The seeds of EFT double copy

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Double copy

Product operation between amplitudes & theories

$$\mathrm{TH}_1 \times \widetilde{\mathrm{TH}}_2 = \mathrm{TH}_3$$

Forms a *web* of somewhat special theories

$\times$	$\widetilde{\mathrm{BAS}}$	$\widetilde{\mathrm{NLSM}}$	$\widetilde{\mathrm{YM}}$
BAS	BAS	NLSM	YM
NLSM		sGal	BI
YM			GR

e.g. $f^{a_1 a_2 a_3} \frac{[12]^3}{[13][23]} \to \left(\frac{[12]^3}{[13][23]}\right)^2$

Facilitates computations

EFT deformations?

What EFTs are valid inputs?

$\equiv$ *single copies* $(\mathcal{A})$

What EFTs obtained as outputs?

$\equiv$ *double copies* $(\mathcal{M})$

Traditional KLT

[Kawai, Lewellen, Tye '86]

- Field-theory limit of a string-theory relation $\quad$ (closed) $=$ (open) $\times$ $(\widetilde{\text{open}})$

- Inputs are vectors of *colour-ordered* amplitudes A

with $\mathrm{Tr}\{T^{a_1}T^{a_2}\ldots\} \equiv (12\ldots)$

for $\quad \mathcal{A} = A \cdot c^{\mathrm{tr}}$

$$c^{\mathrm{tr}}_{n=4} = \begin{pmatrix} (1324)+(4231) \\ (1234)+(4321) \\ (1243)+(3421) \end{pmatrix}$$

satisfy *BCJ relations*, leaving a basis of $(n-3)!$

$$A_{n=4}[1234] = \tfrac{t}{u} A_{n=4}[1243]$$

- Proceeds through a *kernel* matrix

$$\otimes_{n=4} = \begin{pmatrix} \frac{tu}{s} & u & t \\ u & \frac{su}{t} & s \\ t & s & \frac{st}{u} \end{pmatrix}$$

$\mathcal{M} = A \otimes \tilde{A} \qquad$ sums over any two BCJ bases

Traditional CK duality

- Inputs are *adjoint numerators* from $\mathcal{A} = c^{\text{adj}} \cdot P \cdot n^{\text{adj}}$

Sums run over trivalent graphs/topologies.

colour

$c^{\text{adj}} \ni f^{abx} f^{x\dots}$

propagators

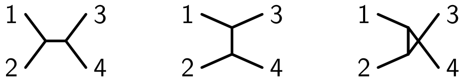
$P_{n=4} = \begin{pmatrix} 1/s & & \\ & 1/t & \\ & & 1/u \end{pmatrix}$

kinematics

$s_{ij},\ p_i \cdot \epsilon_j, \dots$

- n^{adj} must satisfy the same algebraic properties as c^{adj}

e.g. Jacobi identities, for a $f^{abc} A^a_\mu \phi^b \partial^\mu \phi^c$ theory at four points



$$
\begin{array}{lccccccc}
c^{\text{adj}}: & f^{12x} f^{x34} & + & f^{13x} f^{x42} & + & f^{14x} f^{x23} & = & 0 \\
n^{\text{adj}}: & (t-u) & + & (u-s) & + & (s-u) & = & 0
\end{array}
$$

- Exchange colour for kinematics: $\mathcal{M} = n^{\text{adj}} \cdot P \cdot \tilde{n}^{\text{adj}}$

Traditional approaches and EFTs

The field-theory limit of the string KLT
contains a specific tower of operators.

Certain operators are double-copyable out of the box

e.g. $\mathrm{YM} + F^3 \longrightarrow \mathrm{GR} + R^3$ [Brödel, Dixon '12]

Others are not

e.g. $\mathrm{YM} + F^4 \longrightarrow \emptyset$

... double-copy generalisation for EFTs?

The bi-adjoint scalar (BAS)

CK: Exchange kinematics for colour (*zeroth copy*)

$$\mathcal{A}^{\text{BAS}} = c^{\text{adj}} \cdot P \cdot \tilde{c}^{\text{adj}}$$

amplitudes of a $f^{abc} f^{\tilde{a}\tilde{b}\tilde{c}}\, \phi^{a\tilde{a}} \phi^{b\tilde{b}} \phi^{c\tilde{c}}$ theory

KTL: Identity theory $\mathcal{A}^{\text{BAS}} = A^{\text{BAS}} \cdot c^{\text{tr}} = c^{\text{tr}} \cdot m \cdot c^{\text{tr}}$

singly colour ord.

doubly colour ord.

- Kernel satisfying

$$\begin{aligned} A^{\text{BAS}} \otimes A^{\text{BAS}} &= \mathcal{A}^{\text{BAS}} \\ &= c^{\text{tr}} \cdot m \otimes m \cdot c^{\text{tr}} \end{aligned}$$

$\rightarrow$ $\otimes = m^{-1}$ after restriction to a BCJ basis

- BCJ relations encoded in $A \otimes A^{\text{BAS}} = \mathcal{A}$

Generalised KLT

[Chi, Elvang, Herderschee, Jones, Paranjape '21]

- Construct most general higher-derivative m_{hd} and obtain $\otimes_{\text{hd}} = m_{\text{hd}}^{-1}$ (just BAS particle cont.)

- Impose that m_{hd} has same rank as m 'minimal rank' $(n-3)!$
  - observed to be necessary for a sane double copy
  - complicated beyond four points ($n=4$)

- Find double-copyable EFT amplitudes satisfying

$$\mathcal{A}_{\text{hd}} = A_{\text{hd}} \otimes_{\text{hd}} A_{\text{hd}}^{\text{BAS}} \qquad \text{generalised BCJ relations}$$

- Observe that the space of double copies remains the same

$$\{A_{\text{hd}} \otimes_{\text{hd}} \tilde{A}_{\text{hd}}\} = \{A \otimes \tilde{A}\}$$

Generalised CK

[Carrasco, Rodina, Yin, Zekioglu '19, '21]

- Promote $c^{\text{adj}} \rightarrow c^{\text{adj}}_{\text{hd}}(\text{colour}, \text{kinematics})$
  with identical adjoint algebraic properties (e.g. Jacobi)

- Build the full tower of $c^{\text{adj}}_{\text{hd}}$ using composition rules

  $$c^{\text{adj}}_{\text{hd},1} \circ c^{\text{adj}}_{\text{hd},2} = c^{\text{adj}}_{\text{hd},3}$$

  complicated beyond four points

- Obtain double-copyable amplitudes: $\mathcal{A}_{\text{hd}} = n^{\text{adj}} \cdot P \cdot c^{\text{adj}}_{\text{hd}}$
  double copies: $\mathcal{M}_{\text{hd}} = n^{\text{adj}} \cdot P \cdot \tilde{n}^{\text{adj}}$

  ... which are not generalised

Numerator seeds

[Bonnefoy, GD, Grojean, Machado, Roosmale Nepveu '21]

$\rightarrow$ Simplify the construction of double-copyable amps (above four points)

$\rightarrow$ Relate generalised KLT & CK

· Define the map between colour representations: $c^{\text{adj}} = J \cdot c^{\text{tr}}$

e.g. for $n = 4$:

$$\begin{pmatrix} f^{12x} f^{x34} \\ f^{13x} f^{x42} \\ f^{14x} f^{x23} \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} (1324) + (4231) \\ (1234) + (4321) \\ (1243) + (3421) \end{pmatrix}$$

which encodes Jacobi relations

· Construct generalised CK adjoint numerators $c^{\text{adj}}_{\text{hd}} = J \cdot c^{\text{tr}}_{\text{hd}}$

from *seeds* satisfying just *trace* algebraic properties

Four-point scalar results

- Removing redundancies, the most general seed: (single colour trace)

$$c^{\text{tr}}_{\text{hd}} = \overbrace{\begin{pmatrix} g(s,t) & 0 & 0 \\ 0 & g(t,s) & 0 \\ 0 & 0 & g(u,s) \end{pmatrix}}^{G_{\text{hd}}} \cdot c^{\text{tr}} \quad \text{with} \quad g(s,t) = \sum_{i,j} a_{ij} \, \frac{s^i (tu)^j}{\Lambda^{2i+4j}}$$

$u \equiv -s - t$

$$\text{and thus} \quad c^{\text{adj}}_{\text{hd}} = J \cdot c^{\text{tr}}_{\text{hd}} = J \cdot G_{\text{hd}} \cdot c^{\text{tr}}$$

- Most general kinematic numerator: $n^{\text{adj}} = J \cdot \tilde{G}_{\text{hd}} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

→ Most general double-copyable amplitude: $\mathcal{A}_{\text{hd}} = n^{\text{adj}} \cdot P \cdot c^{\text{adj}}_{\text{hd}}$

Four-point KLT from seeds

- CK-generalised BAS amplitudes (more than BAS particle content)

$$\begin{aligned}\mathcal{A}^{\text{BAS}}_{\text{hd}} &= c^{\text{tr}}_{\text{hd}} \cdot m \cdot \tilde{c}^{\text{tr}}_{\text{hd}} \\ &= c^{\text{tr}} \cdot \underbrace{G_{\text{hd}} \cdot m \cdot \tilde{G}_{\text{hd}}}_{\text{rank of } m} \cdot \tilde{c}^{\text{tr}}\end{aligned}$$

- Tentative generalised kernel

$$\begin{aligned}\otimes_{\text{hd}} &= \tilde{G}^{-1}_{\text{hd}} \cdot m^{-1} \cdot G^{-1}_{\text{hd}} \\ &= \tilde{G}^{-1}_{\text{hd}} \otimes G^{-1}_{\text{hd}}\end{aligned}$$

on a BCJ basis
$(n-3)! = 1$

indeed contains the generalised KLT solution,
to all EFT orders!

Four-point generalised KLT double copies

- Generalised double copies formally re-written
  with traditional kernel

$$\begin{aligned}\mathcal{M}_{\mathsf{hd}} &= \tilde{A}_{\mathsf{hd}} \otimes_{\mathsf{hd}} A_{\mathsf{hd}} \\ &= (\tilde{A}_{\mathsf{hd}} \cdot \tilde{G}_{\mathsf{hd}}^{-1}) \otimes (G_{\mathsf{hd}}^{-1} \cdot A_{\mathsf{hd}})\end{aligned}$$

- $G_{\mathsf{hd}}^{-1} \cdot A_{\mathsf{hd}}$ satisfy traditional BCJ relations
  if A_{hd} satisfies generalised ones.

- $G_{\mathsf{hd}}^{-1} \cdot A_{\mathsf{hd}}$ can be rescaled to a well-behaved BAS amplitudes
  if A_{hd} is itself well behaved.

$\rightarrow$ Four-point generalised KLT does not generalise double copies.

Higher points

- n^{adj} constructed from seeds up to six points.
  Five-point count matches [Carrasco et al. '21].

- Five-point seeds shown to reproduce generalised KLT kernel
  at all EFT orders provided in [Chi et al. '21].

- Five-point $G_{\text{hd}}^{-1} \cdot A_{\text{hd}}$ shown to be well behaved
  for a subset of (diagonal) G_{hd}.

The seeds of EFT double copy

New approach to double-copy generalisations for EFTs using *numerator seeds*.

Simpler construction, for all multiplicities, with redundancies.

Relates existing generalisations and helps to understand observed features.

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