Entanglement in SMEFT : Top pair

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Based on

Quantum SMEFT tomography: top quark pair production at the LHC

RA, Eric Madge, Fabio Maltoni and Luca Mantani

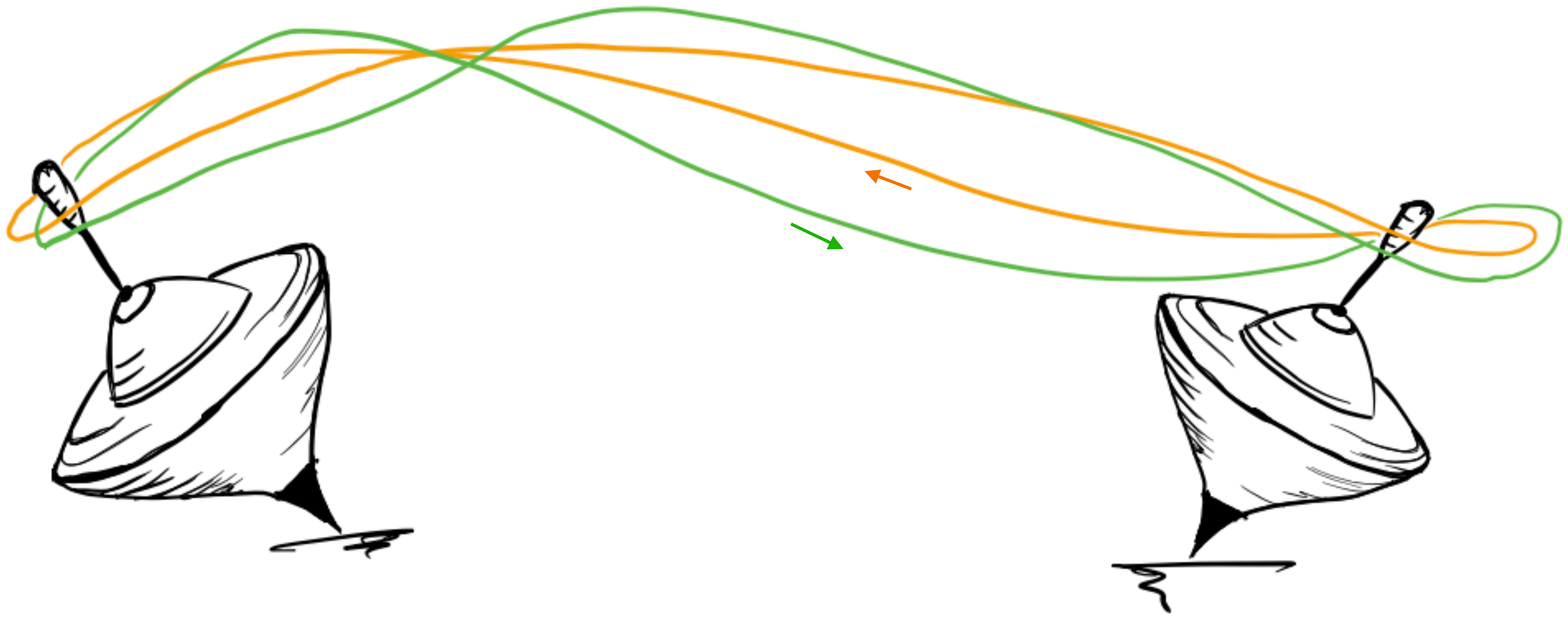
hep-ph/2203.05619

Motivation

- In general, top pair produced entangled

- In the SM, there are two point of maximal entanglement and regions of vanishing of entanglement

- How does SMEFT change these effects?



[Afik and de Nova, 21']

[Fabbrichesi, Floreanini, Panizzo, 21']

[Severi, Degli, Maltoni, Sioli, 21']

[Aoude, Madge, Maltoni, Mantani, 22']

[Afik and de Nova, 22']

[Aguilar-Saavedra, Casas, 22']

Spin production density matrix

[Afik and de Nova, 21']

The state-density matrix is obtained from the R-matrix

$$R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2} \mathcal{M}_{\alpha_1\beta_1}$$

where $\mathcal{M}_{\alpha\beta} \equiv \langle t(k_1,\alpha)\bar{t}(k_2,\beta)|\mathcal{T}|a(p_1)b(p_2)\rangle$

$I = gg, q\bar{q}$

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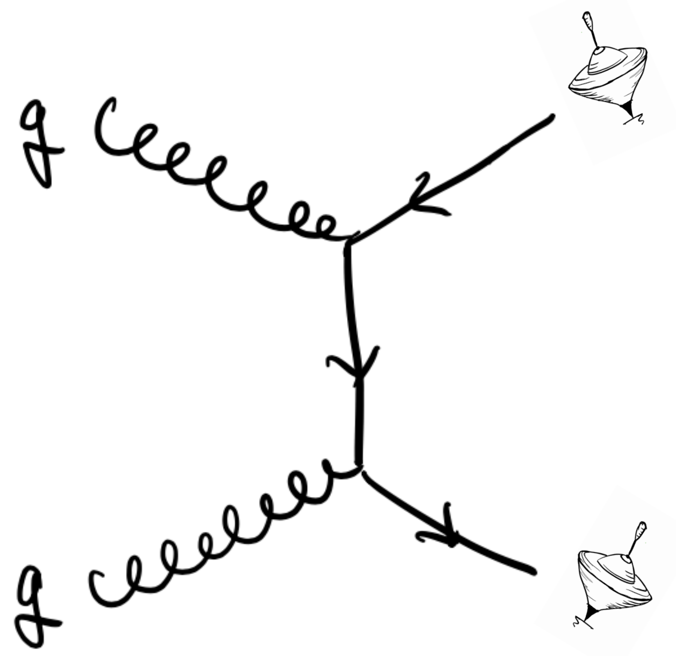
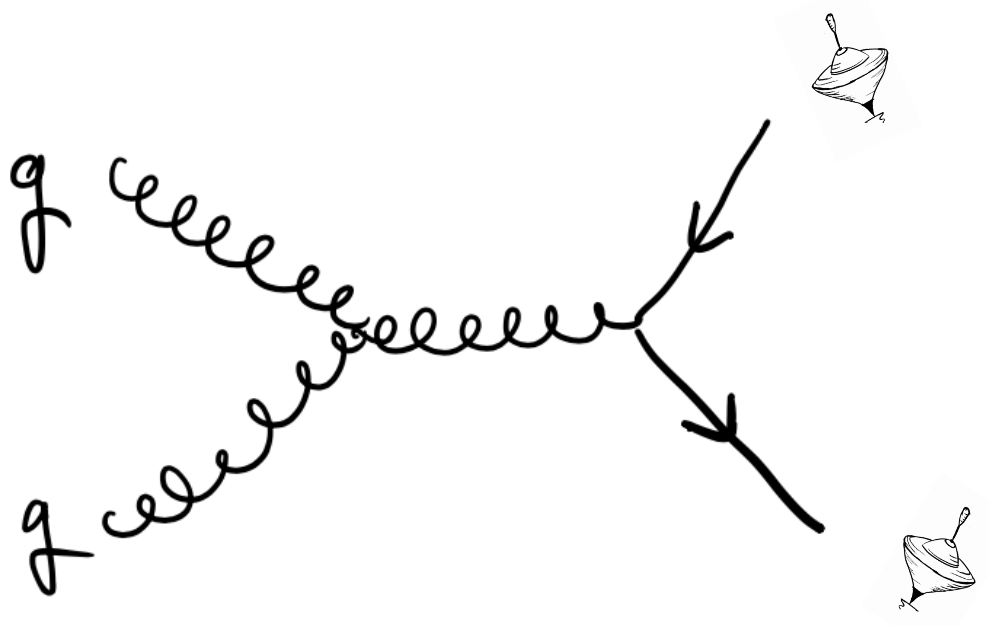
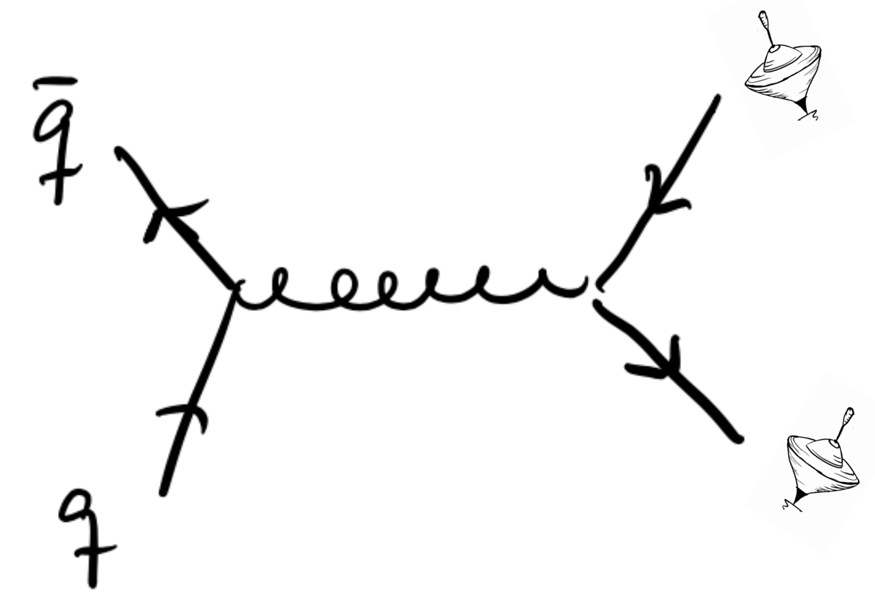
SM:







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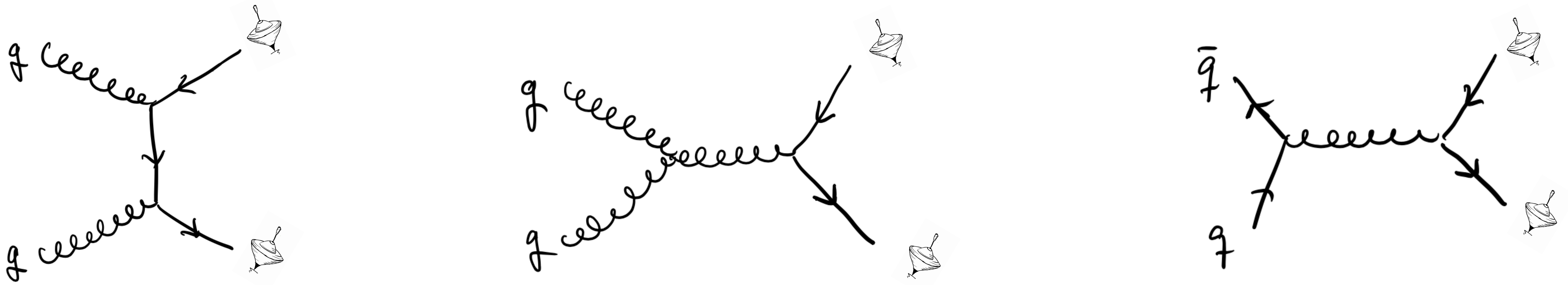

Mixed state of qq and gg initiated channels, weighted by the luminosity functions

$$R(\hat{s},\boldsymbol{k}) = \sum_I L^I(\hat{s}) R^I(\hat{s},\boldsymbol{k})$$

Spin production density matrix

[Afik and de Nova, 21']

4x4 matrix in spin-space of the top pair.

Fano decomposition: (spanned by tensor prod. of Pauli and Identity)

$$R = \tilde{A}\,\mathbb{1}_2 \otimes \mathbb{1}_2 + \tilde{B}_i^{+} \sigma^i \otimes \mathbb{1}_2 + \tilde{B}_i^{-} \mathbb{1}_2 \otimes \sigma^i + \tilde{C}_{ij}\, \sigma^i \otimes \sigma^j \,.$$

16-coefficients where the norm $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega \mathrm{d}\hat{s}} = \frac{\alpha_s^2 \beta}{\hat{s}^2} \tilde{A}\,(\hat{s}, \boldsymbol{k})$

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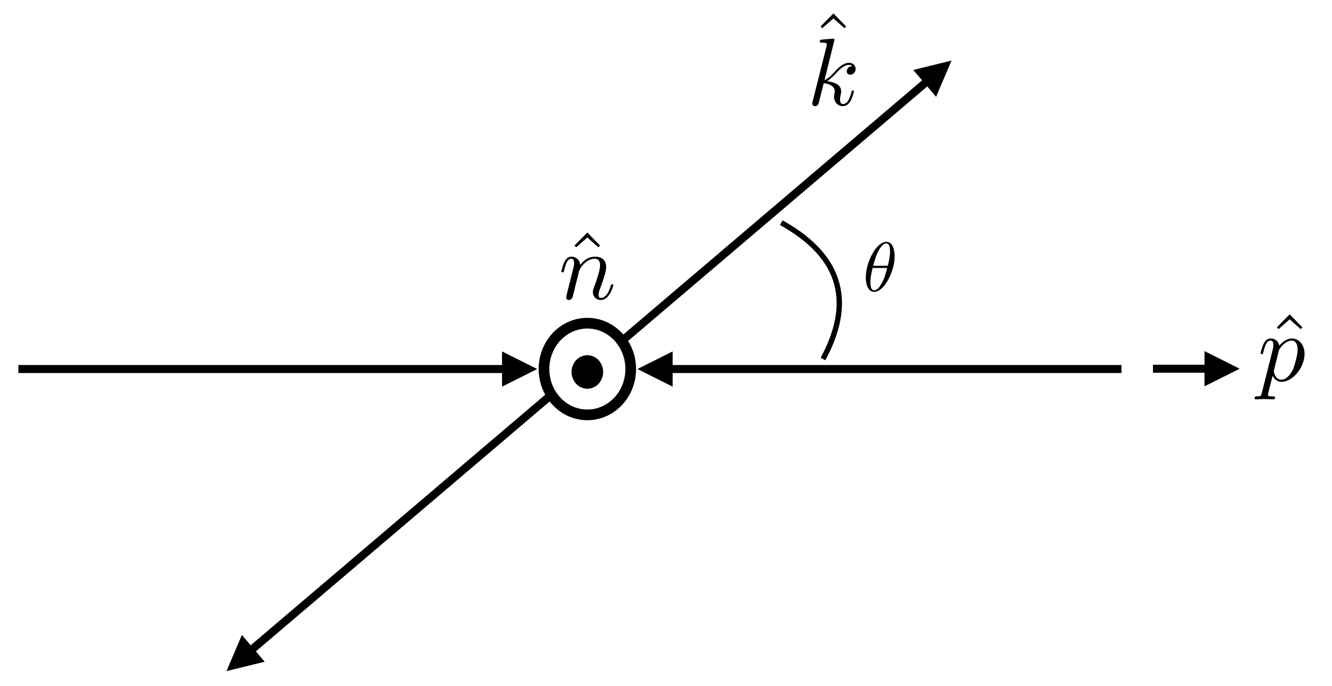
Normalize the state as $\rho = R/\mathrm{tr}(R)$

$$\rho = \frac{\mathbb{1}_2 \otimes \mathbb{1}_2 + B_i^+ \sigma^i \otimes \mathbb{1}_2 + B_i^- \mathbb{1}_2 \otimes \sigma^i + C_{ij}\, \sigma^i \otimes \sigma^j}{4}.$$

Density matrix and helicity-basis

Helicity basis:

$$\{\boldsymbol{k},\boldsymbol{n},\boldsymbol{r}\}: \ \boldsymbol{r} = \frac{(\boldsymbol{p} - z\boldsymbol{k})}{\sqrt{1-z^2}}, \quad \boldsymbol{n} = \boldsymbol{k} \times \boldsymbol{r},$$



To expand in this basis, e.g.

$$C_{nn} = \mathrm{tr}[C_{ij}\,\boldsymbol{n}\otimes\boldsymbol{n}\,]$$

Phase-space parametrized by: $\beta^2 = (1 - 4m_t^2/\hat{s})$ and $\cos\theta$

Entanglement in bipartite systems

Given a bipartite system $\mathcal{H}_{\rm ab} = \mathcal{H}_{\rm a} \otimes \mathcal{H}_{\rm b}$

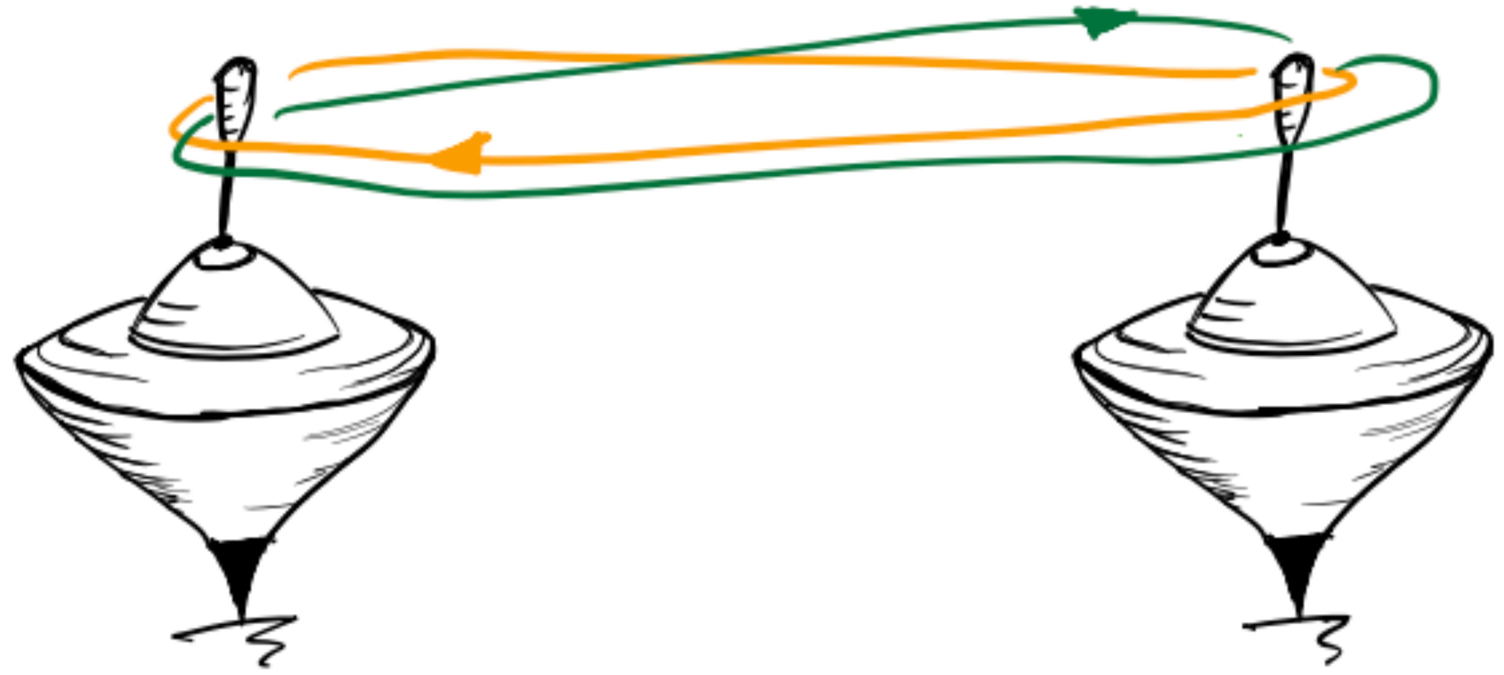
Can you write $|\Psi_{\rm ab}\rangle = |\Psi_{\rm a}\rangle \otimes |\Psi_{\rm b}\rangle$?

No? Then it is entangled.

Or more generally as product (mixed states): $\rho_{\rm ab} = \sum_k p_k \, \rho_{\rm a}^k \otimes \rho_{\rm b}^k$

Maximally entangled states (e.g Bell states):

$$|\Phi^\pm\rangle = \frac{|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle}{\sqrt{2}}$$ or $$|\Psi^\pm\rangle = \frac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}$$



Entanglement in bipartite systems

An entanglement measure is more useful than the previous definition:

- Peres-Horodecki Criterion: $\Delta \equiv -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$

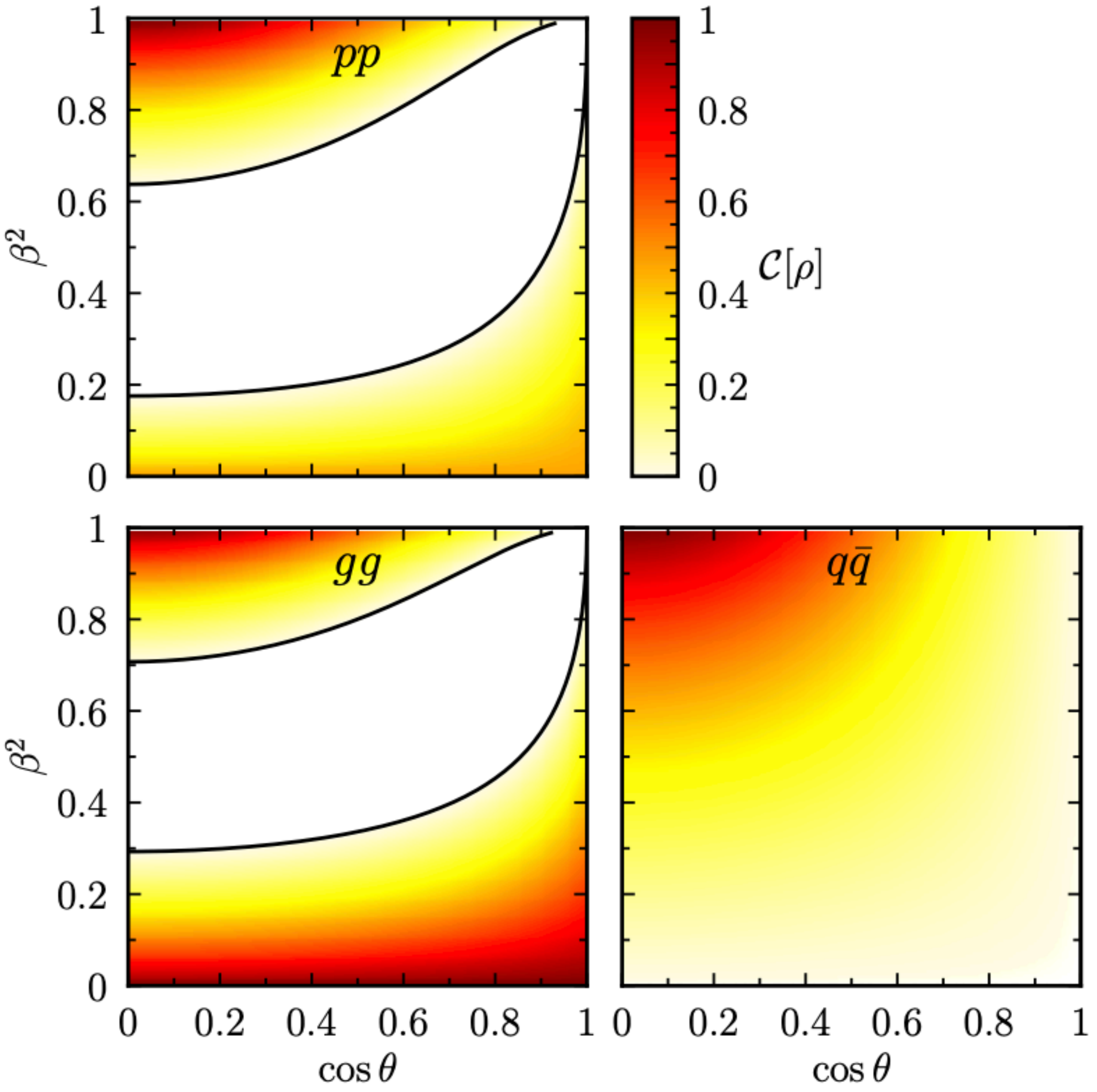
  (in the helicity-basis) (entangled)

- Concurrence: $C[\rho] = \max(\Delta/2, 0)$

  $C[\rho] = 1$ (maximally entangled)

What's the story for the SM?

[Afik and de Nova, 21']



White regions: zero-entanglement

Maximal entanglement points/regions

- At threshold: $\beta^2 = 0, \forall\theta$
- high-E: $\beta^2 \to 1, \cos\theta = 0$

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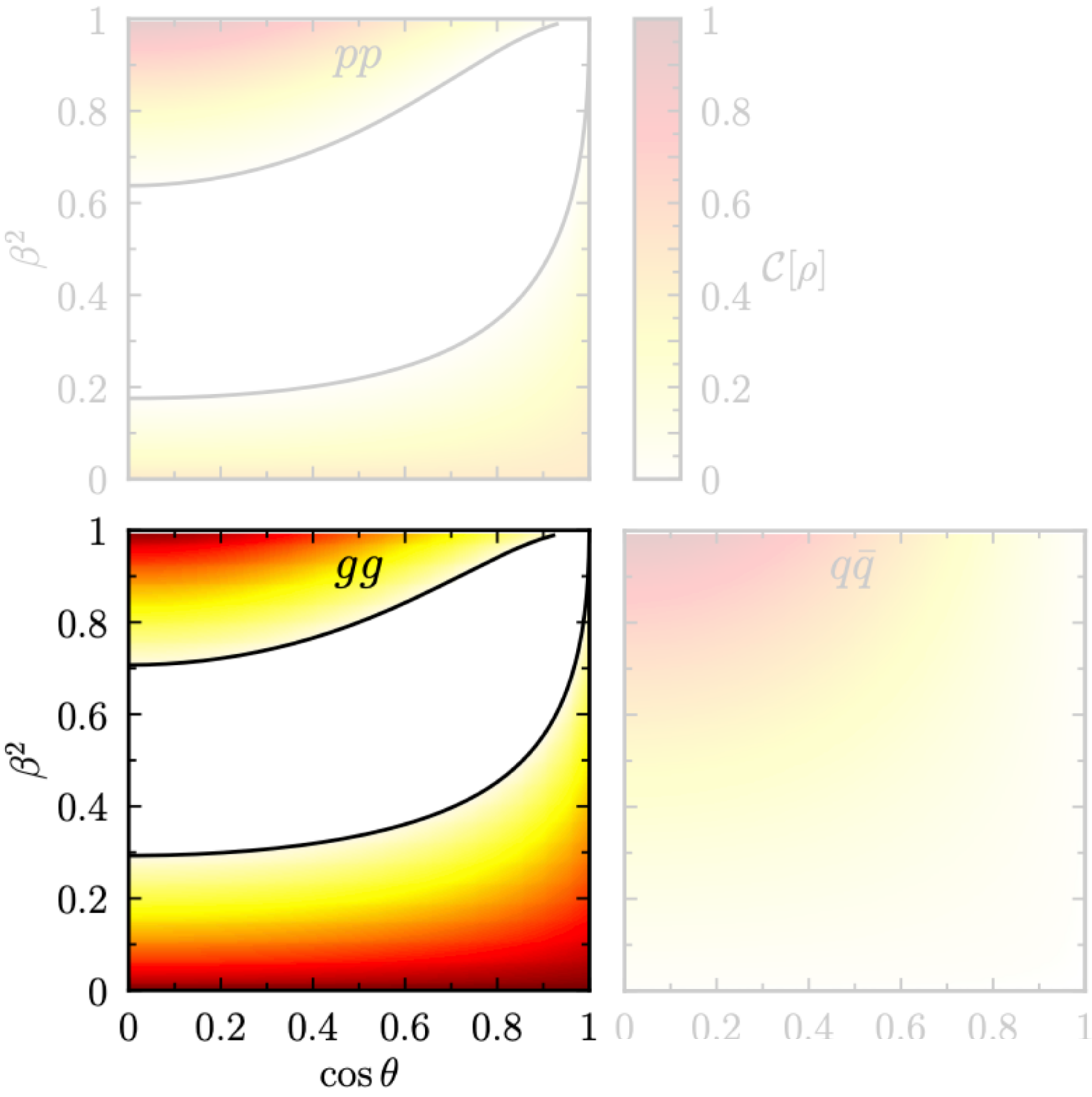

Maximal entanglement points/regions

- At threshold: $\beta^2 = 0, \forall\theta$

(singlet)

$$\rho_{gg}^{\mathrm{SM}}(0, z) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}},$$

- high-E: $\beta^2 \to 1, \cos\theta = 0$

(triplet)

$$\rho_{gg}^{\mathrm{SM}}(1, 0) = |\Psi^-\rangle_{\boldsymbol{n}}\langle\Psi^-|_{\boldsymbol{n}}$$

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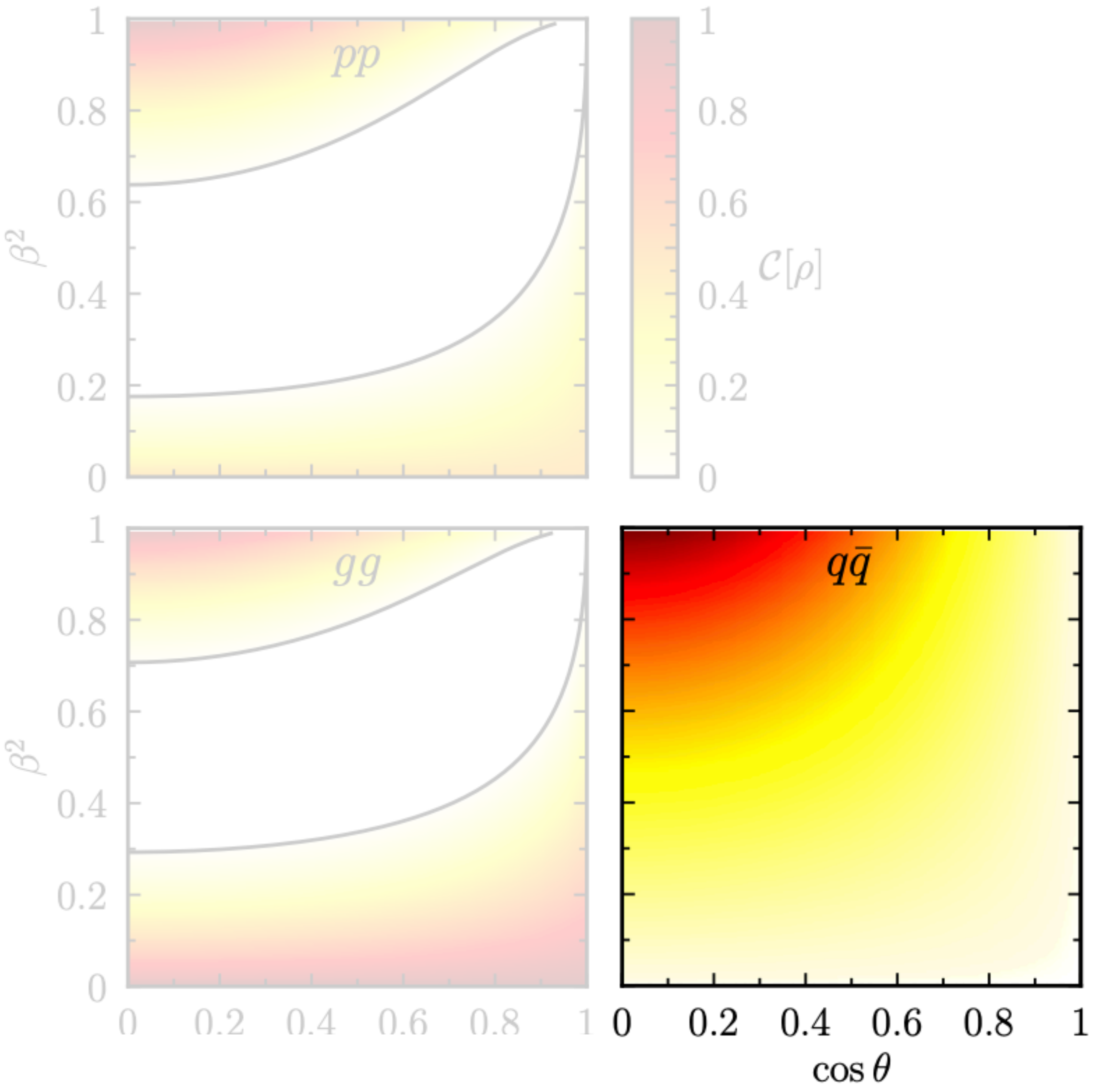

Maximal entanglement points/regions

- At threshold: $\beta^2 = 0, \forall\theta$

  mixed but separable

- high-E: $\beta^2 \to 1, \cos\theta = 0$

  (triplet: same as gg)

$$\rho^{\mathrm{SM}}_{q\bar{q}}(1,0) = |\Psi^-\rangle_{\boldsymbol{n}}\langle\Psi^-|_{\boldsymbol{n}}$$

SMEFT

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda^2}\sum_i c_i \mathcal{O}_i$$

$\mathcal{L}_{\mathrm{UV}}$

Λ

$\mathcal{L}_{\mathrm{SMEFT}}$

LO-QCD in ttbar prod. (SMEFTatNLO)

[Degrande et. al, 08']

$$\mathcal{O}_G = g_s f^{ABC} G_\nu^{A,\mu} G_\rho^{B,\nu} G_\mu^{C,\rho}$$

$$\mathcal{O}_{\varphi G} = \left(\varphi^\dagger \varphi - \frac{v^2}{2}\right) G_A^{\mu\nu} G_{\mu\nu}^A$$

$$\mathcal{O}_{tG} = g_s (\bar{Q}\sigma^{\mu\nu} T^A\, t)\tilde{\varphi} G_{\mu\nu}^A + \mathrm{h.c.}$$

+4F operators

$$\mathcal{O}_{Qq}^{(8,1)}, \mathcal{O}_{Qq}^{(8,3)}, \mathcal{O}_{tu}^{(8)}, \mathcal{O}_{td}^{(8)}, \mathcal{O}_{Qu}^{(8)}, \mathcal{O}_{Qd}^{(8)}, \mathcal{O}_{tq}^{(8)}$$

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Maximal points are affected by SMEFT?

Can SMEFT induce new regions?

SMEFT

Back to the R-matrix… $$R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2} \mathcal{M}_{\alpha_1\beta_1}$$

With dim-six contributions:

$$\mathcal{M}_{\alpha\beta} = \mathcal{M}^{\text{SM}}_{\alpha\beta} + \frac{1}{\Lambda^2}\mathcal{M}^{(\text{d6})}_{\alpha\beta} \longrightarrow \rho = \frac{R^{\text{SM}} + R^{\text{EFT}}}{\text{tr}(R^{\text{SM}}) + \text{tr}(R^{\text{EFT}})}$$

The Fano coefficients $X = X^{(0)} + \frac{1}{\Lambda^2}X^{(1)} + \frac{1}{\Lambda^4}X^{(2)}$ where

$$X = \tilde{A},\ \tilde{C}_{ij} \text{ and } \tilde{B}^{\pm}_i$$

$\mathcal{O}(\Lambda^{-4})$ from dim-6 sq.

SMEFT

Back to the R-matrix…
$$R^{I}_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^{*}_{\alpha_2\beta_2}\,\mathcal{M}_{\alpha_1\beta_1}$$

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At $\mathcal{O}(\Lambda^{-2})$

$$\tilde{C}^{gg,(1)}_{nn} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{-7 g_s^2 v m_t}{12\sqrt{2}} c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2 - (1-\beta^2) m_h^2} c_{\varphi G} + \frac{9 g_s^2 \beta^2 m_t^2 z^2}{8} c_G\right]$$

SMEFT entanglement: gg-initiated

[Aoude, Madge, Maltoni, Mantani, 22']

only $\mathcal{O}_{tG}, \mathcal{O}_{G}, \mathcal{O}_{\varphi G}$ contributes

$$\rho = \frac{R^{\mathrm{SM}} + R^{\mathrm{EFT}}}{\mathrm{tr}(R^{\mathrm{SM}}) + \mathrm{tr}(R^{\mathrm{EFT}})}$$

gg-initiated at threshold $\beta^2 = 0$

- linear interference exactly cancel, maximally entangled state unchanged
- quadratics vanish for $\mathcal{O}_{\varphi G}$ and decreases for $\mathcal{O}_{tG}, \mathcal{O}_{G}$

gg-initiated at high-E: $\beta^2 \to 1$: EFT not valid but $m_t^2 \ll \hat{s} \ll \Lambda^2$

- linear interference: sign dependent
- quadratics always decreases

SMEFT entanglement: qq-initiated

[Aoude, Madge, Maltoni, Mantani, 22']

only $\mathcal{O}_{tG}$ and 4F contributes

$$\rho = \frac{R^{\rm SM} + R^{\rm EFT}}{\mathrm{tr}(R^{\rm SM}) + \mathrm{tr}(R^{\rm EFT})}$$

qq-initiated at threshold $\beta^2 = 0$

- no contributions for linear and quad

qq-initiated at high-E: $m_t^2 \ll \hat{s} \ll \Lambda^2$

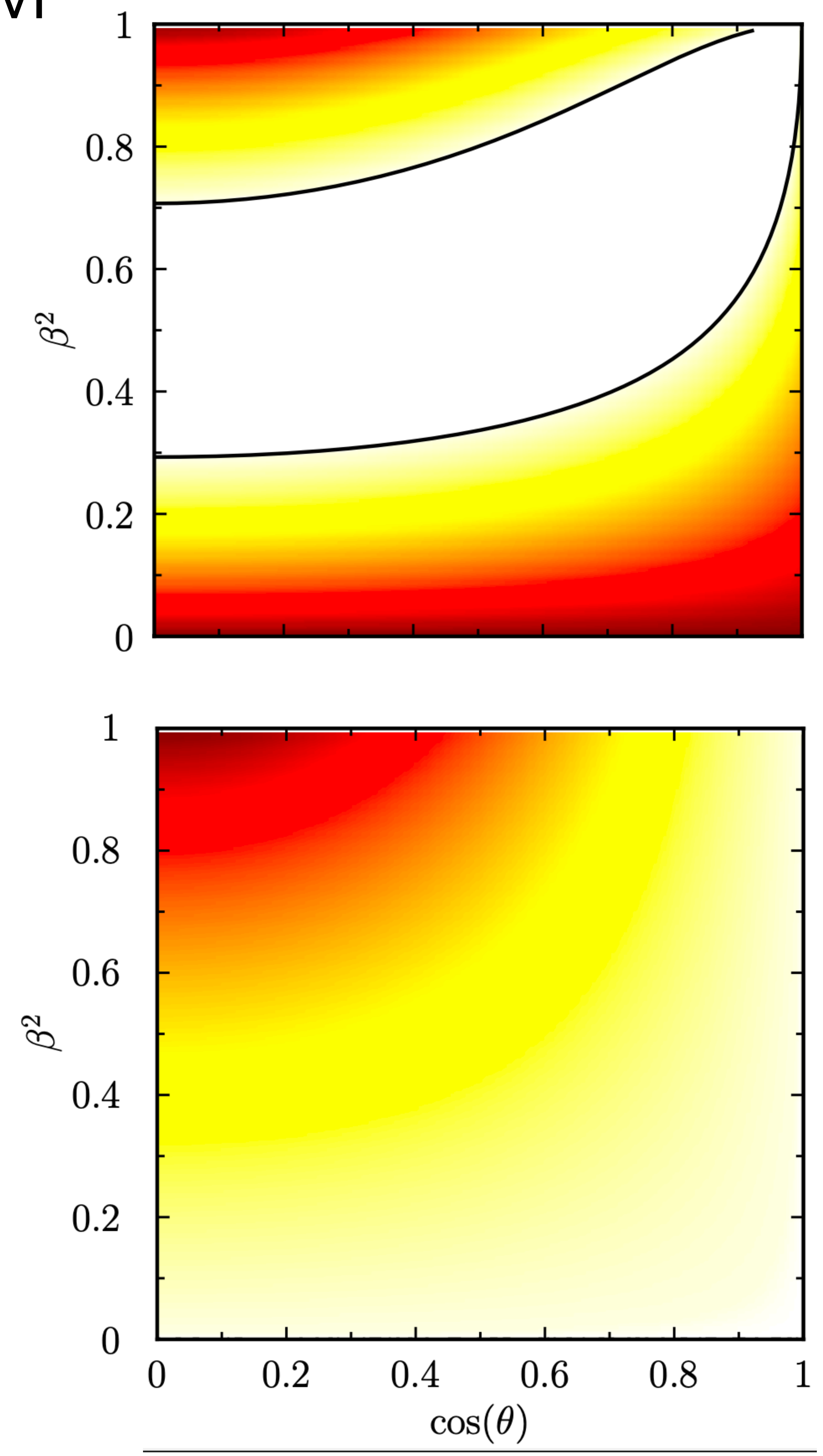
- sign dependent for linear and quadratics always decreases

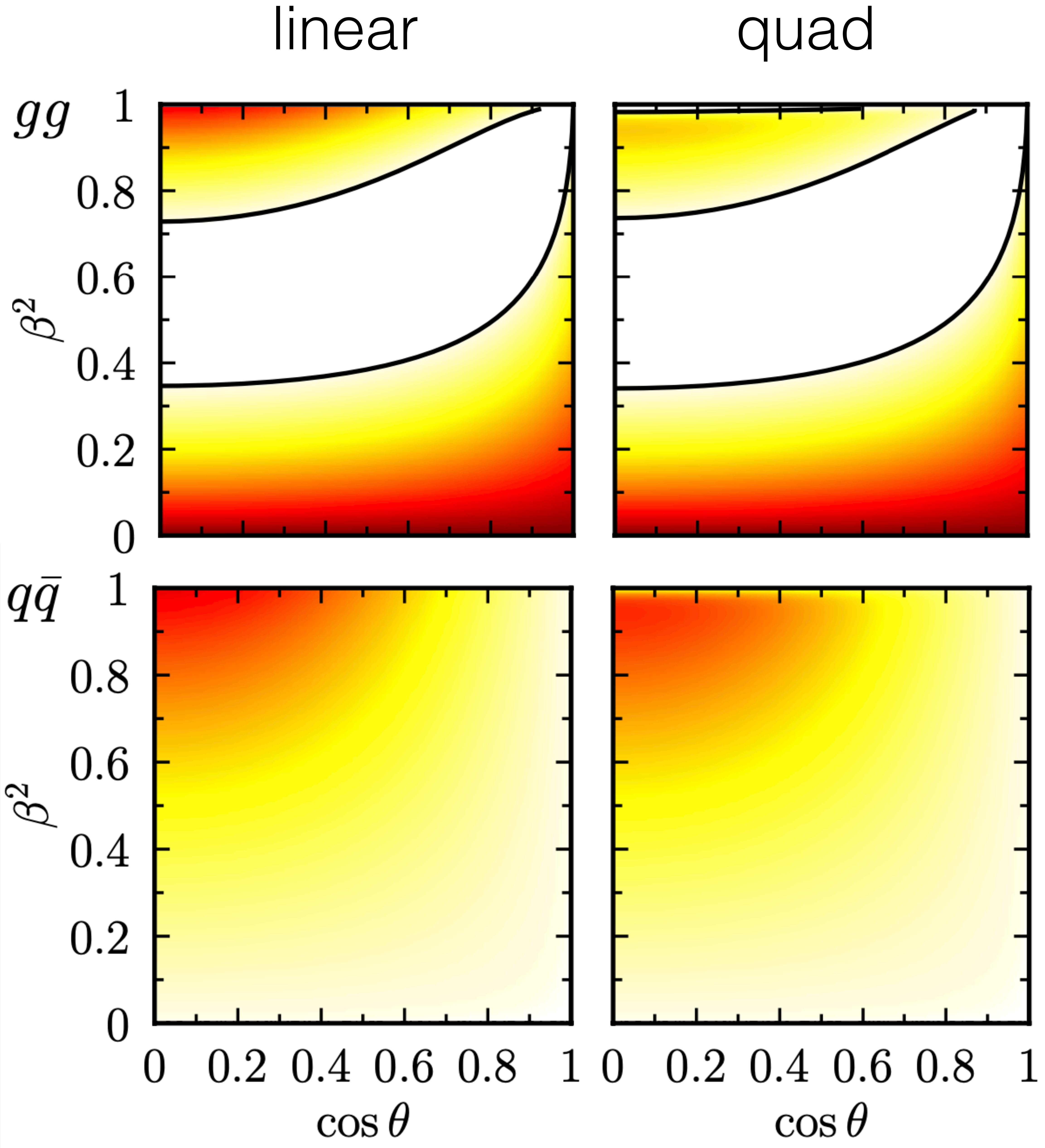
everything gets more involved for pp

SMEFT entanglement

$$\mathcal{O}_{tG} = g_s(\bar{Q}\sigma^{\mu\nu}T^A\,t)\tilde{\varphi}G^A_{\mu\nu} + \text{h.c.}$$

SM





SMEFT entanglement marker

$$\rho = \frac{R^{\mathrm{SM}} + R^{\mathrm{EFT}}}{\mathrm{tr}(R^{\mathrm{SM}}) + \mathrm{tr}(R^{\mathrm{EFT}})}$$

Δ_0 calculated with SM R's

$$\Delta_1 \equiv \Delta - \Delta_0$$

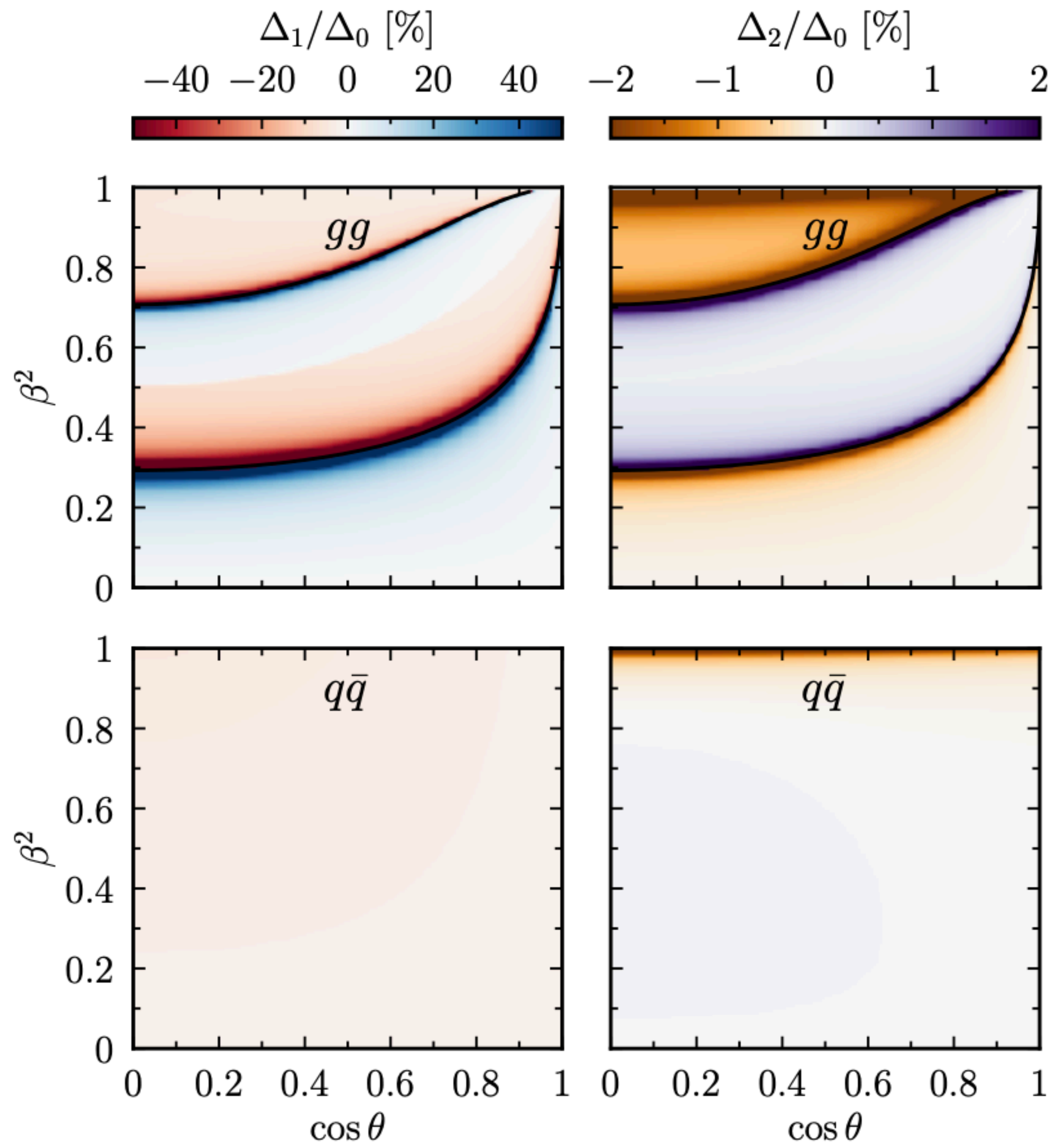
calculated with SMEFT R's up to $\mathcal{O}(\Lambda^{-2})$

$$\Delta_2 \equiv \Delta - \Delta_1 - \Delta_0$$

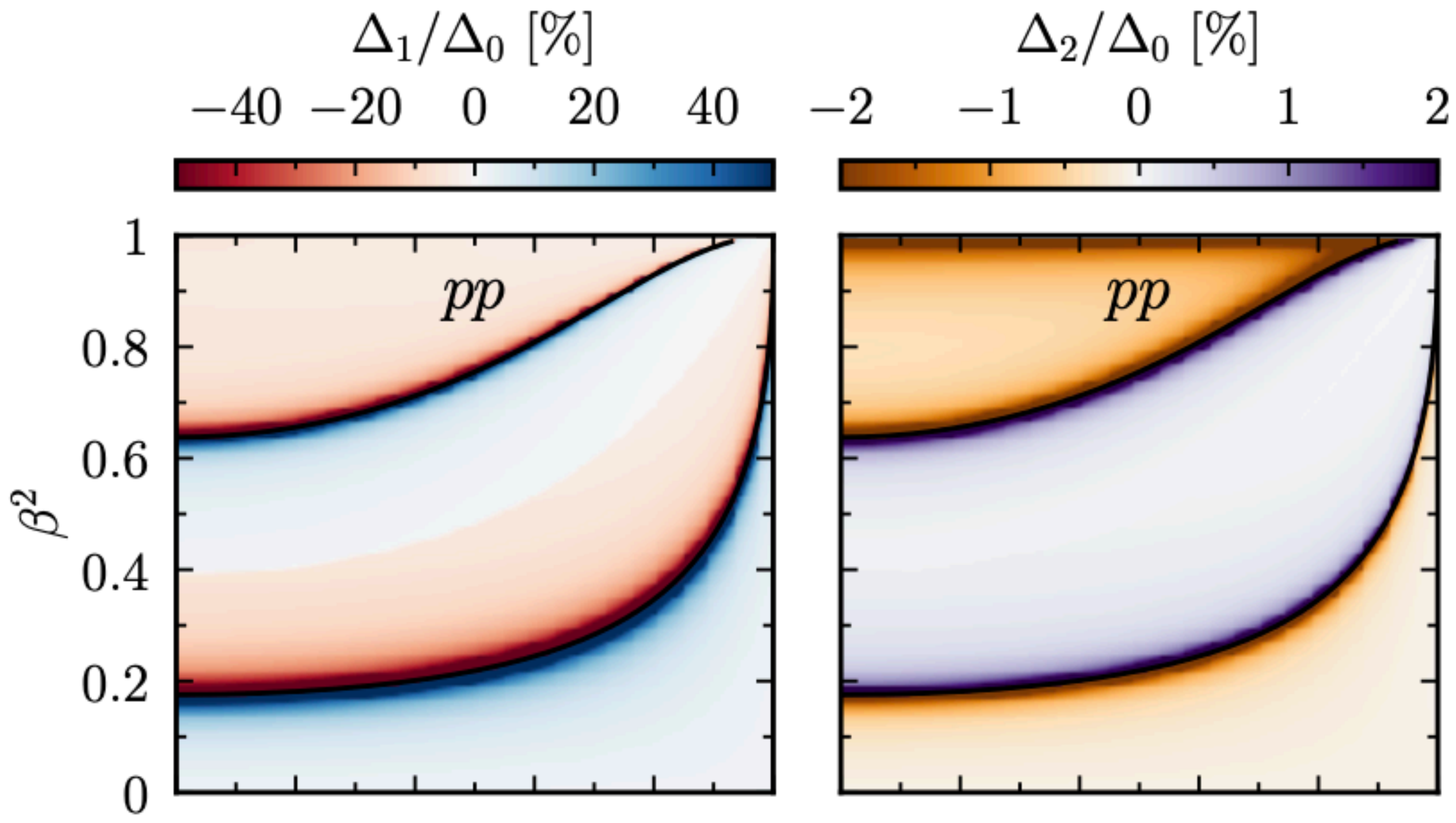
calculated with SMEFT R's up to $\mathcal{O}(\Lambda^{-4})$

SMEFT entanglement marker

separate channels



mixed state



SMEFT averaged concurrence

Average over the solid angle

$$\bar{R} = (4\pi)^{-1} \int \mathrm{d}\Omega \, R(\hat{\boldsymbol{s}}, \boldsymbol{k}) \,, \qquad \longrightarrow$$

PHC implies

$$\delta \equiv -C_z + |2C_\perp| - 1 > 0$$

$$C[\rho] = \max(\delta/2, 0)$$

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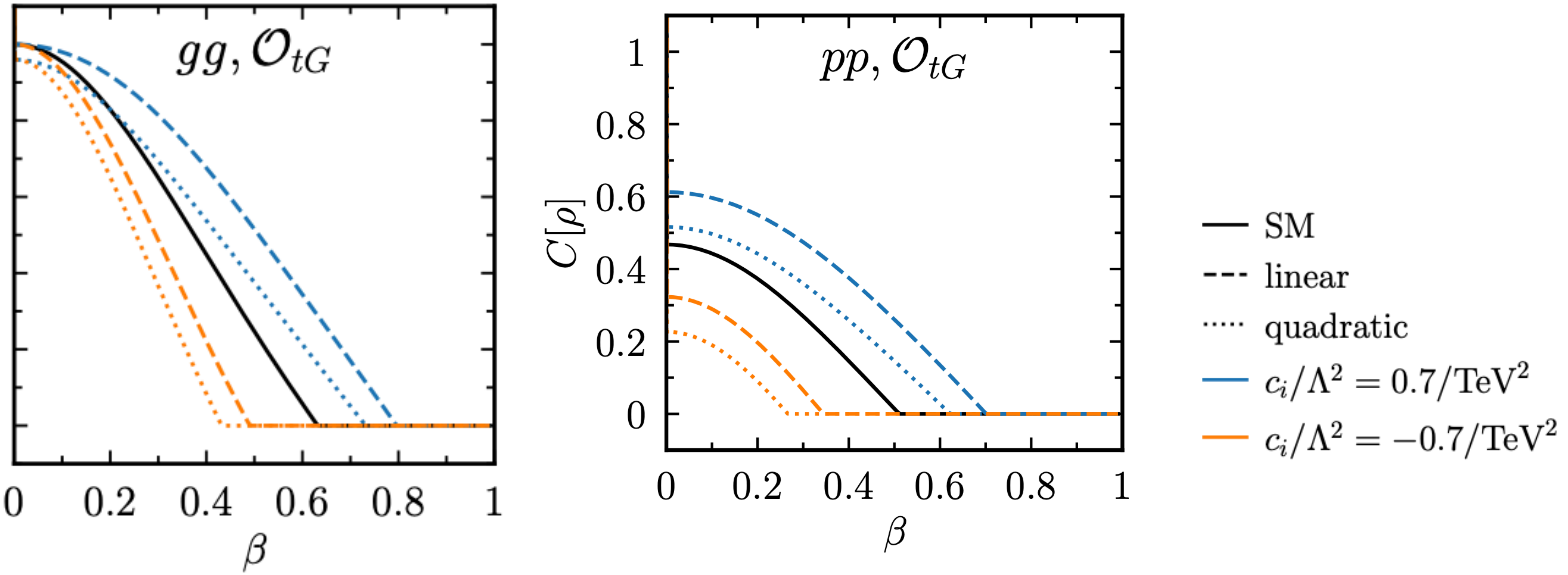


SMEFT quantum state

At threshold

$$\rho_{gg}^{\mathrm{EFT}}(0,z) = p_{gg}|\Psi^-\rangle_{\boldsymbol{p}}\langle\Psi^-|_{\boldsymbol{p}} + (1-p_{gg})|\Psi^+\rangle_{\boldsymbol{p}}\langle\Psi^+|_{\boldsymbol{p}}\,.$$

(Induces a triplet)

$$\rho_{q\bar{q}}^{\mathrm{EFT}}(0,z) = p_{q\bar{q}}\,|\uparrow\uparrow\rangle_{\boldsymbol{p}}\,\langle\uparrow\uparrow|_{\boldsymbol{p}} + (1-p_{q\bar{q}})\,|\downarrow\downarrow\rangle_{\boldsymbol{p}}\,\langle\downarrow\downarrow|_{\boldsymbol{p}}\ ,$$

(changes the mixed state)

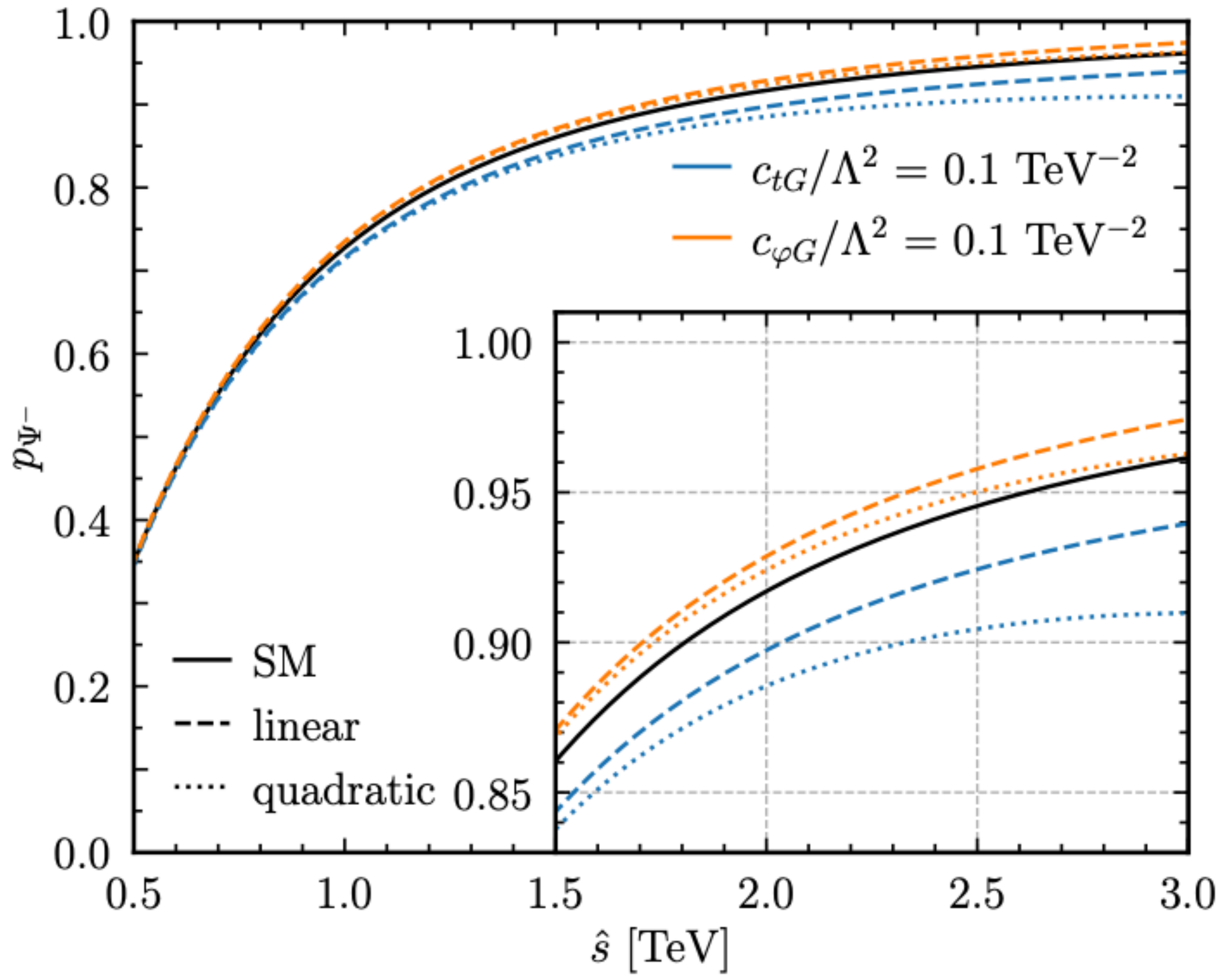
where

$$p_{gg} = \frac{72}{7\Lambda^4} m_t^2 (3\sqrt{2} m_t\, c_G + v\, c_{tG})^2\,,$$

$$p_{q\bar{q}} = \frac{1}{2} - 4\frac{c_{VA}^{(8),u}}{\Lambda^2} + \frac{8m_t^4}{\Lambda^4}\Big(\frac{v\sqrt{2}}{m_t} c_{VA}^{(8),u} c_{tG} - 9 c_{VA}^{(1),u} c_{VV}^{(1),u} + 2 c_{VA}^{(8),u} c_{VV}^{(8),u}\Big),$$

At high-pT

(triplet prob.) $p_{\Psi^-} = \langle\Psi^-|_{\boldsymbol{n}}\,\rho\,|\Psi^-\rangle_{\boldsymbol{n}}$



Conclusions

SM induces maximal entanglement points/regions in ttbar

Purely linear interference SMEFT effects vanish in these regions!

Quadratic interference decrease the entanglement at these points

Missing dim-8 linear interference and double-insertions at $\mathcal{O}(\Lambda^{-4})$

Questions?

QI observables can help contraint SMEFT ops?

Other processes?

All this effects due to approxs? Tree-level, only dim-six, no double insertions

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LO coefficients - gg channel

$$\tilde{A}^{gg,(1)} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{g_s^2 v m_t(9\beta^2 z^2+7)}{12\sqrt{2}}c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2-(1-\beta^2)m_h^2}c_{\varphi G} + \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right],$$

$$\tilde{C}_{nn}^{gg,(1)} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}\left[\frac{-7g_s^2 v m_t}{12\sqrt{2}}c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2-(1-\beta^2)m_h^2}c_{\varphi G} + \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right],$$

$$\begin{aligned}\tilde{C}_{kk}^{gg,(1)} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}&\left[\frac{g_s^2 v m_t\left(9\beta^2 z^2+7\right)\left(\beta^2\left(z^4-z^2-1\right)+1\right)}{12\sqrt{2}\left(\beta^2 z^2-1\right)}c_{tG}\right.\\ &\left.+\frac{\beta^2 m_t^4}{4m_t^2-\left(1-\beta^2\right)m_h^2}c_{\varphi G} - \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right],\end{aligned}$$

$$\begin{aligned}\tilde{C}_{rr}^{gg,(1)} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}&\left[\frac{g_s^2 v m_t\left(-9\beta^4\left(z-z^3\right)^2-7\beta^2\left(z^4-z^2+1\right)+7\right)}{12\sqrt{2}\left(\beta^2 z^2-1\right)}c_{tG}\right.\\ &\left.-\frac{\beta^2 m_t^4}{4m_t^2-\left(1-\beta^2\right)m_h^2}c_{\varphi G} + \frac{9g_s^2\beta^2 m_t^2 z^2}{8}c_G\right],\end{aligned}$$

$$\begin{aligned}\tilde{C}_{rk}^{gg,(1)} = \frac{g_s^2}{\Lambda^2}\frac{1}{1-\beta^2 z^2}&\left[\frac{g_s^2 v m_t\beta^2 z\left(1-z^2\right)\left(9\beta^2+\left(\beta^2-2\right)z^2\left(9\beta^2\left(z^2-1\right)+7\right)-2\right)}{24\sqrt{2}\sqrt{\left(\beta^2-1\right)\left(z^2-1\right)}\left(\beta^2 z^2-1\right)}c_{tG}\right.\\ &\left.+\frac{9g_s^2\beta^2 m_t^2 z}{8}\sqrt{\frac{1-z^2}{1-\beta^2}}c_G\right].\end{aligned}$$

LO coefficients - qq channel

$$\tilde{A}^{q\bar{q},(1)} = \frac{4g_s^2 m_t^2}{9\Lambda^2(1-\beta^2)}\left[\sqrt{2}g_s^2\frac{v}{m_t}(1-\beta^2)c_{tG} + \left(2-(1-z^2)\beta^2\right)c_{VV}^{(8),u} + 2z\beta c_{AA}^{(8),u}\right],$$

$$\tilde{C}_{nn}^{q\bar{q},(1)} = -\frac{g_s^2 m_t^2}{\Lambda^2}\frac{4\beta^2(1-z^2)}{9(1-\beta^2)}c_{VV}^{(8),u},$$

$$\tilde{C}_{kk}^{q\bar{q},(1)} = \frac{2g_s^2 m_t^2}{9\Lambda^2(1-\beta^2)}\left[2\sqrt{2}g_s^2\frac{v}{m_t}(1-\beta^2)z^2 c_{tG} + \left(2+\beta^2-(2-\beta^2)(1-2z^2)\right)c_{VV}^{(8),u} + 4\beta z c_{AA}^{(8),u}\right]$$

$$\tilde{C}_{rr}^{q\bar{q},(1)} = \frac{4g_s^2 m_t^2(1-z^2)}{9\Lambda^2(1-\beta^2)}\left[\sqrt{2}g_s^2\frac{v}{m_t}(1-\beta^2)c_{tG} + (2-\beta^2)c_{VV}^{(8),u}\right],$$

$$\tilde{C}_{rk}^{q\bar{q},(1)} = -\frac{2g_s^2 m_t^2}{9\Lambda^2}\sqrt{\frac{1-z^2}{1-\beta^2}}\left[\sqrt{2}g_s^2\frac{v}{m_t}(2-\beta^2)z c_{tG} + 4z c_{VV}^{(8),u} + 2\beta c_{AA}^{(8),u}\right],$$

$$B_k^{\pm,q\bar{q},(1)} = 4g_s^2\frac{m_t^2}{9\Lambda^2}\frac{1}{1-\beta^2}\left(\beta(z^2+1)c_{AV}^{(8),u} + 2z c_{VA}^{(8),u}\right),$$

$$B_r^{\pm,q\bar{q},(1)} = -4g_s^2\frac{m_t^2}{9\Lambda^2}\sqrt{\frac{1-z^2}{1-\beta^2}}\left(\beta z c_{AV}^{(8),u} + 2c_{VA}^{(8),u}\right).$$

where

$$c_{VV}^{(8),u} = (c_{Qq}^{(8,1)} + c_{Qq}^{(8,3)} + c_{tu}^{(8)} + c_{tq}^{(8)} + c_{Qu}^{(8)})/4, \qquad c_{AA}^{(8),u} = (c_{Qq}^{(8,1)} + c_{Qq}^{(8,3)} + c_{tu}^{(8)} - c_{tq}^{(8)} - c_{Qu}^{(8)})/4,$$

$$c_{AV}^{(8),u} = (-c_{Qq}^{(8,1)} - c_{Qq}^{(8,3)} + c_{tu}^{(8)} + c_{tq}^{(8)} - c_{Qu}^{(8)})/4, \qquad c_{VA}^{(8),u} = (-c_{Qq}^{(8,1)} - c_{Qq}^{(8,3)} + c_{tu}^{(8)} - c_{tq}^{(8)} + c_{Qu}^{(8)})/4,$$

Concurrence

Given the density matrix, build $\omega = \sqrt{\sqrt{\tilde{\rho}}\rho\sqrt{\tilde{\rho}}}$ where

$$\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2)$$

The concurrence (in bipartite systems) is given by

$$C[\rho] = \max\left[0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\right]$$

where λ_i are the increasingly ordered eigenvalues of ω