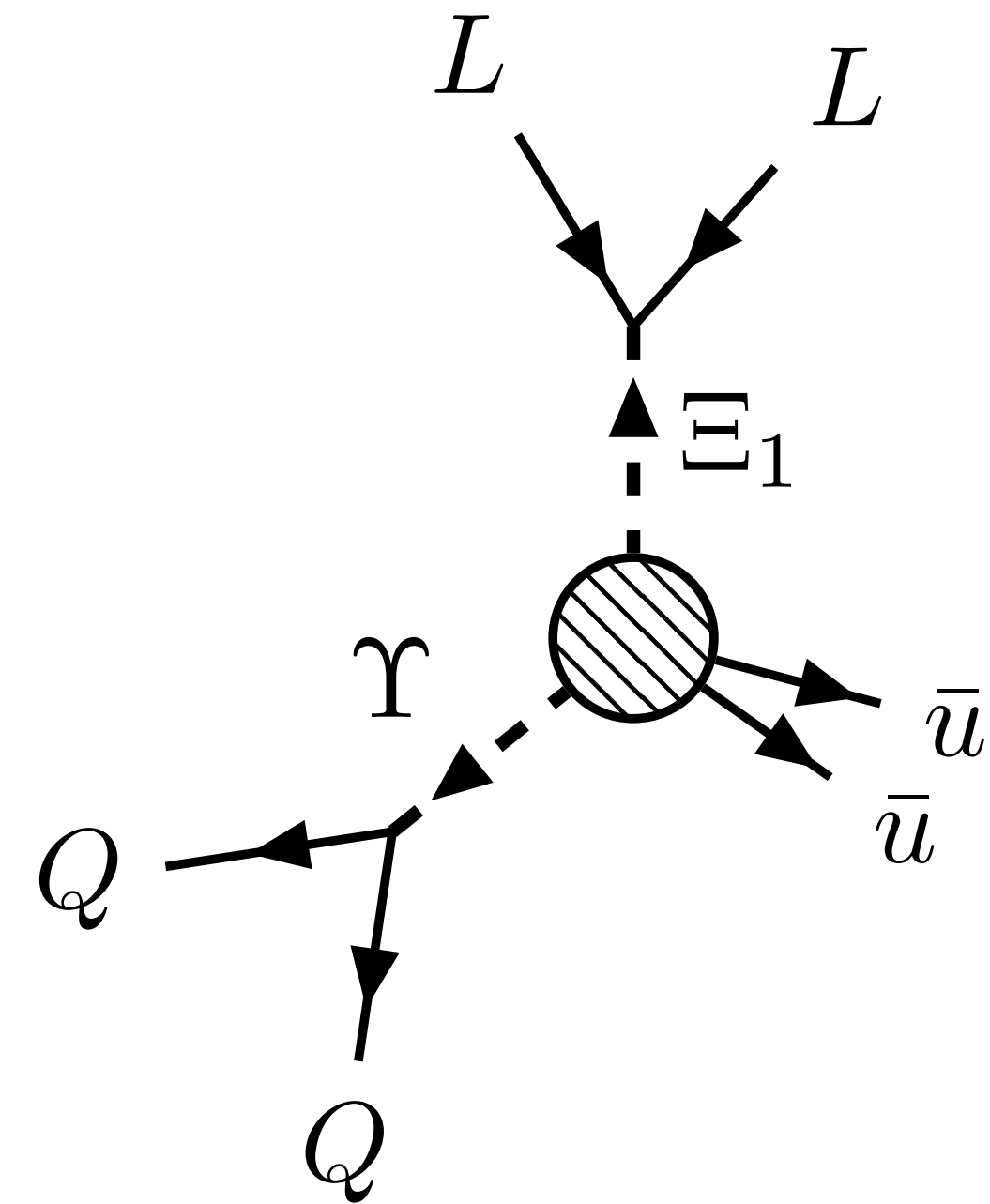
Example of completion algorithm

<i>Gauge</i>	<i>Lorentz</i>
$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	$SU(2)_+ \otimes SU(2)_-$
$\{i, j, \dots\}$	$\{\alpha, \beta, \dots\} \quad \{\dot{\alpha}, \dot{\beta}, \dots\}$

The procedure can be phrased as a kind of **abstract term-rewriting system**

$$\mathcal{L} = y_{QQ} \Upsilon^{\dagger\{ab\}\{ij\}} (Q_a^\dagger Q_b^\dagger) + y_{LL} \Xi_1^{\dagger\{ij\}} (L^i L^j) + C'' \cdot \Xi_1^{\{ij\}} \Upsilon_{ijab} \bar{u}^{\dagger a} \bar{u}^{\dagger b}$$

$$\begin{aligned}
 & L^{\alpha i} L^{\beta j} Q_{ia}^\dagger Q_{jb}^\dagger \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \delta b} \\
 & \rightarrow L^{\alpha i} L^{\beta j} Q_{ia}^\dagger Q_{jb}^\dagger \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \delta b} \quad + (i \leftrightarrow j) \\
 & \quad \underbrace{\hspace{1.5cm}} \quad \cdot \epsilon_{\alpha\beta} \\
 & \Xi_1 \sim (\mathbf{1}, \mathbf{3}, -1)_{(1,1)} \\
 & \rightarrow \Xi^{ij} Q_{ia}^\dagger Q_{jb}^\dagger \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \delta b} \quad + (a \leftrightarrow b) \\
 & \quad \underbrace{\hspace{1.5cm}} \quad \cdot \epsilon_{\dot{\alpha}\dot{\beta}} \\
 & \Upsilon \sim (\bar{\mathbf{6}}, \bar{\mathbf{3}}, -\frac{1}{3})_{(1,1)}
 \end{aligned}$$





Example of completion algorithm

<i>Gauge</i>	<i>Lorentz</i>
$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	$SU(2)_+ \otimes SU(2)_-$
$\{i, j, \dots\}$	$\{\alpha, \beta, \dots\} \quad \{\dot{\alpha}, \dot{\beta}, \dots\}$

The procedure can be phrased as a kind of **abstract term-rewriting system**

$$\mathcal{L} = y_{\bar{u}\bar{u}} \Omega_{4\{ab\}}^\dagger (\bar{u}^{\dagger a} \bar{u}^{\dagger b}) + y_{QQ} \Upsilon^{\dagger\{ab\}\{ij\}} (Q_a^\dagger Q_b^\dagger) + y_{LL} \Xi_{1\{ij\}}^\dagger (L^i L^j) + C''' \cdot \Xi_1^{\{ij\}} \Upsilon_{\{ab\}ij} \Omega_4^{ab}$$

$$L^{\alpha i} L^{\beta j} Q_{ia}^\dagger Q_{jb}^\dagger \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \delta b}$$

+ (i ↔ j)

$$\rightarrow L^{\alpha i} L^{\beta j} Q_{ia}^\dagger Q_{jb}^\dagger \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \delta b} \quad \cdot \epsilon_{\alpha\beta}$$

┌

$$\Xi_1 \sim (\mathbf{1}, \mathbf{3}, -1)_{(1,1)}$$

+ (a ↔ b)

$$\rightarrow \Xi_1^{ij} Q_{ia}^\dagger Q_{jb}^\dagger \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \delta b} \quad \cdot \epsilon_{\dot{\alpha}\dot{\beta}}$$

┌

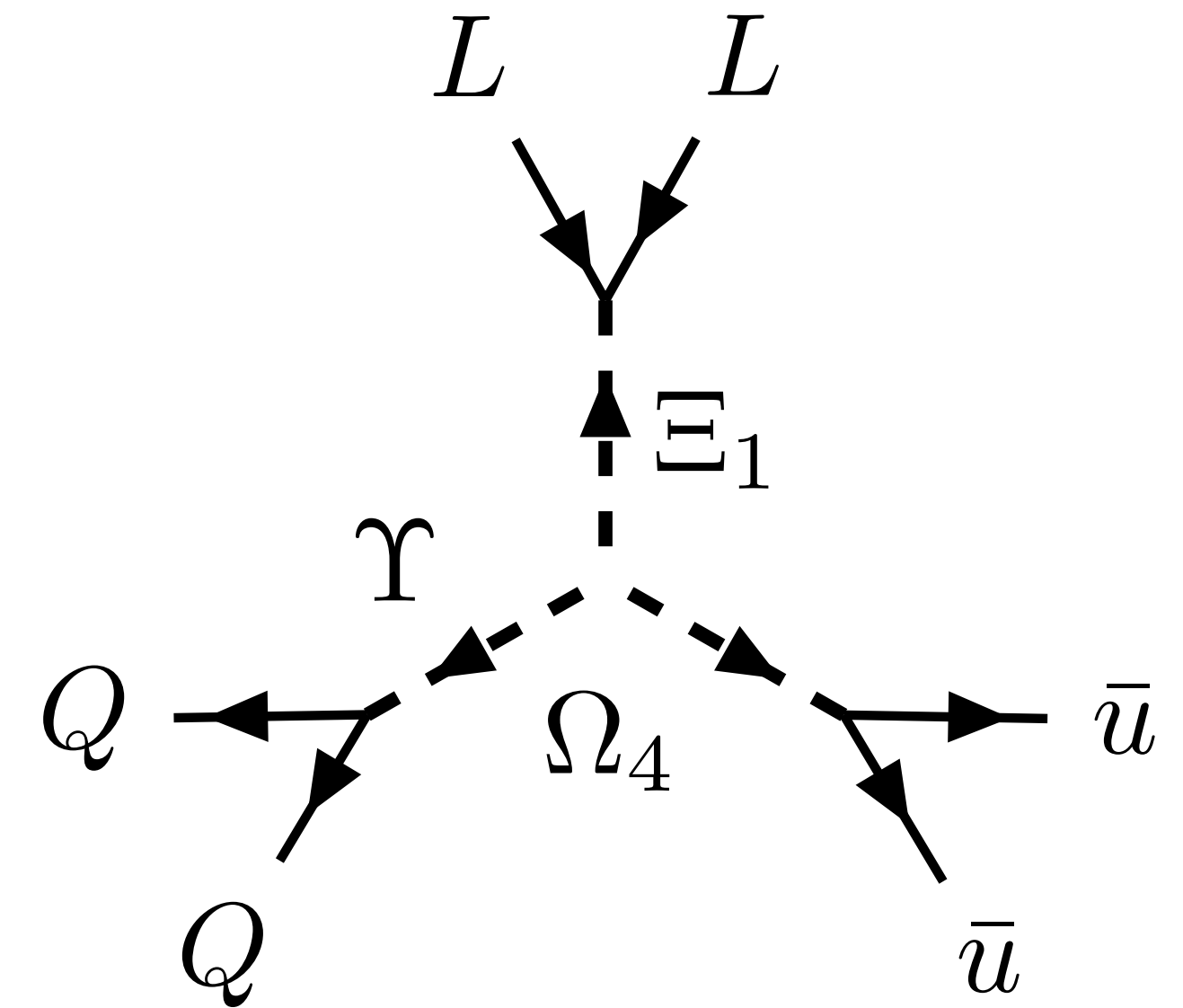
$$\Upsilon \sim (\bar{\mathbf{6}}, \bar{\mathbf{3}}, -\frac{1}{3})_{(1,1)}$$

+ (γ ↔ δ)

$$\rightarrow \Xi_1^{ij} \Upsilon_{abij} \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \delta b} \quad \cdot \epsilon_{\gamma\delta}$$

┌

$$\Omega_4 \sim (\mathbf{6}, \mathbf{1}, -\frac{4}{3})_{(1,1)}$$



$$(L^{\alpha i} L^{\beta j} + i \leftrightarrow j)(Q_{ia}^\dagger Q_{jb}^\dagger + a \leftrightarrow b) \bar{u}^{\dagger \gamma a} \bar{u}^{\dagger \delta b} \cdot \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon_{\gamma\delta}$$

# Example of completion algorithm

Many rules implemented, most important:

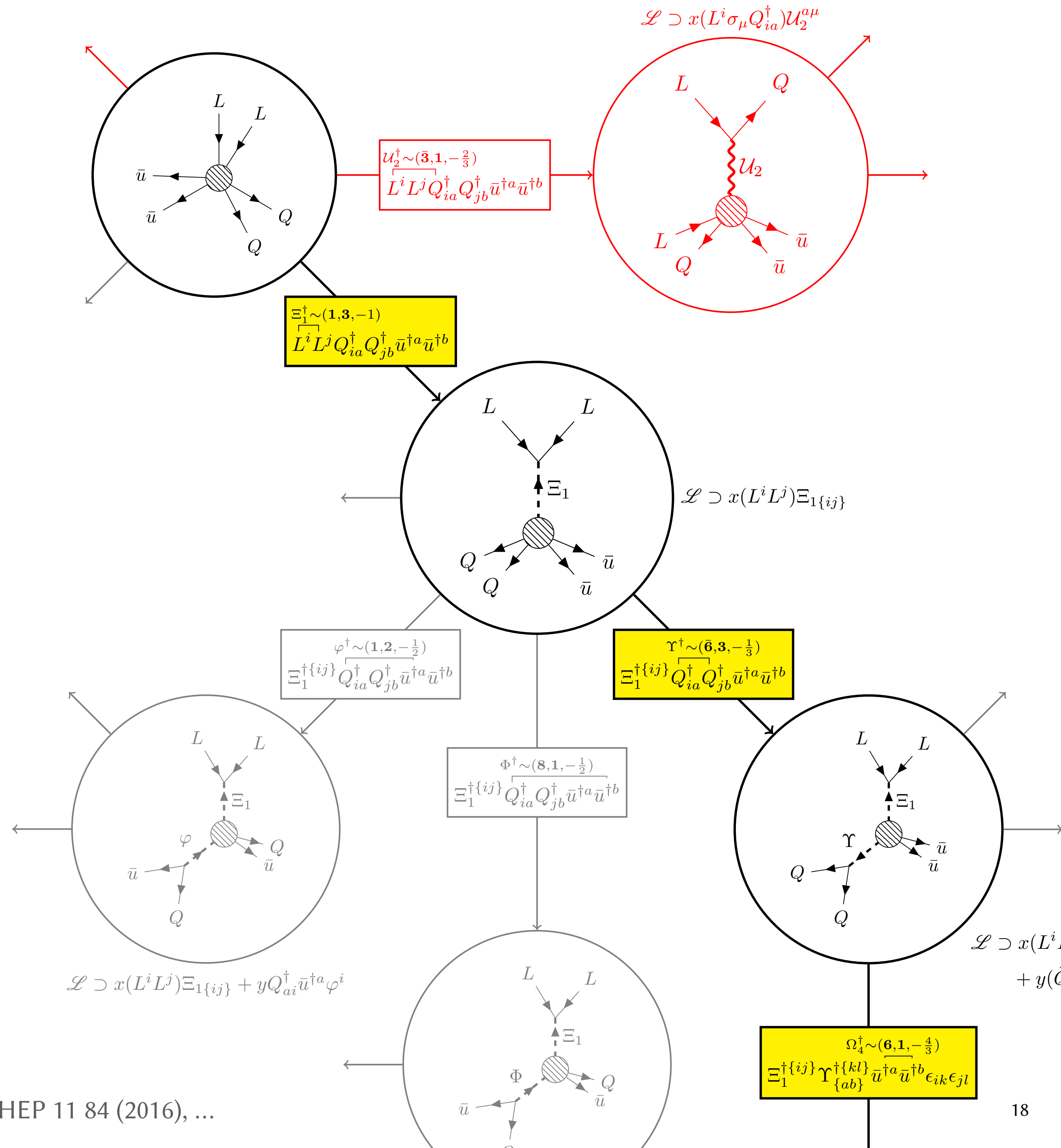
$$\psi^\alpha \phi \rightarrow \Psi^\alpha \quad \alpha \longleftrightarrow \alpha$$

$$\partial^{\alpha\dot{\alpha}} \psi_{\dot{\alpha}}^\dagger \phi \rightarrow \Psi^\alpha \quad \alpha \longleftrightarrow \dot{\alpha}$$

$$\phi_1 \phi_2 \phi_3 \rightarrow \Phi$$

All rules that match should be applied in **all possible ways** at each step of the procedure, leading to a *completion graph*

Validated against dimension-6 SMEFT and other neutrino-mass examples



## *Caveats*

- Currently have only thought about  $SU(2)$  and  $SU(3)$ , with colour structure **not more complicated** than  $\mathbf{3} \otimes \bar{\mathbf{3}}$  and  $(\mathbf{3} \otimes \bar{\mathbf{3}})^2$
- Code is not optimised,  $d = 11$  operators currently **need to be parallelised**
- Exotic spin-1 and spin-3/2 fields are absent by design, **can be included**, but this needs thought

# **Program and model database**

github.com

pytest.ini	Better testing	2 years ago
requirements.txt	Updated README and added pandas dep	2 years ago
setup.py	Updated to patched version	2 years ago

README.org

## Exploding operators for Majorana neutrino masses and beyond 🧨💣

This is the code accompanying the paper "Exploding operators for Majorana neutrino masses and beyond". We don't intend this to be a polished and general purpose implementation of the methods discussed in the paper, but rather an example of how the methods can be used. The code is not optimised for performance and contains many aspects specific to the problem tackled in the paper. The package can be installed through `pip`

```
pip install neutrinomass
```

Please get in touch if you are having trouble install or using the code.

The code is split into three main modules:

1. `tensormethod` provides the field objects and effective operators.
2. `completions` takes the effective operators generated by `tensormethod` and finds their tree-level UV completions with the methods discussed in the paper.
3. `database` provides the filtered database of lepton-number violating models and functions for interacting with it.

### Languages

- Python 72.4%
- Jupyter Notebook 25.8%
- Mathematica 1.7%
- Shell 0.1%

```
> pip install neutrinomass
```

```
In [13]: H = Field("H", "00001", charges={"y": Rational("1/2")})
Q = Field("Q", "10101", charges={"y": Rational("1/6"), "3b": 1})

print(H, Q)
type(H)
```

Tensor manipulations with large dependency on SymPy

```
H(00001)(1/2) Q(10101)(1/6)
```

```
Out[13]: neutrinomass.tensor.method.core.Field
```

```
In [4]: h = H("i0")
q = Q("u0 c0 i1")

type(q)
```

```
Out[4]: neutrinomass.tensor.method.core.IndexedField
```

```
In [14]: print(h * q * eps("-i0 -i1"))

h * q * eps("-i0 -i1")
```

```
H(I_0)*Q(u0, c0, I_1)*metric(-I_0, -I_1)
```

```
Out[14]:  $Q^{aa} H^j \cdot e_{ij}$ 
```

Builds operators in our spanning set automatically

```
In [15]: weinberg = invariants(L, L, H, H)[0]
weinberg
```

```
Out[15]:  $L^{ai} L^{\beta j} H^k H^l \cdot e_{ik} e_{jl}$ 
```

Implements completion procedure, outputs *Completion* objects containing  $\mathcal{L}_{\Delta L=2}$

```
In [17]: model_1, model_2, model_3 = completions(weinberg)
model_1.info()
```



```
In [13]: H = Field("H", "00001", charges={"y": Rational("1/2")})
Q = Field("Q", "10101", charges={"y": Rational("1/6"), "3b": 1})

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In [14]: print(h * q * eps("-i0 -i1"))

h * q * eps("-i0 -i1")

H(I_0)*Q(u0, c0, I_1)*metric(-I_0, -I_1)
```

```
Out[14]:  $Q^{aa} H^j \cdot \epsilon_{ij}$ 
```

```
In [15]: weinberg = invariants(L, L, H, H)[0]
weinberg
```

```
Out[15]:  $L^{ai} L^{\beta j} H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$ 
```

```
In [17]: model_1, model_2, model_3 = completions(weinberg)
model_1.info()
```

Builds operators in our set automatically

Implements completion procedure, outputs  $Cor$  objects containing  $\mathcal{L}_\Delta$

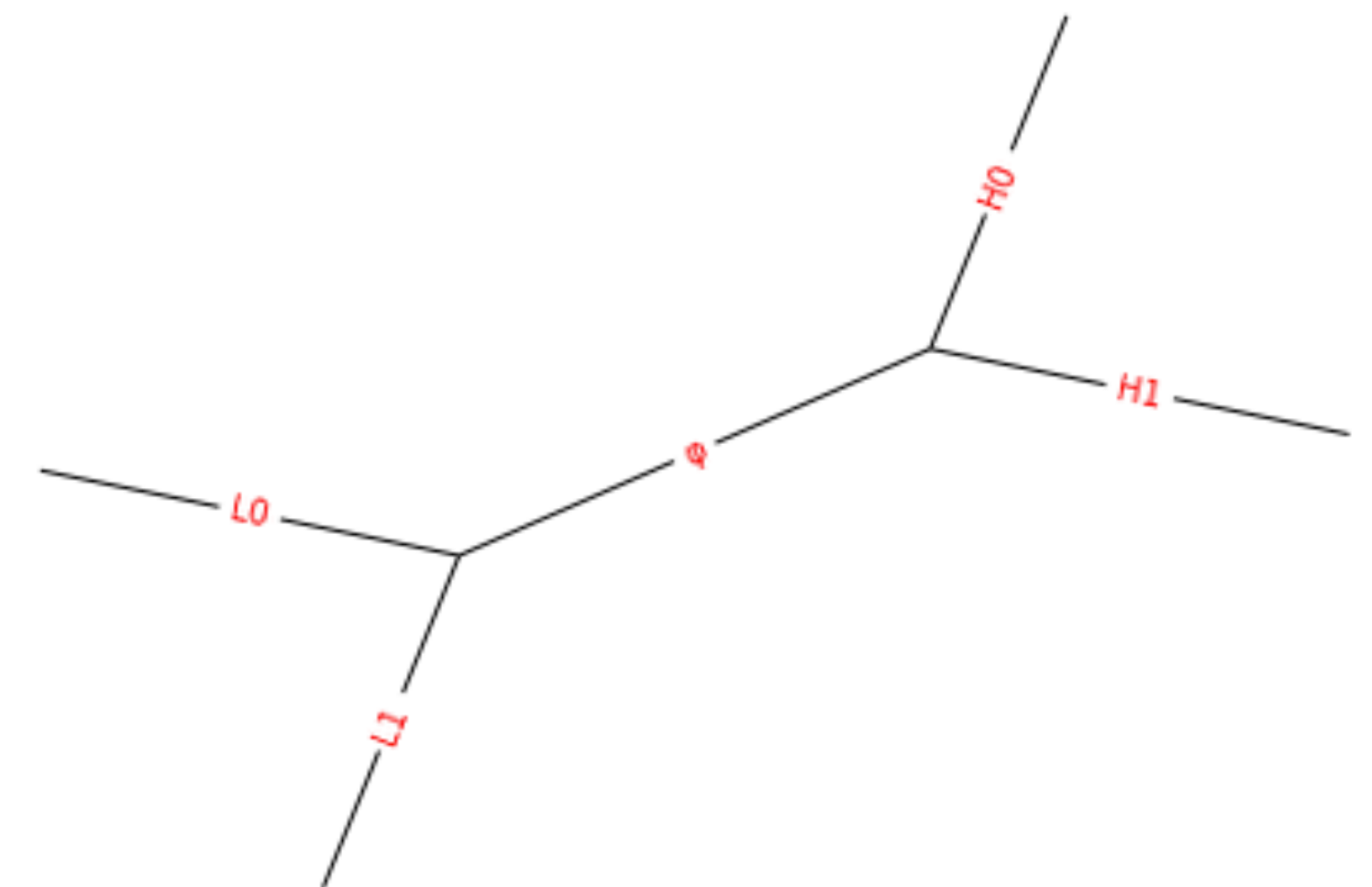
```
Fields:
φ S(1, 3, 1)(0)
```

```
Lagrangian:
```

$$L^{ai} L^{\beta j} \tilde{\phi}^{kl} \cdot \epsilon_{\alpha\beta} \epsilon_{il} \epsilon_{jk}$$

$$H^i H^j \phi^{kl} \cdot \epsilon_{ik} \epsilon_{jl}$$

```
Diagram:
```



Completion procedure produces 430,810 inequivalent  $\Delta L = 2$  Lagrangians. About 70% contain seesaw fields! All published in database: <https://doi.org/10.5281/zenodo.40546183>

Only ~ 3% survive filtering (11,216 models). Published with Python package

The screenshot shows a Jupyter Notebook window with a search query and its results. The query filters for models with fewer than 5 fields and a scale less than 7000 TeV. The results table shows three models, with the third model (index 12772) highlighted. A text box on the right explains that symbolic structures are mapped to prime IDs for numerical querying using Pandas.

```

12772      16179461      8614953467442367  63d  11  37.9148278684193  loop**2*loopv2*v**2*yd*ye/
  Λ      2s6f_4      4      3      1      2

There are many search results. Let's make a more specific query.

In [20]: # Extend the query to look at the models with fewer than 5 fields that need to be at less than 7000 TeV
df[
  (df["democratic_num"] % df.exotics["S,01,0,1/3,-1"] == 0) &
  (df["democratic_num"] % df.exotics["S,00,0,1,0"] == 0) &
  (df["scale"] < 7000) &
  (df["n_fields"] < 5)
]

Out[20]:
   democratic_num  stringent_num  op  dim  scale  symbolic_scale  topology  n_fields  n_scalars  n_fermions  min_loops  max_loops
8387      3379507      30579275025083  10  9  5967.42299748072  loop**2*v**2*yd*ye/Λ  0s6f_1  3  3  0  2
12771      12372529  1378968263787181  63d  11  37.9148278684193  loop**2*loopv2*v**2*yd*ye/
  Λ      2s6f_4  4  3  1  2
12772      16179461  8614953467442367  63d  11  37.9148278684193  loop**2*loopv2*v**2*yd*ye/
  Λ      2s6f_4  4  3  1  2

We find three models. Let's look at one of them. Copy the index on the far left and ask for the completion.

In [21]: comp = df.completion(8387)
comp.info()

```

Symbolic structures mapped to prime IDs, used to query the symbolic database *numerically* (and efficiently!) using *Pandas*



Labels	Operator	Models	Filtered	Loops	$\Lambda$ [TeV]
1	$L^i L^j H^k H^l \cdot \epsilon_{ik} \epsilon_{jl}$	3	3	0	$6 \cdot 10^{11}$
2	$L^i L^j L^k \bar{e} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	8	2	1	$4 \cdot 10^7$
3a	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ij} \epsilon_{kl}$	9	2	2	$2 \cdot 10^5$
3b	$L^i L^j Q^k \bar{d} H^l \cdot \epsilon_{ik} \epsilon_{jl}$	14	5	1	$9 \cdot 10^7$
4a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ik} \epsilon_{jl}$	5	0	1	$4 \cdot 10^9$
4b	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l \cdot \epsilon_{ij} \epsilon_{kl}$	4	2	2	$10 \cdot 10^6$
5a	$L^i L^j Q^k \bar{d} H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	790	36	2	$6 \cdot 10^5$
5b	$\mathcal{O}_1 \cdot Q^i \bar{d} \tilde{H}^j \cdot \epsilon_{ij}$	492	14	1,2	$6 \cdot 10^5$
5c	$\mathcal{O}_{3a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	509	0	2,3	$1 \cdot 10^3$
5d	$\mathcal{O}_{3b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	799	16	1,2	$6 \cdot 10^5$
6a	$L^i L^j \tilde{Q}^k \bar{u}^\dagger H^l H^m \tilde{H}^n \cdot \epsilon_{il} \epsilon_{jn} \epsilon_{km}$	289	14	2	$2 \cdot 10^7$
6b	$\mathcal{O}_1 \cdot \tilde{Q}^i \bar{u}^\dagger \tilde{H}^j \cdot \epsilon_{ij}$	177	0	1,2	$2 \cdot 10^7$
6c	$\mathcal{O}_{4a} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	262	0	1,2	$2 \cdot 10^7$
6d	$\mathcal{O}_{4b} \cdot H^i \tilde{H}^j \cdot \epsilon_{ij}$	208	0	2,3	$6 \cdot 10^4$
7	$L^i \bar{e}^\dagger Q^j \tilde{Q}^k H^l H^m H^n \cdot \epsilon_{il} \epsilon_{jm} \epsilon_{kn}$	240	15	2	$2 \cdot 10^5$
8	$L^i \bar{e}^\dagger \bar{u}^\dagger \bar{d} H^j \cdot \epsilon_{ij}$	5	1	2,3	$2 \cdot 10^1$
9	$L^i L^j L^k L^l \bar{e} \bar{e} \cdot \epsilon_{ik} \epsilon_{jl}$	14	1	2	$3 \cdot 10^3$
10	$L^i L^j L^k \bar{e} Q^l \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	50	1	2	$6 \cdot 10^3$
11a	$L^i L^j Q^k Q^l \bar{d} \bar{d} \cdot \epsilon_{ij} \epsilon_{kl}$	48	0	3	$4 \cdot 10^1$
11b	$L^i L^j Q^k Q^l \bar{d} \bar{d} \cdot \epsilon_{ik} \epsilon_{jl}$	72	16	2	$1 \cdot 10^4$

Some operators produce no filtered models

← Canonical seesaw models

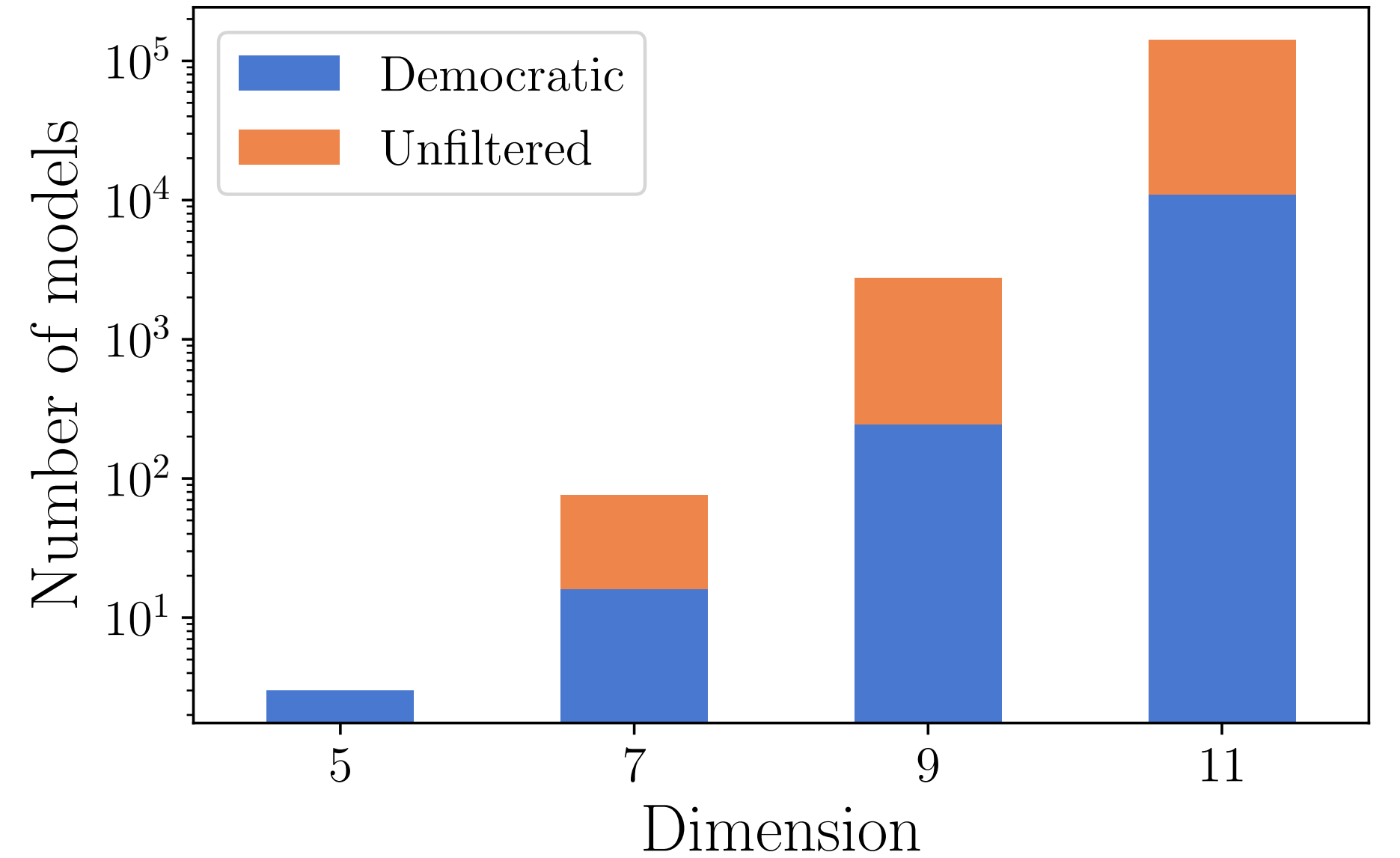
← Not obvious!

← Zee-Babu model

Babu *Phys. Lett. B*203 (1998) 132–136

## Number of models

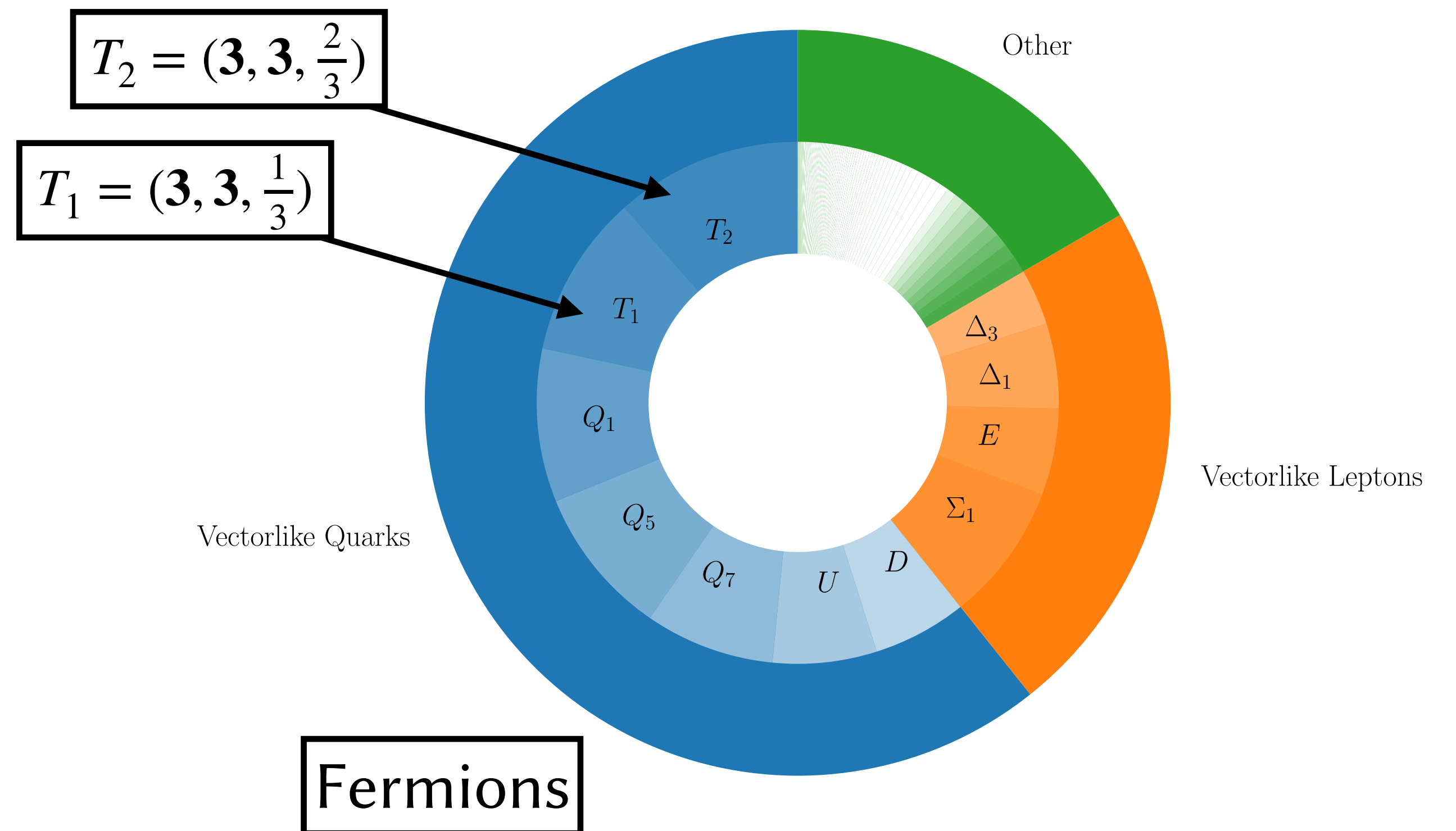
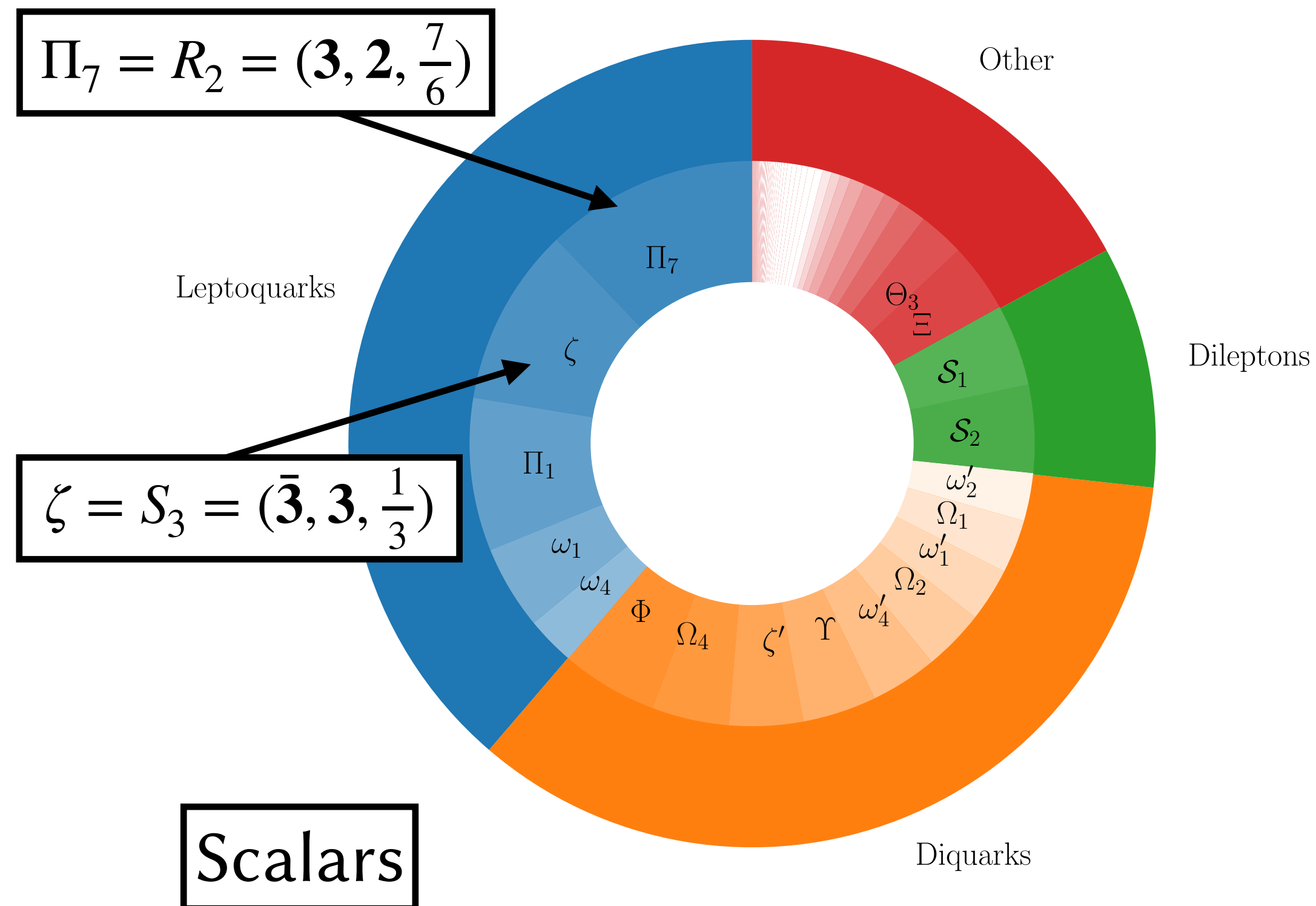
Field	Interactions	$\Delta L = 2$ Lagrangians	Models
$N \sim (\mathbf{1}, \mathbf{1}, 0)_F$	$LHN$	51,245 (11.9%)	17,139 (17.1%)
	Other	12,433 (2.9%)	
$\Sigma \sim (\mathbf{1}, \mathbf{3}, 0)_F$	$LH\Sigma$	87,535 (20.3%)	31,629 (31.5%)
	Other	28,157 (6.5%)	
$\Xi_1 \sim (\mathbf{1}, \mathbf{3}, 1)_S$	$LL\Xi_1$	59,791 (13.0%)	51,576 (51.4%)
	$HH\Xi_1^\dagger$	95,410 (22.1%)	
	Both	10,323 (2.4%)	
	Other	30,761 (7.1%)	

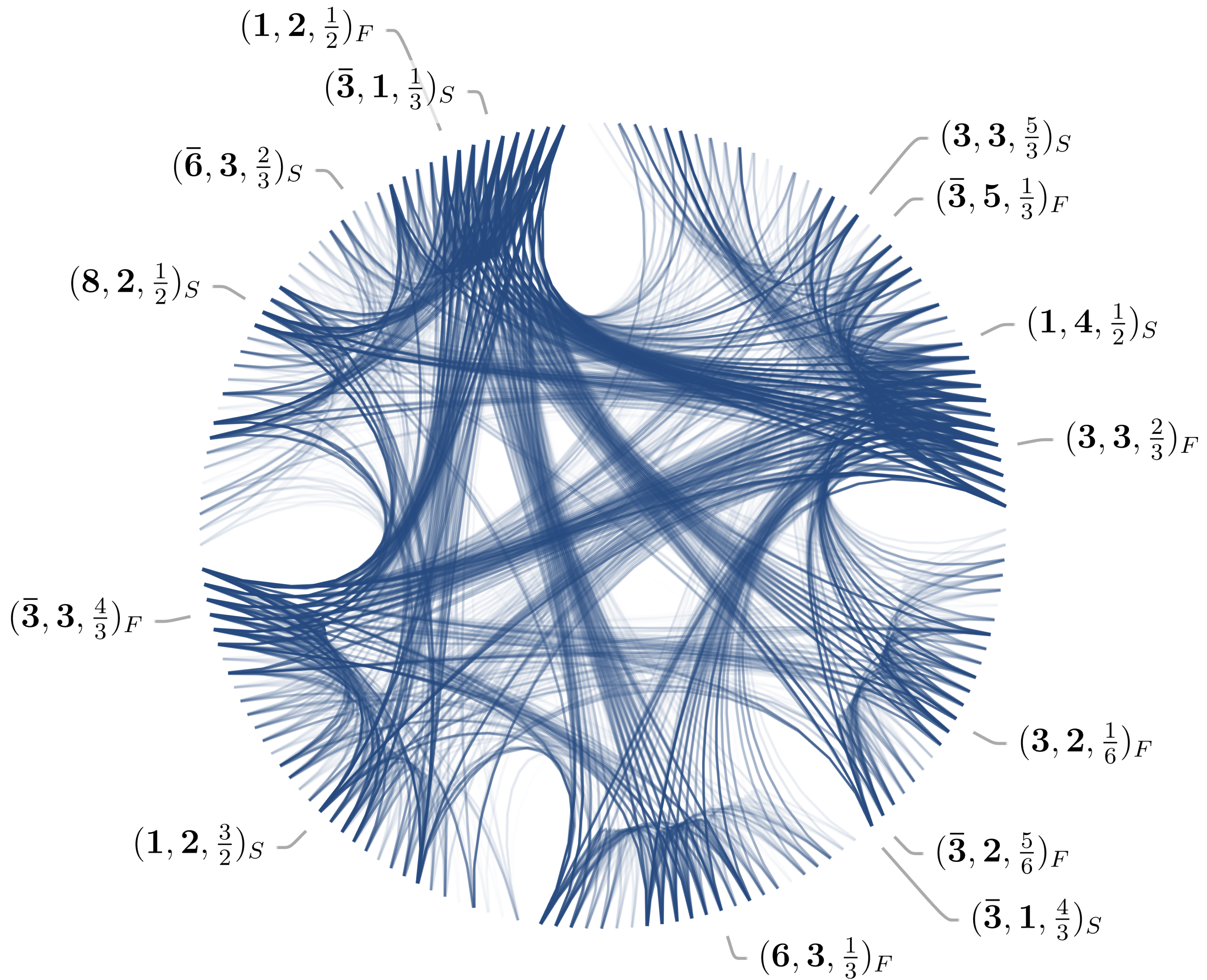


# Exotic fields

In total, 84 different scalar species and 82 different fermions

Most represented class of fields are **leptoquarks**





## *Closing thoughts and future directions*

Our model database is perhaps a good laboratory for **automated phenomenology**

*Can we **automate the production of FeynRules files** from the UV Lagrangians?*

⇒ See talk by Renato Fonseca!

*Can we rule out a large class of models just from their **effects at dimension 6**?*

Symbolic Lagrangian ⇒ MatchMaker/Matchete (talks to follow) ⇒ Flavio?

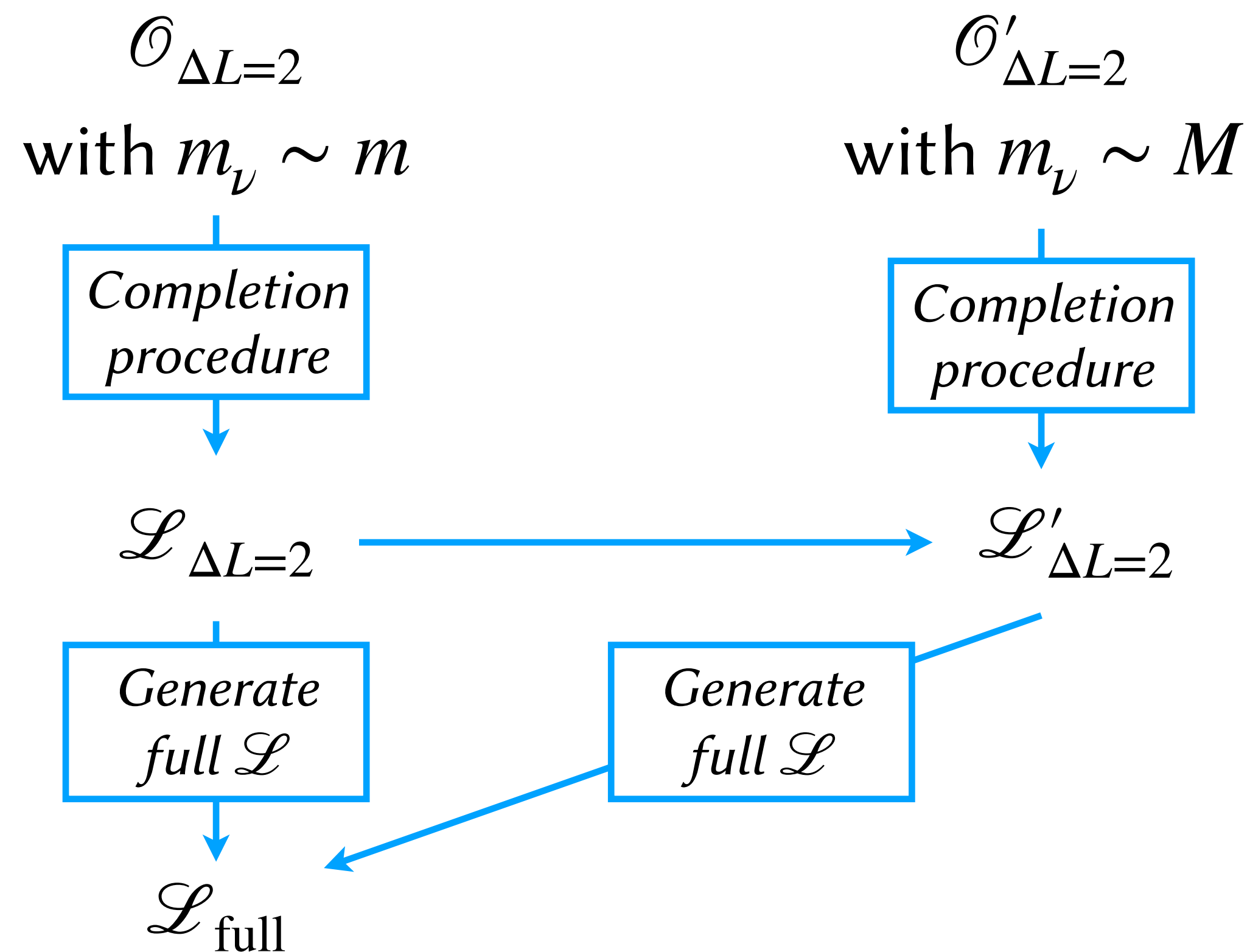
Techniques are more generally applicable!

Is it time to think about a standardised representation for *e.g.* models, Lagrangians, fields?

# Backup

## Model filtering

Want to ensure that each model gives a **dominant contribution to the neutrino mass**, and is not just a small correction to it



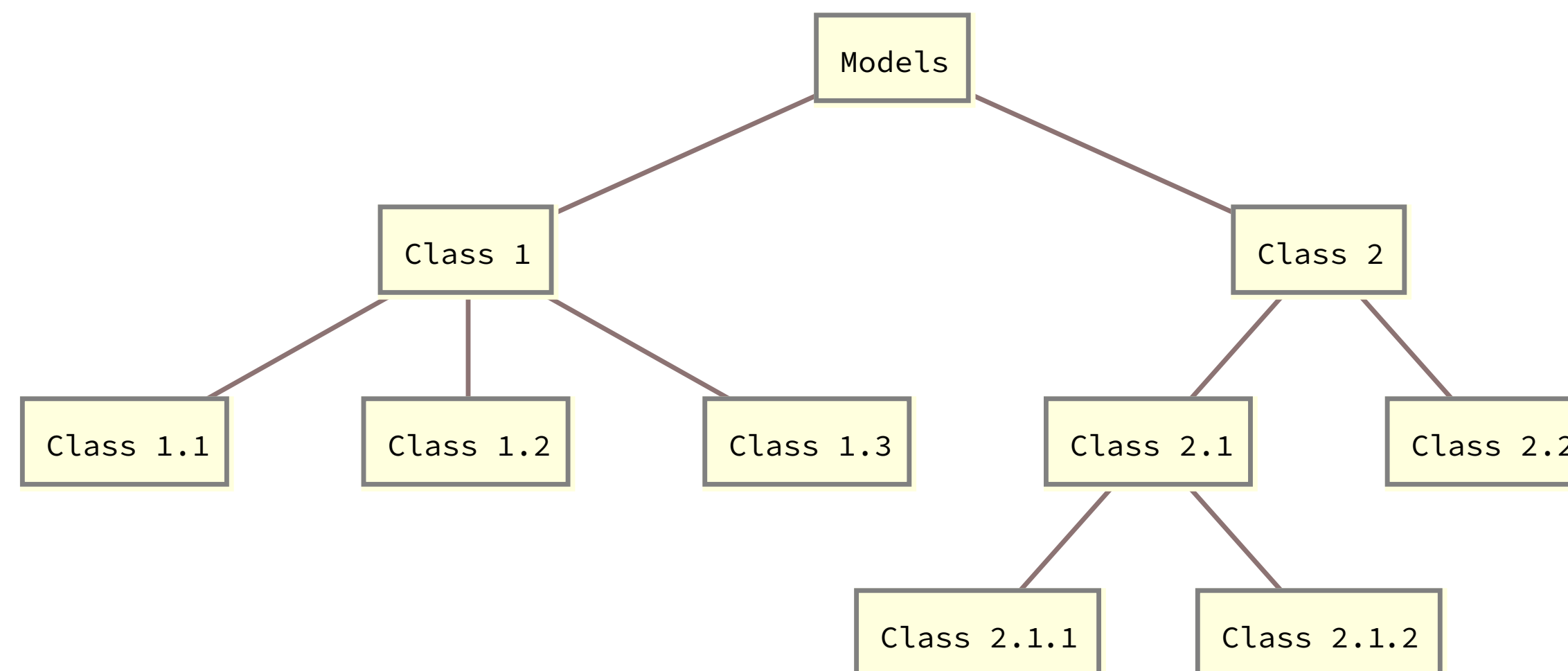
Suppose  $M \gg m$ , then  $\mathcal{L}_{\Delta L=2}$  is only a small correction and should be removed

Take a *democratic* approach: assume **no special hierarchy** between couplings

Often because model **already generates a lower-dimensional operator!**

Neutrino oscillations and the implied non-zero neutrino masses **cannot be accommodated without new physics**. Nice to have:

1. A systematic method for deriving models
2. A(n) (preferably hierarchical) organising principle and classification
  - Can models be organised by testability, complexity, explanatory power?
  - Clustered by common features?



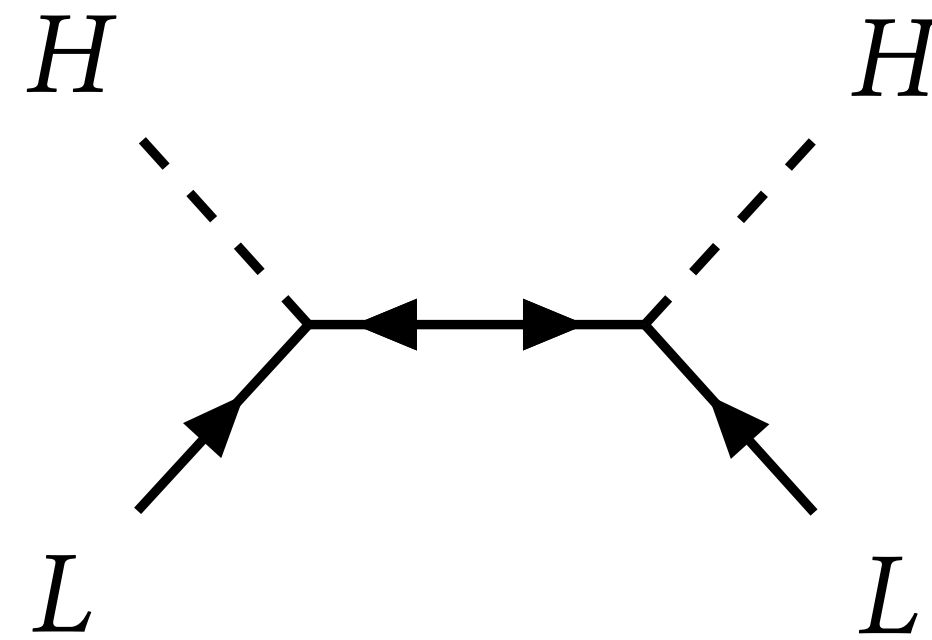


How can we understand the *smallness* of neutrino masses?

1. Large  $\Lambda$  with  $C_5 \sim 1$

*Seesaw models*

$$\frac{v^2}{\Lambda} \sim 0.05 \text{ eV} \Rightarrow \Lambda \sim 10^{14} \text{ GeV}$$



$$\mathcal{L}^{(5)} = \frac{C_5}{\Lambda} \cdot LLHH$$

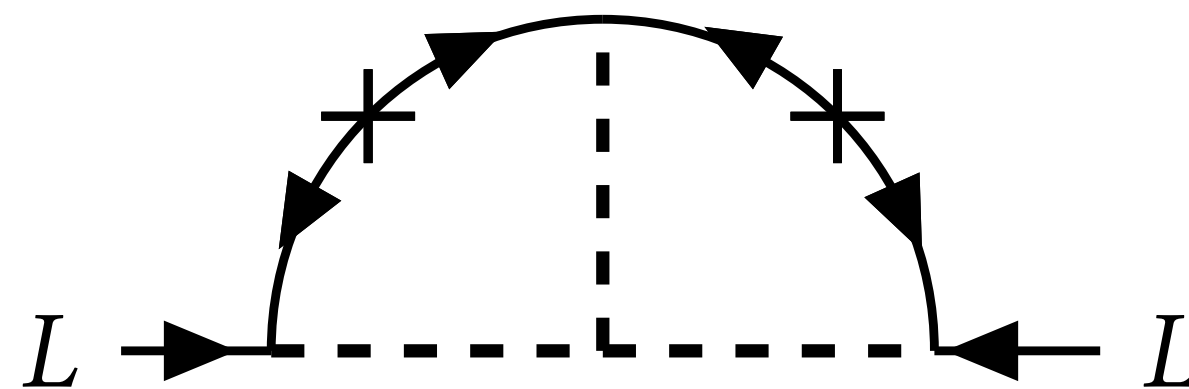
$$\rightarrow C_5 \frac{v^2}{\Lambda} \cdot \nu\nu$$

*Majorana*

2. Small  $C_5$  with  $\Lambda > v$

*Inverse seesaw, radiative models, ...*

$$C_5 \frac{v^2}{\Lambda} \sim \frac{v^2}{\Lambda} \left( \frac{1}{16\pi^2} \right)^n \prod_i y_i$$



Review: Cai et al. Front. Phys. 00063 (2017)

There are *very many* models...

$$G_{\text{SM}} \sim \text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y + \text{SU}(2)_+ \otimes \text{SU}(2)_-$$

Quarks	Leptons	Higgs
$Q \sim (\mathbf{3}, \mathbf{2}, \frac{1}{6})_{(2,1)}$	$L \sim (\mathbf{1}, \mathbf{2}, -\frac{1}{2})_{(2,1)}$	$H \sim (\mathbf{1}, \mathbf{2}, \frac{1}{2})_{(1,1)}$
$\bar{u} \sim (\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{(2,1)}$	$\bar{\nu} \sim (\mathbf{1}, \mathbf{1}, 0)_{(2,1)}$	
$\bar{d} \sim (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})_{(2,1)}$	$\bar{e} \sim (\mathbf{1}, \mathbf{1}, 1)_{(2,1)}$	
Gauge Bosons		
$G \sim (\mathbf{8}, \mathbf{1}, 0)_{(3,1)}$	$W \sim (\mathbf{1}, \mathbf{3}, 0)_{(3,1)}$	$B \sim (\mathbf{1}, \mathbf{1}, 0)_{(3,1)}$

Integrating out a scalar at tree level

$\Phi \rightarrow$  heavy field  
 $\phi, \psi \rightarrow$  light fields

$$\mathcal{L}_{\text{HE}}[\Phi, \phi, \psi]$$

$E > M$

$$\mathcal{L}_{\Phi} = -\Phi^{\dagger}(D^2 + M^2)\Phi + \Phi^{\dagger} \cdot \frac{\partial \mathcal{L}}{\partial \Phi^{\dagger}} + \mathcal{O}(\Phi^3)$$

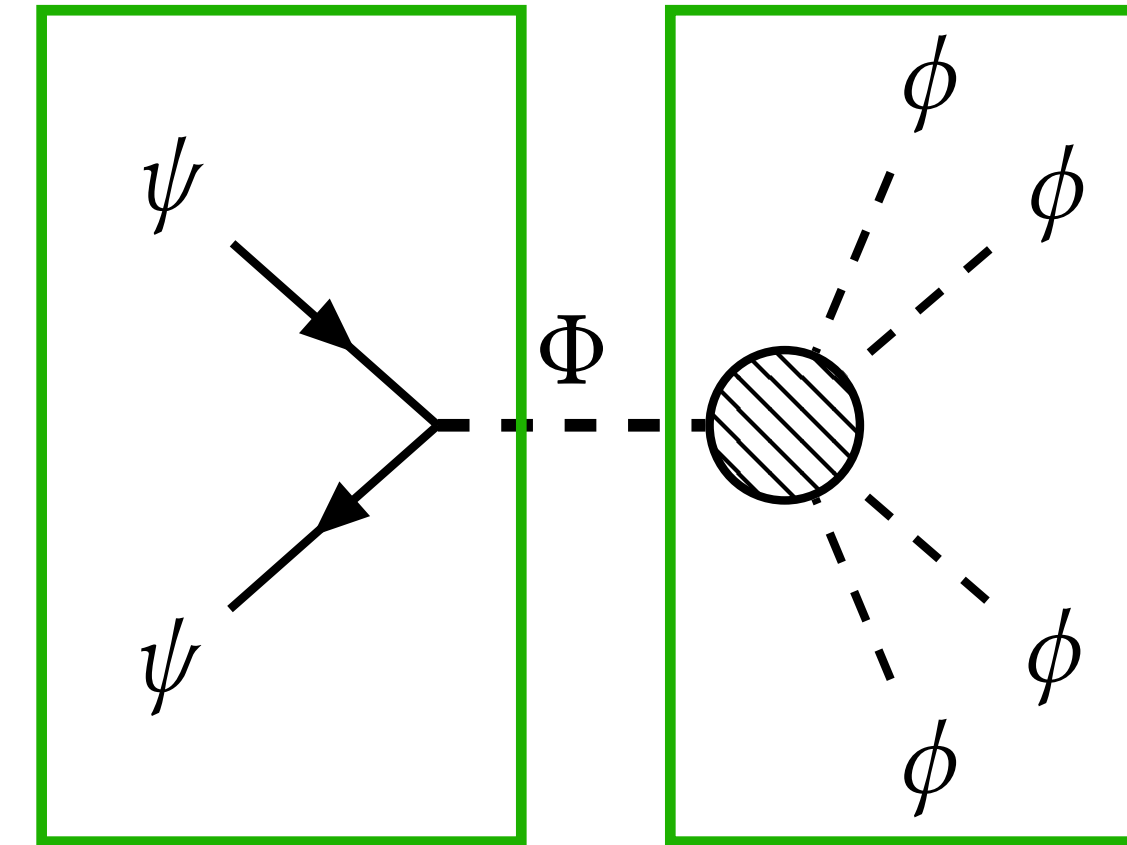
Replace  
by  
EOM

$E < M$

$$\mathcal{L}_{\text{eff}} = \frac{\partial \mathcal{L}}{\partial \Phi} \cdot \frac{1}{M^2} \left[ 1 + \frac{(iD)^2}{M^2} + \dots \right] \cdot \frac{\partial \mathcal{L}}{\partial \Phi^{\dagger}}$$

Fierz  
EOM  
IBP

$$\sum_i c_i \mathcal{O}_i$$



The fields inside  $\frac{\partial \mathcal{L}}{\partial \Phi^{\dagger}}$  must transform opposite to  $\Phi^{\dagger}$

$$\frac{\partial \mathcal{L}}{\partial \Phi^{\dagger}} \otimes \Phi^{\dagger} \sim (1, 1, 0)_{(1,1)}$$

Derivatives can only arise past leading order

Integrating out a **VLF** at tree level

$\Psi, \bar{\Psi} \rightarrow$  heavy fields  
 $\phi, \psi \rightarrow$  light fields

$$\mathcal{L}_{\text{HE}}[\Psi, \bar{\Psi}, \phi, \psi]$$

$E > M$



$$\begin{aligned} \Psi &\sim \Psi_L \\ \bar{\Psi} &\sim \Psi_R^C \end{aligned}$$

$$\begin{aligned} \mathcal{L}_\Psi &= i\Psi^\dagger \bar{\sigma}^\mu D_\mu \Psi + i\bar{\Psi}^\dagger \bar{\sigma}^\mu D_\mu \bar{\Psi} \\ &+ \Psi^\dagger \cdot \frac{\partial \mathcal{L}}{\partial \Psi^\dagger} + \bar{\Psi}^\dagger \cdot \frac{\partial \mathcal{L}}{\partial \bar{\Psi}^\dagger} - M\bar{\Psi}\Psi \end{aligned}$$

Derivatives can arise from *arrow-preserving* fermion propagator **already at leading order**

Replace  
by  
EOM

Fierz  
EOM  
IBP

 $\sum_i c_i \mathcal{O}_i$ 

But not field strengths!

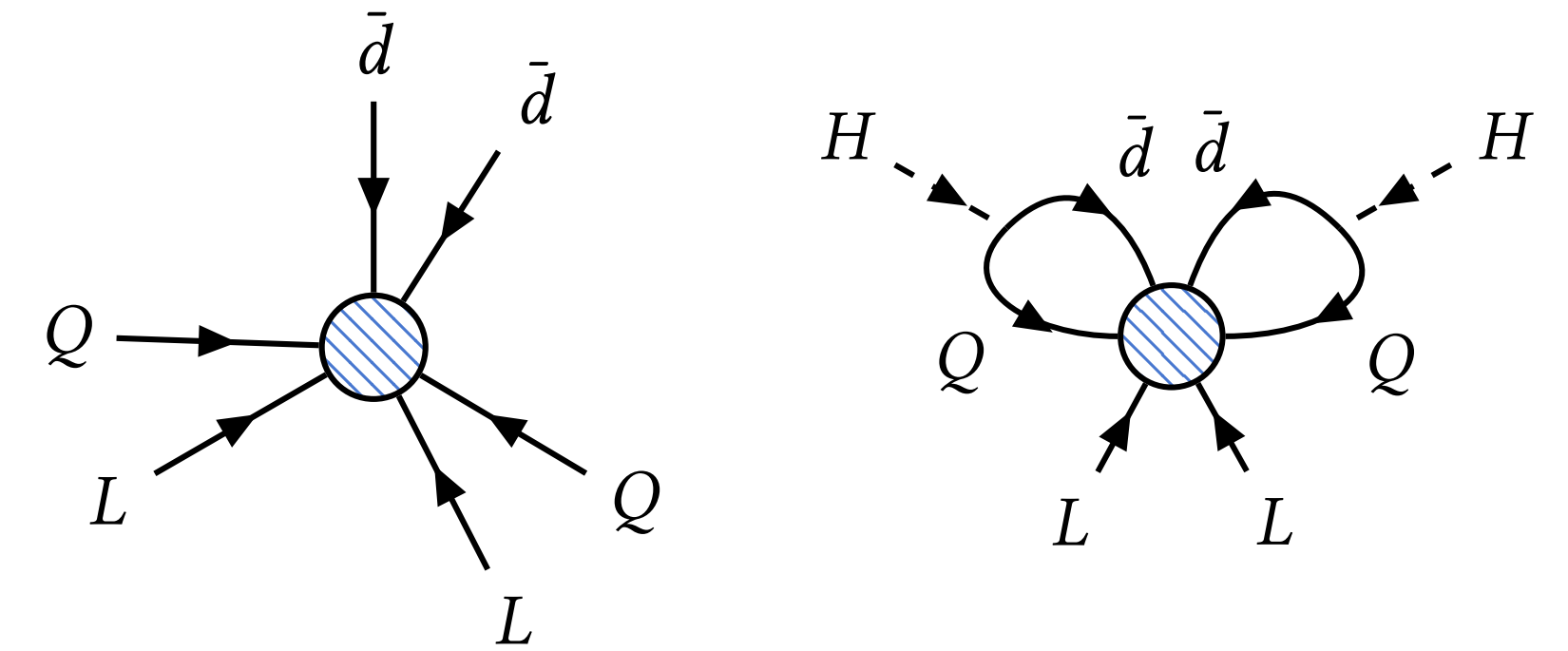
$E < M$

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{\partial \mathcal{L}}{\partial \Psi_\beta} \frac{1}{M^2} \left( \epsilon_{\alpha\beta} + \frac{X_{\alpha\beta} - D^2 \epsilon_{\alpha\beta}}{M^2} + \dots \right) \boxed{iD^{\alpha\dot{\alpha}}} \frac{\partial \mathcal{L}}{\partial \Psi_{\dot{\beta}}^\dagger} \epsilon_{\dot{\alpha}\dot{\beta}} \\ &+ \frac{\partial \mathcal{L}}{\partial \Psi_\beta} \frac{1}{M} \left( \epsilon_{\alpha\beta} + \frac{X_{\alpha\beta} - D^2 \epsilon_{\alpha\beta}}{M^2} + \dots \right) \frac{\partial \mathcal{L}}{\partial \bar{\Psi}_\alpha} \end{aligned}$$

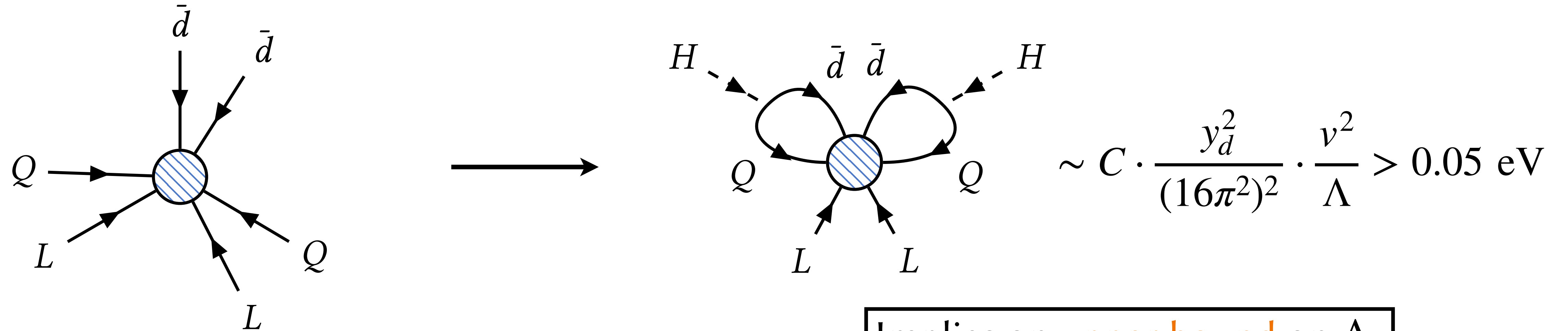


## Summary of approach to neutrino-mass model building

1. Loop off operator into Weinberg(-like) ones
2. Derive **bounded estimates of the new-physics scale from the atmospheric bound** predicted by each operator in newly expanded catalogue up to dimension 11
3. Derive UV models for each operator using algorithm discussed previously
4. *Filter* these models, keeping only those that **contribute dominantly** to the neutrino mass
5. Package these into a convenient computation representation that is easy and efficient to query



*IR considerations: loops and estimates of new-physics scale*



$$\sim C \cdot \frac{y_d^2}{(16\pi^2)^2} \cdot \frac{v^2}{\Lambda} > 0.05 \text{ eV}$$

$$\mathcal{O}_{11b} = L^i L^j \bar{d} Q^k \bar{d} Q^l \cdot \epsilon_{il} \epsilon_{jl}$$

Conservatively assume  $C = 1$  in all estimates

Implies an **upper bound** on  $\Lambda$ :

$$\Lambda \lesssim \frac{C v^2}{0.05 \text{ eV}} \frac{y_d^2}{16\pi^2}$$

$$\approx C \cdot 10^4 \text{ TeV}$$

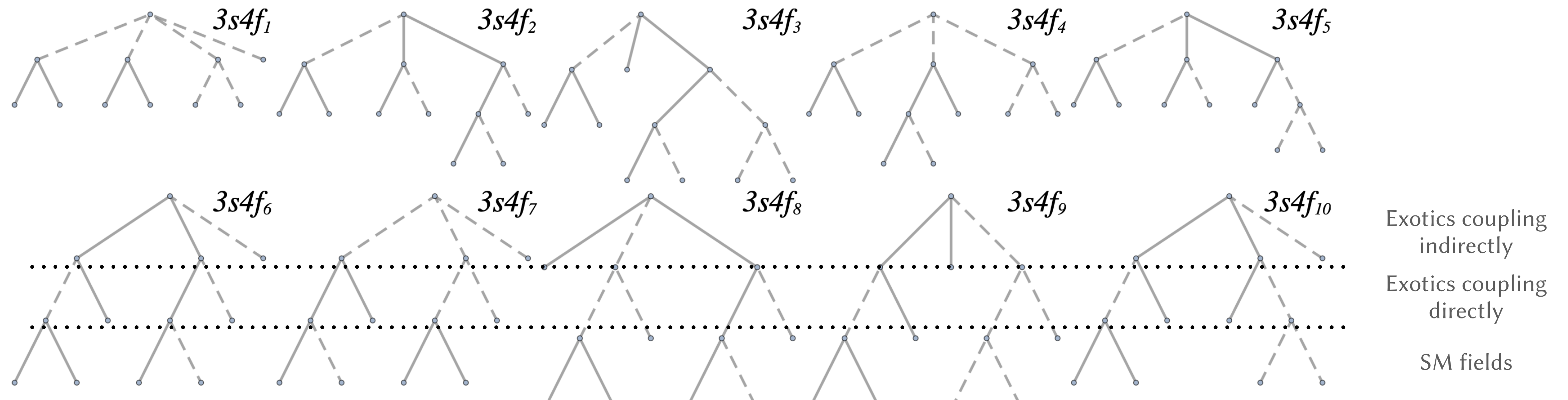
Automate closure procedure and **validate against as many UV examples as possible**

Suppression by SM Yukawa couplings a **very common** feature

# Exotic fields

By far the most common fields are those that generate dimension-6 operators at tree-level, *i.e.* that couple directly to SM fields

de Blas et al. JHEP 03 109 (2018) ; Criado CPC 227 06 2018



## Interesting example models

Database is easy to query on fields, interactions, NP scale, *etc.*

### Example 1: Low-scale, *simple* models

$700 \text{ GeV} < \Lambda < 100 \text{ TeV} \ \&\& \ n\_fields < 4$

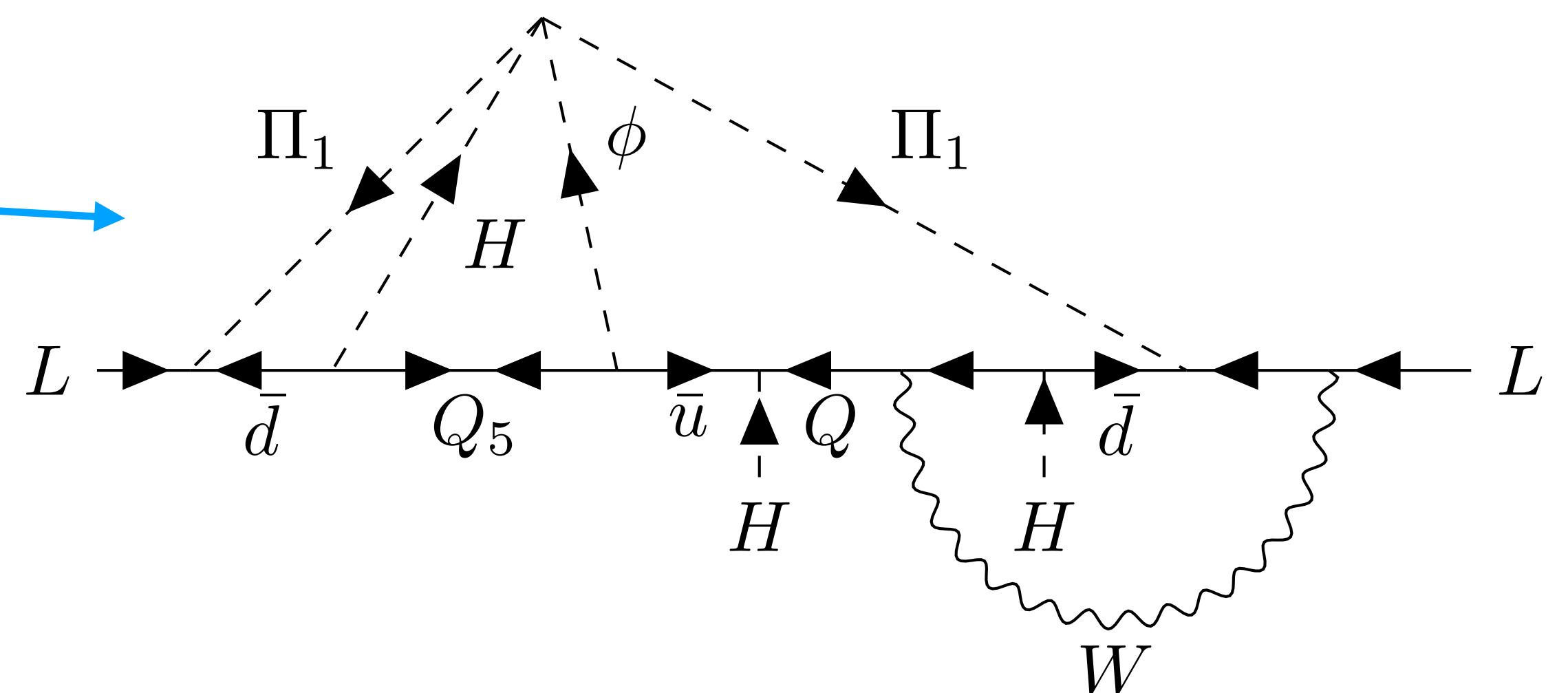
Field content	Operators	$\Lambda$ [TeV]
$(\mathbf{3}, \mathbf{2}, \frac{1}{6})_S, (\mathbf{3}, \mathbf{2}, \frac{7}{6})_F$	8, $D15$	15
$(\bar{\mathbf{6}}, \mathbf{2}, \frac{7}{6})_F, (\mathbf{8}, \mathbf{2}, \frac{1}{2})_S, (\mathbf{3}, \mathbf{2}, \frac{1}{6})_S$	20	0.8
$(\mathbf{6}, \mathbf{1}, \frac{4}{3})_S, (\mathbf{6}, \mathbf{1}, \frac{1}{3})_F, (\mathbf{3}, \mathbf{2}, \frac{1}{6})_S$	20	0.8
$(\mathbf{6}, \mathbf{2}, \frac{5}{6})_S, (\mathbf{3}, \mathbf{2}, \frac{1}{6})_F, (\mathbf{3}, \mathbf{2}, \frac{1}{6})_S$	$50a, b$	10
$(\bar{\mathbf{6}}, \mathbf{2}, \frac{1}{6})_S, (\bar{\mathbf{3}}, \mathbf{2}, \frac{5}{6})_F, (\mathbf{3}, \mathbf{2}, \frac{1}{6})_S$	$50a, b$	10

Sextet fields can be replaced by triplets with different baryon-number assignments

Cai et al. JHEP 02 161 (2016)  
Klein et al. JHEP 03 018 (2019)

Only previously known model

$$\begin{aligned} \Pi_1 &\sim (\mathbf{3}, \mathbf{2}, \frac{1}{6})_S \\ Q_5 &\sim (\mathbf{3}, \mathbf{2}, \frac{1}{6})_F \\ \phi &\sim (\mathbf{6}, \mathbf{2}, \frac{5}{6})_S \end{aligned}$$



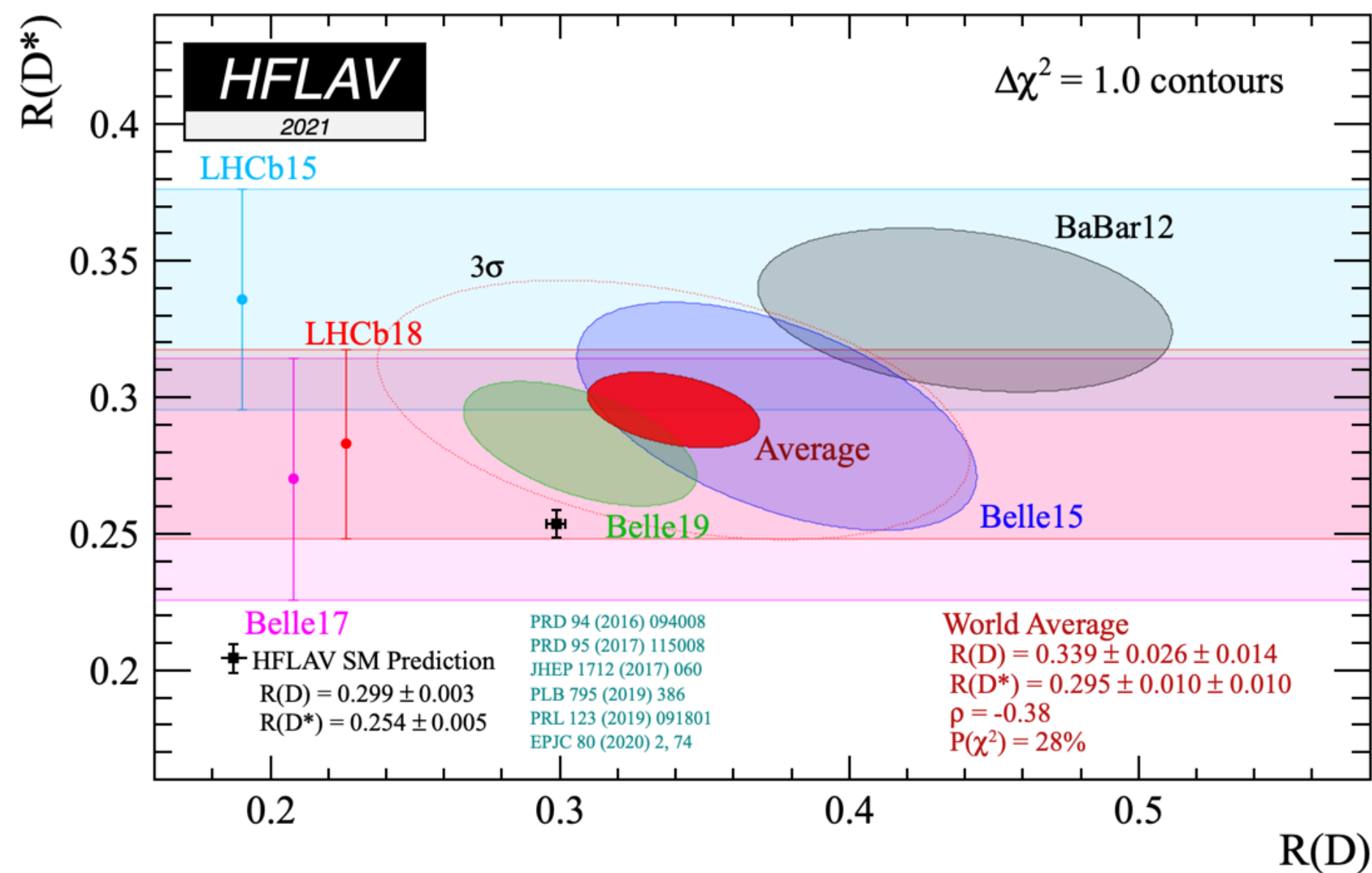


## Interesting example models

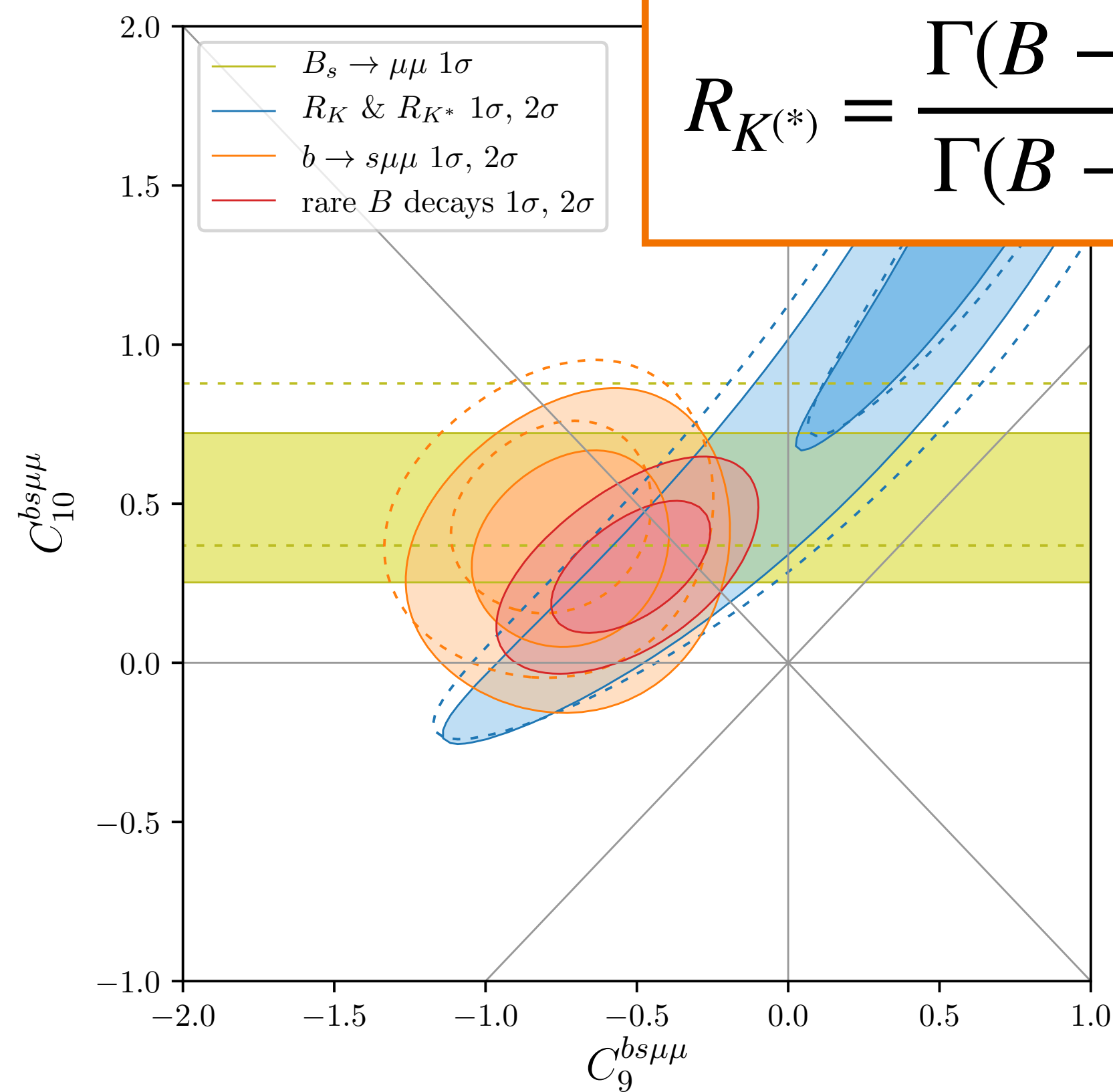
Database is easy to query on fields, interactions, NP scale, *etc.*

### Example 2: Connection with flavour anomalies

$$R_{D^{(*)}} = \frac{\Gamma(B \rightarrow D^{(*)}\tau\nu)}{\Gamma(B \rightarrow D^{(*)}\ell\nu)}$$



$$R_{K^{(*)}} = \frac{\Gamma(B \rightarrow K^{(*)}\mu\mu)}{\Gamma(B \rightarrow K^{(*)}ee)}$$



## Example 2: Connection with flavour anomalies

Very many models contain the appropriate scalar leptoquarks

$$R_{D^{(*)}} = \frac{\Gamma(B \rightarrow D^{(*)}\tau\nu)}{\Gamma(B \rightarrow D^{(*)}\ell\nu)}$$

$$R_{K^{(*)}} = \frac{\Gamma(B \rightarrow K^{(*)}\mu\mu)}{\Gamma(B \rightarrow K^{(*)}ee)}$$

$$\mathcal{L}_{S_1} = f_{rs}L_rQ_sS_1 + g_{rs}\bar{e}_r^\dagger\bar{u}_s^\dagger S_1 + \text{h.c.}$$

$$\mathcal{L}_{R_2} = x_{rs}L_r\bar{u}_sR_2 + y_{rs}\bar{e}_r^\dagger Q_s^\dagger R_2 + \text{h.c.} \leftarrow$$

Most  
represented  
field in  
database!

(LQS1 && euS1\* in interactions)

|| (LuR2 && eQR2\* in interactions)

↓  
None!

$$\mathcal{L}_{S_3} = w_{rs}L_rQ_sS_3 + \text{h.c.} \leftarrow$$

2nd most  
represented  
field in  
database!

S1 in model && S3 in model

R2 in model && S3 in model

88 models containing  $S_1$  and  $S_3$   
178 models containing  $R_2$  and  $S_3$

*IR considerations: loops and estimates of new-physics scale*

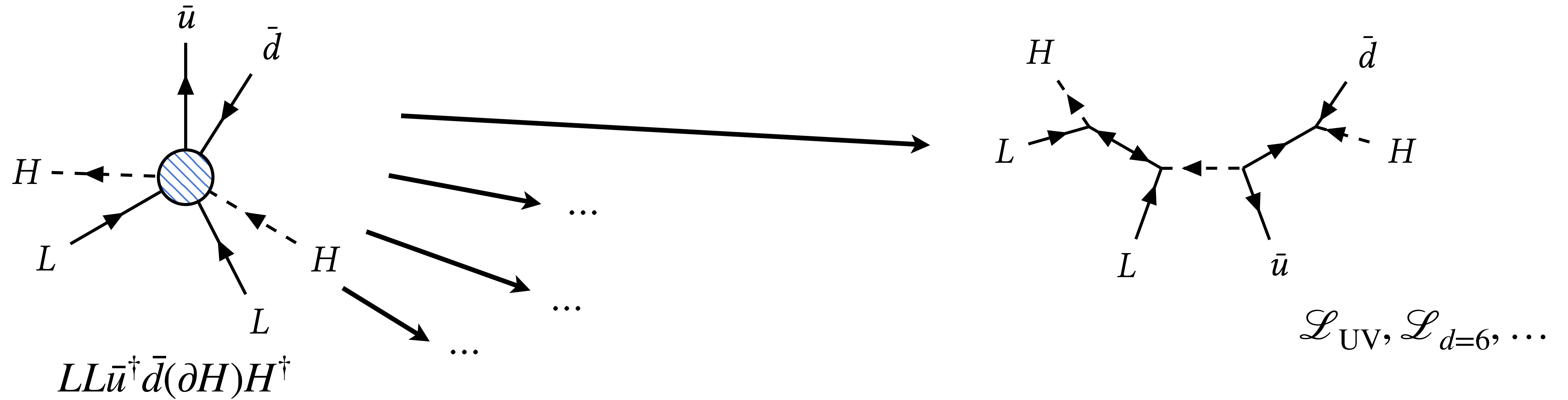
Least suppressed neutrino masses arise from operators like

$$LLHH \cdot \prod_{i=1}^n (\psi_i^\dagger \psi_i) \Rightarrow m_\nu \sim \frac{C}{(16\pi^2)^n} \cdot \frac{v^2}{\Lambda}$$

Applying [atmospheric bound](#) implies  $n \leq 5.7$  for  $C = 1$  and  $\Lambda = v$ , can come from dimension-21 operators of the form

$$LLH(\partial^\mu H)(\psi_0 \sigma_\mu \psi_0^\dagger) \prod_{i=1}^4 \psi_i^\dagger \psi_i$$

What does the UV physics look like?



1. Exotic fields are **vector-like or Majorana fermions, or scalars**

A. Familial structure allowed if necessary

2. Assume SM gauge group

3. Assume  $B$  conservation

