Large N expansion of two dimensional QCD

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August 5, 2022

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- Does SU(3) YM exhibit confinement in 4 dimensions?
- If so, what is the mechanism responsible for this?
- What is the full spectrum of the theory?

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- What is the mesons' spectrum? And their dynamics?

Motivations

Why QCD in 1+1 dimensions?

Old idea. . . but renewed interest

- Field Theory perspective:

- String theory perspective:

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  - Understanding confinement better on the String Theory side
  - Recent success in worldsheet constructions in lower dimensions (e.g. AdS_3 / CFT_2): can we do the same here?

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Overall: simpler and insightful

Quantum Chromodinamics on the line

An SU(N) gauge theory coupled with N_f fundamental & massive fermions:

$$\mathcal{L} = -\frac{1}{4} \mathrm{Tr}\, F \wedge \star F + \overline{\Psi}_A (i \not{D} - m_A) \Psi_A$$

Convenient to use lightcone coordinates: $x_\pm = \frac{1}{\sqrt{2}}(x_1 \pm x_0)$.

Massive simplification: fix gauge redundancy through lightcone gauge: $A_- = A^+ = 0$.

$$\mathcal{L}_{gf} = -\frac{1}{2}(\partial_- A_+)^2 + \overline{\Psi}_A (i \not{\partial} - m_A - g\gamma_- A_+) \Psi_A$$

A confining potential for quarks

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Crucial consequence: Gauge Field are completely non-dynamical

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A linear (confining) potential between quarks!

A large N expansion

A linear potential arises... but this is still not enough!

Goal: showing that there are no free quarks in the spectrum.

Further simplification: large N limit $\quad N \to \infty$ with $g^2 N$ fixed

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- W.r.t. $g^2 N$, diagrams organise in a $1/N$ expansion $\Rightarrow$ Perturbation theory!
- $3 \neq \infty \ldots$ but maybe $3 \gg 1$! Again: toy model for universal features

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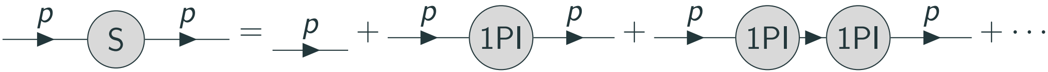
Large N $\longleftrightarrow$ matrix model: double line notation  = 

In $N \to \infty$ only planar diagrams in double line survive

Non-perturbative quark propagator

In this approximation the theory is (almost) solvable

The full, non-perturbative, quark propagator:



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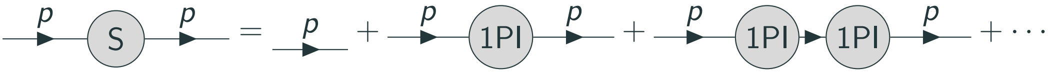

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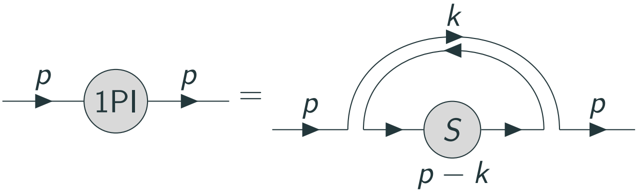

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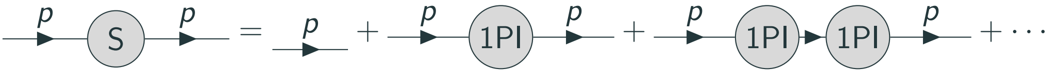

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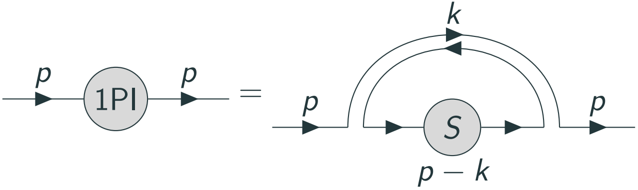
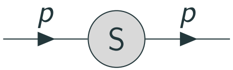


That can be miraculously solved:

$$= \frac{N_c\, g^2}{\pi}\left(\frac{\mathrm{sgn}(p_-)}{\rho} - \frac{1}{p_-}\right)$$

Non-perturbative quark propagator: no free quarks

The resulting full propagator is:



$$= \frac{ip_-\gamma_-}{2p_+p_- - m^2 + N_c\, g^2/\pi - N_c\, g^2|p_-|/(\rho\pi)} = \frac{ip_-\gamma_-}{2p_+p_- - M_{eff}^2}$$

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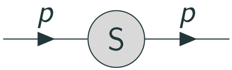

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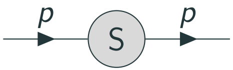

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The position of the poles $2p_+p_- = M_{eff}^2$ gives the spectrum of quarks.

As $\rho \to 0$, poles are pushed to ∞

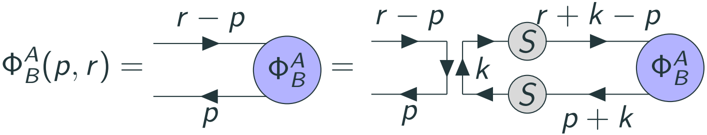
$\Longrightarrow$ **No free quark states in the spectrum!**

The model is indeed confining

Quark-antiquark

Not fully convinced? let's compute the scattering between two quarks.

Boostrappable though Bethe-Salpeter:



From which we obtain the eigenproblem:

$$\mu^2 \phi_B^A(x) = \left(\frac{\alpha_A}{x} + \frac{\alpha_B}{1-x}\right) \phi_B^A(x) - \mathcal{P}\int_0^1 \mathrm{d}y\, \frac{\phi_B^A(y)}{(y-x)^2}, \quad \phi_B^A(x) = \int \mathrm{d}p_+\, \Phi_B^A(p,r)$$

$$\alpha_{A,B} = \frac{\pi m_{A,B}^2}{g^2} - 1, \quad 2r_+ r_- = \frac{g^2}{\pi}\mu^2, \quad x = \frac{p_-}{r_-}, \quad x \in [0,1]$$

Mesons spectrum

$$\mu^2 \phi_B^A(x) = \left(\frac{\alpha_A}{x} + \frac{\alpha_B}{1-x}\right) \phi_B^A(x) - \mathcal{P} \int_0^1 \mathrm{d}y \, \frac{\phi_B^A(y)}{(y-x)^2}$$

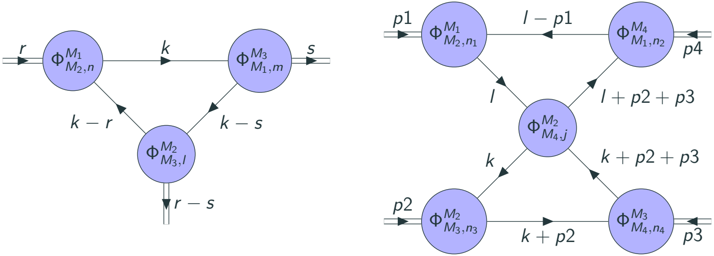
- There is no analytical expression for ϕ
- The spectrum is discrete $\Rightarrow$ No free quarks final states
- Only mesons states with M^2 determined by μ^2
- Resemble QM of pair of particles in a box with linear potential $\mathcal{P} \int_0^1 \mathrm{d}y \, \frac{\phi_B^A(y)}{(y-x)^2} = \mathfrak{F}^{-1} \left[(|p|) \, \tilde{\phi}(p) \right]$
- High energy limit $\to$ neglect linear potential $\sim$ two particle confined in a box $\mu^2 \sim k^2$ (Regge trajectories)

What are we currently doing?

We are studying the scattering of mesons in this model.

Qualitative behaviour: in mesons correlation functions we have only:

- Local interactions
- Mesons exchange



The theory is confining $\rightarrow$ No long range interactions mediated by gluons/quarks

Conclusions and outlook

Summarising:

- QCD_2 is a confining gauge theory
- No propagating dof from gluons: they provide linear potential
- No free quarks in the spectrum
- Pair of quarks can only exists in bound mesons states
- Mesons correlation functions respect the confining structure

Outlook:

- String Theory worldsheet construction?
- Beyond numerics for mesons: any analytical structure?
- Higher orders in $1/N$ expansion
- What about Baryons?

Thank you for you attention!

Questions?