ElectroWeak input schemes in the SMEFT

Tommy Smith

IPPP Durham

Based on work with:
A. Biekötter, B. Pecjak and D. Scott (to appear)

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The motivation and aims

In the SMEFT, one-loop calculations are being used to increase precision of predictions and global fits are being done at LO / NLO

The choice renormalisation / input scheme becomes an integral part of any calculation.

This talk is based on our study into different possible choices of inputs

Inputs in the Standard Model (EFT)

After Electroweak symmetry breaking, the bare Lagrangian is written in terms of a number of free parameters.

In SM(EFT) calculations, to define these, it is common practice to make the following choices:

- C_i are renormalized in the $\overline{MS}$ scheme
- M_H and m_t are renomalised on-shell. All other $m_f = 0$. (Except m_b for $H \to b\bar{b}$ which is renormalised in an $\overline{MS}$-light like α (shown later))
- Approximate $V_{ij} = \delta_{ij}$

This still leaves us with three undetermined parameters

$$\{g_1, g_W, vev\} \to \{\text{input 1, input 2, input 3}\}$$

M_W	$80.433(9) GeV/c^2$
M_Z	$91.1876(21) GeV/c^2$
G_μ	$1.1663787(6) \times 10^{-5} GeV^2$
$\alpha(M_Z)$	$0.007127(2)$

These three inputs the define the renomalisation scheme

Why does the choice of inputs matter?

Attention is necessary when choosing the inputs. There are considerations arising in the SMEFT in addition to those in the SM.

In the SM we should consider:

- Precision of the inputs value
- Convergence of the perturbative series

The additional EFT considerations:

- Number of additional Wilson coefficients appearing at LO and NLO from renormalisation [1]
- The convergence of terms with Wilson coefficients

[1] We are taking the perspective of trying to do fits on unknown Wilson coefficients

Our Work

What have we done?

- Calculated corrections needed for all electroweak process in the SMEFT for 3 common scheme choices (to be introduced)
- We have worked up to dimension 6 at one-loop in the SMEFT. The Warsaw basis was used and no flavour assumptions were made.
- Compared convergence and number of Wilson coefficients appearing for corrections in different schemes
- Allowing insight before hand to which input scheme(s) may be more suitable
- Specific examples of W, Z and H decay in the paper. Biekoetter, Pecjak, Scott, TS (to appear)
- I will present (preliminary) results for these

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The Schemes

The "α_μ scheme" - $\{G_\mu, M_W, M_Z\}$

- M_W and M_Z are renormalised on-shell
- G_μ is the Fermi constant and renormalised through muon decay
- Sometimes called "M_W scheme" in the SMEFT literature

The "α scheme" - $\{\alpha, M_W, M_Z\}$

- M_W and M_Z are renormalised on-shell
- α is the fine structure constant renormalised in a given scheme

The "LEP scheme" - $\{\alpha, G_\mu, M_Z\}$

- Inputs are renormalised as above
- Sometimes called "α scheme" in the SMEFT literature

Renormlisation Calculations

Quick overview of how we have renormalised in each scheme and some notation. The methods were taken from (Denner,Dittmaier [arXiv:1912.06823 [hep-ph]])

Full details in the paper (Biekoetter, Pecjak, Scott, TS (to appear))

For all our calculations, we used an in-house FeynRules model alongside a SMEFTSim (Brivio [arXiv:2012.11343 [hep-ph]]) model file to crosscheck results.

The α_μ scheme - $\{G_\mu, M_W, M_Z\}$

To use the α_μ scheme we write the bare Lagrangian in terms of v_T, M_W and M_Z.

- M_W and M_Z are then renormalised on-shell
- We define the variable v_μ as a substitution for the input G_μ

$$v_\mu = \frac{1}{\sqrt{\sqrt{2}G_\mu}}$$

- v_T is renormalised through

$$\frac{1}{v_{T,0}^2} = \frac{1}{v_\mu^2}\left(1 - v_\mu^2 \Delta v^{(6,0,\mu)} - \frac{1}{v_\mu^2}\Delta v_\mu^{(4,1,\mu)} - \Delta v_\mu^{(6,1,\mu)}\right)$$

Where $\Delta v_\mu^{(i,j,\mu)}$ is the counterterm needed to impose Fermi decay is exact to all orders

The α scheme - $\{\alpha, M_W, M_Z\}$

This scheme differs to the α_μ scheme through how v_T is renormalised.

- v_α is now a derived parameter given by

$$v_\alpha = \frac{2M_W s_w}{\sqrt{4\pi\alpha}}$$

- we write the corrisponding equation relating v_α and v_T

$$\frac{1}{v_{T,0}^2} = \frac{1}{v_\alpha^2}\left[1 - v_\alpha^2 \Delta v_\alpha^{(6,0,\alpha)} - \frac{1}{v_\alpha^2}\Delta v_\alpha^{(4,1,\alpha)} - \Delta v_\alpha^{(6,1,\alpha)}\right]$$

- $\Delta v_\alpha^{(i,j,\mu)}$ are identified by treating the below equation as a relation between bare parameters (using the first equation to substitute out v_α), renormalising and then matching onto the above

$$\frac{1}{v_T^2} = \frac{1}{v_\alpha^2}\left(1 + 2v_\alpha^2 \frac{c_w}{s_w}\left[C_{HWB} + \frac{c_w}{4s_w} C_{HD}\right]\right)$$

The LEP scheme - $\{\alpha, G_\mu, M_Z\}$

The LEP scheme is slightly more difficult to deal with

- v_T is renormalised as in the α_μ scheme
- The renormalised W boson mass is now a derived parameter

$$\hat{M}_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha v_\mu^2}{M_Z^2}}\right)$$

- Again, treating this as a relation between bare parameters we can define the counterterms $\Delta\hat{M}_W^{(i,j,\mu)}$

$$M_{W,0} = \hat{M}_W\Big(1 + v_\mu^2\Delta\hat{M}_W^{(6,0,\mu)}(\hat{M}_W) + \frac{1}{v_\mu^2}\Delta\hat{M}_W^{(4,1,\mu)}(\hat{M}_W) + \Delta\hat{M}_W^{(6,1,\mu)}(\hat{M}_W)\Big)$$

$$\hat{M}_W^{(4,1,\mu)} = \frac{\hat{s}_w^2}{1 - 2\hat{c}_w^2}\left[\frac{1}{2}\hat{\Delta}\alpha^{(4,1,\mu)} + \frac{1}{2}\hat{\Delta}v_\mu^{(4,1,\mu)} - \frac{\hat{c}_w^2}{\hat{s}_w^2}\hat{\Delta}M_Z^{(4,1,\mu)}\right]$$

An aside on α

Our definition of α in this work matches the work of (Cullen, Pecjak and Scott [arXiv:1512.02508 [hep-ph]])

- We use a five flavour QEDxQCD where all particles heavier than the b quark are decoupled and effectively calculated on-shell which we call $\overline{\alpha}^{(\ell)}(\mu)$
- We can show how $\overline{\alpha}^{(\ell)}(M_Z)$ relates to the quantity $\alpha(M_Z)$ by considering the two relations

$$\overline{\alpha}^{(\ell)}(M_Z) = \frac{\alpha(0)}{1 - \Delta\overline{\alpha}^{(\ell)}(M_Z)}, \quad \alpha(M_Z) = \frac{\alpha(0)}{1 - \Delta\alpha(M_Z)}$$

- Eliminating $\alpha(0)$ gives

$$\overline{\alpha}^{(\ell)}(M_Z) = \alpha(M_Z)\left(1 - \Delta\alpha(M_Z) + \Delta\overline{\alpha}^{(\ell)}(M_Z)\right)$$

- Which evaluates to

$$\overline{\alpha}^{(\ell)}(\mu) = \alpha(M_Z)\left[1 + \frac{\alpha(0)}{\pi}\left(\frac{100}{27} - \frac{20}{9}\ln\frac{M_Z^2}{\mu^2}\right)\right]$$

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The operators appearing

By using each of the following as an input, it will introduce the following operators into the calculation if renormalised

Input	LO	NLO
M_W		16 Ops
M_Z	$\mathcal{C}_{HD}, \mathcal{C}_{HWB}$	29 Ops
G_μ	$C^{(3)}_{\underset{11}{Hl}}, C^{(3)}_{\underset{22}{Hl}}, C_{\underset{1221}{ll}}$	13 Ops
α	$\mathcal{C}_{HWB}$	9 Ops

- Need to remember we use three of these inputs at a time
- The operators appearing may/will overlap between inputs and bare matrix elements
- However, the need to introduce a counterterm to M_Z will more than likely introduce a number of new, scheme dependant operators at NLO

v_μ VS v_α

A major difference between schemes is the treatment of v_T. To see the difference, we will look into the size of the SM NLO corrections.

- In the α_μ and LEP schemes we have

$$\frac{1}{v_\mu^2}\left(1-\frac{1}{v_\mu^2}\Delta v_\mu^{(4,1)}\right)=\frac{1}{v_\mu^2}\left(1-0.001-0.049\,[\mathrm{top},\mathrm{tadpole}]\right)$$

- Whereas in the α scheme we find

$$\begin{aligned}\frac{1}{v_\alpha^2}\left(1-\frac{1}{v_\alpha^2}\Delta v^{(4,1)}\right)&=\frac{1}{v_\alpha^2}\left(1-0.046-0.051\,[\mathrm{top},\mathrm{tadpole}]\right)\\&=\frac{1}{v_\mu^2}\left(1-0.014-0.053\,[\mathrm{top},\mathrm{tadpole}]\right).\end{aligned}$$

- Where we have numerically substituted v_α for v_μ
- We see a difference of 1.3% between the two schemes at NLO
- A largish NLO correction in the α scheme brings them down from a 5% difference at LO

v_μ VS v_α

The source of the large corrections in the α scheme is well known and can easily be identified

- The quantity v_α defined in the α scheme is a derived parameter

$$v_\alpha = \frac{2M_W s_w}{\sqrt{4\pi\alpha}}$$

- As mentioned we renormalise v_T through treating the following as a relation between bare parameters

$$\frac{1}{v_T^2} = \frac{1}{v_\alpha^2}\left(1 + 2v_\alpha^2 \frac{c_w}{s_w}\left[C_{HWB} + \frac{c_w}{4s_w} C_{HD}\right]\right)$$

- Doing so leads to the realisation

$$\frac{\delta v_\alpha}{v_\alpha} \equiv \frac{\delta M_W}{M_W} + \frac{\delta \hat{s}_w}{\hat{s}_w} - \frac{\delta e}{e}$$

$$\frac{\delta \hat{s}_w}{\hat{s}_w} = -\frac{c_w^2}{s_w^2}\left(\frac{\delta M_W}{M_W} - \frac{\delta M_Z}{M_Z}\right) \approx -7\left(\frac{\delta M_W}{M_W} - \frac{\delta M_Z}{M_Z}\right)$$

- This $\frac{c_w^2}{s_w^2}$ enhancement leads to a larger corrections in the α scheme in the SM. This also applies to the SMEFT

Caveats of counterterm analysis

Counterterms are unphysical. Definitive conclusions cannot be made.

Despite these limitations, we can establish a few predictions on how each scheme will perform before we see if they do indeed translate to the decay rates.

- The need to renormalize M_Z may lead to additional Wilson Coefficeints appearing at NLO
- The α scheme will receive larger corrections at NLO than the α_μ or the LEP schemes

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Values of input

Table of values for possible inputs

M_H	125 GeV	$\overline{m}_b(M_H)$	3.0 GeV
m_t	175 GeV	$\alpha(M_Z)$	1/128
M_W	80.4 GeV	G_F	$1.17 \times 10^{-5} GeV^2$
M_Z	91.2 GeV	$\alpha_s(M_H)$	0.1

μ, the scale was set to the scale of the process

Results

Plots only including the SM contributions will be given as decay rates

SMEFT results are in terms of fractional correction to tree level result

$$\Delta_{Xab} \equiv \frac{\Gamma_{Xab}}{\Gamma^{(4,0)}_{Xab}} = 1 + \Delta^{(4,1)}_{Xab} + \Delta^{(6,0)}_{Xab} + \Delta^{(6,1)}_{Xab} \, ; \qquad \Delta^{(i,j)}_{Xab} \equiv \frac{\Gamma^{(i,j)}_{Xab}}{\Gamma^{(4,0)}_{Xab}} \, .$$

For the decay

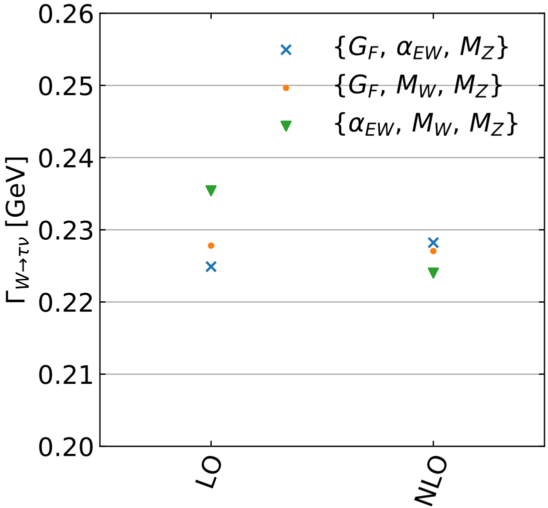
$$X \to ab$$

We will look at the three heavy boson decays

$$W \to \tau \nu_\tau$$
$$Z \to e^+ e^-$$
$$H \to b\bar{b}$$

$W \to \tau\nu_\tau$ - SM

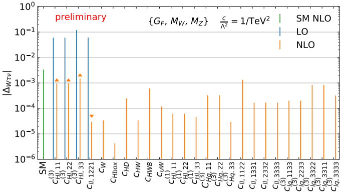
- Large differences ($\sim$ 5%) between the schemes at LO
- NLO result brings the schemes closer together
- Largest correction given to α scheme
- Uncertainty due to precision of input parameters $\sim$ per mille



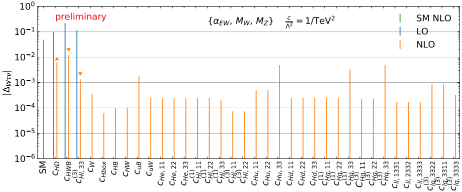
$W \to \tau\nu_\tau$ - SMEFT

$$\mathcal{M}_0^{(4,0)} = \frac{\sqrt{2} M_W \mathrm{DirL}}{v_T}$$

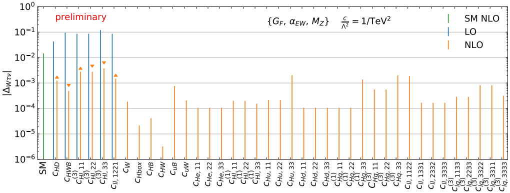
- Differences in coefficients at LO between schemes
- Fewer coefficients appearing in the α_μ scheme at NLO
- Larger corrections appearing at LO and NLO in the α scheme



(α_μ)



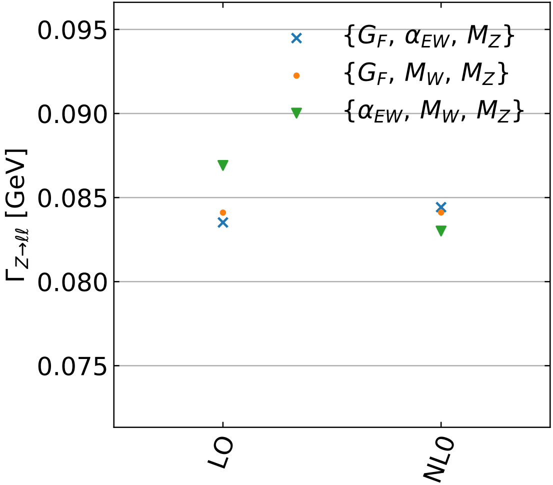
(α)



(LEP)

$Z \to e^+ e^-$ - SM

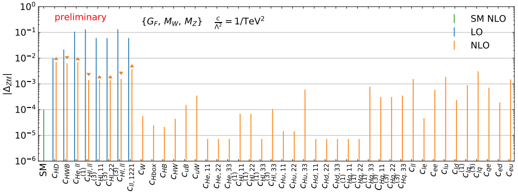
- Same inferences as from W decay
- Large differences ($\sim 5\%$) between the schemes at LO
- NLO result brings the schemes closer together
- Largest correction given to α scheme
- Uncertainty due to precision of input parameters $\sim$ per mille



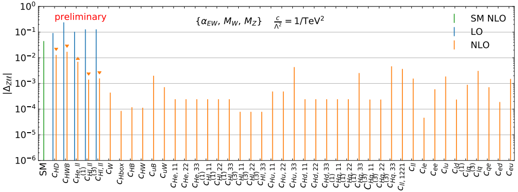
$Z \to e^+ e^-$ - SMEFT

$$\mathcal{M}_0^{(4,0)} = \frac{(\text{DirL} + 2\text{DirR})\, M_Z^2}{M_Z v_T} - \frac{2\,(\text{DirL} + \text{DirR})\, M_W^2}{M_Z v_T}$$

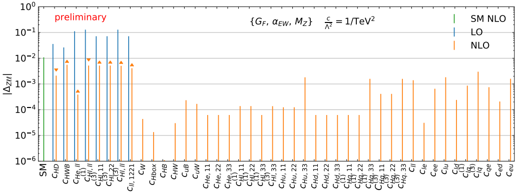
- Same number of operators appear in each scheme at NLO
- Larger corrections appearing at LO and NLO in the α scheme
- α_μ scheme gains particularly small corrections at NLO



(α_μ)



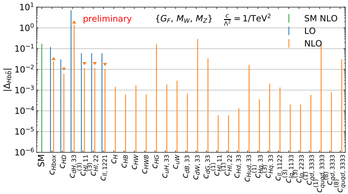
(α)



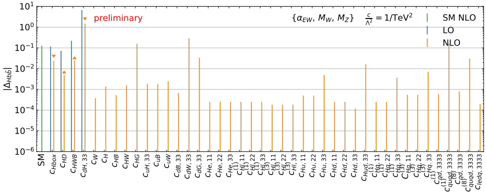
(LEP)

$H \to b\bar{b}$ SMEFT

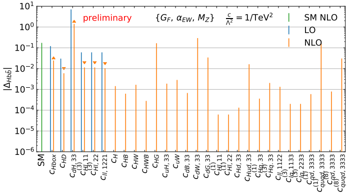
$$\mathcal{M}_0^{(4,0)} = \frac{m_b\,(\mathrm{DirL} + \mathrm{DirR})}{v_T}$$



(α_μ)



(α)



(LEP)

- The LEP and α_μ schemes are identical in this case
- As we have become to expect, the α scheme introduces a lot of operators at NLO

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Summary

We have calculated the corrections needed for any electroweak process in the SMEFT

We have applied these corrections to the examples of heavy boson decays

Here I have presented the results in three scheme choices:

The "α_μ scheme" - $\{G_\mu, M_W, M_Z\}$
The "α scheme" - $\{\alpha, M_W, M_Z\}$
The "LEP scheme" - $\{\alpha, G_\mu, M_Z\}$.

Conclusions and Extra things

From the results presented we can draw a few conclusions

- Large number of Wilson Coefficients appears no matter the scheme especially at NLO.
- Minimising bare parameters appearing at LO may decrease scheme dependent Wilson coefficients appearing at NLO
- The α scheme only introduces 2 scheme dependant Wilson coefficients at LO - potentially useful for global LO fits
- However, in general, convergence is worse for the α scheme and more coefficients are introduced at NLO
- The size of the corrections is will be heavily process dependant however, we can say refraining to use G_μ as an input can have the consequence of inducing larger corrections.

As a byproduct of our calculations, we have derived relations allowing conversion between scheme choices at the level of decay rates. Again, to be in the paper.