



CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Quantifying symmetry breaking with flavor-invariants : CP and Peccei-Quinn

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(CP) arXiv:2112.03889 [hep-ph]
+ work in progress
w/ E. Gendy, C. Grojean and J. Ruderman

arXiv:2206.04182 [hep-ph]
w/ C. Grojean and J. Kley

(PQ)

Collective symmetry breaking : CP in the SM

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Which flavor basis ? Fully specified ?

What are $V_{\text{CKM},13}, \dots$?

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Those questions imply :

$$J_4 = \text{Im Tr} \left[Y_u Y_u^\dagger, Y_d Y_d^\dagger \right]^3 = 6(y_t^2 - y_c^2)(y_t^2 - y_u^2)(y_c^2 - y_u^2)(y_b^2 - y_s^2)(y_b^2 - y_d^2)(y_s^2 - y_d^2) \mathcal{J}$$

$$\mathcal{J} = \text{Im}(V_{\text{CKM},us} V_{\text{CKM},cb} V_{\text{CKM},ub}^* V_{\text{CKM},cs}^*)$$

**CP is conserved in the SM
iff
 $J_4=0$**

[Jarlskog '85]

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Flavour invariant

	$SU(3)_Q$	$SU(3)_u$	$SU(3)_d$	$SU(3)_L$	$SU(3)_e$
Q_L	3	1	1	1	1
Y_u	3	$\bar{3}$	1	1	1
Y_d	3	1	$\bar{3}$	1	1
Y_e	1	1	1	3	$\bar{3}$

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Collective effect

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Collective effect

Explains suppressions

$$\mathcal{J} = \text{Im}(V_{\text{CKM},us} V_{\text{CKM},cb} V_{\text{CKM},ub}^* V_{\text{CKM},cs}^*)$$

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Collective sym. breaking : CP in the SMEFT

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(interference ?)

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$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{C_{HQ,ij}^{(1)}}{\Lambda^2} \left(i H^\dagger \overleftrightarrow{D}_\mu H \right) \overline{Q_{i,L}} \gamma^\mu Q_{j,L}$$

preserves CP at order $1/\Lambda^2$ iff

$$J_4 = 0$$

$$L_1 = \text{ImTr} \left(X_u X_d C_{HQ}^{(1)} \right) = 0$$

$$L_2 = \text{ImTr} \left(X_u^2 X_d^2 C_{HQ}^{(1)} \right) = 0$$

$$L_3 = \text{ImTr} \left(X_u X_d X_u^2 X_d^2 C_{HQ}^{(1)} \right) = 0$$

$$X_u \equiv Y_u Y_u^\dagger$$

$$X_d \equiv Y_d Y_d^\dagger$$

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Collective effect

$$X_u \equiv Y_u Y_u^\dagger$$

$$X_d \equiv Y_d Y_d^\dagger$$

Collective sym. breaking : CP in the SMEFT

699 invariants beyond Jarlskog at dim-6

Bilinears					
Modified Yukawa		Dipole		Current-current	
Q_{eH}	$(H^\dagger H)(\bar{L}_i e_j H)$	Q_{eW}	$(\bar{L}_i \sigma^{\mu\nu} e_j) \tau^I H W_{\mu\nu}^I$	$Q_{HL}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{L}_i \gamma^\mu L_j)$
Q_{uH}	$(H^\dagger H)(\bar{Q}_i u_j \tilde{H})$	Q_{eB}	$(\bar{L}_i \sigma^{\mu\nu} e_j) H B_{\mu\nu}$	$Q_{HL}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{L}_i \tau^I \gamma^\mu L_j)$
Q_{dH}	$(H^\dagger H)(\bar{Q}_i d_j H)$	Q_{uG}	$(\bar{Q}_i \sigma^{\mu\nu} T^A u_j) \tilde{H} G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_i \gamma^\mu e_j)$
		Q_{uW}	$(\bar{Q}_i \sigma^{\mu\nu} u_j) \tau^I \tilde{H} W_{\mu\nu}^I$	$Q_{HQ}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{Q}_i \gamma^\mu Q_j)$
		Q_{uB}	$(\bar{Q}_i \sigma^{\mu\nu} u_j) \tilde{H} B_{\mu\nu}$	$Q_{HQ}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{Q}_i \tau^I \gamma^\mu Q_j)$
		Q_{dG}	$(\bar{Q}_i \sigma^{\mu\nu} T^A d_j) H G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_i \gamma^\mu u_j)$
		Q_{dW}	$(\bar{Q}_i \sigma^{\mu\nu} d_j) \tau^I H W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_i \gamma^\mu d_j)$
		Q_{dB}	$(\bar{Q}_i \sigma^{\mu\nu} d_j) H B_{\mu\nu}$	$Q_{Hud} + \text{h.c.}$	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_i \gamma^\mu d_j)$
4-Fermi					
$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{LL}	$(\bar{L}_i \gamma_\mu L_j)(\bar{L}_k \gamma^\mu L_l)$	Q_{ee}	$(\bar{e}_i \gamma_\mu e_j)(\bar{e}_k \gamma^\mu e_l)$	Q_{Le}	$(\bar{L}_i \gamma_\mu L_j)(\bar{e}_k \gamma^\mu e_l)$
$Q_{QQ}^{(1)}$	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{Q}_k \gamma^\mu Q_l)$	Q_{uu}	$(\bar{u}_i \gamma_\mu u_j)(\bar{u}_k \gamma^\mu u_l)$	Q_{Lu}	$(\bar{L}_i \gamma_\mu L_j)(\bar{u}_k \gamma^\mu u_l)$
$Q_{QQ}^{(3)}$	$(\bar{Q}_i \gamma_\mu \tau^I Q_j)(\bar{Q}_k \gamma^\mu \tau^I Q_l)$	Q_{dd}	$(\bar{d}_i \gamma_\mu d_j)(\bar{d}_k \gamma^\mu d_l)$	Q_{Ld}	$(\bar{L}_i \gamma_\mu L_j)(\bar{d}_k \gamma^\mu d_l)$
$Q_{LQ}^{(1)}$	$(\bar{L}_i \gamma_\mu L_j)(\bar{Q}_k \gamma^\mu Q_l)$	Q_{eu}	$(\bar{e}_i \gamma_\mu e_j)(\bar{u}_k \gamma^\mu u_l)$	Q_{Qe}	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{e}_k \gamma^\mu e_l)$
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		$Q_{ud}^{(1)}$	$(\bar{u}_i \gamma_\mu u_j)(\bar{d}_k \gamma^\mu d_l)$	$Q_{Qu}^{(8)}$	$(\bar{Q}_i \gamma_\mu T^A Q_j)(\bar{u}_k \gamma^\mu T^A u_l)$
		$Q_{ud}^{(8)}$	$(\bar{u}_i \gamma_\mu T^A u_j)(\bar{d}_k \gamma^\mu T^A d_l)$	$Q_{Qd}^{(1)}$	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{d}_k \gamma^\mu d_l)$
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$(\bar{L}R)(\bar{R}L) + \text{h.c.}$		$(\bar{L}R)(\bar{L}R) + \text{h.c.}$			
Q_{LedQ}	$(\bar{L}_i^a e_j)(\bar{d}_k Q_{la})$	$Q_{QuQd}^{(1)}$	$(\bar{Q}_i^a u_j) \epsilon_{ab} (\bar{Q}_k^b d_l)$		
		$Q_{QuQd}^{(8)}$	$(\bar{Q}_i^a T^A u_j) \epsilon_{ab} (\bar{Q}_k^b T^A d_l)$		
		$Q_{LeQu}^{(1)}$	$(\bar{L}_i^a e_j) \epsilon_{ab} (\bar{Q}_k^b u_l)$		
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New sources of CPV :

$$\frac{C_{Hud,mn}}{\Lambda^2} i \tilde{H}^\dagger D_\mu H \bar{u}_{R,m} \gamma^\mu d_{R,n}$$

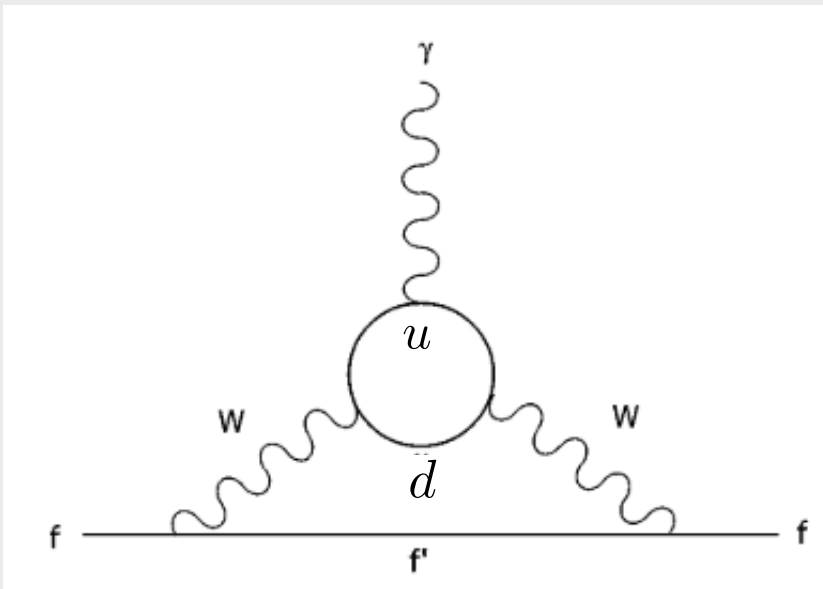
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699 invariants beyond Jarlskog at dim-6

New sources of CPV :

$$\frac{C_{Hud,mn}}{\Lambda^2} i \tilde{H}^\dagger D_\mu H \bar{u}_{R,m} \gamma^\mu d_{R,n}$$



[Kadoyoshi/Oshimo '97]

$$\frac{d}{e} = \sum_{i,j} \text{Im} (V_{CKM,ij} C_{Hud,ij}^*) F(m_{u_i}^2, m_{d_j}^2)$$

Modified Yukawa		Bilinears			
		Dipole		Current-current	
Q_{eH}	$(H^\dagger H)(\bar{L}_i e_j H)$	Q_{eW}	$(\bar{L}_i \sigma^{\mu\nu} e_j) \tau^I H W_{\mu\nu}^I$	$Q_{HL}^{(1)}$	$(H^\dagger i \vec{D}_\mu H)(\bar{L}_i \gamma^\mu L_j)$
Q_{uH}	$(H^\dagger H)(\bar{Q}_i u_j \tilde{H})$	Q_{eB}	$(\bar{L}_i \sigma^{\mu\nu} e_j) H B_{\mu\nu}$	$Q_{HL}^{(3)}$	$(H^\dagger i \vec{D}_\mu^I H)(\bar{L}_i \tau^I \gamma^\mu L_j)$
Q_{dH}	$(H^\dagger H)(\bar{Q}_i d_j H)$	Q_{uG}	$(\bar{Q}_i \sigma^{\mu\nu} T^A u_j) \tilde{H} G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i \vec{D}_\mu H)(\bar{e}_i \gamma^\mu e_j)$
		Q_{uW}	$(\bar{Q}_i \sigma^{\mu\nu} u_j) \tau^I \tilde{H} W_{\mu\nu}^I$	$Q_{HQ}^{(1)}$	$(H^\dagger i \vec{D}_\mu H)(\bar{Q}_i \gamma^\mu Q_j)$
		Q_{uB}	$(\bar{Q}_i \sigma^{\mu\nu} u_j) \tilde{H} B_{\mu\nu}$	$Q_{HQ}^{(3)}$	$(H^\dagger i \vec{D}_\mu^I H)(\bar{Q}_i \tau^I \gamma^\mu Q_j)$
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		$Q_{ud}^{(1)}$	$(\bar{u}_i \gamma_\mu u_j)(\bar{d}_k \gamma^\mu d_l)$	$Q_{Qu}^{(8)}$	$(\bar{Q}_i \gamma_\mu T^A Q_j)(\bar{u}_k \gamma^\mu T^A u_l)$
		$Q_{ud}^{(8)}$	$(\bar{u}_i \gamma_\mu T^A u_j)(\bar{d}_k \gamma^\mu T^A d_l)$	$Q_{Qd}^{(1)}$	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{d}_k \gamma^\mu d_l)$
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	$(\bar{L}R)(\bar{R}L) + \text{h.c.}$		$(\bar{L}R)(\bar{L}R) + \text{h.c.}$		
		$(\bar{d}_k Q_{la})$	$Q_{QuQd}^{(1)}$	$(\bar{Q}_i^a u_j) \epsilon_{ab} (\bar{Q}_k^b d_l)$	
			$Q_{QuQd}^{(8)}$	$(\bar{Q}_i^a T^A u_j) \epsilon_{ab} (\bar{Q}_k^b T^A d_l)$	
			$Q_{LeQu}^{(1)}$	$(\bar{L}_i^a e_j) \epsilon_{ab} (\bar{Q}_k^b u_l)$	
			$Q_{LeQu}^{(3)}$	$(\bar{L}_i^a \sigma_{\mu\nu} e_j) \epsilon_{ab} (\bar{Q}_s^k \sigma^{\mu\nu} u_t)$	

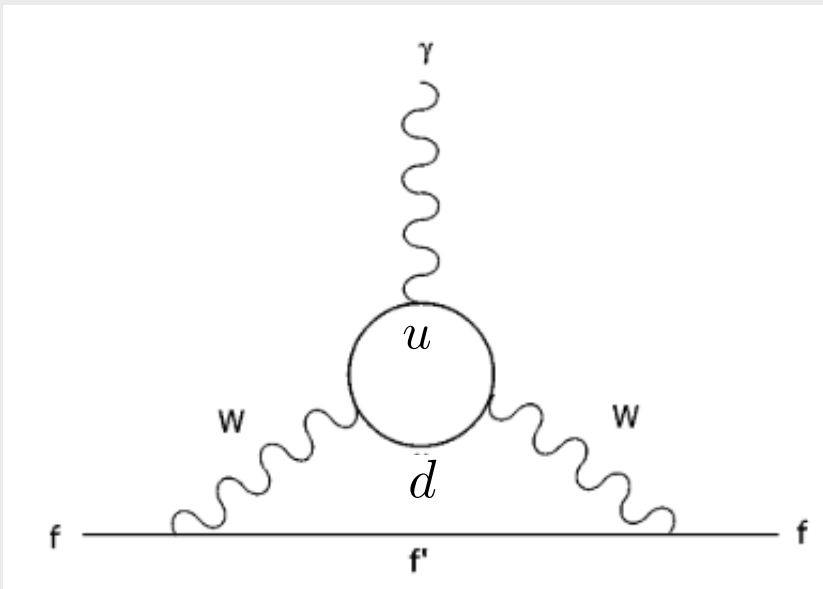
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Collective sym. breaking : CP in the SMEFT

699 invariants beyond Jarlskog at dim-6

New sources of CPV :

$$\frac{C_{Hud,mn}}{\Lambda^2} i \tilde{H}^\dagger D_\mu H \bar{u}_{R,m} \gamma^\mu d_{R,n}$$



[Kadoyoshi/Oshimo '97]

$$\frac{d}{e} = \sum_{i,j} \text{Im} (V_{CKM,ij} C_{Hud,ij}^*) F(m_{u_i}^2, m_{d_j}^2)$$

Bilinears

Modified Yukawa		Dipole		Current-current	
Q_{eH}	$(H^\dagger H)(\bar{L}_i e_j H)$	Q_{eW}	$(\bar{L}_i \sigma^{\mu\nu} e_j) \tau^I H W_{\mu\nu}^I$	$Q_{HL}^{(1)}$	$(H^\dagger i \vec{D}_\mu H)(\bar{L}_i \gamma^\mu L_j)$
Q_{uH}	$(H^\dagger H)(\bar{Q}_i u_j \tilde{H})$	Q_{eB}	$(\bar{L}_i \sigma^{\mu\nu} e_j) H B_{\mu\nu}$	$Q_{HL}^{(3)}$	$(H^\dagger i \vec{D}_\mu^I H)(\bar{L}_i \tau^I \gamma^\mu L_j)$
Q_{dH}	$(H^\dagger H)(\bar{Q}_i d_j H)$	Q_{uG}	$(\bar{Q}_i \sigma^{\mu\nu} T^A u_j) \tilde{H} G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i \vec{D}_\mu H)(\bar{e}_i \gamma^\mu e_j)$
		Q_{uW}	$(\bar{Q}_i \sigma^{\mu\nu} u_j) \tau^I \tilde{H} W_{\mu\nu}^I$	$Q_{HQ}^{(1)}$	$(H^\dagger i \vec{D}_\mu H)(\bar{Q}_i \gamma^\mu Q_j)$
		Q_{uB}	$(\bar{Q}_i \sigma^{\mu\nu} u_j) \tilde{H} B_{\mu\nu}$	$Q_{HQ}^{(3)}$	$(H^\dagger i \vec{D}_\mu^I H)(\bar{Q}_i \tau^I \gamma^\mu Q_j)$
		Q_{dG}	$(\bar{Q}_i \sigma^{\mu\nu} T^A d_j) H G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i \vec{D}_\mu H)(\bar{u}_i \gamma^\mu u_j)$
		Q_{dW}	$(\bar{Q}_i \sigma^{\mu\nu} d_j) \tau^I H W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i \vec{D}_\mu H)(\bar{d}_i \gamma^\mu d_j)$
		Q_{dB}	$(\bar{Q}_i \sigma^{\mu\nu} d_j) H B_{\mu\nu}$	$Q_{Hud} + \text{h.c.}$	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_i \gamma^\mu d_j)$

4-Fermi

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{LL}	$(\bar{L}_i \gamma_\mu L_j)(\bar{L}_k \gamma^\mu L_l)$	Q_{ee}	$(\bar{e}_i \gamma_\mu e_j)(\bar{e}_k \gamma^\mu e_l)$	Q_{Le}	$(\bar{L}_i \gamma_\mu L_j)(\bar{e}_k \gamma^\mu e_l)$
$Q_{QQ}^{(1)}$	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{Q}_k \gamma^\mu Q_l)$	Q_{uu}	$(\bar{u}_i \gamma_\mu u_j)(\bar{u}_k \gamma^\mu u_l)$	Q_{Lu}	$(\bar{L}_i \gamma_\mu L_j)(\bar{u}_k \gamma^\mu u_l)$
$Q_{QQ}^{(3)}$	$(\bar{Q}_i \gamma_\mu \tau^I Q_j)(\bar{Q}_k \gamma^\mu \tau^I Q_l)$	Q_{dd}	$(\bar{d}_i \gamma_\mu d_j)(\bar{d}_k \gamma^\mu d_l)$	Q_{Ld}	$(\bar{L}_i \gamma_\mu L_j)(\bar{d}_k \gamma^\mu d_l)$
$Q_{LQ}^{(1)}$	$(\bar{L}_i \gamma_\mu L_j)(\bar{Q}_k \gamma^\mu Q_l)$	Q_{eu}	$(\bar{e}_i \gamma_\mu e_j)(\bar{u}_k \gamma^\mu u_l)$	Q_{Qe}	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{e}_k \gamma^\mu e_l)$
$Q_{LQ}^{(3)}$	$(\bar{L}_i \gamma_\mu \tau^I L_j)(\bar{Q}_k \gamma^\mu \tau^I Q_l)$	Q_{ed}	$(\bar{e}_i \gamma_\mu e_j)(\bar{d}_k \gamma^\mu d_l)$	$Q_{Qu}^{(1)}$	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{u}_k \gamma^\mu u_l)$
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Collective sym. breaking : CP in the SMEFT

Algebraic approach to the **suppression** associated to **collective effects**

Collective sym. breaking : CP in the SMEFT

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À la Wolfenstein :

$$Y_u = \text{diag} (a_u \lambda^8, a_c \lambda^4, a_t)$$

$$Y_d = V_{\text{CKM}} \text{diag} (a_d \lambda^7, a_s \lambda^5, a_b \lambda^3)$$

$$V_{\text{CKM}} = \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} Aa_b^2 a_t^2 \text{Im} C_{HQ,23}^{(1)} \lambda^8 \\ 0 \\ 0 \end{pmatrix} + \mathcal{O}(\lambda^9)$$

Rank increase at $\mathcal{O}(\lambda^{19})$

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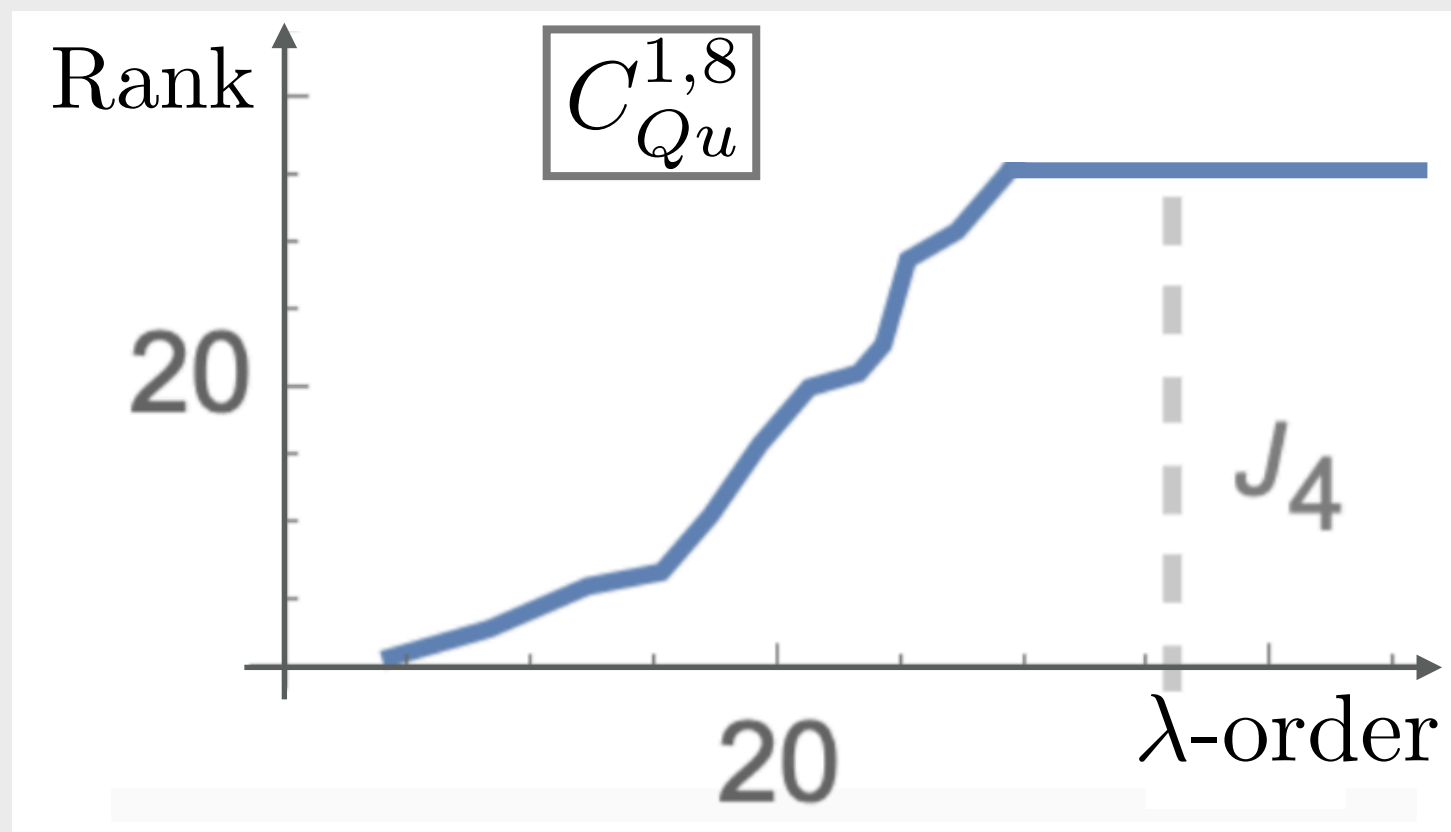
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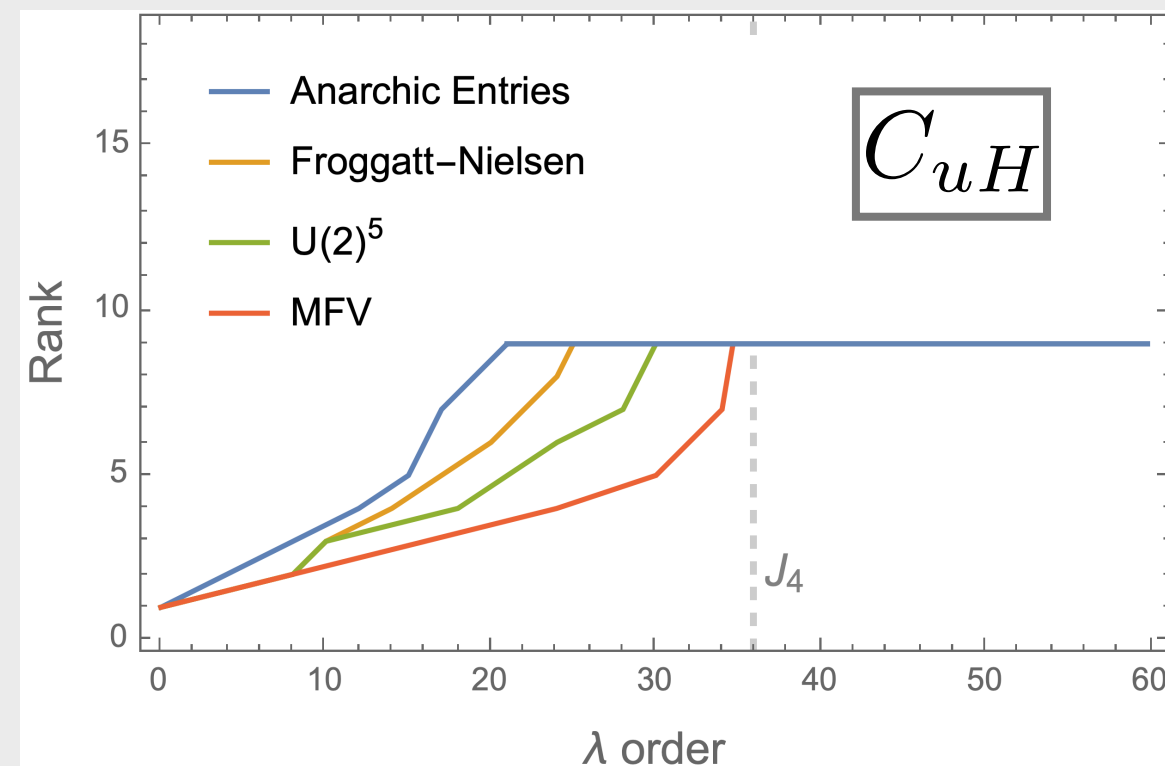
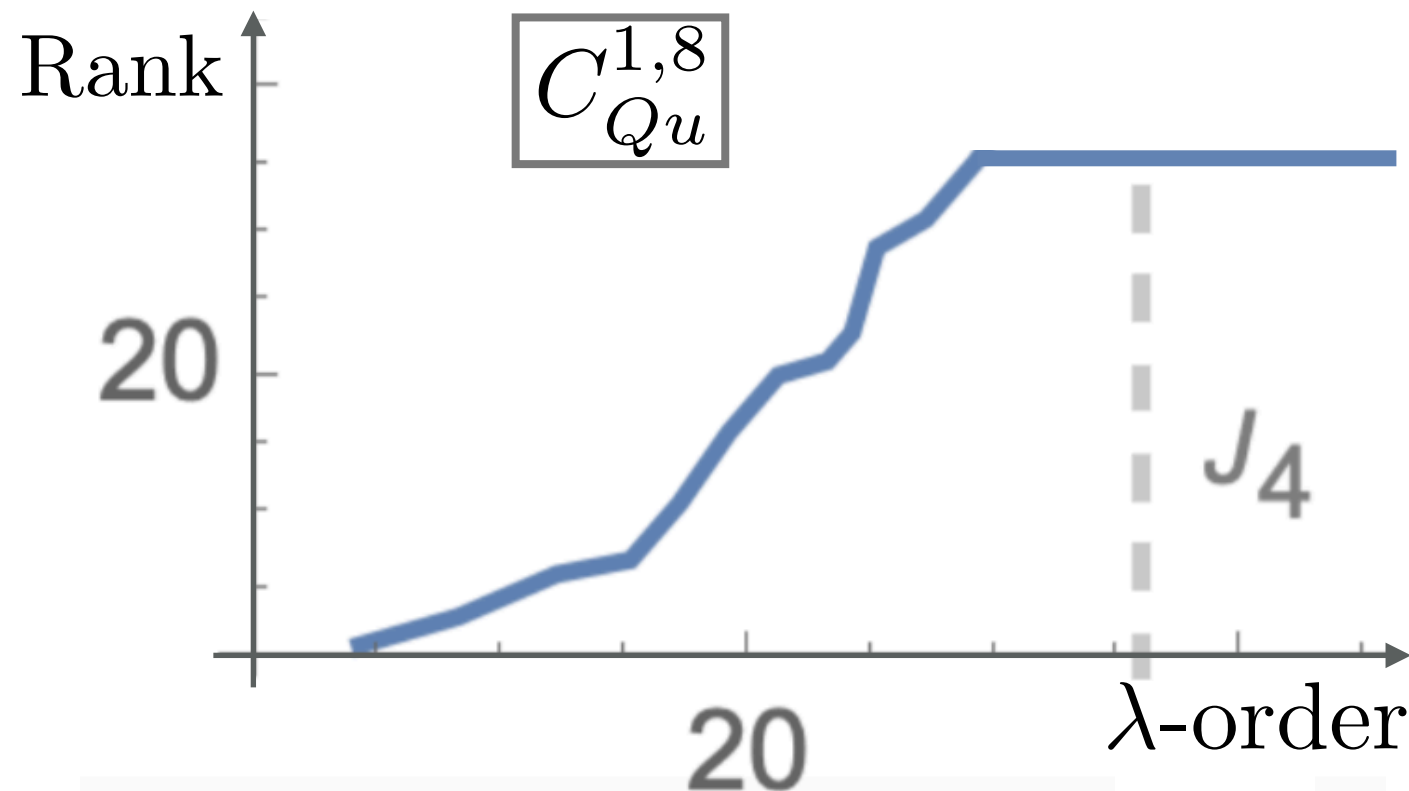
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Collective sym. breaking : Peccei-Quinn

Collective effects : beyond CPV

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$$\frac{a}{f} \bar{Q}_1 \tilde{y}_{uc} \tilde{H} u_2 \quad \mathbf{PQ ?}$$

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PQ ?

Which flavor basis ? Fully specified ?
What are $y_{uc}, \tilde{y}_{ut}, \dots$?

Collective sym. breaking : Peccei-Quinn

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$\frac{a}{f} \left(\bar{Q} \tilde{Y}_u \tilde{H} u + \bar{Q} \tilde{Y}_d H d + \bar{L} \tilde{Y}_e H e + \text{h.c.} \right) \xrightarrow{\text{if PQ}} \frac{\partial_\mu a}{f} \sum_{\psi \in \text{SM}} \bar{\psi} c_\psi \gamma^\mu \psi$
[many authors here]

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approximate ?

Need **order parameters** !

Collective sym. breaking : Peccei-Quinn

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**PQ exact
iff**

$$I_u^{(1)} = \text{ReTr} \left(\tilde{Y}_u Y_u^\dagger \right), \quad I_u^{(2)} = \text{ReTr} \left(X_u \tilde{Y}_u Y_u^\dagger \right), \quad I_u^{(3)} = \text{ReTr} \left(X_u^2 \tilde{Y}_u Y_u^\dagger \right),$$

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$$I_{ud}^{(1)} = \text{ReTr} \left(X_d \tilde{Y}_u Y_u^\dagger + X_u \tilde{Y}_d Y_d^\dagger \right),$$

$$I_{ud,u}^{(2)} = \text{ReTr} \left(X_u^2 \tilde{Y}_d Y_d^\dagger + \{X_u, X_d\} \tilde{Y}_u Y_u^\dagger \right), \quad = \mathbf{0}$$

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Collective sym. breaking : Peccei-Quinn

Collective effects : beyond CPV

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**Collective
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$$I_{ud}^{(4)} = \text{ImTr} \left([X_u, X_d]^2 \left([X_d, \tilde{Y}_u Y_u^\dagger] - [X_u, \tilde{Y}_d Y_d^\dagger] \right) \right)$$

Outlook

Symmetry breaking can be collective

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- **PQ breaking** : also captured by flavor-invariants
 - We built them in the SM+axion EFT at dim-5
 - Match to $\mathbb{P}\mathbb{Q}$ in UV models, closed RGEs
 - Mostly CP-odd (connection to CPV observables ?)

THANK YOU

Warsaw basis

Bilinears

Modified Yukawa		Dipole		Current-current	
Q_{eH}	$(H^\dagger H)(\bar{L}_i e_j H)$	Q_{eW}	$(\bar{L}_i \sigma^{\mu\nu} e_j) \tau^I H W_{\mu\nu}^I$	$Q_{HL}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{L}_i \gamma^\mu L_j)$
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		Q_{uW}	$(\bar{Q}_i \sigma^{\mu\nu} u_j) \tau^I \tilde{H} W_{\mu\nu}^I$	$Q_{HQ}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{Q}_i \gamma^\mu Q_j)$
		Q_{uB}	$(\bar{Q}_i \sigma^{\mu\nu} u_j) \tilde{H} B_{\mu\nu}$	$Q_{HQ}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{Q}_i \tau^I \gamma^\mu Q_j)$
		Q_{dG}	$(\bar{Q}_i \sigma^{\mu\nu} T^A d_j) H G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_i \gamma^\mu u_j)$
		Q_{dW}	$(\bar{Q}_i \sigma^{\mu\nu} d_j) \tau^I H W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_i \gamma^\mu d_j)$
		Q_{dB}	$(\bar{Q}_i \sigma^{\mu\nu} d_j) H B_{\mu\nu}$	$Q_{Hud} + \text{h.c.}$	$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_i \gamma^\mu d_j)$

4-Fermi

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$Q_{LQ}^{(3)}$	$(\bar{L}_i \gamma_\mu \tau^I L_j)(\bar{Q}_k \gamma^\mu \tau^I Q_l)$	Q_{ed}	$(\bar{e}_i \gamma_\mu e_j)(\bar{d}_k \gamma^\mu d_l)$	$Q_{Qu}^{(1)}$	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{u}_k \gamma^\mu u_l)$
		$Q_{ud}^{(1)}$	$(\bar{u}_i \gamma_\mu u_j)(\bar{d}_k \gamma^\mu d_l)$	$Q_{Qu}^{(8)}$	$(\bar{Q}_i \gamma_\mu T^A Q_j)(\bar{u}_k \gamma^\mu T^A u_l)$
		$Q_{ud}^{(8)}$	$(\bar{u}_i \gamma_\mu T^A u_j)(\bar{d}_k \gamma^\mu T^A d_l)$	$Q_{Qd}^{(1)}$	$(\bar{Q}_i \gamma_\mu Q_j)(\bar{d}_k \gamma^\mu d_l)$
				$Q_{Qd}^{(8)}$	$(\bar{Q}_i \gamma_\mu T^A Q_j)(\bar{d}_k \gamma^\mu T^A d_l)$

$(\bar{L}R)(\bar{R}L) + \text{h.c.}$

$$Q_{LedQ} \quad | \quad (\bar{L}_i^a e_j)(\bar{d}_k Q_{la})$$

$(\bar{L}R)(\bar{L}R) + \text{h.c.}$

$$Q_{QuQd}^{(1)} \quad | \quad (\bar{Q}_i^a u_j) \epsilon_{ab} (\bar{Q}_k^b d_l)$$

$$Q_{QuQd}^{(8)} \quad | \quad (\bar{Q}_i^a T^A u_j) \epsilon_{ab} (\bar{Q}_k^b T^A d_l)$$

$$Q_{LeQu}^{(1)} \quad | \quad (\bar{L}_i^a e_j) \epsilon_{ab} (\bar{Q}_k^b u_l)$$

$$Q_{LeQu}^{(3)} \quad | \quad (\bar{L}_i^a \sigma_{\mu\nu} e_j) \epsilon_{ab} (\bar{Q}_s^k \sigma^{\mu\nu} u_t)$$

Invariants

Need a **finite set of algebraically-independent structures**

[Jenkins/Manohar '09]

For fermion bilinears : $\text{ImTr} (X_u^a X_d^b X_u^c X_d^d M)$ with $a, b, c, d = 0, 1, 2$
 $a \neq c, b \neq d$

For 4-Fermi operators : « A-type » $\text{Im} \left([X^{(abcd)}]_{ji} [X^{(efgh)}]_{lk} M_{ijkl} \right)$

« B-type » $\text{Im} \left([X^{(abcd)}]_{jk} [X^{(efgh)}]_{li} M_{ijkl} \right)$

More invariants : fermion bilinears

Wilson coefficient	Number of phases	Minimal set
$C_e \equiv \begin{cases} C_{eH} \\ C_{eW} \\ C_{eB} \end{cases}$	3	$\begin{cases} L_0 \left(C_e Y_e^\dagger \right) \\ L_1 \left(C_e Y_e^\dagger \right) \\ L_2 \left(C_e Y_e^\dagger \right) \end{cases}$
$C_u \equiv \begin{cases} C_{uH} \\ C_{uG} \\ C_{uW} \\ C_{uB} \end{cases}$	9	$\begin{cases} L_{0000} \left(C_u Y_u^\dagger \right) \\ L_{1000} \left(C_u Y_u^\dagger \right) \\ L_{0100} \left(C_u Y_u^\dagger \right) \\ L_{1100} \left(C_u Y_u^\dagger \right) \\ L_{0110} \left(C_u Y_u^\dagger \right) \\ L_{2200} \left(C_u Y_u^\dagger \right) \\ L_{0220} \left(C_u Y_u^\dagger \right) \\ L_{1220} \left(C_u Y_u^\dagger \right) \\ L_{0122} \left(C_u Y_u^\dagger \right) \end{cases}$
$C_d \equiv \begin{cases} C_{dH} \\ C_{dG} \\ C_{dW} \\ C_{dB} \end{cases}$		Same with $C_u Y_u^\dagger \rightarrow C_d Y_d^\dagger$
C_{Hud}		Same with $C_u Y_u^\dagger \rightarrow Y_u C_{Hud} Y_d^\dagger$
$C_{HL}^{1,3}, C_{He}$	0	\emptyset
$C_{HQ}^{(1,3)}$	3	$\begin{cases} L_{1100} \left(C_{HQ}^{(1,3)} \right) \\ L_{2200} \left(C_{HQ}^{(1,3)} \right) \\ L_{1122} \left(C_{HQ}^{(1,3)} \right) \end{cases}$
C_{Hu}		Same with $C_{HQ}^{(1,3)} \rightarrow Y_u C_{Hu} Y_u^\dagger$
C_{Hd}		Same with $C_{HQ}^{(1,3)} \rightarrow Y_d C_{Hd} Y_d^\dagger$

More invariants : 4-Fermi

Wilson coefficient	Number of phases	Minimal set
C_{LL}, C_{ee}	0	\emptyset
C_{Le}	3	$\left\{ \begin{array}{l} B_0^0 (C_{LL\tilde{e}\tilde{e}}) \\ B_0^1 (C_{LL\tilde{e}\tilde{e}}) \\ B_0^2 (C_{LL\tilde{e}\tilde{e}}) \end{array} \right\}$
C_{Qe}	9	$\left\{ \begin{array}{l} A_0^{1100} (C_{QQee}) \\ A_0^{1100} (C_{QQ\tilde{e}\tilde{e}}) \\ A_0^{2200} (C_{QQee}) \\ A_0^{2200} (C_{QQ\tilde{e}\tilde{e}}) \\ A_0^{1122} (C_{QQee}) \\ A_0^{1122} (C_{QQ\tilde{e}\tilde{e}}) \\ A_1^{1100} (C_{QQ\tilde{e}\tilde{e}}) \\ A_1^{2200} (C_{QQ\tilde{e}\tilde{e}}) \\ A_1^{1122} (C_{QQ\tilde{e}\tilde{e}}) \end{array} \right\}$
C_{ed}		Same with $C_{QQee} \rightarrow C_{ee\tilde{d}\tilde{d}}$ (as well as \tilde{e} versions)
C_{eu}		Same with $C_{QQee} \rightarrow C_{ee\tilde{u}\tilde{u}}$ (as well as \tilde{e} versions)
$C_{LQ}^{(1,3)}$		$\left\{ \begin{array}{l} A_{1100}^0 \left(C_{LQ}^{(1,3)} \right) \quad A_{1100}^1 \left(C_{LQ}^{(1,3)} \right) \quad A_{1100}^2 \left(C_{LQ}^{(1,3)} \right) \\ A_{2200}^0 \left(C_{LQ}^{(1,3)} \right) \quad A_{2200}^1 \left(C_{LQ}^{(1,3)} \right) \quad A_{2200}^2 \left(C_{LQ}^{(1,3)} \right) \\ A_{1122}^0 \left(C_{LQ}^{(1,3)} \right) \quad A_{1122}^1 \left(C_{LQ}^{(1,3)} \right) \quad A_{1122}^2 \left(C_{LQ}^{(1,3)} \right) \end{array} \right\}$
C_{Ld}		Same with $C_{LQ}^{(1,3)} \rightarrow C_{LL\tilde{d}\tilde{d}}$
C_{Lu}		Same with $C_{LQ}^{(1,3)} \rightarrow C_{LL\tilde{u}\tilde{u}}$
C_{LeQu}	27	$\left\{ \begin{array}{l} A_{0000}^0 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{0000}^1 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{0000}^2 (C_{L\tilde{e}Q\tilde{u}}) \\ A_{1000}^0 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{1000}^1 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{1000}^2 (C_{L\tilde{e}Q\tilde{u}}) \\ A_{0100}^0 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{0100}^1 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{0100}^2 (C_{L\tilde{e}Q\tilde{u}}) \\ A_{1100}^0 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{1100}^1 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{1100}^2 (C_{L\tilde{e}Q\tilde{u}}) \\ A_{0110}^0 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{0110}^1 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{0110}^2 (C_{L\tilde{e}Q\tilde{u}}) \\ A_{2200}^0 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{2200}^1 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{2200}^2 (C_{L\tilde{e}Q\tilde{u}}) \\ A_{0220}^0 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{0220}^1 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{0220}^2 (C_{L\tilde{e}Q\tilde{u}}) \\ A_{1220}^0 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{1220}^1 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{1220}^2 (C_{L\tilde{e}Q\tilde{u}}) \\ A_{0122}^0 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{0122}^1 (C_{L\tilde{e}Q\tilde{u}}) \quad A_{0122}^2 (C_{L\tilde{e}Q\tilde{u}}) \end{array} \right\}$
C_{LedQ}		Same with $C_{L\tilde{e}Q\tilde{u}} \rightarrow C_{L\tilde{e}\tilde{d}Q}$ and $A_{bcde}^a \rightarrow A_{edcb}^a$

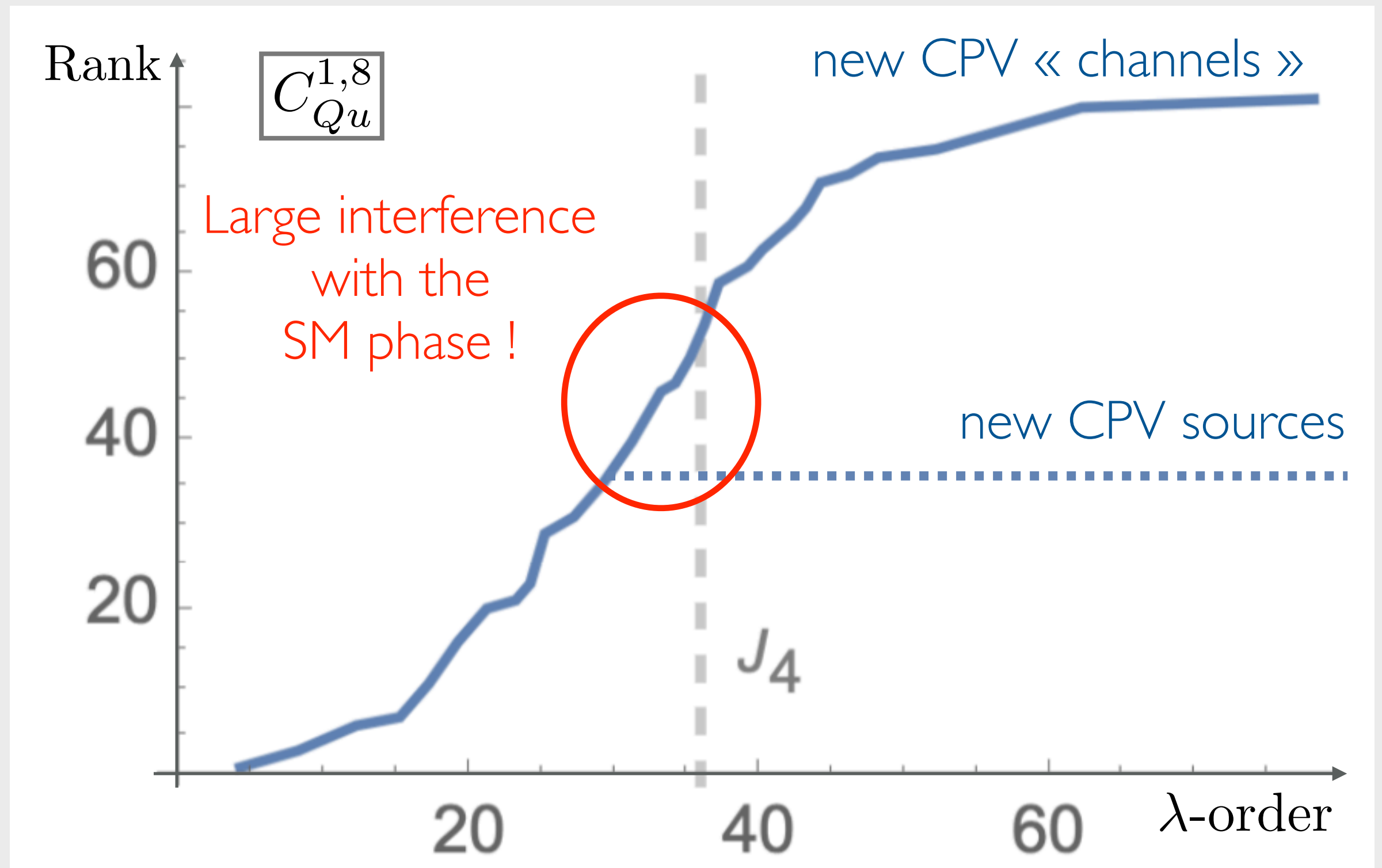
Wilson coefficient	Number of phases	Minimal set
$C_{QQ}^{(1,3)}$	18	$\left\{ \begin{array}{l} A_{1100}^0 (C_{QQQQ}) \quad A_{1100}^{1000} (C_{QQQQ}) \quad A_{1100}^{0100} (C_{QQQQ}) \\ A_{0200}^0 (C_{QQQQ}) \quad A_{1100}^{1100} (C_{QQQQ}) \quad A_{2200}^{1000} (C_{QQQQ}) \\ A_{2200}^{0100} (C_{QQQQ}) \quad A_{1122}^0 (C_{QQQQ}) \quad A_{2200}^{1100} (C_{QQQQ}) \\ A_{2100}^{1200} (C_{QQQQ}) \quad A_{1122}^{1000} (C_{QQQQ}) \quad A_{1122}^{0100} (C_{QQQQ}) \\ A_{1122}^{1100} (C_{QQQQ}) \quad A_{2200}^{1200} (C_{QQQQ}) \quad B_{1100}^0 (C_{QQQQ}) \\ B_{2200}^0 (C_{QQQQ}) \quad B_{1122}^0 (C_{QQQQ}) \quad B_{1122}^{2200} (C_{QQQQ}) \end{array} \right\}$
C_{uu}	18	$\left\{ \begin{array}{l} A_{1100}^0 (C_{uu\tilde{u}\tilde{u}}) \quad A_{1100}^{1000} (C_{uu\tilde{u}\tilde{u}}) \quad A_{1100}^{0100} (C_{uu\tilde{u}\tilde{u}}) \\ A_{2200}^0 (C_{uu\tilde{u}\tilde{u}}) \quad A_{1100}^{1100} (C_{uu\tilde{u}\tilde{u}}) \quad A_{1100}^{0200} (C_{uu\tilde{u}\tilde{u}}) \\ A_{2200}^{0100} (C_{uu\tilde{u}\tilde{u}}) \quad A_{1122}^0 (C_{uu\tilde{u}\tilde{u}}) \quad A_{2200}^{1100} (C_{uu\tilde{u}\tilde{u}}) \\ A_{1100}^{1100} (C_{uu\tilde{u}\tilde{u}}) \quad A_{1122}^{0100} (C_{uu\tilde{u}\tilde{u}}) \quad A_{1100}^{0122} (C_{uu\tilde{u}\tilde{u}}) \\ A_{1200}^{1200} (C_{uu\tilde{u}\tilde{u}}) \quad B_{1100}^0 (C_{uu\tilde{u}\tilde{u}}) \quad B_{1100}^{0100} (C_{uu\tilde{u}\tilde{u}}) \\ B_{2100}^{0200} (C_{uu\tilde{u}\tilde{u}}) \quad B_{1122}^{1200} (C_{uu\tilde{u}\tilde{u}}) \quad B_{1200}^{1000} (C_{uu\tilde{u}\tilde{u}}) \end{array} \right\}$
C_{dd}	18	$\left\{ \begin{array}{l} A_{1100}^0 (C_{dd\tilde{d}\tilde{d}}) \quad A_{1100}^{1000} (C_{dd\tilde{d}\tilde{d}}) \quad A_{2200}^0 (C_{dd\tilde{d}\tilde{d}}) \\ A_{2000}^{1100} (C_{dd\tilde{d}\tilde{d}}) \quad A_{1100}^{0100} (C_{dd\tilde{d}\tilde{d}}) \quad A_{1100}^{1100} (C_{dd\tilde{d}\tilde{d}}) \\ A_{2200}^{1000} (C_{dd\tilde{d}\tilde{d}}) \quad A_{1122}^0 (C_{dd\tilde{d}\tilde{d}}) \quad A_{2200}^{1100} (C_{dd\tilde{d}\tilde{d}}) \\ A_{1100}^{1000} (C_{dd\tilde{d}\tilde{d}}) \quad A_{1122}^{0100} (C_{dd\tilde{d}\tilde{d}}) \quad A_{2110}^{1200} (C_{dd\tilde{d}\tilde{d}}) \\ A_{0122}^{2100} (C_{dd\tilde{d}\tilde{d}}) \quad A_{1220}^{2100} (C_{dd\tilde{d}\tilde{d}}) \quad B_{1100}^0 (C_{dd\tilde{d}\tilde{d}}) \\ B_{2100}^{0100} (C_{dd\tilde{d}\tilde{d}}) \quad B_{1100}^{1000} (C_{dd\tilde{d}\tilde{d}}) \quad B_{2000}^{1200} (C_{dd\tilde{d}\tilde{d}}) \end{array} \right\}$
$C_{Qu}^{(1,8)}$	36	$\left\{ \begin{array}{l} A_0^{1100} (C_{QQqu}) \quad A_{1100}^0 (C_{QQ\tilde{u}\tilde{u}}) \quad A_{1100}^{1000} (C_{QQ\tilde{u}\tilde{u}}) \\ A_{0100}^{1100} (C_{QQ\tilde{u}\tilde{u}}) \quad A_{1100}^{1100} (C_{QQ\tilde{u}\tilde{u}}) \quad A_{0110}^{1100} (C_{QQ\tilde{u}\tilde{u}}) \\ A_{1000}^{1200} (C_{QQ\tilde{u}\tilde{u}}) \quad A_0^{2200} (C_{QQqu}) \quad A_{2200}^{1100} (C_{QQ\tilde{u}\tilde{u}}) \\ A_{0220}^{1100} (C_{QQ\tilde{u}\tilde{u}}) \quad A_{2200}^{0120} (C_{QQ\tilde{u}\tilde{u}}) \quad A_{1100}^{1122} (C_{QQ\tilde{u}\tilde{u}}) \\ A_{1220}^{1200} (C_{QQ\tilde{u}\tilde{u}}) \quad A_{1122}^{2200} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{0100}^0 (C_{QQ\tilde{u}\tilde{u}}) \\ B_{1100}^0 (C_{QQ\tilde{u}\tilde{u}}) \quad B_{0221}^0 (C_{QQ\tilde{u}\tilde{u}}) \quad B_{1000}^{0100} (C_{QQ\tilde{u}\tilde{u}}) \\ B_{1100}^{0100} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{2200}^{0100} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{2110}^{0100} (C_{QQ\tilde{u}\tilde{u}}) \\ B_{2000}^{0200} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{2100}^{0200} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{2110}^{0200} (C_{QQ\tilde{u}\tilde{u}}) \\ B_{0110}^{1000} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{0220}^{1000} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{0221}^{1000} (C_{QQ\tilde{u}\tilde{u}}) \\ B_{1100}^{1100} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{2200}^{1100} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{2100}^{1200} (C_{QQ\tilde{u}\tilde{u}}) \\ B_{2210}^{1200} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{1100}^{2100} (C_{QQ\tilde{u}\tilde{u}}) \quad B_{1210}^{1100} (C_{QQ\tilde{u}\tilde{u}}) \end{array} \right\}$
$C_{Qd}^{(1,8)}$	36	$\left\{ \begin{array}{l} A_0^{1100} (C_{QQdd}) \quad A_{1100}^0 (C_{QQ\tilde{d}\tilde{d}}) \quad A_{1100}^{1000} (C_{QQ\tilde{d}\tilde{d}}) \\ A_{1000}^{1100} (C_{QQ\tilde{d}\tilde{d}}) \quad A_0^{1200} (C_{QQdd}) \quad A_{1100}^{0100} (C_{QQ\tilde{d}\tilde{d}}) \\ A_{2200}^0 (C_{QQ\tilde{d}\tilde{d}}) \quad A_{1100}^{1100} (C_{QQ\tilde{d}\tilde{d}}) \quad A_{2100}^{1100} (C_{QQ\tilde{d}\tilde{d}}) \\ A_0^{1122} (C_{QQdd}) \quad A_{1122}^0 (C_{QQ\tilde{d}\tilde{d}}) \quad A_{2200}^{1100} (C_{QQ\tilde{d}\tilde{d}}) \\ A_{0220}^{1100} (C_{QQ\tilde{d}\tilde{d}}) \quad A_{1100}^{1000} (C_{QQ\tilde{d}\tilde{d}}) \quad A_{1122}^{1100} (C_{QQ\tilde{d}\tilde{d}}) \\ A_{0122}^{2100} (C_{QQ\tilde{d}\tilde{d}}) \quad B_{0100}^0 (C_{QQ\tilde{d}\tilde{d}}) \quad B_{1000}^0 (C_{QQ\tilde{d}\tilde{d}}) \\ B_{0110}^0 (C_{QQ\tilde{d}\tilde{d}}) \quad B_{0220}^0 (C_{QQ\tilde{d}\tilde{d}}) \quad B_{1100}^0 (C_{QQ\tilde{d}\tilde{d}}) \\ B_{0221}^0 (C_{QQ\tilde{d}\tilde{d}}) \quad B_{2200}^0 (C_{QQ\tilde{d}\tilde{d}}) \quad B_{2210}^0 (C_{QQ\tilde{d}\tilde{d}}) \\ B_{1000}^{0100} (C_{QQ\tilde{d}\tilde{d}}) \quad B_{0120}^{0100} (C_{QQ\tilde{d}\tilde{d}}) \quad B_{1100}^{0100} (C_{QQ\tilde{d}\tilde{d}}) \\ B_{2210}^{0100} (C_{QQ\tilde{d}\tilde{d}}) \quad B_{0110}^{1000} (C_{QQ\tilde{d}\tilde{d}}) \quad B_{0220}^{1000} (C_{QQ\tilde{d}\tilde{d}}) \\ B_{2210}^{1000} (C_{QQ\tilde{d}\tilde{d}}) \quad B_{1200}^{1000} (C_{QQ\tilde{d}\tilde{d}}) \quad B_{2200}^{1100} (C_{QQ\tilde{d}\tilde{d}}) \\ B_{2210}^{1100} (C_{QQ\tilde{d}\tilde{d}}) \quad B_{2100}^{1200} (C_{QQ\tilde{d}\tilde{d}}) \quad B_{2211}^{1200} (C_{QQ\tilde{d}\tilde{d}}) \end{array} \right\}$

More invariants : 4-Fermi

Wilson coefficient	Number of phases	Minimal set
$C_{ud}^{(1,8)}$	36	$\left\{ \begin{array}{l} A_0^{1100} (C_{\bar{u}dd}) \quad A_{1100}^0 (C_{u\bar{u}d\bar{d}}) \quad A_{1100}^{1000} (C_{\bar{u}d\bar{d}}) \\ A_{1000}^{1100} (C_{\bar{u}d\bar{d}}) \quad A_0^{2200} (C_{\bar{u}dd}) \quad A_{1100}^{0100} (C_{\bar{u}d\bar{d}}) \\ A_{2200}^0 (C_{u\bar{u}d\bar{d}}) \quad A_{1100}^{1100} (C_{\bar{u}d\bar{d}}) \quad A_{0110}^{1100} (C_{\bar{u}d\bar{d}}) \\ A_{2200}^{1000} (C_{\bar{u}d\bar{d}}) \quad A_{2100}^{1100} (C_{\bar{u}d\bar{d}}) \quad A_0^{1122} (C_{\bar{u}dd}) \\ A_{2200}^{0100} (C_{\bar{u}d\bar{d}}) \quad A_{1122}^0 (C_{u\bar{u}d\bar{d}}) \quad A_{2200}^{1100} (C_{\bar{u}d\bar{d}}) \\ A_{1122}^{1000} (C_{\bar{u}d\bar{d}}) \quad A_{1122}^{0100} (C_{\bar{u}d\bar{d}}) \quad A_{1122}^{1100} (C_{\bar{u}d\bar{d}}) \\ B_{0100}^0 (C_{\bar{u}d\bar{d}}) \quad B_{1000}^0 (C_{\bar{u}d\bar{d}}) \quad B_{0110}^0 (C_{\bar{u}d\bar{d}}) \\ B_{1100}^0 (C_{\bar{u}d\bar{d}}) \quad B_{0221}^0 (C_{\bar{u}d\bar{d}}) \quad B_{2200}^0 (C_{\bar{u}d\bar{d}}) \\ B_{1000}^{0100} (C_{\bar{u}d\bar{d}}) \quad B_{0110}^{0100} (C_{\bar{u}d\bar{d}}) \quad B_{2110}^{0100} (C_{\bar{u}d\bar{d}}) \\ B_{2000}^{0200} (C_{\bar{u}d\bar{d}}) \quad B_{2110}^{0200} (C_{\bar{u}d\bar{d}}) \quad B_{0110}^{1000} (C_{\bar{u}d\bar{d}}) \\ B_{0221}^{1000} (C_{\bar{u}d\bar{d}}) \quad B_{1200}^{1000} (C_{\bar{u}d\bar{d}}) \quad B_{2200}^{1100} (C_{\bar{u}d\bar{d}}) \\ B_{2211}^{1100} (C_{\bar{u}d\bar{d}}) \quad B_{2100}^{1200} (C_{\bar{u}d\bar{d}}) \quad B_{1200}^{2100} (C_{\bar{u}d\bar{d}}) \end{array} \right\}$
$C_{QuQd}^{(1,8)}$	81	$\left\{ \begin{array}{l} A_0^0 (C_{Q\bar{u}Q\bar{d}}) \quad A_{1000}^0 (C_{Q\bar{u}Q\bar{d}}) \quad A_0^{1000} (C_{Q\bar{u}Q\bar{d}}) \\ A_{1000}^{1000} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0100}^0 (C_{Q\bar{u}Q\bar{d}}) \quad A_0^{0100} (C_{Q\bar{u}Q\bar{d}}) \\ A_{1100}^0 (C_{Q\bar{u}Q\bar{d}}) \quad A_{0110}^0 (C_{Q\bar{u}Q\bar{d}}) \quad A_{1000}^{1000} (C_{Q\bar{u}Q\bar{d}}) \\ A_{0100}^{1000} (C_{Q\bar{u}Q\bar{d}}) \quad A_0^{1100} (C_{Q\bar{u}Q\bar{d}}) \quad A_0^{0110} (C_{Q\bar{u}Q\bar{d}}) \\ A_{1100}^{1000} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0110}^{1000} (C_{Q\bar{u}Q\bar{d}}) \quad A_{1000}^{1100} (C_{Q\bar{u}Q\bar{d}}) \\ A_{0100}^{0100} (C_{Q\bar{u}Q\bar{d}}) \quad A_{1100}^{0100} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0110}^{0100} (C_{Q\bar{u}Q\bar{d}}) \\ A_{0110}^{0110} (C_{Q\bar{u}Q\bar{d}}) \quad A_{2200}^0 (C_{Q\bar{u}Q\bar{d}}) \quad A_0^{0220} (C_{Q\bar{u}Q\bar{d}}) \\ A_{2000}^{0200} (C_{Q\bar{u}Q\bar{d}}) \quad A_{1100}^{1100} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0110}^{1100} (C_{Q\bar{u}Q\bar{d}}) \\ A_{0200}^{2000} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0100}^{2100} (C_{Q\bar{u}Q\bar{d}}) \quad A_{1100}^{0110} (C_{Q\bar{u}Q\bar{d}}) \\ A_{0110}^{0110} (C_{Q\bar{u}Q\bar{d}}) \quad A_{1000}^{0210} (C_{Q\bar{u}Q\bar{d}}) \quad A_{1220}^0 (C_{Q\bar{u}Q\bar{d}}) \\ A_{2000}^{1200} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0122}^0 (C_{Q\bar{u}Q\bar{d}}) \quad A_{1220}^{0100} (C_{Q\bar{u}Q\bar{d}}) \\ A_{0122}^{1000} (C_{Q\bar{u}Q\bar{d}}) \quad A_{2200}^{1100} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0220}^{1100} (C_{Q\bar{u}Q\bar{d}}) \\ A_{2100}^{1200} (C_{Q\bar{u}Q\bar{d}}) \quad A_{1200}^{2100} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0210}^{2100} (C_{Q\bar{u}Q\bar{d}}) \\ A_{0110}^{2200} (C_{Q\bar{u}Q\bar{d}}) \quad A_{2200}^{0110} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0220}^{0110} (C_{Q\bar{u}Q\bar{d}}) \\ A_{0112}^{0112} (C_{Q\bar{u}Q\bar{d}}) \quad A_{1100}^{1220} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0112}^{2100} (C_{Q\bar{u}Q\bar{d}}) \\ A_{1220}^{1200} (C_{Q\bar{u}Q\bar{d}}) \quad A_{2200}^{2200} (C_{Q\bar{u}Q\bar{d}}) \quad A_{1122}^{0110} (C_{Q\bar{u}Q\bar{d}}) \\ A_{2100}^{0122} (C_{Q\bar{u}Q\bar{d}}) \quad A_{0220}^{0220} (C_{Q\bar{u}Q\bar{d}}) \quad B_0^0 (C_{Q\bar{u}Q\bar{d}}) \\ B_{0100}^0 (C_{Q\bar{u}Q\bar{d}}) \quad B_{1000}^0 (C_{Q\bar{u}Q\bar{d}}) \quad B_{1100}^0 (C_{Q\bar{u}Q\bar{d}}) \\ B_{2200}^0 (C_{Q\bar{u}Q\bar{d}}) \quad B_{0110}^0 (C_{Q\bar{u}Q\bar{d}}) \quad B_{0122}^0 (C_{Q\bar{u}Q\bar{d}}) \\ B_{0220}^0 (C_{Q\bar{u}Q\bar{d}}) \quad B_0^{0100} (C_{Q\bar{u}Q\bar{d}}) \quad B_{1000}^{0100} (C_{Q\bar{u}Q\bar{d}}) \\ B_{1100}^{0100} (C_{Q\bar{u}Q\bar{d}}) \quad B_{2100}^{0100} (C_{Q\bar{u}Q\bar{d}}) \quad B_{0120}^{0100} (C_{Q\bar{u}Q\bar{d}}) \\ B_{0122}^{0100} (C_{Q\bar{u}Q\bar{d}}) \quad B_{1120}^{0200} (C_{Q\bar{u}Q\bar{d}}) \quad B_0^{1000} (C_{Q\bar{u}Q\bar{d}}) \\ B_{0100}^{1000} (C_{Q\bar{u}Q\bar{d}}) \quad B_{1200}^{1000} (C_{Q\bar{u}Q\bar{d}}) \quad B_{0110}^{1000} (C_{Q\bar{u}Q\bar{d}}) \\ B_{0122}^{1000} (C_{Q\bar{u}Q\bar{d}}) \quad B_{2100}^{1000} (C_{Q\bar{u}Q\bar{d}}) \quad B_0^{1100} (C_{Q\bar{u}Q\bar{d}}) \\ B_{1100}^{1100} (C_{Q\bar{u}Q\bar{d}}) \quad B_{2200}^{1100} (C_{Q\bar{u}Q\bar{d}}) \quad B_{0110}^{1100} (C_{Q\bar{u}Q\bar{d}}) \\ B_{0220}^{1100} (C_{Q\bar{u}Q\bar{d}}) \quad B_{1122}^{1100} (C_{Q\bar{u}Q\bar{d}}) \quad B_{2100}^{1200} (C_{Q\bar{u}Q\bar{d}}) \\ B_{0122}^{2100} (C_{Q\bar{u}Q\bar{d}}) \quad B_0^{2200} (C_{Q\bar{u}Q\bar{d}}) \quad A_{1122}^{2200} (C_{Q\bar{u}Q\bar{d}}) \end{array} \right\}$

Maximal set

Objects of the « Im Tr » kind



Theta QCD

$$\mathcal{L}_{\text{QCD}} \supset -\theta_{\text{QCD}} \frac{g_s^2}{16\pi^2} \text{Tr}(G\tilde{G})$$

	$SU(3)_{Q_L}$	$U(1)_{Q_L}$	$SU(3)_{u_R}$	$U(1)_{u_R}$	$SU(3)_{d_R}$	$U(1)_{d_R}$
Q_L	3	1	1	0	1	0
u_R	1	0	3	1	1	0
d_R	1	0	1	0	3	1
Y_u	3	1	$\bar{3}$	-1	1	0
Y_d	3	1	1	0	$\bar{3}$	-1
$e^{i\theta_{\text{QCD}}}$	1	6	1	-3	1	-3

$$\bar{\theta} \equiv \theta_{\text{QCD}} - \arg \det(Y_u Y_d)$$

$$\text{Im} \left(e^{-i\theta_{\text{QCD}}} \epsilon^{ABC} \epsilon^{abc} \epsilon^{DEF} \epsilon^{def} Y_{u,Aa} Y_{u,Bb} C_{QuQd,CcDd} Y_{d,Ee} Y_{d,Ff} \right)$$

In the UV, suppressed by $e^{-\frac{8\pi^2}{g_s^2}} \approx \lambda^{37-38}$. Relevant in the IR?

Physical parameters at order $1/\Lambda^2$

Type of op.	# of ops	# real	# im.	
bilinears	Yuk.	$3N^2$	$3N^2$	← # physical parameters
		$2N^2 + N$	$2N^2 + N$	← # physical at $\mathcal{O}(1/\Lambda^2)$
	Dipole	$8N^2$	$8N^2$	
		$6N^2 + 2N$	$6N^2 + 2N$	
	curr-curr	$\frac{1}{2}N(9N + 7)$	$\frac{1}{2}N(9N - 7)$	
		$N(3N + 5)$	$N(3N - 2)$	
all bilinears	$\frac{1}{2}N(31N + 7)$	$\frac{1}{2}N(31N - 7)$		
	$N(11N + 8)$	$N(11N + 1)$		
4-Fermi	LLLL	$\frac{1}{4}N^2(7N^2 + 13)$	$\frac{7}{4}N^2(N^2 - 1)$	
		$\frac{1}{2}N^2(N^2 + 2N + 7)$	$\frac{1}{2}N^2(N^2 + 2N - 3)$	
	RRRR	$\frac{1}{8}N(21N^3 + 2N^2 + 31N + 2)$	$\frac{1}{8}N(21N + 2)(N^2 - 1)$	
		$\frac{1}{2}N(3N^3 + 2N^2 + 8N + 1)$	$\frac{1}{2}N^2(3N^2 + 2N - 5)$	
	LLRR	$4N^2(N^2 + 1)$	$4N^2(N^2 - 1)$	
		$\frac{1}{2}N(4N^3 + 3N^2 + 9N + 2)$	$\frac{1}{2}N(4N^3 + 3N^2 - 6N - 1)$	
	LRRL	N^4	N^4	
		N^3	N^3	
	LRLR	$4N^4$	$4N^4$	
		$2N^3(N + 1)$	$2N^3(N + 1)$	
all 4-Fermi	$\frac{1}{8}N(107N^3 + 2N^2 + 89N + 2)$	$\frac{1}{8}N(107N^3 + 2N^2 - 67N - 2)$		
	$\frac{1}{2}N(12N^3 + 13N^2 + 24N + 3)$	$\frac{1}{2}N(12N^3 + 13N^2 - 14N - 1)$		
all	$\frac{1}{8}N(107N^3 + 2N^2 + 213N + 30)$	$\frac{1}{8}N(107N^3 + 2N^2 + 57N - 30)$		
	$\frac{1}{2}N(12N^3 + 13N^2 + 46N + 19)$	$\frac{1}{2}N(12N^3 + 13N^2 + 8N + 1)$		

Flavour symmetries and CPV quantities

Non-generic values			Flavor symmetries of the SM Lagrangian	
Generic case: $\delta_{\text{CKM}} = 0$			$U(1)_B$	
$s_{12} = 0$			$U(1)_B$	
$c_{12} = 0$			$U(1)_B$	
$s_{23} = 0$	-		$U(1)_B$	
	$s_{12} = 0$	-	$U(1)^2$	
	$c_{12} = 0$	-	$U(1)^2$	
$c_{23} = 0$	-		$U(1)_B$	
	$s_{12} = 0$	-	$U(1)^2$	
	$c_{12} = 0$	-	$U(1)^2$	
$s_{13} = 0$	-		$U(1)_B$	
	$s_{12} = 0$	-	$U(1)^2$	
		$s_{23} = 0$	-	$U(1)^3$
	$c_{12} = 0$	$c_{23} = 0$	-	$U(1)^3$
		-	-	$U(1)^2$
		$s_{23} = 0$	-	$U(1)^3$
$c_{23} = 0$		-	$U(1)^3$	
$c_{13} = 0$			$U(1)^2$	
$m_{u_i} = m_{u_j}$	-		$U(1)_B$	
	$s_{12} = 0$	-	$U(1)^2$	
	$c_{12} = 0$	-	$U(1)^2$	
	$s_{13} = 0$	-	-	$U(1)^2$
		$s_{12} = 0$	-	$U(1)^3$
	$c_{13} = 0$	$c_{12} = 0$	-	$U(1)^3$
		-	-	$U(1)^3$
		-	-	$U(1)^2$
	$m_{d_k} = m_{d_l}$	$c_{13} = 0$	-	$U(2) \times U(1)$
		$s_{13} = 0$	-	$U(1)^3$
All m_{u_i} equal	-		$U(1)^3$	
	$m_{d_k} = m_{d_l}$	-	$U(2) \times U(1)$	
	all m_{d_k} equal	-	$U(3)$	

Flavour symmetries and CPV quantities

Flavour symmetries of the SM	C_{eH} C_{eW} C_{eB}	C_{uH} C_{dH} C_{uG} C_{uW} C_{uB} C_{dG} C_{dW} C_{dB} C_{Hud}	$C_{HL}^{1,3}$ C_{He}	$C_{HQ}^{1,3}$ C_{Hu} C_{Hd}	C_{LL} C_{ee}	C_{Le}	$C_{QQ}^{1,3}$ C_{uu} C_{dd}	$C_{LQ}^{1,3}$ C_{eu} C_{ed} C_{Lu} C_{Ld} C_{Qe}	$C_{ud}^{1,8}$ $C_{Qu}^{1,8}$ $C_{Qd}^{1,8}$	C_{LedQ} $C_{LeQu}^{1,3}$	$C_{QuQd}^{1,8}$
$U(1)_B$	3	9	0	3	0	3	18	9	36	27	81
$U(1)^2$	3	5	0	1	0	3	5	3	12	15	33
$U(1)^3$	3	3	0	0	0	3	0	0	3	9	15
$U(2) \times U(1)$	3	2	0	0	0	3	0	0	2	6	6
$U(3)$	3	1	0	0	0	3	0	0	1	3	2
Two degenerate leptons	$\times \frac{2}{3}$	$\times 1$	$\times \frac{2}{3}$	$\times 1$	$\times \frac{2}{3}$	$\times \frac{2}{3}$	$\times 1$	$\times \frac{2}{3}$	$\times 1$	$\times \frac{2}{3}$	$\times 1$
All leptons degenerate	$\times \frac{1}{3}$	$\times 1$	$\times \frac{1}{3}$	$\times 1$	$\times \frac{1}{3}$	$\times \frac{1}{3}$	$\times 1$	$\times \frac{1}{3}$	$\times 1$	$\times \frac{1}{3}$	$\times 1$

New sources of CPV

Must be **proportional to invariants. Which ones?**

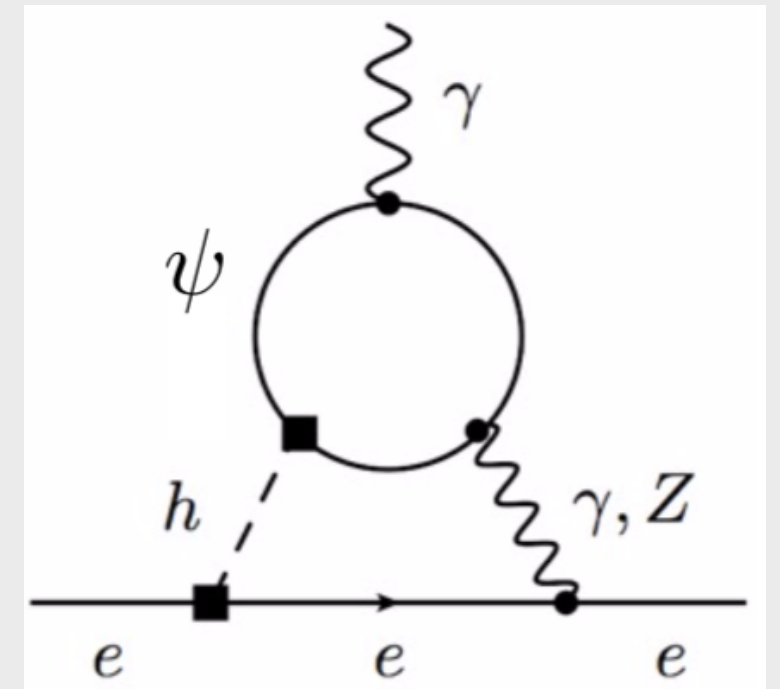
SM CPV could interfere with SMEFT coefficients. Ex : loops over three generations, electroweak and SMEFT vertices.

⇒ « Simple » graphs map to new sources of CPV (for invariant observables, e.g. EDMs)

Example : Barr-Zee two-loop contribution to the electron EDM

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C_{uH}}{\Lambda^2} |H|^2 \overline{Q}_L u_R \tilde{H} + h.c.$$

$$\frac{d_e}{e} = -\frac{1}{48\pi^2} \frac{vm_e m_u}{m_h^2} \frac{\text{Im}(C_{uH})}{\Lambda^2} F_1 \left(\frac{m_u^2}{m_h^2}, 0 \right)$$



[Barr/Zee '90, Brod/Haisch/Zupan '13]

one-to-one correspondance with invariants

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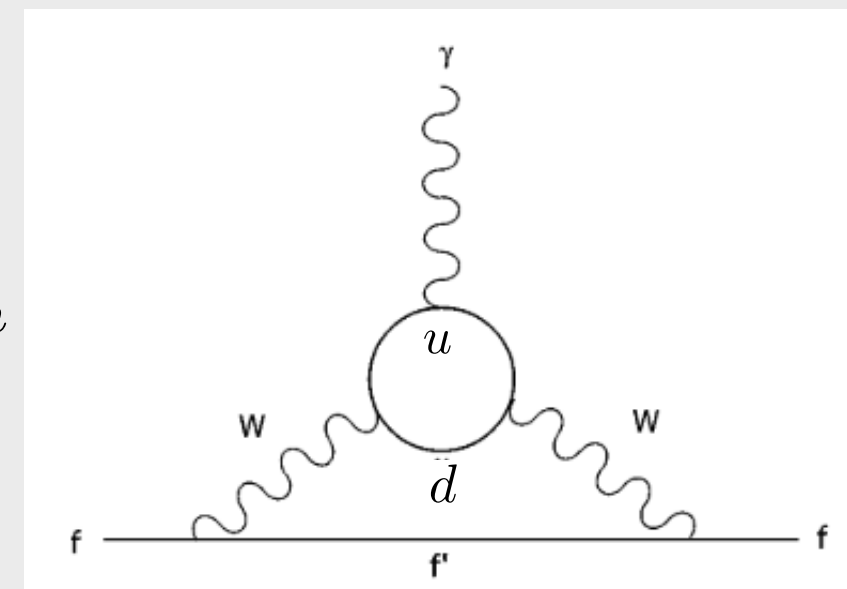
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$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{C_{Hud,mn}}{\Lambda^2} i \tilde{H}^\dagger D_\mu H \bar{u}_{R,m} \gamma^\mu d_{R,n}$$

$$\frac{d}{e} = \sum_{i,j} \text{Im} (V_{CKM,ij} C_{Hud,ij}^*) F(m_{u_i}^2, m_{d_j}^2)$$



[Kadoyoshi/Oshimo '97]

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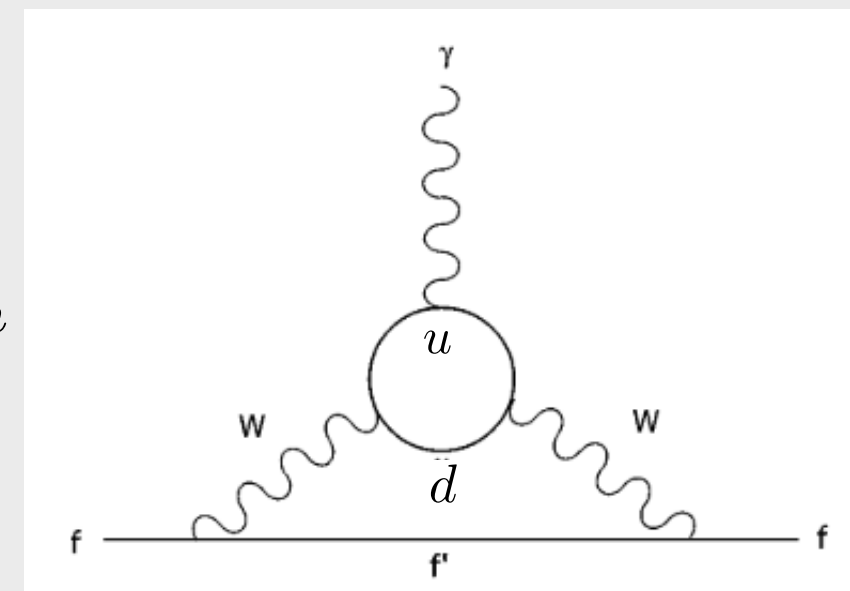
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[Kadoyoshi/Oshimo '97]

$$\frac{d_e}{e} \sim \# \times \text{Im} C_{Hud,23} + \#' \times (\cos \delta_{CKM} \text{Im} C_{Hud,13} + \sin \delta_{CKM} \text{Re} C_{Hud,13}) + \dots$$

UV models

Invariants easily track CPV

Ex : spontaneous CP breaking model

[Bento/Branco/Parada '91]

$$\mathcal{L} \supset -\bar{Q}_L Y_u \tilde{H} u_R - (\bar{Q}_L \quad \bar{D}_L) \begin{pmatrix} H\Gamma & 0 \\ F'S + F'S^\dagger & \mu \end{pmatrix} \begin{pmatrix} d_R \\ D_R \end{pmatrix} + h.c.$$

$\langle S \rangle = V e^{i\alpha}$

$$\mathcal{L}_{\text{SMEFT}} \supset -\bar{Q}_L Y_u \tilde{H} u_R - \bar{Q}_L Y_d H d_R + \frac{(\lambda_D^\dagger \lambda_D Y_d)_{ij}}{2M^2} \mathcal{O}_{dH,ij} - \frac{(\lambda_D^\dagger \lambda_D)_{ij}}{4M^2} \left(\mathcal{O}_{HQ,ij}^{(1)} + \mathcal{O}_{HQ,ij}^{(3)} \right)$$

$$\lambda_D = \frac{V}{M} \mathcal{F} \Gamma^T \quad Y_d Y_d^\dagger = \Gamma \Gamma^T - \frac{V^2}{M^2} \Gamma \mathcal{F}^\dagger \mathcal{F} \Gamma^T \quad (\mathcal{F} = e^{i\alpha} F + e^{-i\alpha} F')$$

What makes CP conserved, at dim-4, 6...? **Algebraic answers**

Ex : can get J4=0 but not the L's, or vice-versa, etc

UV models and PQ breaking

Invariants easily track PQ

Ex : axiflavor/flaxion model

[Ema/Hamaguchi/Moroi/Nakayama '16
Calibbi/Goertz/Redigolo/Ziegler/Zupan '16]

$$-\mathcal{L} = \alpha_{ij}^d \left(\frac{\phi}{M}\right)^{q_{Q_i} - q_{d_j}} \bar{Q}_i H d_j + \alpha_{ij}^u \left(\frac{\phi}{M}\right)^{q_{Q_i} - q_{u_j}} \bar{Q}_i \tilde{H} u_j + \alpha_{ij}^e \left(\frac{\phi}{M}\right)^{q_{L_i} - q_{e_j}} \bar{L}_i H e_j + \text{h.c.}$$

$$\implies I_u^{(1)} = I_u^{(2)} = \dots = 0$$

$$\bullet -\mathcal{L}_{PQ} = \delta_{i1} \delta_{j1} \alpha' \left(\frac{\phi}{M}\right)^{q'_{Q_1} - q'_{u_1}} \bar{Q}_1 \tilde{H} u_1 + \text{h.c.}$$

$$\implies I_u^{(1)}, I_u^{(2)}, \dots \propto q_{Q_1} - q_{u_1} - (q'_{Q_1} - q'_{u_1})$$

$$\bullet \alpha_{ij}^u \left(\frac{\phi}{M}\right)^{q_{Q_i} - q_{u_j}} \bar{Q}_i \tilde{H} u_j \rightarrow \alpha_{ij}^u \left(\frac{\phi}{M}\right)^{q'_{Q_i} - q_{u_j}} \bar{Q}_i \tilde{H} u_j$$

$$\implies I_u^{(1)} = \dots I_d^{(3)} = 0, \quad I_{ud}^{(1)} \propto q_{Q_1} - q_{Q'_1}$$

RGEs of $\overline{\text{PQ}}$ -invariants

$$\dot{I}_u^{(1)} = 2\gamma_u I_u^{(1)} + 6I_u^{(2)} - 3I_{ud}^{(1)} - 2\text{Tr}(X_u) \left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)}) \right),$$

$$\dot{I}_u^{(2)} = 4\gamma_u I_u^{(2)} + 9I_u^{(3)} - 3I_{ud,u}^{(2)} - 2\text{Tr}(X_u^2) \left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)}) \right),$$

$$\dot{I}_u^{(3)} = 6\gamma_u I_u^{(3)} + 12I_u^{(4)} - 3I_u' - 2\text{Tr}(X_u^3) \left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)}) \right),$$

$$\dot{I}_d^{(1)} = 2\gamma_d I_d^{(1)} + 6I_d^{(2)} - 3I_{ud}^{(1)} + 2\text{Tr}(X_d) \left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)}) \right),$$

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$$\dot{I}_{ud}^{(1)} = 2(\gamma_u + \gamma_d) I_{ud}^{(1)},$$

$$\dot{I}_{ud,u}^{(2)} = (4\gamma_u + 2\gamma_d) I_{ud,u}^{(2)} + 3I_u' - 6I_{ud}^{(3)} - 2\text{Tr}(X_u X_d X_u) \left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)}) \right),$$

$$\dot{I}_{ud,d}^{(2)} = (4\gamma_d + 2\gamma_u) I_{ud,d}^{(2)} + 3I_d' - 6I_{ud}^{(3)} + 2\text{Tr}(X_d X_u X_d) \left(I_e^{(1)} + 3(I_d^{(1)} - I_u^{(1)}) \right),$$

$$\dot{I}_{ud}^{(3)} = 4(\gamma_u + \gamma_d) I_{ud}^{(3)},$$

$$\dot{I}_{ud}^{(4)} = 6 \left(\gamma_u + \gamma_d + \frac{1}{2} \text{Tr}(X_u + X_d) \right) I_{ud}^{(4)} - \text{ImTr}([X_u, X_d]^3) (I_u^{(1)} + I_d^{(1)}).$$

Similar

$$I_u^{(4)} = \text{Tr}(X_u) I_u^{(3)}$$

$$- \frac{1}{2} \left((\text{Tr} X_u)^2 - \text{Tr} X_u^2 \right) I_u^{(2)}$$

$$+ \frac{1}{6} \left((\text{Tr} X_u)^3 - 3 \text{Tr} X_u^2 \text{Tr} X_u + 2 \text{Tr} X_u^3 \right) I_u^{(1)}$$