



Higgs couplings and VBF at Muon Colliders

[\[A. Costantini et al. arXiv:2005.10289\]](#)

[\[M. Chiesa et al. arXiv:2003.13628\]](#)

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In collaboration with:

**F. Maltoni, R. Ruiz, A. Costantini, F. De Lillo,
X. Zhao, M. Chiesa, B. Mele, F. Piccinini**



European Research Council

Established by the European Commission

Snowmass contributions

- Muon collider physics summary [\[arXiv:2203.07256\]](#)
- The physics case of a 3 TeV muon collider stage [\[arXiv:2203.07261\]](#)

Higgs Physics and EFT interpretation

- Vector boson fusion at multi-TeV muon colliders [\[arXiv:2005.10289\]](#)
- Two Paths Towards Precision at a Very High Energy Lepton Collider [\[arXiv:2012.11555\]](#)
- Higgs Boson Studies at Future Particle Colliders [\[arXiv:1905.03764\]](#)
- Electroweak couplings of the Higgs boson at a multi-TeV muon collider [\[arXiv:2008.12204\]](#)
- ...

Dark Matter

- Wimps at high energy muon collider [\[arXiv:2009.11287\]](#)
- Hunting wino and higgsino dark matter at the muon collider [\[arXiv:2102.11292\]](#)
- ...

g-2 and B-anomalies

- Muon g-2 at multi-TeV muon collider [\[arXiv:2012.03928\]](#)
- Probing the RK anomaly at a muon collider [\[arXiv:2103.01617\]](#)
- ...

Hadron collider

QCD background

High energy

Initial state unknown

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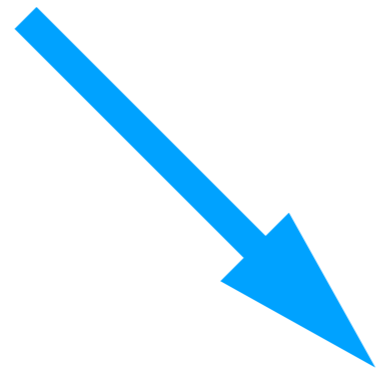
$e^+ e^-$ collider

Synchrotron radiation

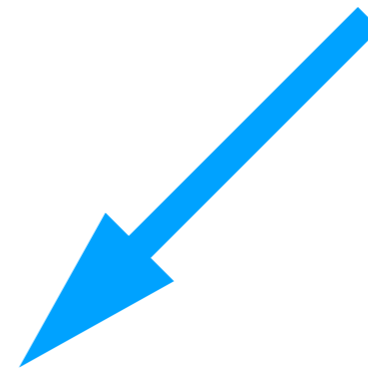
Precision

Known initial state

High energy



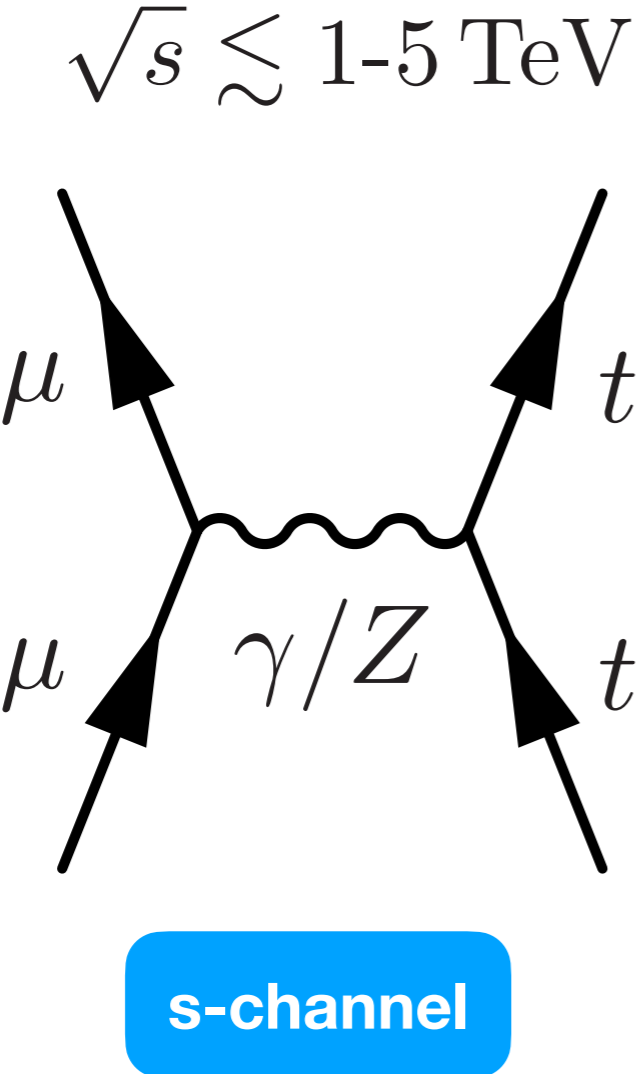
Precision



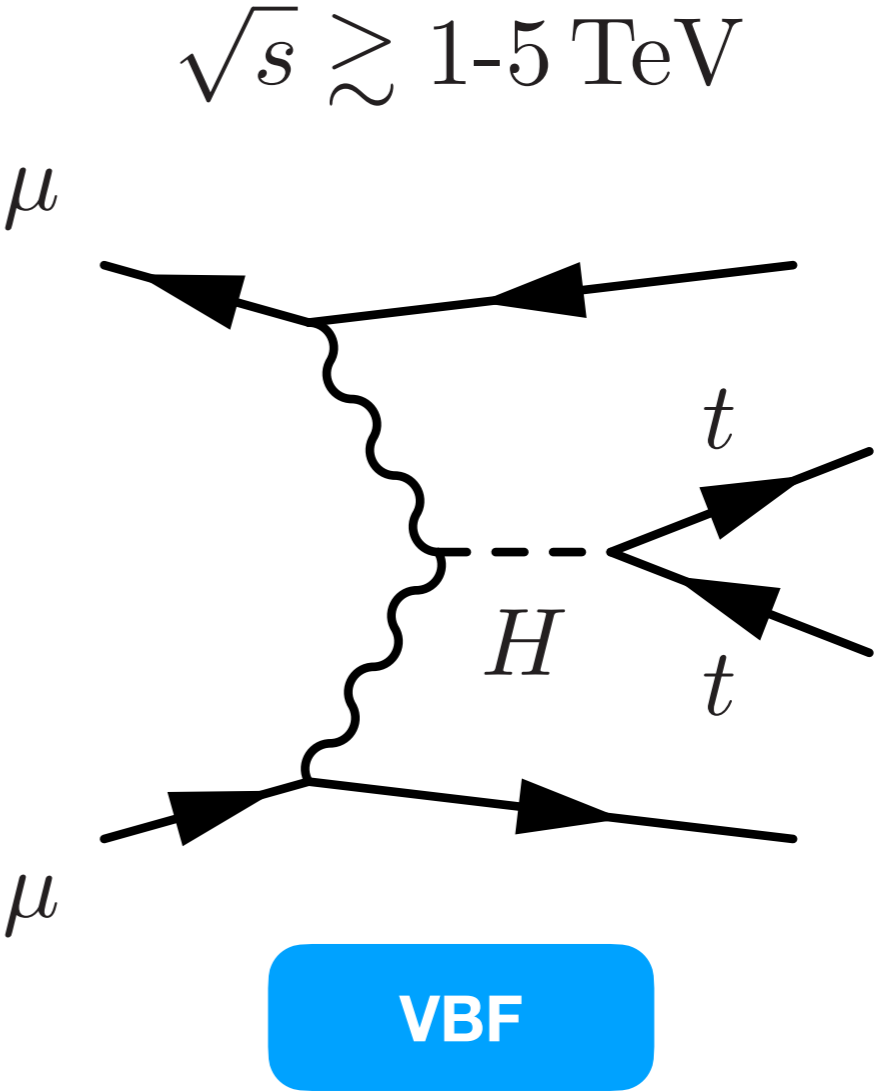
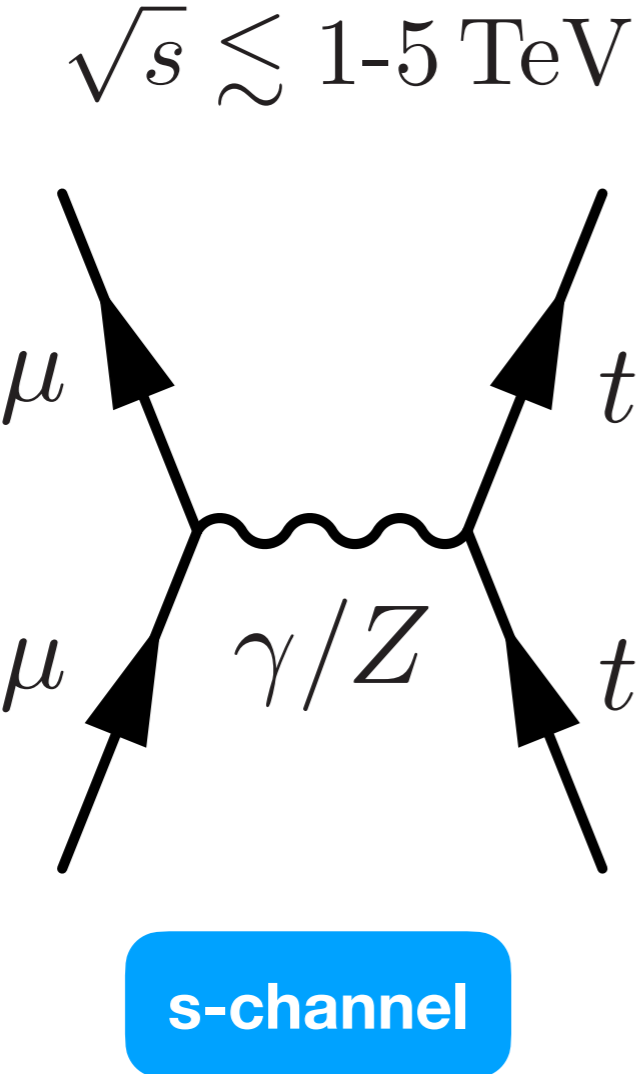
Muon collider

Different mode of production at different energies

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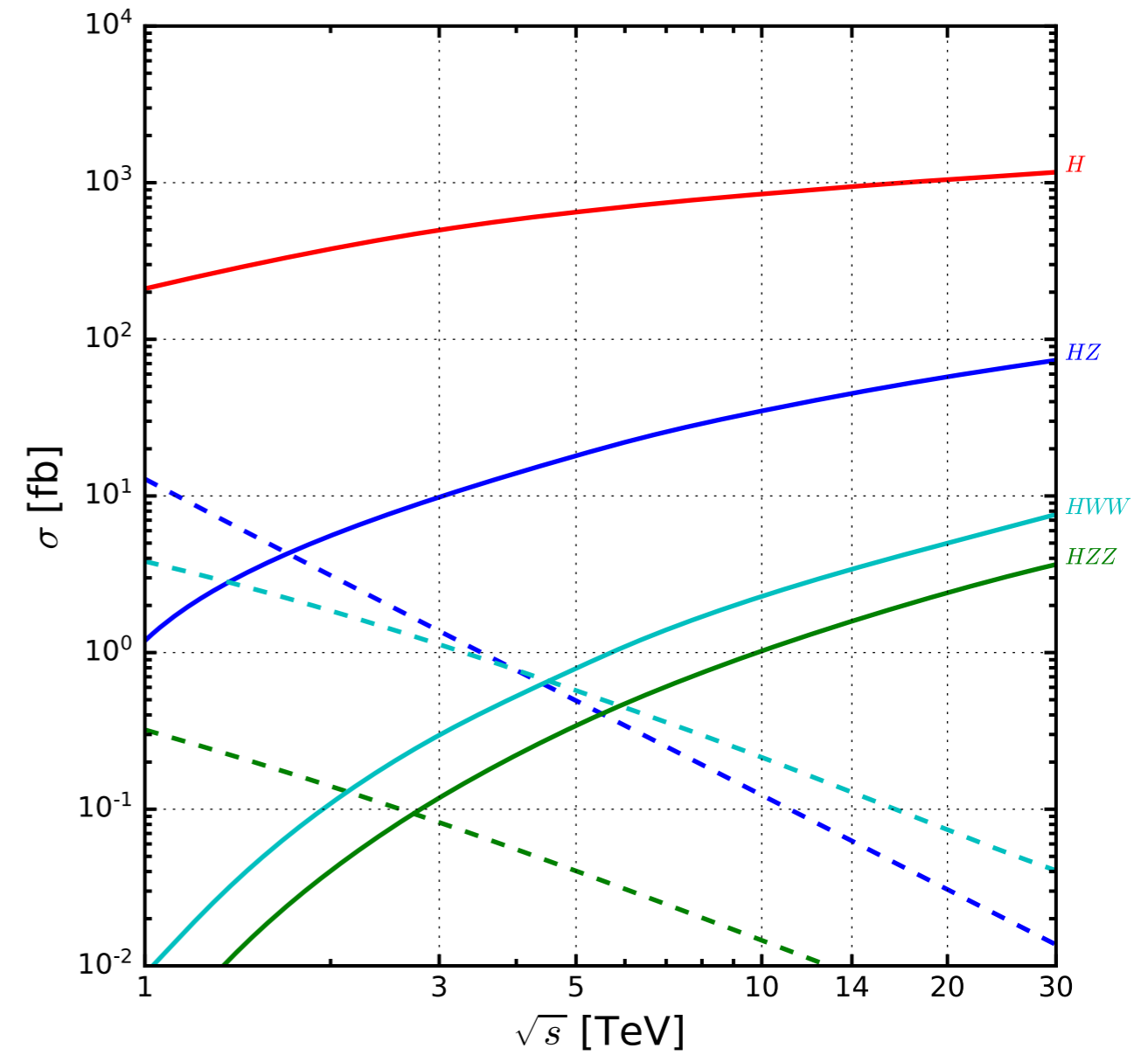
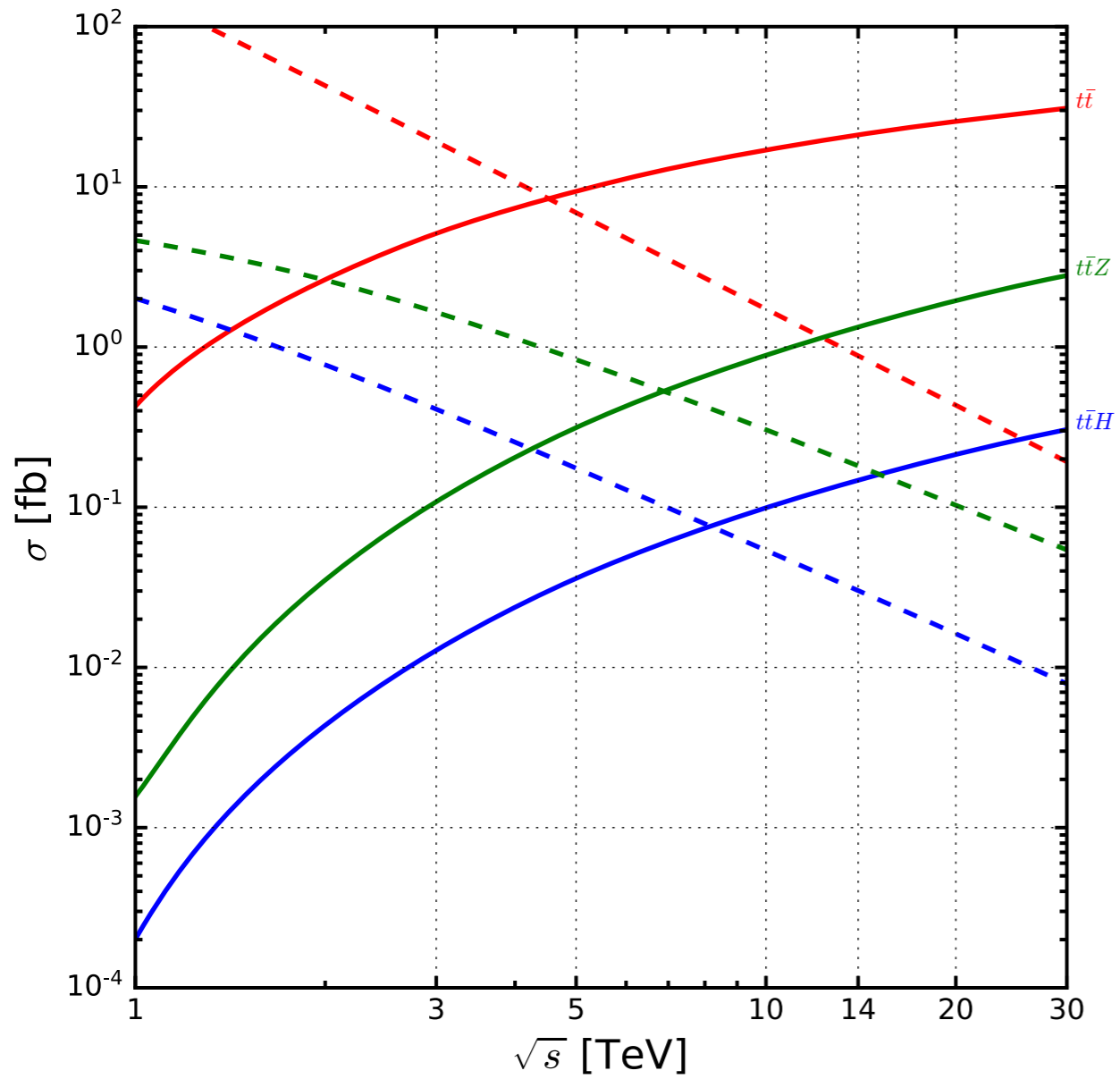


ttX

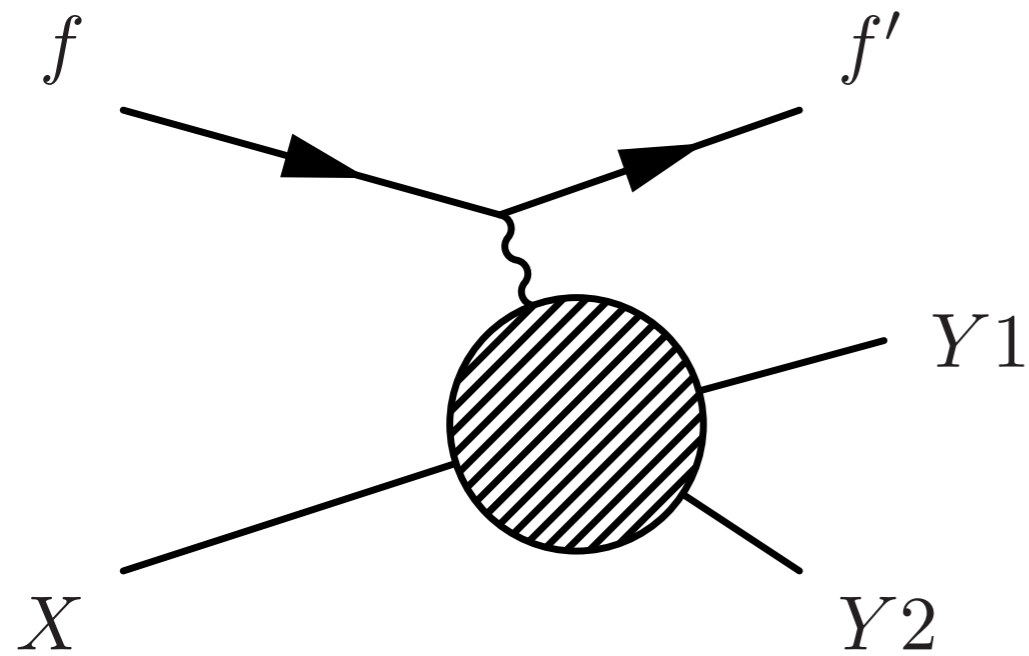
s-channel

VBF

HX



We can have an analytical insight with EWA



$$E \sim xE \sim (1-x)E, \quad \frac{m}{E} \ll 1, \quad \frac{p_{\perp}}{E} \ll 1$$

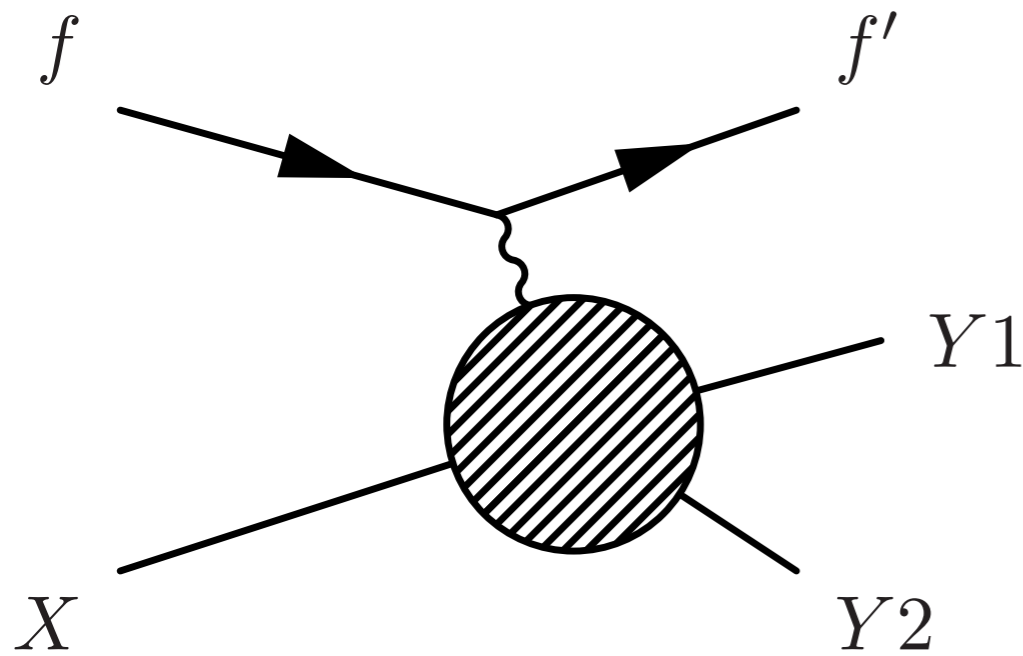
$$f_{+} = \frac{(1-x)^2}{x} \frac{p_{\perp}^3}{(m^2(1-x) + p_{\perp}^2)^2},$$

$$f_{-} = \frac{1}{x} \frac{p_{\perp}^3}{(m^2(1-x) + p_{\perp}^2)^2},$$

$$f_0 = \frac{(1-x)^2}{x} \frac{2m^2 p_{\perp}}{(m^2(1-x) + p_{\perp}^2)^2}.$$

[R. Ruiz et al. arXiv:2111.02442] [P. Borel et al. arXiv:1202.1904]

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$$\frac{d\sigma_{EWA}}{dx dp_{\perp}} (fX \rightarrow f'Y) = \frac{C^2}{2\pi^2} \sum_{i=+,-,0} f_i \times d\sigma(W_i X \rightarrow Y)$$

Weak bosons can be described as partons!

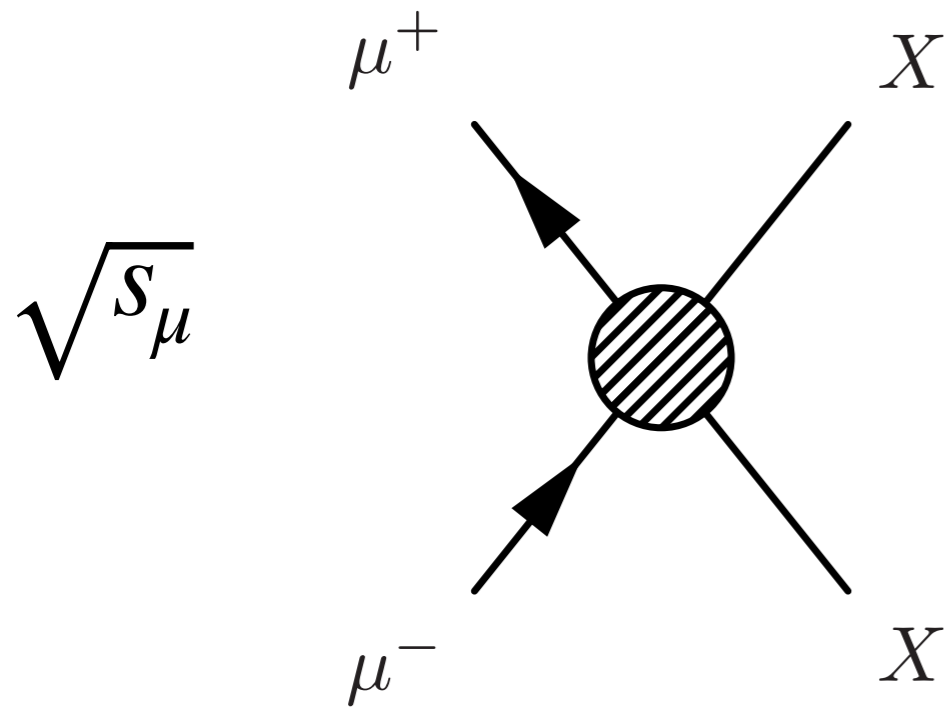
Discovery potential

Pair production

$$M_X = 0.9 \times \sqrt{s_\mu} / 2$$

Pair production

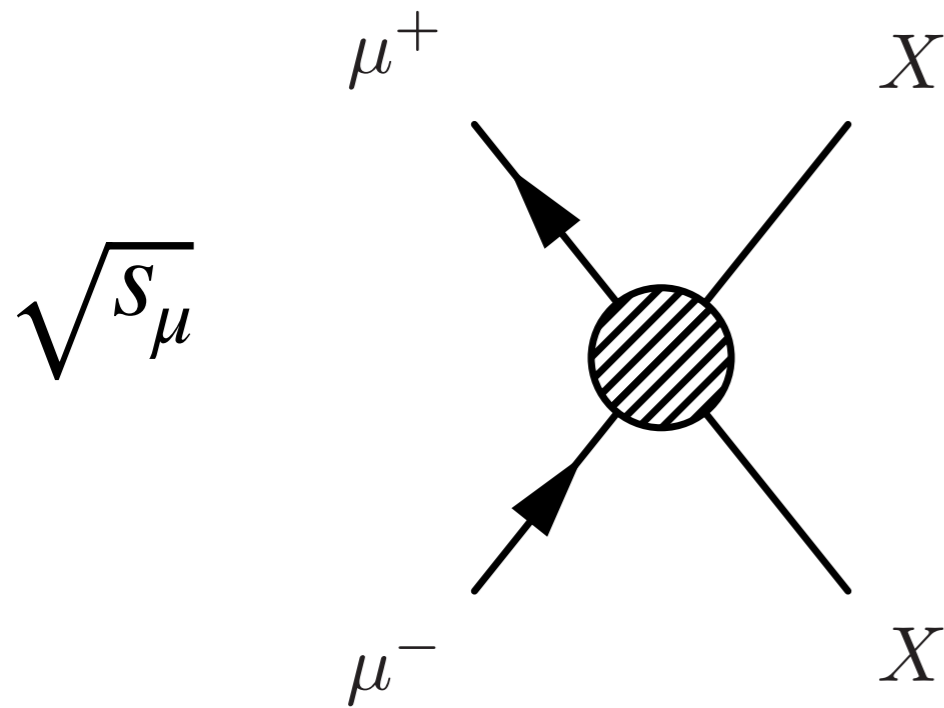
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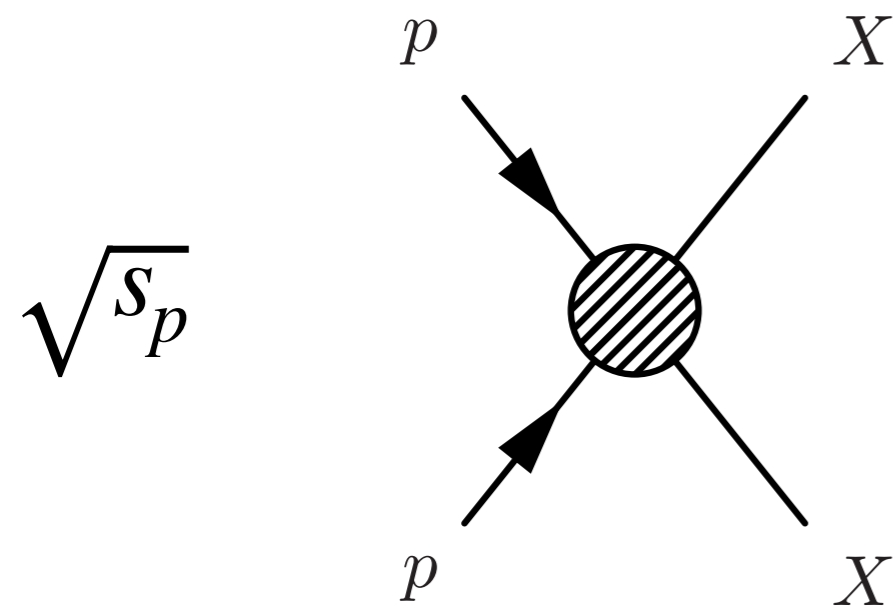
$$\sigma_\mu(s_\mu) = \frac{1}{s_\mu} [\hat{\sigma}\hat{S}]_\mu$$

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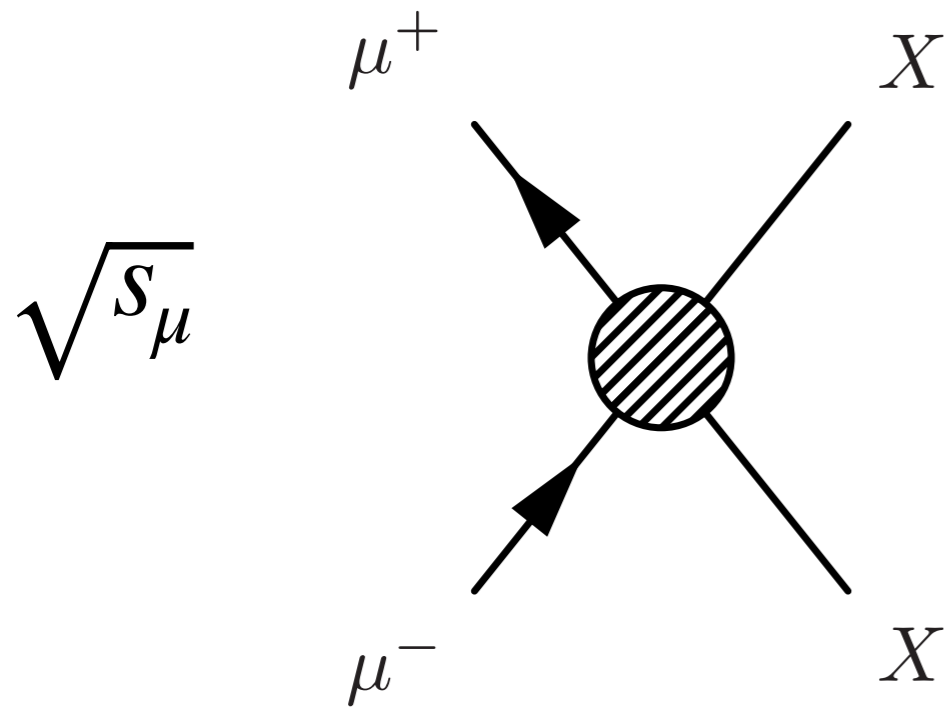
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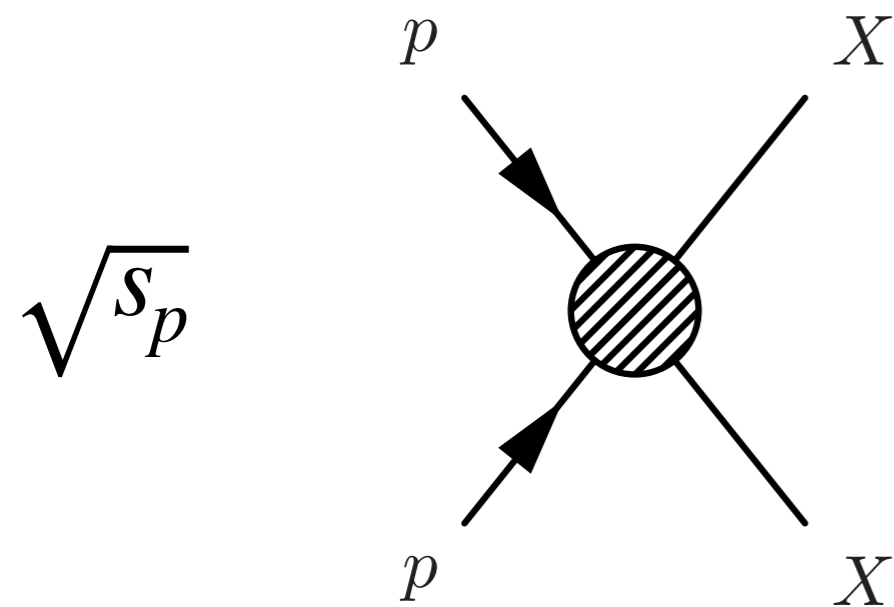
$$\sigma_p(s_p) = \frac{1}{s_p} \int_{\tau_0}^1 d\tau \frac{1}{\tau} \sum_{ij} \Phi_{ij}(\tau, \mu_f) [\hat{\sigma}_{ij} \hat{s}]_p$$

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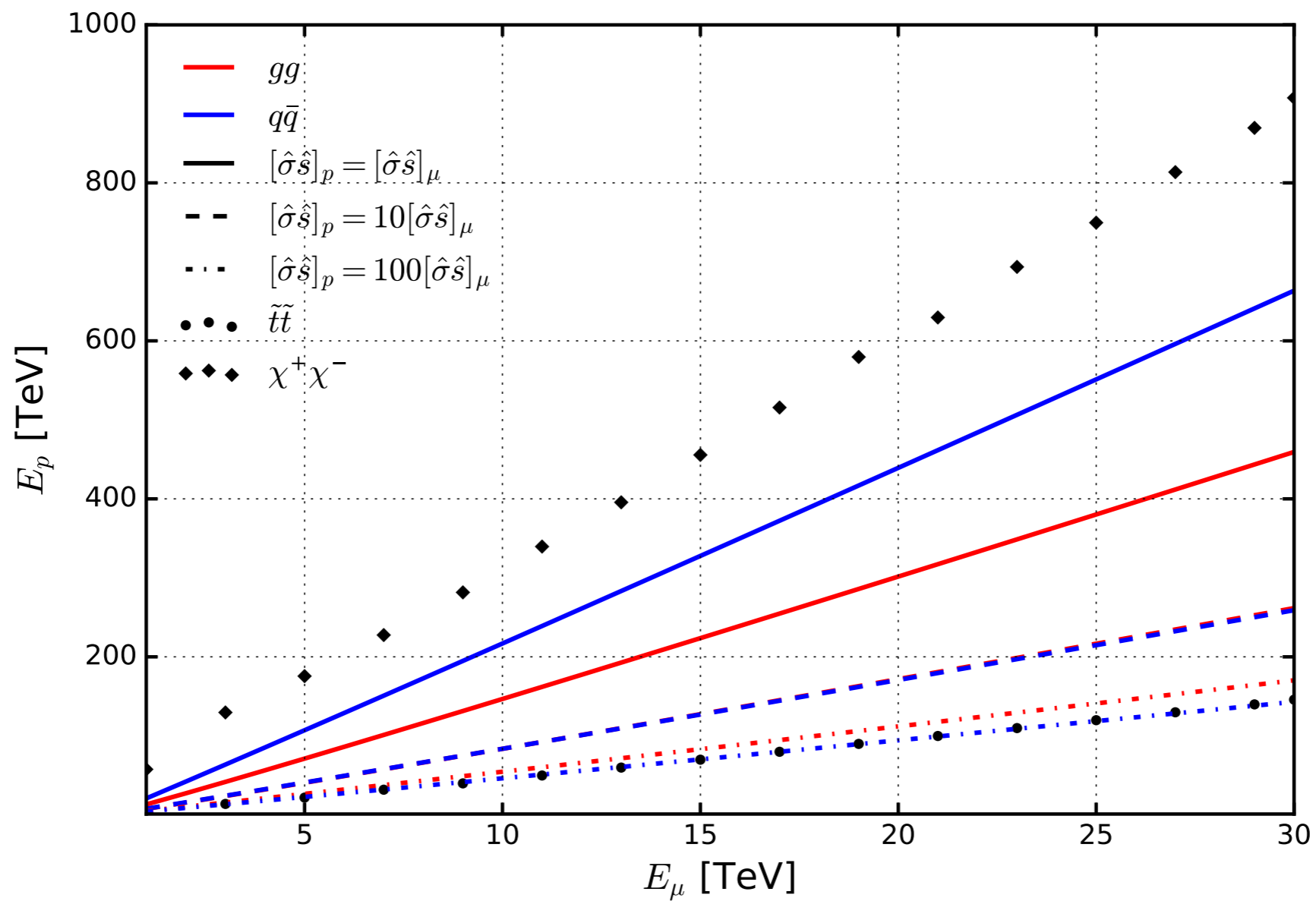


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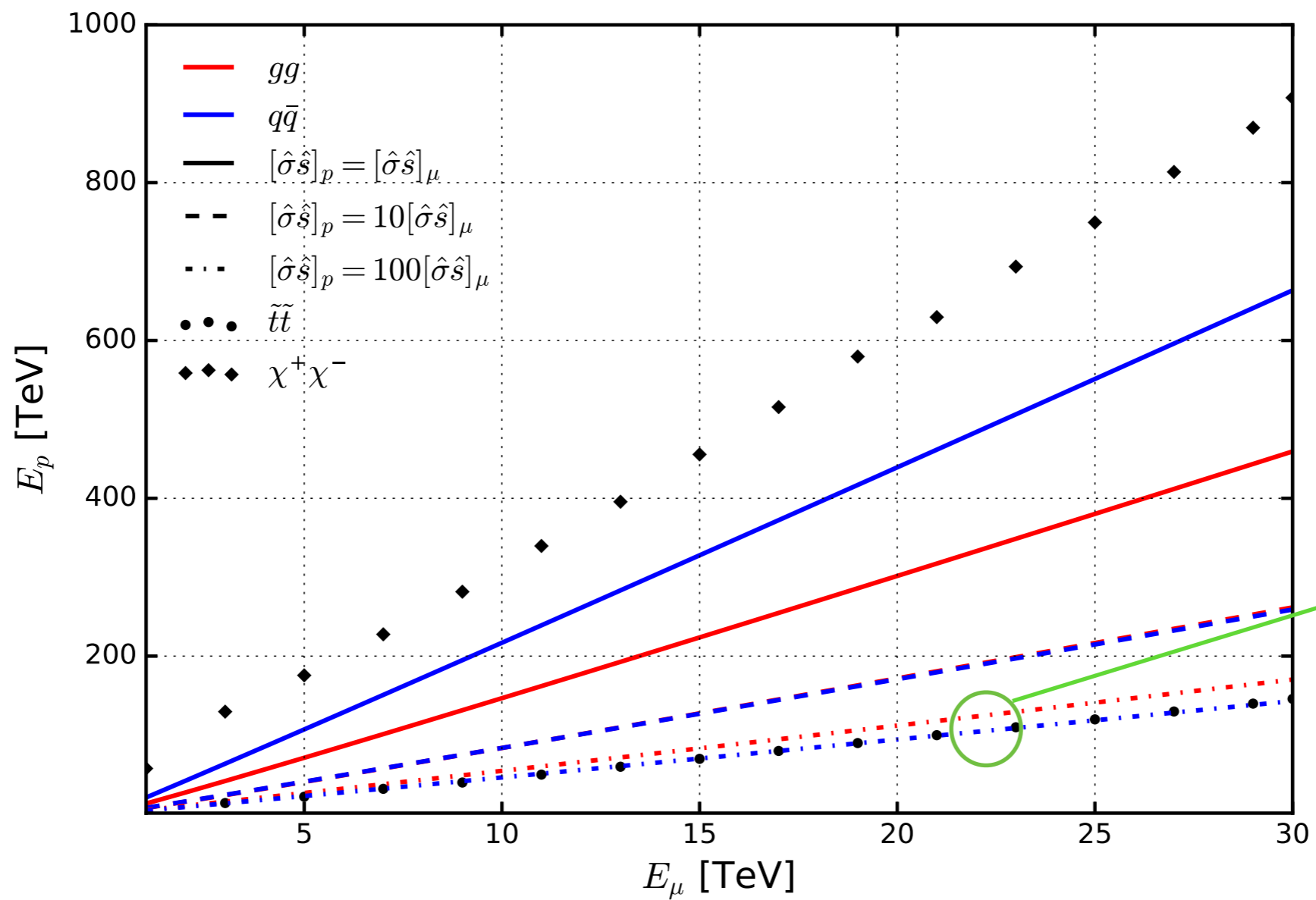
$$\sigma_\mu(s_\mu) = \sigma_p(s_p)$$

$$\frac{s_\mu}{s_p} \int_{\frac{s_\mu}{s_p}}^1 d\tau \frac{1}{\tau} \sum_{ij} \Phi_{ij} \left(\tau, \frac{\sqrt{s_\mu}}{2} \right) = \frac{[\hat{\sigma}\hat{S}]_\mu}{[\hat{\sigma}\hat{S}]_p} \equiv \frac{1}{\beta}$$

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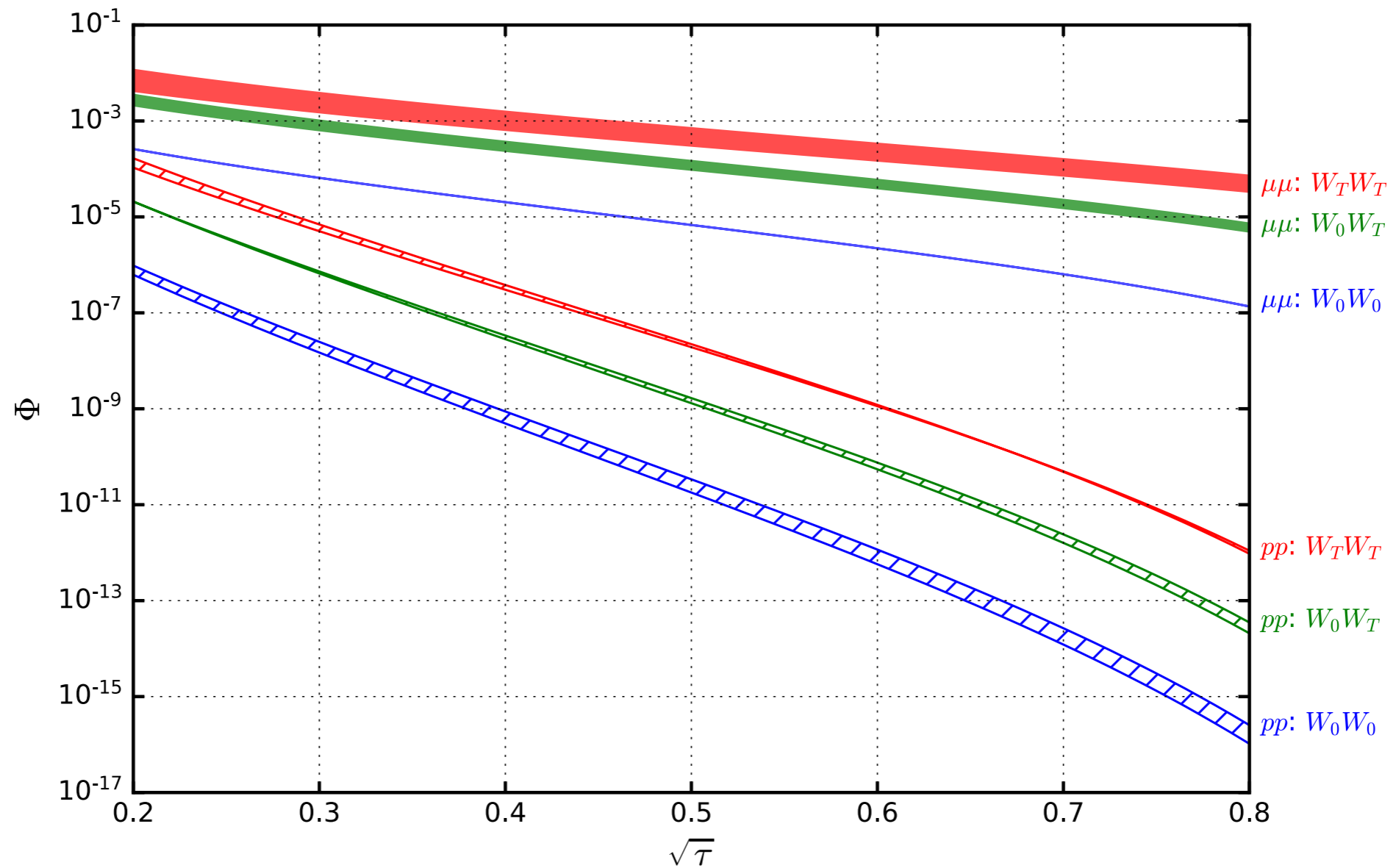


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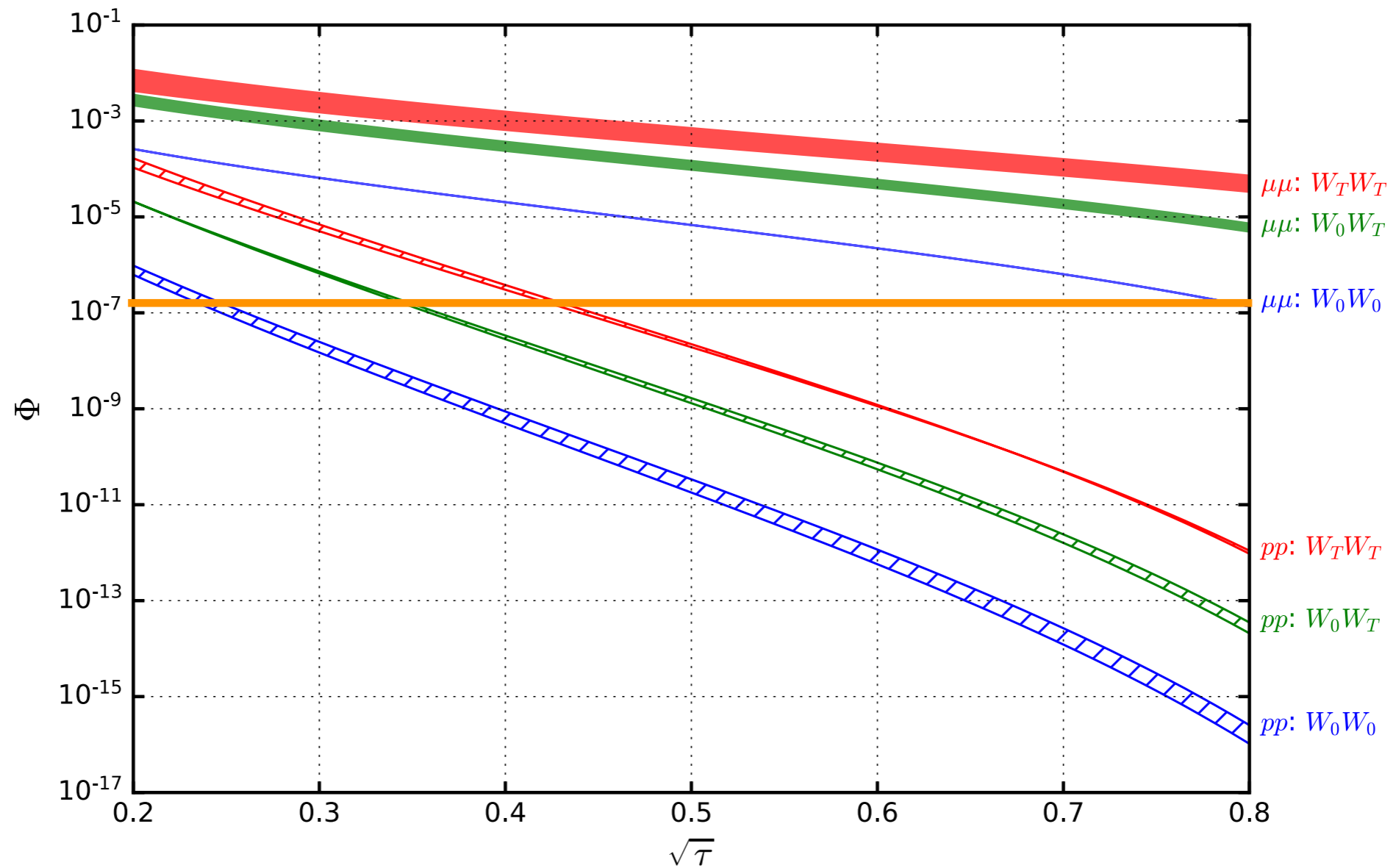


Stop prod
well described

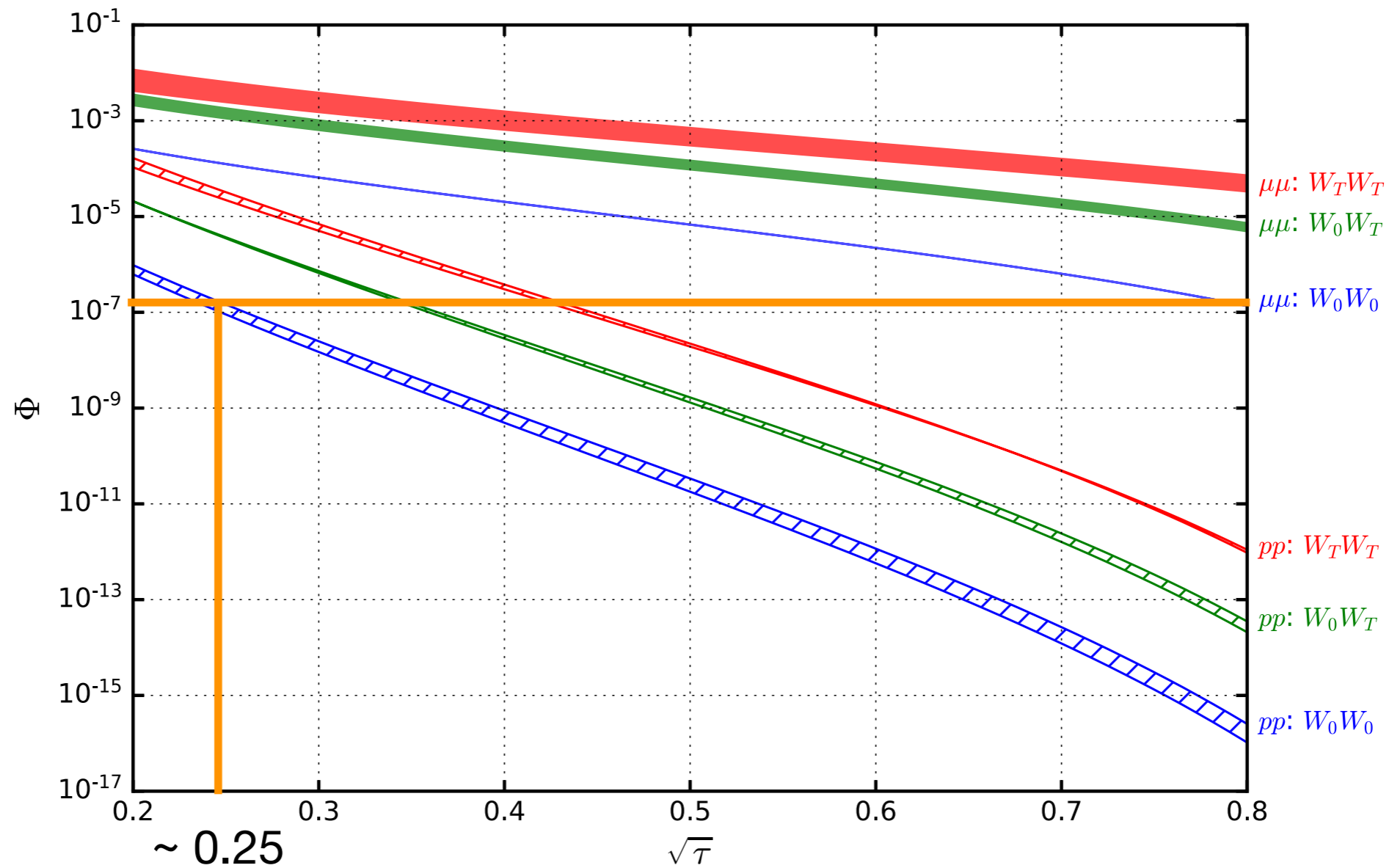
$$\Phi_{W_{\lambda_1}^+ W_{\lambda_2}^-}(\tau, \mu_f) = \int_{\tau}^1 \frac{d\xi}{\xi} f_{W_{\lambda_1}/\mu}(\xi, \mu_f) f_{W_{\lambda_2}/\mu}\left(\frac{\tau}{\xi}, \mu_f\right)$$



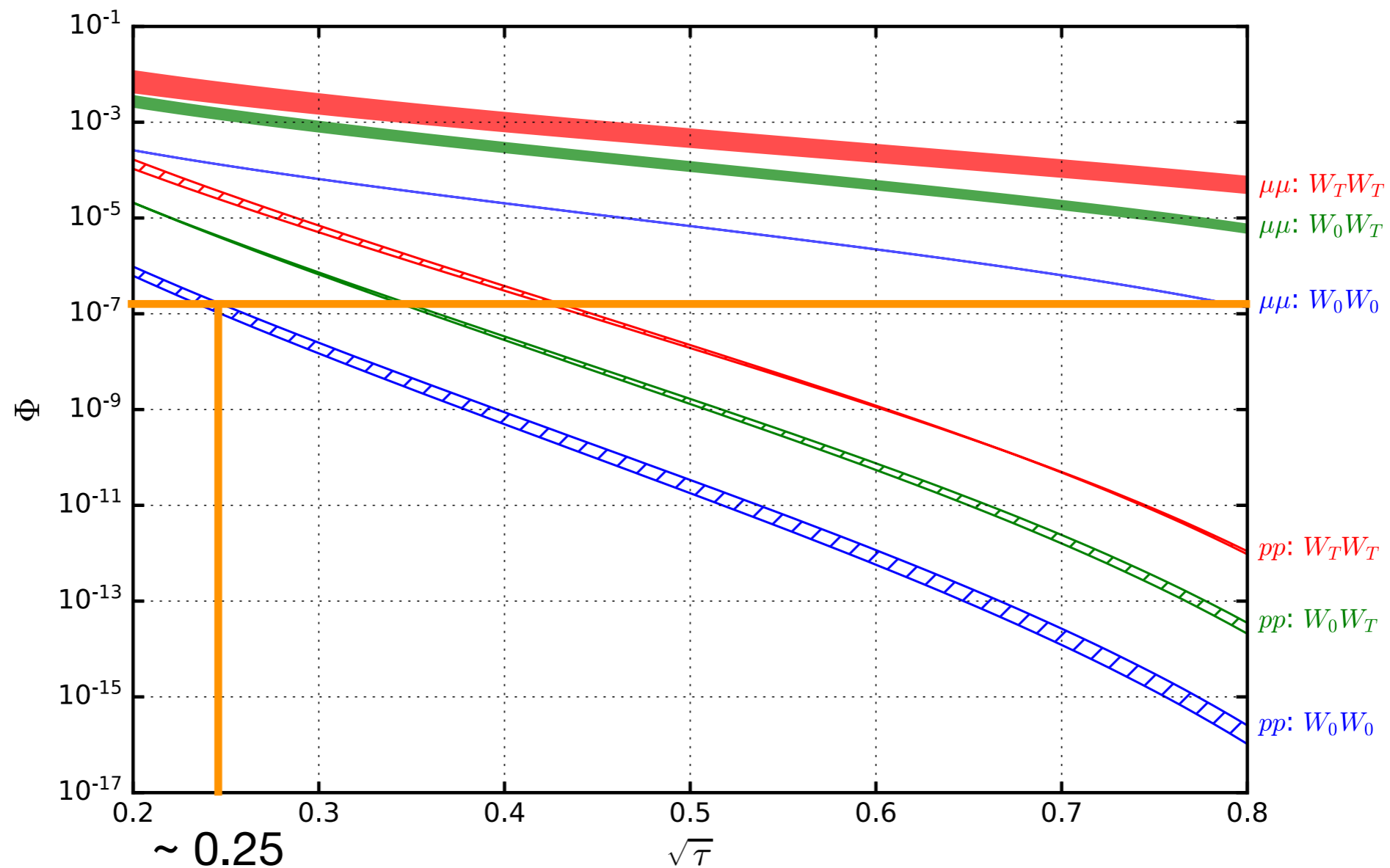
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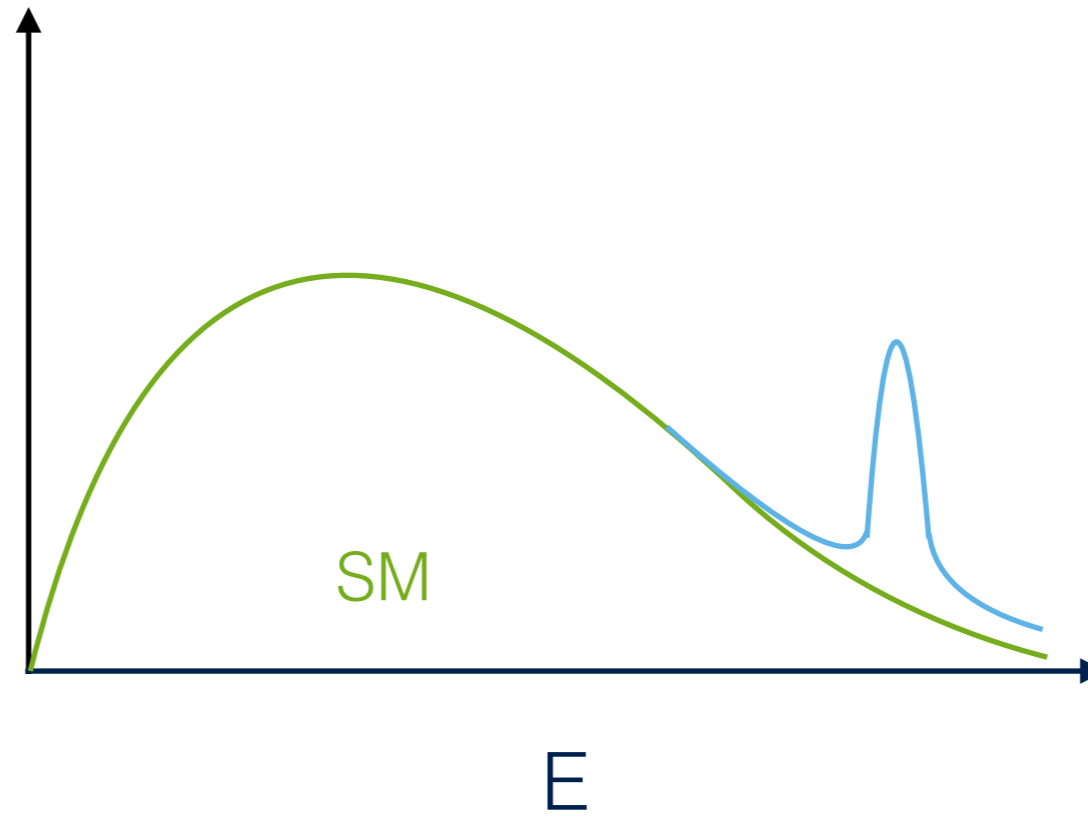
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0.8/0.25 ~ 3.5
Muon 14 TeV ~ Proton 50 TeV

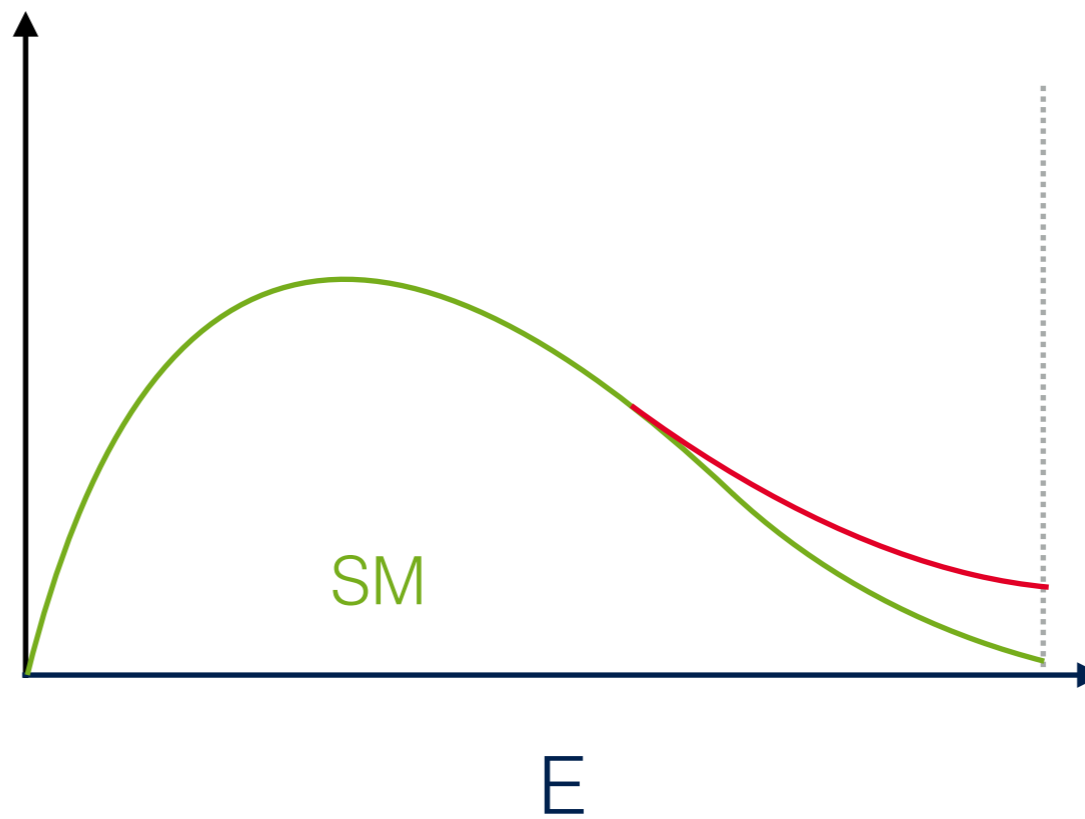
Precision potential

Direct search (Bumps)



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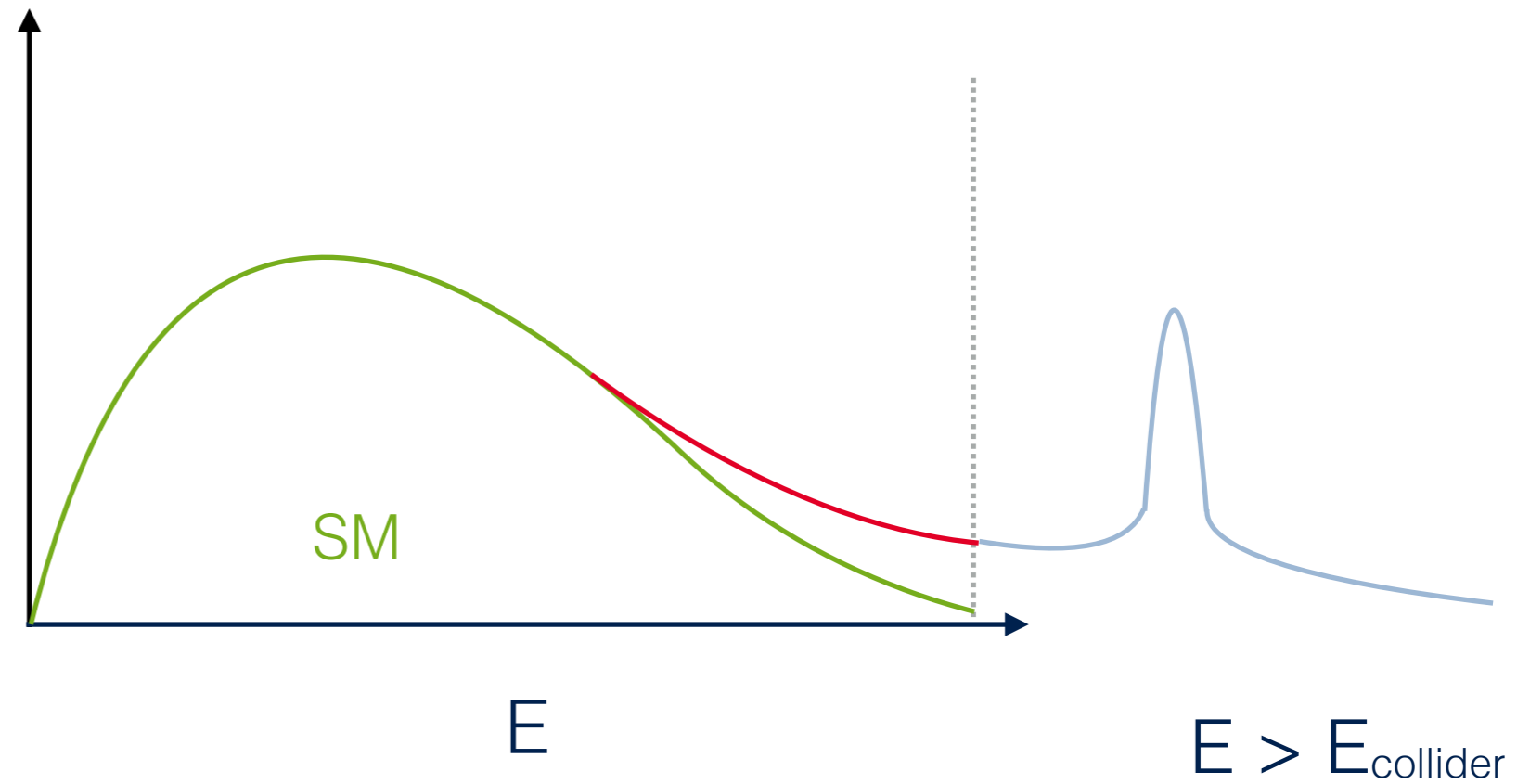
Indirect (scouting tails)



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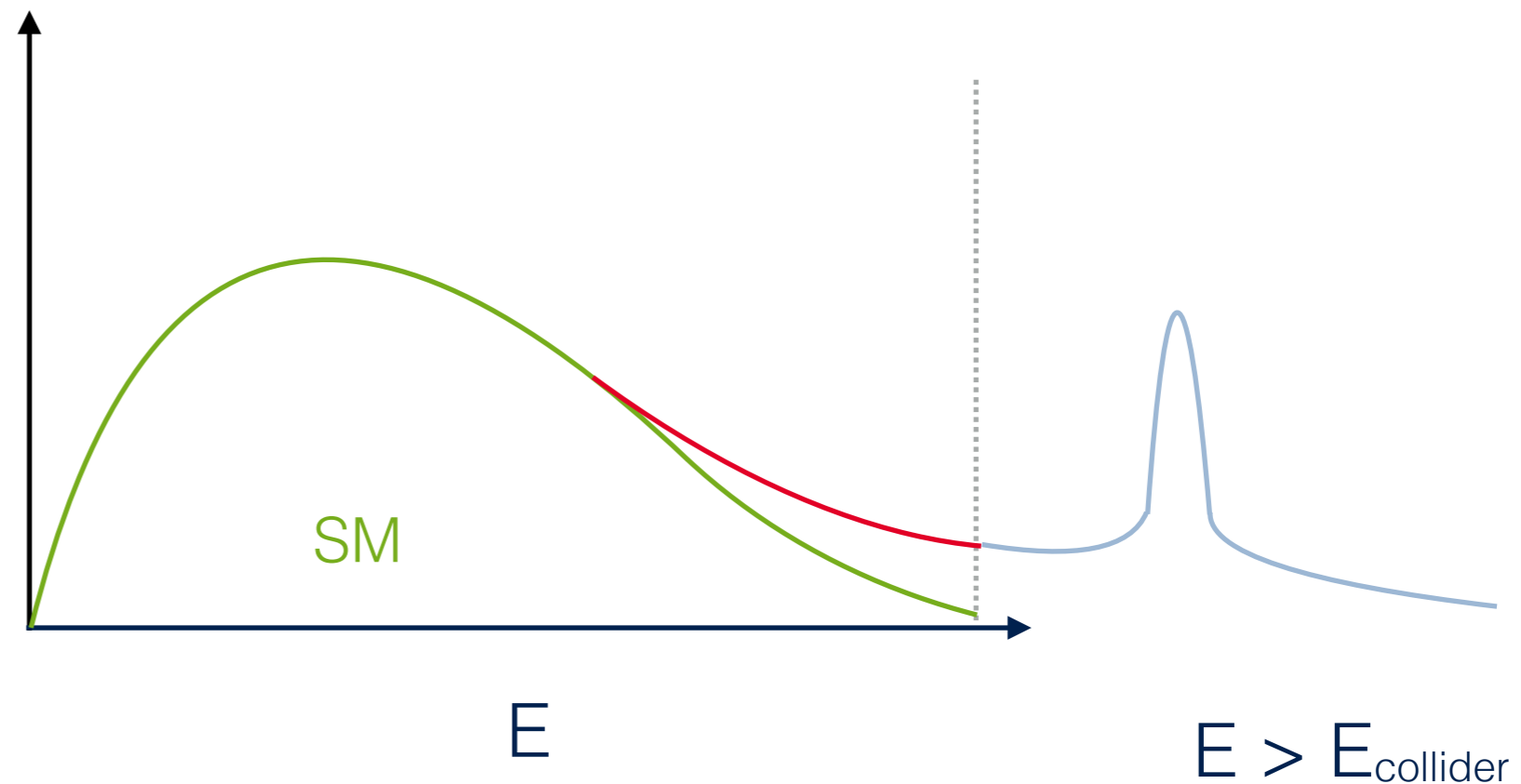
⇒ New physics is heavy



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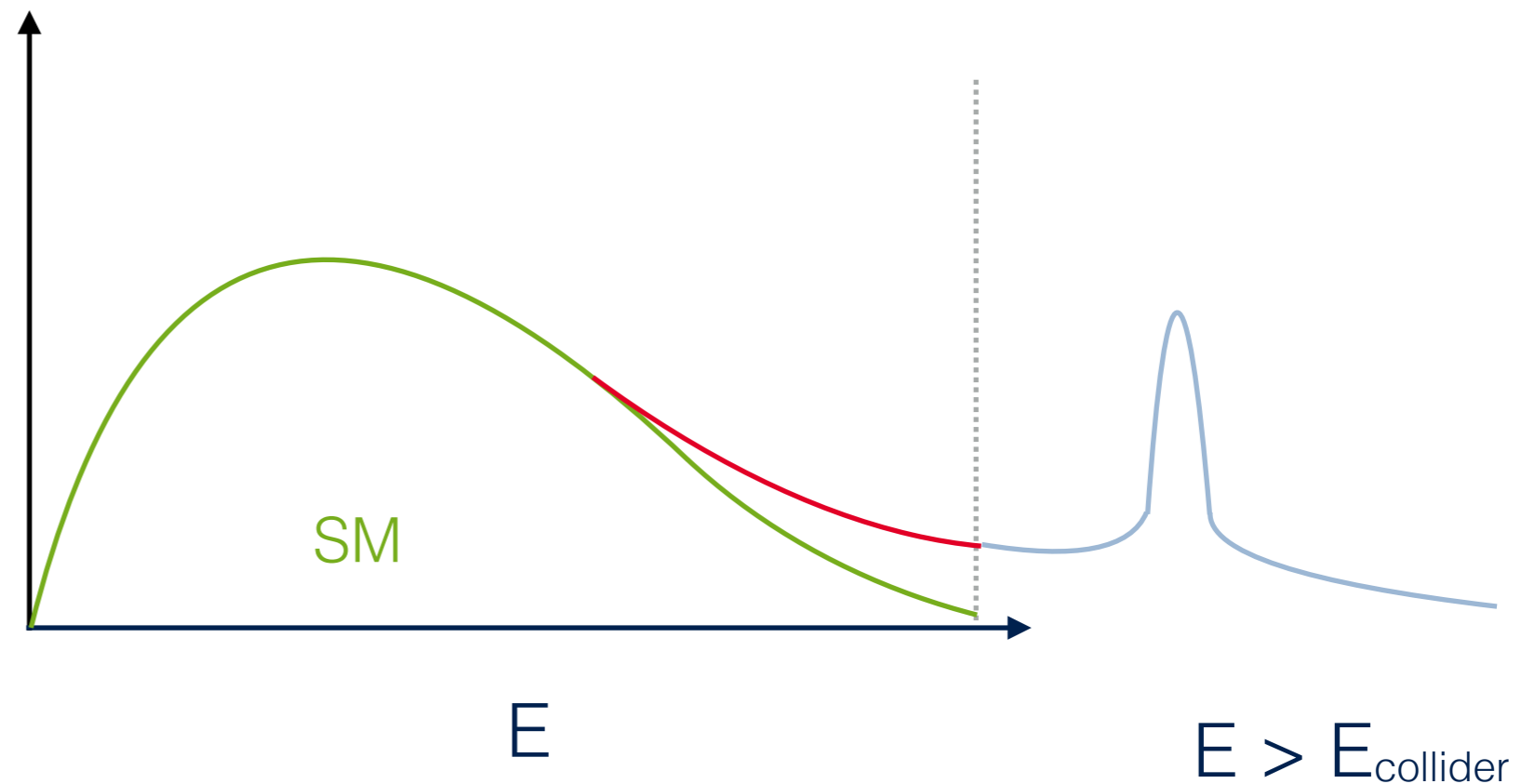


Important to assess the potential of a muon collider in indirect searches

Direct search (Bumps)

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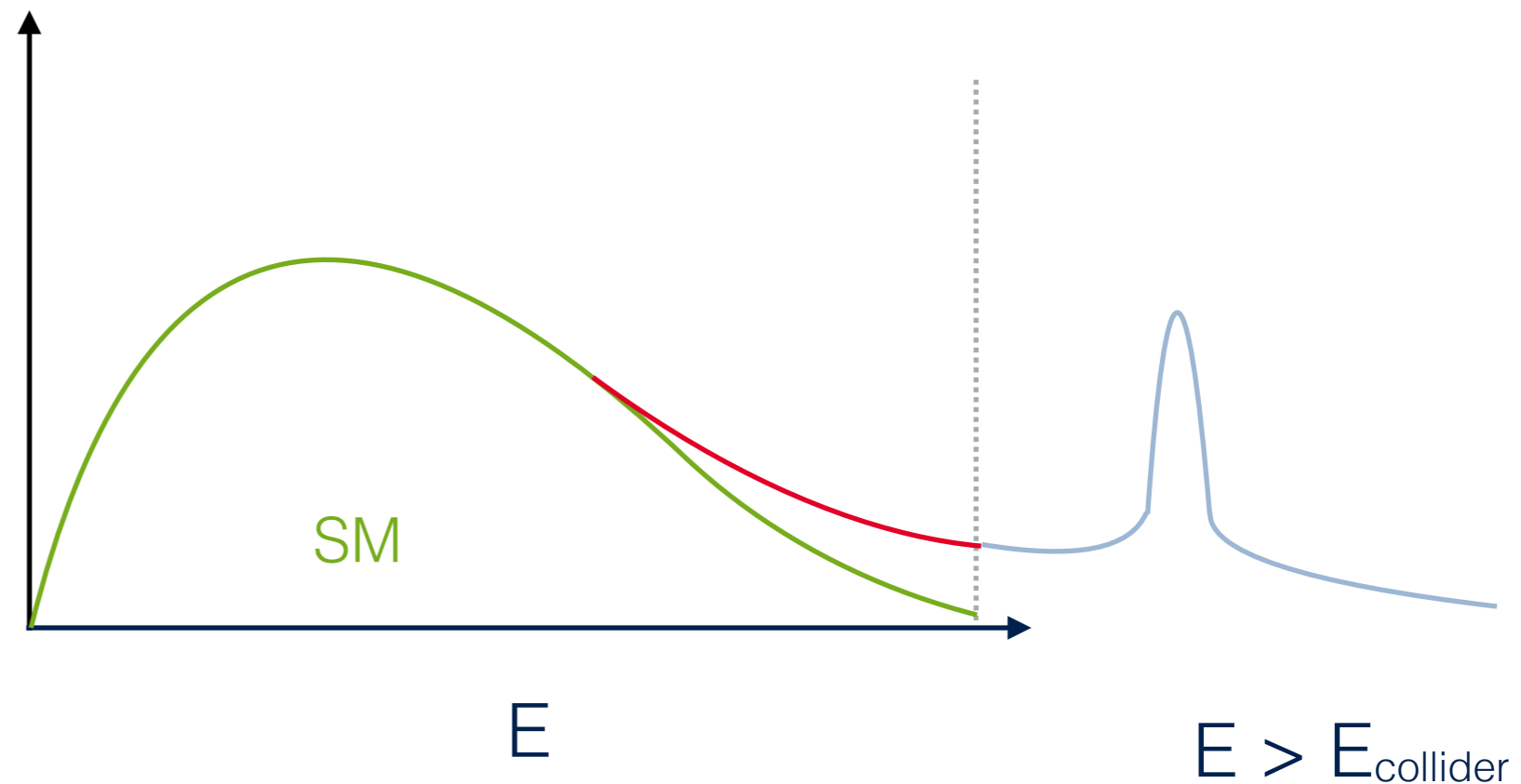
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New physics effects can lead to **unitarity violating behaviours** at high energy

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High energy tails

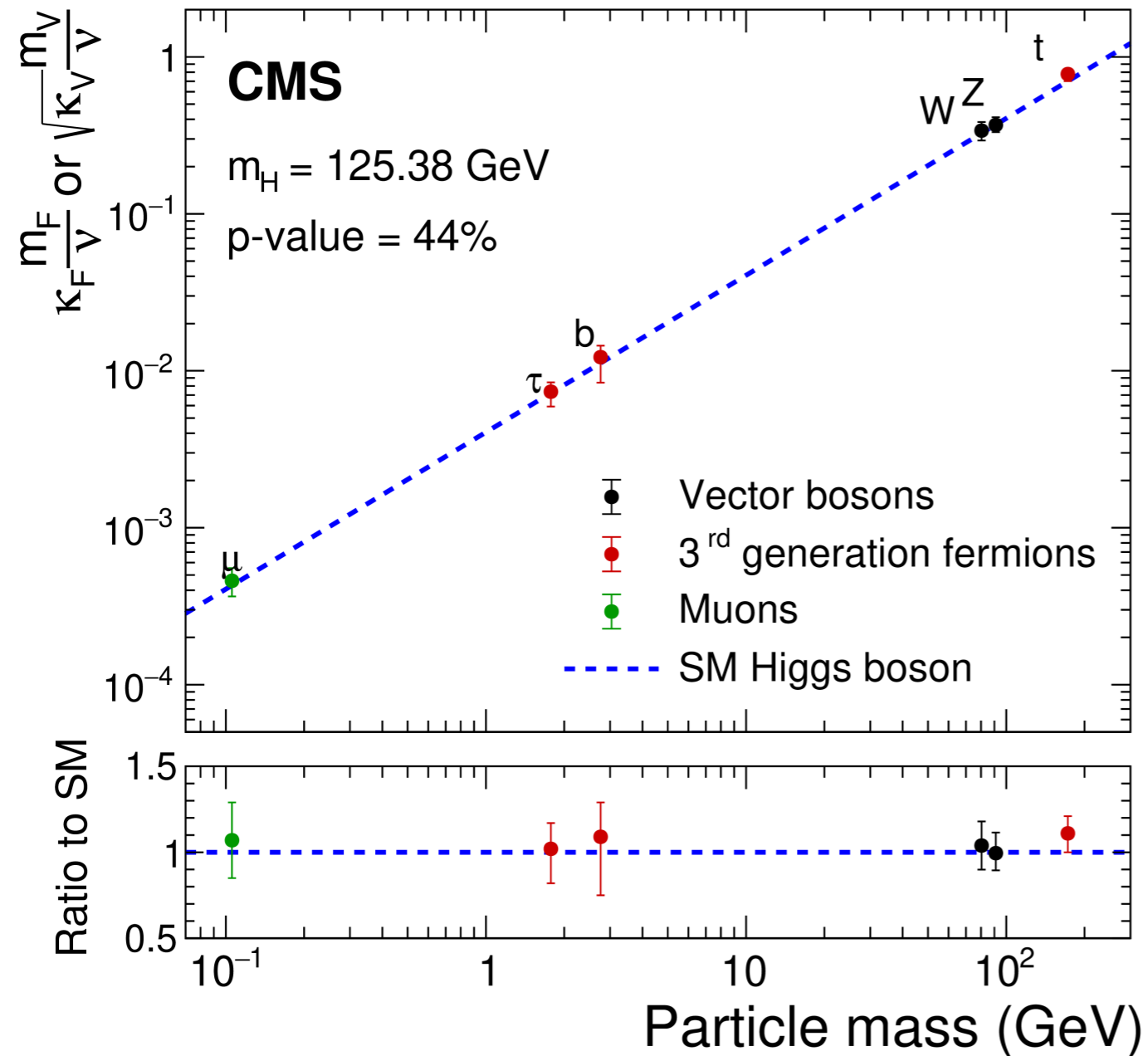


Less statistics, more sensitivity

The LHC has found a scalar particle that behaves like the SM Higgs.

It couples to masses.

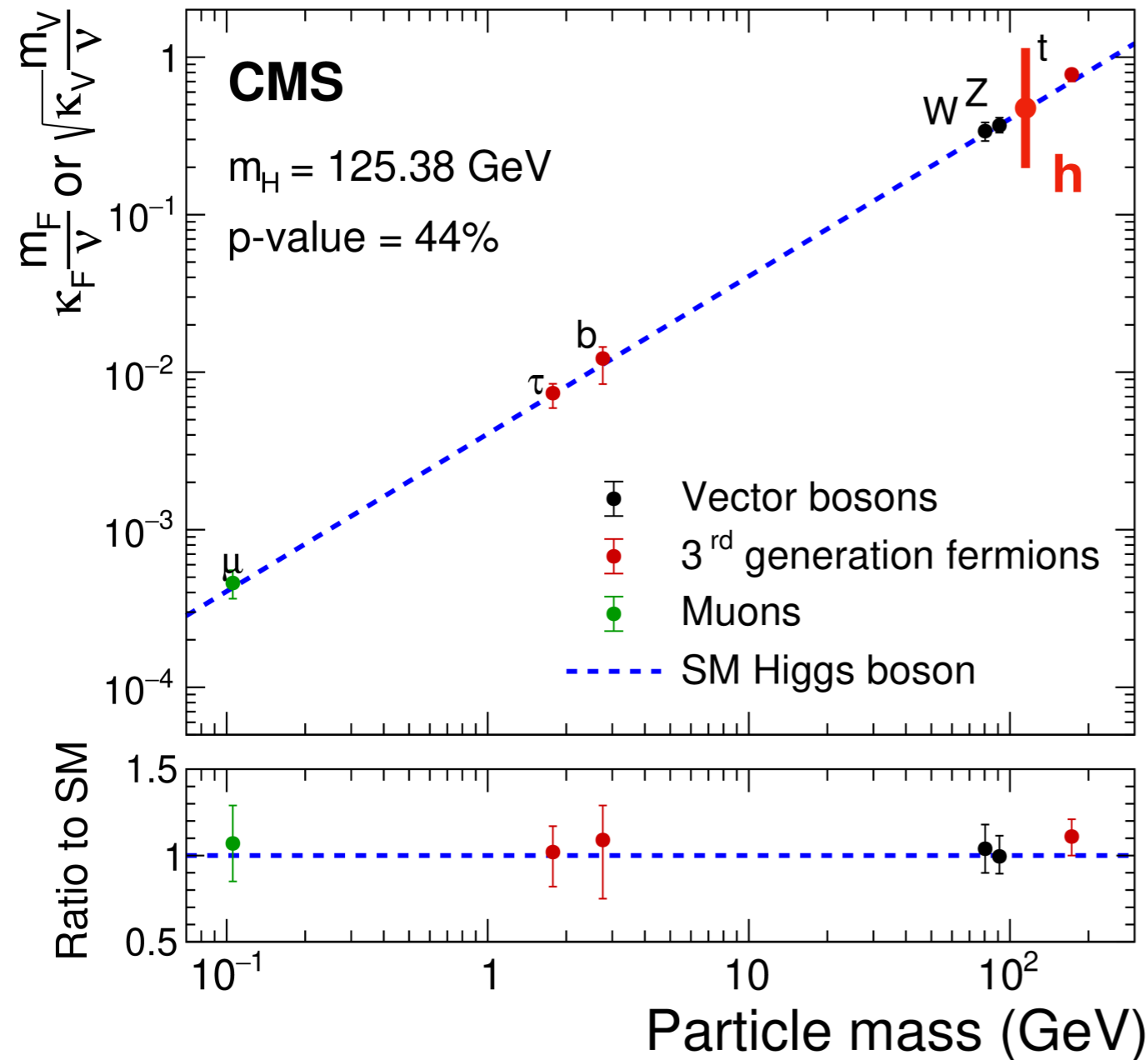
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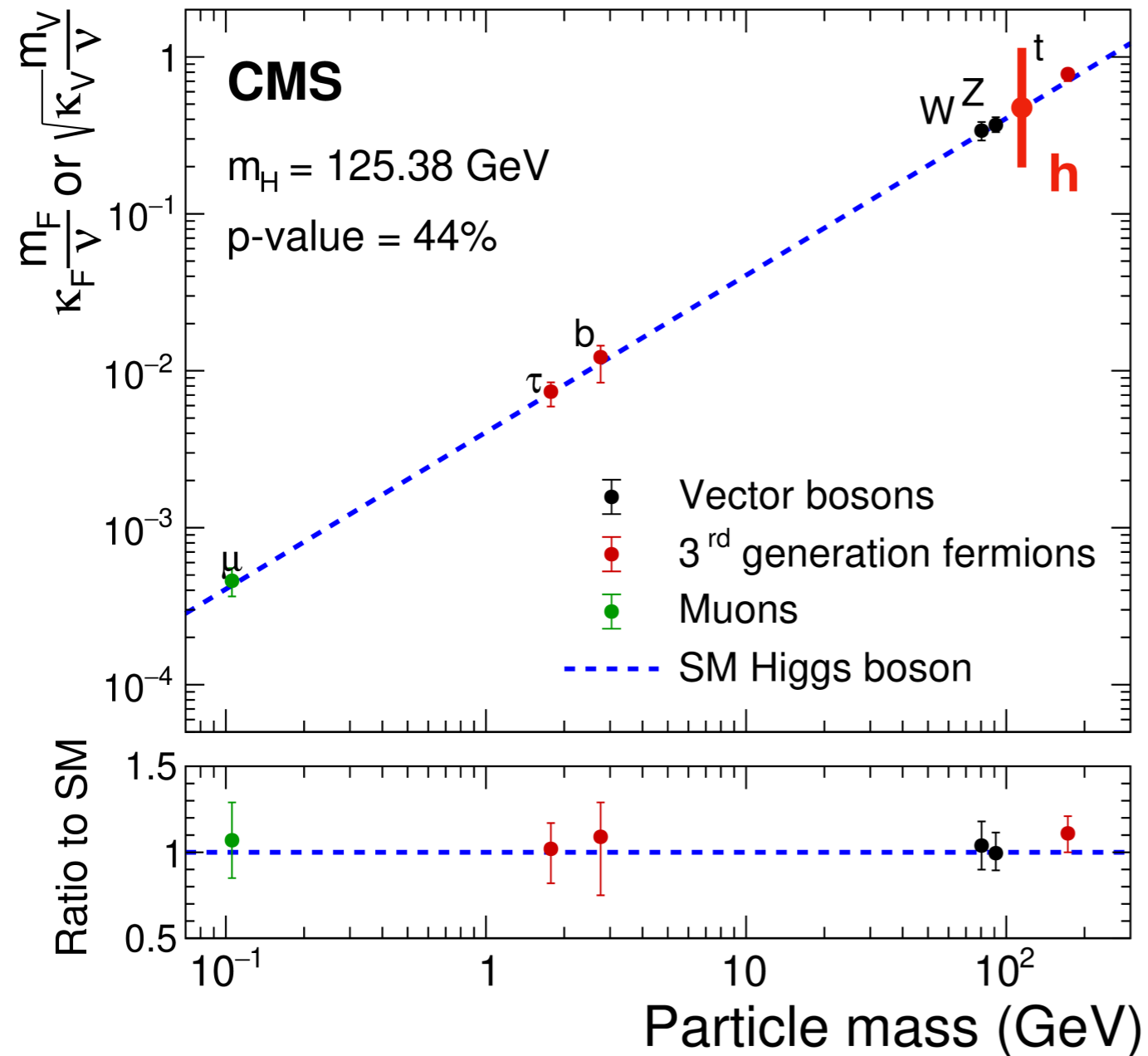


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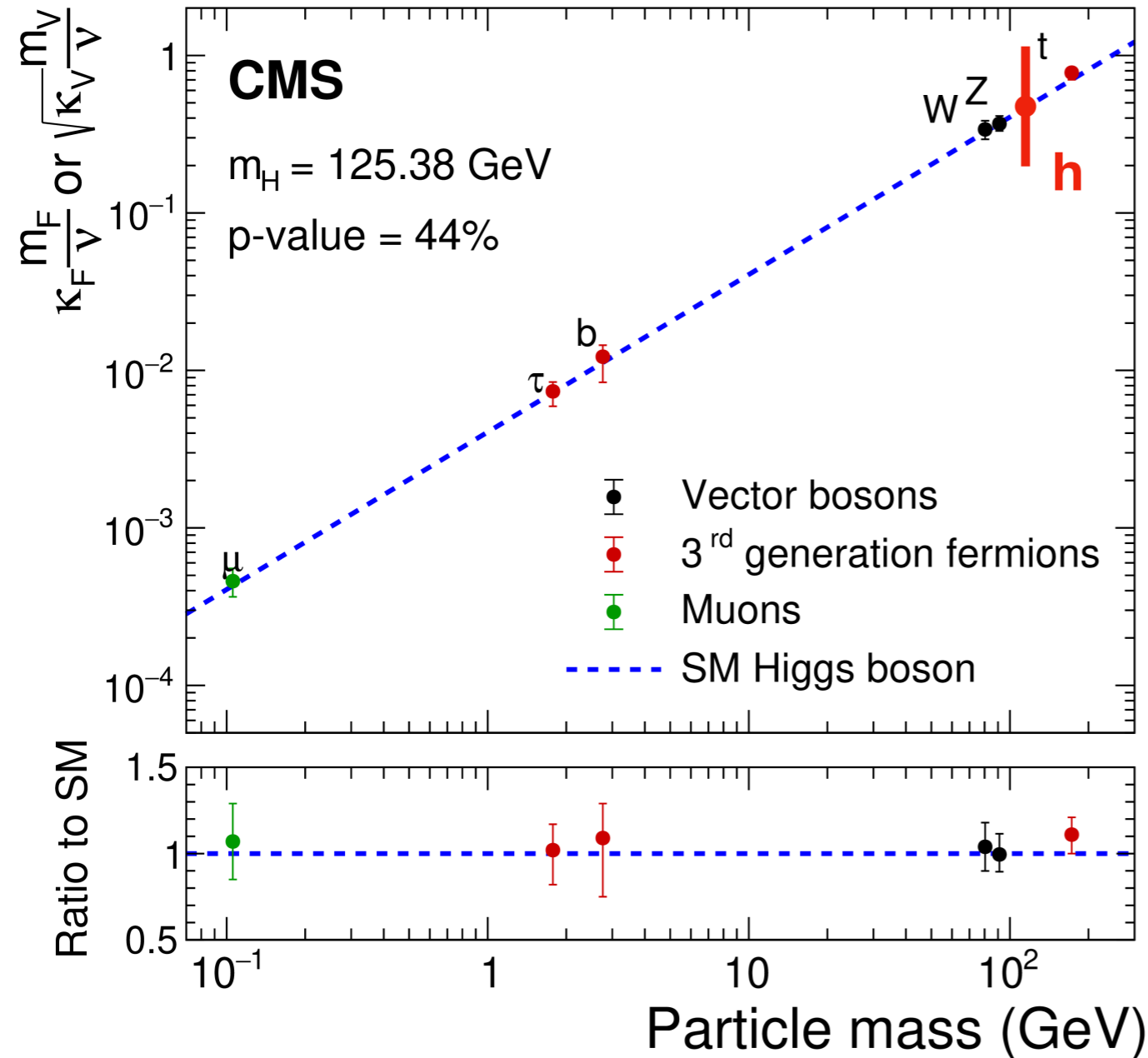
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Is a low energy lepton collider enough?

$$V(H) = \frac{1}{2}m_H^2 H^2 + \lambda_3 v H^3 + \frac{1}{4}\lambda_4 H^4$$

$$\lambda_3 = \lambda_4 = m_H^2/2v^2 \equiv \lambda_{SM}$$

[A. Abada et al.
Eur. Phys. J. C 79 (2019) no.6, 474]

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**HH and radiative corrections to single Higgs
FCC ~ 5%**

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Challenging even at FCC!

$$\text{ILC} \sim [-10, 10]$$

$$\text{CLIC} \sim [-5, 5]$$

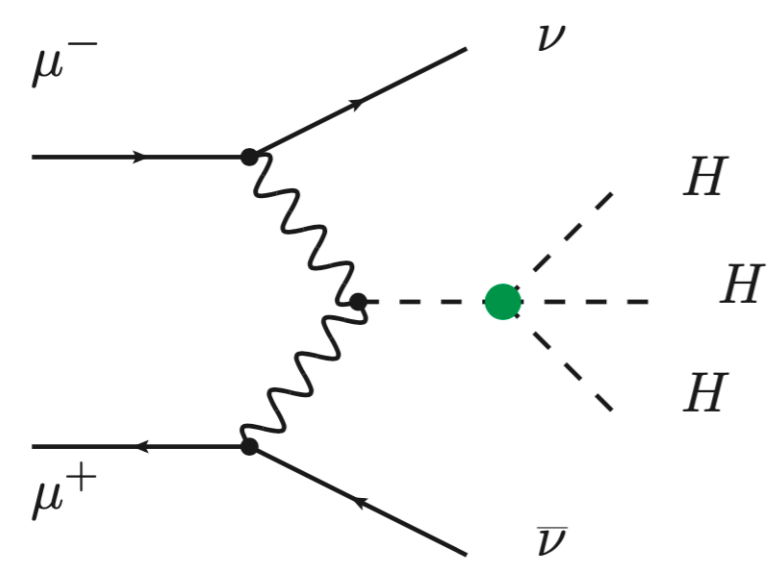
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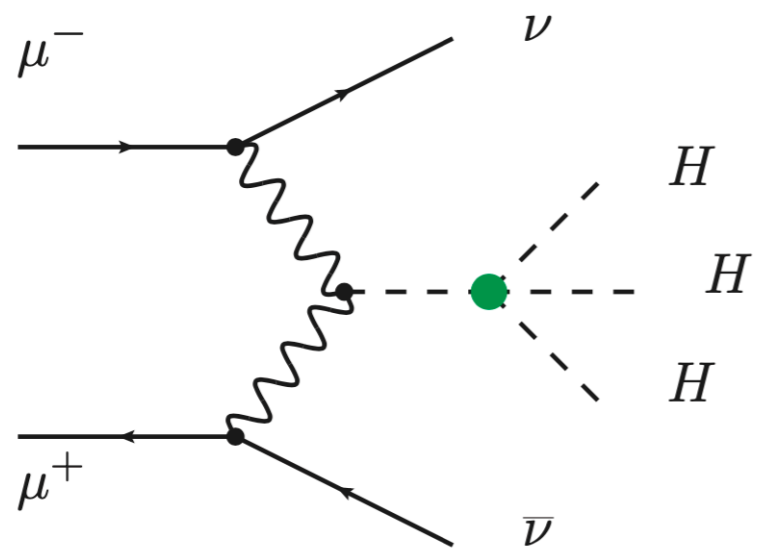
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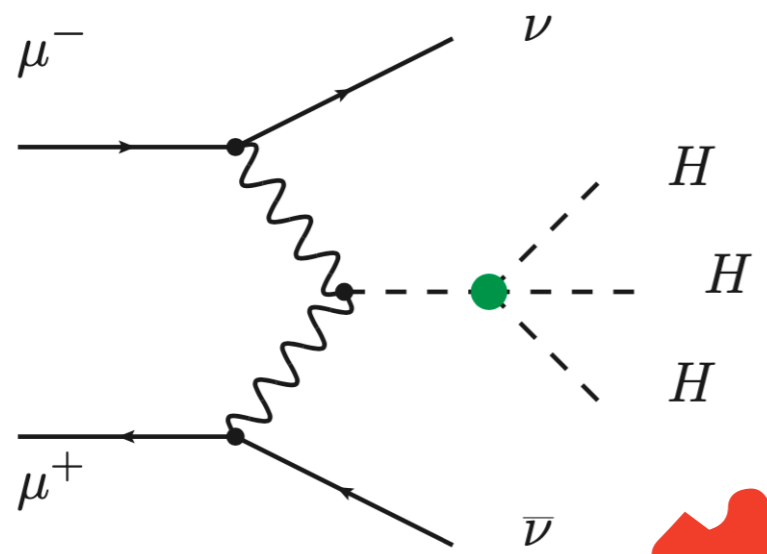
| \sqrt{s} (TeV) / L (ab ⁻¹) | 1.5 / 1.2 | 3 / 4.4 | 6 / 12 | 10 / 20 | 14 / 33 | 30 / 100 |
|------------------------------------------|-----------|----------|-----------|-----------|------------|--------------|
| σ_{SM} (ab) [N_{ev}] | | | | | | |
| σ^{tot} | 0.03 [0] | 0.31 [1] | 1.65 [20] | 4.18 [84] | 7.02 [232] | 18.51 [1851] |
| $\sigma(M_{HHHH} < 3 \text{ TeV})$ | 0.03 [0] | 0.31 [1] | 1.47 [18] | 2.89 [58] | 3.98 [131] | 6.69 [669] |
| $\sigma(M_{HHHH} < 1 \text{ TeV})$ | 0.02 [0] | 0.12 [1] | 0.26 [3] | 0.37 [7] | 0.45 [15] | 0.64 [64] |

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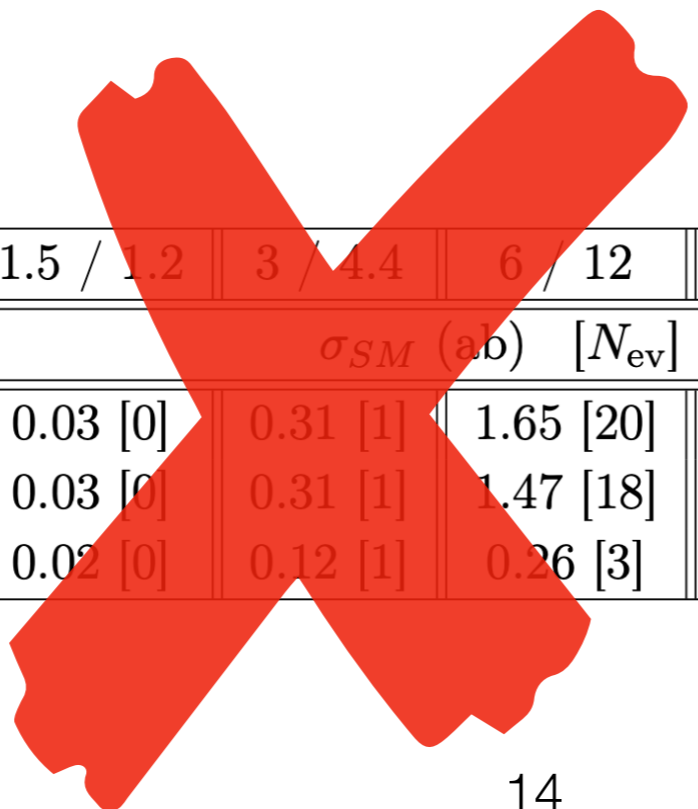
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$$\mathcal{O}_\varphi = \left(\varphi^\dagger \varphi - \frac{v^2}{2} \right)^3$$

SMEFT scenario $\delta_4 = 6 \delta_3$

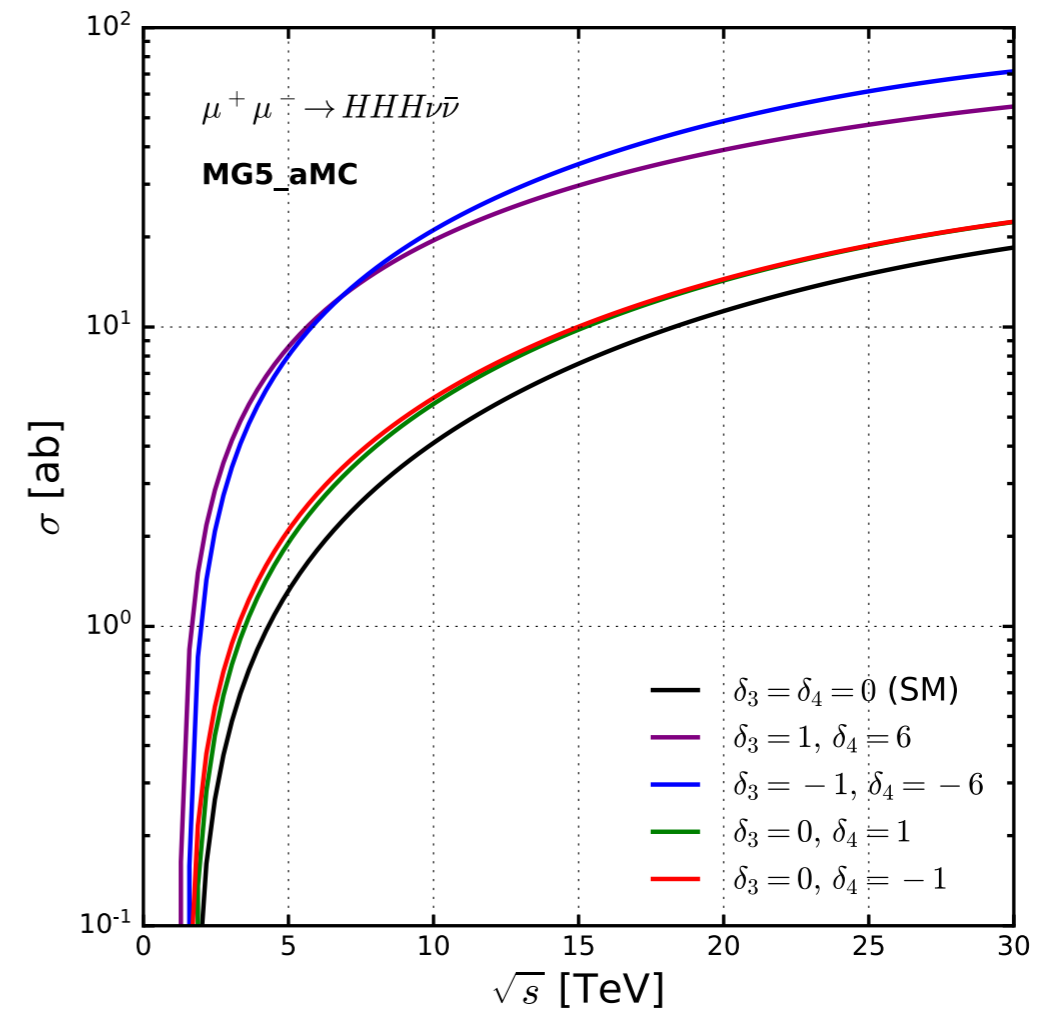
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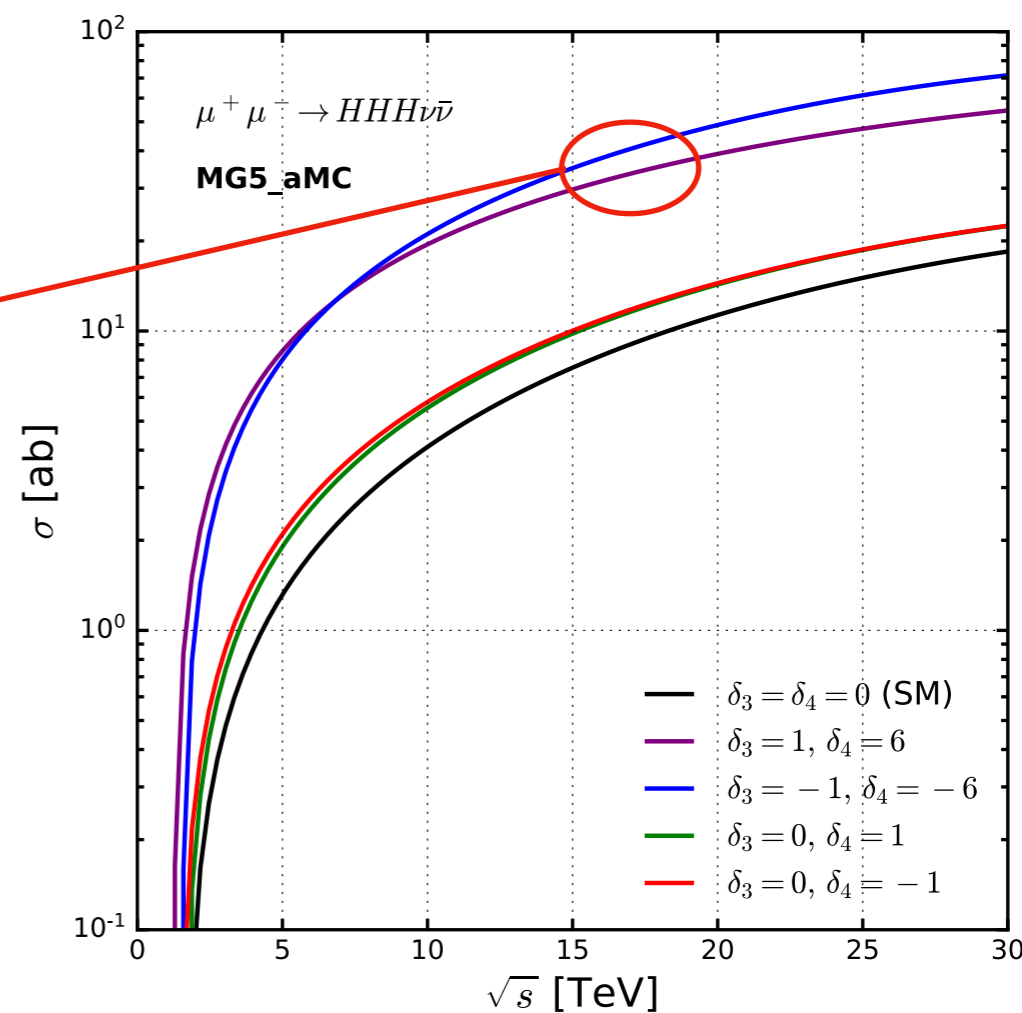
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**Huge effects in SMEFT
caused by trilinear shift**



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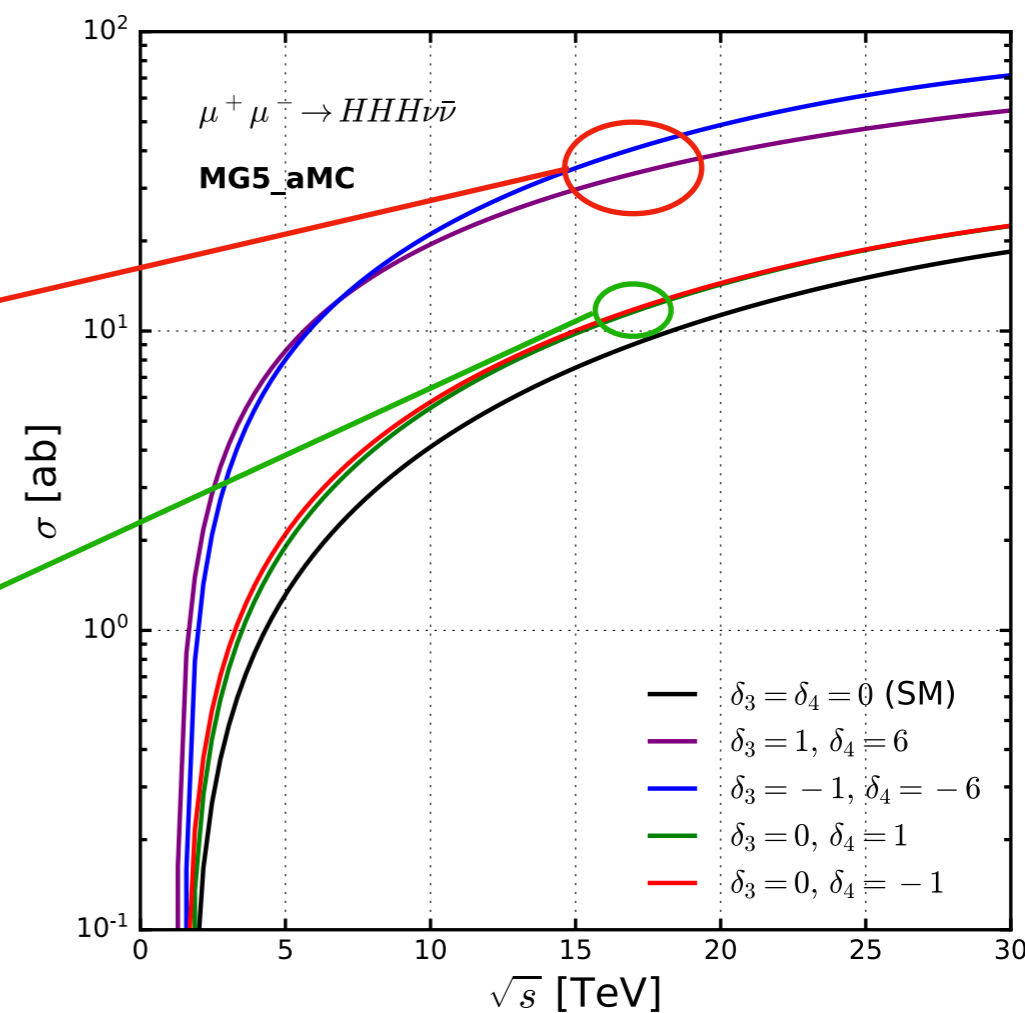
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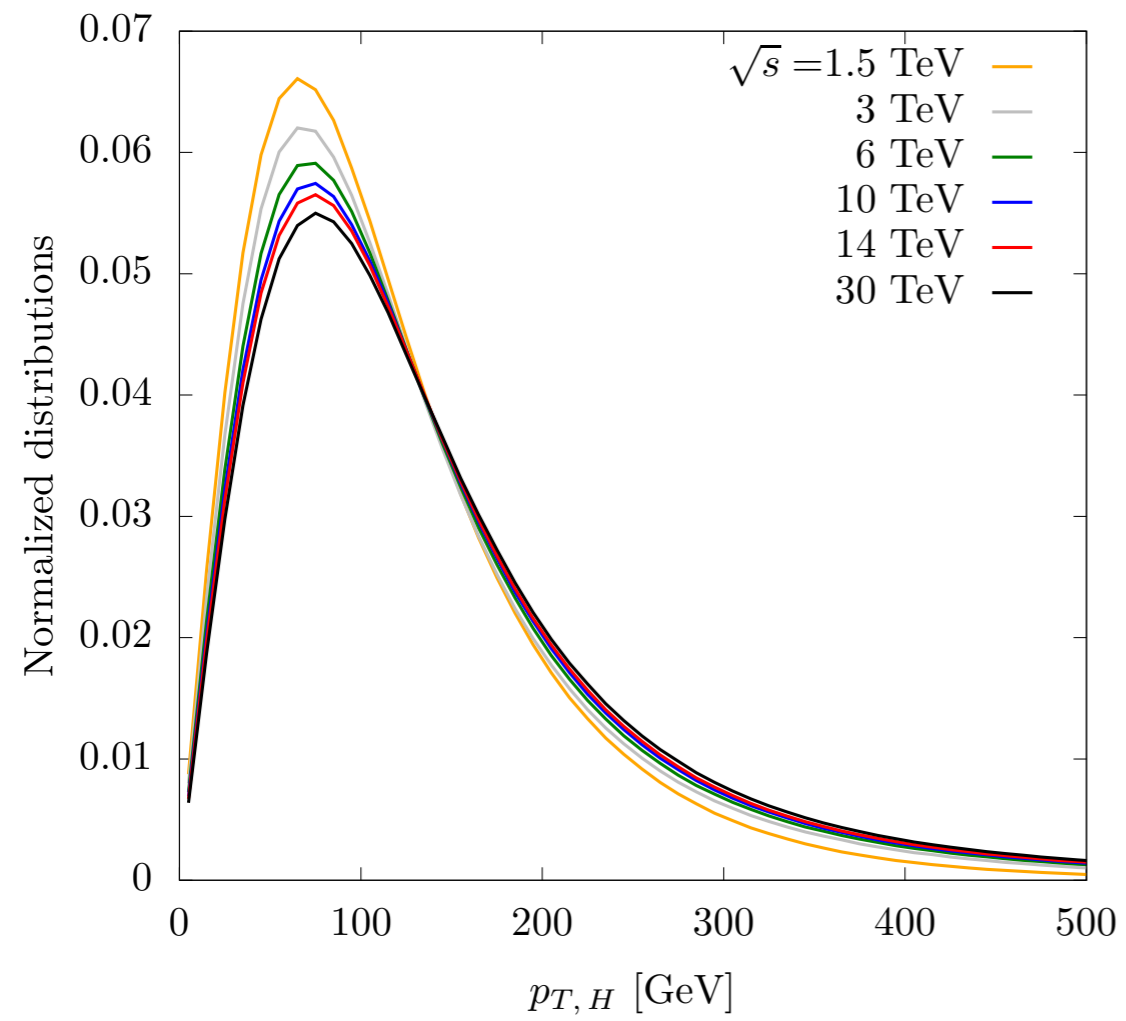
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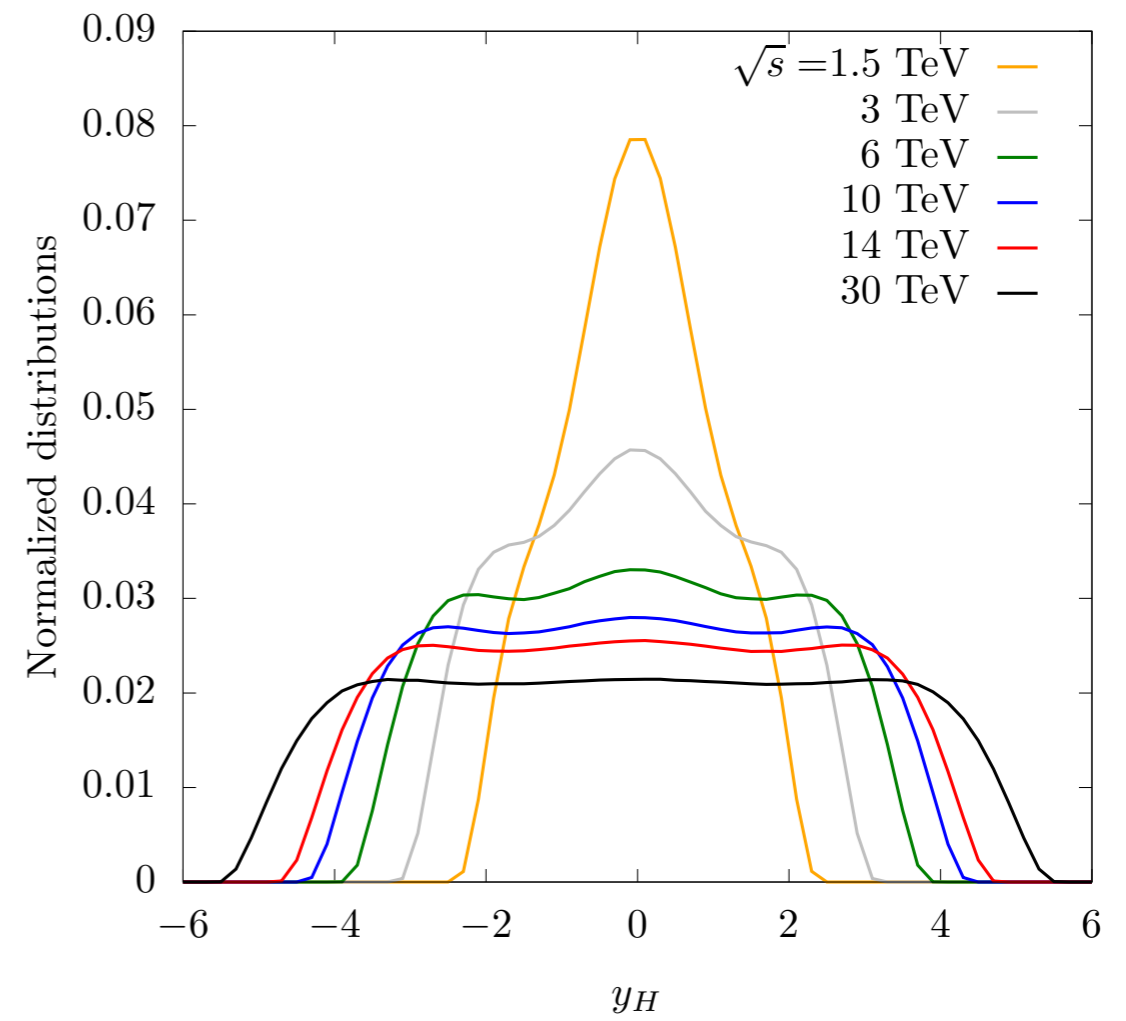
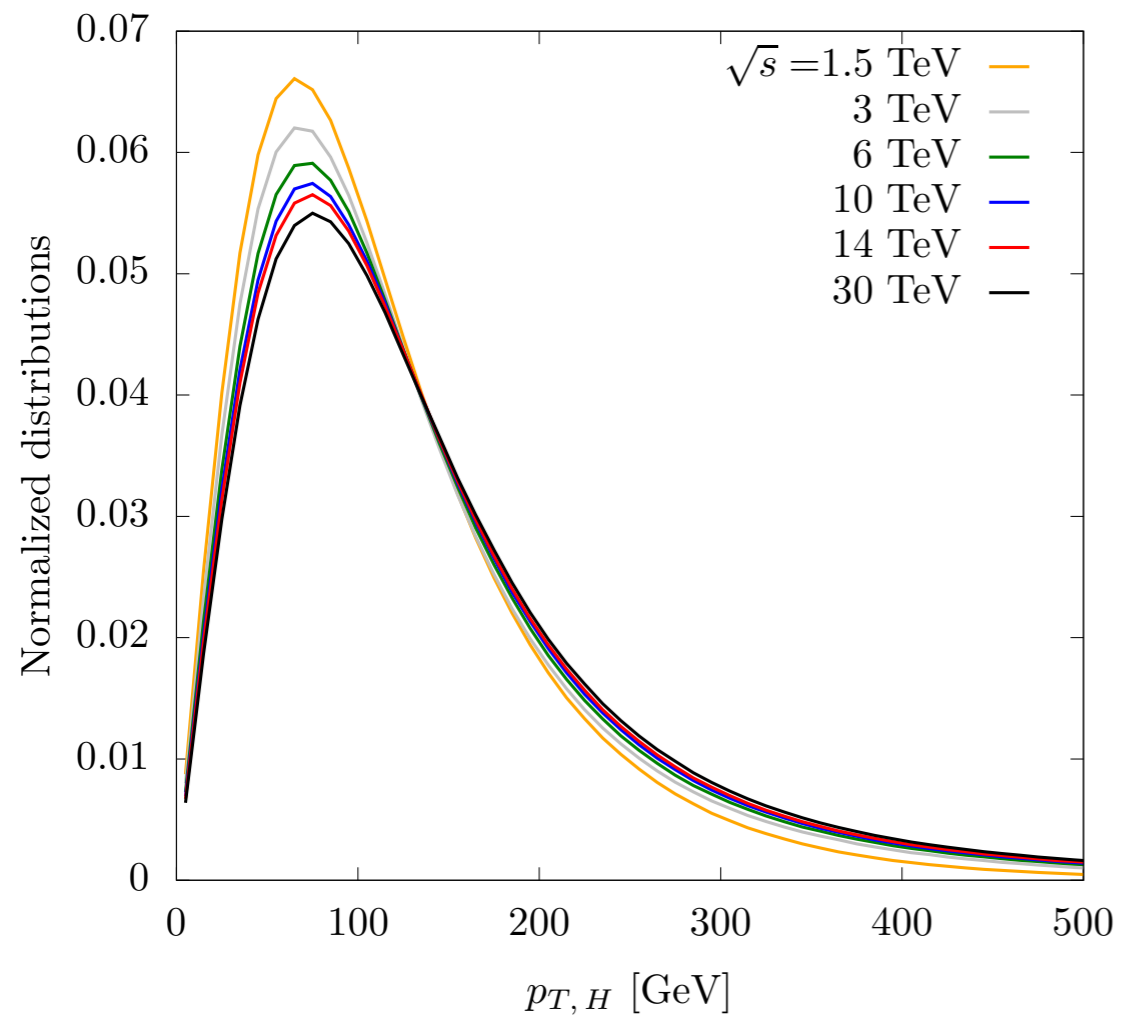
SMEFT scenario $\delta_4 = 6 \delta_3$

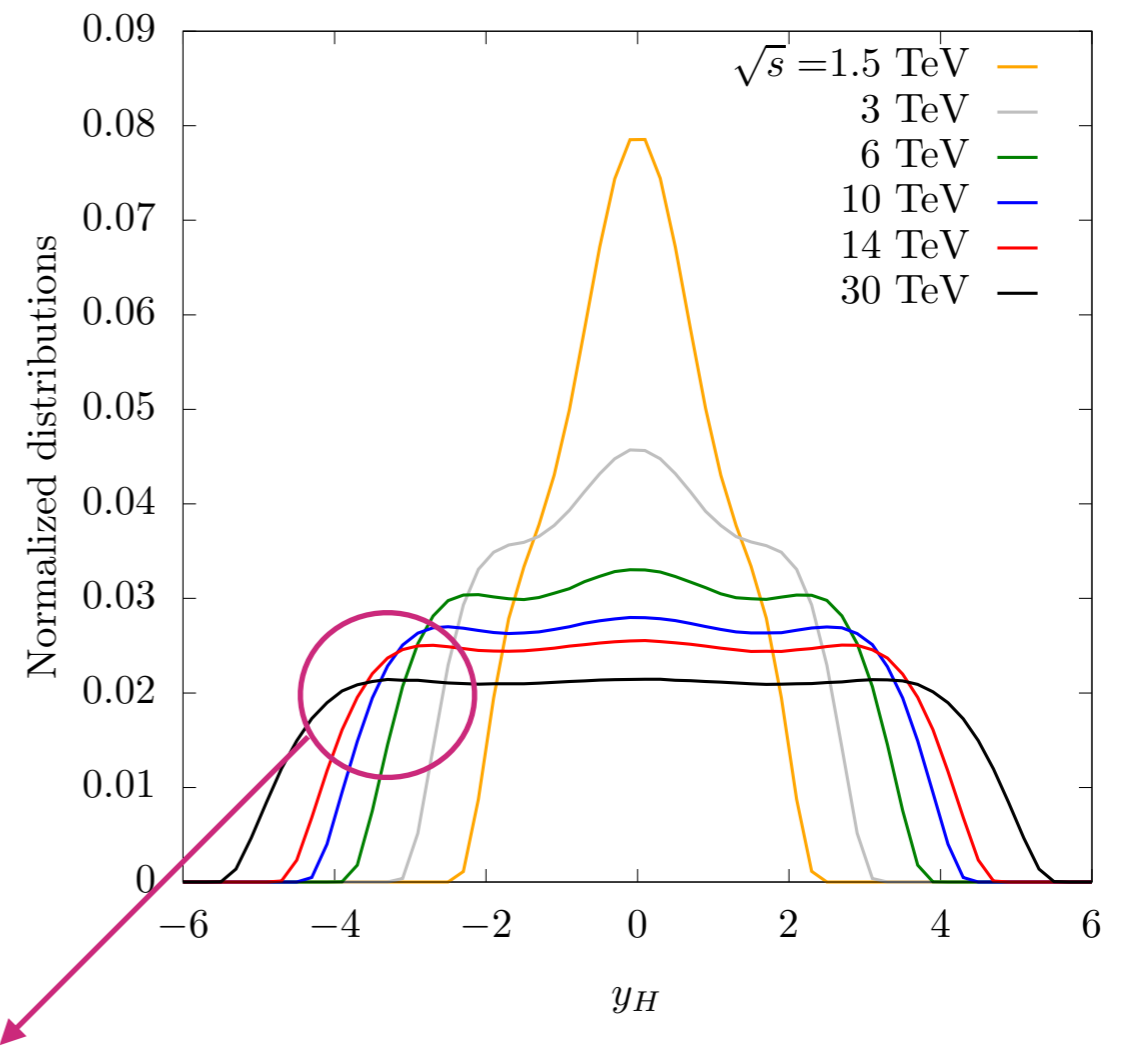
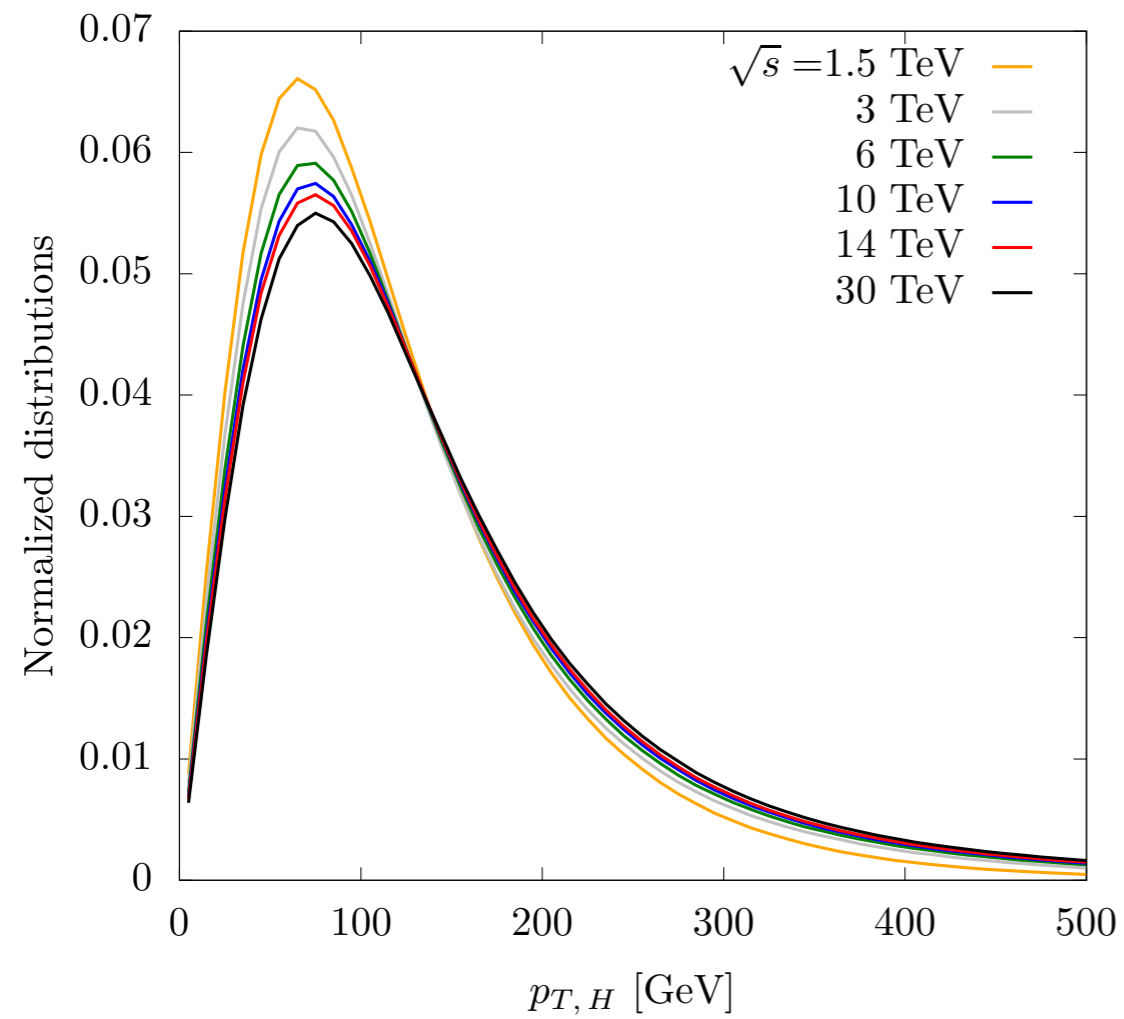
**Huge effects in SMEFT
caused by trilinear shift**

**Quartic deviations less sensitivity
sign hardly distinguishable**

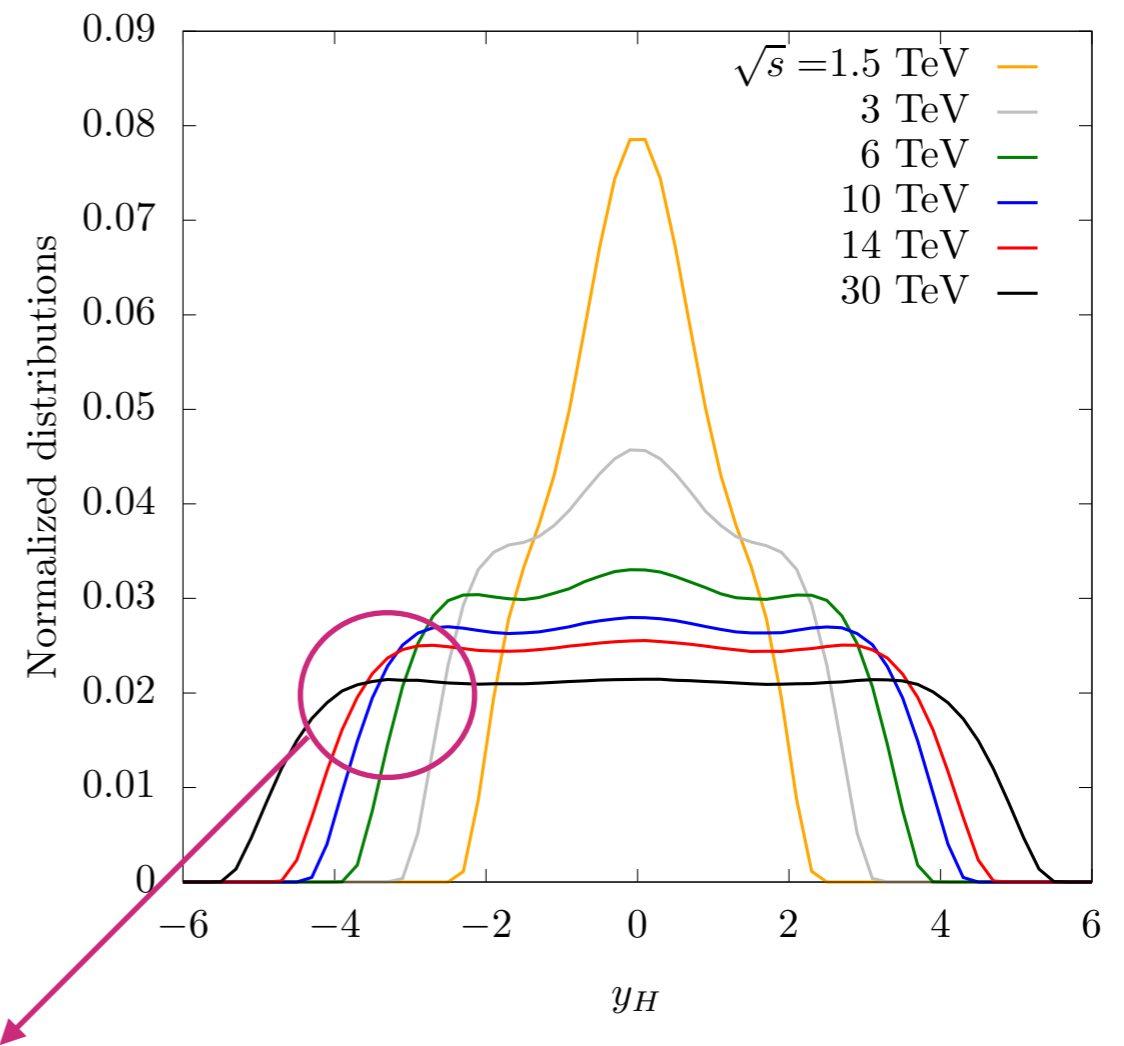
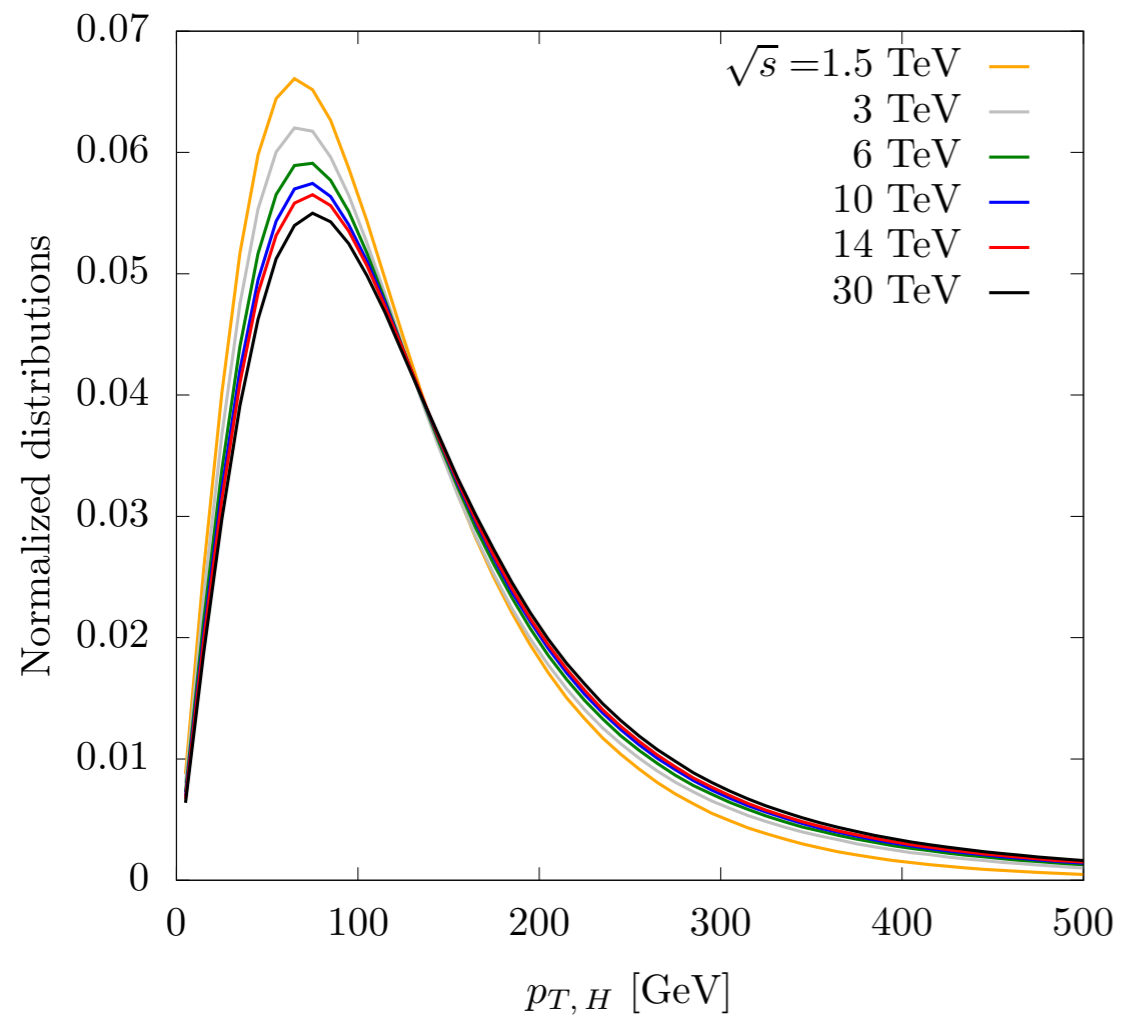






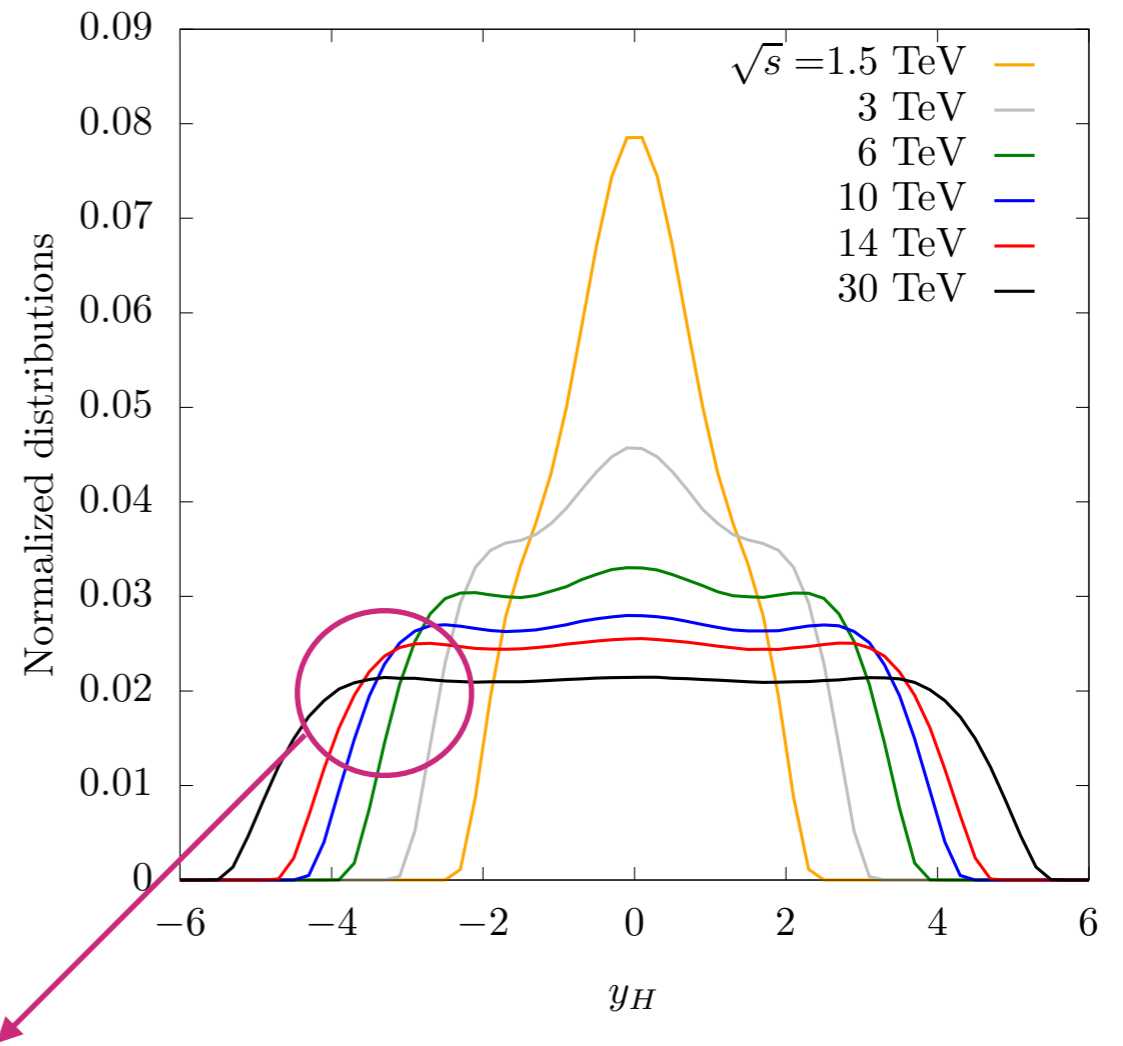
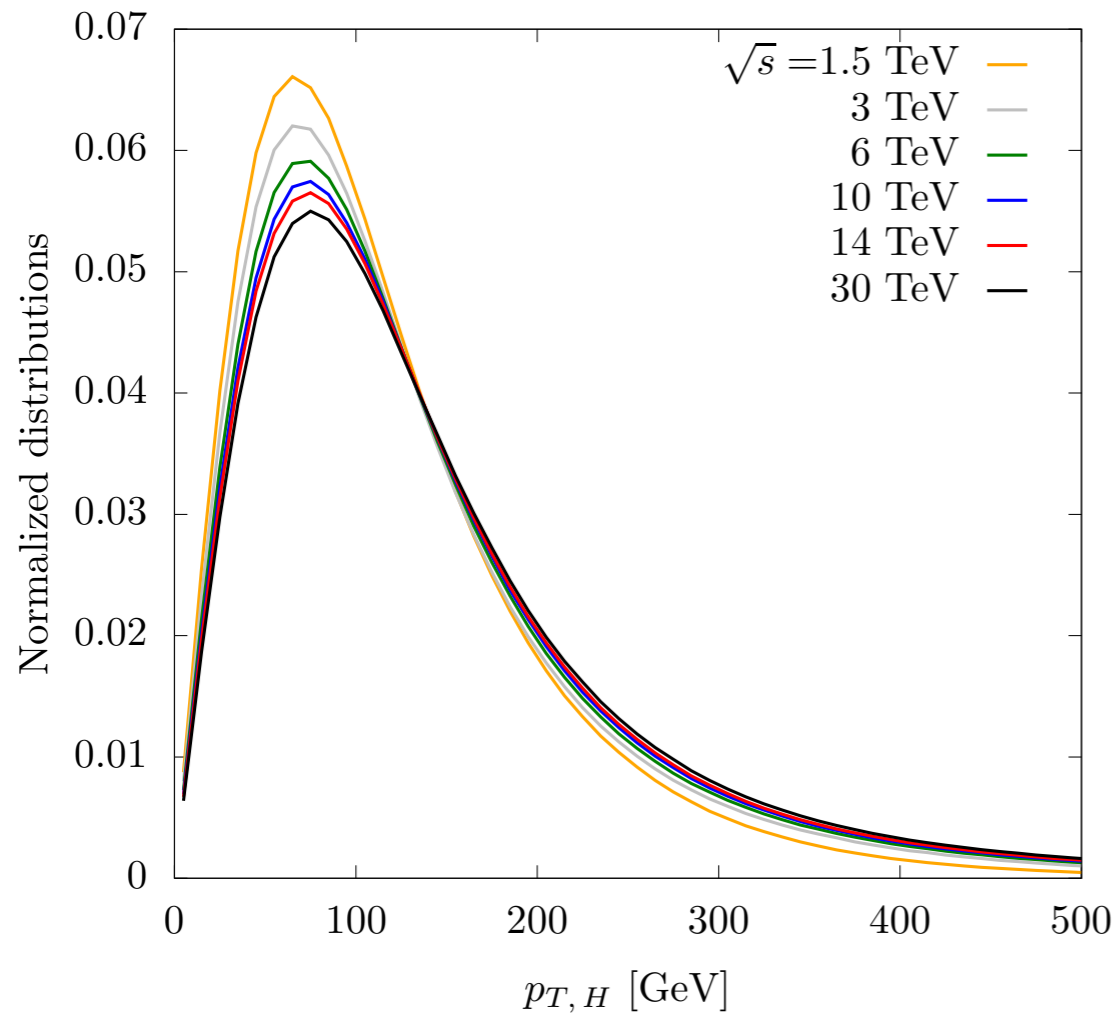


Production in forward region at high energy!



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Detector studies have shields in the forward region due to beam-induced background



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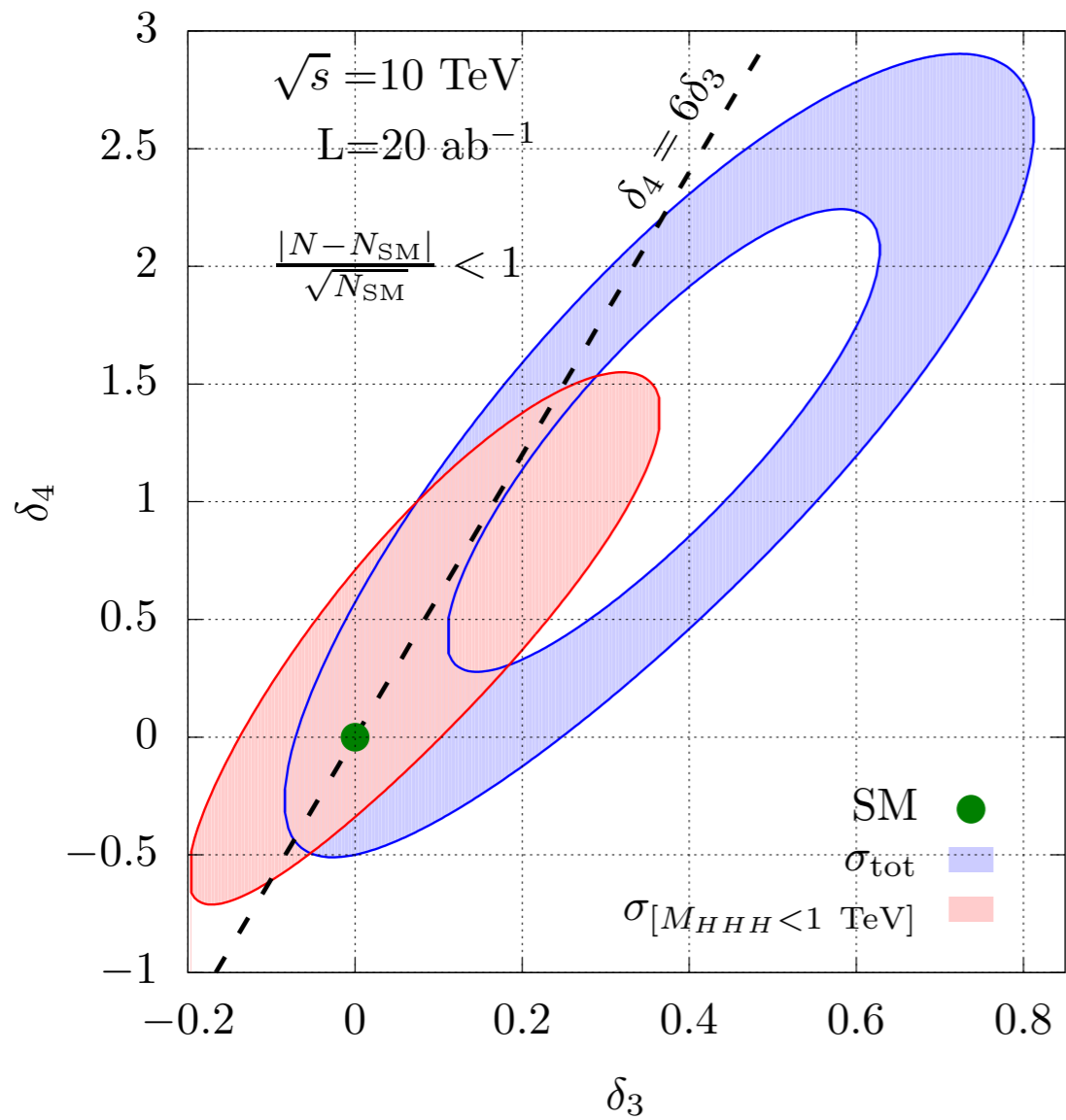
Important to have high rapidity coverage

$$|\eta| < 5$$

$$p_T^b > 20\text{GeV}$$

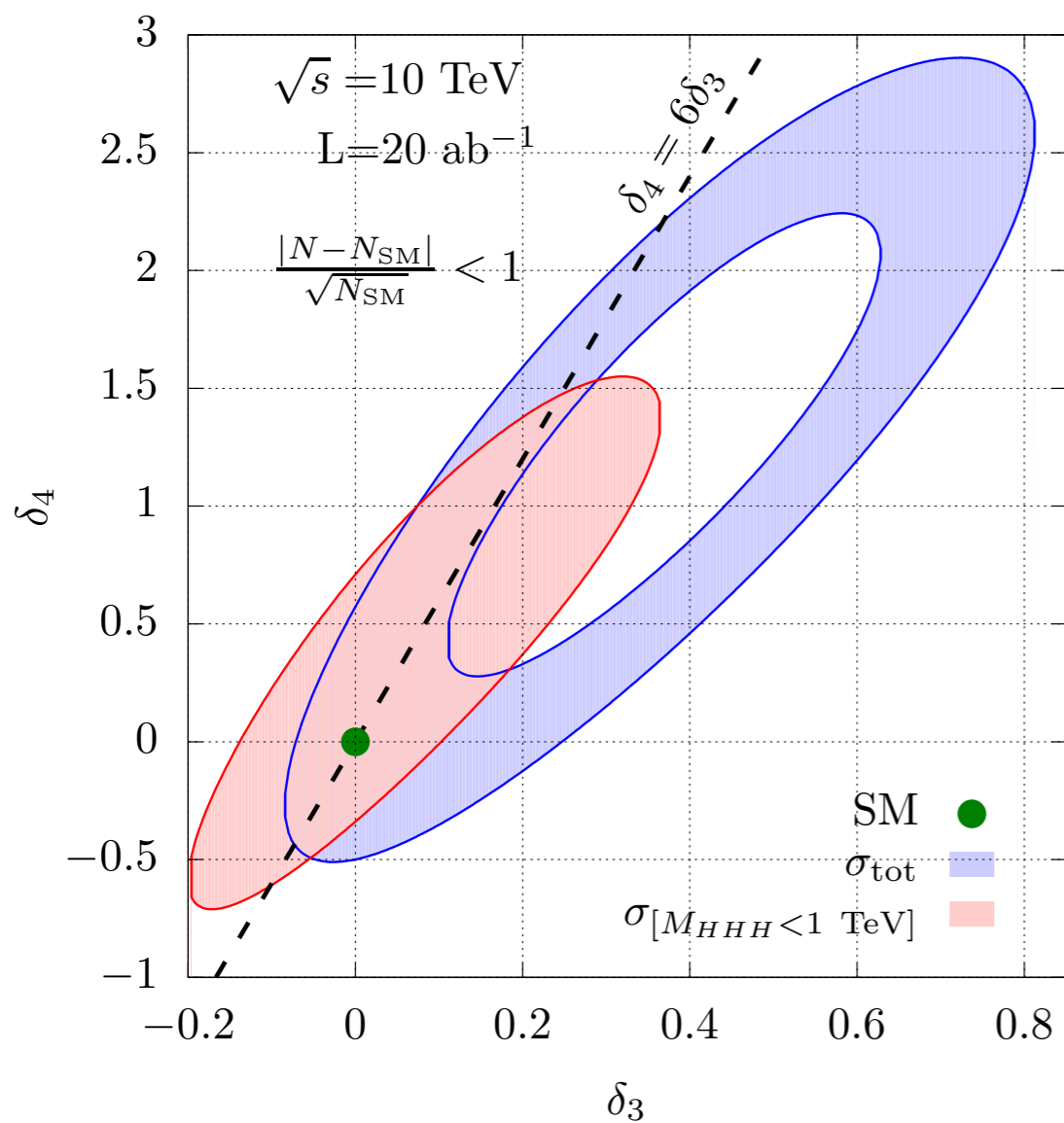
$$A \sim 60 - 70\%$$

$$\frac{S}{\sqrt{B}} = \frac{|\mathcal{L} \cdot (\sigma - \sigma_{SM})|}{\sqrt{\mathcal{L} \cdot \sigma_{SM}}} \leq 1$$

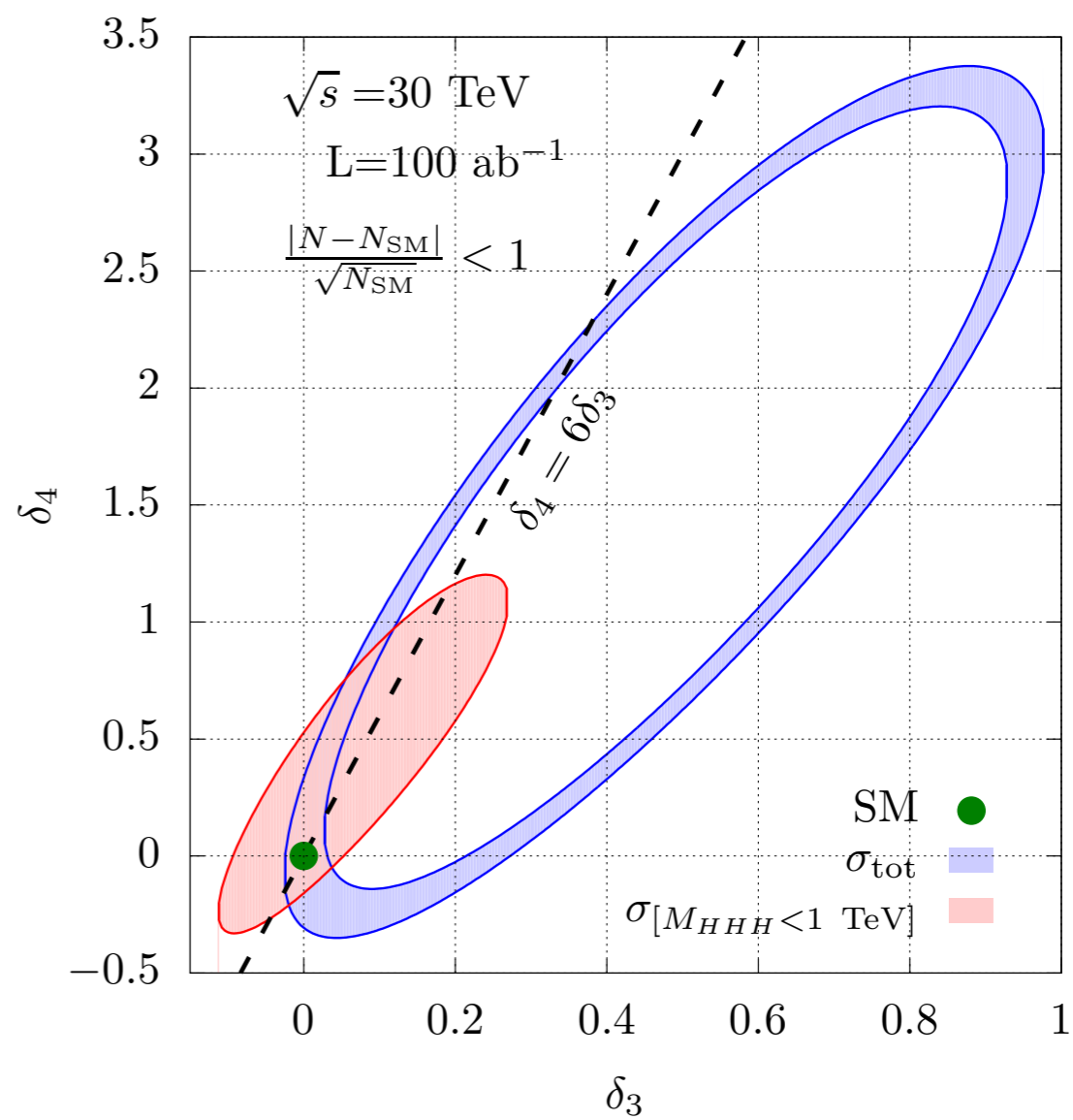


10 TeV $\delta_4 \sim [-0.4, 0.7]$

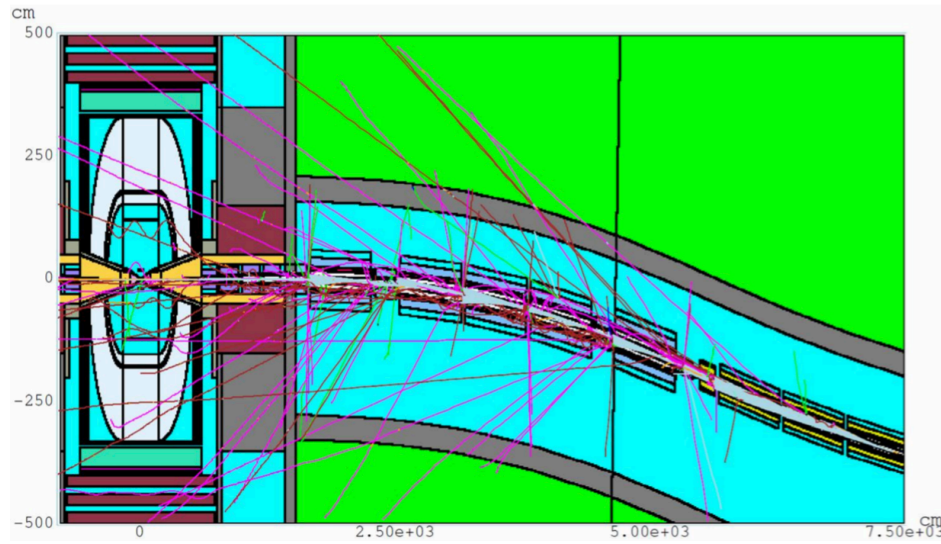
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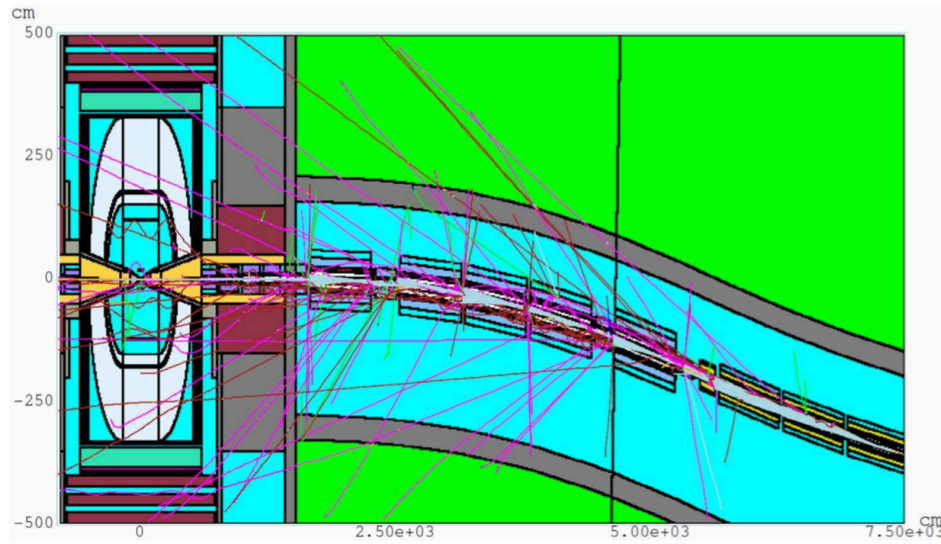
10 TeV $\delta_4 \sim [-0.4, 0.7]$



30 TeV $\delta_4 \sim [-0.2, 0.5]$

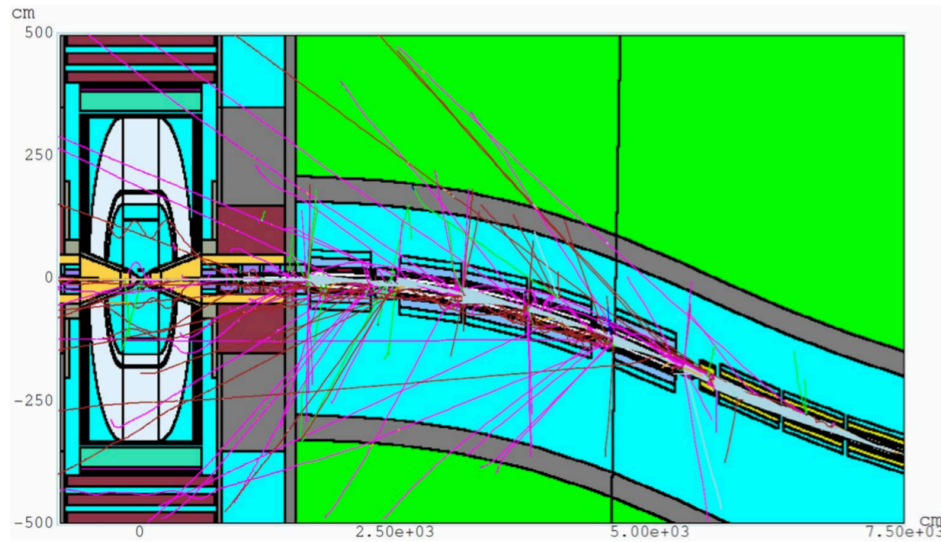


Nozzles:
Detector must be shielded from beam radiation
5-10 degrees blind spot at 3 TeV



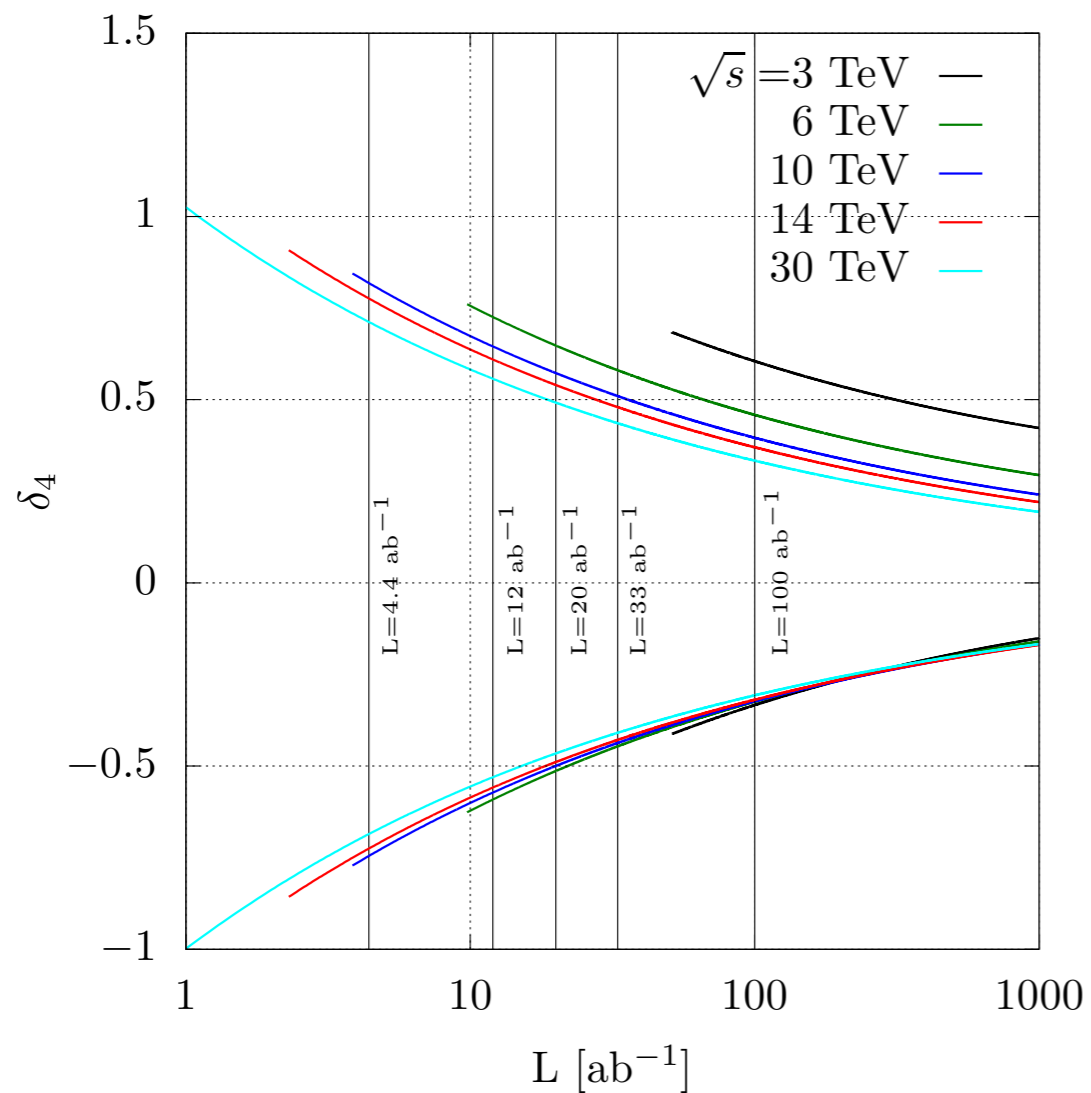
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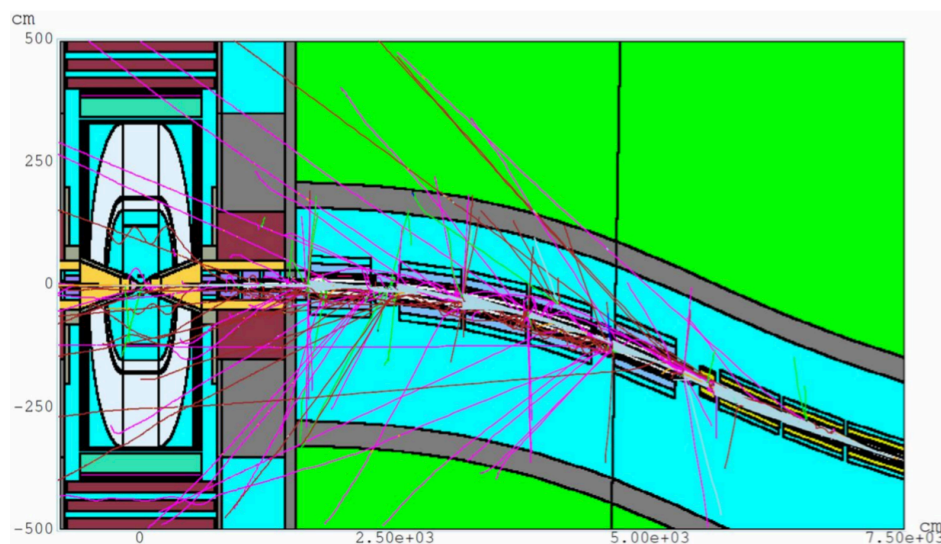
At higher energies the angle can be reduced!



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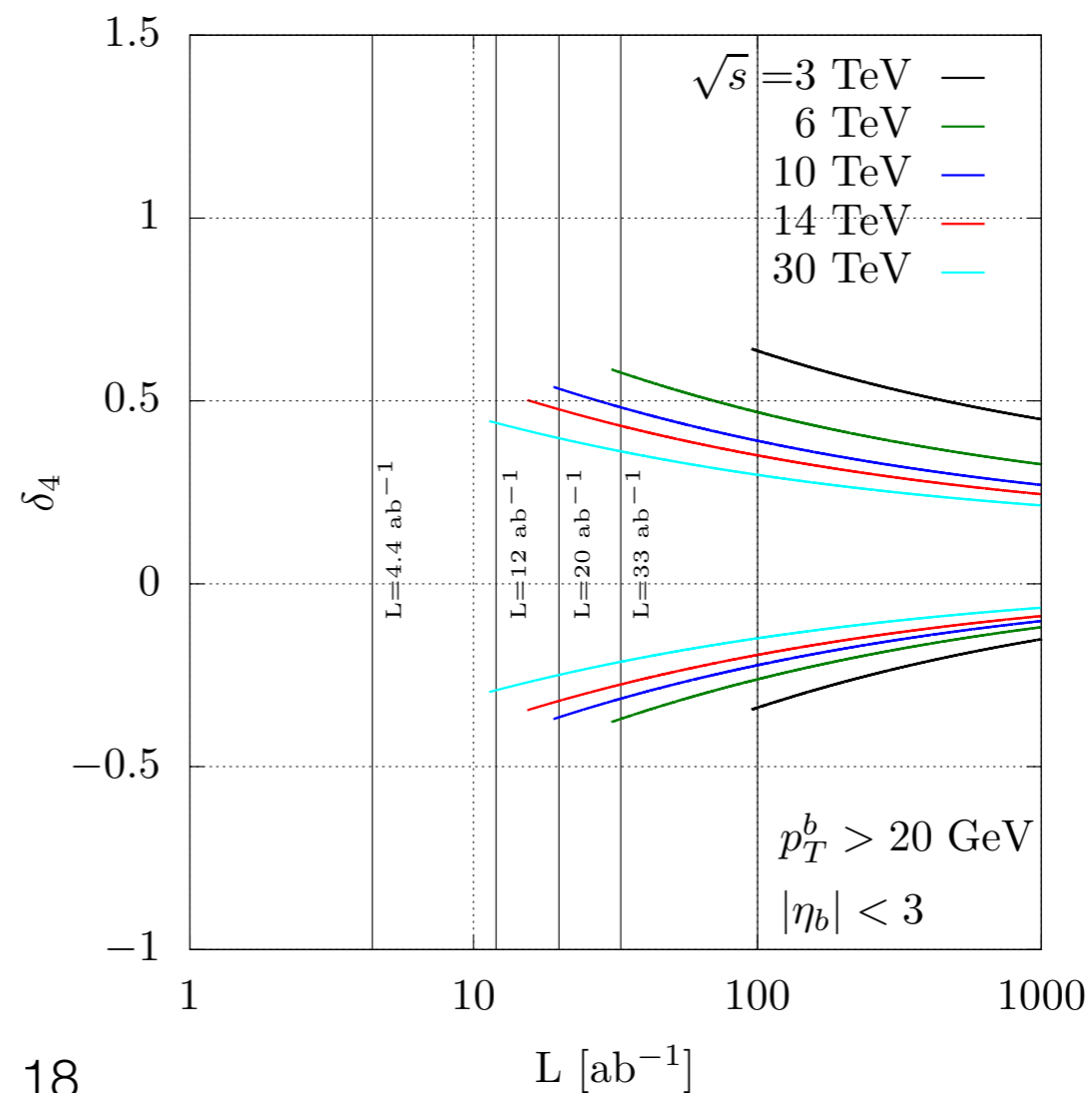
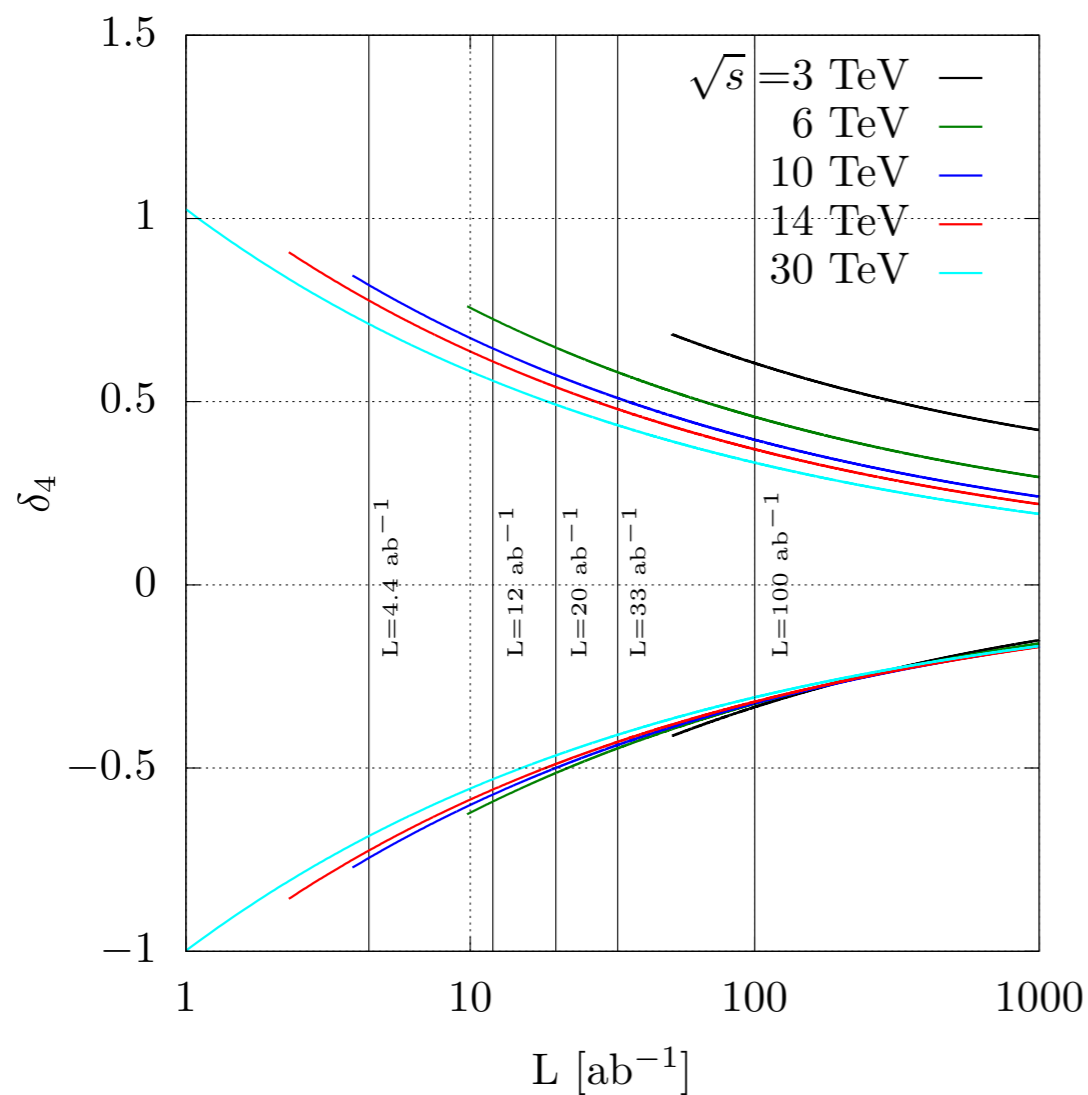
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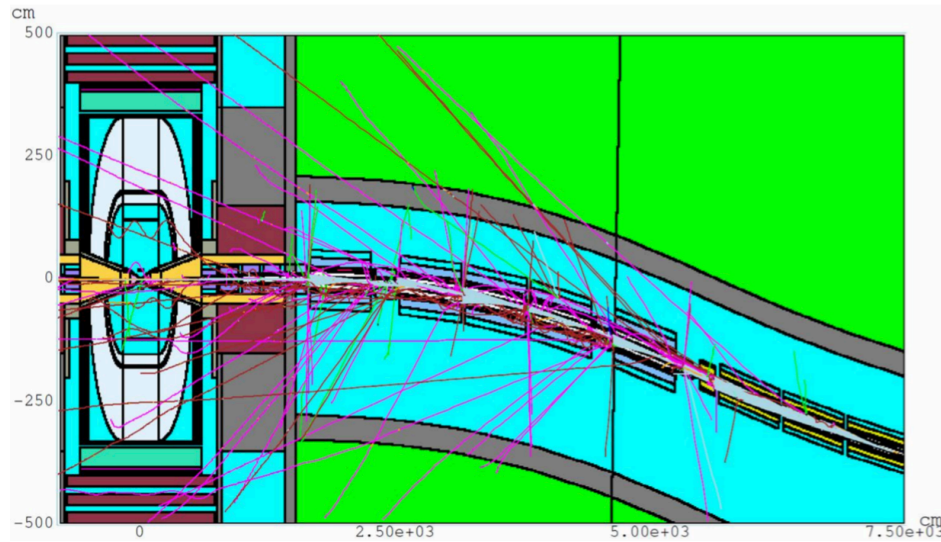




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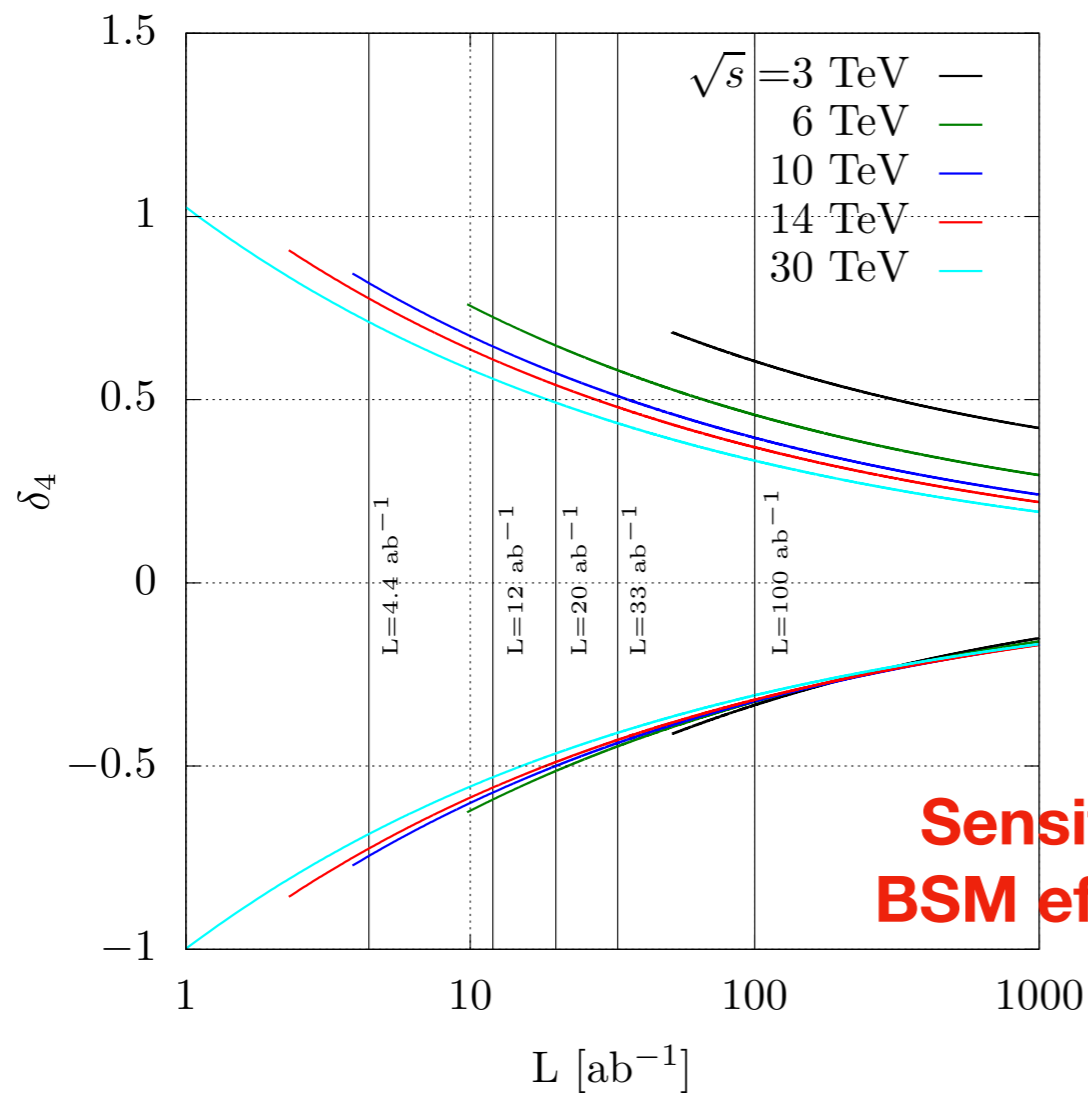
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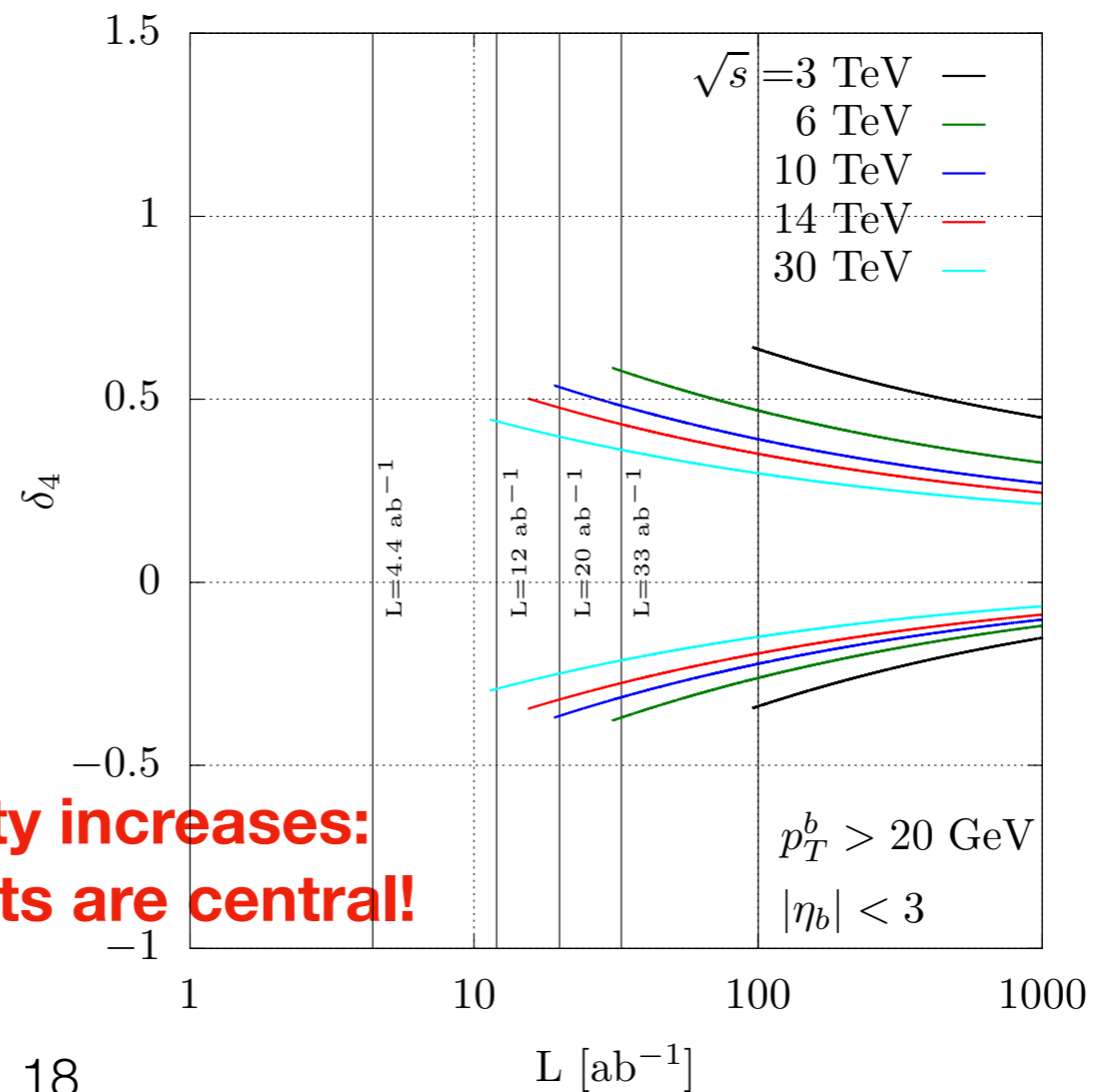


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At higher energies the angle can be reduced!



Sensitivity increases:
BSM effects are central!



$p_T^b > 20 \text{ GeV}$
 $|\eta_b| < 3$

- ❖ **Muon collider is a dream machine**
- ❖ **Both precision and discovery potential are top notch**
- ❖ **A multi-TeV machine would be effectively a EW boson collider**
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- ❖ **For the quartic coupling, factor ~ 10 improvement over FCC**
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Thanks!

Backup

Luminosities are assumed from MAP studies

Typical EW process at muon collider

$$\sigma = \left(\frac{10 \text{ TeV}}{\sqrt{s_\mu}} \right)^2 \cdot 10^3 \text{ ab}$$

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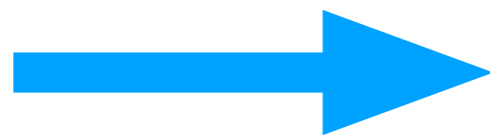
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For $N \sim 10^4$



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| | σ [fb] | \sqrt{s} [TeV] | | σ [fb] | \sqrt{s} [TeV] |
|--------------|---------------------|------------------|--------------------|---------------------|------------------|
| $t\bar{t}$ | $8.4 \cdot 10^0$ | 4.5 | $t\bar{t}ZZ$ | $2.2 \cdot 10^{-2}$ | 8.4 |
| $t\bar{t}Z$ | $5.3 \cdot 10^{-1}$ | 6.9 | $t\bar{t}HZ$ | $7.0 \cdot 10^{-3}$ | 11 |
| $t\bar{t}H$ | $7.6 \cdot 10^{-2}$ | 8.2 | $t\bar{t}HH$ | $5.9 \cdot 10^{-4}$ | 13 |
| $t\bar{t}WW$ | $1.2 \cdot 10^{-1}$ | 15 | $t\bar{t}t\bar{t}$ | $1.6 \cdot 10^{-3}$ | 22 |
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| WW | $2.1 \cdot 10^2$ | 4.8 | WWZ | $1.6 \cdot 10^1$ | 6.2 |
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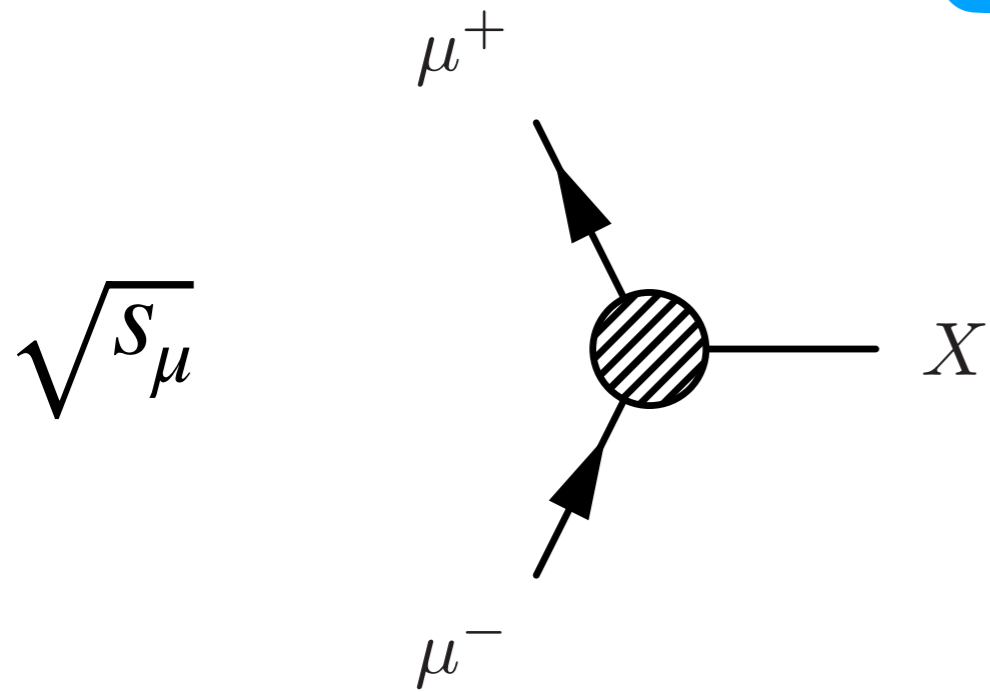
Effectively a EW boson collider!

Resonance production

$$M_X = \sqrt{s_\mu}$$

Resonance production

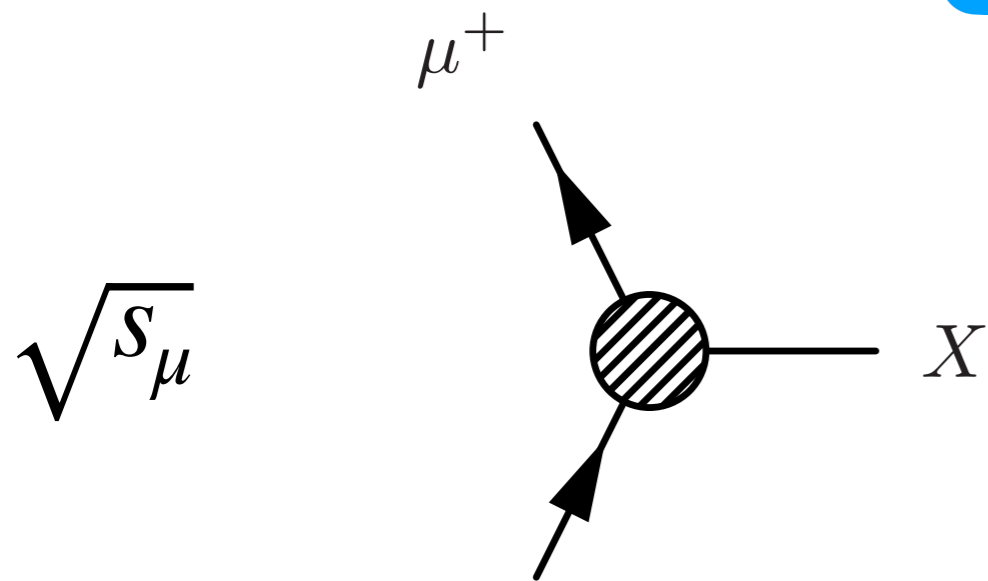
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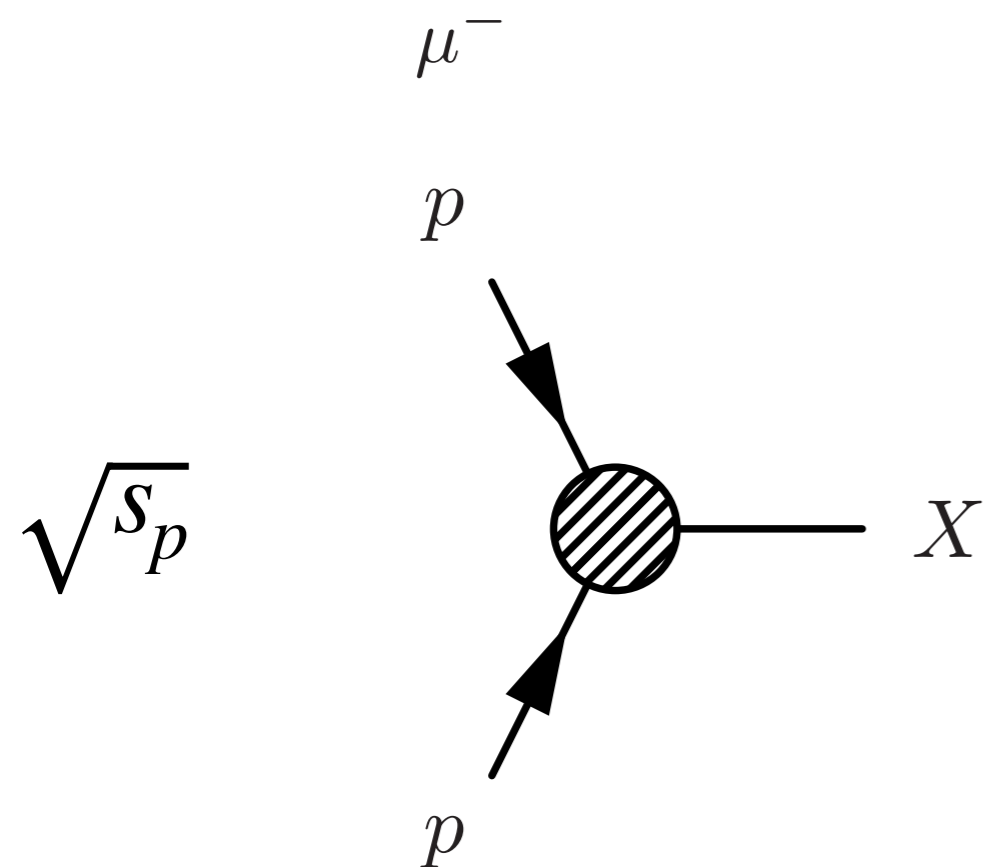
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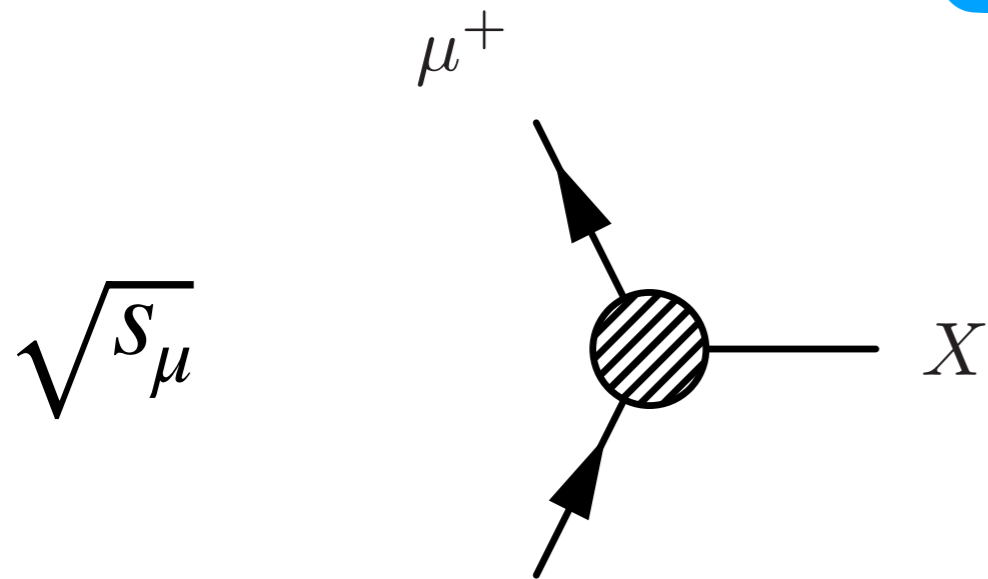
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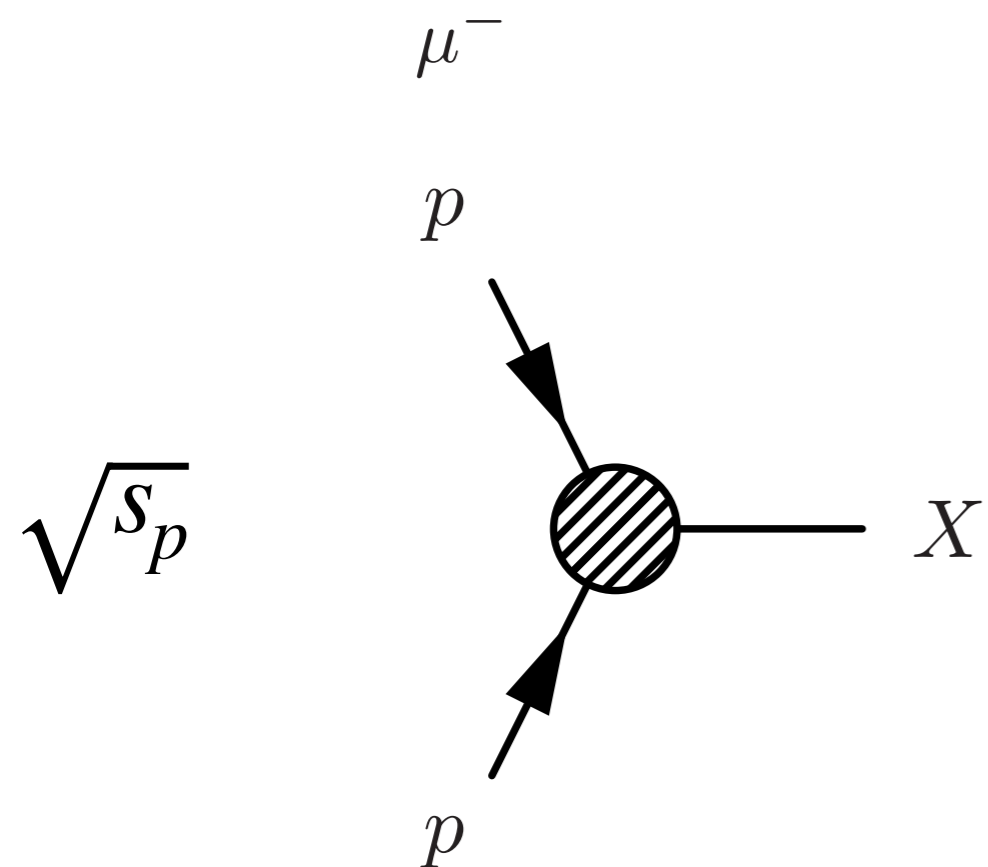
$$\sigma_p(s_p) = \int_{\tau_0}^1 d\tau \sum_{ij} \Phi_{ij}(\tau, \mu_f) [\hat{\sigma}_{ij}]_p \delta\left(\tau - \frac{M_X^2}{s_p}\right)$$

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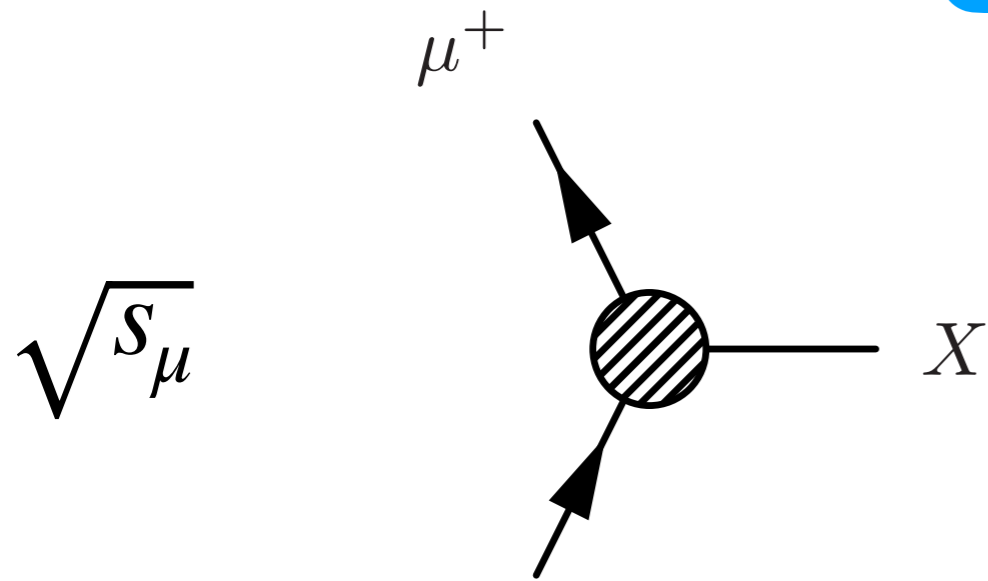


Parton lumi

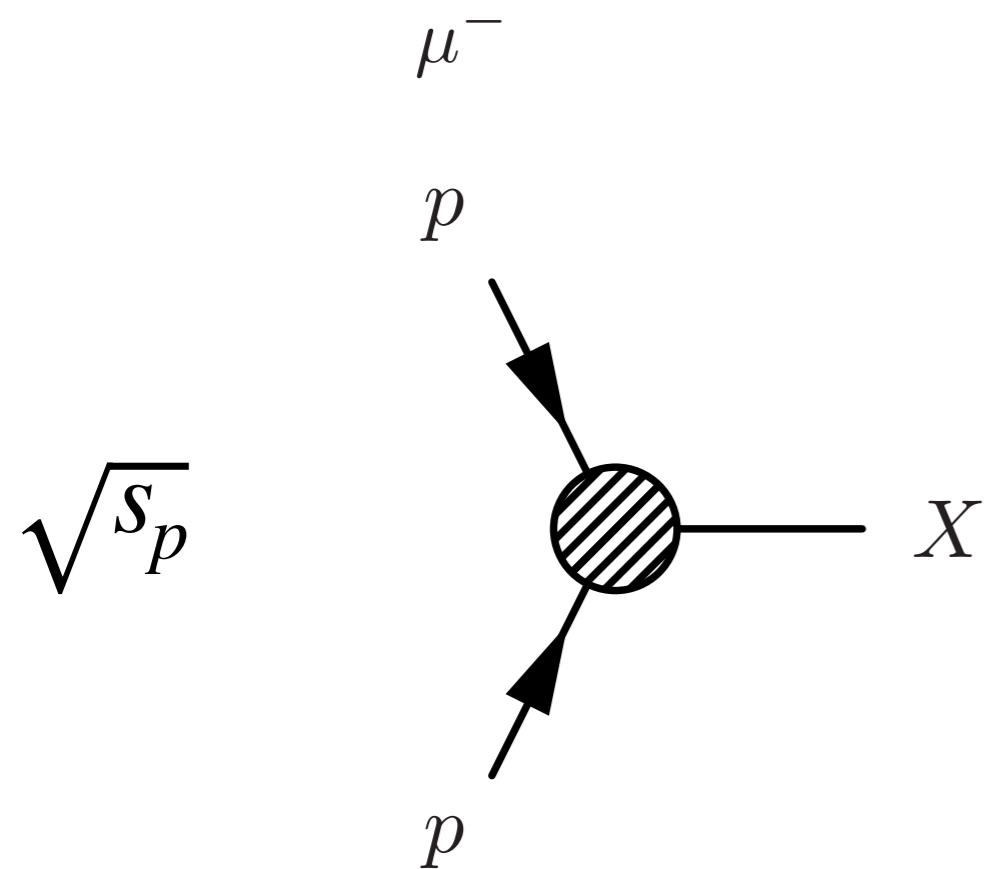
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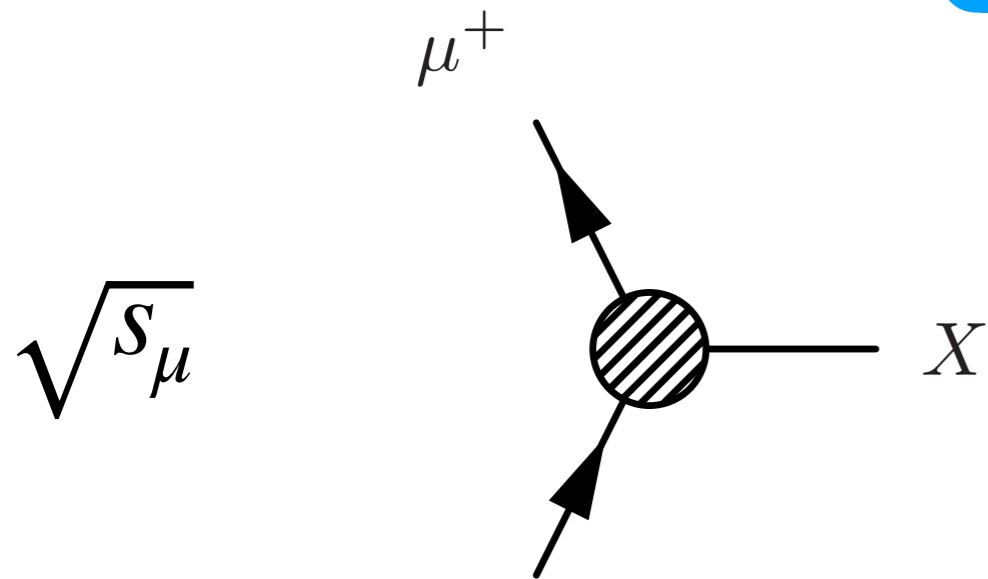
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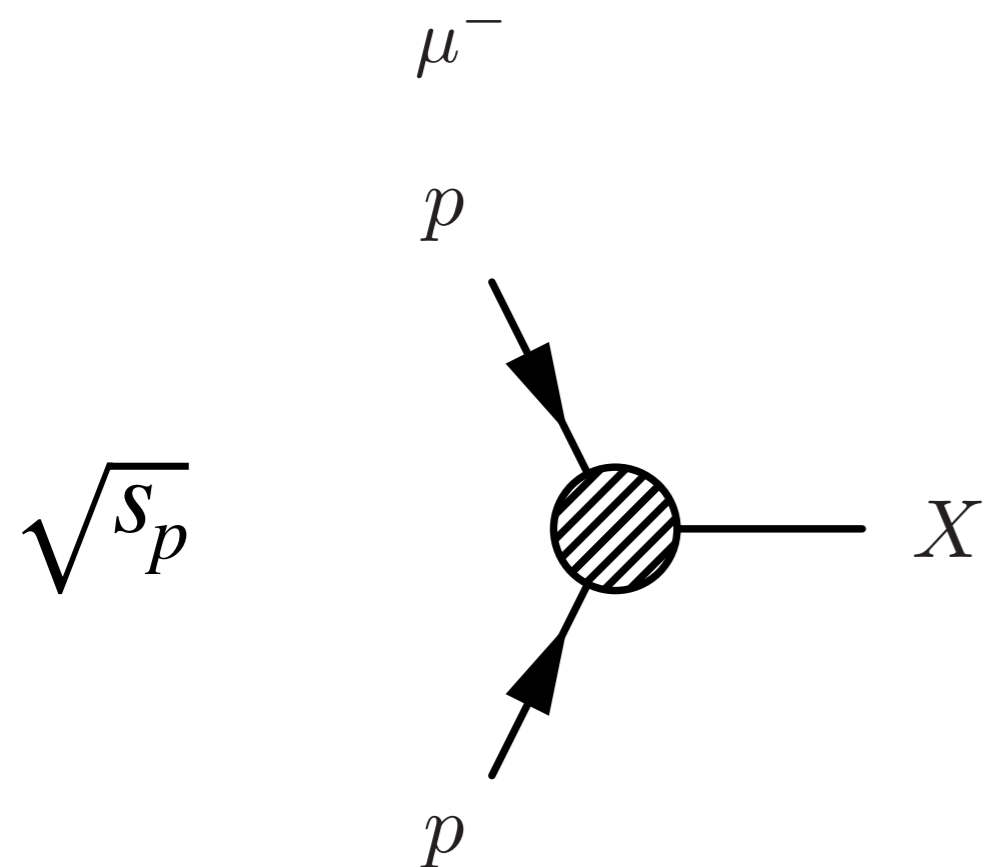
Hard XS

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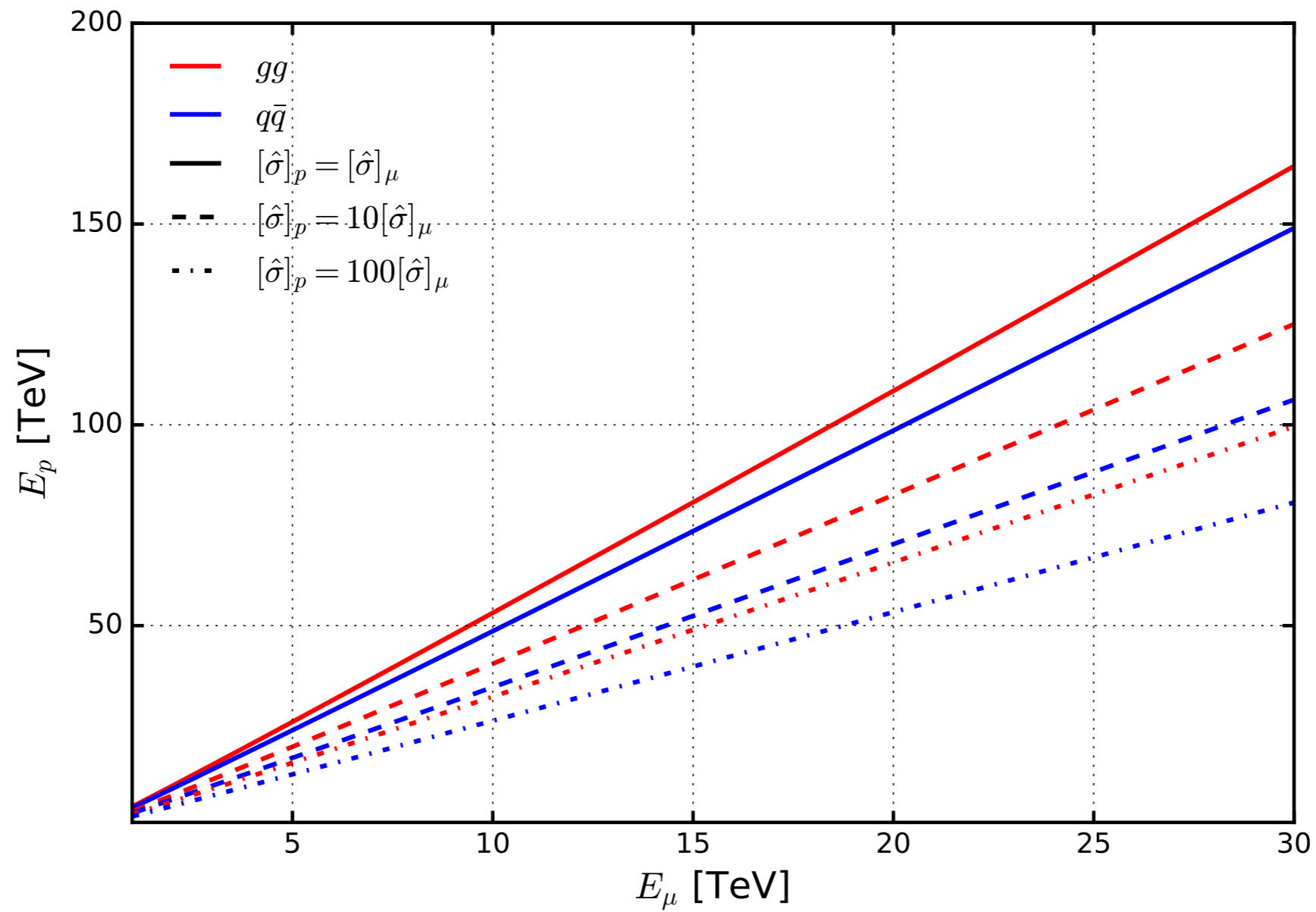
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Hard XS

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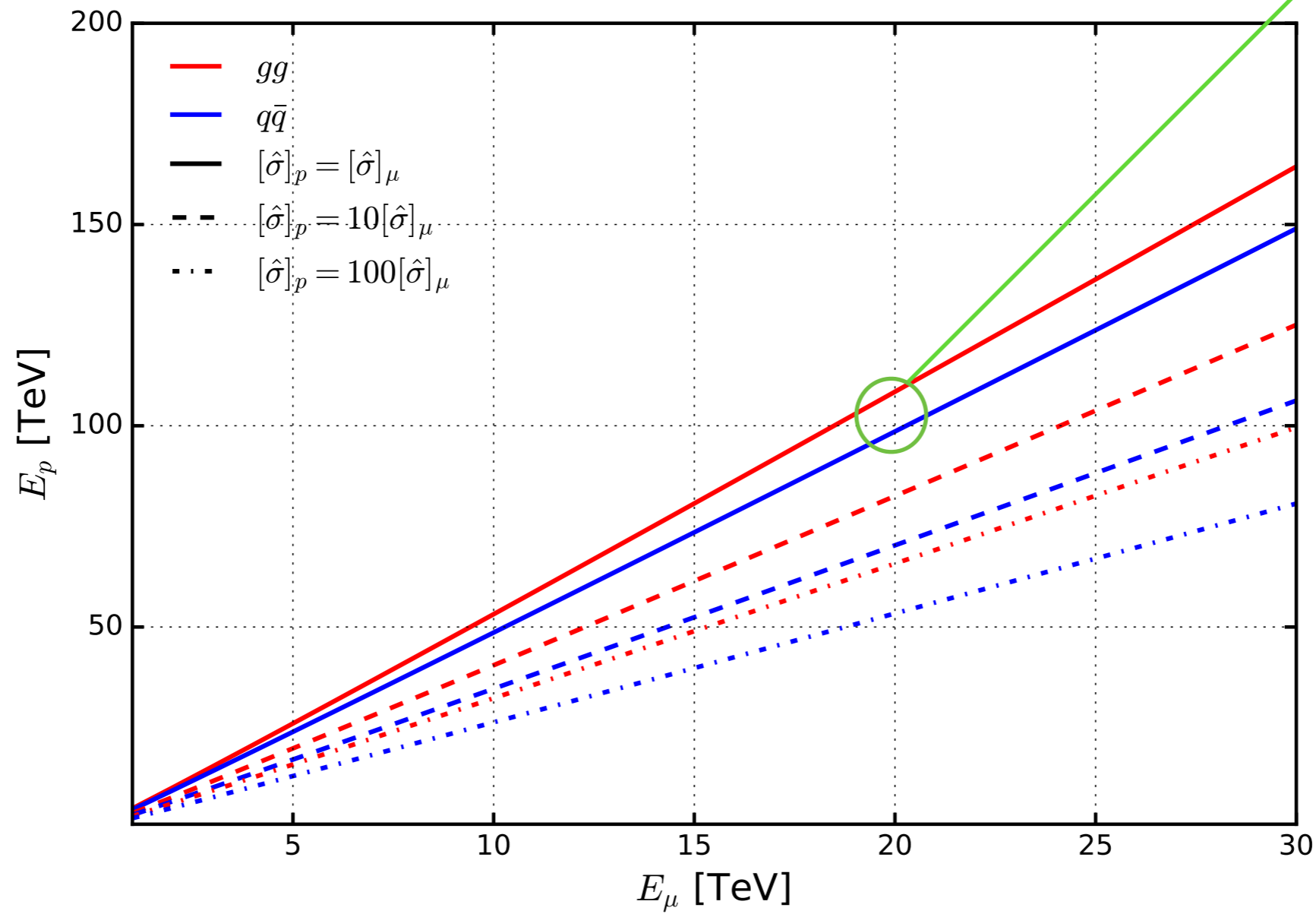
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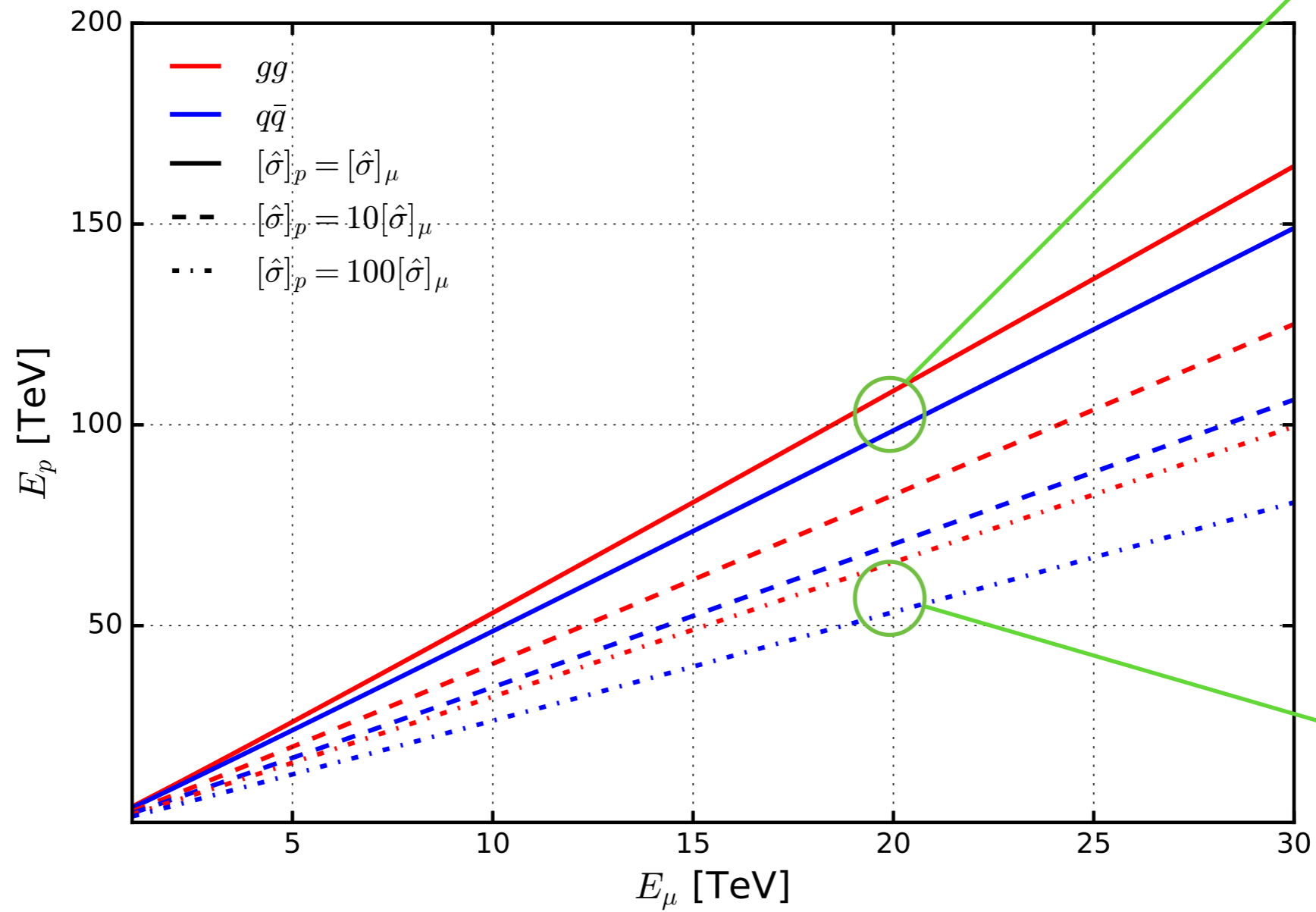


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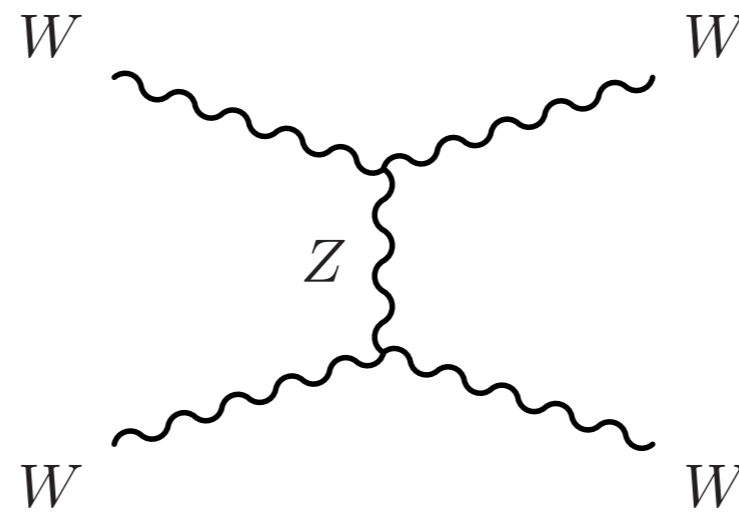
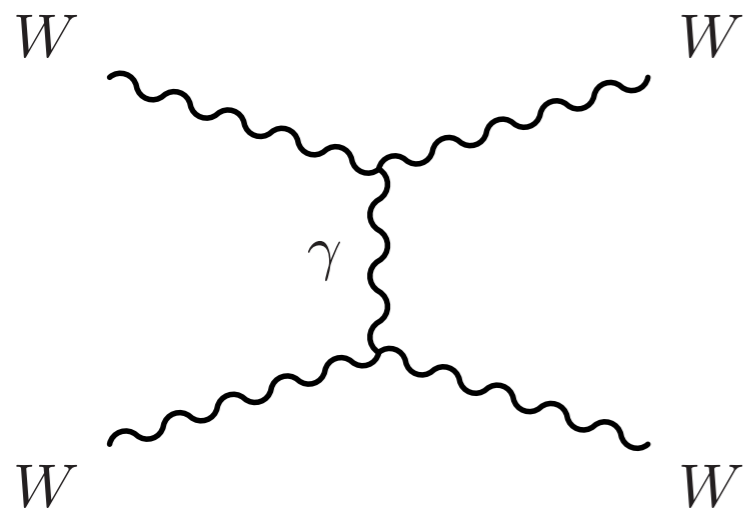


FCC comparable!

Still good!

The most well-known example is longitudinal W-boson scattering

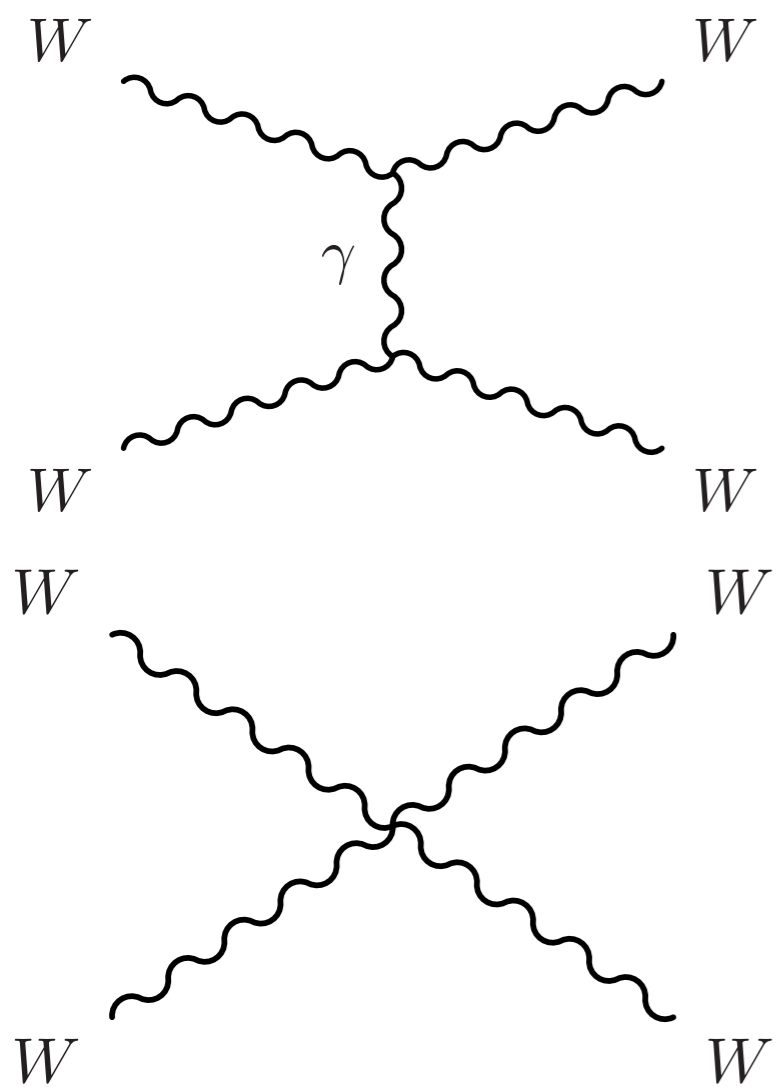
$$W_L W_L \rightarrow W_L W_L$$



$$\propto E^4$$

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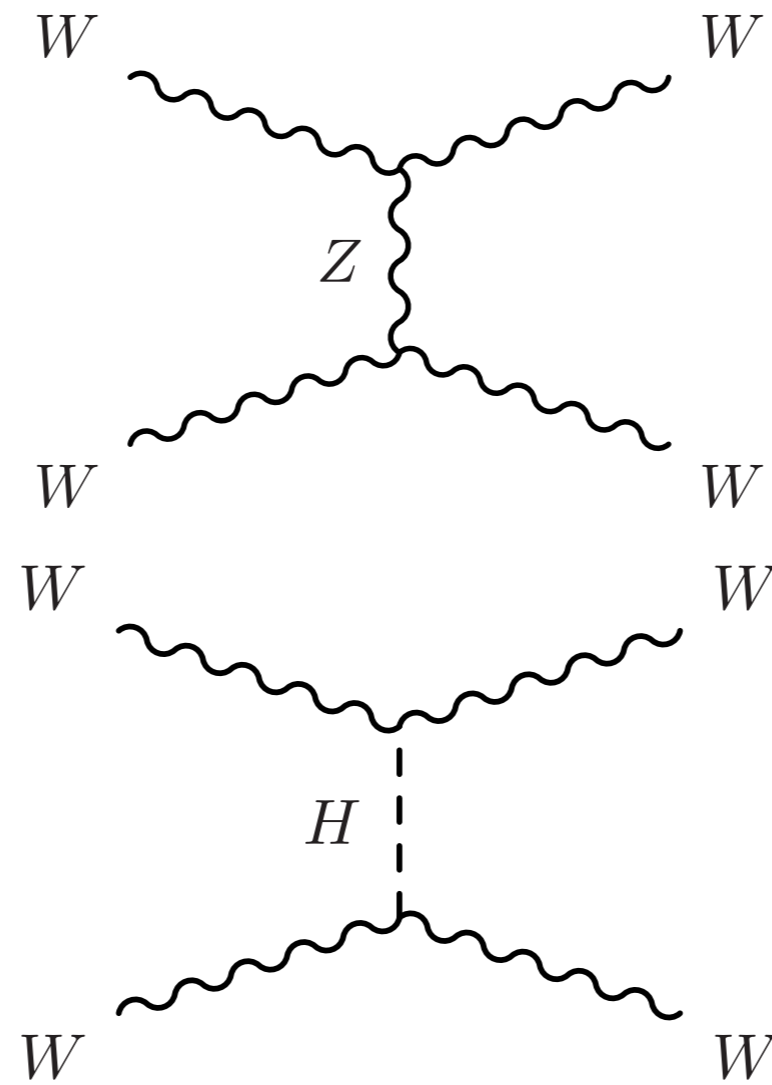
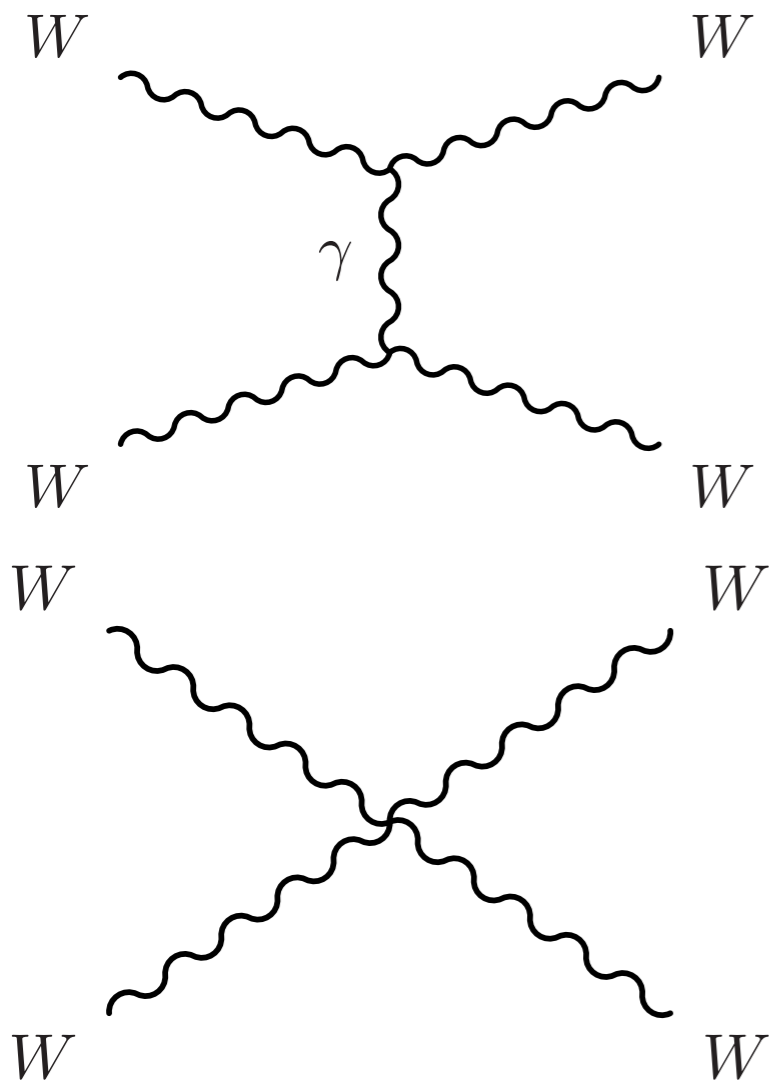
$$W_L W_L \rightarrow W_L W_L$$



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$$\propto E^0$$

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{1}{\Lambda} \mathcal{O}_i^5 + \sum_i \frac{1}{\Lambda^2} \mathcal{O}_i^6 + \dots$$

- ❖ Higher dimensional operators preserve SM symmetries.
- ❖ Mappable to a large class of BSM models.
- ❖ Warsaw basis truncated at dim 6.

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Dim 6 operators introduce energy growing effects

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Not every operator generates energy growing effects!

The maximum energy growth of an amplitude can be guessed from the contact term generated by higher dimension operators.

Let's consider a 2 to N scattering amplitude (mass dim 2-N):

$$\mathcal{L} \supset \frac{1}{\Lambda^{K-4}} \mathcal{O}_K \quad \longrightarrow \quad \mathcal{M} \propto \frac{1}{\Lambda^{K-4}} E^{K-N-2}$$

[F. Maltoni et al. JHEP 10 (2019) 004]

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$$H \quad \longrightarrow \quad v$$

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Dim 6, 2 to 2 $\mathcal{M} \propto \frac{v^m}{\Lambda^2} \frac{E^{2-m+n}}{M_V^n}$

[F. Maltoni et al. JHEP 10 (2019) 004]

Three operators directly affect Higgs self interaction

$$\mathcal{O}_\varphi = \left(\varphi^\dagger \varphi - \frac{v^2}{2} \right)^3 \supset v^3 H^3 + \frac{3}{2} v^2 H^4,$$

$$\mathcal{O}_{\varphi d} = \left(\varphi^\dagger \varphi \right) \square \left(\varphi^\dagger \varphi \right) \supset 2v \left(H \square H^2 + H^2 \square H \right) + H^2 \square H^2,$$

$$\mathcal{O}_{\varphi D} = \left(\varphi^\dagger D_\mu \varphi \right)^\dagger \left(\varphi^\dagger D^\mu \varphi \right) \supset \frac{v}{2} H \partial_\mu H \partial^\mu H + \frac{H^2}{4} \partial_\mu H \partial^\mu H.$$

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Constrained by EWPO

We look at HH and HHH production

$$\sigma = \sigma_{SM} + \sum_i c_i \sigma_{Int}^i + \sum_{i,j} c_{i,j} \sigma_{Sq}^{i,j}$$

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Constrained by EWPO

We look at HH and HHH production

$$\sigma = \sigma_{SM} + \sum_i c_i \sigma_{Int}^i + \sum_{i,j} c_{i,j} \sigma_{Sq}^{i,j}$$

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Sensitivity linear

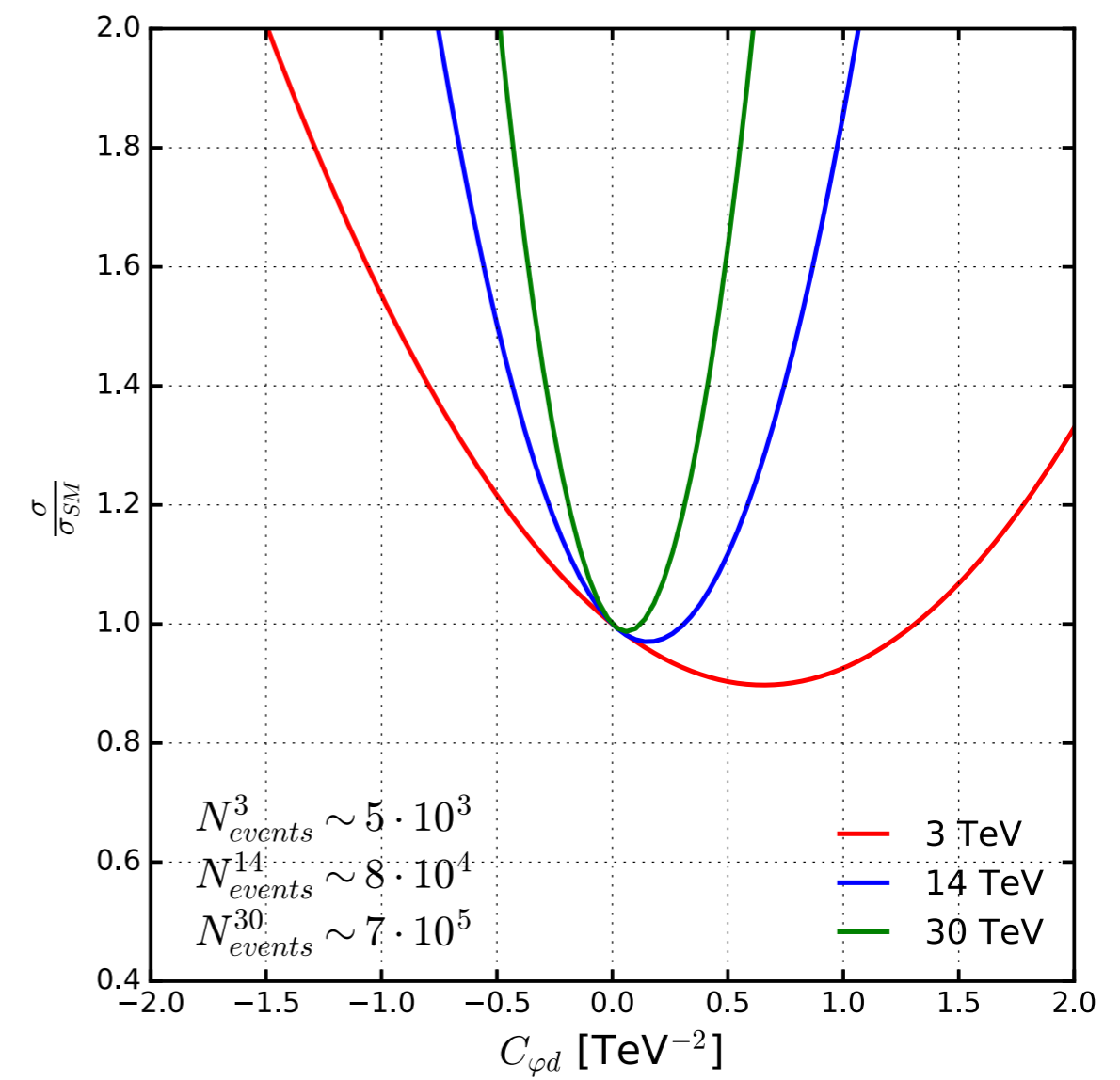
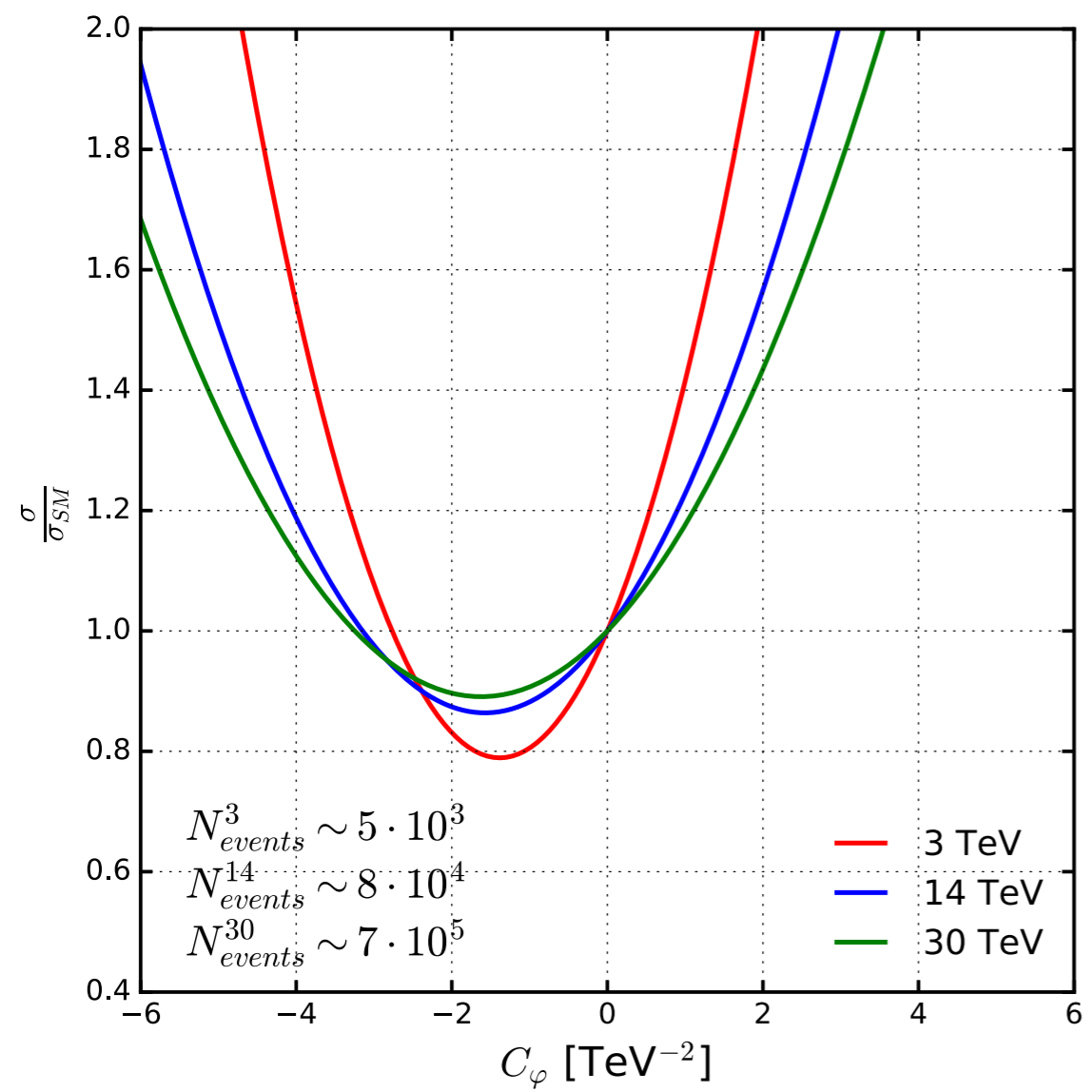
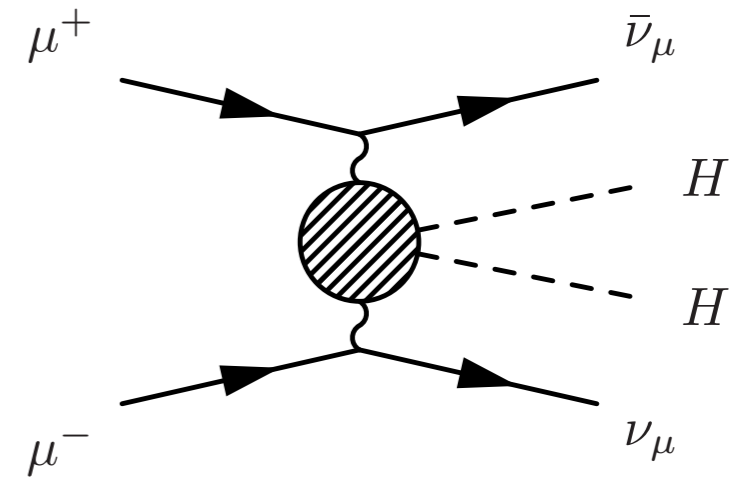
Sensitivity quadratic

$$\mathcal{L}_3 = 6 \text{ ab}^{-1}$$

$$\mathcal{L}_{14} = 20 \text{ ab}^{-1}$$

$$\mathcal{L}_{30} = 100 \text{ ab}^{-1}$$

HH production

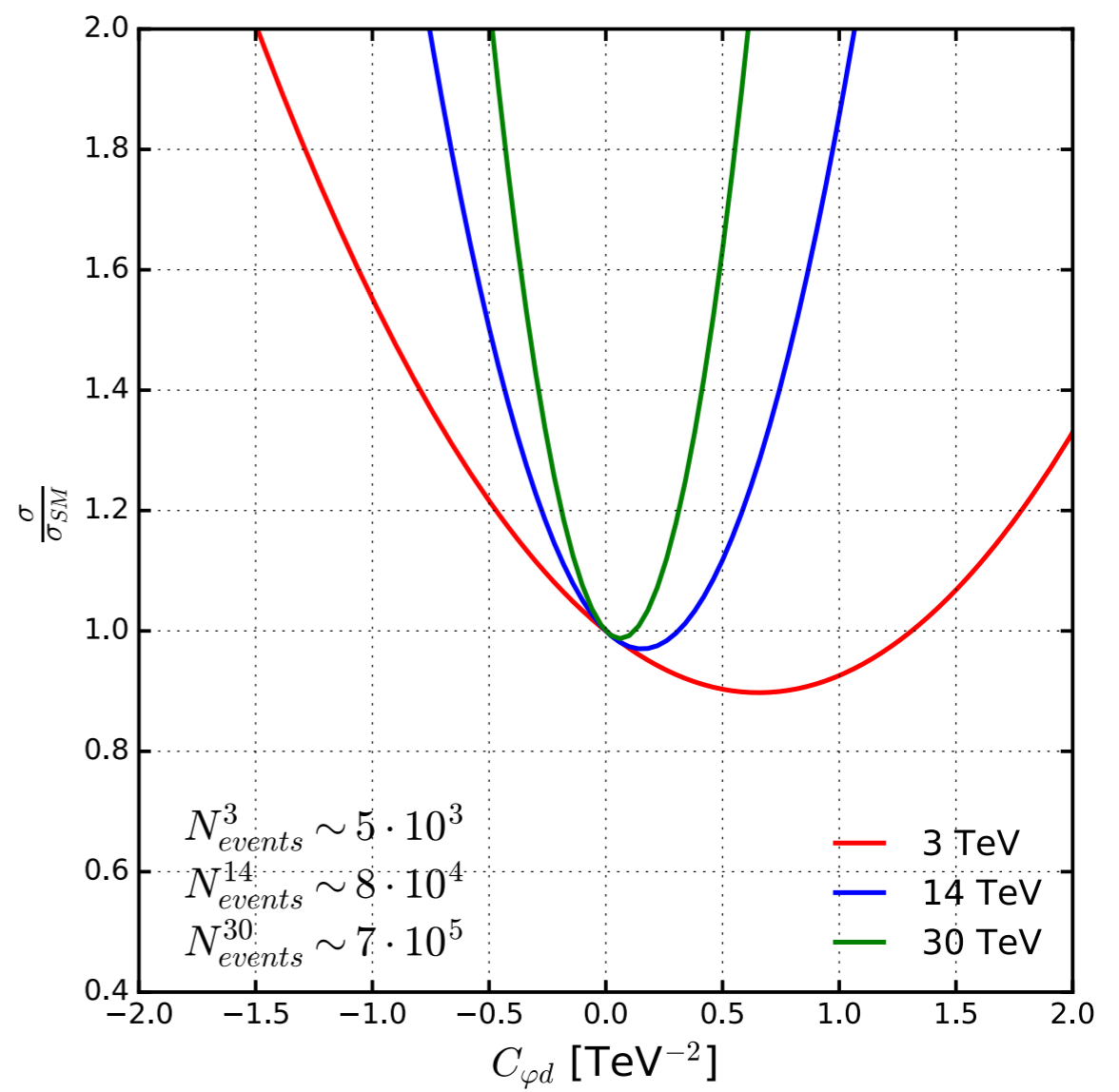
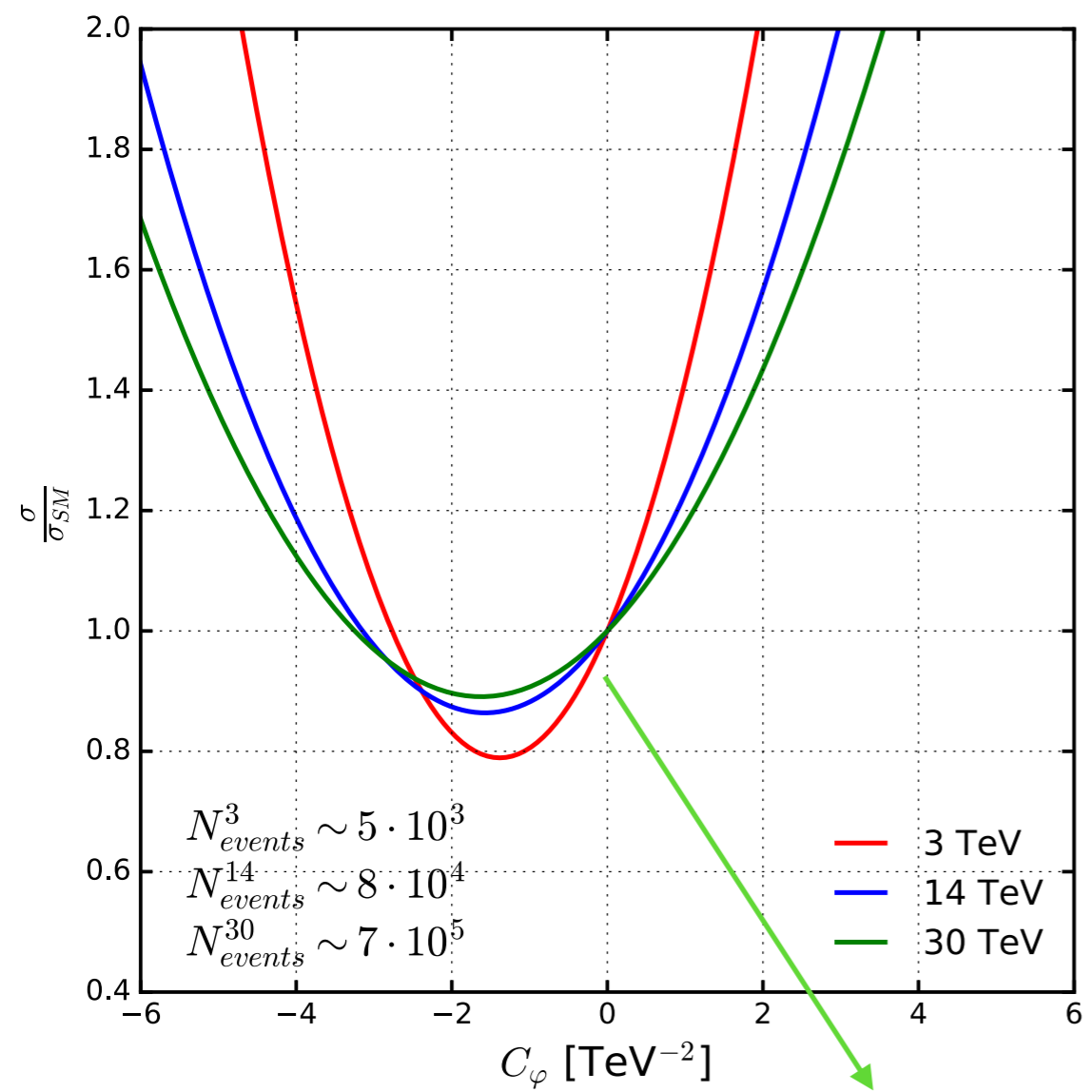
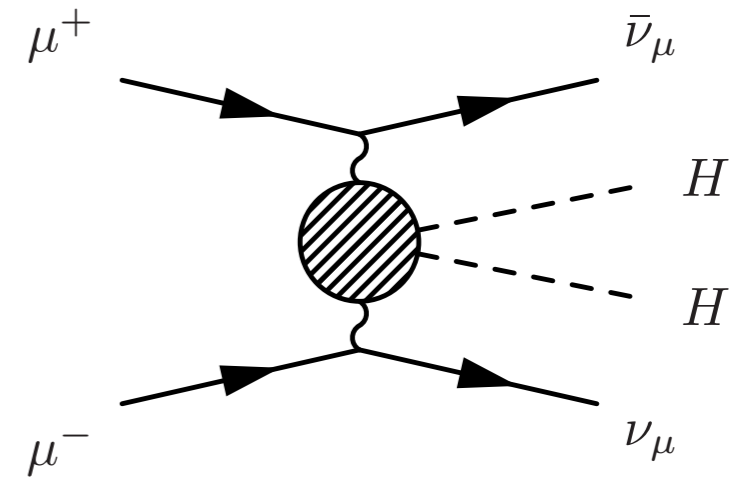


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HH production



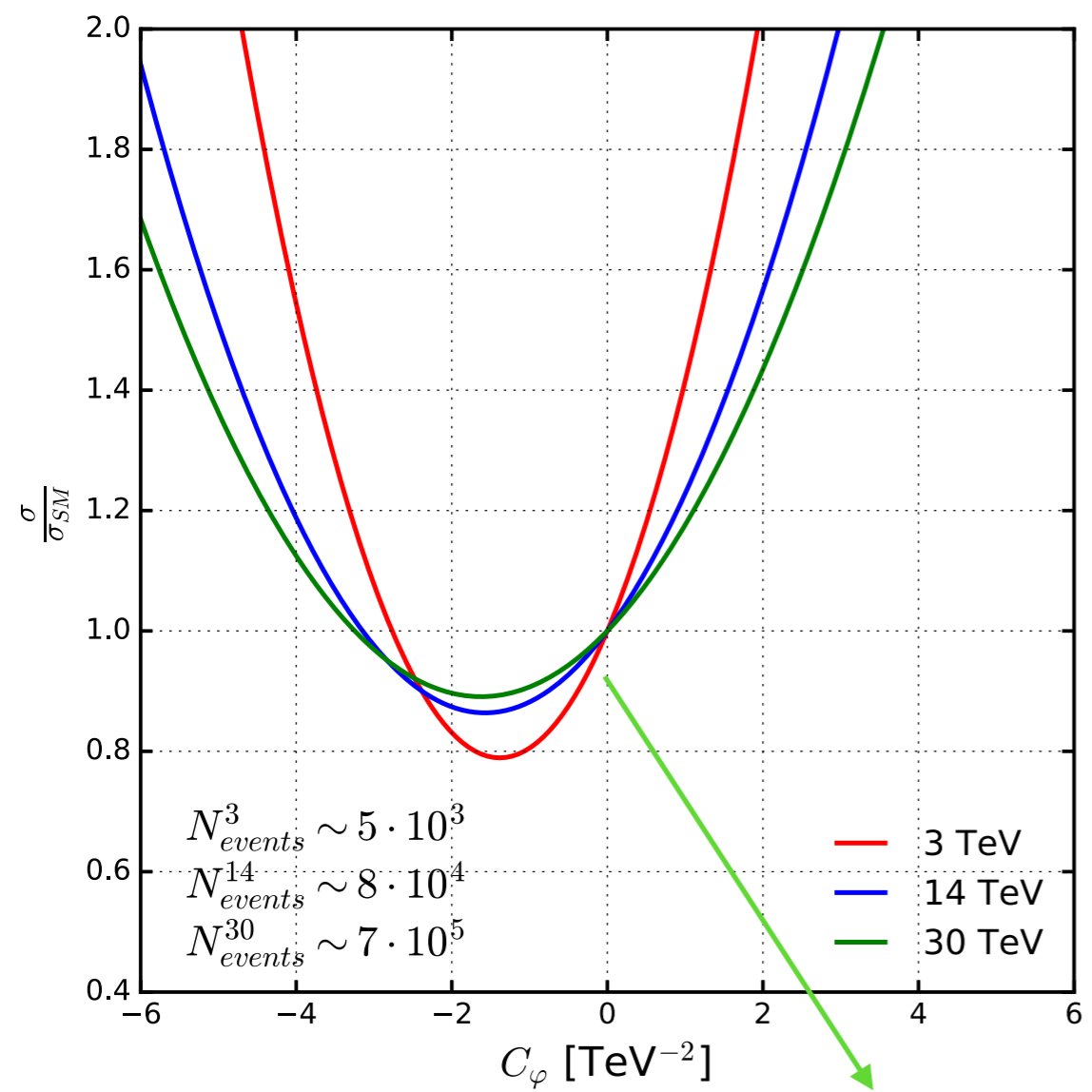
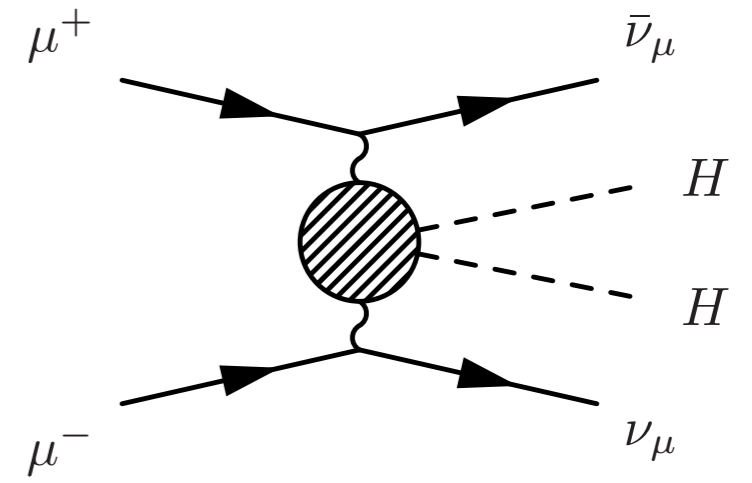
High energy not helping!

$$\mathcal{L}_3 = 6 \text{ ab}^{-1}$$

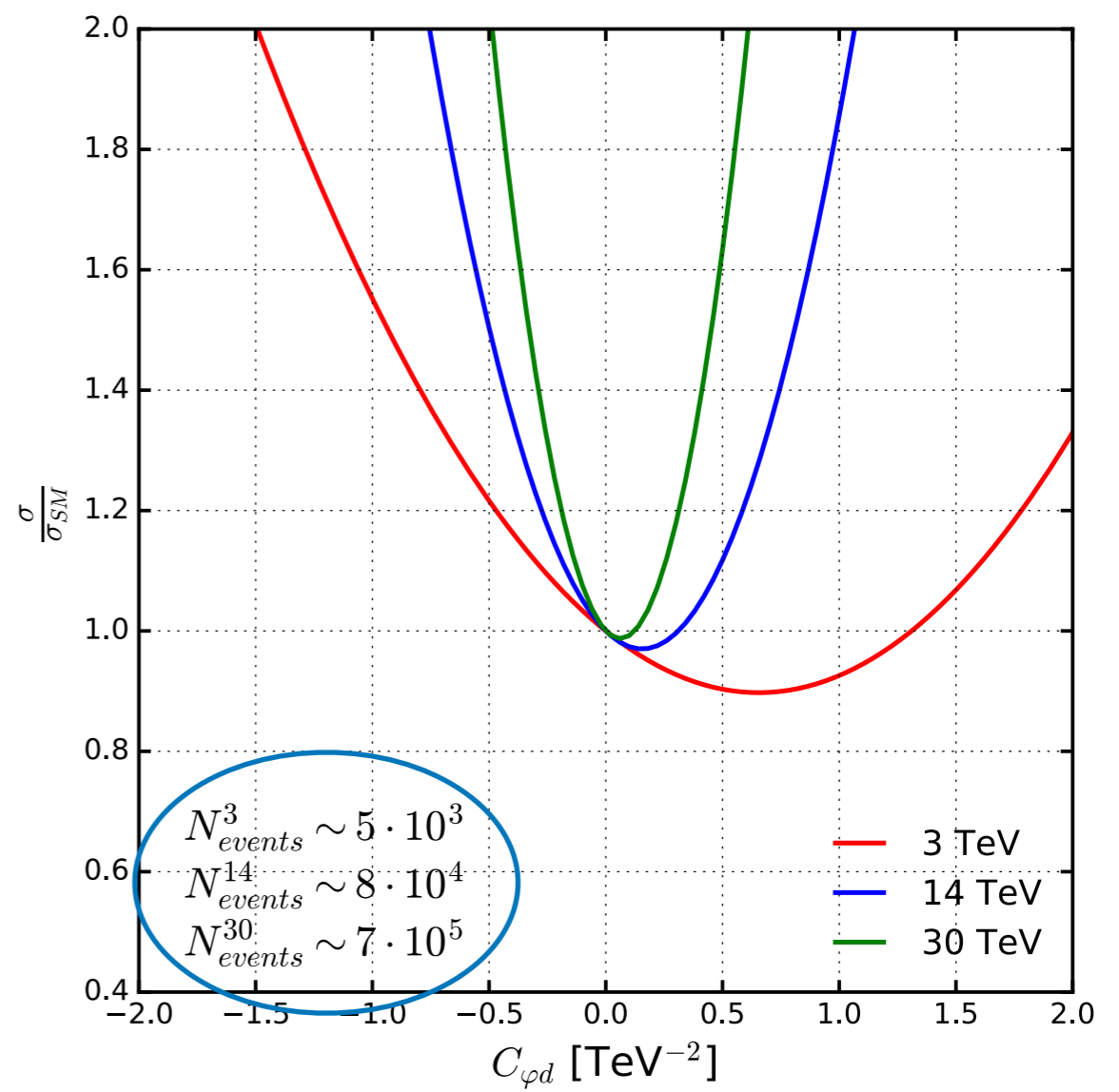
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HH production



High energy not helping!



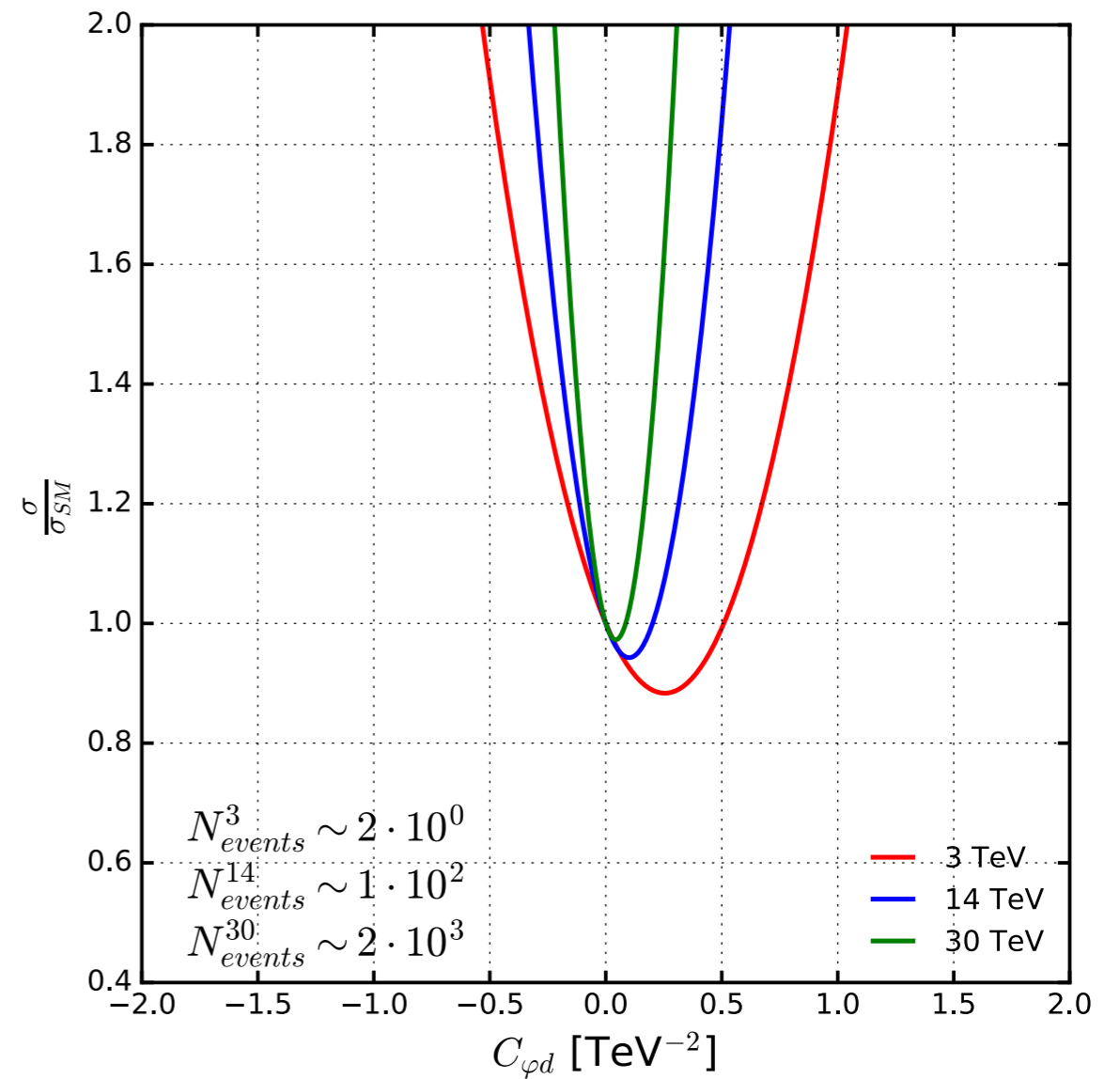
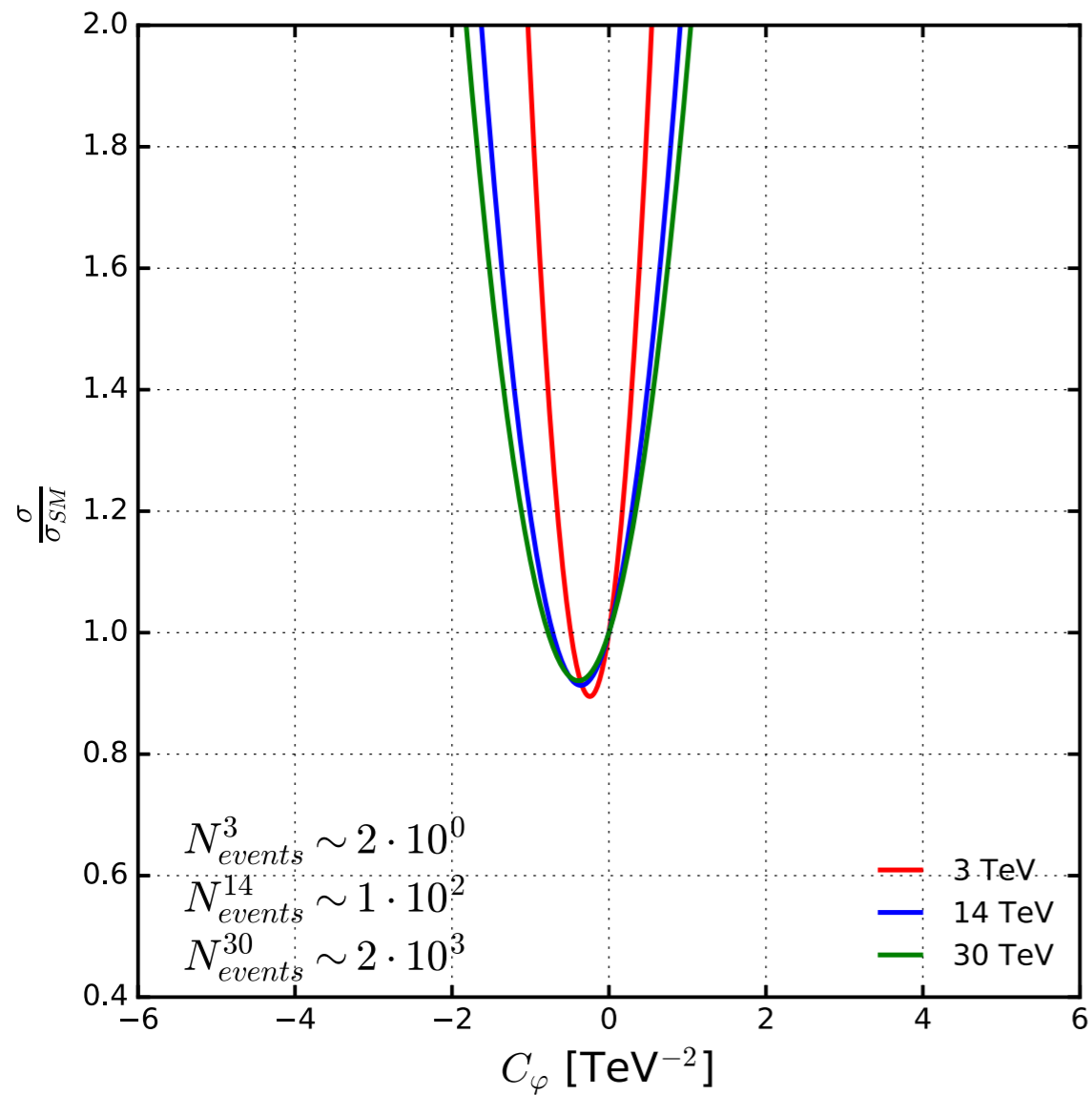
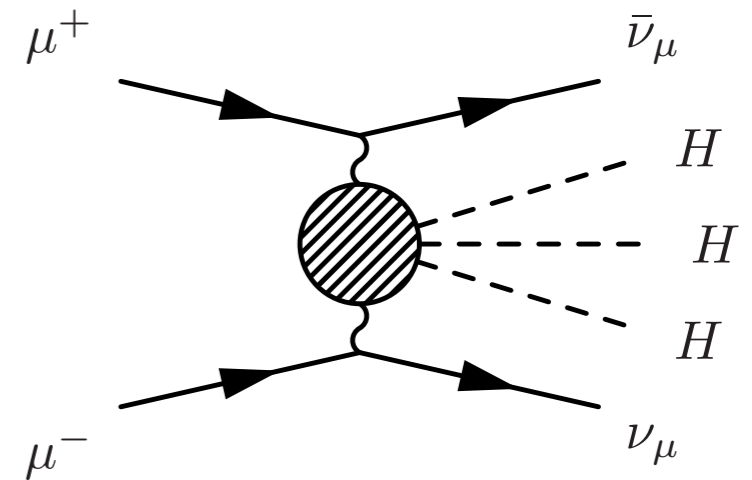
Many events!

$$\mathcal{L}_3 = 6 \text{ ab}^{-1}$$

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HHH production



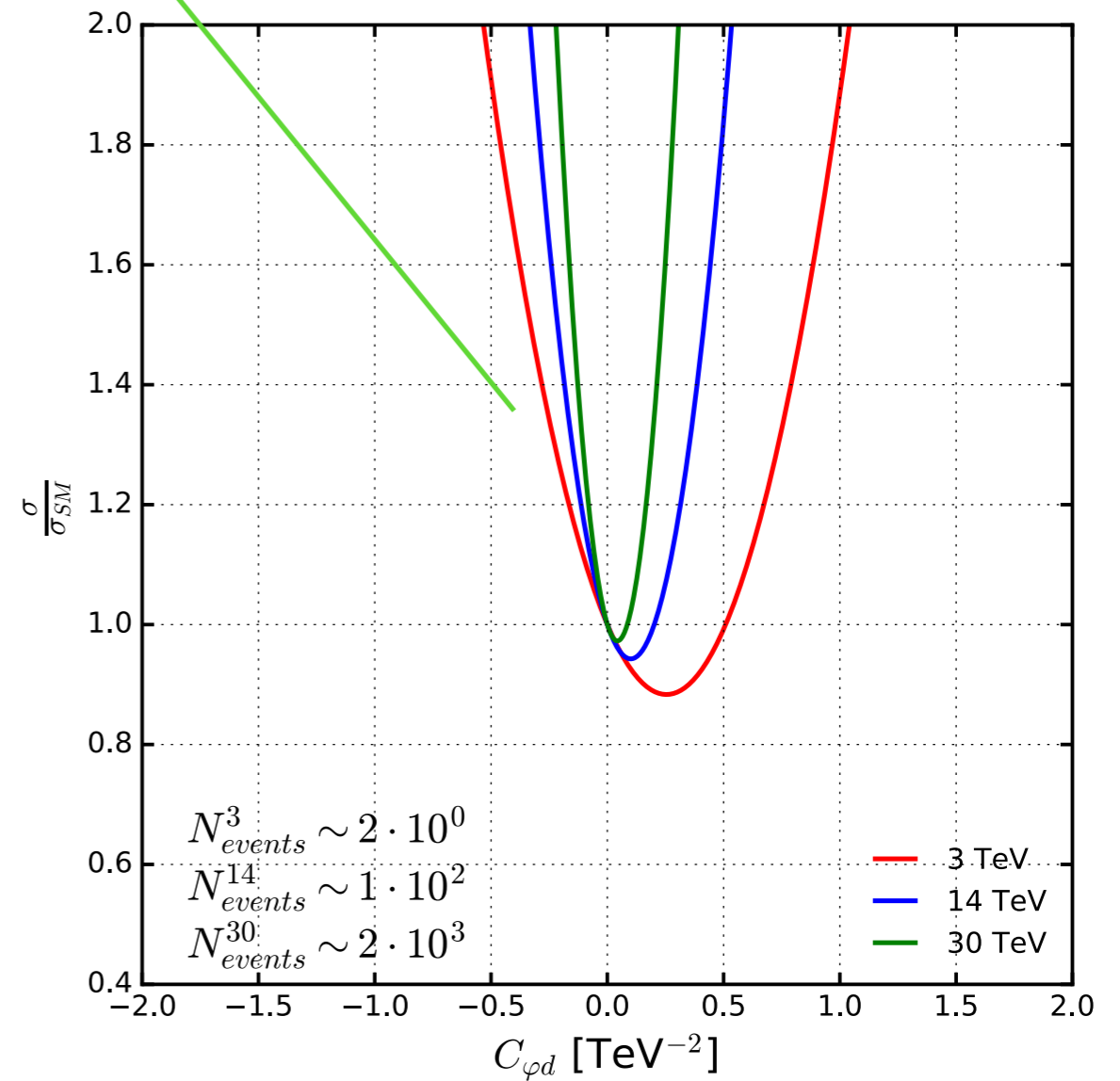
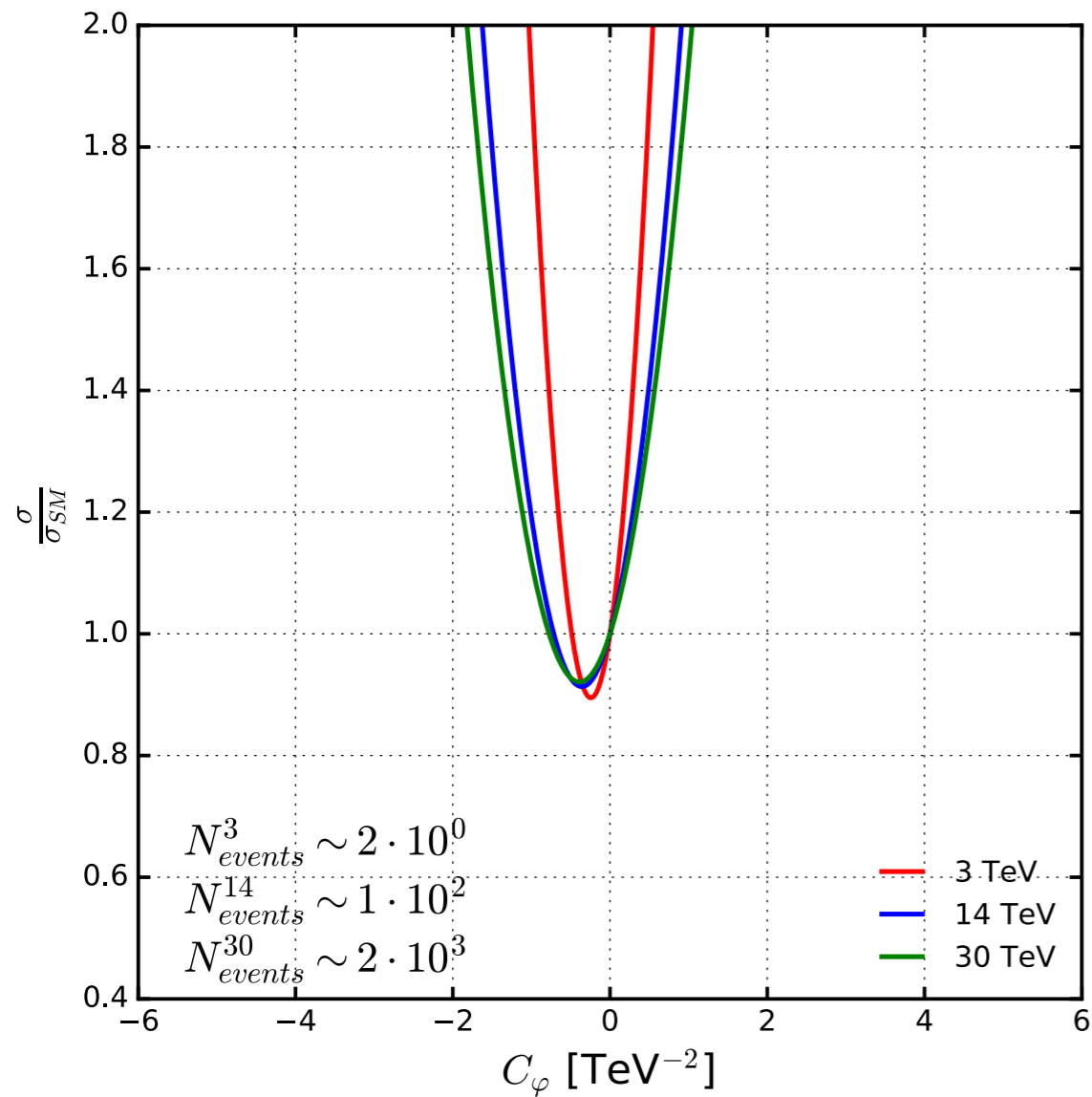
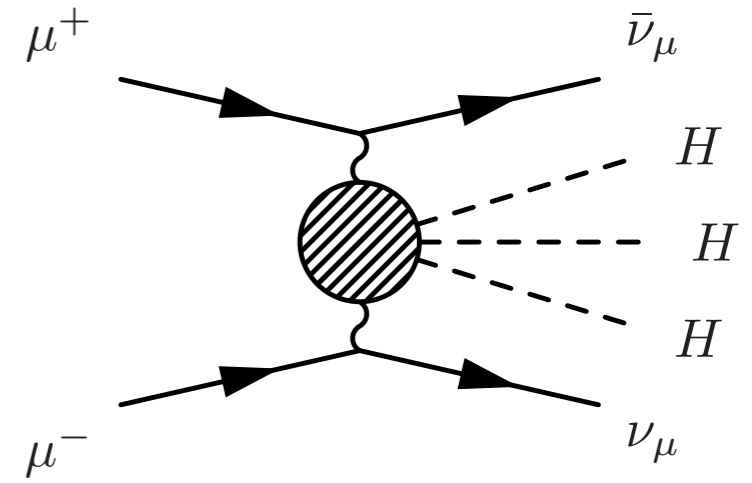
$$\mathcal{L}_3 = 6 \text{ ab}^{-1}$$

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HHH production

Higher sensitivity



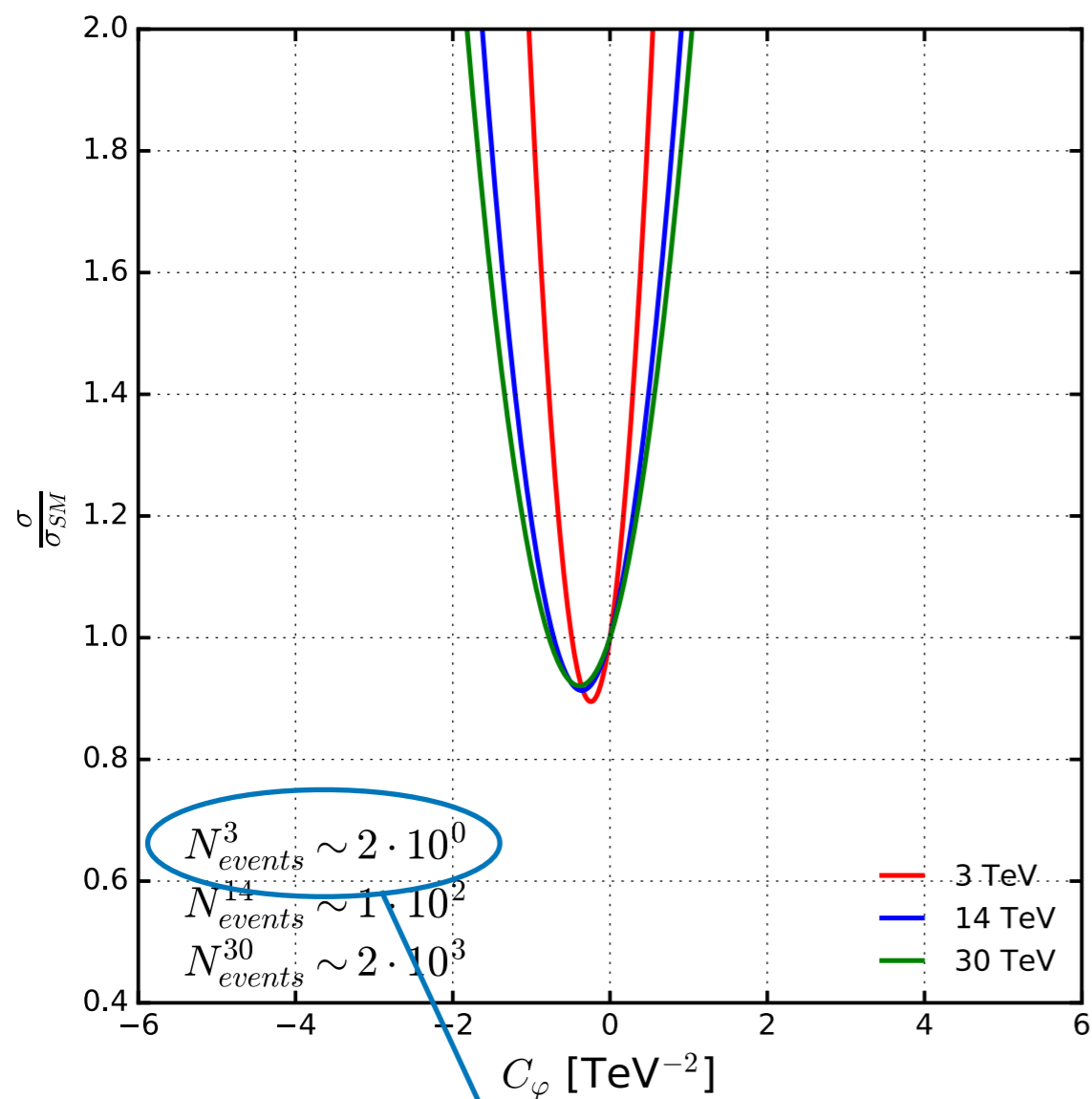
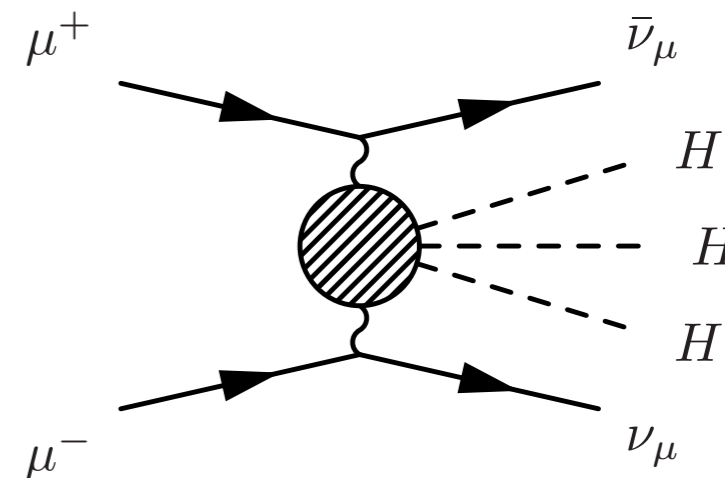
$$\mathcal{L}_3 = 6 \text{ ab}^{-1}$$

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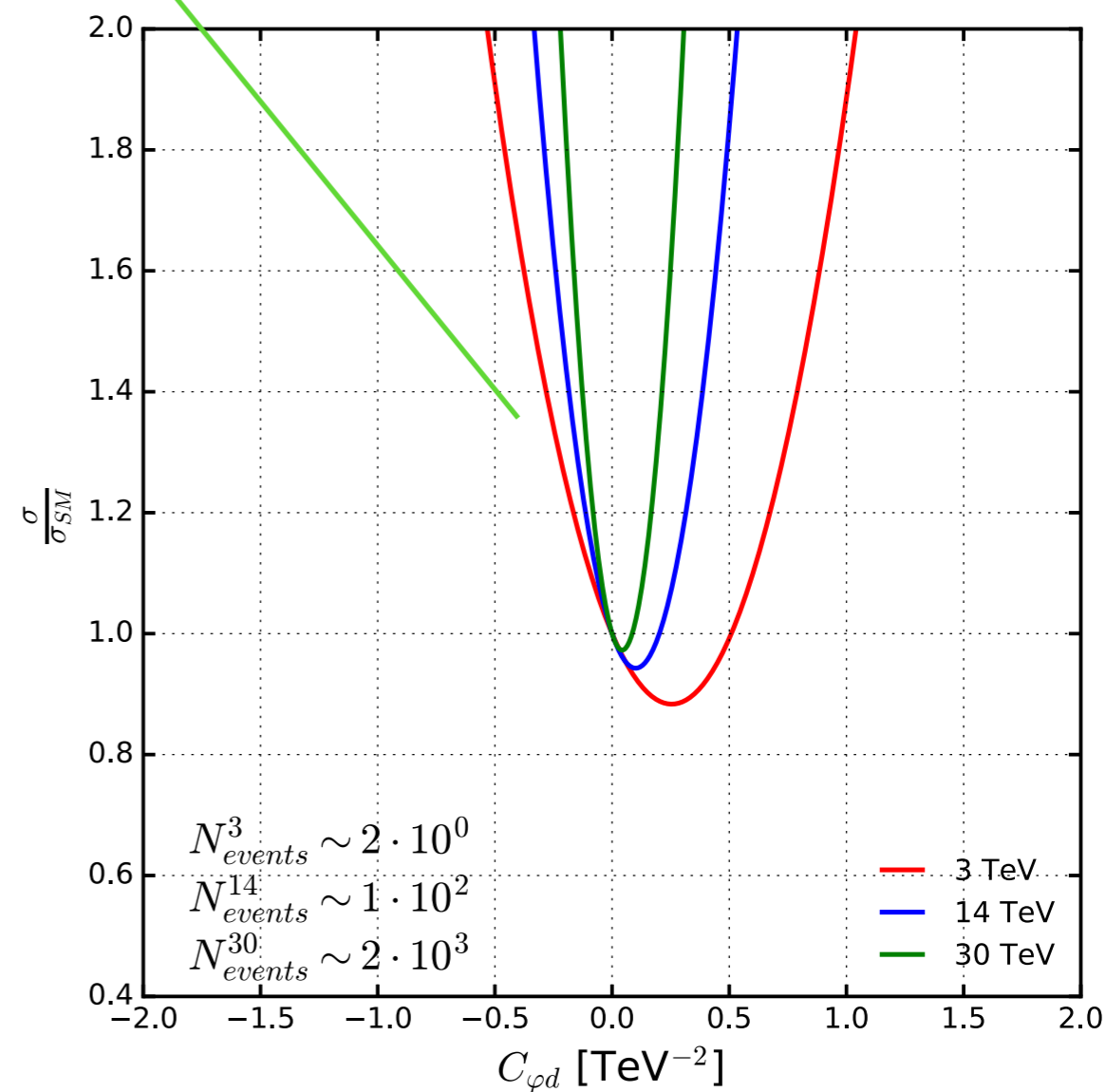
$$\mathcal{L}_{30} = 100 \text{ ab}^{-1}$$

HHH production

Higher sensitivity

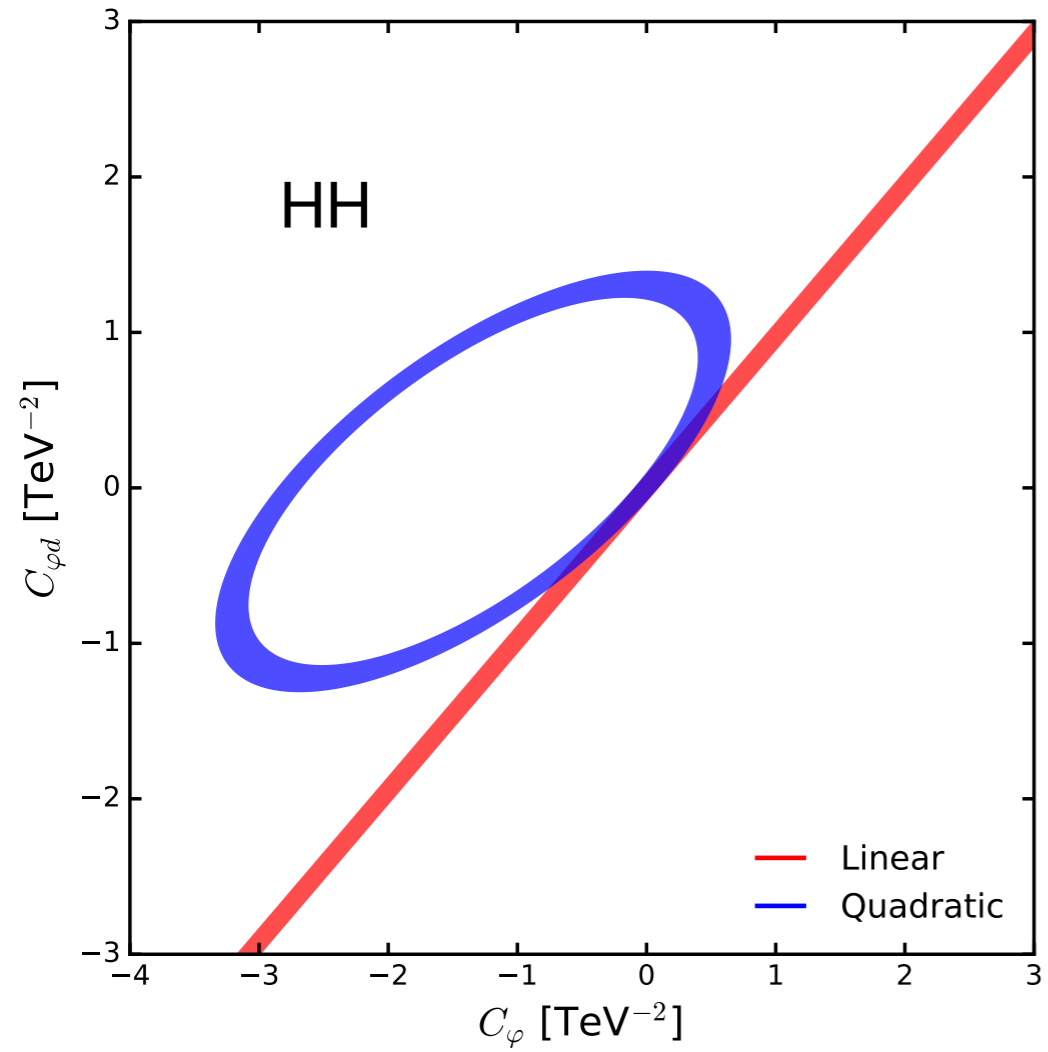


No events



3 TeV collider

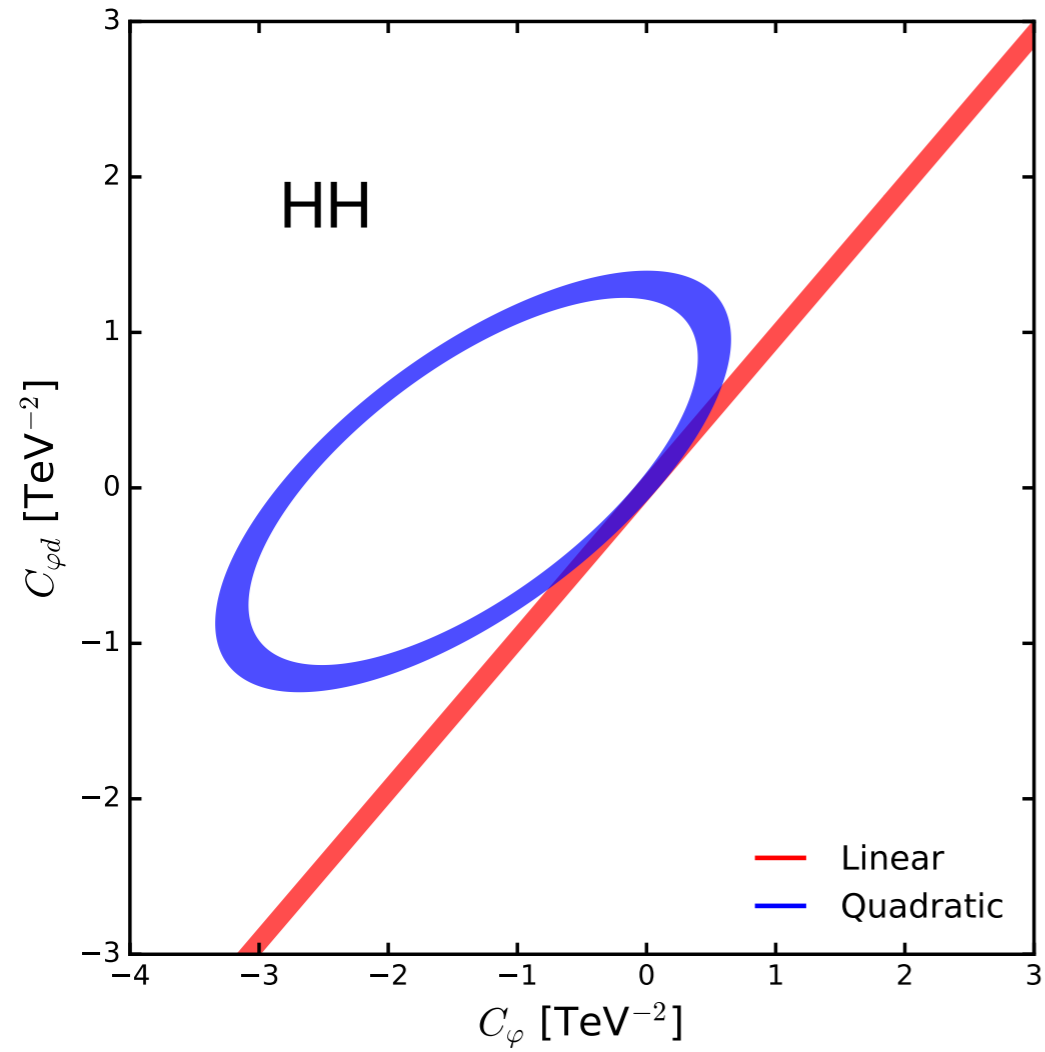
$$\frac{S}{\sqrt{B}} = \frac{|\mathcal{L} \cdot (\sigma - \sigma_{SM})|}{\sqrt{\mathcal{L} \cdot \sigma_{SM}}} \leq 2$$



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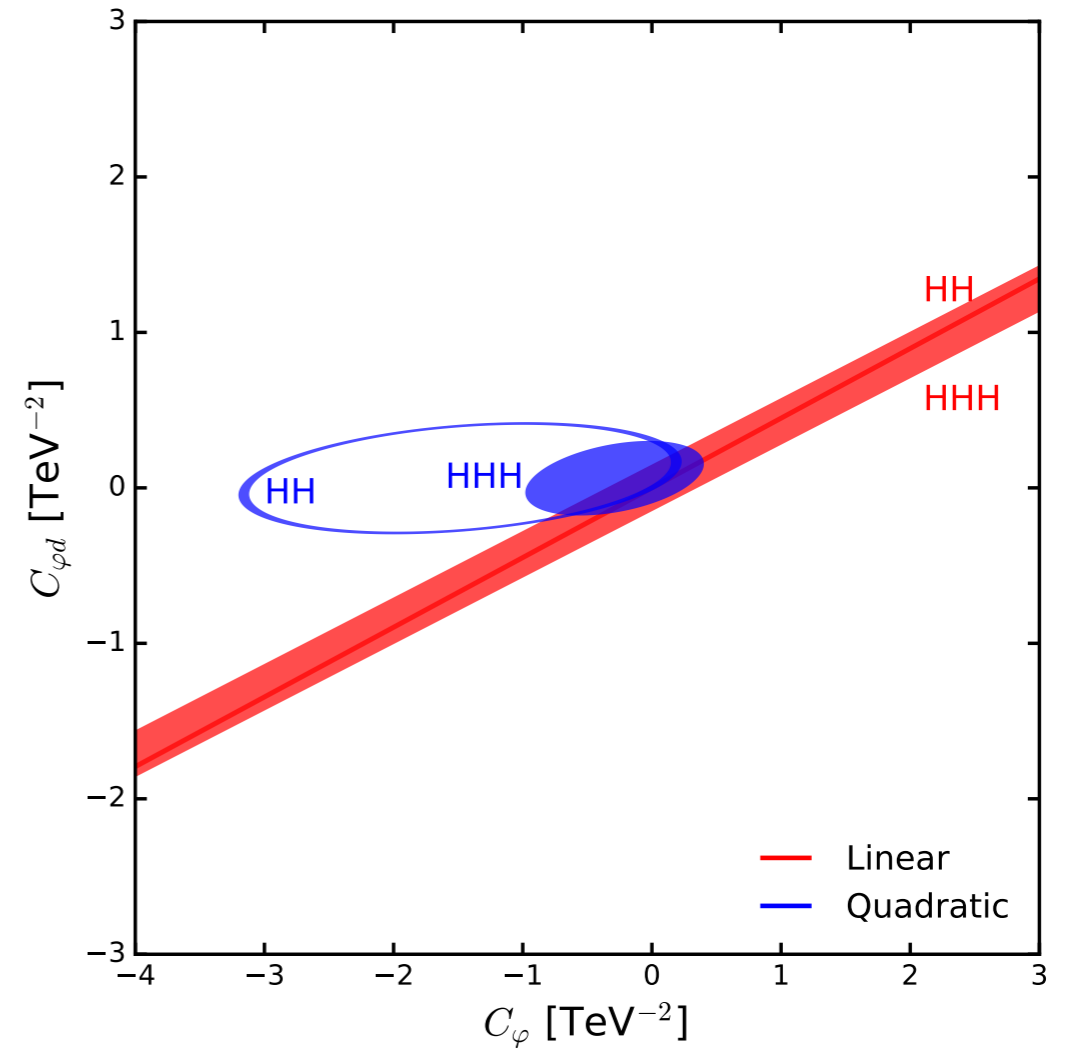
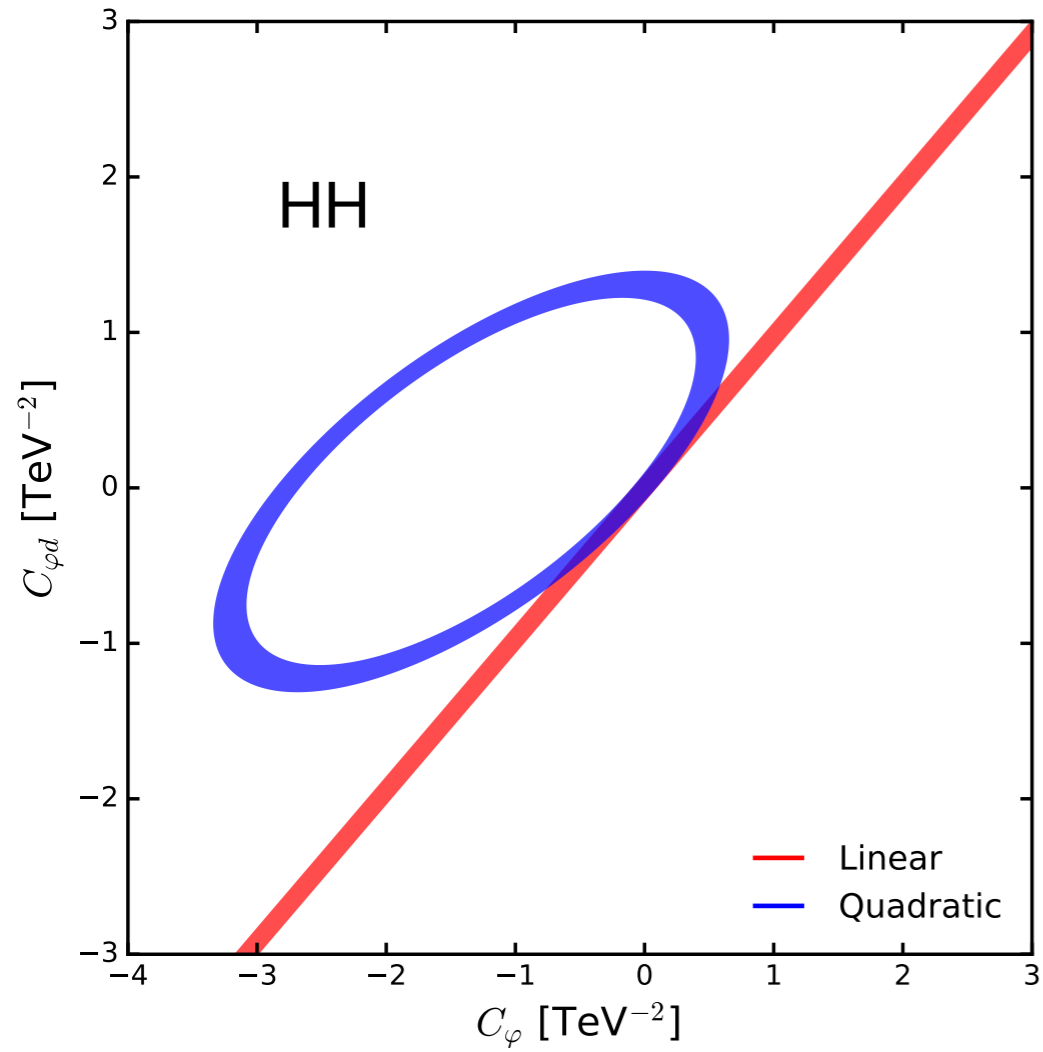
14 TeV collider



3 TeV collider

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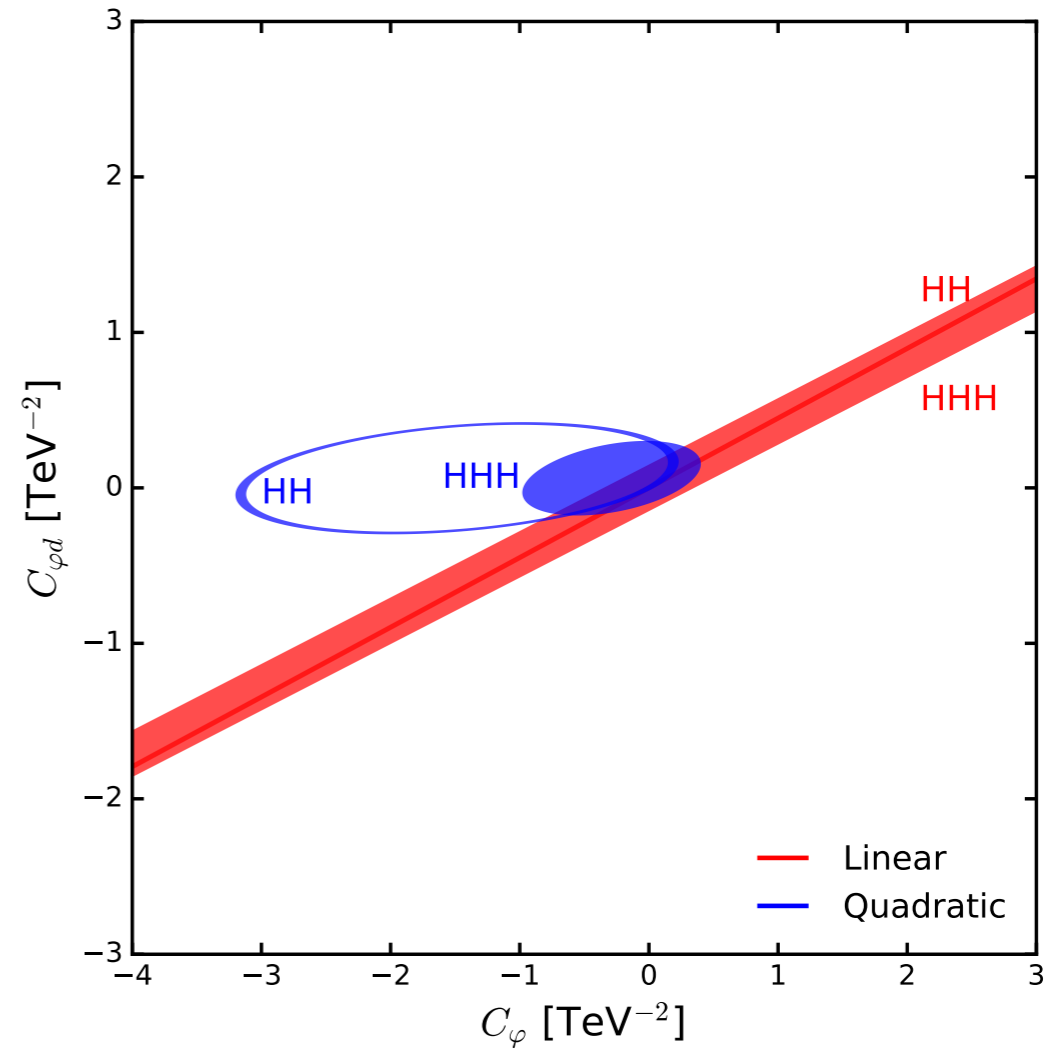
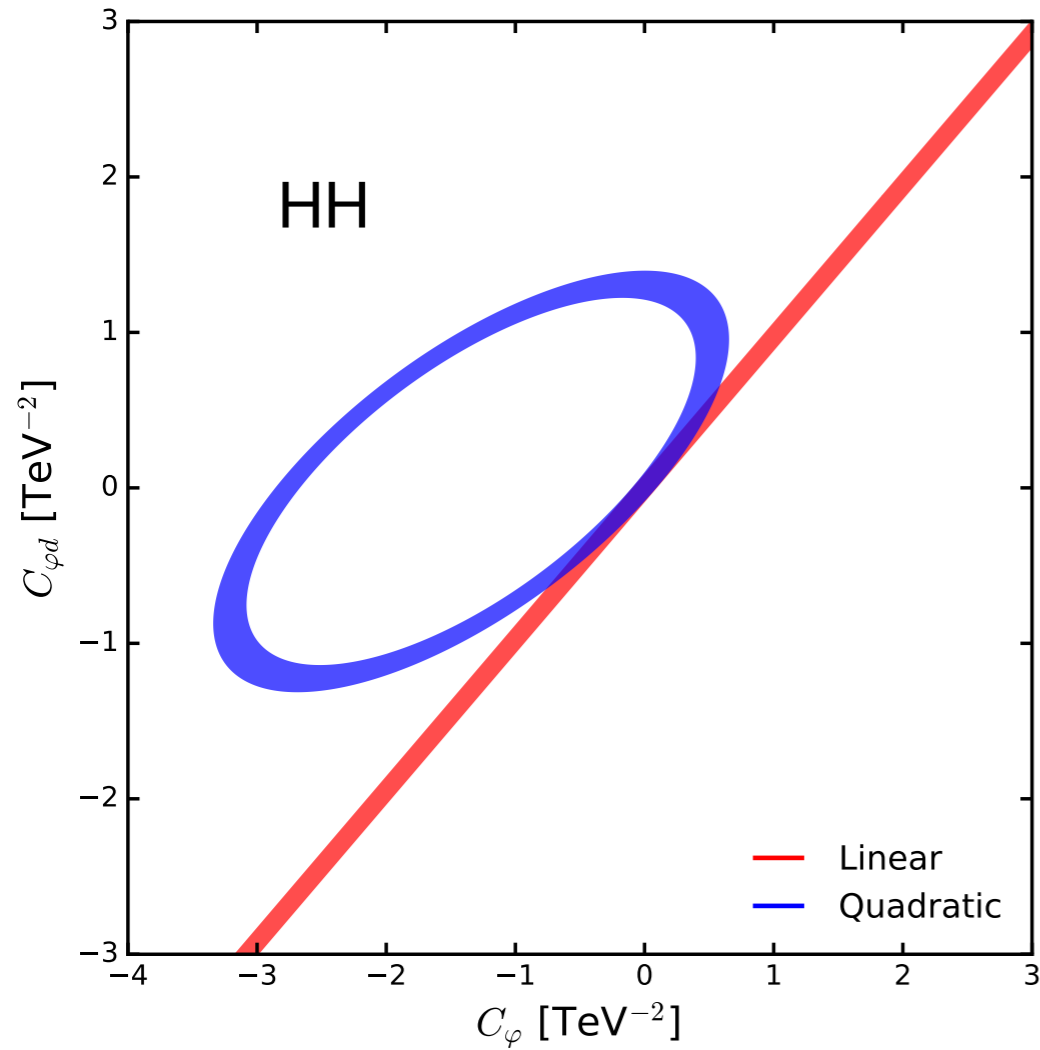
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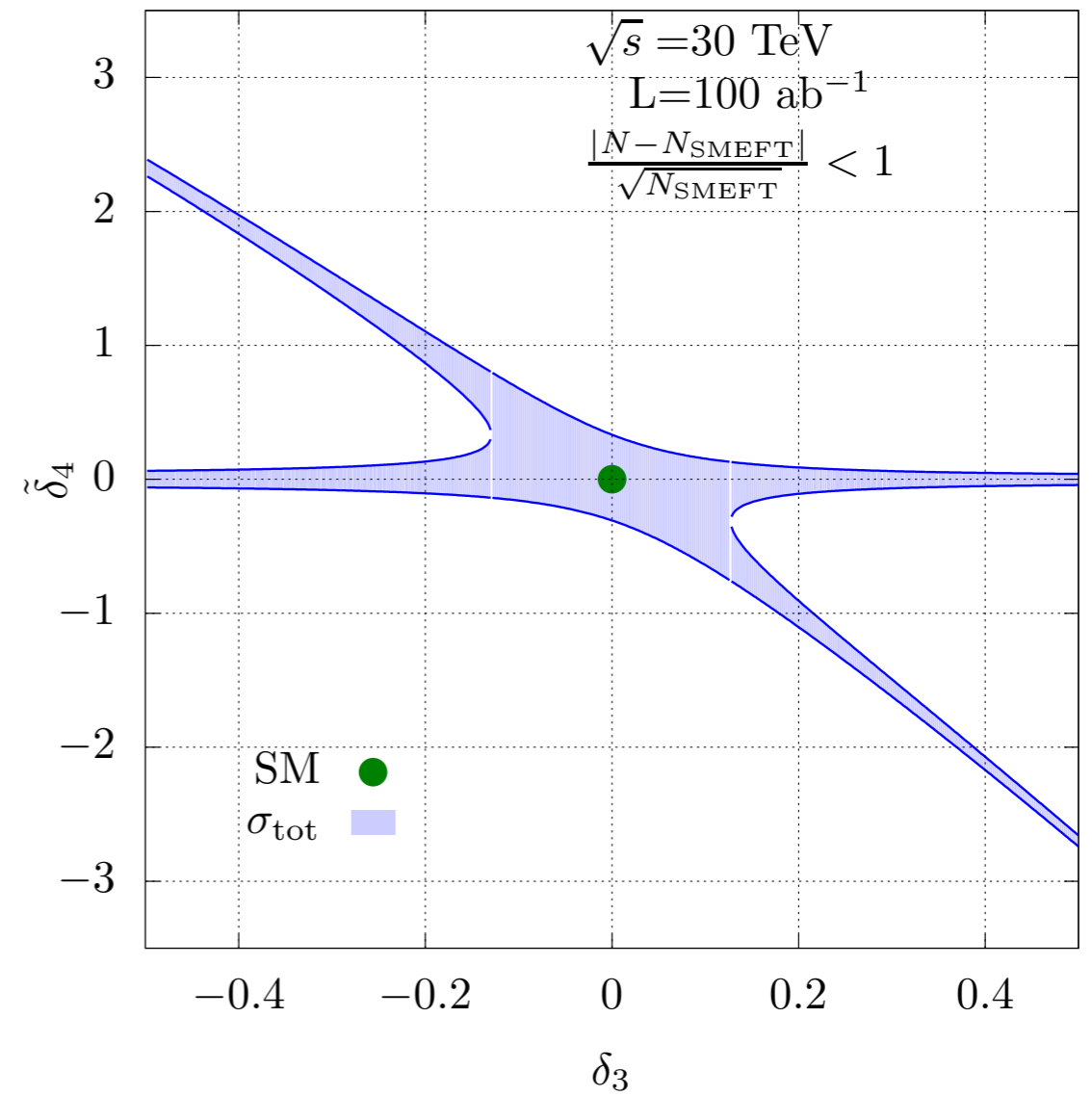
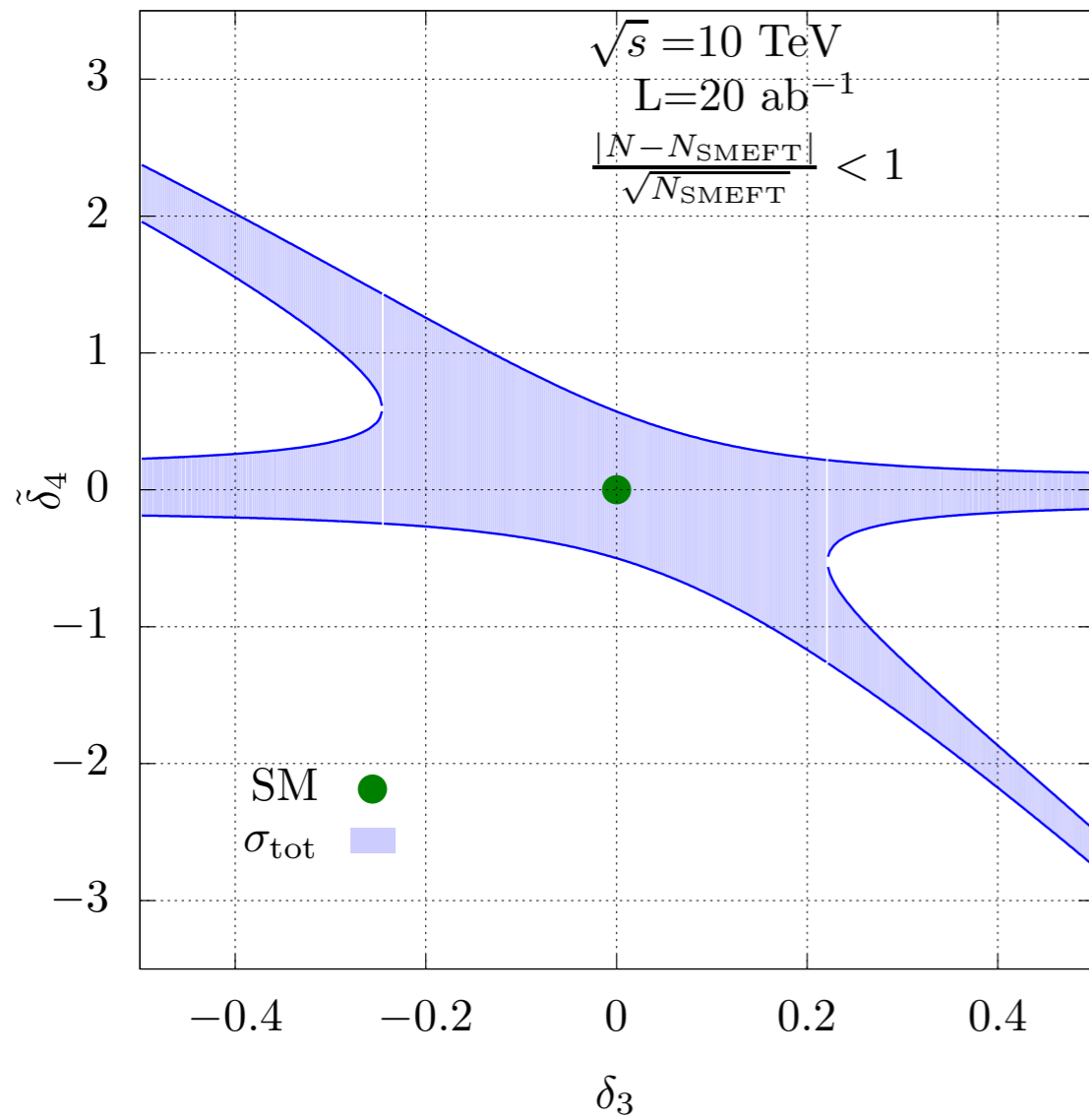
14 TeV collider



| | 3 TeV | 14 TeV |
|-----------------|-----------------|-----------------|
| C_φ | $[-3.33, 0.65]$ | $[-0.66, 0.23]$ |
| $C_{\varphi d}$ | $[-1.31, 1.39]$ | $[-0.17, 0.30]$ |

95% confidence level

$$\tilde{\delta}_4 = \delta_4 - 6\delta_3$$



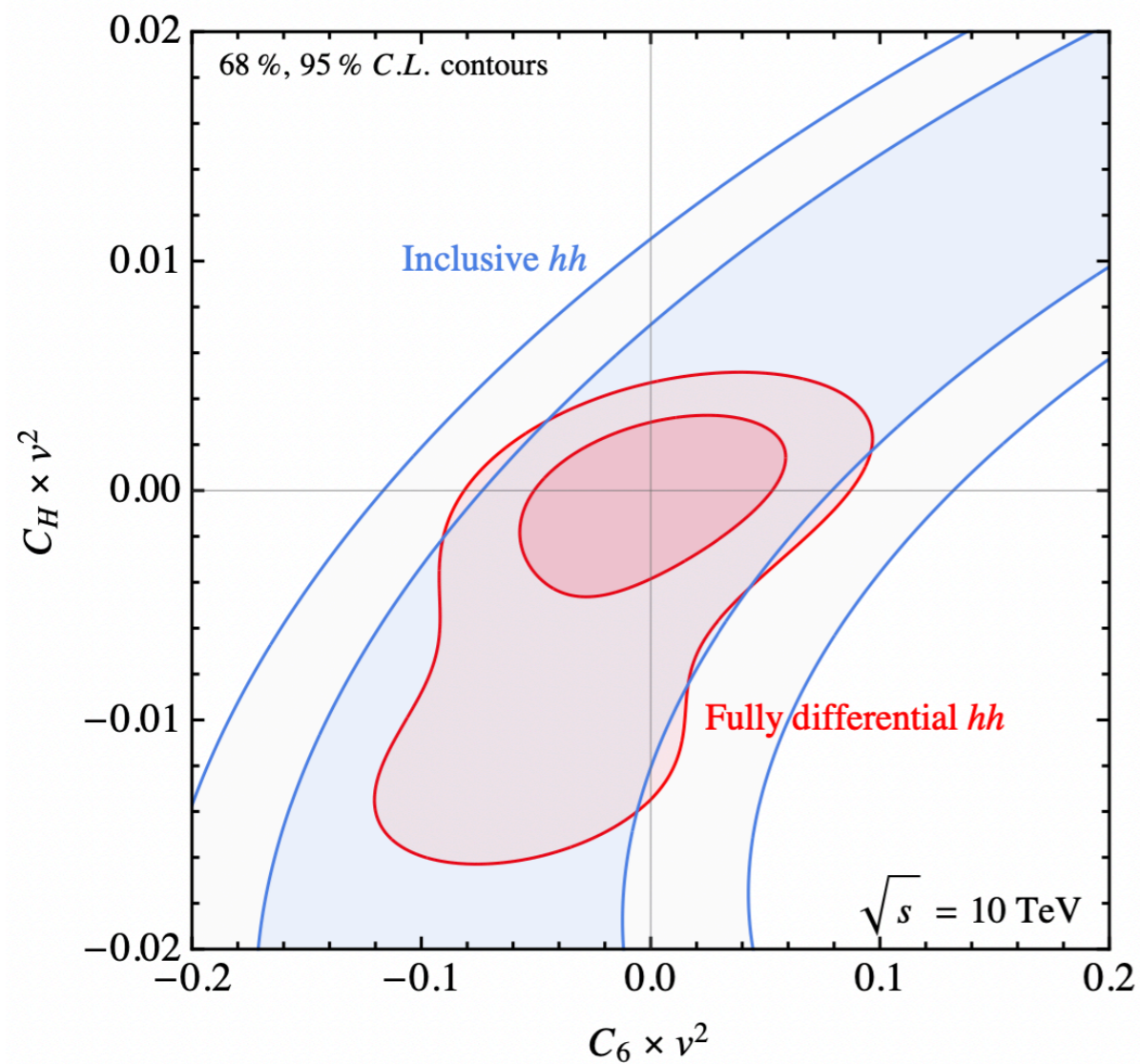
[Buttazzo, Franceschini, Wulzer arXiv:2012.11555]

$$\mathcal{O}_6 = -\frac{m_h^2}{2v^2} \left(H^\dagger H - \frac{v^2}{2} \right)^3,$$

$$\mathcal{O}_H = \frac{1}{2} \left(\partial_\mu (H^\dagger H) \right)^2$$

$$\delta\kappa_3 = v^2 \left(C_6 - \frac{3}{2} C_H \right)$$

$$\kappa = \kappa_V = \kappa_f = 1 - \frac{C_H v^2}{2}$$



10 TeV

