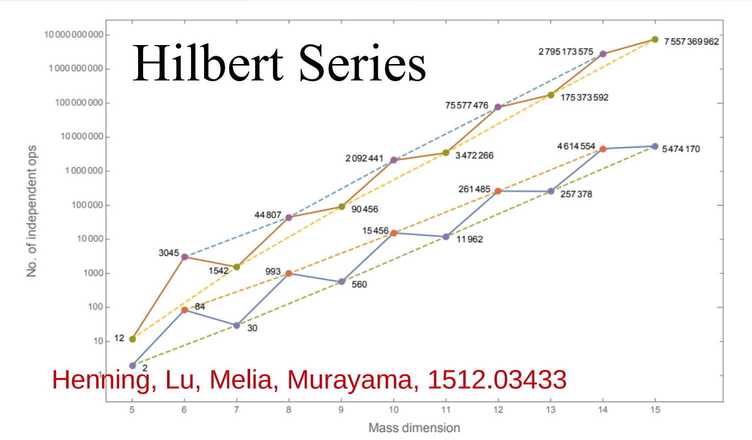
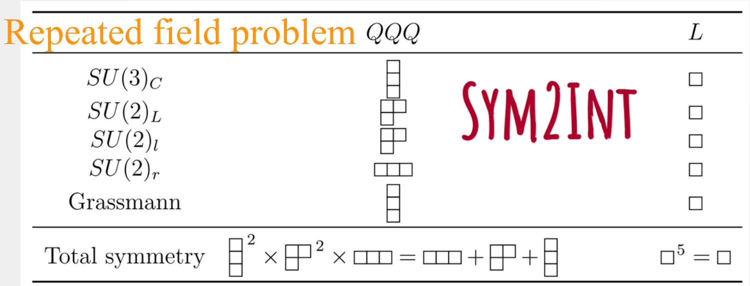
# J-basis Operators and Its Applications

Hao-Lin Li
CP3, UCLouvain
HEFT
2022.06.16 Granada,Spain

# Higher Dim Operators

## Counting




Renato M. Fonseca, 1907.12584, 1703.05221

## Explicit Basis

BasisGen: automatic generation of operator bases

J. C. Criado[1]

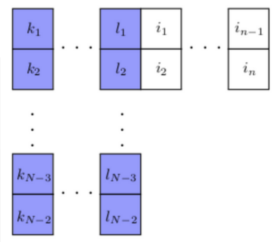
DEFT: A program for operators in EFT

Ben Gripaios[a] and Dave Sutherland[b]

Constructing effective field theories via their harmonics

Brian Henning[1,2] and Tom Melia[3]

$$[j_1 j_2]\epsilon^{j_1 j_2 k_1..k_{N-2}} \ldots [j_{\tilde{n}-1} j_{\tilde{n}}]\epsilon^{j_{\tilde{n}-1} j_{\tilde{n}} l_1..l_{N-2}} \langle i_1 i_2 \rangle \ldots \langle i_{n-1} i_n \rangle,$$



Amplitude operator correspondence

Operators For Generic Effective Field Theory at any Dimension: On-shell Amplitude Basis Construction

Hao-Lin Li[a,c1], Zhe Ren[a,b2], Ming-Lei Xiao[a,d,e3], Jiang-Hao Yu[a,b,f,g,h4], Yu-Hui Zheng[a,b5]

**Amplitude Basis Construction for Effective Field Theories (ABC4EFT)**

# Outline

- Explain the concept various basis in our framework

  **H.-L.Li**, Z.Ren, M.-L.Xiao, J.-H.Yu, Y.-H. Zheng, 2201.04639

- How to obtain the *J-basis*

- Use *J-basis* to find UV orgin for an operator

  **H.-L.Li**, Y.-H.Ni, M.-L.Xiao, J.-H.Yu, 2204.03660

- Dim-8 Contribution to qqWW at LHC

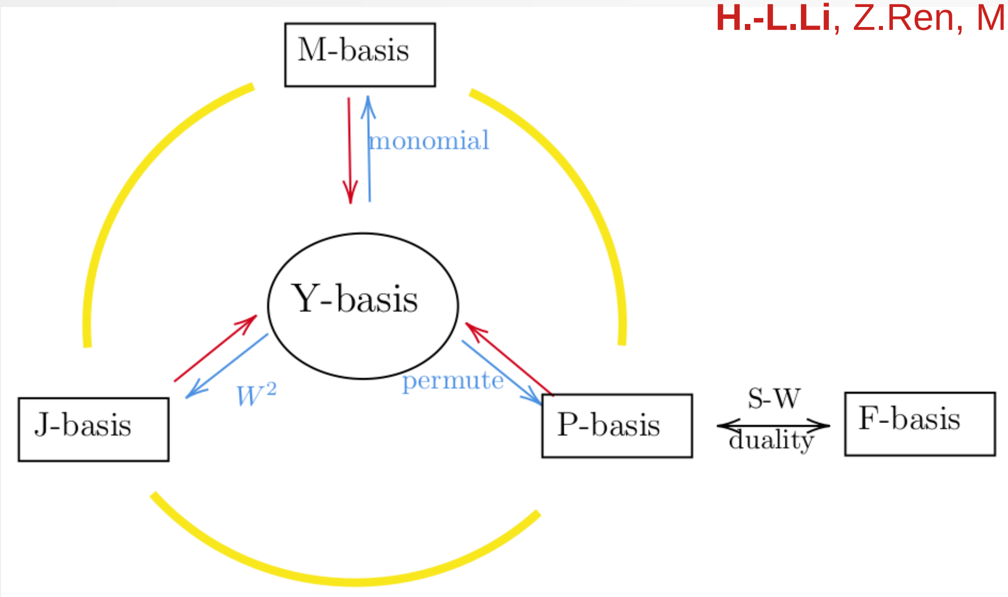
  C. Degrande, **H.-L.Li**, 2206.xxxxx

- Summary and Outlook

# Operator Basis

Operator Type: Fixed field contents and the number of derivative $W_L W_L H H^\dagger D,\ Q^3 L$

**H.-L.Li**, Z.Ren, M.-L.Xiao, J.-H.Yu, Y.-H. Zheng, 2201.04639




- Y-basis: obtained with Young tablueax method and amplitude operator correspondence.

$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array} \qquad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}$$

$$\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_3\alpha_4} \qquad \epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_2\alpha_4}$$

$$\langle 12\rangle\langle 34\rangle \qquad \langle 13\rangle\langle 24\rangle$$

$$\psi_1^\alpha\psi_{2\alpha}\psi_3^\beta\psi_{4\beta} \qquad \psi_1^\alpha\psi_2^\beta\psi_{3\alpha}\psi_{4\beta}.$$

SU(2)

$$\begin{array}{|c|c|}\hline i & j \\ \hline k & l \\ \hline \end{array} \qquad \begin{array}{|c|c|}\hline i & k \\ \hline j & l \\ \hline \end{array}$$

$$\epsilon^{ik}\epsilon^{jl} \qquad \epsilon^{ij}\epsilon^{kl}$$

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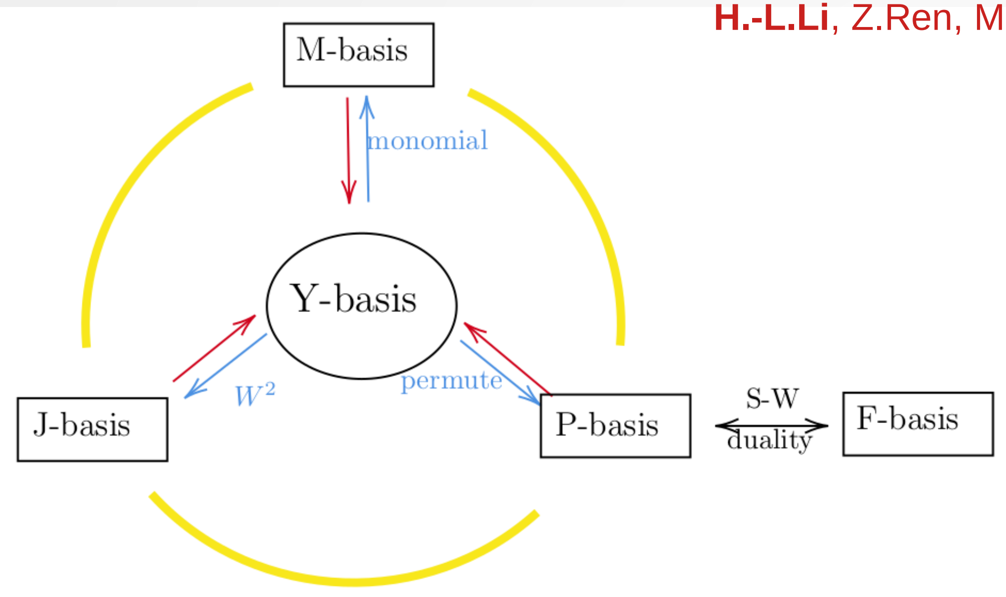


- Y-basis: obtained with Young tablueax method and amplitude operator correspondence.
- M-basis: independent monomial operators

$$\mathcal{B}_1^{(y)} = F_{\mathrm{L}1}{}^{\alpha\beta} F_{\mathrm{L}2\alpha\beta} (D\phi_3)^{\gamma}{}_{\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}$$

$$\mathcal{B}_2^{(y)} = F_{\mathrm{L}1}{}^{\alpha\beta} F_{\mathrm{L}2\alpha}{}^{\gamma} (D\phi_3)_{\beta\dot\alpha} (D\phi_4)_{\gamma}{}^{\dot\alpha}$$

$$\mathcal{B}_1^{(m)} = F_{\mathrm{L}1\nu\mu} F_{\mathrm{L}2}{}^{\mu\nu} \left(D_\lambda \phi_3\right) \left(D^\lambda \phi_4\right) = -\frac{1}{4}\mathcal{B}_1^{(y)}$$

$$\mathcal{B}_2^{(m)} = F_{\mathrm{L}1\mu}{}^{\nu} F_{\mathrm{L}2}{}^{\mu\lambda} \left(D_\lambda \phi_3\right) \left(D_\nu \phi_4\right) = \frac{1}{8}\mathcal{B}_1^{(y)} - \frac{1}{8}\mathcal{B}_2^{(y)}$$

$$\mathcal{B}_1^{(y)} = -4 F_{\mathrm{L}1\nu\mu} F_{\mathrm{L}2}{}^{\mu\nu} \left(D_\lambda \phi_3\right) \left(D^\lambda \phi_4\right)$$

$$\mathcal{B}_2^{(y)} = -4 F_{\mathrm{L}1\mu}{}^{\nu} F_{\mathrm{L}2}{}^{\mu\lambda} \left(D_\lambda \phi_3\right) \left(D_\nu \phi_4\right) + 4 F_{\mathrm{L}1\mu}{}^{\nu} F_{\mathrm{L}2}{}^{\mu\lambda} \left(D_\nu \phi_3\right) \left(D_\lambda \phi_4\right) - 2 F_{\mathrm{L}1\nu\mu} F_{\mathrm{L}2}{}^{\mu\nu} \left(D_\lambda \phi_3\right) \left(D^\lambda \phi_4\right)$$

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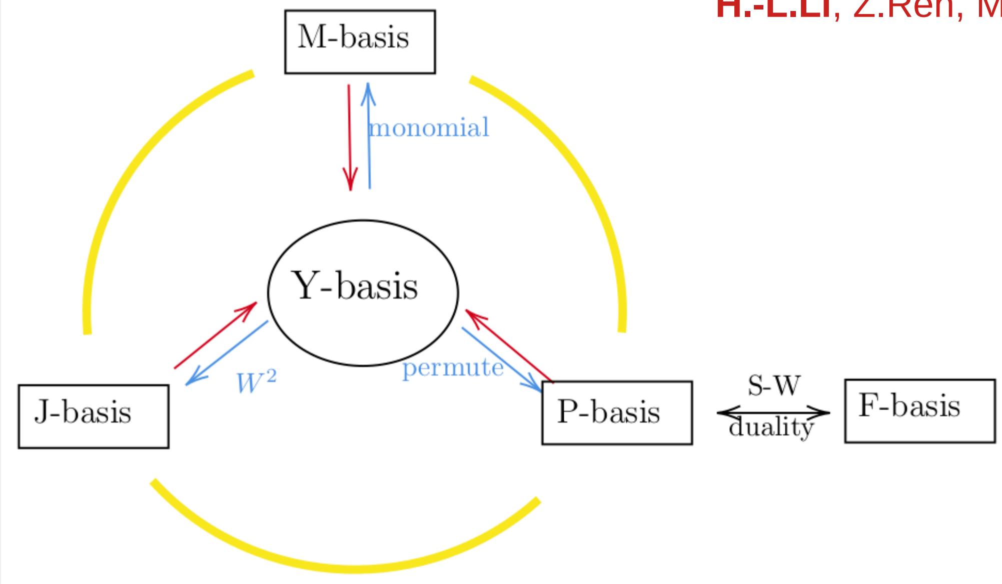


- Y-basis: obtained with Young tablueax method and amplitude operator correspondence.
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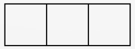
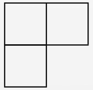
- P-basis: irrep of symmetric group of repeated fields—also irrep of $SU(n_f)$

$$\mathcal{O}_1^{\rm y} = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})$$

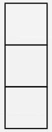
$$\mathcal{Y}\left[\begin{array}{|c|c|c|}\hline r & s & t \\ \hline\end{array}\right] \circ \mathcal{O}_1^{\rm y}$$



$$\mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}\right] \circ \mathcal{O}_1^{\rm y}, \quad (st)\mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}\right] \circ \mathcal{O}_1^{\rm y}$$



$$\mathcal{Y}\left[\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline\end{array}\right] \circ \mathcal{O}_1^{\rm y}$$



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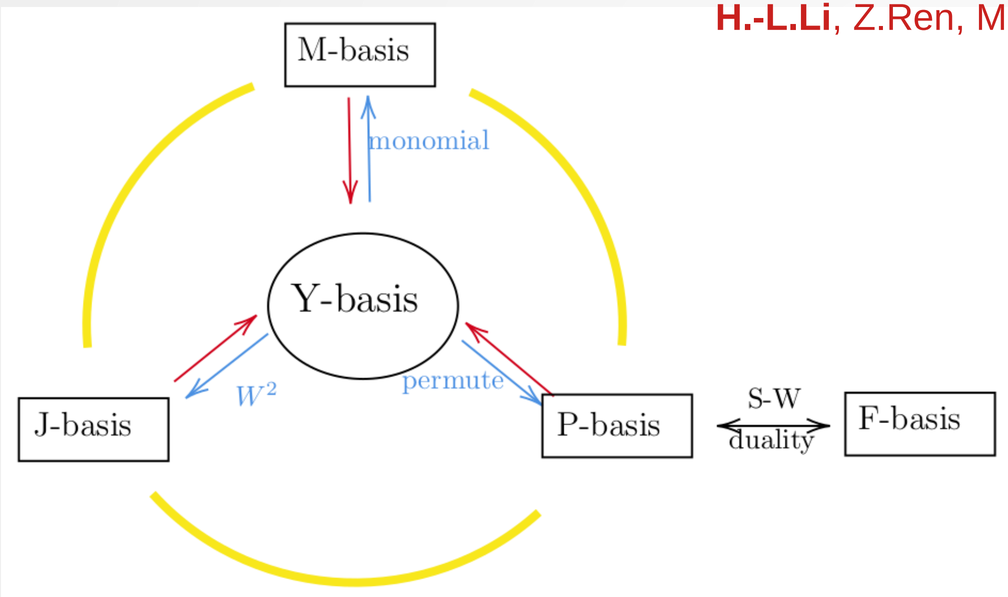


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$$\mathcal{O}^{\rm y}_1 = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})$$

$$\mathcal{Y}\left[\boxed{r\,|\,s\,|\,t}\right]\circ\mathcal{O}^{\rm y}_1$$

$$\mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}\right]\circ\mathcal{O}^{\rm y}_1,\quad (st)\mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}\right]\circ\mathcal{O}^{\rm y}_1$$

$$\mathcal{Y}\left[\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline\end{array}\right]\circ\mathcal{O}^{\rm y}_1$$

$$\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline\end{array}:\quad \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline\end{array},$$

$$\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}:\quad \begin{array}{|c|c|}\hline 1 & 1 \\ \hline 2 \\ \cline{1-1}\end{array},\ \begin{array}{|c|c|}\hline 1 & 1 \\ \hline 3 \\ \cline{1-1}\end{array},\ \begin{array}{|c|c|}\hline 2 & 2 \\ \hline 3 \\ \cline{1-1}\end{array},\ \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 2 \\ \cline{1-1}\end{array},\ \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 3 \\ \cline{1-1}\end{array},\ \begin{array}{|c|c|}\hline 2 & 3 \\ \hline 3 \\ \cline{1-1}\end{array},\ \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array},\ \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array},$$

$$\boxed{r\,|\,s\,|\,t}:\quad \boxed{1|1|1},\ \boxed{1|1|2},\ \boxed{1|1|3},\ \boxed{1|2|2},\ \boxed{1|2|3},$$
$$\boxed{1|3|3},\ \boxed{2|2|2},\ \boxed{2|2|3},\ \boxed{2|3|3},\ \boxed{3|3|3},$$

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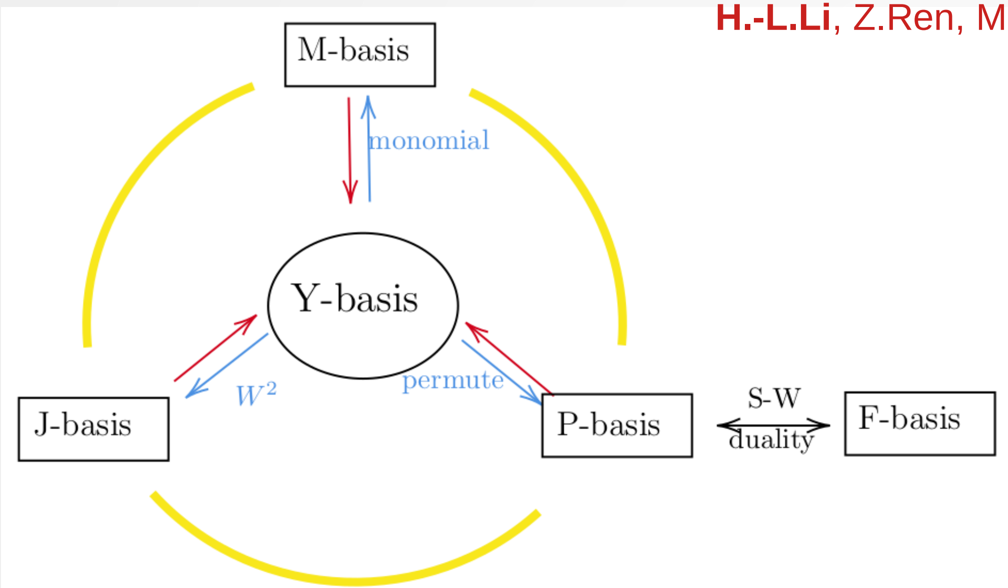


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$$\mathcal{Y}\left[\begin{array}{|c|c|c|}\hline r & s & t \\ \hline\end{array}\right]\circ\mathcal{O}_1^{y}$$

$$\mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}\right]\circ\mathcal{O}_1^{y},\quad (st)\mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}\right]\circ\mathcal{O}_1^{y}$$

$$\mathcal{Y}\left[\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline\end{array}\right]\circ\mathcal{O}_1^{y}$$

Forbidden If $n_f = 2$

$\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline\end{array}$ : $\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline\end{array}$,

$\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}$ : $\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 2 \\ \cline{1-1}\end{array}$, $\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 3 \\ \cline{1-1}\end{array}$, $\begin{array}{|c|c|}\hline 2 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}$, $\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 2 \\ \cline{1-1}\end{array}$, $\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 3 \\ \cline{1-1}\end{array}$, $\begin{array}{|c|c|}\hline 2 & 3 \\ \hline 3 \\ \cline{1-1}\end{array}$, $\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}$, $\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array}$,

$\begin{array}{|c|c|c|}\hline r & s & t \\ \hline\end{array}$ : $\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline\end{array}$, $\begin{array}{|c|c|c|}\hline 1 & 1 & 2 \\ \hline\end{array}$, $\begin{array}{|c|c|c|}\hline 1 & 1 & 3 \\ \hline\end{array}$, $\begin{array}{|c|c|c|}\hline 1 & 2 & 2 \\ \hline\end{array}$, $\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline\end{array}$,
$\begin{array}{|c|c|c|}\hline 1 & 3 & 3 \\ \hline\end{array}$, $\begin{array}{|c|c|c|}\hline 2 & 2 & 2 \\ \hline\end{array}$, $\begin{array}{|c|c|c|}\hline 2 & 2 & 3 \\ \hline\end{array}$, $\begin{array}{|c|c|c|}\hline 2 & 3 & 3 \\ \hline\end{array}$, $\begin{array}{|c|c|c|}\hline 3 & 3 & 3 \\ \hline\end{array}$,

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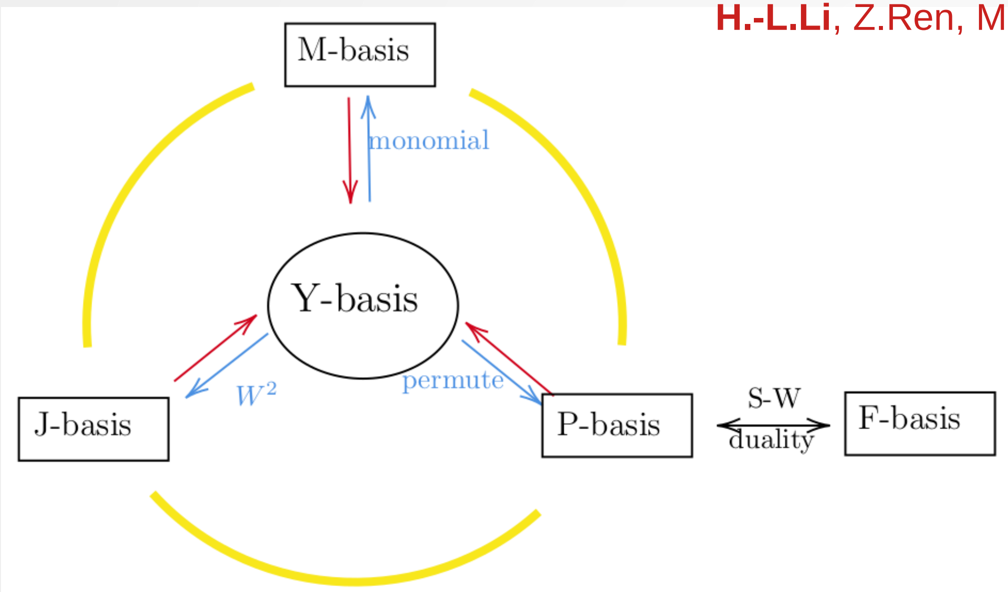


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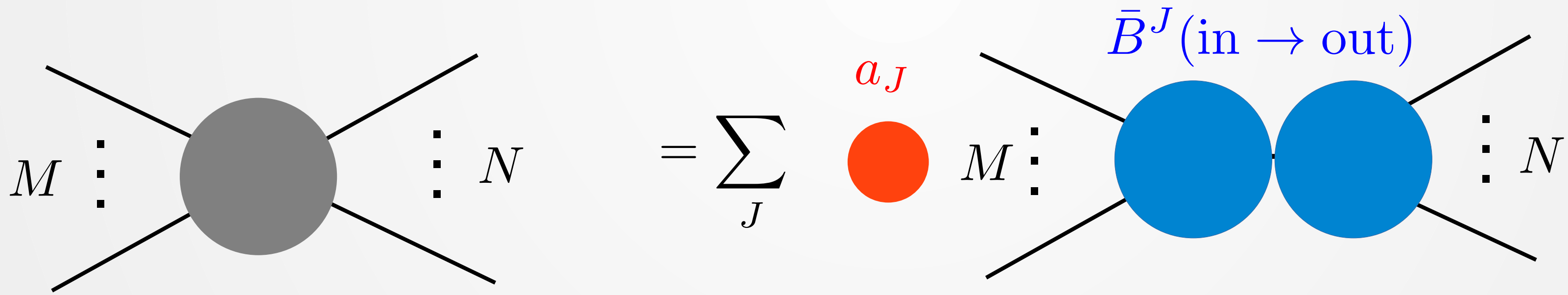
- F-basis: independnent flavor tensor spaces – eliminate the improper and redundant flavor tensors
- J-basis: Eigen-basis of Casimirs

ABC4EFT: powerful to obtain the transformation matrix between different bases

# The origin of the J-basis—Generalizd partial wave basis

$$\langle out|\mathbf{T}|in\rangle = \sum_{J,\sigma}\sum_{J',\sigma'}\int \mathrm{d}P\ \mathrm{d}P' \langle out|P,J,\sigma\rangle\langle P,J,\sigma|\mathbf{T}|P',J',\sigma'\rangle\langle P',J',\sigma'|in\rangle$$

$$\equiv \sum_J a_J \bar{B}^J(\mathrm{in}\to\mathrm{out})\delta^{(4)}\left(p_{out}-p_{in}\right)$$




$$\bar{B}^J(\mathrm{in}\to\mathrm{out}) = \sum_\sigma \langle out|P,J,\sigma\rangle\langle P,J,\sigma|in\rangle$$

A J-basis amplitude for *M-to-N* scattering with specific angular momentum of in and out states

# Casimir of Poincare Symmetry

$$\bar{B}^J(\text{in} \to \text{out}) = \sum_\sigma \langle out|P,J,\sigma\rangle\langle P,J,\sigma|in\rangle$$

Pauli-Lubanski operator $W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}$

$$\mathbf{P}|P,J,\sigma\rangle = P|P,J,\sigma\rangle, \quad \mathbf{W}^2|P,J,\sigma\rangle = -P^2 J(J+1)|P,J,\sigma\rangle$$

$$\langle P,J,\sigma \mid \Psi_1,\ldots,\Psi_N\rangle \equiv \mathcal{C}_N^{J,\sigma}\delta^{(4)}\left(P - \sum_i p_i\right)$$ Poincaré CG coefficients

2-body states (unique):

M.-Y. Jiang, J.Shu, M.-L.Xiao, Y.-H.Zheng, 2001.04481

$$\mathcal{C}^{J,\sigma}_{(h_1,h_2)} \sim [12]^{J+h_1+h_2}\left(\langle 1\chi\rangle^{J-h_1+h_2}\langle 2\chi\rangle^{J+h_1-h_2}\right)^{\{I_1,\ldots,I_{2J}\}}$$

Massive spinor state: $\chi^I(P)$

# Casimir of Poincare Symmetry

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$$\lambda^I_\alpha\lambda^{\alpha J} = m\epsilon^{JI}, \quad \tilde{\lambda}_{\dot{\alpha}I}\tilde{\lambda}^{\dot{\alpha}}_J = m\epsilon_{IJ}$$

$$\bar{B}^J_{1,2\to 3,4} = \sum_\sigma \left[\mathcal{C}^{J,\sigma}(3,4)\right]^* \mathcal{C}^{J,\sigma}(1,2) \sim d^J_{h_2-h_1,h_4-h_3}(\theta)$$

# Casimir of Poincare Symmetry

$$\bar{B}^J(\text{in} \to \text{out}) = \sum_\sigma \langle out|P, J, \sigma\rangle\langle P, J, \sigma|in\rangle$$

Pauli-Lubanski operator $W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}$

$$\mathbf{P}|P, J, \sigma\rangle = P|P, J, \sigma\rangle, \quad \mathbf{W}^2|P, J, \sigma\rangle = -P^2 J(J+1)|P, J, \sigma\rangle$$

$$\langle P, J, \sigma \mid \Psi_1, \ldots, \Psi_N\rangle \equiv \mathcal{C}_N^{J,\sigma}\delta^{(4)}\left(P - \sum_i p_i\right)$$ Poincaré CG coefficients

3-body states (degenerate):

$$\mathcal{C}^{J=1,\sigma,1}_{(0,0,0)} \sim [12]\left\langle 1\chi^{\{I_1}\right\rangle\left\langle 2\chi^{I_2\}}\right\rangle$$

$$\mathcal{C}^{J=1,0,2}_{(0,0,0)} \sim [23]\left\langle 2\chi^{\{I_1}\right\rangle\left\langle 3\chi^{I_2\}}\right\rangle$$

...

Not systematic, need to find some other way to construct $B^J$

# Casimir of Poincare Symmetry

$$\mathbf{P}|P,J,\sigma\rangle = P|P,J,\sigma\rangle, \quad \mathbf{W}^2|P,J,\sigma\rangle = -P^2 J(J+1)|P,J,\sigma\rangle$$

$$\langle P,J,\sigma \mid \Psi_1,\ldots,\Psi_N\rangle \equiv \mathcal{C}_N^{J,\sigma}\delta^{(4)}\left(P - \sum_i p_i\right)$$

$$W^2\mathcal{C}_N^{J,\sigma} \equiv \int \mathrm{d}^4P \left\langle P,J,\sigma \left|\mathbf{W}^2\right| \Psi_1,\ldots,\Psi_N\right\rangle = -sJ(J+1)\mathcal{C}_N^{J,\sigma}$$

Poincaré Algebra for Functions of Spinor variables

$$W^2 = \frac{1}{8}P^2\left(\mathrm{Tr}\left[M^2\right] + \mathrm{Tr}\left[\tilde{M}^2\right]\right) - \frac{1}{4}\mathrm{Tr}\left[P^\top M P \widetilde{M}\right]$$

$$M_{\alpha\beta} = i\sum_{i=1}^{N}\left(\lambda_{i\alpha}\frac{\partial}{\partial\lambda_i^\beta} + \lambda_{i\beta}\frac{\partial}{\partial\lambda_i^\alpha}\right), \quad \widetilde{M}_{\dot\alpha\dot\beta} = i\sum_{i=1}^{N}\left(\tilde\lambda_{i\dot\alpha}\frac{\partial}{\partial\tilde\lambda_i^{\dot\beta}} + \tilde\lambda_{i\dot\beta}\frac{\partial}{\partial\tilde\lambda_i^{\dot\alpha}}\right)$$

E. Witten, hep-th/0312171
M.-Y. Jiang, J.Shu, M.-L.Xiao, Y.-H.Zheng, 2001.04481

# Casimir of Poincare Symmetry

Define the action on the amplitude:

$$W_{\mathcal{I}}{}^2 \bar{B}^J(\mathcal{I} \to \mathcal{I}') \equiv \left(W^2 \mathcal{C}^J_{\mathcal{I}}\right) \cdot \mathcal{C}^J_{\mathcal{I}'} = -s_{\mathcal{I}} J(J+1) \bar{B}^J(\mathcal{I} \to \mathcal{I}')$$

$$M_{\mathcal{I},\alpha\beta} = i \sum_{i \in \mathcal{I}} \left( \lambda_{i\alpha} \frac{\partial}{\partial \lambda_i^{\beta}} + \lambda_{i\beta} \frac{\partial}{\partial \lambda_i^{\alpha}} \right)$$

$$\tilde{M}_{\mathcal{I},\dot{\alpha}\dot{\beta}} = i \sum_{i \in \mathcal{I}} \left( \tilde{\lambda}_{i\dot{\alpha}} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\beta}}} + \tilde{\lambda}_{i\dot{\beta}} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \right)$$

Acting only on the momentum of particle in the part *I*

As one expact the partial wave basis is eigen-basis of the Casimir $W^2$

For a complete set of amplitudes for a specific set of particles and of a fixed dimension—the amplitude generated by a type of operator, we expect to find a representation matrix of the Casimir operator $W^2$:

$$W^2_{\mathcal{I}} \mathcal{B}_i = -s_{\mathcal{I}} \mathcal{W}_i{}^j \mathcal{B}_j$$

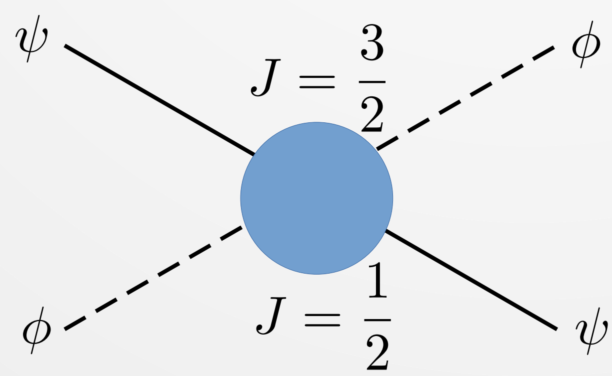
# Casimir of Poincare Symmetry

Take $L_1 L_2 H_3 H_4 D^2$ as an example:

$\mathcal{W}$

$$\mathcal{B}^{y}_{\psi^2\phi^2 D^2} = \begin{pmatrix} s_{34}\langle 12\rangle \\ [34]\langle 13\rangle\langle 24\rangle \end{pmatrix}, \quad W^2_{\{13\}}\mathcal{B}^y = s_{13}\begin{pmatrix} -\frac{15}{4} & 2 \\ 0 & -\frac{3}{4} \end{pmatrix}\mathcal{B}^y, \quad \mathcal{K}^{jy}_{\mathcal{B}} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{B}^j = \mathcal{K}^{jy}_{\mathcal{B}}\mathcal{B}^y = \begin{cases} 3s_{34}\langle 12\rangle + 2[34]\langle 13\rangle\langle 24\rangle & J = \frac{3}{2} \\ \langle 13\rangle\langle 24\rangle & J = \frac{1}{2} \end{cases}$$



# Gauge Amplitude

One can do the same thing for the gauge amplitude

$$\begin{aligned} \mathbb{C}_2 &= \mathbb{T}^a\mathbb{T}^a, \text{ for both } SU(2) \text{ and } SU(3), \\ \mathbb{C}_3 &= d^{abc}\mathbb{T}^a\mathbb{T}^b\mathbb{T}^c, \text{ for } SU(3) \text{ only}, \end{aligned}$$

$$\underset{\mathcal{I}}{\mathbb{T}}^A \circ \Theta_{I_1 I_2 \dots I_N} = \sum_{i\in\mathcal{I}}^N (T^A_{r_i})^Z_{I_i} \Theta_{I_1\dots I_{i-1} Z I_{i+1} I_N}$$

Representation of the i-th index

$$\underset{\mathbb{S}}{\mathbb{C}} \circ \mathcal{T}_i = \sum_j \mathcal{T}_j C_{ji}$$

A complete set of invariant tensors

Representation matrix of Casimir

# Gauge Amplitude

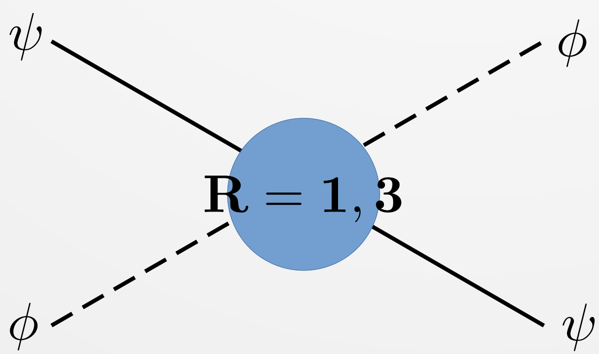
Take again $L_1 L_2 H_3 H_4 D^2$ as an example:

$$\mathcal{T}^m_{LLHH} = \begin{pmatrix} \epsilon^{ik}\epsilon^{jl} \\ \epsilon^{ij}\epsilon^{kl} \end{pmatrix}, \quad \underset{\{13\}}{\mathbb{C}_2} \circ \mathcal{T}^m = (\underset{\{13\}}{C_2})^{\mathrm{T}}.\mathcal{T}^m = \begin{pmatrix} 0 & 0 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} \epsilon^{ik}\epsilon^{jl} \\ \epsilon^{ij}\epsilon^{kl} \end{pmatrix}.$$

$C_2(\mathbf{1})$ $C_2(\mathbf{3})$

$$\mathcal{K}^{jm}_G.(\underset{\{13\}}{C_2})^{\mathrm{T}}(\mathcal{K}^{jm}_G)^{-1} = \begin{pmatrix} 0 & 0 \\ 0 & 6 \end{pmatrix} \text{ with } \mathcal{K}^{jm}_G = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$$

$$\Rightarrow \mathcal{T}^j = \mathcal{K}^{jm}_G \mathcal{T}^m = \begin{cases} \epsilon^{ik}\epsilon^{jl} & \mathbf{R} = \mathbf{1} \\ \epsilon^{ik}\epsilon^{jl} - 2\epsilon^{ij}\epsilon^{kl} & \mathbf{R} = \mathbf{3} \end{cases}$$



# Definition of J-basis operator

Amplitude operator correspondence defines J-basis operators

$$\mathcal{O}^{J,\mathbf{R}}_{\mathcal{I}\to\mathcal{I}'} \sim \mathcal{T}(\mathbf{R})\bar{B}^{J}\,(\mathcal{I}\to\mathcal{I}') \quad \begin{cases} W_{\mathcal{I}}^2\bar{B}^J = -s_{\mathcal{I}}J(J+1)\bar{B}^J \\ \mathbb{C}_{\mathcal{I}}\mathcal{T}(\mathbf{R}) = C(\mathbf{R})\mathcal{T}(\mathbf{R}) \end{cases}$$

It annihilates multiparticle states with fixed angular momentum and Gauge quantum numbers

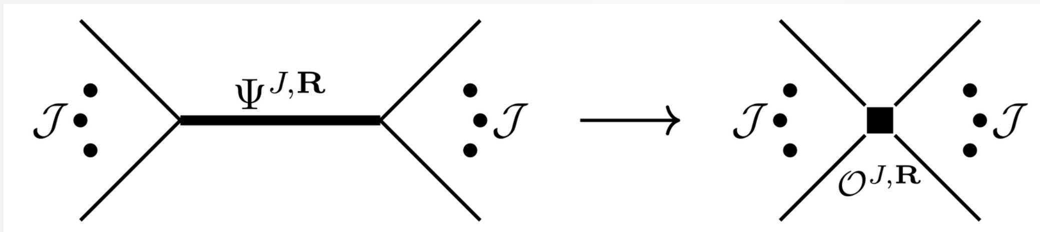
$$\mathcal{O}^{J,\mathbf{R}}_{\mathcal{I}\to\mathcal{I}'}\,|\Psi_{\mathcal{I}}\rangle^{J',\mathbf{R}'} \sim \delta^{JJ'}\delta^{\mathbf{R}\mathbf{R}'}$$

Automatic generation of J-basis operator for a given type in our package

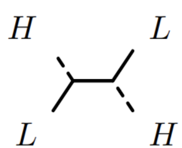
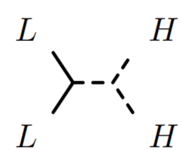
```
In[4]:= LoadModel["SMEFT.m"];

In[9]:= GetJBasisForType[SMEFT, "ec" "L" "Q" "uc" "D"^2, {{1, 3}, {2, 4}}]

Out[9]= ⟨|basis → {ϵ^ij (ec_p L_ri) ((D_μ Q_saj) (D^μ uc_t^a)), ϵ^ij (ec_p Q_saj) ((D_μ L_ri) (D^μ uc_t^a)), i ϵ^ij (ec_p σ_μν L_ri) ((D^μ Q_saj) (D^ν uc_t^a))},
  groups → {SU3c, SU2w, Spin}, j-basis → {<|{L_2, uc_4} → {{0, 1}, {1}, 2}, {ec_1, Q_3} → {{1, 0}, {1}, 2}|> → {{-6, -2, -6}},
    <|{L_2, uc_4} → {{0, 1}, {1}, 1}, {ec_1, Q_3} → {{1, 0}, {1}, 1}|> → {{2, -2, -2}},
    <|{L_2, uc_4} → {{0, 1}, {1}, 0}, {ec_1, Q_3} → {{1, 0}, {1}, 0}|> → {{0, 2, 0}}}|⟩
```

# Application 1: Finding UV Origin

Top Down: $\mathcal{L}_{\mathrm{UV}} \supset \Psi^{J,\mathrm{R}}_{\mathrm{heavy}} \cdot \mathcal{J}_{\mathrm{light}} \xrightarrow{\mathrm{CDE}} \mathcal{J}_{\mathrm{light}} \cdot \mathcal{J}_{\mathrm{light}}$




Bottom up: $\mathcal{O}^{J,\mathrm{R}} \longrightarrow \Psi^{J,\mathrm{R}}_{\mathrm{heavy}}$ we can exhaust all the tree level resonance without UV models

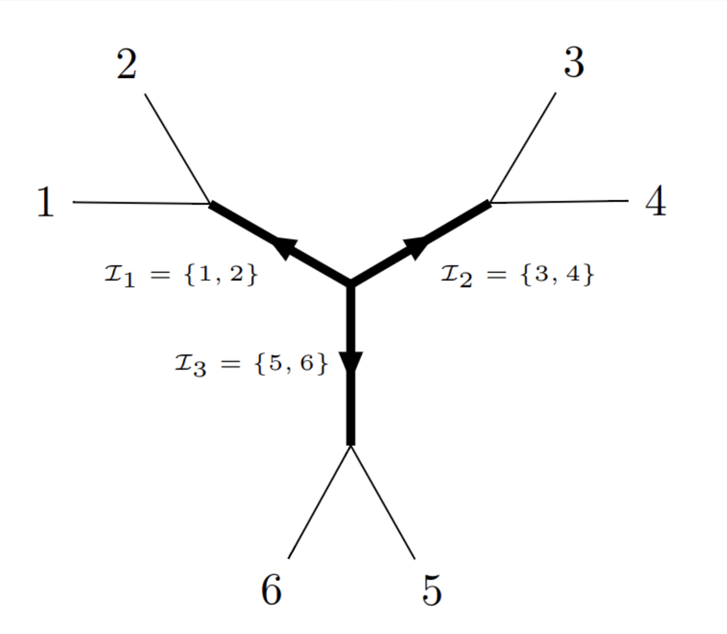
| Topology | j-basis | Quantum numbers $\{J, \mathbf{R}, Y\}$ | Model |
|---|---|---|---|
|   | $\mathcal{O}^{J=1/2,\mathbf{R}=1}_{\{13\}} = \mathcal{O}^p_1 + \mathcal{O}^p_2.$ | $\{\frac{1}{2}, 1, 0\}$ | Type I |
| | $\mathcal{O}^{J=1/2,\mathbf{R}=3}_{\{13\}} = -\mathcal{O}^p_1 + 3\mathcal{O}^p_2,$ | $\{\frac{1}{2}, 3, 0\}$ | Type III |
|   | $\mathcal{O}^{J=0,\mathbf{R}=3}_{\{12\}} = -2\mathcal{O}^p_1,$ | $\{0, 3, -1\}$ | Type II |
| | $\mathcal{O}^{J=0,\mathbf{R}=1}_{\{12\}} = 2\mathcal{O}^p_2.$ | $\{0, 1, -1\}$ | N/A |

$$\mathcal{O}^p_{LLHH} = \begin{pmatrix} \mathcal{O}^p_{LLHH,1} \\ \mathcal{O}^p_{LLHH,2} \end{pmatrix} = \begin{pmatrix} \frac{1}{4}\mathcal{Y}[\,\boxed{p}\boxed{r}\,]\mathcal{Y}[\,\boxed{\phantom{x}}\boxed{\phantom{x}}\,_H]\epsilon^{ik}\epsilon^{jl}H_k H_l(L_{pi}L_{rj}) \\ \frac{1}{4}\mathcal{Y}[\,\begin{smallmatrix}\boxed{p}\\\boxed{r}\end{smallmatrix}\,]\mathcal{Y}[\,\begin{smallmatrix}\boxed{\phantom{x}}\\\boxed{\phantom{x}}\end{smallmatrix}\,_H]\epsilon^{ik}\epsilon^{jl}H_k H_l(L_{pi}L_{rj}) \end{pmatrix}$$

Forbidden, as Higgs have only one flavor

# Application 1: Finding UV Origin

So far we only discuss two partite channel, extension to multi-partite is straight forward




$$[W^2_{\mathcal{I}_1}, W^2_{\mathcal{I}_2}], [W^2_{\mathcal{I}_1}, W^2_{\mathcal{I}_3}], [W^2_{\mathcal{I}_3}, W^2_{\mathcal{I}_2}]$$

$$[\mathbb{C}_{\mathcal{I}_1}, \mathbb{C}_{\mathcal{I}_2}], [\mathbb{C}_{\mathcal{I}_1}, \mathbb{C}_{\mathcal{I}_3}], [\mathbb{C}_{\mathcal{I}_3}, \mathbb{C}_{\mathcal{I}_2}]$$

Can Find simultanous eigenbasis for each Casimir operator

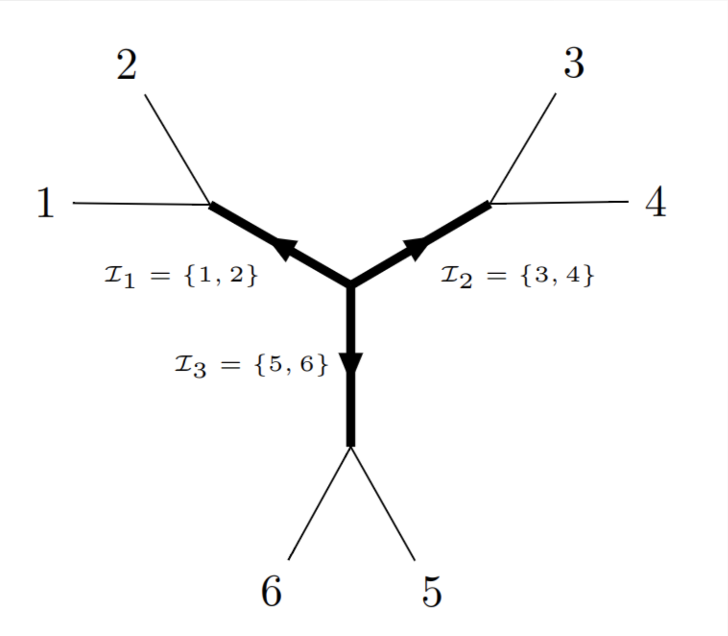
$$W^2_{\mathcal{I}_i} \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}} = -s_{\mathcal{I}_i} J_i \left(J_i + 1\right) \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}}$$

$$\mathbb{C}_{\mathcal{I}_i} \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}} = C\left(\mathbf{R}_i\right) \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}}$$

$\mathcal{O}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}} \sim \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}}$ Can be obtained by integrating out heavy fields $\{\Psi^{J_i,\mathbf{R}_i}\}$

# Application 1: Finding UV Origin

So far we only discuss two partite channel, extension to multi-partite is straight forward




$$[W^2_{\mathcal{I}_1}, W^2_{\mathcal{I}_2}], [W^2_{\mathcal{I}_1}, W^2_{\mathcal{I}_3}], [W^2_{\mathcal{I}_3}, W^2_{\mathcal{I}_2}]$$

$$[\mathbb{C}_{\mathcal{I}_1}, \mathbb{C}_{\mathcal{I}_2}], [\mathbb{C}_{\mathcal{I}_1}, \mathbb{C}_{\mathcal{I}_3}], [\mathbb{C}_{\mathcal{I}_3}, \mathbb{C}_{\mathcal{I}_2}]$$

Can Find simultanous eigenbasis for each Casimir operator

$$W^2_{\mathcal{I}_i} \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}} = -s_{\mathcal{I}_i} J_i \left(J_i + 1\right) \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}}$$

$$\mathbb{C}_{\mathcal{I}_i} \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}} = C\left(\mathbf{R}_i\right) \mathcal{B}^{\{J_i\},\{\mathbf{R}_i\}}_{\mathcal{P}}$$

Three steps finding tree-level UV origins of an operator type:

1) Finding all the tree topologies and partitions for fixed number of external legs,
2) Finding all possible J-basis for all the partitions of the given topologies.
3) Expand J-basis with P-basis, and find out those contribute to allowed permutation symmetry

# Application 2: Analysis Dim-8 Contritution to qqWW

C. Degrande, **H.-L.Li**, 2206.xxxxx

Motivation: Dim-6 Operator inteference effects maybe suppressed:

$$
\begin{aligned}
|\mathcal{A}|^2 &= \left|\mathcal{A}_{SM} + \frac{\mathcal{A}_6}{\Lambda^2} + \frac{\mathcal{A}_8}{\Lambda^4}\right|^2 \\
&= |\mathcal{A}_{SM}|^2 + \frac{1}{\Lambda^2}\mathcal{A}_6\mathcal{A}_{SM}^* + \frac{1}{\Lambda^4}|\mathcal{A}_6|^2 + \frac{1}{\Lambda^4}\mathcal{A}_8\mathcal{A}_{SM}^* + \ldots
\end{aligned}
$$

May vanishes due to helicity selection rules

A. Azatov, et.al.,1607.05236

If one considers the dim-6 square contribution, it better to taking into accout the dim-8 inteference for consistency

# Application 2: Analysis Dim-8 Contritution to qqWW

The dim-8 interference amplitude at most scales as $\frac{E^4}{\Lambda^4}$

$$O_8 = iW^{I\mu}{}_{\nu}W^{I\nu}{}_{\lambda}(\bar{d}_{\mathrm{R}p}\gamma^{\lambda}\overleftrightarrow{D}_{\mu}d_{\mathrm{R}r}),$$

$$O_9 = iW^{I\mu}{}_{\nu}W^{I\nu}{}_{\lambda}(\bar{u}_{\mathrm{R}p}\gamma^{\lambda}\overleftrightarrow{D}_{\mu}u_{\mathrm{R}r}),$$

$$O_{10} = iW^{I\mu}{}_{\nu}W^{I\nu}{}_{\lambda}\left(\bar{q}_{\mathrm{L}r}\gamma^{\lambda}\overleftrightarrow{D}_{\mu}q_{\mathrm{L}p}\right),$$

$$O_{11} = i\epsilon^{IJK}W^{I\mu}{}_{\nu}W^{J\nu}{}_{\lambda}\left(\bar{q}^{i}_{\mathrm{L}p}\gamma^{\lambda}\left(\tau^{K}\right)_{i}{}^{j}\overleftrightarrow{D}_{\mu}q_{\mathrm{L}rj}\right),$$

CP Violating

$$O_{12} = i\epsilon^{IJK}\tilde{W}^{I\mu}{}_{\nu}W^{J\nu}{}_{\lambda}\left(\bar{q}^{i}_{\mathrm{L}p}\gamma^{\lambda}\left(\tau^{K}\right)_{i}{}^{j}\overleftrightarrow{D}_{\mu}q_{\mathrm{L}rj}\right),$$

$$O_{13} = i\epsilon^{IJK}W^{I\mu}{}_{\nu}\tilde{W}^{J\nu}{}_{\lambda}\left(\bar{q}^{i}_{\mathrm{L}p}\gamma^{\lambda}\left(\tau^{K}\right)_{i}{}^{j}\overleftrightarrow{D}_{\mu}q_{\mathrm{L}rj}\right).$$

Other dim-8 operators at least suppressed by $\frac{v}{\Lambda}$

# Application 2: Analysis Dim-8 Contritution to qqWW

Scaling beheaviour of each operator as high energy

| Operator | Scaling of $\sum_{\{h_i\}} 2\,\mathrm{Re}(\mathcal{A}^{\rm SM}_{h_i}\mathcal{A}^{\rm dim-8*}_{h_i})$ | $\int_{-1}^{1} d\cos\theta\,\mathcal{A}^2$ |
|---|---|---|
| $\mathcal{O}_8$ | $d\bar{d}:\ [B_8 S + C_8]$ | 0 |
| $\mathcal{O}_9$ | $\bar{u}u:\ [B_9 S + C_9]$ | 0 |
| $\mathcal{O}_{10}$ | $u\bar{u}:\ A^u_{10}\cdot S^2 + B^u_{10}\cdot S + C^u_{10}$ | $\overline{A}^u_{10}\cdot S^2 + \overline{B}^u_{10}\cdot S + \overline{C}^u_{10}$ |
| | $d\bar{d}:\ A^d_{10}\cdot S^2 + B^d_{10}\cdot S + C^d_{10}$ | $\overline{A}^d_{10}\cdot S^2 + \overline{B}^d_{10}\cdot S + \overline{C}^d_{10}$ |
| $\mathcal{O}_{11}$ | 0 | 0 |
| $\mathcal{O}_{12}$ | $u\bar{u}:\ A^u_{12}S^2 + B^u_{12}S + C^u_{12}$ | $u\bar{u}:\ \overline{A}^u_{12}S^2 + \overline{B}^u_{12}S + \overline{C}^u_{12} + \overline{D}^u_{12}\log S$ |
| | $d\bar{d}:\ A^d_{12}S^2 + B^d_{12}S + C^d_{12}$ | $d\bar{d}:\ \overline{A}^d_{12}S^2 + \overline{B}^d_{12}S + \overline{C}^d_{12} + \overline{D}^d_{12}\log S$ |
| $\mathcal{O}_{13}$ | $u\bar{u}:\ A^u_{13}S^2 + B^u_{13}S + C^u_{13}$ | $u\bar{u}:\ \overline{A}^u_{13}S^2 + \overline{B}^u_{13}S + \overline{C}^u_{13} + \overline{D}^u_{13}\log S$ |
| | $d\bar{d}:\ A^d_{13}S^2 + B^d_{13}S + C^d_{13}$ | $d\bar{d}:\ \overline{A}^d_{13}S^2 + \overline{B}^d_{13}S + \overline{C}^d_{13} + \overline{D}^d_{13}\log S$ |

Zero can be explained by J-basis analysis

# Application 2: Analysis Dim-8 Contritution to qqWW

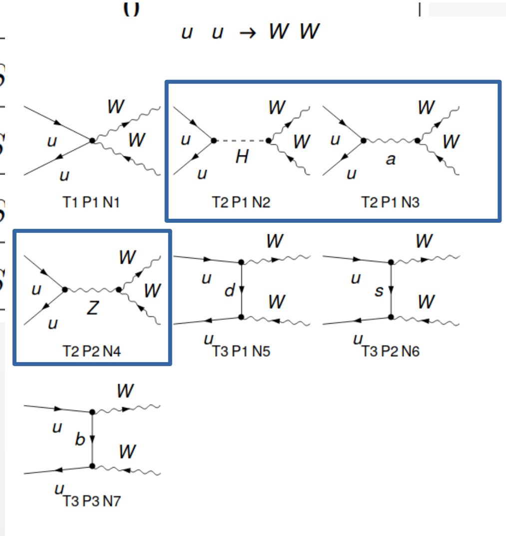
## Scaling beheaviour of each operator as high energy

| Operator | Scaling of $\sum_{\{h_i\}} 2\,\mathrm{Re}(\mathcal{A}^{\mathrm{SM}}_{h_i}\mathcal{A}^{\mathrm{dim-8}*}_{h_i})$ | $\int_{-1}^{1} d\cos\theta\,\mathcal{A}^2$ |
|---|---|---|
| $\mathcal{O}_8$ | $d\bar{d}:\ [B_8 S + C_8]$ | 0 |
| $\mathcal{O}_9$ | $\bar{u}u:\ [B_9 S + C_9]$ | 0 |
| $\mathcal{O}_{10}$ | $u\bar{u}:\ A^u_{10}\cdot S^2 + B^u_{10}\cdot S + C^u_{10}$ | |
| | $d\bar{d}:\ A^d_{10}\cdot S^2 + B^d_{10}\cdot S + C^d_{10}$ | |
| $\mathcal{O}_{11}$ | 0 | 0 |
| $\mathcal{O}_{12}$ | $u\bar{u}:\ A^u_{12}S^2 + B^u_{12}S + C^u_{12}$ | $u\bar{u}:\ \overline{A}^u_{12}S$ |
| | $d\bar{d}:\ A^d_{12}S^2 + B^d_{12}S + C^d_{12}$ | $d\bar{d}:\ \overline{A}^d_{12}S$ |
| $\mathcal{O}_{13}$ | $u\bar{u}:\ A^u_{13}S^2 + B^u_{13}S + C^u_{13}$ | $u\bar{u}:\ \overline{A}^u_{13}S$ |
| | $d\bar{d}:\ A^d_{13}S^2 + B^d_{13}S + C^d_{13}$ | $d\bar{d}:\ \overline{A}^d_{13}S$ |

$$\mathcal{O}_9 = iW^{I\mu}{}_{\nu}W^{I\nu}{}_{\lambda}(\bar{u}_{\mathrm{R}p}\gamma^{\lambda}\overleftrightarrow{D}_{\mu}u_{\mathrm{R}r})$$

```
GetJBasisForType[SMEFT, "uc" "uc†" "WL" "WR" "D", {{1, 4}, {2, 3}}]

⟨|basis → {i WR^{Iλμ} WL^I_λ^ν (uc_p^a σ_μ (D_ν uc†_ra))}, groups → {SU3c, SU2w, Spin},
 j-basis → {<|{uc_2, uc†_3} → {{0, 0}, {0}, 2}, {WL_1, WR_4} → {{0, 0}, {0}, 2}|> → {{-4}}}|⟩
```

$J = 2$

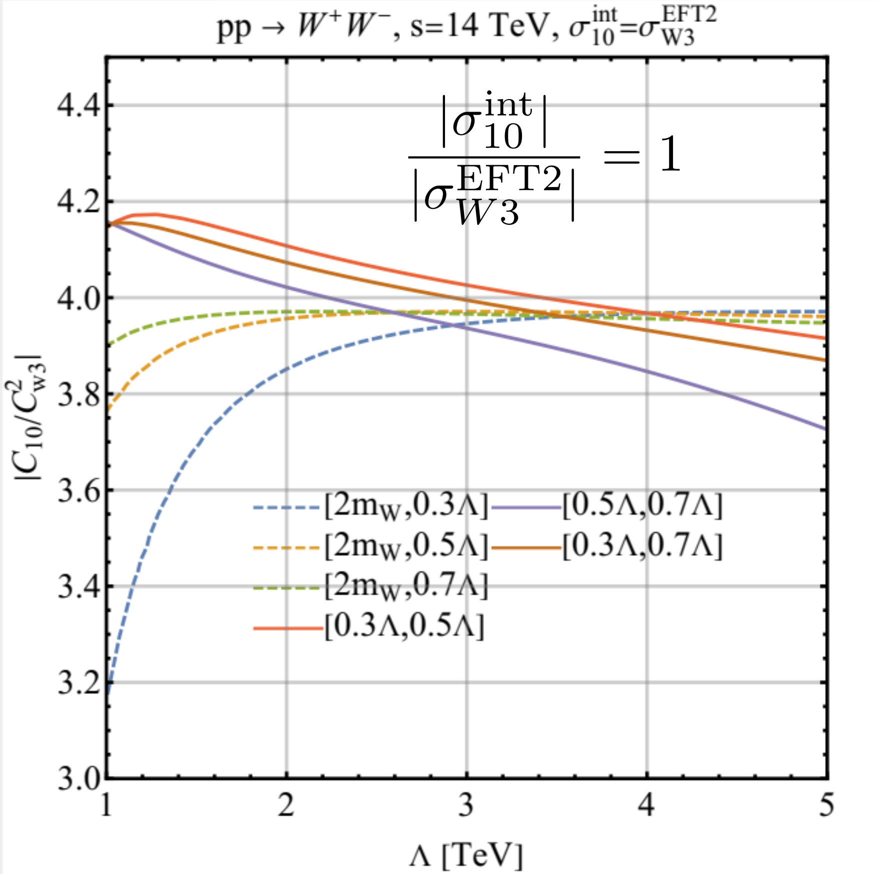


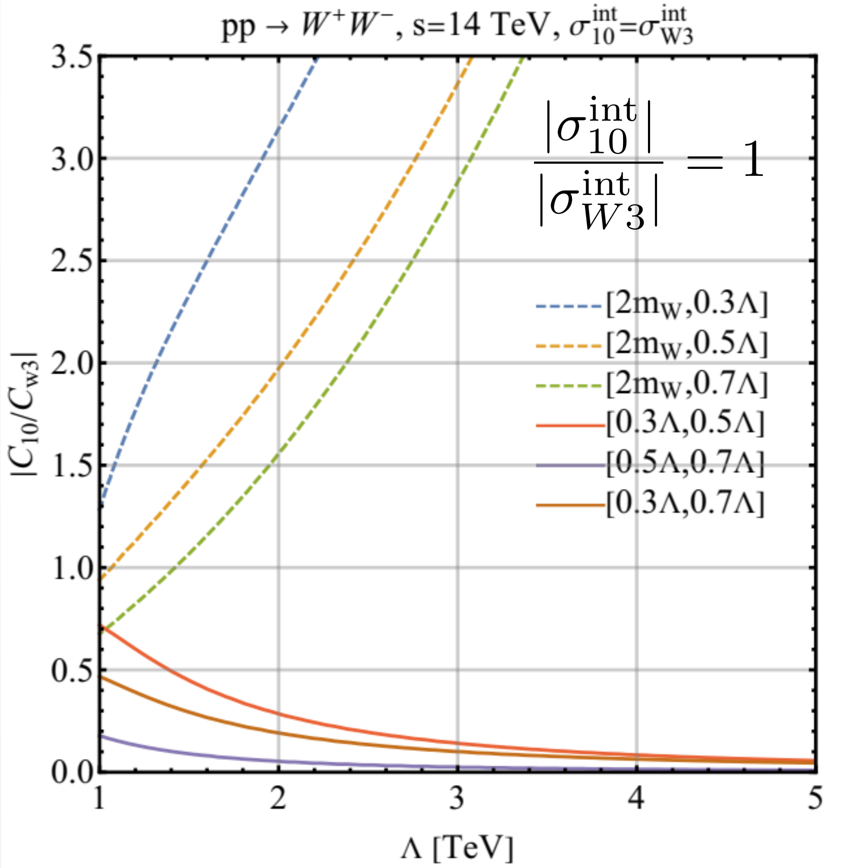

Only *J=1* & *J=0* channel contribute to right-handed u qark scattering

# Application 2: Analysis Dim-8 Contritution to qqWW

Numerical result: comparison with dim-6 EFT² and inteference

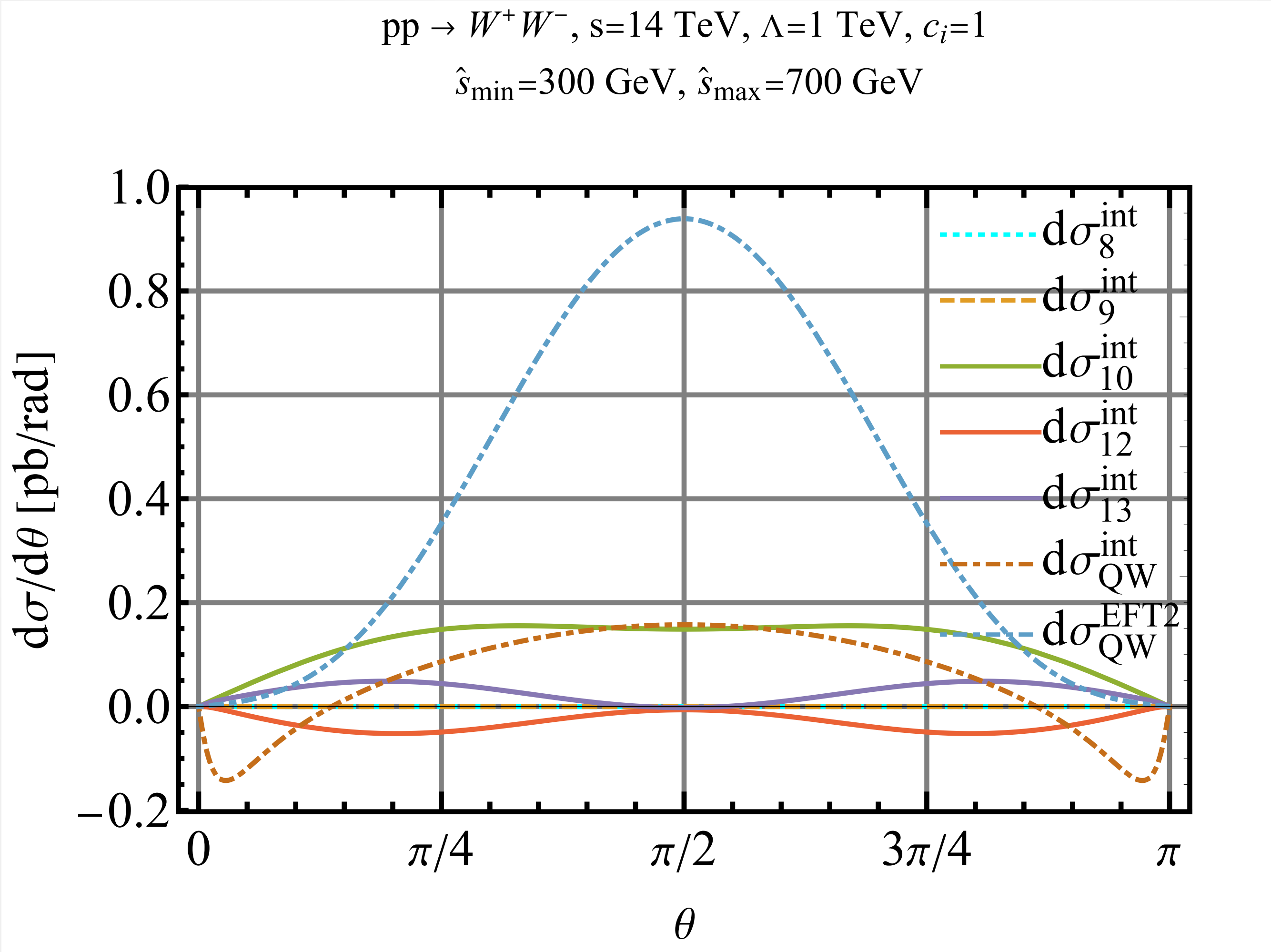
$\frac{C_{W3}}{\Lambda^2}$ $\mathcal{O}_{W3} = \varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$ $\frac{C_{10}}{\Lambda^4}$ $\mathcal{O}_{10} = i W^{I\mu}{}_\nu W^{I\nu}{}_\lambda \left(\bar{q}_{Lr} \gamma^\lambda \overleftrightarrow{D}_\mu q_{Lp}\right)$



# Application 2: Analysis Dim-8 Contritution to qqWW

## Angular distribution



## Summary and Outlook:

1) Our package automate the enumeration and convertion between operator basis

2) J-basis is useful in many applications: UV origins, Anomolus Dimension, vanishing interefrence, etc.

3) Dim-8 contrituion to the qqWW is significant.

4) Angular distribution of dim-8 contribution is different.

5) Bring back the EOM information.

6) Positivity and Unitarity bound on certain dim-8 operators.

7)Enumeration of ChPT or ALP EFT with Adler Zero condition.

# Back up slides

$$\sigma_{\rm QW}^{\rm EFT2}(\hat{s}) = -0.01143 + \frac{326}{\hat{s}} + 8.142 \times 10^{-7}\hat{s}$$

$$\sigma_{10}^{\rm int}(\hat{s}) - 0.00346 + \frac{5.392 \times 10^{-15}}{\hat{s}} + 3.619 \times 10^{-7}\hat{s}$$

$M_V$ 


| Hel | NP | SM |
|---|---|---|
| −+ | $\text{C8 S } (-2\,\text{MW2} + \text{S})\ \text{Sin}\left[\frac{\theta}{2}\right]^2 \text{Sin}[\theta]\ \text{SUNT}[\text{Col1, Col2}]$ | 0 |
| −− | $\text{C8 MW2 S Cos}[\theta]\ \text{Sin}[\theta]\ \text{SUNT}[\text{Col1, Col2}]$ | $\frac{4\,\text{Alfa MZ2}\,\pi \sqrt{1-\frac{4\,\text{MW2}}{\text{S}}}\ \text{Sin}[\theta]\ \text{SUNT}[\text{Col1,Col2}]}{3\,(-\text{MZ2}+\text{S})}$ |
| −0 | $\frac{\text{C8 MW2 S}^{3/2}\,(1+2\,\text{Cos}[\theta])\ \text{Sin}\left[\frac{\theta}{2}\right]^2 \text{SUNT}[\text{Col1,Col2}]}{\sqrt{2}\ \text{MW}}$ | $\frac{4\,\text{Alfa MZ2}\,\pi \sqrt{-8\,\text{MW2}+2\,\text{S}}\ \text{Sin}\left[\frac{\theta}{2}\right]^2 \text{SUNT}[\text{Col1,Col2}]}{-3\,\text{MW MZ2}+3\,\text{MW S}}$ |
| ++ | $\text{C8 MW2 S Cos}[\theta]\ \text{Sin}[\theta]\ \text{SUNT}[\text{Col1, Col2}]$ | $\frac{4\,\text{Alfa MZ2}\,\pi \sqrt{1-\frac{4\,\text{MW2}}{\text{S}}}\ \text{Sin}[\theta]\ \text{SUNT}[\text{Col1,Col2}]}{3\,(-\text{MZ2}+\text{S})}$ |
| +− | $-2\,\text{C8 S } (-2\,\text{MW2} + \text{S})\ \text{Cos}\left[\frac{\theta}{2}\right]^3 \text{Sin}\left[\frac{\theta}{2}\right]\ \text{SUNT}[\text{Col1, Col2}]$ | 0 |
| +0 | $\frac{\text{C8 MW2 S}^{3/2}\,\text{Cos}\left[\frac{\theta}{2}\right]^2 (-1+2\,\text{Cos}[\theta])\ \text{SUNT}[\text{Col1,Col2}]}{\sqrt{2}\ \text{MW}}$ | $\frac{4\,\text{Alfa MZ2}\,\pi \sqrt{-8\,\text{MW2}+2\,\text{S}}\ \text{Cos}\left[\frac{\theta}{2}\right]^2 \text{SUNT}[\text{Col1,Col2}]}{-3\,\text{MW MZ2}+3\,\text{MW S}}$ |
| 0+ | $-\frac{\text{C8 MW2 S}^{3/2}\,(1+2\,\text{Cos}[\theta])\ \text{Sin}\left[\frac{\theta}{2}\right]^2 \text{SUNT}[\text{Col1,Col2}]}{\sqrt{2}\ \text{MW}}$ | $\frac{4\sqrt{2}\ \text{Alfa MZ2}\,\pi \sqrt{-4\,\text{MW2}+\text{S}}\ \text{Sin}\left[\frac{\theta}{2}\right]^2 \text{SUNT}[\text{Col1,Col2}]}{3\,\text{MW MZ2}-3\,\text{MW S}}$ |
| 0− | $\frac{\text{C8 MW2 S}^{3/2}\,\text{Cos}\left[\frac{\theta}{2}\right]^2 (1-2\,\text{Cos}[\theta])\ \text{SUNT}[\text{Col1,Col2}]}{\sqrt{2}\ \text{MW}}$ | $\frac{4\sqrt{2}\ \text{Alfa MZ2}\,\pi \sqrt{-4\,\text{MW2}+\text{S}}\ \text{Cos}\left[\frac{\theta}{2}\right]^2 \text{SUNT}[\text{Col1,Col2}]}{3\,\text{MW MZ2}-3\,\text{MW S}}$ |
| 00 | $-2\,\text{C8 MW2 S}\left(\text{Cos}\left[\frac{\theta}{2}\right] + \text{Cos}\left[\frac{3\theta}{2}\right]\right) \text{Sin}\left[\frac{\theta}{2}\right]\ \text{SUNT}[\text{Col1, Col2}]$ | $\frac{2\,\text{Alfa MZ2}\,\pi \sqrt{1-\frac{4\,\text{MW2}}{\text{S}}}\ (2\,\text{MW2}+\text{S})\ \text{Sin}[\theta]\ \text{SUNT}[\text{Col1,Col2}]}{3\,\text{MW2}\,(\text{MZ2}-\text{S})}$ |

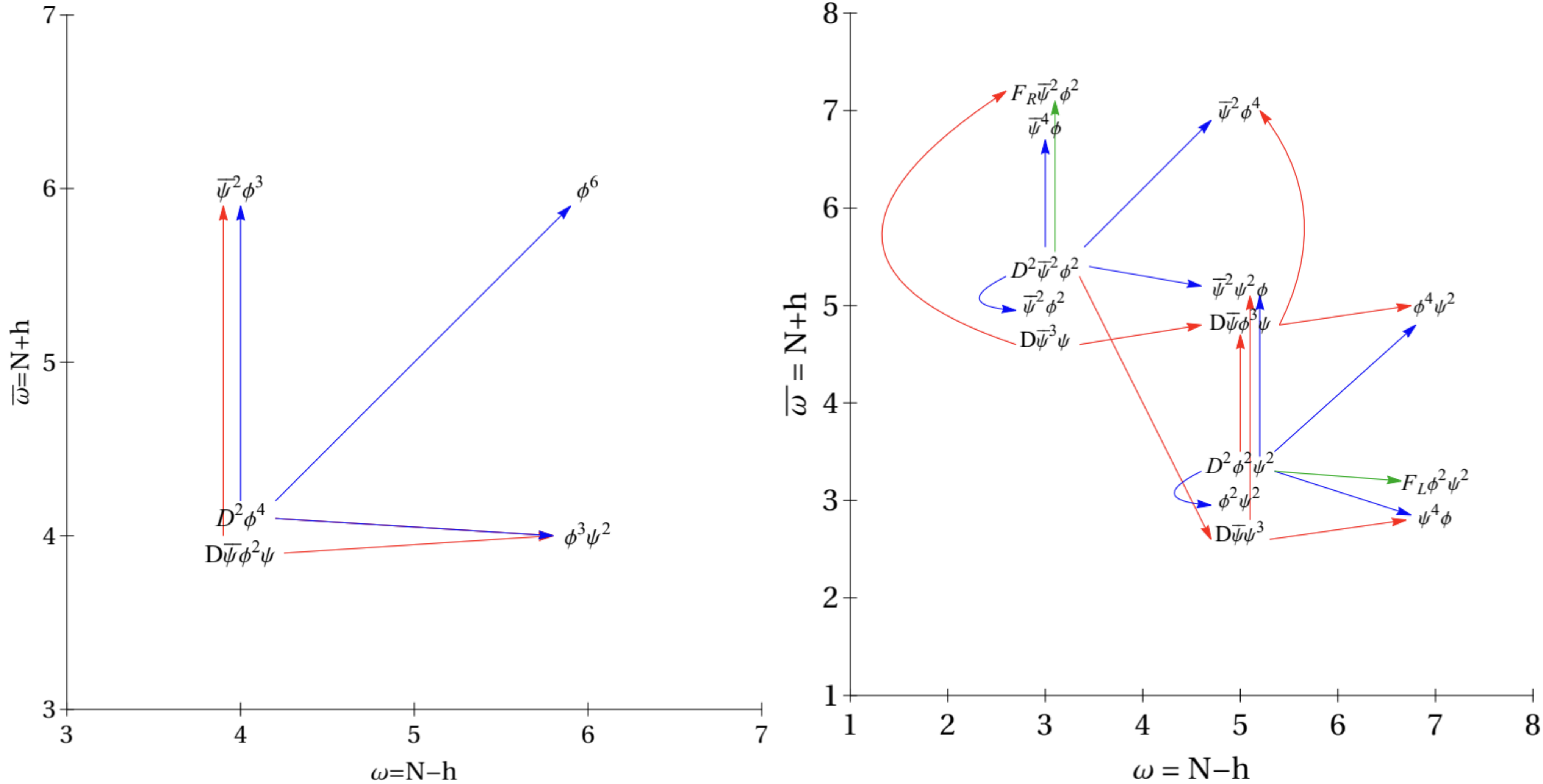
# Lorentz Structure – Conventions

## Using Spinor Indices

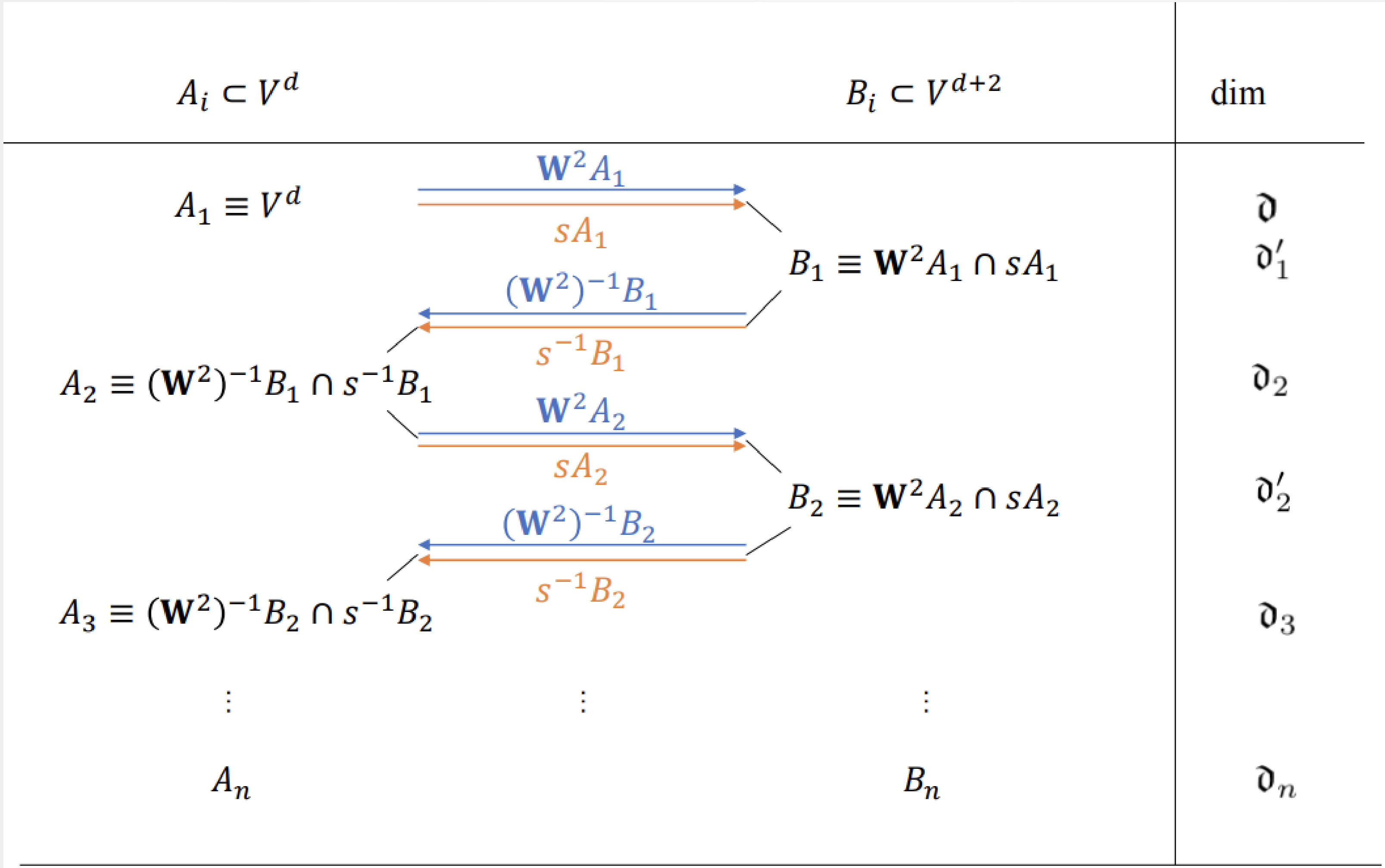
| SO(3,1) | SL(2,C) $\sim$ $\mathrm{SU}(2)_l \times \mathrm{SU}(2)_r$ | Spinor Helicity Variables |
|---|---|---|
| $\phi$ | $\phi \sim (0,0)$ | N.A. |
| $\psi$ | $\psi_\alpha \sim (1/2,0)$<br>$\psi^\dagger_{\dot\alpha} \sim (0,1/2)$ | $\lambda_\alpha, \quad \lambda_{\dot\alpha}$ |
| $F_{\mu\nu}$ | $F_{\mathrm{L}\alpha\beta} = \frac{i}{2} F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \sim (1,0)$<br>$F_{\mathrm{R}\dot\alpha\dot\beta} = -\frac{i}{2} F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \sim (0,1)$ | $\lambda_\alpha\lambda_\beta, \quad \lambda_{\dot\alpha}\lambda_{\dot\beta}$ |
| $D_\mu$ | $D_\mu \sigma^\mu_{\alpha\dot\alpha} \sim (1/2,1/2)$ | $\lambda_\alpha\lambda_{\dot\alpha}$ |

Field Building Block: $D^{r_i-|h_i|}\Phi_i \sim (\frac{r_i-h_i}{2}, \frac{r_i+h_i}{2}) \oplus \text{lower weights}$

$$D\psi = (D_\mu \sigma^\mu \psi)_{\alpha\dot\alpha\beta} \to h_i = -1/2, \; r_i = 1/2$$



$\overline{\omega}$=N+h
$\omega$=N−h
$\overline{\psi}^2\phi^3$
$\phi^6$
$D^2\phi^4$
$\mathrm{D}\overline{\psi}\phi^2\psi$
$\phi^3\psi^2$
$\overline{\omega}$ = N+h
$\omega$ = N−h
$F_R\overline{\psi}^2\phi^2$
$\overline{\psi}^4\phi$
$\overline{\psi}^2\phi^4$
$D^2\overline{\psi}^2\phi^2$
$\overline{\psi}^2\phi^2$
$\overline{\psi}^2\psi^2\phi$
$\mathrm{D}\overline{\psi}\phi^3\psi$
$\phi^4\psi^2$
$\mathrm{D}\overline{\psi}^3\psi$
$D^2\phi^2\psi^2$
$F_L\phi^2\psi^2$
$\phi^2\psi^2$
$\psi^4\phi$
$\mathrm{D}\overline{\psi}\psi^3$




$B_n = \mathbf{W}^2 A_n = sA_n$

$$W_{\mathcal{I}}^2 \tilde{V}^d = s_{\mathcal{I}} \tilde{V}^d \subset V^{d+2}, \quad \tilde{V}^d = \mathrm{span}\{\mathcal{B}_i^{\mathrm{j}}\} \subset V^d.$$

$$s_{134}^{-3/2}\mathcal{B}^{y}_{\psi^2\psi^{\dagger 4},2} = s_{134}^{-3/2}\langle 12\rangle[35][46] = \frac{1}{3}\overline{\mathcal{B}}_1^{J=1/2} - \frac{1}{4}\overline{\mathcal{B}}_2^{J=1/2} + \overline{\mathcal{B}}^{J=3/2}$$

| | $\mathcal{B}^{y}_{\psi^2\psi^{\dagger 4}}$ | $s_{134}\mathcal{B}^{y}_{\psi^2\psi^{\dagger 4}}$ | $W_{134}^2\mathcal{B}^{y}_{\psi^2\psi^{\dagger 4}}$ |
|---|---|---|---|
| $d=9$<br>$\psi^2\psi^{\dagger 4}$ | $[34][56]\langle 12\rangle,\ [35][46]\langle 12\rangle.$ | | |
| $d=11$<br>$\psi^2\psi^{\dagger 4}D^2$ | $-[34][56]s_{56}\langle 12\rangle,\ -[35][46]s_{56}\langle 12\rangle,$<br>$[34][56]s_{46}\langle 12\rangle,\ -[34][56]s_{45}\langle 12\rangle,$<br>$[35][46]s_{46}\langle 12\rangle,\ -[35][46]s_{45}\langle 12\rangle,$<br>$-[35][46]s_{36}\langle 12\rangle,\ [35][46]s_{35}\langle 12\rangle,$<br>$-[34][56]s_{36}\langle 12\rangle,\ [34][56]s_{35}\langle 12\rangle,$<br>$-[34][56]s_{34}\langle 12\rangle,\ -[35][46]s_{34}\langle 12\rangle,$<br>$[34][56]^2\langle 15\rangle\langle 26\rangle,\ -[34][45][46]\langle 14\rangle\langle 24\rangle,$<br>$-[36][46][56]\langle 16\rangle\langle 26\rangle,\ -[35][45][56]\langle 15\rangle\langle 25\rangle,$<br>$[35][46][56]\langle 15\rangle\langle 26\rangle,\ -[34][46][56]\langle 14\rangle\langle 26\rangle,$<br>$[34][45][56]\langle 14\rangle\langle 25\rangle,\ -[35][46]^2\langle 14\rangle\langle 26\rangle,$<br>$[35][45][46]\langle 14\rangle\langle 25\rangle,\ -[34][35][36]\langle 13\rangle\langle 23\rangle,$<br>$[35][36][46]\langle 13\rangle\langle 26\rangle,\ -[35]^2[46]\langle 13\rangle\langle 25\rangle,$<br>$[34][36][56]\langle 13\rangle\langle 26\rangle,\ -[34][35][56]\langle 13\rangle\langle 25\rangle,$<br>$[34]^2[56]\langle 13\rangle\langle 24\rangle,\ [34][35][46]\langle 13\rangle\langle 24\rangle.$ | $[34][35][56]\langle 13\rangle\langle 25\rangle$<br>$+[34][45][56]\langle 14\rangle\langle 25\rangle$<br>$+[34][36][56]\langle 13\rangle\langle 26\rangle$<br>$+[34][46][56]\langle 14\rangle\langle 26\rangle,$<br><br>$[35]^2[46]\langle 13\rangle\langle 25\rangle$<br>$+[35][36][46]\langle 13\rangle\langle 26\rangle$<br>$+[35][45][46]\langle 14\rangle\langle 25\rangle$<br>$+[35][46]^2\langle 14\rangle\langle 26\rangle.$ | $-\frac{3}{4}[34][35][56]\langle 13\rangle\langle 25\rangle$<br>$-\frac{3}{4}[34][36][56]\langle 13\rangle\langle 26\rangle$<br>$-\frac{3}{4}[34][45][56]\langle 14\rangle\langle 25\rangle$<br>$-\frac{3}{4}[34][46][56]\langle 14\rangle\langle 26\rangle,$<br><br>$2[34][35][56]\langle 13\rangle\langle 25\rangle$<br>$-\frac{15}{4}[35]^2[46]\langle 13\rangle\langle 25\rangle$<br>$+[34][36][56]\langle 13\rangle\langle 26\rangle$<br>$-\frac{15}{4}[35][36][46]\langle 13\rangle\langle 26\rangle$<br>$+[34][45][56]\langle 14\rangle\langle 25\rangle$<br>$-\frac{15}{4}[35][45][46]\langle 14\rangle\langle 25\rangle$<br>$+2[34][46][56]\langle 14\rangle\langle 26\rangle$<br>$-\frac{15}{4}[35][46]^2\langle 14\rangle\langle 26\rangle.$ |
| | | | $PW \equiv W_{134}^2[\mathcal{B}^{y}_{\psi^2\psi^{\dagger 4}}] \cap s_{134}[\mathcal{B}^{y}_{\psi^2\psi^{\dagger 4}}] =$<br>$(-[34][35][56]\langle 13\rangle\langle 25\rangle - [34][36][56]\langle 13\rangle\langle 26\rangle$<br>$-[34][45][56]\langle 14\rangle\langle 25\rangle - [34][46][56]\langle 14\rangle\langle 26\rangle).$ |

# Convertion of non-SSYT

- Decompose into the SSYT basis (y-basis) $\mathcal{B} = \sum_i \mathcal{K}_i \mathcal{B}_i$
  1. Replace $p_1$:

$$\langle i1\rangle[1j] = -\sum_{k=2}^{N}\langle ik\rangle[kj]. \qquad [i|p_1|j\rangle \sim \begin{array}{c}\begin{array}{|c|c|}\hline 2 & 1 \\ \hline 3 & j \\ \hline\end{array}\\ \vdots \\ \begin{array}{|c|}\hline N \\ \hline\end{array}\end{array}$$

  2. Replace $p_2$ and $p_3$ in the following situation

$$[1|p_2|k\rangle \sim \begin{array}{c}\begin{array}{|c|c|}\hline 3 & 2 \\ \hline 4 & k \\ \hline\end{array}\\ \vdots \\ \begin{array}{|c|}\hline N \\ \hline\end{array}\end{array}, \quad [k|p_2|1\rangle \sim \begin{array}{c}\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 3 & 2 \\ \hline\end{array}\\ \vdots \\ \begin{array}{|c|}\hline N \\ \hline\end{array}\end{array} \quad [1|p_3|2\rangle \sim \begin{array}{c}\begin{array}{|c|c|}\hline 2 & 2 \\ \hline 4 & 3 \\ \hline\end{array}\\ \vdots \\ \begin{array}{|c|}\hline N \\ \hline\end{array}\end{array}, \quad [2|p_3|1\rangle \sim \begin{array}{c}\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 4 & 3 \\ \hline\end{array}\\ \vdots \\ \begin{array}{|c|}\hline N \\ \hline\end{array}\end{array}$$

  3. Replace $s_{23}$

$$s_{23} \sim \begin{array}{c}\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 4 & 3 \\ \hline\end{array}\\ \vdots \\ \begin{array}{|c|}\hline N \\ \hline\end{array}\end{array} \qquad s_{23} = -\sum_{i=4}^{N}\left[s_{2i} + s_{3i} + \sum_{j>i} s_{ji}\right]$$

  4. Schouten for $i < j < k < l$

$$\langle il\rangle\langle jk\rangle = \langle ik\rangle\langle jl\rangle - \langle ij\rangle\langle kl\rangle, \;)$$
$$[il][jk] = [ik][jl] - [ij][kl].$$

Automated in the program

$$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 4 & 3 \\ \hline\end{array} = -\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array} + \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array},$$

$$\epsilon^{\alpha_1\alpha_4}\epsilon^{\alpha_2\alpha_3} = -\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_3\alpha_4} + \epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_2\alpha_4},$$

$$\langle 14\rangle\langle 23\rangle = -\langle 12\rangle\langle 34\rangle \quad \langle 13\rangle\langle 24\rangle$$

Non-SSYT Amp → # of Fields →

```
AmpReduce[ab[2, 3] × ab[1, 4], 4]

ab[1, 3] × ab[2, 4] - ab[1, 2] × ab[3, 4]
```

$$\mathcal{M} = \sum \mathcal{K}_{ij}\mathcal{M}^{y}_{j}$$

Credit to Ming-Lei Xiao

# Repeated Field: Y-Basis to P-Basis

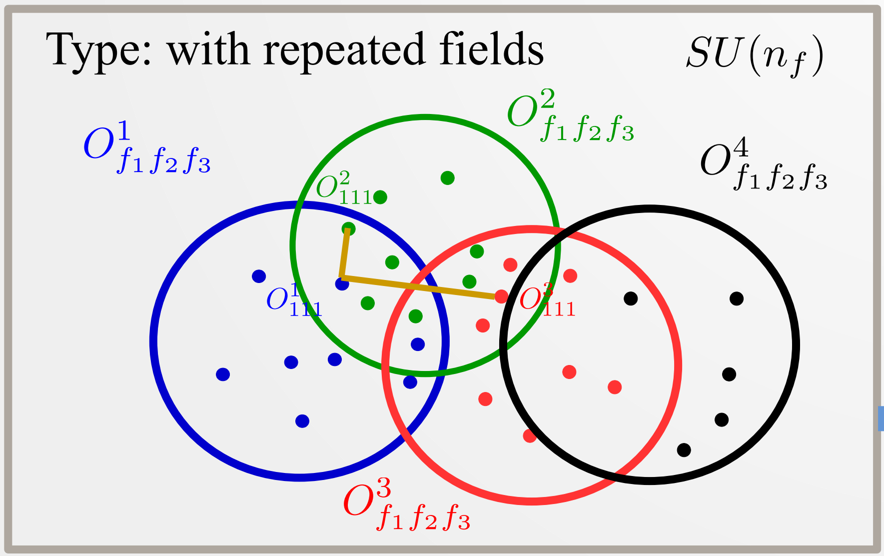
Flavor-Blind $\{ \ O_i^{\mathrm{y}} \ \}$ Y-Basis
$i = 1,2,\ldots, d_G \cdot d_B$

$S_3$

$O^1_{f_1 f_2 f_3}$ ▲ $O^2_{f_1 f_2 f_3}$ ▲ $O^3_{f_1 f_2 f_3}$ ▲ $O^4_{f_1 f_2 f_3}$ ▲

$$O^i_{\pi(f_1 f_2 f_3)} = \sum_j D(\pi)_{ji} O^j_{f_1 f_2 f_3}$$

reducible




$$O^1_{111} = x O^2_{111} + y O^3_{111}$$

Type with repeated fields:

$$Q^3 L$$

$$T_G = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}, \ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}$$

$$\mathcal{M} = (L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}), \ (L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})$$

↓

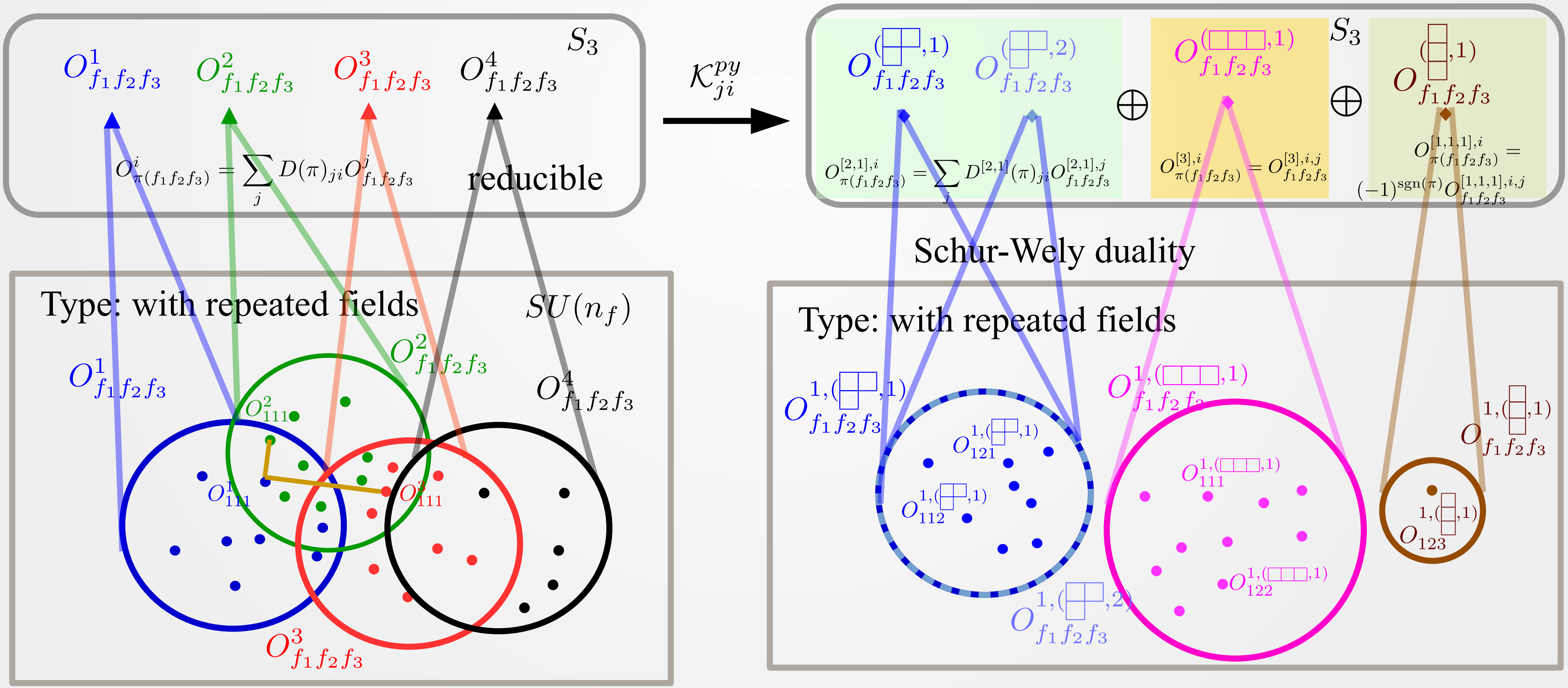
$$\mathcal{O}^{\mathrm{y}}_1 = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})$$
$$\mathcal{O}^{\mathrm{y}}_2 = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})$$
$$\mathcal{O}^{\mathrm{y}}_3 = \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})$$
$$\mathcal{O}^{\mathrm{y}}_4 = \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})$$

Obstancle finding independent flavor entries

# Repeated Field: Y-Basis to P-Basis

Flavor-Blind $\{ O_i^{y} \}$ Y-Basis
$i = 1,2,\ldots, d_G \cdot d_B$

Flavor-Blind $\{ O_j^{p} \}$ P-Basis
$i = 1,2,\ldots, d_G \cdot d_B$




$$O^1_{111} = xO^2_{111} + yO^3_{111}$$

# Permute transfer

An operator point of view

$$
\begin{aligned}
\underbrace{\pi \circ \mathcal{O}^{\{f_k,\dots\}}}_{\text{permute flavor}} &= T_{\mathrm{SU3}}^{\{g_k,\dots\}} T_{\mathrm{SU2}}^{\{h_k,\dots\}} \mathcal{M}_{\{g_k,\dots\},\{h_k,\dots\}}^{\{f_{\pi(k)},\dots\}} \\
&= T_{\mathrm{SU3}}^{\{g_{\pi(k)},\dots\}} T_{\mathrm{SU2}}^{\{h_{\pi(k)},\dots\}} \mathcal{M}_{\{g_{\pi(k)},\dots\},\{h_{\pi(k)},\dots\}}^{\{f_{\pi(k)},\dots\}} \\
&= \underbrace{\left(\pi \circ T_{\mathrm{SU3}}^{\{g_k,\dots\}}\right)\left(\pi \circ T_{\mathrm{SU2}}^{\{h_k,\dots\}}\right)}_{\text{permute gauge}} \underbrace{\left(\pi \circ \mathcal{M}_{\{g_k,\dots\},\{h_k,\dots\}}^{\{f_k,\dots\}}\right)}_{\text{permute Lorentz}}
\end{aligned}
$$

Rename the dummy indices

Example: $LLHH$

$$T_{SU(2)}^{i_1 i_2, j_1 j_2} = (\epsilon^{i_1 j_1}\epsilon^{i_2 j_2} + \epsilon^{i_2 j_1}\epsilon^{i_1 j_2})$$

$$
\begin{aligned}
(12) \circ \mathcal{O}^{f_1 f_2} &= \mathcal{O}^{f_2 f_1} \\
&= T_{SU2}^{i_1 i_2, j_1 j_2} \epsilon^{\alpha_1 \alpha_2} L_{\alpha_1, i_1}^{f_2} L_{\alpha_2, i_2}^{f_1} H_{j_1} H_{j_2} \\
&= T_{SU(2)}^{i_2 i_1, j_1 j_2} \epsilon^{\alpha_2 \alpha_1} L_{\alpha_2, i_1}^{f_1} L_{\alpha_1, i_2}^{f_2} H_{j_1} H_{j_2} \\
&= \left(\pi \circ T_{SU2}^{i_1 i_2, j_1 j_2}\right)\left(\pi \circ \mathcal{M}_{\{i_1 i_2, j_1 j_2\}}^{\{f_1 f_2, 11\}}\right)
\end{aligned}
$$

Allowed irreps of flavor is determined by irreps of gauge and Lorentz

$$\lambda_f = \lambda_G \odot \lambda_{\mathcal{M}}$$

# Example of LQQQ

$$\mathcal{Y}\left[\begin{array}{|c|c|c|}\hline r & s & t \\ \hline\end{array}\right] = \begin{pmatrix} -\frac{1}{6} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{6} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{6} \end{pmatrix}$$

$$\mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}\right] = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$\mathcal{Y}\left[\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline\end{array}\right] O_1^{\mathrm{y}} = 1/2 O_1^{\mathrm{y}} - 1/2 O_4^{\mathrm{y}}$$

$$\mathcal{Y}\left[\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline\end{array}\right] = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\mathcal{Y}\left[\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline\end{array}\right] O_{2,3}^{\mathrm{y}} = 0$$

$$\mathcal{O}_1^{\mathrm{y}} = \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})$$

Final result:

$$\mathcal{Y}\left[\begin{array}{|c|c|c|}\hline r & s & t \\ \hline\end{array}\right] \circ \mathcal{O}_1^{\mathrm{y}}$$

$$\mathcal{Y}\left[\begin{array}{|c|c|}\hline r & s \\ \hline t \\ \cline{1-1}\end{array}\right] \circ \mathcal{O}_1^{\mathrm{y}}$$

$$\mathcal{Y}\left[\begin{array}{|c|}\hline r \\ \hline s \\ \hline t \\ \hline\end{array}\right] \circ \mathcal{O}_1^{\mathrm{y}}$$

# The use of the package

- Define the Model

```
ModelIni[SMEFT];
```

Initiate the name of the model

```
AddGroup[SMEFT, "U1b"];
AddGroup[SMEFT, "U1l"];
AddGroup[SMEFT, "SU3c", GaugeBoson -> "G"];
AddGroup[SMEFT, "SU2w", GaugeBoson -> "W"];
AddGroup[SMEFT, "U1y", GaugeBoson -> "B"];
```

Adding Global and Gauge group

Setting number of flavor

```
nf = 3;
AddField[SMEFT, "Q", -1 / 2,
  {"SU3c" -> {1, 0}, "SU2w" -> {1}, "U1y" -> 1 / 6, "U1b" -> 1 / 3}, Flavor -> nf];
AddField[SMEFT, "uc", -1 / 2, {"SU3c" -> {0, 1}, "U1y" -> -2 / 3, "U1b" -> -1 / 3},
  Flavor -> nf];
AddField[SMEFT, "dc", -1 / 2, {"SU3c" -> {0, 1}, "U1y" -> 1 / 3, "U1b" -> -1 / 3},
  Flavor -> nf];
AddField[SMEFT, "L", -1 / 2, {"SU2w" -> {1}, "U1y" -> -1 / 2, "U1l" -> 1}, Flavor -> nf];
AddField[SMEFT, "ec", -1 / 2, {"U1y" -> 1, "U1l" -> -1}, Flavor -> nf];
AddField[SMEFT, "H", 0, {"SU2w" -> {1}, "U1y" -> 1 / 2}];
```

Adding Fields

# The use of the package

- Define Group Profile (SU(3) and SU(2) templates so far)

```
(* Initialization *)
If[MatchQ[groupList, _List], AppendTo[groupList, "SU2"], groupList = {"SU2"}];
AssocIni[tRep, tOut, tList, tasList, INDEX, tVal, tYDcol, tSimp, tY2M, tM2Y];
tList[SU2] = {del2, eps2a, eps2f, τ, del3n, eps3n};
tasList[SU2] = {eps2a, eps2f, eps3n};
tVal[SU2] = {del2 -> IdentityMatrix[2], eps2f -> LeviCivitaTensor[2], eps2a -> LeviCivitaTensor[2], τ -> GellMann[2], del3n -> IdentityMatrix[3], eps3n -> LeviCivitaTensor[3]};
tYDcol[SU2] = eps2a;
If[! IntegerQ[dummyIndexCount], dummyIndexCount = 0];
```

```
(* Define invariant tensors *)
AppendTo[tAssumptions, del2 ∈ Arrays[{2, 2}, Reals]];
tRep[del2] = {{-1}, {1}};
tOut[del2] = PrintTensor[<|"tensor" -> "δ", "upind" -> {#1}, "downind" -> {#2}|>] &;
TensorConj[del2[a_, b_]] := del2[b, a]

AppendTo[tAssumptions, eps2a ∈ Arrays[{2, 2}, Reals, Antisymmetric[{1, 2}]]];
tRep[eps2a] = {{-1}, {-1}};
tOut[eps2a] = PrintTensor[<|"tensor" -> "ϵ", "upind" -> {#1, #2}|>] &;
TensorConj[eps2a[x__]] := eps2f[x]

AppendTo[tAssumptions, eps2f ∈ Arrays[{2, 2}, Reals, Antisymmetric[{1, 2}]]];
tRep[eps2f] = {{1}, {1}};
tOut[eps2f] = PrintTensor[<|"tensor" -> "ϵ", "downind" -> {#1, #2}|>] &;
TensorConj[eps2f[x__]] := eps2a[x]

AppendTo[tAssumptions, τ ∈ Arrays[{3, 2, 2}, Reals]];
tRep[τ] = {{2}, {1}, {-1}};
tOut[τ] = PrintTensor[<|"tensor" ->  PrintTensor[<|"tensor" -> "τ", "upind" -> {#1}|>], "upind" -> {#3
TensorConj[τ[I_, a_, b_]] := τ[I, b, a]

AppendTo[tAssumptions, del3n ∈ Arrays[{3, 3}, Reals, Symmetric[{1, 2}]]];
tRep[del3n] = {{2}, {2}};
tOut[del3n] = PrintTensor[<|"tensor" -> "δ", "upind" -> {#1, #2}|>] &;
TensorConj[del3n[x__]] := del3n[x]

AppendTo[tAssumptions, eps3n ∈ Arrays[{3, 3, 3}, Reals, Antisymmetric[{1, 2, 3}]]];
tRep[eps3n] = {{2}, {2}, {2}};
tOut[eps3n] = PrintTensor[<|"tensor" -> "ϵ", "upind" -> {#1, #2, #3}|>] &;
TensorConj[eps3n[x__]] := eps3n[x]
```

```
AssociateTo[tY2M, {
τ[a_, j_, k_] × τ[b_, k_, m_] :> Module[{dummy = Unique[]}, I eps3n[a, b, dummy] × τ[dummy, j, m] + del3n[a, b] × del2[m, j]]
}];
AssociateTo[tM2Y, {
eps3n[a_, b_, c_] :> Module[{d1 = Unique[], d2 = Unique[], d3 = Unique[]}, - (I / 4)  τ[a, d1, d2] (τ[b, d2, d3] × τ[c, d3, d1] - τ[c, d2, d3] × τ[b, d3, d1])]
}];
```

```
tSimp[SU2] = Hold[Block[{},
del2[i_, j_] × del2[j_, k_] := del2[i, k];
del2[i_, i_] := 2;
del3n[i_, i_] := 3;
del3n[a_, c_] × del3n[a_, b_] := del3n[c, b];
del3n[a_, b_] × del3n[b_, c_] := del3n[a, c];
del3n[a_, c_] × del3n[b_, c_] := del3n[a, b];
del3n[b_, c_] × del3n[a_, b_] := del3n[a, c];
del3n[a_, b_]^2 := 3;
del2[a_, c_] × τ[J_, a_, b_] := τ[J, c, b];
del2[c_, a_] × τ[J_, b_, a_] := τ[J, b, c];
τ[i_, j_, j_] := 0;
τ[a_, i_, j_] × τ[a_, k_, l_] := 2 del2[l, i] × del2[j, k] - del2[l, k] × del2[j, i];
eps2a[x_, y_] × eps2f[w_, z_] := del2[x, w] × del2[y, z] - del2[x, z] × del2[y, w];
eps3n[i_, j_, k_] × eps3n[l_, m_, n_] := Det@Outer[del3n, {i, j, k}, {l, m, n}];
del3n[a_, d_] × eps3n[a_, b_, c_] := eps3n[d, b, c];
del3n[a_, d_] × eps3n[b_, a_, c_] := eps3n[b, d, c];
del3n[a_, d_] × eps3n[c_, b_, a_] := eps3n[c, b, d];
eps2f[i_, j_] × del2[i_, k_] := eps2f[k, j];
eps2f[i_, j_] × del2[j_, k_] := eps2f[i, k];
eps2a[i_, j_] × del2[k_, i_] := eps2a[k, j];
eps2a[i_, j_] × del2[k_, j_] := eps2a[i, k];
]]
```

```
ConvertToFundamental[model_, groupname_, {0}] := If[CheckGroup[model, groupname] == SU2, 1, Message[ConvertToFundamental::name, groupname, {1}]]
ConvertToFundamental[model_, groupname_, {1}] := If[CheckGroup[model, groupname] == SU2, {1, eps2f[a[1], aa[1]]}, Message[ConvertToFundamental::name, groupname, {1}]]
ConvertToFundamental[model_, groupname_, {2}] := If[CheckGroup[model, groupname] == SU2, dummyIndexCount++;
τ[A[1], aa[1], dummyIndex[dummyIndexCount]] × eps2f[dummyIndex[dummyIndexCount], aa[2]], Message[ConvertToFundamental::name, groupname, {2}]]

CF[{0}, num_, ind_] := 1
CF[{1}, num_, ind_] := del2[ind, Subscript[num,  1]]
CF[{-1}, num_, ind_] := eps2f[Subscript[num,  1], ind]
CF[{2}, num_, ind_] := TensorContract[eps2f⊗τ, {{1, 5}}][Subscript[num,  1], ind, Subscript[num,  2]]
```

# The use of the package

- Basic Counting and Enumeration

**Counting**

```
SMEFTstat8 = StatResult[SMEFT, 8];
```

Done! time used: 0.577472

number of real types → 541

number of real terms → 1266

number of real operators → 44 807

```
SMEFTstat9 = StatResult[SMEFT, 9];
```

Done! time used: 0.607295

number of real types → 296

number of real terms → 1256

number of real operators → 90 456

**Changing number of the flavor**

```
(* change flavor number *)
SetNflavor[SMEFT, #, 1] & /@ {"Q", "uc", "dc", "L", "ec"};
PresentStat[SMEFTstat8, SMEFT]
```

number of real types → 521

number of real terms → 993

number of real operators → 993

**Enumeration**

```
GetBasisForType[SMEFT, "Q"^3 "L" "WL"]
```

$$\Big\langle \Big| \{Q\to\{3\}\} \to \{i\,\tau^{Ij}{}_{m}\epsilon^{abc}\epsilon^{il}\epsilon^{km}WL^{I\mu\nu}(L_{pi}\sigma_{\mu\nu}Q_{raj})(Q_{sbk}Q_{tcl}),\ i\,\tau^{Ij}{}_{m}\epsilon^{abc}\epsilon^{il}\epsilon^{km}WL^{I\mu\nu}(Q_{raj}\sigma_{\mu\nu}Q_{sbk})(L_{pi}Q_{tcl})\},$$

$$\{Q\to\{2,1\}\}\to\{i\,\tau^{Ij}{}_{m}\epsilon^{abc}\epsilon^{il}\epsilon^{km}WL^{I\mu\nu}(L_{pi}\sigma_{\mu\nu}Q_{raj})(Q_{sbk}Q_{tcl}),$$

$$i\,\tau^{Ij}{}_{m}\epsilon^{abc}\epsilon^{il}\epsilon^{km}WL^{I\mu\nu}(L_{pi}\sigma_{\mu\nu}Q_{sbk})(Q_{raj}Q_{tcl}),\ i\,\tau^{Ij}{}_{m}\epsilon^{abc}\epsilon^{il}\epsilon^{km}WL^{I\mu\nu}(Q_{raj}\sigma_{\mu\nu}Q_{sbk})(L_{pi}Q_{tcl})\},$$

$$\{Q\to\{1,1,1\}\}\to\{i\,\tau^{Ij}{}_{m}\epsilon^{abc}\epsilon^{il}\epsilon^{km}WL^{I\mu\nu}(L_{pi}\sigma_{\mu\nu}Q_{raj})(Q_{sbk}Q_{tcl})\}\Big|\Big\rangle$$

```
GetBasisForType[SMEFT, "Q"^3 "L" "WL", DeSym → False]
```

$$\Big\langle\Big| \text{m-basis}\to\{i\,\tau^{Ij}{}_{m}\epsilon^{abc}\epsilon^{il}\epsilon^{km}WL^{I\mu\nu}(L_{pi}\sigma_{\mu\nu}Q_{raj})(Q_{sbk}Q_{tcl}),$$

$$i\,\tau^{Ij}{}_{m}\epsilon^{abc}\epsilon^{il}\epsilon^{km}WL^{I\mu\nu}(L_{pi}\sigma_{\mu\nu}Q_{sbk})(Q_{raj}Q_{tcl}),\ i\,\tau^{Ij}{}_{m}\epsilon^{abc}\epsilon^{il}\epsilon^{km}WL^{I\mu\nu}(Q_{raj}\sigma_{\mu\nu}Q_{sbk})(L_{pi}Q_{tcl}),$$

$$i\,\tau^{Ii}{}_{m}\epsilon^{abc}\epsilon^{jl}\epsilon^{km}WL^{I\mu\nu}(L_{pi}\sigma_{\mu\nu}Q_{raj})(Q_{sbk}Q_{tcl}),\ i\,\tau^{Ii}{}_{m}\epsilon^{abc}\epsilon^{jl}\epsilon^{km}WL^{I\mu\nu}(L_{pi}\sigma_{\mu\nu}Q_{sbk})(Q_{raj}Q_{tcl}),$$

$$i\,\tau^{Ii}{}_{m}\epsilon^{abc}\epsilon^{jl}\epsilon^{km}WL^{I\mu\nu}(Q_{raj}\sigma_{\mu\nu}Q_{sbk})(L_{pi}Q_{tcl}),\ i\,\tau^{Ii}{}_{m}\epsilon^{abc}\epsilon^{jm}\epsilon^{kl}WL^{I\mu\nu}(L_{pi}\sigma_{\mu\nu}Q_{raj})(Q_{sbk}Q_{tcl}),$$

$$i\,\tau^{Ii}{}_{m}\epsilon^{abc}\epsilon^{jm}\epsilon^{kl}WL^{I\mu\nu}(L_{pi}\sigma_{\mu\nu}Q_{sbk})(Q_{raj}Q_{tcl}),\ i\,\tau^{Ii}{}_{m}\epsilon^{abc}\epsilon^{jm}\epsilon^{kl}WL^{I\mu\nu}(Q_{raj}\sigma_{\mu\nu}Q_{sbk})(L_{pi}Q_{tcl})\},$$

$$\text{p-basis}\to\langle|\{Q\to\{3\}\}\to\{1,3\},\ \{Q\to\{2,1\}\}\to\{1,2,3\},\ \{Q\to\{1,1,1\}\}\to\{1\}|\rangle,$$

$$\text{Kpm}\to\Big\{\{0,0,0,-\tfrac{1}{6},\tfrac{1}{3},0,\tfrac{1}{3},-\tfrac{1}{6},0\},\ \{-\tfrac{1}{3},-\tfrac{1}{3},1,0,\tfrac{1}{3},-\tfrac{1}{3},\tfrac{1}{3},0,-\tfrac{1}{3}\},$$

$$\{\tfrac{1}{3},\tfrac{1}{3},0,\tfrac{1}{3},-\tfrac{1}{3},0,-\tfrac{1}{3},\tfrac{1}{3},0\},\ \{-\tfrac{2}{3},\tfrac{1}{3},0,0,-\tfrac{1}{3},0,\tfrac{2}{3},-\tfrac{1}{3},0\},$$

$$\{\tfrac{2}{3},\tfrac{2}{3},0,0,-\tfrac{1}{3},0,-\tfrac{1}{3},0,0\},\ \{-\tfrac{4}{3},\tfrac{2}{3},0,\tfrac{1}{3},-\tfrac{1}{3},0,\tfrac{2}{3},-\tfrac{1}{3},0\},\ \{\tfrac{1}{3},\tfrac{1}{3},0,0,-\tfrac{1}{3},$$

$$\tfrac{1}{3},-\tfrac{1}{3},0,\tfrac{1}{3}\},\ \{-\tfrac{2}{3},\tfrac{1}{3},0,0,-\tfrac{1}{3},\tfrac{1}{3},\tfrac{2}{3},0,-\tfrac{2}{3}\},\ \{0,0,0,\tfrac{1}{6},0,0,0,-\tfrac{1}{6},0\}\Big\}\Big|\Big\rangle$$

# Example of LQQQ

$$T^{\mathrm{y}}_{\mathrm{SU(2)},1} = \epsilon^{ik}\epsilon^{jl}, \; T^{\mathrm{y}}_{\mathrm{SU(2)},2} = \epsilon^{ij}\epsilon^{kl}$$

$$(i \to 1, j \to 2, k \to 3, l \to 4)$$

$$D_{\mathrm{SU(2)}}[(12)] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$D_{\mathrm{SU(2)}}[(123)] = \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix}$$

$$B^{\mathrm{y}}_1 = \epsilon^{\alpha\beta}\epsilon^{\gamma\delta} L_{pi\alpha} Q_{raj\beta} Q_{sbk\gamma} Q_{tcl\delta}$$

$$B^{\mathrm{y}}_2 = \epsilon^{\alpha\gamma}\epsilon^{\beta\delta} L_{pi\alpha} Q_{sbk\gamma} Q_{raj\beta} Q_{tcl\delta}$$

$$(\beta \to 1, \gamma \to 2, \delta \to 3)$$

$$D_{\mathrm{L}}[(12)] = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$D_{\mathrm{L}}[(123)] = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$$

$$T^{\mathrm{y}}_{\mathrm{SU(3)},1} = \epsilon^{abc}$$

$$(a \to 1, b \to 2, c \to 3)$$

$$D_{\mathrm{SU(3)}}[(12)] = -1$$

$$D_{\mathrm{SU(3)}}[(123)] = 1$$

$$\mathcal{O}^{\mathrm{y}}_i = \{\epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi} Q_{raj})(Q_{sbk} Q_{tcl}),\; \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi} Q_{sbk})(Q_{raj} Q_{tcl}),\; \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi} Q_{raj})(Q_{sbk} Q_{tcl}),\; \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi} Q_{sbk})(Q_{raj} Q_{tcl})\}$$

$$D_{\mathcal{O}}[(12)] = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \qquad D_{\mathcal{O}}[(123)] = \begin{pmatrix} 0 & 1 & 0 & -1 \\ -1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$$

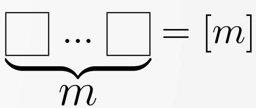
# Clarification

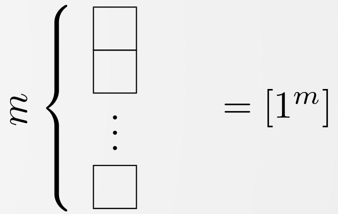
An amplitude point of view

Spin-Stat

$$\pi \circ Amp(1,2,...,m,...,N) = Amp(\pi(1),\pi(2),...,\pi(m),...,N) = \pm Amp(1,2,...,m,...,N)$$

*Amp* transforms as 1-dim irrep of the $S_m$




$$Amp(1,...,N) \sim C_{f_1,...,f_N} T_G^{a_1,...,a_N} B^{(d)}(h_1,\ldots,h_N)$$

Each factor spans a representation of the $S_m$ group

$$\lambda_{Amp} = [m] \text{ or } [1^m] \in \lambda_C \odot \lambda_G \odot \lambda_B$$

$$[2,1]^T = [2,1]$$

$$\lambda_C^{(T)} = \lambda_G \odot \lambda_B$$